Modeling and Optimization with OPL 2 Introduction to OPL

Andreas Popp



These slides are licensed under a Creative Commons Attribution-ShareAlike 4.0 International License. 2 Introduction to OPL

> CC-BY-SA A. Popp

.1 Structure of an OPL project

2.2 Basic data types and operators

Data type

Operator

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

Content

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

Operators

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

2 Introduction to OPL

> CC-BY-SA A. Popp

.1 Structure of an OPL project

2.2 Basic data types and operators

Data type:

Operator

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

2/28 《□▶ 《□▶ 《三▶ 《三▶ 三三 · 의식⊙

About CPLEX

- solver (software with implementent solution methods) for optimization problems distributed since 1988
- first sold by CPLEX Optimization Inc., then ILOG, then sold to IBM
- widely spread in science and industry
- interfaces for well known programming languages like C++, Java or C#
- own modeling language: OPL (Optimization Programming Language)
- cost free for academic purposes as part of the IBM Academic Iniative

2 Introduction to OPL

> CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data type

Operator

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.1 Structure of an OPL project

2 Introduction to OPL

> CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

Operator

2.3 Mathematica models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

4/28 《□▶ 《□▶ 《三▶ 《三▶ 三三 · 의익⊙

Types of OPL files

model files description of a generic optimization model
 (extension: .mod)
 data files data for instantiation of an OPL model
 (extension: .dat)
settings files settings for the solver (extension: .ops)

2 Introduction to OPL

> CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

Operator

2.3 Mathematica models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

5/28 《□▶ 《□▶ 《三▶ 《三▶ 三三 · 의식⊙

Types of OPL files

model files description of a generic optimization model (extension: .mod) data files data for instantiation of an OPL model (extension: .dat)

settings files settings for the solver (extension: .ops)

2 Introduction to OPL

> CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

Operator

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

Structure of an OPL project



2 Introduction to OPL

> CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

Operator

2.3 Mathematica models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

6/28 《 🗆 🕨 《 🗇 》 《 🖹 》 《 🖹 》 🖹 🛛 옛 ९ 📀

2 Introduction to OPL

CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data type

Operator

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

2.2 Basic data types and operators

Structure of a simple assignment command



2 Introduction to OPL

> CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data type

Operator

2.3 Mathematica models in OPL syntax

2.4 The CPLEX Studio IDE

Primitive data types

- int short for: "integer", an integer value with arbitrary sign. Example literals: 0, 1, -2, -786
- float floating point number with arbitrary sign. Example literals: 0.0, 1.0, 3.14, -7.86
- boolean technically a logical value; as decision variable a 0-1-variable.

2 Introduction to OPL

CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

```
2.3 Mathematica
models in OPL
syntax
```

2.4 The CPLEX Studio IDE

Derived data types

Set an ordered set of elements of (i.a.) primitve data types, e.g. {string} Locations = {"Ansbach", "Berlin", "Cottbus"}; Array a tuple of (i.a.) primitive data type, sets and other arrays indexed over a set, e.g. float Fixcosts[Locations] = [27.4, 58.3, 30.0]: access by index, z.B.: Fixcosts["Cottbus"] $\rightarrow 30.0$

2 Introduction to OPL

> CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

2.3 Mathematica models in OPL syntax

2.4 The CPLEX Studio IDE

Derived data types: mutiple arrays

 Arrays can be nested into one another to represent multiple indexes, e.g.

```
float Entf[Locations][Locations] =
  [[0.0, 5.05, 4.89],
  [5.05, 0.0, 1.22],
  [4.89, 1.22, 0.0]];
```

Mapping rule: from left to right, from outer to inner

2 Introduction to OPL

CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

11/28 ◀ □ ▸ ◀ @ ▸ ◀ 볼 ▸ ◀ 볼 ▸ _ 볼 _ ∽ 익 ೕ ♡

Simple Operators

- assignment operator =
- arithmetic operators
 - + addition
 - subtraction
 - multiplication
 - / division (rare in linear models)
- comparison operator (for linear models)
 - == equal
 - <= less or equal
 - >= greater or equal

2 Introduction to OPL

CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

Operators

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

Indexed operators

sum operator

$\sum_{i \in I} \ldots \rightarrow \operatorname{sum}(i \text{ in } I)(\ldots)$

universal quantifier

 $\forall i \in I \rightarrow \text{forall}(i \text{ in } I)$

2 Introduction to OPL

CC-BY-SA A. Popp

.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

Operators

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

2.3 Mathematical models in OPL syntax

OPL

CC-BY-SA A. Popp

2 Introduction to

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data type:

Operator

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

Example: Production problem - index sets

Mathematical model

Index sets:

- / set of products
- R set of ressources

Model file

//Index sets
{string} I = ...; //products
{string} R = ...; //ressources

Data file

```
//Index sets
I = {"product 1", "product 2", "product 3"};
R = {"machine A", "machine B"};
```

2 Introduction to OPL

> CC-BY-SA A. Popp

.1 Structure of an OPL project

2.2 Basic data types and operators

Data type

Operator

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

15/28 ◀ □ ▸ ◀ @ ▸ ◀ 볼 ▸ ◀ 볼 ▸ = 볼 = ∽ ೕ

Example: Production problem - parameters

Mathematical model

Parameters:

- p_i price of product $i \in I$
- c_r capacity of ressource $r \in R$
- v_{ri} capacity consumption of product $i \in I$ on ressource $r \in R$

Model file

```
//Parameter
float p[I] = ...; //price
float c[R] = ...; //capacity
float v[R][I] = ...; //capacity consumption
```

2 Introduction to OPL

> CC-BY-SA A. Popp

.1 Structure of an OPL project

2.2 Basic data types and operators

Data type

Operator

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

16/28 ◀ □ ▸ ◀ @ ▸ ◀ 볼 ▸ ◀ 볼 ▸ = 볼 = ∽ ೕ

Example: Production problem - parameters

Mathematical model

Parameters:

- p_i price of product $i \in I$
- c_r capacity of ressource $r \in R$
- v_{ri} capacity consumption of product $i \in I$ on ressource $r \in R$

Data file

```
//Parameters
p = [2.9, 3.3, 2.2];
c = [64.0, 48.0];
v = [
   [5.3, 2.9, 2.5],
   [3.9, 4.8, 3.1]
];
```

2 Introduction to OPL

> CC-BY-SA A. Popp

.1 Structure of an OPL project

2.2 Basic data types and operators

Data type

Operator

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

17/28 《 ㅁ › 《 @ › 《 볼 › 《 볼 › 월 · ' 의 옷 (?

Example: Production problem - decision variables

Mathematical model

Decision variables:

 x_i production quantity of product $i \in I$

[...]

```
x_i \ge 0 \quad \forall i \in I
```

Model file

//Decision variables
dvar float+ x[I]; //production quantity

2 Introduction to OPL

> CC-BY-SA A. Popp

.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

Operator

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

18/28 《 ㅁ › 《 @ › 《 볼 › 《 볼 › 월 · ' 의 오 야

Example: Production problem – objective function

Mathematical model

 $\max \quad \sum_{i \in I} p_i \cdot x_i$

Model file

//objective function
maximize sum(i in I)(p[i]*x[i]);

2 Introduction to OPL

> CC-BY-SA A. Popp

.1 Structure of an OPL project

2.2 Basic data types and operators

Data type

Operator

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

19/28 ◀ □ ▸ ◀ @ ▸ ◀ 볼 ▸ ◀ 볼 ▸ 볼 - ∽ ↔

Example: Production problem - constraints

Mathematical model

s.t.
$$\sum_{i \in I} v_{ri} \cdot x_i \leq c_r \quad \forall r \in R$$

Model file

```
//constraints
subject to{
   //capacity constraints
   forall(r in R)
      sum(i in I)(v[r,i]*x[i]) <= c[r];
}</pre>
```

2 Introduction to OPL

> CC-BY-SA A. Popp

.1 Structure of an OPL project

2.2 Basic data types and operators

Data type

Operator

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

Example: Produktionsproblem.mod

```
1 //index sets
2 {string} I = ...; //products
3 {string} R = ...; //ressources
4
5 //parameters
6 float p[I] = ...; //price
7 float c[R] = ...; //capacity
8 float v[R][I] = ...; //capacity consumption
9
10 //decision variables
11 dvar float+ x[I]; //production quantity
12
  //objective function
13
14 maximize sum(i in I)(p[i] * x[i]);
15
16 //constraints
  subject to{
18
    //capacity constraints
19
    forall(r in R)
20
       sum(i in I)(v[r][i]*x[i]) <= c[r]:</pre>
21
22
23 }
```

2 Introduction to OPL

> CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data type

Operator

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

21/28 《□▶ 《□▶ 《토▶ 《토▶] 토 - '○٩?'

Beispiel: LewigSanstetten.dat

```
1 //index sets
2 I = {"product_1", "product_2", "product_3"};
3 R = {"machine_A", "machine_B"};
4
5 //parameters
6 p = [2.9, 3.3, 2.2];
7 c = [64.0, 48.0];
8 v = [
9 [5.3, 2.9, 2.5],
10 [3.9, 4.8, 3.1]
11 ];
```

2 Introduction to OPL

CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data type

Operator

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

Solution of model instance

> oplrun -v Produktionsproblem.mod LewigSanstetten.dat

. . .

. . .

OBJECTIVE: 35.61677

 $x = [11.737 \ 0 \ 0.71856];$

\leftarrow optimal value

 \leftarrow optimal solution

2 Introduction to OPL

CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data type

Operator

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

23/28 ◀ □ ▶ ◀ @ ▶ ◀ 볼 ▶ ◀ 볼 ▶ 월 - ∽) < @

2 Introduction to OPL

> CC-BY-SA A. Popp

.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

Operator

2.3 Mathematica models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL



2 Introduction to OPL

CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Operators

2.3 Mathematica models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL



2 Introduction to OPL

CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

Operators

2.3 Mathematica models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL



2 Introduction to OPL

CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

0-----

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL



2 Introduction to OPL

CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

Operators

2.3 Mathematica models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL



2 Introduction to OPL

CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Operators

2.3 Mathematica models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

25/28 《□▶ 《□▶ 《■▶ 《필▶ 《필▶] 필 - ∽) < . . .



2 Introduction to OPL

CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Operators

2.3 Mathematica models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

2.5 Errors and warnings in OPL

2 Introduction to OPL

> CC-BY-SA A. Popp

1.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

Operator

2.3 Mathematica models in OPL syntax

2.4 The CPLEX Studio IDE

2.5 Errors and warnings in OPL

26/28 《 □ ▶ 《 🗇 ▶ 《 볼 ▶ 《 볼 ▶ 🛛 볼 🛛 외익 🗠

Types of errors

Classification by severity

- Error prevents the succesful completion of the solution run
- Warning does not prevent the solution run, but can cause unexpected results. Sometimes clue to mistakes in the code.

Classification by time of occurence

compiler errors occur during the translation of the problem for the solver. Will be recognized by the IDE. runtime errors occur during solver runtime. Will not be recognized by the IDE but displayed after a solution run. CC-BY-SA A. Popp

2.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

Operators

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE

Common error messages during first tries

syntax error, unexpected ... (compiler error)

- Compiler does not understand the command after "unexpected" here
- missing semi-colon?
- syntax error, unexpected = (compiler error)
 - special case of above
 - often mix-up of the assignment operator = and the comparison operator ==
- The type ... cannot be used for ... (compiler error)
 - data type mix-up
- index out of bound for array ... (runtime error)
 - tried to access an array with a wrong index value

2 Introduction to OPL

> CC-BY-SA A. Popp

.1 Structure of an OPL project

2.2 Basic data types and operators

Data types

Operators

2.3 Mathematical models in OPL syntax

2.4 The CPLEX Studio IDE