

Inverse Z transform: Example 3 (directly invertible)

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February 3, 2013

Given the following system function of a causal system:

$$H(z) = \frac{-5 - 3z + 2z^{-1}}{1 - 2z^{-1}} \quad \text{ROC} \equiv |z| > 2 \quad (1)$$

Find the impulse response $h[n]$ of the system.

SOLUTION:

Although the numerator of $H(z)$ is a polynomial of greater degree than the denominator you do not need to perform long division. You should notice that this Z-transform is already directly invertible using the shifting property of the Z-transform (without needing to compute residuals or long division):

$$H(z) = -5 \underbrace{\frac{1}{1 - 2z^{-1}}}_{G(z)} - 3z \underbrace{\frac{1}{1 - 2z^{-1}}}_{G(z)} + 2z^{-1} \underbrace{\frac{1}{1 - 2z^{-1}}}_{G(z)}$$

So we finally obtain that:

$$h[n] = -5(2)^n \mu[n] - 3(2)^{n+1} \mu[n+1] + 2(2)^{n-1} \mu[n-1] \quad (2)$$