EXERCISE 3

SGN-1156 Signal Processing Techniques
http://www.cs.tut.fi/courses/SGN-1156
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PROBLEM 1: Consider the system of Figure 1, where the input unit-energy continuous-time signal $x_a(t)$ has a band-limited spectrum $X_a(j\Omega)$, as sketched in Figure 2(a), and is being sampled at the Nyquist rate. The discrete-time processor is an ideal lowpass filter with a frequency response $H(e^{j\omega})$, as shown in Figure 2(b), and has a cutoff frequency $\omega_c = \Omega_m T_s/2$ where T_s is the sampling period. Sketch as accurately as possible the spectrum $Y_a(j\Omega)$ of the output continuous-time signal $y_a(t)$. What is the energy of the output signal $y_a(t)$?

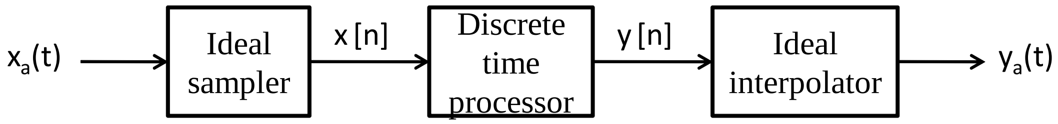

Figure 1: Block diagram of the system.

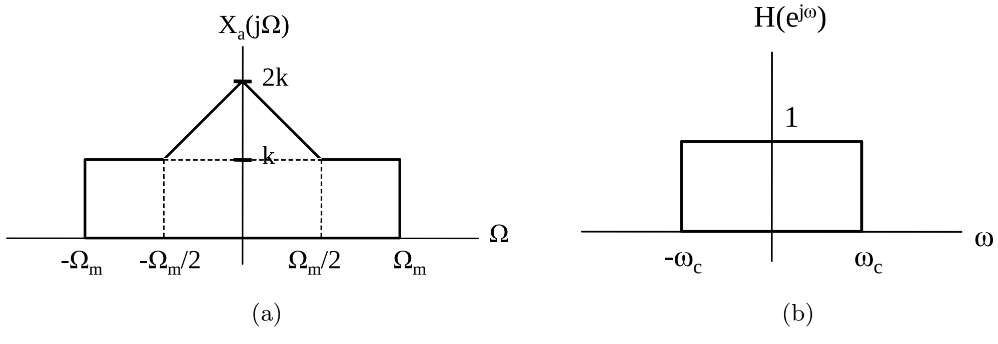

Figure 2: Spectrum of the input and frequency response of the digital filter. The scalar value k on the left figure is unknown.

PROBLEM 2: There is a given signal and noise mixture $x_a(t) = s_a(t) + \epsilon_a(t)$. The signal of interest has a bandwidth of 1000 Hz and the unwanted noise contamination is a sinusoid of frequency equal to 60 Hz, i.e. $\epsilon_a(t) = A\sin(120\pi t)$. Design a discrete-time system, which will suppress the noise component ϵ_a in the mixture $x_a(t)$. The system should include an ideal A/D converter, a discrete-time filter and an ideal D/A converter. The A/D converter should convert the analog signal into a discrete one $x[n] = x_a(nT_s)$, with an appropriate sampling period T_s. The discrete-time block should filter out the unwanted noise components by the following difference equation:

$$y[n] = x[n] + ax[n-1] + bx[n-2]$$

Determine the sampling frequency f_s of the A/D and D/A converters as well as the parameters a and b of the discrete-time filter.

PROBLEM 3: The following *bandpass* signals are to be sampled and fully reconstructed from their sampled versions by using an appropriate bandpass filter. Determine the *smallest possible* sampling frequency Ω_s for each of them, and sketch the frequency response of the reconstruction filters.

(a) $x_a(t)$ is real, with Fourier transform $X_a(j\Omega)$ nonzero for $2\pi \cdot 9000 \leq |\Omega| \leq 2\pi \cdot 12000$.

(b) $x_a(t)$ is real, with Fourier transform $X_a(j\Omega)$ nonzero for $2\pi \cdot 18000 \leq |\Omega| \leq 2\pi \cdot 22000$.

PROBLEM 4: Diagrammed in Figure 3 is a hybrid digital-analog system. The discrete-time system $H_{LPF}(e^{j\omega})$ is a low-pass filter:

$$H_{LPF}(e^{j\omega}) = \begin{cases} A & |\omega| \leq \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

and the analog system is a high-pass filter with a frequency response shown in Figure 4. The input analog signal is bandlimited to $\Omega_0 = 2\pi \cdot 4000$, and the sampling period of the ideal C/D and D/C converters is $T_s = 10^{-4}$ s Find values for A and ω_0 that will result in perfect reconstruction of $x(t)$, i.e. values for which $y(t) = x(t)$.

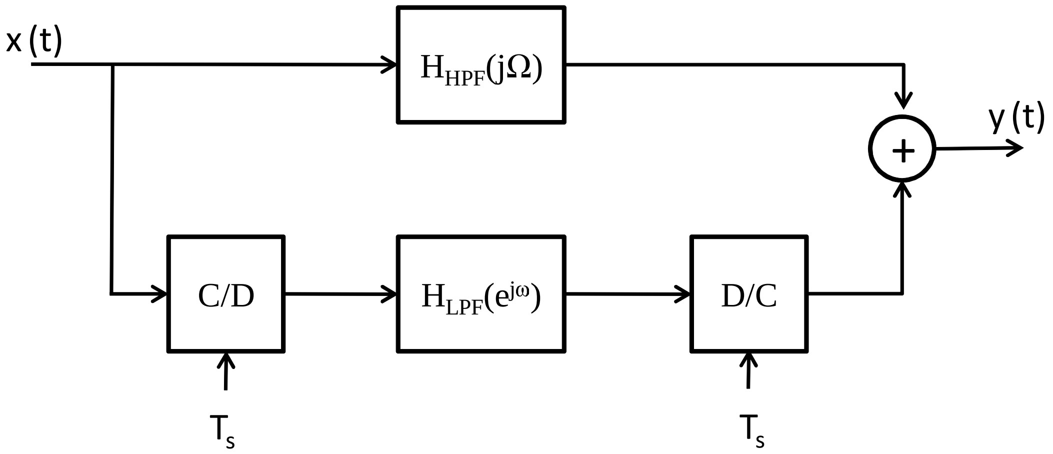

Figure 3: Hybrid digital-analog system.

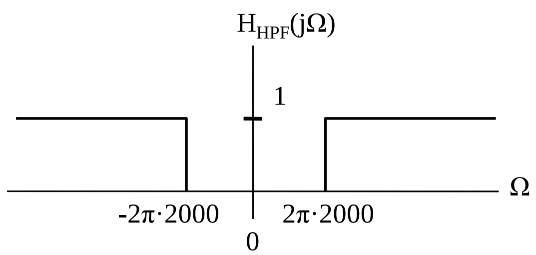

Figure 4: Hybrid digital-analog system.