Bouncing Balls

In this note, we describe the collision between two perfectly elastic balls moving in one dimension.

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            m1        m2
            O->       O->
Before:     u1        u2
After:      v1        v2
```

We have two balls, with mass m_1 and m_2. Let their velocities before the collision be u_1 and u_2, and their velocities after be v_1 and v_2.

We know the following:

Conservation of Momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Conservation of Energy

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

These two equations can be solved (for example, by eliminating one variable, then solving the resultant quadratic equation). If $u_1 \neq u_2$ this equation has two solutions, one for the solution before the collision, the other for the solution after.

For the solution after collisions we obtain:

$$v_2 - v_1 = -(u_2 - u_1)$$

that is, the relative velocity changes signs at the collision.

So, define the following values I (the total momentum) and R (the relative velocity before the collision):

$$I = m_1 u_1 + m_2 u_2$$
$$R = u_2 - u_1$$

We can then obtain v_1 and v_2 from the system of equations:

$$m1v1 + m2v2 = I$$
$$v2 - v1 = -R$$

which is easier to solve.