

Exercise problems on probability basics

Q1. In one year, three awards (research, teaching, and service) will be given to a class of 25 graduate students in a statistics department. If each student can receive at most one award, how many possible selections are there?

Q2. A president and a treasurer are to be chosen from a student club consisting of 50 people. How many different choices of officers are possible if

- (a) There are no restrictions;
- (b) A will serve only if he is president;
- (c) B and C will serve together or not at all.

Q3. In a college football training session, the defensive coordinator needs to have 10 players standing in a row. Among these 10 players, there are 1 freshman, 2 sophomores, 4 juniors, and 3 seniors. How many different ways can they be arranged in a row if only their class level will be distinguished?

Q4. How many different letter arrangements can be made from the letters in the word *STATISTICS*?

Q5. A coin is tossed twice. What is the probability that at least 1 head occurs?

Q6. A statistics class for engineers consists of 25 industrial, 10 mechanical, 10 electrical and 8 civil engineering students. If a person is randomly selected by the instructor to answer a question, find the probability that the student chosen is:

- (a) an industrial engineering major and
- (b) a civil engineering or an electrical engineering major.

Q7. John is going to graduate from an industrial engineering department in a university by the end of the semester. After being interviewed at two companies he likes, he assesses that his probability of getting an offer from company A is 0.8, and his probability of getting an offer from company B is 0.6. If he believes that the probability that he will get offers from both companies is 0.5, what is the probability that he will get at least one offer from these two companies?

Q8. If the probabilities are, respectively, 0.09, 0.15, 0.21, and 0.23 that a person purchasing a new automobile will choose the color green, white, red, or blue, what is the probability that a given buyer will purchase a new automobile that comes in one of those colors?

Q9. According to *Consumer Digest* (July/August 1996), the probable location of personal computers (PC) in the home is as follows:

Adult bedroom: 0.03

Child bedroom: 0.15

Other bedroom: 0.14

Office or den: 0.40

Other rooms: 0.28

- (a) What is the probability that a PC is in a bedroom?
- (b) What is the probability that it is not in a bedroom?
- (c) Suppose a household is selected at random from households with a PC; in what room would you expect to find a PC?

Q10. The probability that a regularly scheduled flight departs on time is $P(D) = 0.83$; the probability that it arrives on time is $P(A) = 0.82$; and the probability that it departs and arrives on time is $P(D \cap A) = 0.78$. Find the probability that a plane:

- (a) arrives on time, given that it departed on time, and
- (b) departed on time, given that it has arrived on time.

Q11. Suppose that we have a fuse box containing 20 fuses, of which 5 are defective. If 2 fuses are selected at random and removed from the box in succession without replacing the first, what is the probability that both fuses are defective?

Q12. A small town has one fire engine and one ambulance available for emergencies. The probability that the fire engine is available when needed is 0.98, and the probability that the ambulance is available when called is 0.92. In the event of an injury resulting from a burning building, find the probability that both the ambulance and the fire engine will be available, assuming they operate independently.

Q13. A random sample of 200 adults are classified below by sex and their level of education attained.

Education	Male	Female
Elementary	38	45
Secondary	28	50
College	22	17

If a person is picked at random from this group, find the probability that

- (a) the person is a male, given that the person has a secondary education;
- (b) the person does not have a college degree, given that the person is a female.

Q14. In a certain assembly plant, three machines, B_1 , B_2 , and B_3 , make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished

product is randomly selected. What is the probability that it is defective? If the product was chosen randomly and found to be defective, what is the probability that it was made by machine $B3$?

Q15. A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for 30%, 20%, and 50% of the products, respectively. The defect rate is different for the three procedures which are 0.01, 0.03 and 0.02 respectively. If a random product was observed and found to be defective, which plan was most likely used and thus responsible?

Q16. The probability that a certain kind of component will survive a shock test is $3/4$. Find the probability that exactly 2 of the next 4 components tested survive.

Q17. The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that (a) at least 10 survive, (b) from 3 to 8 survive, and (c) exactly 5 survive?

Q18. It is conjectured that an impurity exists in 30% of all drinking wells in a certain rural community. In order to gain some insight into the true extent of the problem, it is determined that some testing is necessary. It is too expensive to test all of the wells in the area, so 10 are randomly selected for testing.

- (a) Using the binomial distribution, what is the probability that exactly 3 wells have the impurity, assuming that the conjecture is correct?
- (b) What is the probability that more than 3 wells are impure?
- (c) Find the mean and variance of the binomial random variable and then use Chebyshev's theorem to interpret the interval $\mu \pm 2\sigma$.

Q19. According to *Chemical Engineering Progress* approximately 30% of all pipework failures in chemical plants are caused by operator error.

- (a) What is the probability that out of the next 20 pipework failures at least 10 are due to operator error?
- (b) What is the probability that no more than 4 out of 20 such failures are due to operator error?
- (c) Suppose, for a particular plant, that out of the random sample of 20 such failures, exactly 5 are due to operator error. Do you feel that the 30% figure stated above applies to this plant?

Q20. In an NBA (National Basketball Association) championship series, the team that wins four games out of seven is the winner. Suppose that teams A and B face each other in the championship games and that team A has probability 0.55 of winning a game over team B .

- (a) What is the probability that team A will win the series in 6 games?
- (b) What is the probability that team A will win the series?
- (c) If teams A and B were facing each other in a regional playoff series, which is decided by winning three out of five games, what is the probability that team A would win the series?

Q21. At a “busy time,” a telephone exchange is very near capacity, so callers have difficulty placing their calls. It may be of interest to know the number of attempts necessary in order to make a connection. Suppose that we let $p = 0.05$ be the probability of a connection during a busy time.

Q22. Changes in airport procedures require considerable planning. Arrival rates of aircraft are important factors that must be taken into account. Suppose small aircraft arrive at a certain airport, according to a Poisson process, at the rate of 6 per hour.

- (a) What is the probability that exactly 4 small aircraft arrive during a 1-hour period?
- (b) What is the probability that at least 4 arrive during a 1-hour period?
- (c) If we define a working day as 12 hours, what is the probability that at least 75 small aircraft arrive during a working day?

Q23. Given that X has a normal distribution with $\mu = 300$ and $\sigma = 50$, find the probability that X assumes a value greater than 362.

Q24. The marks of students of BTech computer science engineering is normally distributed random variable with mean 76.5 and standard deviation 6.5.

- a. What is the probability that mean marks is more than 80?
- b. What is the probability that mean marks is less than 60?
- c. What is the probability that the mean marks lie between 65 and 85?
- d. What is the probability that the mean marks lie between 85 and 95?
- e. What is the probability that mean marks lie between 50 and 60?

Q25. An electrical firm manufactures light bulbs that have a life, before burn-out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.

Q26. Gauges are used to reject all components for which a certain dimension is not within the specification $1.50 \pm d$. It is known that this measurement is normally distributed with mean 1.50 and standard deviation 0.2. Determine the value d such that the specifications “cover” 95% of the measurements.

Q27. Consider that the random variable X follows exponential probability distribution with average 8 per minute.

- a) Find $P(x \leq 6)$
- b) Find $P(x \leq 4)$
- c) Find $P(x \geq)$
- d) Find $P(4 \leq x \leq 6)$

Q28. Internet Magazines monitors Internet service providers (ISPs) and provides statistics on their performance. The average time to download a web page for free ISP is approximately 20 seconds. Assume the time to download a Web page follows an exponential distribution.

- a. What is the probability that it will take less than 10 seconds?
- b. What is the probability that it will take more than 30 seconds?
- c. What is the probability that it will take between 10 and 30 seconds?

Q29.

The placement department of a management school estimated the probability distribution of placement of its students based on past experience. Compute the expected number of students that will be placed in the long run. Also calculate the variance from the data given below.

No. of Students Placed	Probability
40	0.4
55	0.1
32	0.2
48	0.15
45	0.15

Q30. Compute the expected value and variance of the following discrete probability distribution

x	Probability (x)
1	0.1
2	0.2
3	0.3
4	0.15
5	0.25

Q31. In a binomial distribution where $n=20$ and $p = 0.35$. Compute the probability of the following exactly:

- a) $P(X=1)$
- b) $P(X=4)$
- c) $P(3 \leq X \leq 8)$

Q32. In a binomial probability distribution where $n=15$ and $p = 0.65$. Determine the probability of exactly a) $P(X=0)$ b) $P(X = 5)$ c) $P(X > 3)$

Q33. A recent survey revealed 80 percent graduating students of CSE in Thapar gets placed. Suppose if you pick up a random sample of 10 CSE graduating students.

- a) What is the probability that exactly two students will get placed?
- b) What is the probability that exactly five students will get placement?
- c) What is the expected number of students that get placement?

Q34. Sania is a personal attendant at Jyoti Clinic. She wants to determine the number of patients arrive every 30 minutes. After studying three months previous data, she arrived at the conclusion that average number of patients arrive in clinic every 30 minutes is 10. Find the probability of arriving exactly 2 and 4 patients every 30 minutes. Also find the $P(X>5)$.

Q35. It is known that 3% of people whose luggage is screened at an airport have questionable objects in their luggage. What is the probability that a string of 15 people pass through screening successfully before an individual is caught with a questionable object? What is the expected number of people to pass through before an individual is stopped?

Q36. Computer technology has produced an environment in which robots operate with the use of microprocessors. The probability that a robot fails during any 6-hour shift is 0.10. What is the probability that a robot will operate through at most 5 shifts before it fails?