

Combining relation (1) and (2);

we have $\sqrt{A} < x_1 < x_0$

By induction, we can establish

$$\sqrt{A} < x_{m+1} < x_m < \dots < x_0$$

& hence $x_m \rightarrow \sqrt{A}$ as $m \rightarrow \infty$ b/c

$\{x_m\}$ is a decreasing sequence that is bounded from below!

iii) If $0 < x_0 < \sqrt{A}$;

$$0 < (x_0 - \sqrt{A})^2 = x_0^2 - 2x_0\sqrt{A} + A$$

$$\Rightarrow 2x_0\sqrt{A} < x_0^2 + A$$

$$\Rightarrow \sqrt{A} < \frac{x_0}{2} + \frac{A}{2x_0} = x_1$$

So now we have $x_1 > \sqrt{A}$ & if we proceed w/ the iteration scheme $g(x_n) = x_{n+1}$, we know from case ii) that we will be able to establish the following

$$\sqrt{A} < x_{m+1} < x_m < \dots < x_2 < x_1$$

& $\therefore x_m \rightarrow \sqrt{A}$ as $m \rightarrow \infty$.

b) If $x_0 < 0$, $x_m \rightarrow -\sqrt{A}$ as $m \rightarrow \infty$.