

Module 1: Thinking in Probability

Topsy in a Circus

Circular Random Walk

A circus named *Apollo* designed a performance for a kangaroo named *Topsy* to hop on a circular track. The track is a one-dimensional grid of a length of 60 units (labeling from 1 to 60 units). *Topsy* is excited and jumps around in the given circular track. At any given instance, he decides to hop to the left or to the right independent of his past moves.

In this project, we will use calculations based on the principles of conditional probability, the law of total probability, and the law of total expectation to predict what are the odds, he will bounce on the circular track, his left-sided moves balancing out his right-sided moves on an average, and can end his performance once he reaches at 60th grid position, that is, the stopping position of his movement. Given that he can start from any position on the circular track, what is his expectancy in terms of the total number of hops reaching the stopping position from his first move? Does a certain starting position on the track give him the best chance to hop for a longer duration? Here, we will build a computer simulation of his actions by applying a random number generator using computer software in order to mimic *Topsy*'s mental choices to hop either to the left or to the right independent of his past moves.

Exercise 1. Consider a one-dimensional circular grid track of the length of 60 units (labeling from 1 to 60) and the starting position is 10 units. Create a simulation of *Topsy* hopping on the circular track over time.

Exercise 2. Does the kangaroo eventually reaches the stopping position or does he just bounce on and off the track in a never-ending fashion? Play your simulation and justify your answer.

Exercise 3. Does your answer above depend on the size of the circular track or the starting position of the kangaroo? Repeat your experiment with different grid sizes and starting positions to justify your answer.

Exercise 4. What is the life expectancy of the performance of the kangaroo? Does your answer tally with the theoretically predicted expected number of hops? Is there a discrepancy? Explain why.