

Full Name: \_\_\_\_\_

UID: \_\_\_\_\_

*Instructions:* You must **not** be in possession of any cheat sheet, notes, or electronic devices like laptops or calculators inside the examination hall. Answer **all five multiple-choice questions (MCQs)**. The score allotted to each question is **one**. There will be a penalty of **0.5 marks** for each wrong answer and a penalty of **0.25 marks** for each un-attempted question. **Maximum score is 5**. Tick against the correct option. Only one option is correct in every question.

=====START OF QUESTIONS=====

1. Let  $A$  be a null matrix of order  $3 \times 3$ , then the eigenspace (the set of all eigenvectors of  $A$ ) corresponding to the eigenvalues of matrix  $A$  is

$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$      $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$      $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$     *None of these*

2. Let  $V$  be the subspace of  $\mathbb{R}^4$  spanned by  $u = \{1, 1, 1, 1\}$  and  $v = \{1, 9, 9, 1\}$ . The orthonormal bases of  $V$  obtained by the Gram-Schmidt orthonormalization process are:

$\left\{ \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right), \left( -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right) \right\}$   
  $\{(1, 1, 0, 0), (1, 0, 1, 0)\}$   
  $\left\{ \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right), \left( \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right) \right\}$   
  $\left\{ \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right), \left( \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right) \right\}$

3. Let  $T$  be a linear transformation which is a projection of the space  $\mathbb{R}^3$  to the  $xy$ -plane embedded in  $\mathbb{R}^3$ . What are the eigenvalues of the matrix representation of  $T$ ?

0,0,1    0,1,1    1,1,1,    0,0,0

4. Find the matrix  $A^3$ , if matrix the  $A$  is diagonalizable such that  $D = S^{-1}AS$ , where matrix  $S = \begin{bmatrix} 2 & -1 \\ 5 & 1 \end{bmatrix}$

and matrix  $D = \begin{bmatrix} 6 & 0 \\ 0 & -1 \end{bmatrix}$ ?

$\begin{bmatrix} 61 & 62 \\ 156 & 154 \end{bmatrix}$   
  $\begin{bmatrix} 8 & -1 \\ 125 & 1 \end{bmatrix}$   
  $\begin{bmatrix} 61 & 62 \\ 155 & 154 \end{bmatrix}$   
  $\begin{bmatrix} 216 & 0 \\ 0 & -1 \end{bmatrix}$

5. The set of linearly independent eigenvectors corresponding to the eigenvalues of the matrix  $\begin{bmatrix} 2 & -1 \\ 2 & 4 \end{bmatrix}$  is:

$\{(i, 1), (1, i)\}$   
  $\{(i, -1), (-i, -1)\}$   
  $\{(1, -1 - i), (1, -1 + i)\}$   
 None of these