Q.1 (a)

Suppose $g(x) = \frac{x}{2} + \frac{A}{2x}$

Plug in $x = \sqrt{A}$ in $g(x)$

$$g(\sqrt{A}) = \frac{\sqrt{A}}{2} + \frac{A}{2\sqrt{A}} = \frac{\sqrt{A}}{2} + \frac{\sqrt{A}}{2}$$
$$= \sqrt{A}$$

So $\sqrt{A}$ is a f.p. of $g(x)$.

Now, note $g'(x) = \frac{1}{2} - \frac{A}{2x^2}$; $x \neq 0$

If $x > \sqrt{A}$; then the $\underline{2^{nd}\text{ term}}$ is less than the 1^{st} term ($1/2$) & $\underline{\underline{g'(x) > 0}}$

Now analyze the following 3 cases :-

i) if $x_0 = \sqrt{A}$ & b/c $\sqrt{A}$ is a f.p. of $g(x)$;
$\Rightarrow x_m = \sqrt{A}$ $\forall m$ and $\lim_{m \to \infty} x_m = \sqrt{A}$ & we are done

ii) if $x_0 > \sqrt{A}$, then

$$x_1 - \sqrt{A} = g(x_0) - g(\sqrt{A}) \quad \text{b/c } x_{n+1} = g(x_n)$$
$$\overset{MVT}{=} g'(\xi)(x_0 - \sqrt{A}) > 0 \quad \text{b/c } g'(\xi) > 0,\ x_0 > \sqrt{A}$$

So $x_1 > \sqrt{A}$ —— (1)

Also $x_1 = \frac{x_0}{2} + \frac{A}{2x_0} < \frac{x_0}{2} + \frac{A}{2\sqrt{A}}$ b/c $x_0 > \sqrt{A}$

$$= \frac{1}{2}(x_0 + \sqrt{A}) < \frac{1}{2}(x_0 + x_0)$$

$\Rightarrow x_1 < x_0$ —— (2)