

Database Systems, CSCI 4380-01  
**Homework # 1 Extra Credit**

Due Tuesday, February 3, 2026 at 7:59:59 PM EDT

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**Question 3 [30 points].** For the following relations, find and list all the keys.

1.  $R1(A, B, C, D, E, F, G)$ ,  $\mathcal{F} = \{BDG \rightarrow FG, AF \rightarrow DE, EF \rightarrow A\}$
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**Solution**

- To start, ensuring that the fd (functional dependency) is in its minimal basis makes it significantly easier to find the candidate keys. The four rules for transforming a fd into a minimal basis are as follows:
  - Convert all fd's RHS (right-hand side) to a single attribute).
  - Remove trivial functional dependencies ( $A \rightarrow A$ ).
  - No redundant dependencies;  
No extraneous attributes on the LHS (left-hand side).

- (a) Single attribute RHS:

	<u>Previous</u>	<u>After</u>
$\rightarrow \mathcal{F} = \{BDG \rightarrow FG\}$	$\Rightarrow$	$BDG \rightarrow F, BDG \rightarrow G$
$AF \rightarrow DE$	$\Rightarrow$	$AF \rightarrow D, AF \rightarrow E$
$EF \rightarrow A$	$\Rightarrow$	$EF \rightarrow A$ (unchanged)

$\rightarrow$  New Functional Dependency:  
 $\Rightarrow \mathcal{F} = \{BDG \rightarrow F, BDG \rightarrow G, AF \rightarrow D, AF \rightarrow E, EF \rightarrow A\}$

- (b) Removing trivial fds:

$\rightarrow \mathcal{F} = \{BDG \rightarrow F, BDG \rightarrow G, AF \rightarrow D, AF \rightarrow E, EF \rightarrow A\}$   
+ + + +  
(*G is trivial*)

$\rightarrow$  New Functional Dependency:  
 $\Rightarrow \mathcal{F} = \{BDG \rightarrow F, AF \rightarrow D, AF \rightarrow E, EF \rightarrow A\}$

- (c) Removing redundant and extraneous attributes:

$\rightarrow \mathcal{F} = \{BDG \rightarrow F, AF \rightarrow D, AF \rightarrow E, EF \rightarrow A\}$   
 $\rightarrow$  There is a shortcut we can take here. Each RHS attribute is produced by exactly one fd, so none can be redundant. (In other words, removing one of the LHS attributes in a fd or removing an entire fd will make the RHS attribute inaccessible.)

- Finding the keys using a minimal basis(s):
  - Find all unreachable attributes on the RHS side (cannot be derived).
  - Build a key starting from unreachable attributes and adding RHS attributes from  $\mathcal{F}$ . Check every key found to ensure it is not a superkey.
- $\mathcal{F} = \{BDG \rightarrow F, AF \rightarrow D, AF \rightarrow E, EF \rightarrow A\}$
- (a) Inaccessible Attributes:  $B, C, G$  (*must appear in all candidate keys*)  
 $\rightarrow BCG+ = \{B, C, G\}$  (*not a key*)
- (b) Example: Building the key with  $B, C, G$ :
  - $\rightarrow (BCG)$  Let's add attribute  $D$ , based on the functional dependency  $BDG \rightarrow F$ :  
 $BCGD+ = \{B, C, D, G\}$  (*missing A, E, F*)
  - $(BCGD)$  Next, try adding attribute  $E$ , based on the functional dependency  $EF \rightarrow A$ :  
 $BCGDE+ = \{A, B, C, D, E, F, G\}$  (*works!*)
  - Check if  $BCGDE$  is a superkey:  
 $CGDE+ = \{A, B, C, D, E, G\}$  (*missing F*)  
 $BGDE+ = \{A, B, D, E, F, G\}$  (*missing C*)  
 $BCDE+ = \{A, B, C, D, E, G\}$  (*missing F*)  
 $BCGE+ = \{A, B, C, D, E, G\}$  (*missing F*)  
 $BCGD+ = \{B, C, D, E, F, G\}$  (*missing A*)
  - Since this cannot be reduced, we conclude that  $\{B, C, D, G, E\}$  is a **minimal key**.
  - $(BCGD)$  Let's try adding  $A$  based on  $AF \rightarrow D, AF \rightarrow E$ :  
 $BCGAD+ = \{A, B, C, D, E, F, G\}$  (*works!*)
  - Check if  $BCGAD$  is a superkey:  
 $CGAD+ = \{A, B, C, D, E, G\}$  (*missing F*)  
 $BGAD+ = \{A, B, D, E, F, G\}$  (*missing C*)  
 $BCAD+ = \{A, B, C, D, E, G\}$  (*missing F*)  
 $BCGD+ = \{B, C, D, E, F, G\}$  (*missing A*)  
 $BCGA+ = \{A, B, C, D, E, G\}$  (*missing F*)
  - $\{A, B, C, D, G\}$  is a **minimal key** as it cannot be reduced.
  - $(BCGD)$  There are no other LHS attributes in the fds that we can try adding (the other attributes already exist in  $BCGD+$ ). This means we move back to checking  $BCG$ .
    - $\rightarrow (BCG)$  Add attributes  $AF$  based on the fd  $AF \rightarrow D, AF \rightarrow E$ :  
 However, we already found a minimal key with  $AF$ . We can skip this fd.
    - $\rightarrow (BCG)$  Add attributes  $EF$  based on the last fd  $EF \rightarrow A$ :  
 $BCGEF+ = \{A, B, C, D, G, F, G\} = R1$  (*works!*)
    - Check if  $BCGEF$  is a superkey:  
 $CGEF+ = \{A, C, D, E, F\}$  (*missing B*)  
 $BGEF+ = \{A, B, D, E, F\}$  (*missing C*)  
 $BCEF+ = \{B, C, D, E, F\}$  (*missing A*)  
 $BCGF+ = \{B, C, D, E, F\}$  (*missing A*)  
 $BCGE+ = \{A, B, C, D, E\}$  (*missing F*)
    - We conclude that  $\{B, C, E, F, G\}$  is a **minimal key**.

- **Answer:** Our minimal keys are  $\{B, C, D, G, E\}, \{A, B, C, D, G\}, \{B, C, E, F, G\}$ .