

Database Systems, CSCI 4380-01
Homework # 1 Extra Credit
Due Tuesday, February 3, 2026 at 7:59:59 PM EDT

Question 3 [30 points]. For the following relations, find and list all the keys.

1. $R1(A, B, C, D, E, F, G)$, $\mathcal{F} = \{BDG \rightarrow FG, AF \rightarrow DE, EF \rightarrow A\}$
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Solution

- To start, ensuring that the fd (functional dependency) is in it's minimal basis makes it significantly easier to find the candidate keys. The four rules for transforming a fd into a minimal basis are as follows:
 - a) Convert all fd's RHS (right-hand side) to a single attribute).
 - b) Remove trivial functional dependencies ($A \rightarrow A$).
 - c) No redundant dependencies;
No extraneous attributes on the LHS (left-hand side).

(a) Single attribute RHS:

<u>Previous</u>	<u>After</u>
$\rightarrow \mathcal{F} = \{BDG \rightarrow FG$	$\Rightarrow BDG \rightarrow F, BDG \rightarrow G$
$AF \rightarrow DE$	$\Rightarrow AF \rightarrow D, AF \rightarrow E$
$EF \rightarrow A$	$\Rightarrow EF \rightarrow A$ <i>(unchanged)</i>

\rightarrow New Functional Dependency:
 $\Rightarrow \mathcal{F} = \{BDG \rightarrow F, BDG \rightarrow G, AF \rightarrow D, AF \rightarrow E, EF \rightarrow A\}$

(b) Removing trivial fds:

$$\rightarrow \mathcal{F} = \{BDG \rightarrow F, BDG \rightarrow G, AF \rightarrow D, AF \rightarrow E, EF \rightarrow A\}$$

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(G is trivial)

\rightarrow New Functional Dependency:
 $\Rightarrow \mathcal{F} = \{BDG \rightarrow F, AF \rightarrow D, AF \rightarrow E, EF \rightarrow A\}$

(c) Removing redundant and extraneous attributes:

$$\rightarrow \mathcal{F} = \{BDG \rightarrow F, AF \rightarrow D, AF \rightarrow E, EF \rightarrow A\}$$

- \rightarrow There is a shortcut we can take here. Each RHS attribute is produced by exactly one fd, so none can be redundant. (In other words, removing one of the LHS attributes in a fd or removing an entire fd will make the RHS attribute inaccessible.)

- Finding the keys using a minimal basis(s):
 - a) Find all unreachable attributes on the RHS side (cannot be derived).
 - b) Build a key starting from unreachable attributes and adding RHS attributes from \mathcal{F} .
Check every key found to ensure it is not a superkey.
- $\mathcal{F} = \{BDG \rightarrow F, AF \rightarrow D, AF \rightarrow E, EF \rightarrow A\}$

(a) Inaccessible Attributes: B, C, G (*must appear in all candidate keys*)
 $\rightarrow BCG+ = \{B, C, G\}$ (*not a key*)

(b) Example: Building the key with B, C, G :

$\rightarrow (BCG)$ Let's add attribute D , based on the functional dependency $BDG \rightarrow F$:
 $BCGD+ = \{B, C, D, G\}$ (*missing A, E, F*)

$(BCGD)$ Next, try adding attribute E , based on the functional dependency $EF \rightarrow A$:
 $BCGDE+ = \{A, B, C, D, E, F, G\}$ (*works!*)

Check if $BCGDE$ is a superkey:

$CGDE+ = \{A, B, C, D, E, G\}$ (*missing F*)

$BGDE+ = \{A, B, D, E, F, G\}$ (*missing C*)

$BCDE+ = \{A, B, C, D, E, G\}$ (*missing F*)

$BCGE+ = \{A, B, C, D, E, G\}$ (*missing F*)

$BCGD+ = \{B, C, D, E, F, G\}$ (*missing A*)

Since this cannot be reduced, we conclude that $\{B, C, D, G, E\}$ is a **minimal key**.

$(BCGD)$ Let's try adding A based on $AF \rightarrow D, AF \rightarrow E$:

$BCGAD+ = \{A, B, C, D, E, F, G\}$ (*works!*)

Check if $BCGAD$ is a superkey:

$CGAD+ = \{A, B, C, D, E, G\}$ (*missing F*)

$BGAD+ = \{A, B, D, E, F, G\}$ (*missing C*)

$BCAD+ = \{A, B, C, D, E, G\}$ (*missing F*)

$BCGD+ = \{B, C, D, E, F, G\}$ (*missing A*)

$BCGA+ = \{A, B, C, D, E, G\}$ (*missing F*)

$\{A, B, C, D, G\}$ is a **minimal key** as it cannot be reduced.

$(BCGD)$ There are no other LHS attributes in the fds that we can try adding (the other attributes already exist in $BCGD+$). This means we move back to checking BCG .

$\rightarrow (BCG)$ Add attributes AF based on the fd $AF \rightarrow D, AF \rightarrow E$:

However, we already found a minimal key with AF . We can skip this fd.

$\rightarrow (BCG)$ Add attributes EF based on the last fd $EF \rightarrow A$:

$BCGEF+ = \{A, B, C, D, G, F, G\} = R1$ (*works!*)

Check if $BCGEF$ is a superkey:

$CGEF+ = \{A, C, D, E, F\}$ (*missing B*)

$BGEF+ = \{A, B, D, E, F\}$ (*missing C*)

$BCEF+ = \{B, C, D, E, F\}$ (*missing A*)

$BCGF+ = \{B, C, D, E, F\}$ (*missing A*)

$BCGE+ = \{A, B, C, D, E\}$ (*missing F*)

We conclude that $\{B, C, E, F, G\}$ is a **minimal key**.

- **Answer: Our minimal keys are** $\{B, C, D, G, E\}, \{A, B, C, D, G\}, \{B, C, E, F, G\}$.