Asymptote: the Vector Graphics Language

For version 2.10



This file documents Asymptote, version 2.10.

http://asymptote.sourceforge.net

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1 Description

Asymptote is a powerful descriptive vector graphics language that provides a mathematical coordinate-based framework for technical drawings. Labels and equations are typeset with LaTeX, for overall document consistency, yielding the same high-quality level of typesetting that LaTeX provides for scientific text. By default it produces PostScript output, but it can also generate any format that the ImageMagick package can produce.

A major advantage of Asymptote over other graphics packages is that it is a high-level programming language, as opposed to just a graphics program: it can therefore exploit the best features of the script (command-driven) and graphical-user-interface (GUI) methods for producing figures. The rudimentary GUI xasy included with the package allows one to move script-generated objects around. To make Asymptote accessible to the average user, this GUI is currently being developed into a full-fledged interface that can generate objects directly. However, the script portion of the language is now ready for general use by users who are willing to learn a few simple Asymptote graphics commands (see Chapter 4 [Drawing commands], page 14).

Asymptote is mathematically oriented (e.g. one can use complex multiplication to rotate a vector) and uses LaTeX to do the typesetting of labels. This is an important feature for scientific applications. It was inspired by an earlier drawing program (with a weaker syntax and capabilities) called MetaPost.

The Asymptote vector graphics language provides:

- a standard for typesetting mathematical figures, just as TEX/LaTeX is the de-facto standard for typesetting equations.
- LaTeX typesetting of labels, for overall document consistency;
- the ability to generate and embed 3D vector PRC graphics within PDF files;
- a natural coordinate-based framework for technical drawings, inspired by MetaPost, with a much cleaner, powerful C++-like programming syntax;
- compilation of figures into virtual machine code for speed, without sacrificing portability;
- the power of a script-based language coupled to the convenience of a GUI;
- customization using its own C++-like graphics programming language;
- sensible defaults for graphical features, with the ability to override;
- a high-level mathematically oriented interface to the PostScript language for vector graphics, including affine transforms and complex variables;
- functions that can create new (anonymous) functions;
- deferred drawing that uses the simplex method to solve overall size constraint issues between fixed-sized objects (labels and arrowheads) and objects that should scale with figure size;

Many of the features of Asymptote are written in the Asymptote language itself. While the stock version of Asymptote is designed for mathematics typesetting needs, one can write Asymptote modules that tailor it to specific applications. A scientific graphing module has already been written (see Section 8.27 [graph], page 95). Examples of Asymptote code and output, including animations, are available at

http://asymptote.sourceforge.net/gallery/.

Links to many external resources, including an excellent user-written ${\tt Asymptote}$ tutorial can be found at

http://asymptote.sourceforge.net/links.html.

2 Installation

After following the instructions for your specific distribution, please see also Section 2.4 [Configuring], page 4.

We recommend subscribing to new release announcements at

```
http://freshmeat.net/projects/asy
```

Users may also wish to monitor the Asymptote forum:

http://sourceforge.net/projects/asymptote/forums/forum/409349

2.1 UNIX binary distributions

We release both tgz and RPM binary distributions of Asymptote. The root user can install the Linux i386 tgz distribution of version x.xx of Asymptote with the commands:

```
tar -C / -zxf asymptote-x.xx.i386.tgz
texhash
```

The texhash command, which installs LaTeX style files, is optional. The executable file will be /usr/local/bin/asy) and example code will be installed by default in /usr/local/share/doc/asymptote/examples.

Fedora users can easily install the most recent version of Asymptote with the command yum --enablerepo=rawhide install asymptote

To install the latest version of Asymptote on a Debian-based distribution (e.g. Ubuntu, Mepis, Linspire) follow the instructions for compiling from UNIX source (see Section 2.6 [Compiling from UNIX source], page 6). Alternatively, Debian users can install one of Hubert Chan's prebuilt Asymptote binaries from

http://ftp.debian.org/debian/pool/main/a/asymptote

2.2 MacOS X binary distributions

MacOS X users can either compile the UNIX source code (see Section 2.6 [Compiling from UNIX source], page 6) or install the contributed Asymptote binary available at

```
http://www.hmug.org/pub/MacOS_X/X/Applications/Publishing/asymptote/
```

Because these preconfigured binary distributions have strict architecture and library dependencies that many installations do not satisfy, we recommend installing Asymptote directly from the official source:

```
http://sourceforge.net/project/showfiles.php?group_id=120000
```

Note that many MacOS X (and FreeBSD) systems lack the GNU readline library. For full interactive functionality, GNU readline version 4.3 or later must be installed.

2.3 Microsoft Windows

Users of the Microsoft Windows operating system can install the self-extracting Asymptote executable asymptote-x.xx-setup.exe, where x.xx denotes the latest version.

A working TEX implementation (such as the one available at http://www.miktex.org) will be required to typeset labels. You will also need to install GPL Ghostscript from http://sourceforge.net/projects/ghostscript/.

To view the default PostScript output, you can install the program gsview available from http://www.cs.wisc.edu/~ghost/gsview/. A better (and free) PostScript viewer available at http://psview.sourceforge.net/ (which in particular works properly in interactive mode) unfortunately currently requires some manual configuration. Specifically, if version x.xx of psview is extracted to the directory c:\Program Files one needs to put import settings;

```
psviewer="c:\Program Files\psview-x.xx\psv.exe"; in the optional Asymptote configuration file; see [configuration file], page 159).

The ImageMagick package from
```

http://www.imagemagick.org/script/binary-releases.php

is required to support output formats other than EPS, PDF, SVG, and PNG (see [convert], page 159). The Python interpreter from http://www.python.org is only required if you wish to try out the graphical user interface (see Chapter 11 [GUI], page 163).

Example code will be installed by default in the examples subdirectory of the installation directory (by default, C:\Program Files\Asymptote).

2.4 Configuring

In interactive mode, or when given the -V option (the default when running Asymptote on a single file under MSDOS), Asymptote will automatically invoke the PostScript viewer gv (under UNIX) or gsview (under MSDOS to display graphical output. These defaults may be overridden with the configuration variable psviewer. The PostScript viewer should be capable of automatically redrawing whenever the output file is updated. The default UNIX PostScript viewer gv supports this (via a SIGHUP signal). Version gv-3.6.3 or later (from http://ftp.gnu.org/gnu/gv/) is required for interactive mode to work properly. Users of ggv will need to enable Watch file under Edit/Postscript Viewer Preferences. Users of gsview will need to enable Options/Auto Redisplay (however, under MSDOS it is still necessary to click on the gsview window; under UNIX one must manually redisplay by pressing the r key). A better (and free) multiplatform alternative to gsview is psview (see [psview], page 4).

Configuration variables are most easily set as Asymptote variables in an optional configuration file config.asy see [configuration file], page 159). Here are the default values of several important configuration variables under UNIX:

```
import settings;
psviewer="gv";
pdfviewer="acroread";
gs="gs";
```

Under MSDOS, the (installation-dependent) default values of these configuration variables are determined automatically from the Microsoft Windows registry. Viewer settings (such as psviewer and pdfviewer) can be set to the string cmd to request the application normally associated with the corresponding file type.

For PDF format output, the gs setting specifies the location of the PostScript-to-PDF processor Ghostscript, available from http://sourceforge.net/projects/ghostscript/.

The setting pdfviewer specifies the location of the PDF viewer. On UNIX systems, to support automatic document reloading in Adobe Reader, we recommend

copying the file reload.js from the Asymptote system directory (by default, /usr/local/share/asymptote under UNIX to ~/.adobe/Acrobat/x.x/JavaScripts/, where x.x represents the appropriate Adobe Reader version number. The automatic document reload feature must then be explicitly enabled by putting

```
import settings;
pdfreload=true;
pdfreloadOptions="-tempFile";
```

in the Asymptote configuration file. This reload feature is not useful under MSDOS since the document cannot be updated anyway on that operating system until it is first closed by Adobe Reader.

The configuration variable dir can be used to adjust the search path (see Section 2.5 [Search paths], page 6).

By default, Asymptote attempts to center the figure on the page, assuming that the paper type is letter. The default paper type may be changed to a4 with the configuration variable papertype. Alignment to other paper sizes can be obtained by setting the configuration variables paperwidth and paperheight.

The following configuration variables normally do not require adjustment:

```
texpath
texcommand
dvips
dvisvgm
convert
display
animate
```

Warnings (such as "writeoverloaded") may be enabled or disabled with the functions warn(string s);
nowarn(string s);

or by directly modifying the string array settings.suppress, which lists all disabled warnings.

Configuration variables may also be set or overwritten with a command-line option:

```
asy -psviewer=gsview -V venn
```

Alternatively, system environment versions of the above configuration variables may be set in the conventional way. The corresponding environment variable name is obtained by converting the configuration variable name to upper case and prepending ASYMPTOTE_: for example, to set the environment variable

ASYMPTOTE_PSVIEWER="C:\Program Files\Ghostgum\gsview\gsview32.exe";

under Microsoft Windows XP:

- 1. Click on the Start button;
- 2. Right-click on My Computer;
- 3. Choose View system information;
- 4. Click the Advanced tab;
- 5. Click the Environment Variables button.

2.5 Search paths

In looking for Asymptote system files, asy will search the following paths, in the order listed:

- 1. The current directory;
- 2. A list of one or more directories specified by the configuration variable dir (separated by : under UNIX and ; under MSDOS);
- 3. The directory specified by the environment variable ASYMPTOTE_HOME; if this variable is not set, the directory .asy in the user's home directory (%USERPROFILE%\.asy under MSDOS) is used:
- 4. The Asymptote system directory (by default, /usr/local/share/asymptote under UNIX and C:\Program Files\Asymptote under MSDOS).

2.6 Compiling from UNIX source

To compile and install a UNIX executable from a source release x.xx, first execute the commands:

```
gunzip asymptote-x.xx.src.tgz
tar -xf asymptote-x.xx.src.tar
cd asymptote-x.xx
```

make install

By default the system version of the Boehm garbage collector will be used; if it is old we recommend first putting http://www.hpl.hp.com/personal/Hans_Boehm/gc/gc_source/gc-7.1.tar.gz in the Asymptote source directory.

If your graphics card supports multisampling, we recommend using version 2.6.0 (or later) of freeglut to support antialiasing in Asymptote's adaptive OpenGL 3D renderer (MacOS X users can skip this step since Asymptote is configured to use the native glut library on that platform). Download

```
http://prdownloads.sourceforge.net/freeglut/freeglut-2.6.0.tar.gz
and type (as the root user):
tar -zxf freeglut-2.6.0.tar.gz
cd freeglut-2.6.0
./configure --prefix=/usr
make install
cd ..
Then compile Asymptote with the commands
./configure
make all
```

Be sure to use GNU make (on non-GNU systems this command may be called gmake). To build the documentation, you may need to install the texinfo-tex package. If you get errors from a broken texinfo or pdftex installation, simply put

```
http://asymptote.sourceforge.net/asymptote.pdf
```

in the directory doc and repeat the command make all.

For a (default) system-wide installation, the last command should be done as the root user. To install without root privileges, change the ./configure command to

```
./configure --prefix=$HOME/asymptote
```

One can disable use of the Boehm garbage collector by configuring with ./configure --disable-gc. For a list of other configuration options, say ./configure --help. For example, one can tell configure to look for header files and libraries in nonstandard locations:

```
./configure CFLAGS=-I/opt/usr/include LDFLAGS=-L/opt/usr/lib
```

If you are compiling Asymptote with gcc, you will need a relatively recent version (e.g. 3.4.4 or later). For full interactive functionality, you will need version 4.3 or later of the GNU readline library. The file gcc3.3.2curses.patch in the patches directory can be used to patch the broken curses.h header file (or a local copy thereof in the current directory) on some AIX and IRIX systems.

The FFTW library is only required if you want Asymptote to be able to take Fourier transforms of data (say, to compute an audio power spectrum). The GSL library is only required if you require the special functions that it supports.

If you don't want to install Asymptote system wide, just make sure the compiled binary asy and GUI script xasy are in your path and set the configuration variable dir to point to the directory base (in the top level directory of the Asymptote source code).

2.7 Editing modes

Users of emacs can edit Asymptote code with the mode asy-mode, after enabling it by putting the following lines in their .emacs initialization file, replacing ASYDIR with the location of the Asymptote system directory (by default, /usr/local/share/asymptote or C:\Program Files\Asymptote under MSDOS):

```
(add-to-list 'load-path "ASYDIR")
(autoload 'asy-mode "asy-mode.el" "Asymptote major mode." t)
(autoload 'lasy-mode "asy-mode.el" "hybrid Asymptote/Latex major mode." t)
(autoload 'asy-insinuate-latex "asy-mode.el" "Asymptote insinuate LaTeX." t)
(add-to-list 'auto-mode-alist '("\\.asy$" . asy-mode))
```

Particularly useful key bindings in this mode are C-c C-c, which compiles and displays the current buffer, and the key binding C-c?, which shows the available function prototypes for the command at the cursor. For full functionality you should also install the Apache Software Foundation package two-mode-mode:

```
http://www.dedasys.com/freesoftware/files/two-mode-mode.el
```

Once installed, you can use the hybrid mode lasy-mode to edit a LaTeX file containing embedded Asymptote code (see Chapter 7 [LaTeX usage], page 82). This mode can be enabled within latex-mode with the key sequence M-x lasy-mode <RET>. On UNIX systems, additional keywords will be generated from all asy files in the space-separated list of directories specified by the environment variable ASYMPTOTE_SITEDIR. Further documentation of asy-mode is available within emacs by pressing the sequence keys C-h f asy-mode <RET>.

Fans of vim can customize vim for Asymptote with

```
cp /usr/local/share/asymptote/asy.vim ~/.vim/syntax/asy.vim
and add the following to their ~/.vimrc file:
augroup filetypedetect
```

```
au BufNewFile,BufRead *.asy setf asy
augroup END
filetype plugin on
```

If any of these directories or files don't exist, just create them. To set vim up to run the current asymptote script using :make just add to ~/.vim/ftplugin/asy.vim:

```
setlocal makeprg=asy\ %
setlocal errorformat=%f:\ %1.%c:\ %m
```

Syntax highlighting support for the KDE editor Kate can be enabled by running asy-kate.sh in the /usr/local/share/asymptote directory and putting the generated asymptote.xml file in ~/.kde/share/apps/katepart/syntax/.

2.8 Subversion (SVN)

The following commands are needed to install the latest development version of Asymptote using Subversion:

```
svn co http://asymptote.svn.sourceforge.net/svnroot/asymptote/trunk/asymptote
cd asymptote
./autogen.sh
./configure
make all
make install
```

To compile without optimization, use the command make CFLAGS=-g.

2.9 Uninstall

To uninstall an Linux i386 binary distribution, use the commands tar -zxvf asymptote-x.xx.i386.tgz | xargs --replace=% rm /% texhash

To uninstall all Asymptote files installed from a source distribution, use the command make uninstall

3 Tutorial

3.1 Drawing in batch mode

To draw a line from coordinate (0,0) to coordinate (100,100), create a text file test.asy containing

```
draw((0,0)--(100,100));
```

Then execute the command

Alternatively, MSDOS users can drag and drop test.asy onto the Desktop asy icon (or make Asymptote the default application for the extension asy).

This method, known as *batch mode*, outputs a PostScript file test.eps. If you prefer PDF output, use the command line

```
asy -V -f pdf test
```

In either case, the -V option opens up a viewer window so you can immediately view the result:



Here, the -- connector joins the two points (0,0) and (100,100) with a line segment.

3.2 Drawing in interactive mode

Another method is *interactive mode*, where Asymptote reads individual commands as they are entered by the user. To try this out, enter Asymptote's interactive mode by clicking on the Asymptote icon or typing the command asy. Then type

```
draw((0,0)--(100,100));
```

followed by Enter, to obtain the above image. At this point you can type further draw commands, which will be added to the displayed figure, erase to clear the canvas,

```
input test;
```

to execute all of the commands contained in the file test.asy, or quit to exit interactive mode. You can use the arrow keys in interactive mode to edit previous lines. The tab key will automatically complete unambiguous words; otherwise, hitting tab again will show the possible choices. Further commands specific to interactive mode are described in Chapter 10 [Interactive mode], page 161.

3.3 Figure size

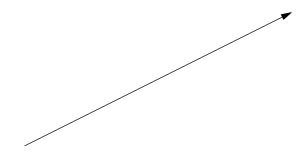
In Asymptote, coordinates like (0,0) and (100,100), called *pairs*, are expressed in PostScript "big points" (1 bp = 1/72 inch) and the default line width is 0.5bp. However, it is often inconvenient to work directly in PostScript coordinates. The next example produces identical output to the previous example, by scaling the line (0,0)--(1,1) to fit a rectangle of width 100.5 bp and height 100.5 bp (the extra 0.5bp accounts for the line width):

```
size(100.5,100.5);
draw((0,0)--(1,1));
```



One can also specify the size in pt (1 pt = 1/72.27 inch), cm, mm, or inches. Two nonzero size arguments (or a single size argument) restrict the size in both directions, preserving the aspect ratio. If 0 is given as a size argument, no restriction is made in that direction; the overall scaling will be determined by the other direction (see [size], page 46):

```
size(0,100.5);
draw((0,0)--(2,1),Arrow);
```



To connect several points and create a cyclic path, use the cycle keyword:

```
size(3cm);
draw((0,0)--(1,0)--(1,1)--(0,1)--cycle);
```



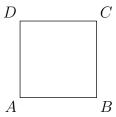
For convenience, the path (0,0)--(1,0)--(1,1)--(0,1)--cycle may be replaced with the predefined variable unitsquare, or equivalently, box((0,0),(1,1)).

To make the user coordinates represent multiples of exactly 1cm: unitsize(1cm); draw(unitsquare);

3.4 Labels

Adding labels is easy in Asymptote; one specifies the label as a double-quoted LaTeX string, a coordinate, and an optional alignment direction:

```
size(3cm);
draw(unitsquare);
label("$A$",(0,0),SW);
label("$B$",(1,0),SE);
label("$C$",(1,1),NE);
label("$D$",(0,1),NW);
```

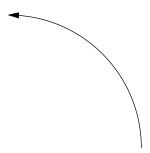


Asymptote uses the standard compass directions E=(1,0), N=(0,1), NE=unit(N+E), and ENE=unit(E+NE), etc., which along with the directions up, down, right, and left are defined as pairs in the Asymptote base module plain (a user who has a local variable named E may access the compass direction E by prefixing it with the name of the module where it is defined: plain.E).

3.5 Paths

This example draws a path that approximates a quarter circle, terminated with an arrow-head:

```
size(100,0);
draw((1,0){up}..{left}(0,1),Arrow);
```



Here the directions up and left in braces specify the incoming and outgoing directions at the points (1,0) and (0,1), respectively.

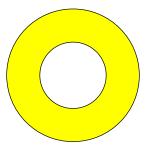
In general, a path is specified as a list of points (or other paths) interconnected with --, which denotes a straight line segment, or ..., which denotes a cubic spline (see Chapter 5

[Bezier curves], page 22). Specifying a final ..cycle creates a cyclic path that connects smoothly back to the initial node, as in this approximation (accurate to within 0.06%) of a unit circle:

```
path unitcircle=E..N..W..S..cycle;
```

An Asymptote path, being connected, is equivalent to a Postscript subpath. The ^^ binary operator, which requests that the pen be moved (without drawing or affecting endpoint curvatures) from the final point of the left-hand path to the initial point of the right-hand path, may be used to group several Asymptote paths into a path[] array (equivalent to a PostScript path):

```
size(0,100);
path unitcircle=E..N..W..S..cycle;
path g=scale(2)*unitcircle;
filldraw(unitcircle^^g,evenodd+yellow,black);
```



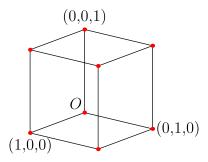
The PostScript even-odd fill rule here specifies that only the region bounded between the two unit circles is filled (see [fillrule], page 41). In this example, the same effect can be achieved by using the default zero winding number fill rule, if one is careful to alternate the orientation of the paths:

```
filldraw(unitcircle^reverse(g), yellow, black);
```

The ^^ operator is used by the box(triple, triple) function in the module three.asy to construct the edges of a cube unitbox without retracing steps (see Section 8.29 [three], page 128):

```
import three;
currentprojection=orthographic(5,4,2,center=true);
size(5cm);
size3(3cm,5cm,8cm);
draw(unitbox);
dot(unitbox,red);
label("$0$",(0,0,0),NW);
label("(1,0,0)",(1,0,0),S);
label("(0,1,0)",(0,1,0),E);
```

label("(0,0,1)",(0,0,1),Z);



See section Section 8.27 [graph], page 95 (or the online Asymptote gallery and external links posted at http://asymptote.sourceforge.net) for further examples, including two-dimensional and interactive three-dimensional scientific graphs. Additional examples have been posted by Philippe Ivaldi at http://www.piprime.fr/asymptote. A user-written Asymptote tutorial is available at

http://www.artofproblemsolving.com/Wiki/index.php/Asymptote:_Basics

4 Drawing commands

All of Asymptote's graphical capabilities are based on four primitive commands. The three PostScript drawing commands draw, fill, and clip add objects to a picture in the order in which they are executed, with the most recently drawn object appearing on top. The labeling command label can be used to add text labels and external EPS images, which will appear on top of the PostScript objects (since this is normally what one wants), but again in the relative order in which they were executed. After drawing objects on a picture, the picture can be output with the shipout function (see [shipout], page 47).

If you wish to draw PostScript objects on top of labels (or verbatim tex commands; see [tex], page 51), the layer command may be used to start a new PostScript/LaTeX layer:

```
void layer(picture pic=currentpicture);
```

The layer function gives one full control over the order in which objects are drawn. Layers are drawn sequentially, with the most recent layer appearing on top. Within each layer, labels, images, and verbatim tex commands are always drawn after the PostScript objects in that layer.

While some of these drawing commands take many options, they all have sensible default values (for example, the picture argument defaults to current picture).

4.1 draw

Draw the path g on the picture pic using pen p for drawing, with optional drawing attributes (Label L, explicit label alignment align, arrows and bars arrow and bar, margins margin, legend, and markers marker). Only one parameter, the path, is required. For convenience, the arguments arrow and bar may be specified in either order. The argument legend is a Label to use in constructing an optional legend entry.

Bars are useful for indicating dimensions. The possible values of bar are None, BeginBar, EndBar (or equivalently Bar), and Bars (which draws a bar at both ends of the path). Each of these bar specifiers (except for None) will accept an optional real argument that denotes the length of the bar in PostScript coordinates. The default bar length is barsize(pen).

The possible values of arrow are None, Blank (which draws no arrows or path), BeginArrow, MidArrow, EndArrow (or equivalently Arrow), and Arrows (which draws an arrow at both ends of the path). All of the arrow specifiers except for None and Blank may be given the optional arguments arrowhead arrowhead (one of the predefined arrowhead styles DefaultHead, SimpleHead, HookHead, TeXHead), real size (arrowhead size in PostScript coordinates), real angle (arrowhead angle in degrees), filltype filltype (one of FillDraw, Fill, NoFill, UnFill, Draw) and (except for MidArrow and Arrows) a real position (in the sense of point(path p, real t)) along the path where the tip of the arrow should be placed. The default arrowhead size when drawn with a pen p is arrowsize(p). There are also arrow versions with slightly modified default values of size

and angle suitable for curved arrows: BeginArcArrow, EndArcArrow (or equivalently ArcArrow), MidArcArrow, and ArcArrows.

Margins can be used to shrink the visible portion of a path by labelmargin(p) to avoid overlap with other drawn objects. Typical values of margin are NoMargin, BeginMargin, EndMargin (or equivalently Margin), and Margins (which leaves a margin at both ends of the path). One may use Margin(real begin, real end) to specify the size of the beginning and ending margin, respectively, in multiples of the units labelmargin(p) used for aligning labels. Alternatively, BeginPenMargin, EndPenMargin (or equivalently PenMargin), PenMargins, PenMargin(real begin, real end) specify a margin in units of the pen line width, taking account of the pen line width when drawing the path or arrow. For example, use DotMargin, an abbreviation for PenMargin(-0.5*dotfactor,0.5*dotfactor), to draw from the usual beginning point just up to the boundary of an end dot of width dotfactor*linewidth(p). The qualifiers BeginDotMargin, EndDotMargin, and DotMargins work similarly. The qualifier TrueMargin(real begin, real end) allows one to specify a margin directly in PostScript units, independent of the pen line width.

The use of arrows, bars, and margins is illustrated by the examples Pythagoras.asy, sqrtx01.asy, and triads.asy.

The legend for a picture pic can be fit and aligned to a frame with the routine:

Here xmargin and ymargin specify the surrounding x and y margins, perline specifies the number of entries per line (default 1; 0 means choose this number automatically), linelength specifies the length of the path lines, hskip and vskip specify the line skip (as a multiple of the legend entry size), maxwidth and maxheight specify optional upper limits on the width and height of the resulting legend (0 means unlimited), hstretch and vstretch allow the legend to stretch horizontally or vertically, and p specifies the pen used to draw the bounding box. The legend frame can then be added and aligned about a point on a picture dest using add or attach (see [add about], page 50).

To draw a dot, simply draw a path containing a single point. The dot command defined in the module plain draws a dot having a diameter equal to an explicit pen line width or the default line width magnified by dotfactor (6 by default), using the specified filltype (see [filltype], page 48):

```
filltype filltype=Fill);
```

If the variable Label is given as the Label argument to the second routine, the format argument will be used to format a string based on the dot location (here defaultformat is "\$%.4g\$"). The third routine draws a dot at every point of a pair array z. One can also draw a dot at every node of a path:

See [pathmarkers], page 105 and Section 8.9 [markers], page 89 for more general methods for marking path nodes.

To draw a fixed-sized object (in PostScript coordinates) about the user coordinate origin, use the routine

4.2 fill

```
void fill(picture pic=currentpicture, path g, pen p=currentpen);
```

Fill the interior region bounded by the cyclic path g on the picture pic, using the pen p.

There is also a convenient filldraw command, which fills the path and then draws in the boundary. One can specify separate pens for each operation:

This fixed-size version of fill allows one to fill an object described in PostScript coordinates about the user coordinate origin:

void fill(pair origin, picture pic=currentpicture, path g, pen p=currentpen); This is just a convenient abbreviation for the commands:

```
picture opic;
fill(opic,g,p);
add(pic,opic,origin);
```

The routine

void filloutside(picture pic=currentpicture, path g, pen p=currentpen); fills the region exterior to the path g, out to the current boundary of picture pic.

Lattice gradient shading varying smoothly over a two-dimensional array of pens p, using fill rule fillrule, can be produced with

If stroke=true, the region filled is the same as the region that would be drawn by draw(pic,g,fillrule+zerowinding); in this case the path g need not be cyclic. The pens in p must belong to the same color space. One can use the functions rgb(pen) or cmyk(pen) to promote pens to a higher color space, as illustrated in the example file latticeshading.asy.

Axial gradient shading varying smoothly from pena to penb in the direction of the line segment a--b can be achieved with

Radial gradient shading varying smoothly from pena on the circle with center a and radius ra to penb on the circle with center b and radius rb is similar:

Illustrations of radial shading are provided in the example files shade.asy, ring.asy, and shadestroke.asy.

Gouraud shading using fill rule $\verb|fillrule|$ and the vertex colors in the pen array $\verb|p|$ on a triangular lattice defined by the vertices $\verb|z|$ and edge flags $\verb|edges|$ is implemented with

In the second form, the elements of **z** are taken to be successive nodes of path **g**. The pens in **p** must belong to the same color space. Illustrations of Gouraud shading are provided in the example file **Gouraud.asy** and in the solid geometry module **solids.asy**. The edge flags used in Gouraud shading are documented here:

```
http://partners.adobe.com/public/developer/en/ps/sdk/TN5600.SmoothShading.pdf.
```

Tensor product shading using fill rule fillrule on patches bounded by the n cyclic paths of length 4 in path array b, using the vertex colors specified in the $n \times 4$ pen array p and internal control points in the $n \times 4$ array z, is implemented with

If the array z is empty, Coons shading, in which the color control points are calculated automatically, is used. The pens in p must belong to the same color space. A simpler interface for the case of a single patch (n = 1) is also available:

One can also smoothly shade the regions between consecutive paths of a sequence using a given array of pens:

Illustrations of tensor product and Coons shading are provided in the example files tensor.asy, Coons.asy, BezierSurface.asy, and rainbow.asy.

More general shading possibilities are available with the pdflatex, context, and pdftex TFX engines: the routine

shades on picture pic the interior of path g according to fill rule fillrule using the PostScript calculator routine specified by the string shader; this routine takes 2 arguments, each in [0,1], and returns colors(fillrule).length color components. Function shading is illustrated in the example functionshading.asy.

The following routine uses evenodd clipping together with the ^^ operator to unfill a region:

void unfill(picture pic=currentpicture, path g);

4.3 clip

Clip the current contents of picture pic to the region bounded by the path g, using fill rule fillrule (see [fillrule], page 41). If stroke=true, the clipped portion is the same as the region that would be drawn with draw(pic,g,fillrule+zerowinding); in this case the path g need not be cyclic. For an illustration of picture clipping, see the first example in Chapter 7 [LaTeX usage], page 82.

4.4 label

Draw Label L on picture pic using pen p. If align is NoAlign, the label will be centered at user coordinate position; otherwise it will be aligned in the direction of align and displaced from position by the PostScript offset align*labelmargin(p). The constant Align can be used to align the bottom-left corner of the label at position. If p is nullpen, the pen specified within the Label, which defaults to currentpen, will be used. The Label L can either be a string or the structure obtained by calling one of the functions

The text of a Label can be scaled, slanted, rotated, or shifted by multiplying it on the left by an affine transform (see Section 6.4 [Transforms], page 45). For example, rotate(45)*xscale(2)*L first scales L in the x direction and then rotates it counterclockwise by 45 degrees. The final position of a Label can also be shifted by a PostScript coordinate translation: shift(10,0)*L. The embed argument determines how the Label should transform with the embedding picture:

Shift only shift with embedding picture;

Rotate only shift and rotate with embedding picture (default);

Rotate(pair z)
rotate with (picture-transformed) vector z.

Slant only shift, rotate, slant, and reflect with embedding picture;

To add a label to a path, use

Scale

shift, rotate, slant, reflect, and scale with embedding picture.

By default the label will be positioned at the midpoint of the path. An alternative label position (in the sense of point(path p, real t)) may be specified as a real value for position in constructing the Label. The position Relative(real) specifies a location relative to the total arclength of the path. These convenient abbreviations are predefined:

```
position BeginPoint=Relative(0);
position MidPoint=Relative(0.5);
position EndPoint=Relative(1);
```

Path labels are aligned in the direction align, which may be specified as an absolute compass direction (pair) or a direction Relative(pair) measured relative to a north axis in the local direction of the path. For convenience LeftSide, Center, and RightSide are defined as Relative(W), Relative((0,0)), and Relative(E), respectively. Multiplying LeftSide, Center, RightSide on the left by a real scaling factor will move the label further away from or closer to the path.

A label with a fixed-size arrow of length arrowlength pointing to b from direction dir can be produced with the routine

If no alignment is specified (either in the Label or as an explicit argument), the optional Label will be aligned in the direction dir, using margin margin.

The function string graphic(string name, string options="") returns a string that can be used to include an encapsulated PostScript (EPS) file. Here, name is the name of the file to include and options is a string containing a comma-separated list of optional bounding box (bb=llx lly urx ury), width (width=value), height (height=value), rotation (angle=value), scaling (scale=factor), clipping (clip=bool), and draft mode (draft=bool) parameters. The layer() function can be used to force future objects to be drawn on top of the included image:

```
label(graphic("file.eps","width=1cm"),(0,0),NE);
layer();
```

The string baseline(string s, string template="\strut") function can be used to enlarge the bounding box of labels to match a given template, so that their baselines will be typeset on a horizontal line. See Pythagoras.asy for an example.

One can prevent labels from overwriting one another with the overwrite pen attribute (see [overwrite], page 44).

The structure object defined in plain_Label.asy allows Labels and frames to be treated in a uniform manner. A group of objects may be packed together into single frame with the routine

frame pack(pair align=2S ... object inset[]);

To draw or fill a box (or ellipse or other path) around a Label and return the bounding object, use one of the routines

object draw(picture pic=currentpicture, Label L, envelope e,

real xmargin=0, real ymargin=xmargin, pen p=currentpen,

filltype filltype=NoFill, bool above=true);

object draw(picture pic=currentpicture, Label L, envelope e, pair position,

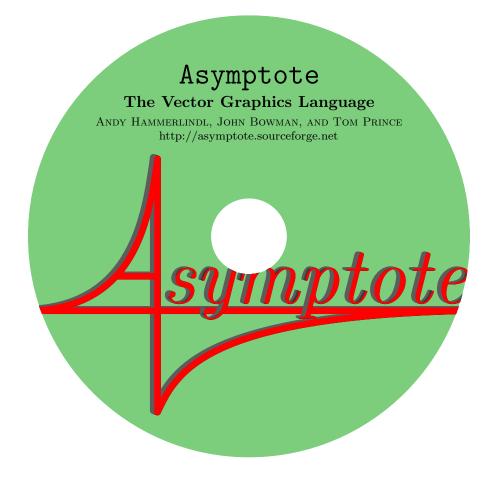
real xmargin=0, real ymargin=xmargin, pen p=currentpen,

filltype filltype=NoFill, bool above=true);

Here envelope is a boundary-drawing routine such as box, roundbox, or ellipse defined in plain_boxes.asy (see [envelope], page 46).

The function path[] texpath(Label L) returns the path array that T_EX would fill to draw the Label L.

The string minipage(string s, width=100pt) function can be used to format string s into a paragraph of width width. This example uses minipage, clip, and graphic to produce a CD label:



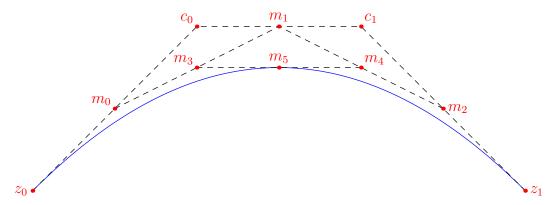
5 Bezier curves

Each interior node of a cubic spline may be given a direction prefix or suffix {dir}: the direction of the pair dir specifies the direction of the incoming or outgoing tangent, respectively, to the curve at that node. Exterior nodes may be given direction specifiers only on their interior side.

A cubic spline between the node z_0 , with postcontrol point c_0 , and the node z_1 , with precontrol point c_1 , is computed as the Bezier curve

$$(1-t)^3 z_0 + 3t(1-t)^2 c_0 + 3t^2(1-t)c_1 + t^3 z_1 \qquad 0 \le t \le 1.$$

As illustrated in the diagram below, the third-order midpoint (m_5) constructed from two endpoints z_0 and z_1 and two control points c_0 and c_1 , is the point corresponding to t = 1/2 on the Bezier curve formed by the quadruple (z_0, c_0, c_1, z_1) . This allows one to recursively construct the desired curve, by using the newly extracted third-order midpoint as an endpoint and the respective second- and first-order midpoints as control points:



Here m_0 , m_1 and m_2 are the first-order midpoints, m_3 and m_4 are the second-order midpoints, and m_5 is the third-order midpoint. The curve is then constructed by recursively applying the algorithm to (z_0, m_0, m_3, m_5) and (m_5, m_4, m_2, z_1) .

In fact, an analogous property holds for points located at any fraction t in [0,1] of each segment, not just for midpoints (t = 1/2).

The Bezier curve constructed in this manner has the following properties:

- It is entirely contained in the convex hull of the given four points.
- It starts heading from the first endpoint to the first control point and finishes heading from the second control point to the second endpoint.

The user can specify explicit control points between two nodes like this: draw((0,0)..controls (0,100) and (100,100)..(100,0));

However, it is usually more convenient to just use the .. operator, which tells Asymptote to choose its own control points using the algorithms described in Donald Knuth's monograph, The MetaFontbook, Chapter 14. The user can still customize the guide (or path) by specifying direction, tension, and curl values.

The higher the tension, the straighter the curve is, and the more it approximates a straight line. One can change the spline tension from its default value of 1 to any real value

greater than or equal to 0.75 (cf. John D. Hobby, Discrete and Computational Geometry 1, 1986):

```
draw((100,0)..tension 2 ..(100,100)..(0,100));
draw((100,0)..tension 2 and 1 ..(100,100)..(0,100));
draw((100,0)..tension atleast 1 ..(100,100)..(0,100));
```

The curl parameter specifies the curvature at the endpoints of a path (0 means straight; the default value of 1 means approximately circular):

```
draw((100,0){curl 0}..(100,100)..{curl 0}(0,100));
```

The MetaPost ... path connector, which requests, when possible, an inflection-free curve confined to a triangle defined by the endpoints and directions, is implemented in Asymptote as the convenient abbreviation :: for ..tension atleast 1 .. (the ellipsis ... is used in Asymptote to indicate a variable number of arguments; see Section 6.11.3 [Rest arguments], page 65). For example, compare

 $draw((0,0){up}..(100,25){right}..(200,0){down});$



with

 $draw((0,0){up}::(100,25){right}::(200,0){down});$



The --- connector is an abbreviation for ..tension atleast infinity.. and the & connector concatenates two paths, after first stripping off the last node of the first path (which normally should coincide with the first node of the second path).

6 Programming

Here is a short introductory example to the Asymptote programming language that highlights the similarity of its control structures with those of C, C++, and Java:

```
// This is a comment.
// Declaration: Declare x to be a real variable;
real x;
// Assignment: Assign the real variable x the value 1.
x=1.0;
// Conditional: Test if x equals 1 or not.
if(x == 1.0) {
  write("x equals 1.0");
} else {
  write("x is not equal to 1.0");
// Loop: iterate 10 times
for(int i=0; i < 10; ++i) {
  write(i);
}
  Asymptote supports while, do, break, and continue statements just as in C/C++. It
also supports the Java-style shorthand for iterating over all elements of an array:
// Iterate over an array
int[] array={1,1,2,3,5};
for(int k : array) {
  write(k);
}
```

In addition, it supports many features beyond the ones found in those languages.

6.1 Data types

Asymptote supports the following data types (in addition to user-defined types):

void The void type is used only by functions that take or return no arguments.

a boolean type that can only take on the values true or false. For example: bool b=true:

defines a boolean variable **b** and initializes it to the value **true**. If no initializer is given:

bool b;

the value false is assumed.

an extended boolean type that can take on the values true, default, or false. A bool3 type can be cast to or from a bool. The default initializer for bool3 is default.

int an integer type; if no initializer is given, the implicit value 0 is assumed. The minimum allowed value of an integer is intMin and the maximum value is intMax.

a real number; this should be set to the highest-precision native floating-point type on the architecture. The implicit initializer for reals is 0.0. Real numbers have precision realEpsilon, with realDigits significant digits. The smallest positive real number is realMin and the largest positive real number is realMax.

pair complex number, that is, an ordered pair of real components (x,y). The real and imaginary parts of a pair z can read as z.x and z.y. We say that x and y are virtual members of the data element pair; they cannot be directly modified, however. The implicit initializer for pairs is (0.0,0.0).

There are a number of ways to take the complex conjugate of a pair:

```
pair z=(3,4);
z=(z.x,-z.y);
z=z.x-I*z.y;
z=conj(z);
```

Here I is the pair (0,1). A number of built-in functions are defined for pairs:

```
pair conj(pair z)
```

returns the conjugate of z;

real length(pair z)

returns the complex modulus |z| of its argument z. For example,

```
pair z=(3,4);
length(z);
```

returns the result 5. A synonym for length(pair) is abs(pair);

real angle(pair z, bool warn=true)

returns the angle of z in radians in the interval [-pi,pi] or 0 if warn is false and z=(0,0) (rather than producing an error);

real degrees(pair z, bool warn=true)

returns the angle of z in degrees in the interval [0,360) or 0 if warn is false and z=(0,0) (rather than producing an error);

pair unit(pair z)

returns a unit vector in the direction of the pair z;

pair expi(real angle)

returns a unit vector in the direction angle measured in radians;

pair dir(real degrees)

returns a unit vector in the direction degrees measured in degrees;

real xpart(pair z) returns z.x;

triple

```
real ypart(pair z)
           returns z.y;
pair realmult(pair z, pair w)
           returns the element-by-element product (z.x*w.x,z.y*w.y);
real dot(explicit pair z, explicit pair w)
           returns the dot product z.x*w.x+z.y*w.y;
pair minbound(pair z, pair w)
           returns (min(z.x,w.x),min(z.y,w.y));
pair maxbound(pair z, pair w)
           returns (\max(z.x,w.x),\max(z.y,w.y)).
an ordered triple of real components (x,y,z) used for three-dimensional draw-
ings. The respective components of a triple v can read as v.x, v.y, and v.z.
The implicit initializer for triples is (0.0,0.0,0.0).
Here are the built-in functions for triples:
real length(triple v)
           returns the length |v| of the vector v. A synonym for
           length(triple) is abs(triple);
real polar(triple v, bool warn=true)
           returns the colatitude of v measured from the z axis in radians or
           0 if warn is false and v=0 (rather than producing an error);
real azimuth(triple v, bool warn=true)
           returns the longitude of v measured from the x axis in radians or 0
           if warn is false and v.x=v.y=0 (rather than producing an error);
real colatitude(triple v, bool warn=true)
           returns the colatitude of v measured from the z axis in degrees or
           0 if warn is false and v=0 (rather than producing an error);
real latitude(triple v, bool warn=true)
           returns the latitude of v measured from the xy plane in degrees or
           0 if warn is false and v=0 (rather than producing an error);
real longitude(triple v, bool warn=true)
           returns the longitude of v measured from the x axis in degrees or 0
           if warn is false and v.x=v.y=0 (rather than producing an error);
triple unit(triple v)
           returns a unit triple in the direction of the triple v;
triple expi(real polar, real azimuth)
           returns a unit triple in the direction (polar, azimuth) measured
           in radians;
triple dir(real colatitude, real longitude)
           returns a unit triple in the direction (colatitude, longitude)
```

measured in degrees;

string

a character string, implemented using the STL string class.

Strings delimited by double quotes (") are subject to the following mappings to allow the use of double quotes in TeX (e.g. for using the babel package, see Section 8.22 [babel], page 94):

- \" maps to "
- \\ maps to \\

Strings delimited by single quotes (') have the same mappings as character strings in ANSI C:

- \' maps to '
- \" maps to "
- \? maps to ?
- \\ maps to backslash
- \a maps to alert
- \b maps to backspace
- \f maps to form feed
- \n maps to newline
- \r maps to carriage return
- \t maps to tab
- \v maps to vertical tab
- \0-\377 map to corresponding octal byte
- \x0-\xFF map to corresponding hexadecimal byte

The implicit initializer for strings is the empty string "". Strings may be concatenated with the + operator. In the following string functions, position 0 denotes the start of the string:

int length(string s)

returns the length of the string s;

int find(string s, string t, int pos=0)

returns the position of the first occurrence of string t in string s at or after position pos, or -1 if t is not a substring of s;

int rfind(string s, string t, int pos=-1)

returns the position of the last occurrence of string t in string s at or before position pos (if pos=-1, at the end of the string s), or -1 if t is not a substring of s;

string insert(string s, int pos, string t)

returns the string formed by inserting string t at position pos in s;

string erase(string s, int pos, int n)

returns the string formed by erasing the string of length n (if n=-1, to the end of the string s) at position pos in s;

string substr(string s, int pos, int n=-1)

returns the substring of s starting at position pos and of length n (if n=-1, until the end of the string s);

string reverse(string s)

returns the string formed by reversing string s;

string replace(string s, string before, string after)

returns a string with all occurrences of the string before in the string s changed to the string after;

string replace(string s, string[][] table)

returns a string constructed by translating in string s all occurrences of the string before in an array table of string pairs {before,after} to the corresponding string after;

string[] split(string s, string delimiter="")

returns an array of strings obtained by splitting s into substrings delimited by delimiter (an empty delimiter signifies a space, but with duplicate delimiters discarded);

string format(string s, int n)

returns a string containing n formatted according to the C-style format string s using the current locale;

string format(string s=defaultformat, real x, string locale="")

returns a string containing x formatted according to the C-style format string s using locale locale (or the current locale if an empty string is specified), following the behaviour of the C function fprintf), except that only one data field is allowed, trailing zeros are removed by default (unless # is specified), and (if the format string specifies math mode) TeX is used to typeset scientific notation;

string locale(string s="")

sets the locale to the given string, if nonempty, and returns the current locale.

string time(string format="%a %b %d %T %Z %Y")

returns the current time formatted by the ANSI C routine strftime according to the string format using the current locale. Thus time():

time("%a %b %d %H:%M:%S %Z %Y");

are equivalent ways of returning the current time in the default format used by the UNIX date command;

int seconds(string t="", string format="")

returns the time measured in seconds after the Epoch (Thu Jan 01 00:00:00 UTC 1970) as determined by the ANSI C routine strptime according to the string format using the current locale, or the current time if t is the empty string. Note that the "%Z" extension to the POSIX strptime specification is ignored by the current GNU C Library. If an error occurs, the value -1 is returned. Here are some examples:

```
seconds("Mar 02 11:12:36 AM PST 2007","%b %d %r PST %Y");
seconds(time("%b %d %r %z %Y"),"%b %d %r %z %Y");
seconds(time("%b %d %r %Z %Y"),"%b %d %r "+time("%Z")+" %Y");
1+(seconds()-seconds("Jan 1","%b %d"))/(24*60*60);
```

The last example returns today's ordinal date, measured from the beginning of the year.

string time(int seconds, string format="%a %b %d %T %Z %Y")

returns the time corresponding to seconds seconds after the Epoch (Thu Jan 01 00:00:00 UTC 1970) formatted by the ANSI C routine strftime according to the string format using the current locale. For example, to return the date corresponding to 24 hours ago:

time(seconds()-24*60*60);

void abort(string s="")

aborts execution (with a non-zero return code in batch mode); if string **s** is nonempty, a diagnostic message constructed from the source file, line number, and **s** is printed;

void assert(bool b, string s="")

aborts execution with an error message constructed from **s** if b=false;

void exit()

exits (with a zero error return code in batch mode);

As in C/C++, complicated types may be abbreviated with typedef (see the example in Section 6.11 [Functions], page 62).

6.2 Paths and guides

path a cubic spline resolved into a fixed path. The implicit initializer for paths is nullpath.

For example, the routine circle(pair c, real r), which returns a Bezier curve approximating a circle of radius r centered on c, is based on unitcircle (see [unitcircle], page 12):

```
path circle(pair c, real r)
{
  return shift(c)*scale(r)*unitcircle;
}
```

If high accuracy is needed, a true circle may be produced with the routine Circle defined in the module graph.asy:

```
import graph;
path Circle(pair c, real r, int n=nCircle);
```

A circular arc consistent with circle centered on c with radius r from angle1 to angle2 degrees, drawing counterclockwise if angle2 >= angle1, can be constructed with

```
path arc(pair c, real r, real angle1, real angle2);
```

One may also specify the direction explicitly:

path arc(pair c, real r, real angle1, real angle2, bool direction); Here the direction can be specified as CCW (counter-clockwise) or CW (clockwise). For convenience, an arc centered at c from pair z1 to z2 (assuming |z2-c|=|z1-c|) in the may also be constructed with

If high accuracy is needed, true arcs may be produced with routines in the module graph.asy that produce Bezier curves with n control points:

An ellipse can be drawn with the routine

```
path ellipse(pair c, real a, real b)
  return shift(c)*scale(a,b)*unitcircle;
This example illustrates the use of all five guide connectors discussed in
Chapter 3 [Tutorial], page 9 and Chapter 5 [Bezier curves], page 22:
size(300,0);
pair[] z=new pair[10];
z[0]=(0,100); z[1]=(50,0); z[2]=(180,0);
for(int n=3; n <= 9; ++n)
  z[n]=z[n-3]+(200,0);
path p=z[0]..z[1]---z[2]::{up}z[3]
\&z[3]..z[4]--z[5]::\{up\}z[6]
&z[6]::z[7]---z[8]..\{up\}z[9];
draw(p,grey+linewidth(4mm));
dot(z);
Here are some useful functions for paths:
int length(path p);
           This is the number of (linear or cubic) segments in path p. If p is
           cyclic, this is the same as the number of nodes in p.
int size(path p);
           This is the number of nodes in the path p. If p is cyclic, this is the
           same as length(p).
bool cyclic(path p);
           returns true iff path p is cyclic.
bool straight(path p, int i);
           returns true iff the segment of path p between node i and node
           i+1 is straight.
bool piecewisestraight(path p)
           returns true iff the path p is piecewise straight.
pair point(path p, int t);
           If p is cyclic, return the coordinates of node t mod length(p).
           Otherwise, return the coordinates of node t, unless t < 0 (in
```

which case point(0) is returned) or t > length(p) (in which case point(length(p)) is returned). pair point(path p, real t); This returns the coordinates of the point between node floor(t) and floor(t)+1 corresponding to the cubic spline parameter tfloor(t) (see Chapter 5 [Bezier curves], page 22). If t lies outside the range [0,length(p)], it is first reduced modulo length(p) in the case where p is cyclic or else converted to the corresponding endpoint of p. pair dir(path p, int t, int sign=0, bool normalize=true); If sign < 0, return the direction (as a pair) of the incoming tangent to path p at node t; if sign > 0, return the direction of the outgoing tangent. If sign=0, the mean of these two directions is returned. pair dir(path p, real t, bool normalize=true); returns the direction of the tangent to path p at the point between node floor(t) and floor(t)+1 corresponding to the cubic spline parameter t-floor(t) (see Chapter 5 [Bezier curves], page 22). pair dir(path p) returns dir(p, length(p)). pair dir(path p, path g) returns unit(dir(p)+dir(g)). pair accel(path p, int t, int sign=0); If sign < 0, return the acceleration of the incoming path p at node t; if sign > 0, return the acceleration of the outgoing path. If sign=0, the mean of these two accelerations is returned. pair accel(path p, real t); returns the acceleration of the path p at the point t. real radius(path p, real t); returns the radius of curvature of the path p at the point t. pair precontrol(path p, int t); returns the precontrol point of p at node t. pair precontrol(path p, real t); returns the effective precontrol point of p at parameter t. pair postcontrol(path p, int t); returns the postcontrol point of p at node t. pair postcontrol(path p, real t); returns the effective postcontrol point of p at parameter t. real arclength(path p);

returns the length (in user coordinates) of the piecewise linear or

cubic curve that path p represents.

real arctime(path p, real L);

returns the path "time", a real number between 0 and the length of the path in the sense of point(path p, real t), at which the cumulative arclength (measured from the beginning of the path) equals L.

real dirtime(path p, pair z);

returns the first "time", a real number between 0 and the length of the path in the sense of point(path, real), at which the tangent to the path has the direction of pair z, or -1 if this never happens.

real reltime(path p, real 1);

returns the time on path p at the relative fraction 1 of its arclength.

pair relpoint(path p, real 1);

returns the point on path p at the relative fraction 1 of its arclength.

pair midpoint(path p);

returns the point on path p at half of its arclength.

path reverse(path p);

returns a path running backwards along p.

path subpath(path p, int a, int b);

returns the subpath of p running from node a to node b. If a < b, the direction of the subpath is reversed.

path subpath(path p, real a, real b);

returns the subpath of p running from path time a to path time b, in the sense of point(path, real). If a < b, the direction of the subpath is reversed.

real[] intersect(path p, path q, real fuzz=-1);

If p and q have at least one intersection point, return a real array of length 2 containing the times representing the respective path times along p and q, in the sense of point(path, real), for one such intersection point (as chosen by the algorithm described on page 137 of The MetaFontbook). The computations are performed to the absolute error specified by fuzz, or if fuzz < 0, to machine precision. If the paths do not intersect, return a real array of length 0.

real[][] intersections(path p, path q, real fuzz=-1);

Return all (unless there are infinitely many) intersection times of paths p and q as a sorted array of real arrays of length 2 (see [sort], page 71). The computations are performed to the absolute error specified by fuzz, or if fuzz < 0, to machine precision.

real[] intersections(path p, explicit pair a, explicit pair b, real fuzz=-1);

Return all (unless there are infinitely many) intersection times of path p with the (infinite) line through points a and b as a sorted

array. The intersections returned are guaranteed to be correct to within the absolute error specified by fuzz, or if fuzz < 0, to machine precision.

real[] times(path p, real x)

returns all intersection times of path p with the vertical line through (x,0).

real[] times(path p, explicit pair z)

returns all intersection times of path p with the horizontal line through (0,z.y).

real[] mintimes(path p)

returns an array of length 2 containing times at which path p reaches its minimal horizontal and vertical extents, respectively.

real[] maxtimes(path p)

returns an array of length 2 containing the times at which path p reaches its maximal horizontal and vertical extents, respectively.

pair intersectionpoint(path p, path q, real fuzz=-1);

returns the intersection point point(p,intersect(p,q,fuzz)[0]).

pair[] intersectionpoints(path p, path q, real fuzz=-1);

returns an array containing all intersection points of the paths p and q.

pair extension(pair P, pair Q, pair p, pair q);

returns the intersection point of the extensions of the line segments P--Q and p--q, or if the lines are parallel, (infinity,infinity).

slice cut(path p, path knife, int n);

returns the portions of path p before and after the nth intersection of p with path knife as a structure slice (if no intersection exist is found, the entire path is considered to be 'before' the intersection):

```
struct slice {
  path before,after;
}
```

The argument n is treated as modulo the number of intersections.

slice firstcut(path p, path knife);

equivalent to cut(p,knife,0); Note that firstcut.after plays the role of the MetaPost cutbefore command.

slice lastcut(path p, path knife);

equivalent to cut(p,knife,-1); Note that lastcut.before plays the role of the MetaPost cutafter command.

path buildcycle(... path[] p);

This returns the path surrounding a region bounded by a list of two or more consecutively intersecting paths, following the behaviour of the MetaPost buildcycle command.

guide

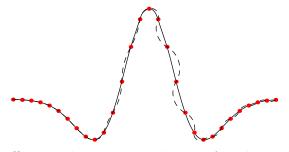
real width=2a/n;

```
pair min(path p);
           returns the pair (left, bottom) for the path bounding box of path p.
pair max(path p);
           returns the pair (right,top) for the path bounding box of path p.
int windingnumber(path p, pair z);
           returns the winding number of the cyclic path p relative to the point
           z. The winding number is positive if the path encircles z in the
           counterclockwise direction. If z lies on p the constant undefined
           (defined to be the largest odd integer) is returned.
bool interior(int windingnumber, pen fillrule)
           returns true if windingnumber corresponds to an interior point ac-
           cording to fillrule.
bool inside(path p, pair z, pen fillrule=currentpen);
           returns true iff the point z lies inside or on the edge of the region
           bounded by the cyclic path p according to the fill rule fillrule
           (see [fillrule], page 41).
int inside(path p, path q, pen fillrule=currentpen);
           returns 1 if the cyclic path p strictly contains q according to the
           fill rule fillrule (see [fillrule], page 41), -1 if the cyclic path q
           strictly contains p, and 0 otherwise.
pair inside(path p, pen fillrule=currentpen);
           returns an arbitrary point strictly inside a cyclic path p according
           to the fill rule fillrule (see [fillrule], page 41).
path[] strokepath(path g, pen p=currentpen);
           returns the path array that PostScript would fill in drawing path
           g with pen p.
an unresolved cubic spline (list of cubic-spline nodes and control points). The
implicit initializer for a guide is nullpath; this is useful for building up a guide
within a loop.
A guide is similar to a path except that the computation of the cubic spline is
deferred until drawing time (when it is resolved into a path); this allows two
guides with free endpoint conditions to be joined together smoothly. The solid
curve in the following example is built up incrementally as a guide, but only
resolved at drawing time; the dashed curve is incrementally resolved at each
iteration, before the entire set of nodes (shown in red) is known:
size(200);
real mexican(real x) {return (1-8x^2)*exp(-(4x^2));}
int n=30;
real a=1.5;
```

```
guide hat;
path solved;

for(int i=0; i < n; ++i) {
  real t=-a+i*width;
  pair z=(t,mexican(t));
  hat=hat..z;
  solved=solved..z;
}

draw(hat);
dot(hat,red);
draw(solved,dashed);</pre>
```



We point out an efficiency distinction in the use of guides and paths:

```
guide g;
for(int i=0; i < 10; ++i)
  g=g--(i,i);
path p=g;
runs in linear time, whereas
path p;
for(int i=0; i < 10; ++i)
  p=p--(i,i);</pre>
```

runs in quadratic time, as the entire path up to that point is copied at each step of the iteration.

The following routines can be used to examine the individual elements of a guide without actually resolving the guide to a fixed path (except for internal cycles, which are resolved):

guide reverse(guide g);

Analogous to reverse(path p). If g is cyclic and also contains a secondary cycle, it is first solved to a path, then reversed. If g is not cyclic but contains an internal cycle, only the internal cycle is solved before reversal. If there are no internal cycles, the guide is reversed but not solved to a path.

pair[] dirSpecifier(guide g, int i);

This returns a pair array of length 2 containing the outgoing (in element 0) and incoming (in element 1) direction specifiers (or (0,0) if none specified) for the segment of guide g between nodes i and i+1.

pair[] controlSpecifier(guide g, int i);

If the segment of guide g between nodes i and i+1 has explicit outgoing and incoming control points, they are returned as elements 0 and 1, respectively, of a two-element array. Otherwise, an empty array is returned.

tensionSpecifier tensionSpecifier(guide g, int i);

This returns the tension specifier for the segment of guide g between nodes i and i+1. The individual components of the tensionSpecifier type can be accessed as the virtual members in, out, and atLeast.

real[] curlSpecifier(guide g);

This returns an array containing the initial curl specifier (in element 0) and final curl specifier (in element 1) for guide g.

As a technical detail we note that a direction specifier given to nullpath modifies the node on the other side: the guides

```
a..{up}nullpath..b;
c..nullpath{up}..d;
e..{up}nullpath{down}..f;
are respectively equivalent to
a..nullpath..{up}b;
c{up}..nullpath..d;
e{down}..nullpath..{up}f;
```

6.3 Pens

In Asymptote, pens provide a context for the four basic drawing commands (see Chapter 4 [Drawing commands], page 14). They are used to specify the following drawing attributes: color, line type, line width, line cap, line join, fill rule, text alignment, font, font size, pattern, overwrite mode, and calligraphic transforms on the pen nib. The default pen used by the drawing routines is called currentpen. This provides the same functionality as the MetaPost command pickup. The implicit initializer for pens is defaultpen.

Pens may be added together with the nonassociative binary operator +. This will add the colors of the two pens. All other non-default attributes of the rightmost pen will override those of the leftmost pen. Thus, one can obtain a yellow dashed pen by saying dashed+red+green or red+green+dashed or red+dashed+green. The binary operator * can be used to scale the color of a pen by a real number, until it saturates with one or more color components equal to 1.

• Colors are specified using one of the following colorspaces:

pen gray(real g);

This produces a grayscale color, where the intensity g lies in the interval [0,1], with 0.0 denoting black and 1.0 denoting white.

pen rgb(real r, real g, real b);

This produces an RGB color, where each of the red, green, and blue intensities \mathbf{r} , \mathbf{g} , \mathbf{b} , lies in the interval [0,1].

pen cmyk(real c, real m, real y, real k);

This produces a CMYK color, where each of the cyan, magenta, yellow, and black intensities c, m, y, k, lies in the interval [0,1].

pen invisible;

This special pen writes in invisible ink, but adjusts the bounding box as if something had been drawn (like the \phantom command in TEX). The function bool invisible(pen) can be used to test whether a pen is invisible.

The default color is black; this may be changed with the routine defaultpen(pen). The function colorspace(pen p) returns the colorspace of pen p as a string ("gray", "rgb", "cmyk", or "").

The function real[] colors(pen) returns the color components of a pen. The functions pen gray(pen), pen rgb(pen), and pen cmyk(pen) return new pens obtained by converting their arguments to the respective color spaces. The function colorless(pen=currentpen) returns a copy of its argument with the color attributes stripped (to avoid color mixing).

A 6-character RGB hexidecimal string can be converted to a pen with the routine

pen rgb(string s);

A pen can be converted to a hexidecimal string with

• string hex(pen p);

Various shades and mixtures of the grayscale primary colors black and white, RGB primary colors red, green, and blue, and RGB secondary colors cyan, magenta, and yellow are defined as named colors, along with the CMYK primary colors Cyan, Magenta, Yellow, and Black, in the module plain:



The standard 140 RGB X11 colors can be imported with the command import x11colors;

and the standard 68 CMYK $T_{E\!X}$ colors can be imported with the command import texcolors;

Note that there is some overlap between these two standards and the definitions of some colors (e.g. Green) actually disagree.

Asymptote also comes with a asycolors.sty LaTeX package that defines to LaTeX CMYK versions of Asymptote's predefined colors, so that they can be used directly within LaTeX strings. Normally, such colors are passed to LaTeX via a pen argument; however, to change the color of only a portion of a string, say for a slide presentation, (see Section 8.18 [slide], page 94) it may be desirable to specify the color directly to LaTeX. This file can be passed to LaTeX with the Asymptote command

```
usepackage("asycolors");
```

The structure hsv defined in plain_pens.asy may be used to convert between HSV and RGB spaces, where the hue h is an angle in [0, 360) and the saturation s and value v lie in [0,1]:

• Line types are specified with the function pen linetype(real[] a, real offset=0, bool scale=true, bool adjust=true), where a is an array of real array numbers. The optional parameter offset specifies where in the pattern to begin. The first

number specifies how far (if scale is true, in units of the pen line width; otherwise in PostScript units) to draw with the pen on, the second number specifies how far to draw with the pen off, and so on. If adjust is true, these spacings are automatically adjusted by Asymptote to fit the arclength of the path. Here are the predefined line types:

```
pen solid=linetype(new real[]);
pen dotted=linetype(new real[] {0,4});
pen dashed=linetype(new real[] {8,8});
pen longdashed=linetype(new real[] {24,8});
pen dashdotted=linetype(new real[] {8,8,0,8});
pen longdashdotted=linetype(new real[] {24,8,0,8});
pen Dotted=dotted+1.0;
pen Dotted(pen p=currentpen) {return dotted+2*linewidth(p);}
```

The default line type is solid; this may be changed with defaultpen(pen). The line type of a pen can be determined with the functions real[] linetype(pen p=currentpen), real offset(pen p), bool scale(pen p), and bool adjust(pen p).

• The pen line width is specified in PostScript units with pen linewidth(real). The default line width is 0.5 bp; this value may be changed with defaultpen(pen). The line width of a pen is returned by real linewidth(pen p=currentpen). For convenience, in the module plain we define

```
static void defaultpen(real w) {defaultpen(linewidth(w));}
static pen operator +(pen p, real w) {return p+linewidth(w);}
static pen operator +(real w, pen p) {return linewidth(w)+p;}
so that one may set the line width like this:
defaultpen(2);
pen p=red+0.5;
```

• A pen with a specific PostScript line cap is returned on calling linecap with an integer argument:

```
pen squarecap=linecap(0);
pen roundcap=linecap(1);
pen extendcap=linecap(2);
```

The default line cap, roundcap, may be changed with defaultpen(pen). The line cap of a pen is returned by int linecap(pen p=currentpen).

• A pen with a specific PostScript join style is returned on calling linejoin with an integer argument:

```
pen miterjoin=linejoin(0);
pen roundjoin=linejoin(1);
pen beveljoin=linejoin(2);
```

The default join style, roundjoin, may be changed with defaultpen(pen). The join style of a pen is returned by int linejoin(pen p=currentpen).

- A pen with a specific PostScript miter limit is returned by calling miterlimit(real). The default miterlimit, 10.0, may be changed with defaultpen(pen). The miter limit of a pen is returned by real miterlimit(pen p=currentpen).
- A pen with a specific PostScript fill rule is returned on calling fillrule with an integer argument:

```
pen zerowinding=fillrule(0);
pen evenodd=fillrule(1);
```

The fill rule, which identifies the algorithm used to determine the insideness of a path or array of paths, only affects the clip, fill, and inside functions. For the zerowinding fill rule, a point z is outside the region bounded by a path if the number of upward intersections of the path with the horizontal line z--z+infinity minus the number of downward intersections is zero. For the evenodd fill rule, z is considered to be outside the region if the total number of such intersections is even. The default fill rule, zerowinding, may be changed with defaultpen(pen). The fill rule of a pen is returned by int fillrule(pen p=currentpen).

• A pen with a specific text alignment setting is returned on calling basealign with an integer argument:

```
pen nobasealign=basealign(0);
pen basealign=basealign(1);
```

The default setting, nobasealign, which may be changed with defaultpen(pen), causes the label alignment routines to use the full label bounding box for alignment. In contrast, basealign requests that the TEX baseline be respected. The base align setting of a pen is returned by int basealigin(pen p=currentpen).

• The font size is specified in T_EX points (1 pt = 1/72.27 inches) with the function pen fontsize(real size, real lineskip=1.2*size). The default font size, 12pt, may be changed with defaultpen(pen). Nonstandard font sizes may require inserting import fontsize;

at the beginning of the file (this requires the type1cm package available from

```
http://www.ctan.org/tex-archive/macros/latex/contrib/type1cm/
```

and included in recent LaTeX distributions). The font size and line skip of a pen can be examined with the routines real fontsize(pen p=currentpen) and real lineskip(pen p=currentpen), respectively.

• A pen using a specific LaTeX NFSS font is returned by calling the function pen font(string encoding, string family, string series, string shape). The default setting, font("OT1","cmr","m","n"), corresponds to 12pt Computer Modern Roman; this may be changed with defaultpen(pen). The font setting of a pen is returned by string font(pen p=currentpen). Support for standardized international characters is provided by the unicode package (see Section 8.20 [unicode], page 94).

Alternatively, one may select a fixed-size TEX font (on which fontsize has no effect) like "cmr12" (12pt Computer Modern Roman) or "pcrr" (Courier) using the function pen font(string name). An optional size argument can also be given to scale the font to the requested size: pen font(string name, real size).

add("brick",brick());

A nonstandard font command can be generated with pen fontcommand(string).

A convenient interface to the following standard PostScript fonts is also provided:

```
pen AvantGarde(string series="m", string shape="n");
pen Bookman(string series="m", string shape="n");
pen Courier(string series="m", string shape="n");
pen Helvetica(string series="m", string shape="n");
pen NewCenturySchoolBook(string series="m", string shape="n");
pen Palatino(string series="m", string shape="n");
pen TimesRoman(string series="m", string shape="n");
pen ZapfChancery(string series="m", string shape="n");
pen Symbol(string series="m", string shape="n");
pen ZapfDingbats(string series="m", string shape="n");
```

• The transparency of a pen can be changed with the command:

```
pen opacity(real opacity=1, string blend="Compatible");
```

The opacity can be varied from 0 (fully transparent) to the default value of 1 (opaque), and blend specifies one of the following foreground-background blending operations:

```
"Compatible", "Normal", "Multiply", "Screen", "Overlay", "SoftLight",
"HardLight", "ColorDodge", "ColorBurn", "Darken", "Lighten", "Difference",
"Exclusion", "Hue", "Saturation", "Color", "Luminosity",
as described in
```

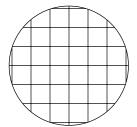
http://partners.adobe.com/public/developer/en/pdf/PDFReference16.pdf.

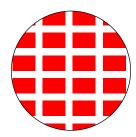
Since PostScript does not support transparency, this feature is only effective with the -f pdf output format option; other formats can be produced from the resulting PDF file with the ImageMagick convert program. Labels are always drawn with an opacity of 1. A simple example of transparent filling is provided in the example file transparency.asy.

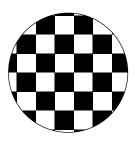
• PostScript commands within a picture may be used to create a tiling pattern, identified by the string name, for fill and draw operations by adding it to the global PostScript frame currentpatterns, with optional left-bottom margin 1b and right-top margin rt.

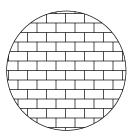
```
import patterns;
void add(string name, picture pic, pair lb=0, pair rt=0);
To fill or draw using pattern name, use the pen pattern("name"). For example, rectangular tilings can be constructed using the routines picture tile(real Hx=5mm, real Hy=0, pen p=currentpen, filltype filltype=NoFill), picture checker(real Hx=5mm, real Hy=0, pen p=currentpen), and picture brick(real Hx=5mm, real Hy=0, pen p=currentpen) defined in patterns.asy:
size(0,90);
import patterns;
add("tile",tile());
add("filledtilewithmargin",tile(6mm,4mm,red,Fill),(1mm,1mm),(1mm,1mm));
add("checker",checker());
```

```
real s=2.5;
filldraw(unitcircle,pattern("tile"));
filldraw(shift(s,0)*unitcircle,pattern("filledtilewithmargin"));
filldraw(shift(2s,0)*unitcircle,pattern("checker"));
filldraw(shift(3s,0)*unitcircle,pattern("brick"));
```









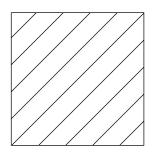
Hatch patterns can be generated with the routines picture hatch(real H=5mm, pair dir=NE, pen p=currentpen), picture crosshatch(real H=5mm, pen p=currentpen):

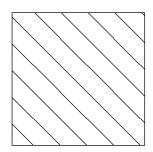
```
size(0,100);
import patterns;

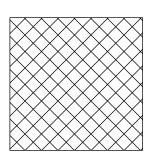
add("hatch",hatch());
add("hatchback",hatch(NW));
add("crosshatch",crosshatch(3mm));

real s=1.25;
filldraw(unitsquare,pattern("hatch"));
filldraw(shift(s,0)*unitsquare,pattern("hatchback"));
```

filldraw(shift(2s,0)*unitsquare,pattern("crosshatch"));







You may need to turn off aliasing in your PostScript viewer for patterns to appear correctly. Custom patterns can easily be constructed, following the examples in patterns.asy. The tiled pattern can even incorporate shading (see [gradient shading], page 16), as illustrated in this example (not included in the manual because not all printers support PostScript 3):

```
size(0,100);
import patterns;
```

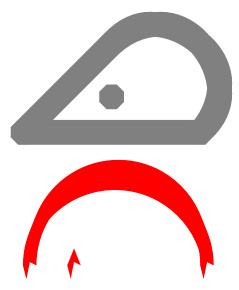
real d=4mm;

```
picture tiling;
path square=scale(d)*unitsquare;
axialshade(tiling,square,white,(0,0),black,(d,d));
fill(tiling,shift(d,d)*square,blue);
add("shadedtiling",tiling);
filldraw(unitcircle,pattern("shadedtiling"));
```

• One can specify a custom pen nib as an arbitrary polygonal path with pen makepen(path); this path represents the mark to be drawn for paths containing a single point. This pen nib path can be recovered from a pen with path nib(pen). Unlike in MetaPost, the path need not be convex:

```
size(200);
pen convex=makepen(scale(10)*polygon(8))+grey;
draw((1,0.4),convex);
draw((0,0)---(1,1)..(2,0)--cycle,convex);

pen nonconvex=scale(10)*
    makepen((0,0)--(0.25,-1)--(0.5,0.25)--(1,0)--(0.5,1.25)--cycle)+red;
draw((0.5,-1.5),nonconvex);
draw((0,-1.5)..(1,-0.5)..(2,-1.5),nonconvex);
```



The value nullpath represents a circular pen nib (the default); an elliptical pen can be achieved simply by multiplying the pen by a transform: yscale(2)*currentpen.

• One can prevent labels from overwriting one another by using the pen attribute overwrite, which takes a single argument:

Allow labels to overwrite one another. This is the default behaviour (unless overridden with defaultpen(pen).

Suppress Suppress, with a warning, each label that would overwrite another label.

SuppressQuiet

Suppress, without warning, each label that would overwrite another label.

Move

Move a label that would overwrite another out of the way and issue a warning. As this adjustment is during the final output phase (in PostScript coordinates) it could result in a larger figure than requested.

MoveQuiet

Move a label that would overwrite another out of the way, without warning. As this adjustment is during the final output phase (in PostScript coordinates) it could result in a larger figure than requested.

The routine defaultpen() returns the current default pen attributes. Calling the routine resetdefaultpen() resets all pen default attributes to their initial values.

6.4 Transforms

Asymptote makes extensive use of affine transforms. A pair (x,y) is transformed by the transform t=(t.x,t.y,t.xx,t.xy,t.yx,t.yy) to (x',y'), where

```
x' = t.x + t.xx * x + t.xy * y

y' = t.y + t.yx * x + t.yy * y
```

This is equivalent to the PostScript transformation [t.xx t.yx t.xy t.xy t.x t.y].

Transforms can be applied to pairs, guides, paths, pens, strings, transforms, frames, and pictures by multiplication (via the binary operator *) on the left (see [circle], page 30 for an example). Transforms can be composed with one another and inverted with the function transform inverse(transform t); they can also be raised to any integer power with the ^ operator.

The built-in transforms are:

```
transform identity();
           the identity transform;
transform shift(pair z);
           translates by the pair z;
transform shift(real x, real y);
           translates by the pair (x,y);
transform xscale(real x);
           scales by x in the x direction;
transform yscale(real y);
           scales by y in the y direction;
transform scale(real s);
           scale by s in both x and y directions;
transform scale(real x, real y);
           scale by x in the x direction and by y in the y direction;
transform slant(real s);
           maps (x,y) \rightarrow (x+s*y,y);
```

The implicit initializer for transforms is identity(). The routines shift(transform t) and shiftless(transform t) return the transforms (t.x,t.y,0,0,0,0) and (0,0,t.xx,t.xy,t.yx,t.yy) respectively.

6.5 Frames and pictures

frame

Frames are canvases for drawing in PostScript coordinates. While working with frames directly is occasionally necessary for constructing deferred drawing routines, pictures are usually more convenient to work with. The implicit initializer for frames is newframe. The function bool empty(frame f) returns true only if the frame f is empty. A frame may be erased with the erase(frame) routine. The functions pair min(frame) and pair max(frame) return the (left,bottom) and (right,top) coordinates of the frame bounding box, respectively. The contents of frame src may be appended to frame dest with the command

```
void add(frame dest, frame src);
or prepended with
void prepend(frame dest, frame src);
```

A frame obtained by aligning frame f in the direction align, in a manner analogous to the align argument of label (see Section 4.4 [label], page 18), is returned by

```
frame align(frame f, pair align);
```

To draw or fill a box or ellipse around a label or frame and return the boundary as a path, use one of the predefined envelope routines

picture

Pictures are high-level structures (see Section 6.8 [Structures], page 56) defined in the module plain that provide canvases for drawing in user coordinates. The default picture is called currentpicture. A new picture can be created like this:

```
picture pic;
```

Anonymous pictures can be made by the expression new picture.

The size routine specifies the dimensions of the desired picture:

If the x and y sizes are both 0, user coordinates will be interpreted as PostScript coordinates. In this case, the transform mapping pic to the final output frame is identity().

If exactly one of x or y is 0, no size restriction is imposed in that direction; it will be scaled the same as the other direction.

If keepAspect is set to Aspect or true, the picture will be scaled with its aspect ratio preserved such that the final width is no more than x and the final height is no more than y.

If keepAspect is set to IgnoreAspect or false, the picture will be scaled in both directions so that the final width is x and the height is y.

To make the user coordinates of picture pic represent multiples of x units in the x direction and y units in the y direction, use

void unitsize(picture pic=currentpicture, real x, real y=x);

When nonzero, these x and y values override the corresponding size parameters of picture pic.

The routine

forces the final picture scaling to map the user coordinates box(min,max) to a region of width xsize and height ysize (when these parameters are nonzero). Alternatively, calling the routine

will cause picture pic to use a fixed scaling to map user coordinates in box(min,max) to the (already specified) picture size, taking account of the width of pen p. A warning will be issued if the final picture exceeds the specified size.

A picture pic can be fit to a frame and output to a file prefix.format using image format format by calling the shipout function:

The default output format, PostScript, may be changed with the -f or -tex command-line options. The options, script, and projection parameters are only relevant for 3D pictures. If defaultfilename is an empty string, the prefix outprefix() will be used.

A shipout() command is added implicitly at file exit if no previous shipout commands have been executed. The default page orientation is Portrait; this may be modified by changing the variable orientation. To output in landscape mode, simply set the variable orientation=Landscape or issue the command

```
shipout(Landscape);
```

To rotate the page by -90 degrees, use the orientation Seascape. The orientation UpsideDown rotates the page by 180 degrees.

A picture pic can be explicitly fit to a frame by calling

The default size and aspect ratio settings are those given to the size command (which default to 0, 0, and true, respectively). The transformation that would currently be used to fit a picture pic to a frame is returned by the member function pic.calculateTransform().

In certain cases (e.g. 2D graphs) where only an approximate size estimate for pic is available, the picture fitting routine

(which scales the resulting frame, including labels and fixed-size objects) will enforce perfect compliance with the requested size specification, but should not normally be required.

To draw a bounding box with margins around a picture, fit the picture to a frame using the function

Here filltype specifies one of the following fill types:

FillDraw Fill the interior and draw the boundary.

FillDraw(real xmargin=0, real ymargin=xmargin, pen fillpen=nullpen, pen drawpen=nullpen) If fillpen is nullpen, fill with the drawing pen; otherwise fill with pen fillpen. If drawpen is nullpen, draw the boundary with fillpen; otherwise with drawpen. An optional margin of xmargin and ymargin can be specified.

Fill Fill the interior.

Fill(real xmargin=0, real ymargin=xmargin, pen p=nullpen)

If p is nullpen, fill with the drawing pen; otherwise fill with pen p. An optional margin of xmargin and ymargin can be specified.

NoFill Do not fill.

Draw Only the boundary.

Draw(real xmargin=0, real ymargin=xmargin, pen p=nullpen)

If p is nullpen, draw the boundary with the drawing pen; otherwise draw with pen p. An optional margin of xmargin and ymargin can be specified.

UnFill Clip the region.

```
UnFill(real xmargin=0, real ymargin=xmargin)
```

Clip the region and surrounding margins xmargin and ymargin.

RadialShade(pen penc, pen penr)

Fill varying radially from penc at the center of the bounding box to penr at the edge.

RadialShadeDraw(real xmargin=0, real ymargin=xmargin, pen penc, pen penr, pen drawpen=nullpen) Fill with RadialShade and draw the boundary.

For example, to draw a bounding box around a picture with a 0.25 cm margin and output the resulting frame, use the command:

```
shipout(bbox(0.25cm));
```

A picture may be fit to a frame with the background color pen p, using the function bbox(p,Fill).

The functions

```
pair min(picture pic, user=false);
pair max(picture pic, user=false);
pair size(picture pic, user=false);
```

calculate the bounds that picture pic would have if it were currently fit to a frame using its default size specification. If user is false the returned value is in PostScript coordinates, otherwise it is in user coordinates.

The function

pair point(picture pic=currentpicture, pair dir, bool user=true); is a convenient way of determining the point on the bounding box of pic in the direction dir relative to its center, ignoring the contributions from fixed-size objects (such as labels and arrowheads). If user is true the returned value is in user coordinates, otherwise it is in PostScript coordinates.

The function

pair truepoint(picture pic=currentpicture, pair dir, bool user=true); is identical to point, except that it also accounts for fixed-size objects, using the scaling transform that picture pic would have if currently fit to a frame using its default size specification. If user is true the returned value is in user

Sometimes it is useful to draw objects on separate pictures and add one picture to another using the add function:

coordinates, otherwise it is in PostScript coordinates.

The first example adds src to currentpicture; the second one adds src to dest. The group option specifies whether or not the graphical user interface xasy should treat all of the elements of src as a single entity (see Chapter 11 [GUI], page 163), filltype requests optional background filling or clipping, and above specifies whether to add src above or below existing objects.

There are also routines to add a picture or frame src specified in postscript coordinates to another picture dest (or currentpicture) about the user coordinate position:

The optional align argument in the last form specifies a direction to use for aligning the frame, in a manner analogous to the align argument of label (see Section 4.4 [label], page 18). However, one key difference is that when align is not specified, labels are centered, whereas frames and pictures are aligned so that their origin is at position. Illustrations of frame alignment can be found in the examples [errorbars], page 106 and [image], page 124. If you want to align three or more subpictures, group them two at a time:

```
picture pic1;
real size=50;
size(pic1,size);
fill(pic1,(0,0)--(50,100)--(100,0)--cycle,red);

picture pic2;
size(pic2,size);
fill(pic2,unitcircle,green);

picture pic3;
size(pic3,size);
fill(pic3,unitsquare,blue);

picture pic;
add(pic,pic1.fit(),(0,0),N);
add(pic,pic2.fit(),(0,0),10S);

add(pic.fit(),(0,0),10S);
```



Alternatively, one can use attach to automatically increase the size of picture dest to accommodate adding a frame src about the user coordinate position:

```
void attach(picture dest=currentpicture, frame src,
```

```
pair position=0, bool group=true,
filltype filltype=NoFill, bool above=true);
void attach(picture dest=currentpicture, frame src,
pair position, pair align, bool group=true,
filltype filltype=NoFill, bool above=true);
```

To erase the contents of a picture (but not the size specification), use the function

```
void erase(picture pic=currentpicture);
```

To save a snapshot of currentpicture, currentpen, and currentprojection, use the function save().

To restore a snapshot of currentpicture, currentpen, and currentprojection, use the function restore().

Many further examples of picture and frame operations are provided in the base module plain.

It is possible to insert verbatim PostScript commands in a picture with one of the routines

Here min and max can be used to specify explicit bounds associated with the resulting PostScript code.

Verbatim TEX commands can be inserted in the intermediate LaTeX output file with one of the functions

```
void tex(picture pic=currentpicture, string s);
void tex(picture pic=currentpicture, string s, pair min, pair max)
Here min and max can be used to specify explicit bounds associated with the
resulting TFX code.
```

To issue a global TEX command (such as a TEX macro definition) in the TEX preamble (valid for the remainder of the top-level module) use:

```
void texpreamble(string s);
The TEX environment can be reset to its initial state, clearing all macro definitions, with the function
void texreset();
The routine
void usepackage(string s, string options="");
provides a convenient abbreviation for
texpreamble("\usepackage["+options+"]{"+s+"}");
that can be used for importing LaTeX packages.
```

6.6 Files

Asymptote can read and write text files (including comma-separated value) files and portable XDR (External Data Representation) binary files.

An input file must first be opened with input(string name, bool check=true, string comment="#"); reading is then done by assignment:

```
file fin=input("test.txt");
real a=fin;
```

If the optional boolean argument check is false, no check will be made that the file exists. If the file does not exist or is not readable, the function bool error(file) will return true. The first character of the string comment specifies a comment character. If this character is encountered in a data file, the remainder of the line is ignored. When reading strings, a comment character followed immediately by another comment character is treated as a single literal comment character.

One can change the current working directory for read operations to the contents of the string s with the function string cd(string s), which returns the new working directory. If string s is empty, the path is reset to the value it had at program startup.

When reading pairs, the enclosing parenthesis are optional. Strings are also read by assignment, by reading characters up to but not including a newline. In addition, Asymptote provides the function string getc(file) to read the next character (treating the comment character as an ordinary character) and return it as a string.

A file named name can be open for output with file output(string name, bool update=false);

If update=false, any existing data in the file will be erased and only write operations can be used on the file. If update=true, any existing data will be preserved, the position will be set to the end-of-file, and both reading and writing operations will be enabled. For security reasons, writing to files in directories other than the current directory is allowed only if the -globalwrite (or -nosafe) command-line option is specified. The function string mktemp(string s) may be used to create and return the name of a unique temporary file in the current directory based on the string s.

There are two special files: stdin, which reads from the keyboard, and stdout, which writes to the terminal. The implicit initializer for files is null.

Data of a built-in type T can be written to an output file by calling one of the functions

```
write(string s="", T x, suffix suffix=endl ... T[]);
write(file file, string s="", T x, suffix suffix=none ... T[]);
write(file file=stdout, string s="", explicit T[] x ... T[][]);
write(file file=stdout, T[][]);
write(file file=stdout, T[][][]);
write(suffix suffix=endl);
write(file file, suffix suffix=none);
```

If file is not specified, stdout is used and terminated by default with a newline. If specified, the optional identifying string s is written before the data x. An arbitrary number of data values may be listed when writing scalars or one-dimensional arrays. The suffix may be one of the following: none (do nothing), flush (output buffered data), end1 (terminate with a newline and flush), newl (terminate with a newline), DOSendl (terminate with a DOS newline and flush), DOSnewl (terminate with a DOS newline), tab (terminate with a tab), or comma (terminate with a comma). Here are some simple examples of data output:

A file may also be opened with xinput or xoutput, instead of input or output, to read or write double precision (64-bit) reals and single precision (32-bit) integers in Sun Microsystem's XDR (External Data Representation) portable binary format (available on all UNIX platforms). Alternatively, a file may also be opened with binput or boutput to read or write double precision reals and single precision integers in the native (nonportable) machine binary format. The virtual member functions file singlereal(bool b=true) and file singleint(bool b=true) be used to change the precision of real and integer I/O operations, respectively, for an XDR or binary file f. Similarly, the function file signedint(bool b=true) can be used to modify the signedness of integer reads and writes for an XDR or binary file f.

The virtual members name, mode, singlereal, singleint, and signedint may be used to query the respective parameters for a given file.

One can test a file for end-of-file with the boolean function eof(file), end-of-line with eol(file), and for I/O errors with error(file). One can flush the output buffers with flush(file), clear a previous I/O error with clear(file), and close the file with close(file). The function int precision(file file=stdout, int digits=0) sets the number of digits of output precision for file to digits, provided digits is nonzero, and returns the previous precision setting. The function int tell(file) returns the current position in a file relative to the beginning. The routine seek(file file, int pos) can be used to change this position, where a negative value for the position pos is interpreted as relative to the end-of-file. For example, one can rewind a file file with the command seek(file,0) and position to the final character in the file with seek(file,-1). The command seekeof(file) sets the position to the end of the file.

Assigning settings.scroll=n for a positive integer n requests a pause after every n output lines to stdout. One may then press Enter to continue to the next n output lines, s followed by Enter to scroll without further interruption, or q followed by Enter to quit the current output operation. If n is negative, the output scrolls a page at a time (i.e. by

one less than the current number of display lines). The default value, settings.scroll=0, specifies continuous scrolling.

The routines

defined in the module plain may be used to prompt for a value from stdin using the GNU readline library. If store=true, the history of values for name is stored in the file ".asy_history_"+name (see [history], page 161). The most recent value in the history will be used to provide a default value for subsequent runs. The default value (initially default) is displayed after prompt. These functions are based on the internal routines

```
string readline(string prompt="", string name="", bool tabcompletion=false);
void saveline(string name, string value, bool store=true);
```

Here, readline prompts the user with the default value formatted according to prompt, while saveline is used to save the string value in a local history named name, optionally storing the local history in a file ".asy_history_"+name.

The routine history(string name, int n=1) can be used to look up the n most recent values (or all values up to historylines if n=0) entered for string name. The routine history(int n=0) returns the interactive history. For example,

```
write(output("transcript.asy"), history());
outputs the interactive history to the file transcript.asy.
```

The function int delete(string s) deletes the file named by the string s. Unless the -globalwrite (or -nosafe) option is enabled, the file must reside in the current directory. The function int rename(string from, string to) may be used to rename file from to file to. Unless the -globalwrite (or -nosafe) option is enabled, this operation is restricted to the current directory. The functions

```
int convert(string args="", string file="", string format="");
int animate(string args="", string file="", string format="");
```

call the ImageMagick commands convert and animate, respectively, with the arguments args and the file name constructed from the strings file and format. If the setting safe is false, then the functions int system(string s) and int system(string[] s) can be used to call the arbitrary system command s.

6.7 Variable initializers

A variable can be assigned a value when it is declared, as in int x=3; where the variable x is assigned the value 3. As well as literal constants such as 3, arbitary expressions can be used as initializers, as in real x=2*sin(pi/2);.

A variable is not added to the namespace until after the initializer is evaluated, so for example, in

```
int x=2;
int x=5*x;
```

the x in the initializer on the second line refers to the variable x declared on the first line. The second line, then, declares a variable x shadowing the original x and initializes it to the value 10.

Variables of most types can be declared without an explicit initializer and they will be initialized by the default initializer of that type:

- Variables of the numeric types int, real, and pair are all initialized to zero; variables of type triple are initialized to O=(0,0,0).
- boolean variables are initialized to false.
- string variables are initialized to the empty string.
- transform variables are initialized to the identity transformation.
- path and guide variables are initialized to nullpath.
- pen variables are initialized to the default pen.
- frame and picture variables are initialized to empty frames and pictures, respectively.
- file variables are initialized to null.

The default initializers for user-defined array, structure, and function types are explained in their respective sections. Some types, such as code, do not have default initializers. When a variable of such a type is introduced, the user must initialize it by explicitly giving it a value.

The default initializer for any type T can be redeclared by defining the function T operator init(). For instance, int variables are usually initialized to zero, but in

```
int operator init() {
  return 3;
}
int y;
```

the variable y is initialized to 3. This example was given for illustrative purposes; redeclaring the initializers of built-in types is not recommended. Typically, operator init is used to define sensible defaults for user-defined types.

The special type var may be used to infer the type of a variable from its initializer. If the initializer is an expression of a unique type, then the variable will be defined with that type. For instance,

```
var x=5;
var y=4.3;
var reddash=red+dashed;
is equivalent to
int x=5;
real y=4.3;
pen reddash=red+dashed;
```

var may also be used with the extended for loop syntax.

```
int[] a = {1,2,3};
for (var x : a)
  write(x);
```

6.8 Structures

Users may also define their own data types as structures, along with user-defined operators, much as in C++. By default, structure members are public (may be read and modified anywhere in the code), but may be optionally declared restricted (readable anywhere but writeable only inside the structure where they are defined) or private (readable and writable only inside the structure). In a structure definition, the keyword this can be used as an expression to refer to the enclosing structure. Any code at the top-level scope within the structure is executed on initialization.

Variables hold references to structures. That is, in the example:

```
struct T {
   int x;
}

T foo=new T;
T bar=foo;
bar.x=5;
```

The variable foo holds a reference to an instance of the structure T. When bar is assigned the value of foo, it too now holds a reference to the same instance as foo does. The assignment bar.x=5 changes the value of the field x in that instance, so that foo.x will also be equal to 5.

The expression new T creates a new instance of the structure T and returns a reference to that instance. In creating the new instance, any code in the body of the record definition is executed. For example:

```
int Tcount=0;
struct T {
  int x;
  ++Tcount;
}
```

T foo=new T;

Here, the expression new T will produce a new instance of the class, but will also cause Tcount to be incremented, so that it keeps track of the number of instances produced.

The expression null can be cast to any structure type to yield a null reference, a reference that does not actually refer to any instance of the structure. Trying to use a field of a null reference will cause an error.

The function bool alias(T,T) checks to see if two structure references refer to the same instance of the structure (or both to null). For example, in the example code at the start of the section, alias(foo,bar) would return true, but alias(foo,new T) would return false, as new T creates a new instance of the structure T. The boolean operators == and != are by default equivalent to alias and !alias respectively, but may be overwritten for a particular type (for example, to do a deep comparison).

After the definition of a structure T, a variable of type T is initialized to a new instance (new T) by default. During the definition of the structure, however, variables of type T are initialized to null by default. This special behaviour is to avoid infinite recursion of creating new instances in code such as

```
struct tree {
  int value;
  tree left;
  tree right;
}
   Here is a simple example that illustrates the use of structures:
struct S {
  real a=1;
  real f(real a) {return a+this.a;}
Ss;
                                   // Initializes s with new S;
write(s.f(2));
                                   // Outputs 3
S operator + (S s1, S s2)
  S result;
  result.a=s1.a+s2.a;
  return result;
}
write((s+s).f(0));
                                   // Outputs 2
   It is often convenient to have functions that construct new instances of a structure. Say
we have a Person structure:
struct Person {
  string firstname;
  string lastname;
}
Person joe=new Person;
joe.firstname="Joe";
joe.lastname="Jones";
```

Creating a new Person is a chore; it takes three lines to create a new instance and to initialize its fields (that's still considerably less effort than creating a new person in real life, though).

We can reduce the work by defining a constructor function Person(string, string):

```
struct Person {
   string firstname;
   string lastname;
```

```
static Person Person(string firstname, string lastname) {
   Person p=new Person;
   p.firstname=firstname;
   p.lastname=lastname;
   return p;
}
```

Person joe=Person.Person("Joe", "Jones");

While it is now easier than before to create a new instance, we still have to refer to the constructor by the qualified name Person. Person. If we add the line

```
from Person unravel Person;
```

immediately after the structure definition, then the constructor can be used without qualification: Person joe=Person("Joe", "Jones");.

The constructor is now easy to use, but it is quite a hassle to define. If you write a lot of constructors, you will find that you are repeating a lot of code in each of them. Fortunately, your friendly neighbourhood Asymptote developers have devised a way to automate much of the process.

If, in the body of a structure, Asymptote encounters the definition of a function of the form void operator init(args), it implicitly defines a constructor function of the arguments args that uses the void operator init function to initialize a new instance of the structure. That is, it essentially defines the following constructor (assuming the structure is called Foo):

```
static Foo Foo(args) {
   Foo instance=new Foo;
   instance.operator init(args);
   return instance;
}
```

This constructor is also implicitly copied to the enclosing scope after the end of the structure definition, so that it can used subsequently without qualifying it by the structure name. Our Person example can thus be implemented as:

```
struct Person {
   string firstname;
   string lastname;

   void operator init(string firstname, string lastname) {
     this.firstname=firstname;
     this.lastname=lastname;
   }
}
Person joe=Person("Joe", "Jones");
```

The use of operator init to implicitly define constructors should not be confused with its use to define default values for variables (see Section 6.7 [Variable initializers], page 54).

Indeed, in the first case, the return type of the operator init must be void while in the second, it must be the (non-void) type of the variable.

The function cputime() returns a structure cputime with cumulative CPU times broken down into the fields parent.user, parent.system, child.user, and child.system. For convenience, the incremental fields change.user and change.system indicate the change in the corresponding total parent and child CPU times since the last call to cputime(). The function

displays the incremental user cputime followed by "u", the incremental system cputime followed by "s", the total user cputime followed by "U", and the total system cputime followed by "S".

Much like in C++, casting (see Section 6.13 [Casts], page 76) provides for an elegant implementation of structure inheritance, including virtual functions:

```
struct parent {
  real x;
  void operator init(int x) {this.x=x;}
  void virtual(int) {write(0);}
  void f() {virtual(1);}
}
void write(parent p) {write(p.x);}
struct child {
  parent parent;
  real y=3;
  void operator init(int x) {parent.operator init(x);}
  void virtual(int x) {write(x);}
  parent.virtual=virtual;
  void f()=parent.f;
}
parent operator cast(child child) {return child.parent;}
parent p=parent(1);
child c=child(2);
write(c):
                                // Outputs 2;
                                // Outputs 0;
p.f();
                                // Outputs 1;
c.f();
write(c.parent.x);
                                // Outputs 2;
write(c.y);
                                // Outputs 3;
```

For further examples of structures, see Legend and picture in the Asymptote base module plain.

6.9 Operators

6.9.1 Arithmetic & logical operators

Asymptote uses the standard binary arithmetic operators. However, when one integer is divided by another, both arguments are converted to real values before dividing and a real quotient is returned (since this is usually what is intended). The function int quotient(int x, int y) returns the greatest integer less than or equal to x/y. In all other cases both operands are promoted to the same type, which will also be the type of the result:

- + addition
- subtraction
- * multiplication
- / division
- % modulo; the result always has the same sign as the divisor. In particular, this makes q*quotient(p,q)+p%q == p for all integers p and nonzero integers q.
- power; if the exponent (second argument) is an int, recursive multiplication is used; otherwise, logarithms and exponentials are used (** is a synonym for ^).

The usual boolean operators are also defined:

```
equals
!=
            not equals
            less than
<
<=
            less than or equals
            greater than or equals
>=
>
            greater than
            and (with conditional evaluation of right-hand argument)
&&
&
            and
            or (with conditional evaluation of right-hand argument)
| | |
            or
            xor
!
            not
```

Asymptote also supports the C-like conditional syntax:

```
bool positive=(pi >= 0) ? true : false;
```

The function T interp(T a, T b, real t) returns (1-t)*a+t*b for nonintegral built-in arithmetic types T. If a and b are pens, they are first promoted to the same color space.

Asymptote also defines bitwise functions int AND(int,int), int OR(int,int), int XOR(int,int), int NOT(int), int CLZ(int) (count leading zeros), and int CTZ(int) (count trailing zeros).

6.9.2 Self & prefix operators

As in C, each of the arithmetic operators +, -, *, /, %, and $^$ can be used as a self operator. The prefix operators ++ (increment by one) and -- (decrement by one) are also defined. For example,

```
int i=1;
i += 2;
int j=++i;
is equivalent to the code
int i=1;
i=i+2;
int j=i=i+1;
```

However, postfix operators like i++ and i-- are not defined (because of the inherent ambiguities that would arise with the -- path-joining operator). In the rare instances where i++ and i-- are really needed, one can substitute the expressions (++i-1) and (--i+1), respectively.

6.9.3 User-defined operators

The following symbols may be used with operator to define or redefine operators on structures and built-in types:

```
- + * / % ^ ! < > == != <= >= & | ^^ .. :: -- --- ++ << >> $ $$ @ @@
```

The operators on the second line have precedence one higher than the boolean operators <, >, <=, and >=.

Guide operators like .. may be overloaded, say, to write a user function that produces a new guide from a given guide:

```
guide dots(... guide[] g)=operator ..;
guide operator ..(... guide[] g) {
 guide G;
  if(g.length > 0) {
    write(g[0]);
    G=g[0];
  }
  for(int i=1; i < g.length; ++i) {</pre>
    write(g[i]);
    write();
    G=dots(G,g[i]);
  }
  return G;
}
guide g=(0,0)\{up\}..\{SW\}(100,100)\{NE\}..\{curl 3\}(50,50)..(10,10);
write("g=",g);
```

6.10 Implicit scaling

If a numeric literal is in front of certain types of expressions, then the two are multiplied:

```
int x=2:
real y=2.0;
real cm=72/2.540005;
write(3x);
write(2.5x);
write(3y);
write(-1.602e-19 y);
write(0.5(x,y));
write(2x^2);
write(3x+2y);
write(3(x+2y));
write(3sin(x));
write(3(sin(x))^2);
write(10cm);
  This produces the output
6
5
-3.204e-19
(1,1)
10
18
2.72789228047704
2.48046543129542
283.464008929116
```

6.11 Functions

Asymptote functions are treated as variables with a signature (non-function variables have null signatures). Variables with the same name are allowed, so long as they have distinct signatures.

Functions arguments are passed by value. To pass an argument by reference, simply enclose it in a structure (see Section 6.8 [Structures], page 56).

Here are some significant features of Asymptote functions:

1. Variables with signatures (functions) and without signatures (nonfunction variables) are distinct:

2. Traditional function definitions are allowed:

```
int sqr(int x)
     return x*x;
  sqr=null;
                        // but the function is still just a variable.
3. Casting can be used to resolve ambiguities:
   int a, a(), b, b(); // Valid: creates four variables.
                        // Invalid: assignment is ambiguous.
  a=b;
  a=(int) b;
                        // Valid: resolves ambiguity.
   (int) (a=b);
                        // Valid: resolves ambiguity.
   (int) a=b;
                        // Invalid: cast expressions cannot be L-values.
  int c();
                        // Valid: only one possible assignment.
  c=a;
4. Anonymous (so-called "high-order") functions are also allowed:
  typedef int intop(int);
  intop adder(int m)
     return new int(int n) {return m+n;};
  intop addby7=adder(7);
  write(addby7(1));
                        // Writes 8.
5. One may redefine a function f, even for calls to f in previously declared functions, by
  assigning another (anonymous or named) function to it. However, if f is overloaded
  by a new function definition, previous calls will still access the original version of f, as
  illustrated in this example:
  void f() {
     write("hi");
  void g() {
     f();
  }
  g(); // writes "hi"
  f=new void() {write("bye");};
  g(); // writes "bye"
  void f() {write("overloaded");};
  f(); // writes "overloaded"
  g(); // writes "bye"
```

6. Anonymous functions can be used to redefine a function variable that has been declared

(and implicitly initialized to the null function) but not yet explicitly defined:

```
void f(bool b);

void g(bool b) {
   if(b) f(b);
   else write(b);
}

f=new void(bool b) {
   write(b);
   g(false);
};

g(true); // Writes true, then writes false.
```

Asymptote is the only language we know of that treats functions as variables, but allows overloading by distinguishing variables based on their signatures.

Functions are allowed to call themselves recursively. As in C++, infinite nested recursion will generate a stack overflow (reported as a segmentation fault, unless a fully working version of the GNU library libsigsegv (e.g. 2.4 or later) is installed at configuration time).

6.11.1 Default arguments

Asymptote supports a more flexible mechanism for default function arguments than C++: they may appear anywhere in the function prototype. Because certain data types are implicitly cast to more sophisticated types (see Section 6.13 [Casts], page 76) one can often avoid ambiguities by ordering function arguments from the simplest to the most complicated. For example, given

```
real f(int a=1, real b=0) {return a+b;} then f(1) returns 1.0, but f(1.0) returns 2.0.
```

The value of a default argument is determined by evaluating the given Asymptote expression in the scope where the called function is defined.

6.11.2 Named arguments

It is sometimes difficult to remember the order in which arguments appear in a function declaration. Named (keyword) arguments make calling functions with multiple arguments easier. Unlike in the C and C++ languages, an assignment in a function argument is interpreted as an assignment to a parameter of the same name in the function signature, not within the local scope. The command-line option -d may be used to check Asymptote code for cases where a named argument may be mistaken for a local assignment.

When matching arguments to signatures, first all of the keywords are matched, then the arguments without names are matched against the unmatched formals as usual. For example,

```
int f(int x, int y) {
  return 10x+y;
}
write(f(4,x=3));
```

outputs 34, as x is already matched when we try to match the unnamed argument 4, so it gets matched to the next item, y.

For the rare occasions where it is desirable to assign a value to local variable within a function argument (generally *not* a good programming practice), simply enclose the assignment in parentheses. For example, given the definition of **f** in the previous example,

```
int x;
write(f(4,(x=3)));
is equivalent to the statements
int x;
x=3;
write(f(4,3));
and outputs 43.
```

Parameters can be specified as "keyword-only" by putting keyword immediately before the parameter name, as in int f(int keyword x) or int f(int keyword x=77). This forces the caller of the function to use a named argument to give a value for this parameter. That is, f(x=42) is legal, but f(25) is not. Keyword-only parameters must be listed after normal parameters in a function definition.

As a technical detail, we point out that, since variables of the same name but different signatures are allowed in the same scope, the code

```
int f(int x, int x()) {
  return x+x();
}
int seven() {return 7;}
```

is legal in Asymptote, with f(2, seven) returning 9. A named argument matches the first unmatched formal of the same name, so f(x=2,x=seven) is an equivalent call, but f(x=seven,2) is not, as the first argument is matched to the first formal, and int () cannot be implicitly cast to int. Default arguments do not affect which formal a named argument is matched to, so if f were defined as

```
int f(int x=3, int x()) {
  return x+x();
}
```

then f(x=seven) would be illegal, even though f(seven) obviously would be allowed.

6.11.3 Rest arguments

Rest arguments allow one to write functions that take a variable number of arguments:

```
// This function sums its arguments.
int sum(... int[] nums) {
  int total=0;
  for(int i=0; i < nums.length; ++i)
     total += nums[i];
  return total;
}
sum(1,2,3,4);  // returns 10</pre>
```

```
sum();
                                        // returns 0
// This function subtracts subsequent arguments from the first.
int subtract(int start ... int[] subs) {
  for(int i=0; i < subs.length; ++i)</pre>
    start -= subs[i];
  return start;
}
                                       // returns 7
subtract(10,1,2);
subtract(10);
                                        // returns 10
subtract();
                                       // illegal
  Putting an argument into a rest array is called packing. One can give an explicit list of
arguments for the rest argument, so subtract could alternatively be implemented as
int subtract(int start ... int[] subs) {
  return start - sum(... subs);
}
  One can even combine normal arguments with rest arguments:
sum(1,2,3 ... new int[] {4,5,6}); // returns 21
This builds a new six-element array that is passed to sum as nums. The opposite operation,
unpacking, is not allowed:
subtract(... new int[] {10, 1, 2});
is illegal, as the start formal is not matched.
```

If no arguments are packed, then a zero-length array (as opposed to null) is bound to the rest parameter. Note that default arguments are ignored for rest formals and the rest argument is not bound to a keyword.

In some cases, keyword-only parameters are helpful to avoid arguments intended for the rest parameter to be assigned to other parameters. For example, here the use of keyword is to avoid pnorm(1.0,2.0,0.3) matching 1.0 to p.

```
real pnorm(real keyword p=2.0 ... real[] v)
{
  return sum(v^p)^(1/p);
}
```

The overloading resolution in Asymptote is similar to the function matching rules used in C++. Every argument match is given a score. Exact matches score better than matches with casting, and matches with formals (regardless of casting) score better than packing an argument into the rest array. A candidate is maximal if all of the arguments score as well in it as with any other candidate. If there is one unique maximal candidate, it is chosen; otherwise, there is an ambiguity error.

```
int f(path g);
int f(guide g);
f((0,0)--(100,100)); // matches the second; the argument is a guide
int g(int x, real y);
```

6.11.4 Mathematical functions

Asymptote has built-in versions of the standard libm mathematical real(real) functions sin, cos, tan, asin, acos, atan, exp, log, pow10, log10, sinh, cosh, tanh, asinh, acosh, atanh, sqrt, cbrt, fabs, expm1, log1p, as well as the identity function identity. Asymptote also defines the order n Bessel functions of the first kind Jn(int n, real) and second kind Yn(int n, real), as well as the gamma function gamma, the error function erf, and the complementary error function erfc. The standard real(real, real) functions atan2, hypot, fmod, remainder are also included.

The functions degrees(real radians) and radians(real degrees) can be used to convert between radians and degrees. The function Degrees(real radians) returns the angle in degrees in the interval [0,360). For convenience, Asymptote defines variants Sin, Cos, Tan, aSin, aCos, and aTan of the standard trigonometric functions that use degrees rather than radians. We also define complex versions of the sqrt, sin, cos, exp, log, and gamma functions.

The functions floor, ceil, and round differ from their usual definitions in that they all return an int value rather than a real (since that is normally what one wants). The functions Floor, Ceil, and Round are respectively similar, except that if the result cannot be converted to a valid int, they return intMax for positive arguments and intMin for negative arguments, rather than generating an integer overflow. We also define a function sgn, which returns the sign of its real argument as an integer (-1, 0, or 1).

There is an abs(int) function, as well as an abs(real) function (equivalent to fabs(real)), an abs(pair) function (equivalent to length(pair)).

Random numbers can be seeded with srand(int) and generated with the int rand() function, which returns a random integer between 0 and the integer randMax. The unitrand() function returns a random number uniformly distributed in the interval [0,1]. A Gaussian random number generator Gaussrand and a collection of statistics routines, including histogram, are provided in the base file stats.asy. The functions factorial(int n), which returns n!, and choose(int n, int k), which returns n!/(k!(n-k)!), are also defined.

When configured with the GNU Scientific Library (GSL), available from http://www.gnu.org/software/gsl/, Asymptote contains an internal module gsl that defines the airy functions Ai(real), Bi(real), Ai_deriv(real), Bi_deriv(real), zero_Ai(int), zero_Bi(int), zero_Ai_deriv(int), zero_Bi_deriv(int), the Bessel functions I(int, real), K(int, real), j(int, real), y(int, real), i_scaled(int, real), k_scaled(int, real), J(real, real), Y(real, real), I(real, real), K(real, real), zero_J(real, int), the elliptic functions F(real, real), E(real, real), and P(real, real), the exponential/trigonometric integrals Ei, Si, and Ci, the Legendre polynomials Pl(int, real), and the Riemann zeta function zeta(real). For example, to compute the sine integral Si of 1.0:

```
import gsl;
write(Si(1.0));
```

Asymptote also provides a few general purpose numerical routines:

real newton(int iterations=100, real f(real), real fprime(real), real x,
bool verbose=false);

Use Newton-Raphson iteration to solve for a root of a real-valued differentiable function f, given its derivative fprime and an initial guess x. Diagnostics for each iteration are printed if verbose=true. If the iteration fails after the maximum allowed number of loops (iterations), realMax is returned.

real newton(int iterations=100, real f(real), real fprime(real), real x1,
real x2, bool verbose=false);

Use bracketed Newton-Raphson bisection to solve for a root of a real-valued differentiable function f within an interval [x1,x2] (on which the endpoint values of f have opposite signs), given its derivative fprime. Diagnostics for each iteration are printed if verbose=true. If the iteration fails after the maximum allowed number of loops (iterations), realMax is returned.

real simpson(real f(real), real a, real b, real acc=realEpsilon, real dxmax=b-a)

returns the integral of f from a to b using adaptive Simpson integration.

6.12 Arrays

Appending [] to a built-in or user-defined type yields an array. The array element i of an array A can be accessed as A[i]. By default, attempts to access or assign to an array element using a negative index generates an error. Reading an array element with an index beyond the length of the array also generates an error; however, assignment to an element beyond the length of the array causes the array to be resized to accommodate the new element. One can also index an array A with an integer array B: the array A[B] is formed by indexing array A with successive elements of array B. A convenient Java-style shorthand exists for iterating over all elements of an array; see [array iteration], page 24.

The declaration

```
real[] A;
```

initializes A to be an empty (zero-length) array. Empty arrays should be distinguished from null arrays. If we say

```
real[] A=null;
```

then A cannot be dereferenced at all (null arrays have no length and cannot be read from or assigned to).

Arrays can be explicitly initialized like this:

```
real[] A=\{0,1,2\};
```

Array assignment in Asymptote does a shallow copy: only the pointer is copied (if one copy if modified, the other will be too). The copy function listed below provides a deep copy of an array.

Every array A of type T[] has the virtual members

- int length,
- int cyclic,
- int[] keys,
- T push(T x),
- void append(T[] a),
- T pop(),
- void insert(int i ... T[] x),
- void delete(int i, int j=i),
- void delete(), and
- bool initialized(int n).

The member A.length evaluates to the length of the array. Setting A.cyclic=true signifies that array indices should be reduced modulo the current array length. Reading from or writing to a nonempty cyclic array never leads to out-of-bounds errors or array resizing.

The member A.keys evaluates to an array of integers containing the indices of initialized entries in the array in ascending order. Hence, for an array of length n with all entries initialized, A.keys evaluates to {0,1,...,n-1}. A new keys array is produced each time A.keys is evaluated.

The functions A.push and A.append append their arguments onto the end of the array, while A.insert(int i ... T[] x) inserts x into the array at index i. For convenience A.push returns the pushed item. The function A.pop() pops and returns the last element, while A.delete(int i, int j=i) deletes elements with indices in the range [i,j], shifting the position of all higher-indexed elements down. If no arguments are given, A.delete() provides a convenient way of deleting all elements of A. The routine A.initialized(int n) can be used to examine whether the element at index n is initialized. Like all Asymptote functions, push, append, pop, insert, delete, and initialized can be "pulled off" of the array and used on their own. For example,

The [] suffix can also appear after the variable name; this is sometimes convenient for declaring a list of variables and arrays of the same type:

real a,A[];

This declares a to be real and implicitly declares A to be of type real[].

In the following list of built-in array functions, T represents a generic type. Note that the internal functions alias, array, copy, concat, sequence, map, and transpose, which depend on type T[], are defined only after the first declaration of a variable of type T[].

new T[] returns a new empty array of type T[];

new T[] {list}

returns a new array of type T[] initialized with list (a comma delimited list of elements).

new T[n] returns a new array of n elements of type T[]. These n array elements are not initialized unless they are arrays themselves (in which case they are each initialized to empty arrays).

T[] array(int n, T value, int depth=intMax)

returns an array consisting of n copies of value. If value is itself an array, a deep copy of value is made for each entry. If depth is specified, this deep copying only recurses to the specified number of levels.

int[] sequence(int n)

if $n \ge 1$ returns the array $\{0,1,\ldots,n-1\}$ (otherwise returns a null array);

int[] sequence(int n, int m)

if $m \ge n$ returns an array $\{n, n+1, \ldots, m\}$ (otherwise returns a null array);

T[] sequence(T f(int), int n)

if $n \ge 1$ returns the sequence $\{f_i : i=0,1,...n-1\}$ given a function T f(int) and integer int n (otherwise returns a null array);

T[] map(T f(T), T[] a)

returns the array obtained by applying the function f to each element of the array a. This is equivalent to sequence(new T(int i) {return f(a[i]);},a.length).

int[] reverse(int n)

if $n \ge 1$ returns the array $\{n-1, n-2, \ldots, 0\}$ (otherwise returns a null array);

int[] complement(int[] a, int n)

returns the complement of the integer array a in {0,1,2,...,n-1}, so that b[complement(a,b.length)] yields the complement of b[a].

```
real[] uniform(real a, real b, int n)
           if n >= 1 returns a uniform partition of [a,b] into n subintervals (otherwise
           returns a null array);
int find(bool[], int n=1)
           returns the index of the nth true value or -1 if not found. If n is negative,
           search backwards from the end of the array for the -nth value;
int search(T[] a, T key)
           For built-in ordered types T, searches a sorted array a of n elements for k,
           returning the index i if a[i] <= key < a[i+1], -1 if key is less than all elements
           of a, or n-1 if key is greater than or equal to the last element of a.
int search(T[] a, T key, bool less(T i, T j))
           searches an array a sorted in ascending order such that element i precedes
           element j if less(i,j) is true;
T[] copy(T[] a)
           returns a copy of the array a;
T[][] copy(T[][] a)
           returns a copy of the array a;
T[][] [] copy(T[][] [] a)
           returns a copy of the array a;
T[] concat(... T[][] a)
           returns a new array formed by concatenating the arrays given as arguments;
bool alias(T[] a, T[] b)
           returns true if the arrays a and b are identical;
T[] sort(T[] a)
           For built-in ordered types T, returns a copy of a sorted in ascending order;
T[][] sort(T[][] a)
           For built-in ordered types T, returns a copy of a with the rows sorted by the
           first column, breaking ties with successively higher columns. For example:
           string[][] a={{"bob","9"},{"alice","5"},{"pete","7"},
                            {"alice","4"}};
           // Row sort (by column 0, using column 1 to break ties):
           write(sort(a));
           produces
           alice
                     5
           alice
           bob
                     9
           pete
T[] sort(T[] a, bool less(T i, T j))
           returns a copy of a stably sorted in ascending order such that element i precedes
           element j if less(i,j) is true.
T[][] transpose(T[][] a)
```

returns the transpose of a.

```
T[][][] transpose(T[][][] a, int[] perm)
           returns the 3D transpose of a obtained by applying the permutation perm of
           new int[]{0,1,2} to the indices of each entry.
T sum(T[] a)
           For arithmetic types T, returns the sum of a. In the case where T is bool, the
           number of true elements in a is returned.
T min(T[] a)
T min(T[][] a)
T min(T[][][] a)
           For built-in ordered types T, returns the minimum element of a.
T max(T[] a)
T max(T[][] a)
T max(T[][][] a)
           For built-in ordered types T, returns the maximum element of a.
T[] min(T[] a, T[] b)
           For built-in ordered types T, and arrays a and b of the same length, returns an
           array composed of the minimum of the corresponding elements of a and b.
T[] max(T[] a, T[] b)
           For built-in ordered types T, and arrays a and b of the same length, returns an
           array composed of the maximum of the corresponding elements of a and b.
pair[] pairs(real[] x, real[] y);
           For arrays x and y of the same length, returns the pair array sequence (new
           pair(int i) {return (x[i],y[i]);},x.length).
pair[] fft(pair[] a, int sign=1)
           returns the Fast Fourier Transform of a (if the optional FFTW package is in-
           stalled), using the given sign. Here is a simple example:
           int n=4;
           pair[] f=sequence(n);
           write(f);
           pair[] g=fft(f,-1);
           write();
           write(g);
           f=fft(g,1);
           write();
           write(f/n);
real dot(real[] a, real[] b)
           returns the dot product of the vectors a and b.
pair dot(pair[] a, pair[] b)
           returns the complex dot product sum(a*conj(b)) of the vectors a and b.
real[] tridiagonal(real[] a, real[] b, real[] c, real[] f);
           Solve the periodic tridiagonal problem Lx = f and return the solution x, where
           f is an n vector and L is the n \times n matrix
```

```
[ b[0] c[0] a[0] ]
[ a[1] b[1] c[1] ]
[ a[2] b[2] c[2] ]
[ ... ]
[ c[n-1] a[n-1] b[n-1] ]
```

For Dirichlet boundary conditions (denoted here by u[-1] and u[n]), replace f[0] by f[0]-a[0]u[-1] and f[n-1]-c[n-1]u[n]; then set a[0]=c[n-1]=0.

real[] solve(real[][] a, real[] b, bool warn=true)

Solve the linear equation $\mathbf{a}x = \mathbf{b}$ by LU decomposition and return the solution x, where \mathbf{a} is an $n \times n$ matrix and \mathbf{b} is an array of length n. For example:

```
import math;
real[][] a={{1,-2,3,0},{4,-5,6,2},{-7,-8,10,5},{1,50,1,-2}};
real[] b={7,19,33,3};
real[] x=solve(a,b);
write(a); write();
write(b); write();
write(x); write();
write(a*x);
```

If a is a singular matrix and warn is false, return an empty array. If the matrix a is tridiagonal, the routine tridiagonal provides a more efficient algorithm (see [tridiagonal], page 72).

real[][] solve(real[][] a, real[][] b, bool warn=true)

Solve the linear equation $\mathbf{a}x = \mathbf{b}$ and return the solution x, where \mathbf{a} is an $n \times n$ matrix and \mathbf{b} is an $n \times m$ matrix. If \mathbf{a} is a singular matrix and warn is false, return an empty matrix.

real[][] identity(int n);

returns the $n \times n$ identity matrix.

real[][] diagonal(... real[] a)

returns the diagonal matrix with diagonal entries given by a.

real[][] inverse(real[][] a)

returns the inverse of a square matrix a.

real[] quadraticroots(real a, real b, real c);

This numerically robust solver returns the real roots of the quadratic equation $ax^2 + bx + c = 0$, in ascending order. Multiple roots are listed separately.

pair[] quadraticroots(explicit pair a, explicit pair b, explicit pair c);

This numerically robust solver returns the complex roots of the quadratic equation $ax^2 + bx + c = 0$.

```
real[] cubicroots(real a, real b, real c, real d);
```

This numerically robust solver returns the real roots of the cubic equation $ax^3 + bx^2 + cx + d = 0$. Multiple roots are listed separately.

Asymptote includes a full set of vectorized array instructions for arithmetic (including self) and logical operations. These element-by-element instructions are implemented in C++ code for speed. Given

```
real[] a={1,2};
real[] b={3,2};
```

then a == b and a >= 2 both evaluate to the vector {false, true}. To test whether all components of a and b agree, use the boolean function all(a == b). One can also use conditionals like (a >= 2)? a: b, which returns the array {3,2}, or write((a >= 2)? a: null, which returns the array {2}.

All of the standard built-in libm functions of signature real(real) also take a real array as an argument, effectively like an implicit call to map.

As with other built-in types, arrays of the basic data types can be read in by assignment. In this example, the code

```
file fin=input("test.txt");
real[] A=fin;
```

reads real values into A until the end-of-file is reached (or an I/O error occurs).

The virtual members dimension, line, csv, word, and read of a file are useful for reading arrays. For example, if line mode is set with file line(bool b=true), then reading will stop once the end of the line is reached instead

```
file fin=input("test.txt");
real[] A=fin.line();
```

Since string reads by default read up to the end of line anyway, line mode normally has no effect on string array reads. However, there is a white-space delimiter mode for reading strings, file word(bool b=true), which causes string reads to respect white-space delimiters, instead of the default end-of-line delimiter:

```
file fin=input("test.txt").line().word();
real[] A=fin;
```

Another useful mode is comma-separated-value mode, file csv(bool b=true), which causes reads to respect comma delimiters:

```
file fin=csv(input("test.txt"));
real[] A=fin;
```

To restrict the number of values read, use the file dimension(int) function:

```
file fin=input("test.txt");
real[] A=dimension(fin,10);
```

This reads 10 values into A, unless end-of-file (or end-of-line in line mode) occurs first. Attempting to read beyond the end of the file will produce a runtime error message. Specifying a value of 0 for the integer limit is equivalent to the previous example of reading until end-of-file (or end-of-line in line mode) is encountered.

Two- and three-dimensional arrays of the basic data types can be read in like this:

```
file fin=input("test.txt");
real[][] A=fin.dimension(2,3);
real[][][] B=fin.dimension(2,3,4);
```

Again, an integer limit of zero means no restriction.

Sometimes the array dimensions are stored with the data as integer fields at the beginning of an array. Such 1, 2, or 3 dimensional arrays can be read in with the virtual member functions read(1), read(2), or read(3), respectively:

```
file fin=input("test.txt");
real[] A=fin.read(1);
real[][] B=fin.read(2);
real[][][] C=fin.read(3);
```

One, two, and three-dimensional arrays of the basic data types can be output with the functions write(file,T[]), write(file,T[][]), respectively.

6.12.1 Slices

Asymptote allows a section of an array to be addressed as a slice using a Python-like syntax. If A is an array, the expression A[m:n] returns a new array consisting of the elements of A with indices from m up to but not including n. For example,

```
int[] x={0,1,2,3,4,5,6,7,8,9};
int[] y=x[2:6]; // y={2,3,4,5};
int[] z=x[5:10]; // z={5,6,7,8,9};
```

If the left index is omitted, it is taken be 0. If the right index is omitted it is taken to be the length of the array. If both are omitted, the slice then goes from the start of the array to the end, producing a non-cyclic deep copy of the array. For example:

```
int[] x={0,1,2,3,4,5,6,7,8,9};
int[] y=x[:4]; // y={0,1,2,3}
int[] z=x[5:]; // z={5,6,7,8,9}
int[] w=x[:]; // w={0,1,2,3,4,5,6,7,8,9}, distinct from array x.
```

If A is a non-cyclic array, it is illegal to use negative values for either of the indices. If the indices exceed the length of the array, however, they are politely truncated to that length.

For cyclic arrays, the slice A[m:n] still consists of the cells with indices in the set [m,n), but now negative values and values beyond the length of the array are allowed. The indices simply wrap around. For example:

```
int[] x={0,1,2,3,4,5,6,7,8,9};
x.cyclic=true;
int[] y=x[8:15]; // y={8,9,0,1,2,3,4}.
int[] z=x[-5:5]; // z={5,6,7,8,9,0,1,2,3,4}
int[] w=x[-3:17]; // w={7,8,9,0,1,2,3,4,5,6,7,8,9,0,1,2,3,4,5,6}
```

Notice that with cyclic arrays, it is possible to include the same element of the original array multiple times within a slice. Regardless of the original array, arrays produced by slices are always non-cyclic.

If the left and right indices of a slice are the same, the result is an empty array. If the array being sliced is empty, the result is an empty array. Any slice with a left index greater than its right index will yield an error.

Slices can also be assigned to, changing the value of the original array. If the array being assigned to the slice has a different length than the slice itself, elements will be inserted or removed from the array to accommodate it. For instance:

```
string[] toppings={"mayo", "salt", "ham", "lettuce"};
toppings[0:2]=new string[] {"mustard", "pepper"};
    // Now toppings={"mustard", "pepper", "ham", "lettuce"}
```

```
toppings[2:3]=new string[] {"turkey", "bacon" };
    // Now toppings={"mustard", "pepper", "turkey", "bacon", "lettuce"}
toppings[0:3]=new string[] {"tomato"};
    // Now toppings={"tomato", "bacon", "lettuce"}
```

If an array is assigned to a slice of itself, a copy of the original array is assigned to the slice. That is, code such as x[m:n]=x is equivalent to x[m:n]=copy(x). One can use the shorthand x[m:m]=y to insert the contents of the array y into the array x starting at the location just before x[m].

For a cyclic array, a slice is bridging if it addresses cells up to the end of the array and then continues on to address cells at the start of the array. For instance, if A is a cyclic array of length 10, A[8:12], A[-3:1], and A[5:25] are bridging slices whereas A[3:7], A[7:10], A[-3:0] and A[103:107] are not. Bridging slices can only be assigned to if the number of elements in the slice is exactly equal to the number of elements we are assigning to it. Otherwise, there is no clear way to decide which of the new entries should be A[0] and an error is reported. Non-bridging slices may be assigned an array of any length.

For a cyclic array A an expression of the form A[A.length:A.length] is equivalent to the expression A[0:0] and so assigning to this slice will insert values at the start of the array. A.append() can be used to insert values at the end of the array.

It is illegal to assign to a slice of a cyclic array that repeats any of the cells.

6.13 Casts

Asymptote implicitly casts int to real, int to pair, real to pair, pair to path, pair to guide, path to guide, guide to path, real to pen, pair[] to guide[], pair[] to path[], path to path[], and guide to path[], along with various three-dimensional casts defined in three.asy. Implicit casts are automatically attempted on assignment and when trying to match function calls with possible function signatures. Implicit casting can be inhibited by declaring individual arguments explicit in the function signature, say to avoid an ambiguous function call in the following example, which outputs 0:

```
int f(pair a) {return 0;}
int f(explicit real x) {return 1;}

write(f(0));
   Other conversions, say real to int or real to string, require an explicit cast:
int i=(int) 2.5;
string s=(string) 2.5;

real[] a={2.5,-3.5};
int[] b=(int []) a;
write(stdout,b);  // Outputs 2,-3
   Casting to user-defined types is also possible using operator cast:
struct rpair {
   real radius;
   real angle;
}
```

```
pair operator cast(rpair x) {
   return (x.radius*cos(x.angle),x.radius*sin(x.angle));
}

rpair x;
x.radius=1;
x.angle=pi/6;

write(x); // Outputs (0.866025403784439,0.5)
```

One must use care when defining new cast operators. Suppose that in some code one wants all integers to represent multiples of 100. To convert them to reals, one would first want to multiply them by 100. However, the straightforward implementation

```
real operator cast(int x) {return x*100;}
```

is equivalent to an infinite recursion, since the result x*100 needs itself to be cast from an integer to a real. Instead, we want to use the standard conversion of int to real:

```
real convert(int x) {return x*100;}
real operator cast(int x)=convert;
```

Explicit casts are implemented similarly, with operator ecast.

6.14 Import

While Asymptote provides many features by default, some applications require specialized features contained in external Asymptote modules. For instance, the lines

```
access graph;
graph.axes();
```

draw x and y axes on a two-dimensional graph. Here, the command looks up the module under the name graph in a global dictionary of modules and puts it in a new variable named graph. The module is a structure, and we can refer to its fields as we usually would with a structure.

Often, one wants to use module functions without having to specify the module name. The code

```
from graph access axes;
```

adds the axes field of graph into the local name space, so that subsequently, one can just write axes(). If the given name is overloaded, all types and variables of that name are added. To add more than one name, just use a comma-separated list:

```
from graph access axes, xaxis, yaxis;
```

Wild card notation can be used to add all non-private fields and types of a module to the local name space:

```
from graph access *;
```

Similarly, one can add the non-private fields and types of a structure to the local environment with the unravel keyword:

```
struct matrix {
  real a,b,c,d;
```

```
real det(matrix m) {
  unravel m;
  return a*d-b*c;
}
  Alternatively, one can unravel selective fields:
real det(matrix m) {
  from m unravel a,b,c as C,d;
  return a*d-b*C;
}
  The command
import graph;
  is a convenient abbreviation for the commands
access graph;
unravel graph;
```

That is, import graph first loads a module into a structure called graph and then adds its non-private fields and types to the local environment. This way, if a member variable (or function) is overwritten with a local variable (or function of the same signature), the original one can still be accessed by qualifying it with the module name.

Wild card importing will work fine in most cases, but one does not usually know all of the internal types and variables of a module, which can also change as the module writer adds or changes features of the module. As such, it is prudent to add import commands at the start of an Asymptote file, so that imported names won't shadow locally defined functions. Still, imported names may shadow other imported names, depending on the order in which they were imported, and imported functions may cause overloading resolution problems if they have the same name as local functions defined later.

To rename modules or fields when adding them to the local environment, use as:

```
access graph as graph2d;
from graph access xaxis as xline, yaxis as yline;
  The command
import graph as graph2d;
  is a convenient abbreviation for the commands
access graph as graph2d;
unravel graph2d;
```

Except for a few built-in modules, such as settings, all modules are implemented as Asymptote files. When looking up a module that has not yet been loaded, Asymptote searches the standard search paths (see Section 2.5 [Search paths], page 6) for the matching file. The file corresponding to that name is read and the code within it is interpreted as the body of a structure defining the module.

```
If the file name contains nonalphanumeric characters, enclose it with quotation marks: access "/usr/local/share/asymptote/graph.asy" as graph; from "/usr/local/share/asymptote/graph.asy" access axes;
```

```
import "/usr/local/share/asymptote/graph.asy" as graph;
```

It is an error if modules import themselves (or each other in a cycle). The module name to be imported must be known at compile time.

However, you can import an Asymptote module determined by the string s at runtime like this:

```
eval("import "+s,true);
```

To conditionally execute an array of asy files, use

```
void asy(string format, bool overwrite ... string[] s);
```

The file will only be processed, using output format format, if overwrite is true or the output file is missing.

One can evaluate an Asymptote expression (without any return value, however) contained in the string s with:

```
void eval(string s, bool embedded=false);
```

It is not necessary to terminate the string s with a semicolon. If embedded is true, the string will be evaluated at the top level of the current environment. If embedded is false (the default), the string will be evaluated in an independent environment, sharing the same settings module (see [settings], page 159).

One can evaluate arbitrary Asymptote code (which may contain unescaped quotation marks) with the command

```
void eval(code s, bool embedded=false);
```

Here code is a special type used with quote {} to enclose Asymptote code like this:

```
real a=1;
code s=quote {
  write(a);
};
eval(s,true); // Outputs 1
```

To include the contents of a file graph verbatim (as if the contents of the file were inserted at that point), use one of the forms:

```
include graph;
```

```
include "/usr/local/share/asymptote/graph.asy";
```

To list all global functions and variables defined in a module named by the contents of the string s, use the function

```
void list(string s, bool imports=false);
```

Imported global functions and variables are also listed if imports is true.

6.15 Static

Static qualifiers allocate the memory address of a variable in a higher enclosing level.

For a function body, the variable is allocated in the block where the function is defined; so in the code

```
struct s {
  int count() {
    static int c=0;
```

```
++c;
    return c;
  }
}
there is one instance of the variable c for each object s (as opposed to each call of count).
   Similarly, in
int factorial(int n) {
  int helper(int k) {
    static int x=1;
    x *= k;
    return k == 1 ? x : helper(k-1);
  return helper(n);
there is one instance of x for every call to factorial (and not for every call to helper), so
this is a correct, but ugly, implementation of factorial.
   Similarly, a static variable declared within a structure is allocated in the block where
the structure is defined. Thus,
struct A {
  struct B {
    static pair z;
  }
}
creates one object z for each object of type A created.
   In this example,
int pow(int n, int k) {
  struct A {
    static int x=1;
    void helper() {
      x *= n;
    }
  }
  for(int i=0; i < k; ++i) {</pre>
    Aa;
    a.helper();
  }
  return A.x;
}
there is one instance of x for each call to pow, so this is an ugly implementation of expo-
nentiation.
   Loop constructs allocate a new frame in every iteration. This is so that higher-order
```

functions can refer to variables of a specific iteration of a loop:

void f();

for(int i=0; i < 10; ++i) {

```
int x=i;
if(x==5) {
   f=new void () { write(x); }
}
f();
```

Here, every iteration of the loop has its own variable x, so f() will write 5. If a variable in a loop is declared static, it will be allocated where the enclosing function or structure was defined (just as if it were declared static outside of the loop). For instance, in:

```
void f() {
  static int x;
  for(int i=0; i < 10; ++i) {
    static int y;
  }
}</pre>
```

both x and y will be allocated in the same place, which is also where f is also allocated.

Statements may also be declared static, in which case they are run at the place where the enclosing function or structure is defined. Declarations or statements not enclosed in a function or structure definition are already at the top level, so static modifiers are meaningless. A warning is given in such a case.

Since structures can have static fields, it is not always clear for a qualified name whether the qualifier is a variable or a type. For instance, in:

```
struct A {
   static int x;
}
pair A;
int y=A.x;
```

does the A in A.x refer to the structure or to the pair variable. It is the convention in Asymptote that, if there is a non-function variable with the same name as the qualifier, the qualifier refers to that variable, and not to the type. This is regardless of what fields the variable actually possesses.

7 LaTeX usage

Asymptote comes with a convenient LaTeX style file asymptote.sty that makes LaTeX Asymptote-aware. Entering Asymptote code directly into the LaTeX source file, at the point where it is needed, keeps figures organized and avoids the need to invent new file names for each figure. Simply add the line \usepackage{asymptote} at the beginning of your file and enclose your Asymptote code within a \begin{asy}...\end{asy} environment. As with the LaTeX comment environment, the \end{asy} command must appear on a line by itself, with no trailing commands/comments.

The sample LaTeX file below, named latexusage.tex, can be run as follows:

```
latex latexusage
asy latexusage-*.asy
latex latexusage
or
pdflatex latexusage
asy latexusage-*.asy
pdflatex latexusage
```

To switch between using inline Asymptote code with latex and pdflatex you may first need to remove the files latexusage-*.tex.

An even better method for processing a LaTeX file with embedded Asymptote code is to use the latexmk utility from

```
http://www.ctan.org/tex-archive/support/latexmk/
after putting the rules
sub asy {return system("asy '$_[0]'");}
add_cus_dep("asy","eps",0,"asy");
add_cus_dep("asy","pdf",0,"asy");
add_cus_dep("asy","tex",0,"asy");
in a file latexmkrc in the same directory. The command
```

latexmk -pdf latexusage

will then call Asymptote automatically, recompiling only the figures that have changed. Since each figure is compiled in a separate system process, this method also tends to use less memory. External Asymptote code in filename.asy should be included with

```
\asyinclude[<options>]{<filename.asy>}
```

so that latexmk will recognize when the code is changed. Note that latemk requires perl, available from http://www.perl.org/.

One can specify width, height, keepAspect, viewportwidth, viewportheight, attach, and inline. keyval-style options to the asy and asyinclude environments. Three-dimensional PRC files may either be embedded within the page (the default) or attached as annotated (but printable) attachments, using the attach option and the attachfile2 (or older attachfile) LaTeX package. The default value of viewportwidth is \the\linewidth for inline 3D figures and 0 for attachments. The inline option generates inline LaTeX code instead of EPS or PDF files. This makes 2D LaTeX symbols visible to the \begin{asy}...\end{asy} environment. In this mode, Asymptote correctly

aligns 2D LaTeX symbols defined outside of \begin{asy}...\end{asy}, but treats their size as zero; an optional second string can be given to Label to provide an estimate of the unknown label size.

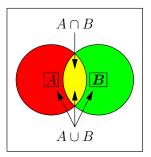
Note that if the latex TEX engine is used with the inline option, labels might not show up in DVI viewers that cannot handle raw PostScript code. One can use dvips/dvipdf to produce PostScript/PDF output (we recommend using the modified version of dvipdf in the Asymptote patches directory, which accepts the dvips -z hyperdvi option).

```
Here now is latexusage.tex:
\documentclass[12pt]{article}
\% Use this form to include EPS (latex) or PDF (pdflatex) files:
\usepackage{asymptote}
% Use this form with latex or pdflatex to include inline LaTeX code by default:
%\usepackage[inline]{asymptote}
\% Use this form with latex or pdflatex to create PDF attachments by default:
%\usepackage[attach]{asymptote}
% Enable this line to support the attach option:
%\usepackage[dvips]{attachfile2}
\begin{document}
% Optional subdirectory for asy files (no spaces):
\def\asydir{}
\begin{asydef}
// Global Asymptote definitions can be put here.
import three;
usepackage("bm");
texpreamble("\def\V#1{\bm{#1}}");
// One can globally override the default toolbar settings here:
// settings.toolbar=true;
\end{asydef}
Here is a venn diagram produced with Asymptote, drawn to width 4cm:
\left( A\{A\} \right)
\left( B_{V\{B\}} \right)
%\begin{figure}
\begin{center}
\begin{asy}
size(4cm,0);
pen colour1=red;
```

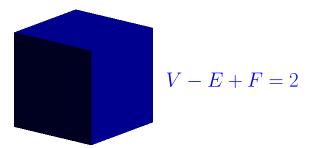
```
pen colour2=green;
pair z0=(0,0);
pair z1=(-1,0);
pair z2=(1,0);
real r=1.5;
path c1=circle(z1,r);
path c2=circle(z2,r);
fill(c1,colour1);
fill(c2,colour2);
picture intersection=new picture;
fill(intersection,c1,colour1+colour2);
clip(intersection,c2);
add(intersection);
draw(c1);
draw(c2);
//draw("$\A$",box,z1);
                                   // Requires [inline] package option.
//draw(Label("$\B$","$B$"),box,z2); // Requires [inline] package option.
draw("$A$",box,z1);
draw("$\V{B}$",box,z2);
pair z=(0,-2);
real m=3;
margin BigMargin=Margin(0,m*dot(unit(z1-z),unit(z0-z)));
draw(Label("$A\cap B$",0),conj(z)--z0,Arrow,BigMargin);
draw(Label("$A\cup B$",0),z--z0,Arrow,BigMargin);
draw(z--z1,Arrow,Margin(0,m));
draw(z--z2,Arrow,Margin(0,m));
shipout(bbox(0.25cm));
\end{asy}
%\caption{Venn diagram}\label{venn}
\end{center}
%\end{figure}
Each graph is drawn in its own environment. One can specify the width
and height to \LaTeX\ explicitly. This 3D example can be viewed
interactively either with Adobe Reader or Asymptote's fast OpenGL-based
renderer. To support {\tt latexmk}, 3D figures should specify
\verb+inline=true+. It is sometimes desirable to embed 3D files as annotated
attachments; this requires the \verb+attach=true+ option as well as the
\verb+attachfile2+ \LaTeX\ package.
```

```
\begin{center}
\begin{asy} [height=4cm,inline=true,attach=false]
currentprojection=orthographic(5,4,2);
draw(unitcube,blue);
label("$V-E+F=2$",(0,1,0.5),3Y,blue+fontsize(17pt));
\end{asy}
\end{center}
One can also scale the figure to the full line width:
\begin{center}
\begin{asy}[width=\the\linewidth,inline=true]
pair z0=(0,0);
pair z1=(2,0);
pair z2=(5,0);
pair zf=z1+0.75*(z2-z1);
draw(z1--z2);
dot(z1,red+0.15cm);
dot(z2,darkgreen+0.3cm);
label("$m$",z1,1.2N,red);
label("$M$",z2,1.5N,darkgreen);
label("$\hat{\}$",zf,0.2*S,fontsize(24pt)+blue);
pair s=-0.2*I;
draw("$x$",z0+s--z1+s,N,red,Arrows,Bars,PenMargins);
s=-0.5*I;
draw("$\bar{x}$",z0+s--zf+s,blue,Arrows,Bars,PenMargins);
s=-0.95*I;
draw("$X$",z0+s--z2+s,darkgreen,Arrows,Bars,PenMargins);
\end{asy}
\end{center}
\end{document}
```

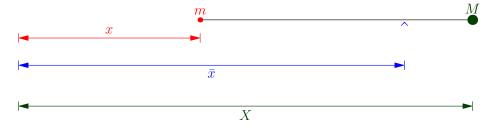
Here is a venn diagram produced with Asymptote, drawn to width 4cm:



Each graph is drawn in its own environment. One can specify the width and height to LATEX explicitly. This 3D example can be viewed interactively either with Adobe Reader or Asymptote's fast OpenGL-based renderer. To support latexmk, 3D figures should specify inline=true. It is sometimes desirable to embed 3D files as annotated attachments; this requires the attach=true option as well as the attachfile2 LATEX package.



One can also scale the figure to the full line width:



8 Base modules

Asymptote currently ships with the following base modules:

8.1 plain

This is the default Asymptote base file, which defines key parts of the drawing language (such as the picture structure).

By default, an implicit private import plain; occurs before translating a file and before the first command given in interactive mode. This also applies when translating files for module definitions (except when translating plain, of course). This means that the types and functions defined in plain are accessible in almost all Asymptote code. Use the -noautoplain command-line option to disable this feature.

8.2 simplex

This package solves the two-variable linear programming problem using the simplex method. It is used by the module plain for automatic sizing of pictures.

8.3 math

This package extends Asymptote's mathematical capabilities with useful functions such as

void drawline(picture pic=currentpicture, pair P, pair Q, pen p=currentpen); draw the visible portion of the (infinite) line going through P and Q, without altering the size of picture pic, using pen p.

real intersect(triple P, triple Q, triple n, triple Z);
returns the intersection time of the extension of the line segment PQ with the
plane perpendicular to n and passing through Z.

triple intersectionpoint(triple n0, triple P0, triple n1, triple P1);

Return any point on the intersection of the two planes with normals n0 and n1 passing through points P0 and P1, respectively. If the planes are parallel, return (infinity, infinity, infinity).

pair[] quarticroots(real a, real b, real c, real d, real e); returns the four complex roots of the quartic equation $ax^4+bx^3+cx^2+dx+e=0$.

pair[][] fft(pair[][] a, int sign=1)
 returns the two-dimensional Fourier transform of a using the given sign.

real time(path g, real x, int n=0)
returns the nth intersection time of path g with the vertical line through x.

real time(path g, explicit pair z, int n=0)
returns the nth intersection time of path g with the horizontal line through
(0,z.y).

real value(path g, real x, int n=0)
 returns the nth y value of g at x.

```
real value(path g, real x, int n=0)
           returns the nth x value of g at y=z.y.
real slope(path g, real x, int n=0)
           returns the nth slope of g at x.
real slope(path g, explicit pair z, int n=0)
```

returns the nth slope of g at y=z.y.

int[][] segment(bool[] b) returns the indices of consecutive true-element segments of bool[] b.

real[] partialsum(real[] a)

returns the partial sums of a real array a.

real[] partialsum(real[] a, real[] dx)

returns the partial dx-weighted sums of a real array a.

bool increasing(real[] a, bool strict=false) returns, if strict=false, whether i > j implies a[i] >= a[j], or if strict=true, whether i > j implies implies a[i] > a[j].

int unique(real[] a, real x)

if the sorted array a does not contain x, insert it sequentially, returning the index of x in the resulting array.

bool lexorder(pair a, pair b)

returns the strict lexicographical partial order of a and b.

bool lexorder(triple a, triple b)

returns the strict lexicographical partial order of a and b.

8.4 interpolate

This module implements Lagrange, Hermite, and standard cubic spline interpolation in Asymptote, as illustrated in the example interpolate1.asy.

8.5 geometry

This module, written by Philippe Ivaldi, provides an extensive set of geomsymbols including perpendicular a triangle and etrvLink to the documentation for the geometry module are posted here: http://asymptote.sourceforge.net/links.html, including an extensive set of examples, http://www.piprime.fr/files/asymptote/geometry/, and an index:

http://www.piprime.fr/files/asymptote/geometry/modules/geometry.asy.index.type.html

8.6 trembling

This module, written by Philippe Ivaldi and illustrated in the example floatingdisk.asy, allows one to draw wavy lines, as if drawn by hand. Further examples are posted at http://www.piprime.fr/files/asymptote/trembling/

8.7 stats

This package implements a Gaussian random number generator and a collection of statistics routines, including histogram and leastsquares.

8.8 patterns

This package implements Postscript tiling patterns and includes several convenient pattern generation routines.

8.9 markers

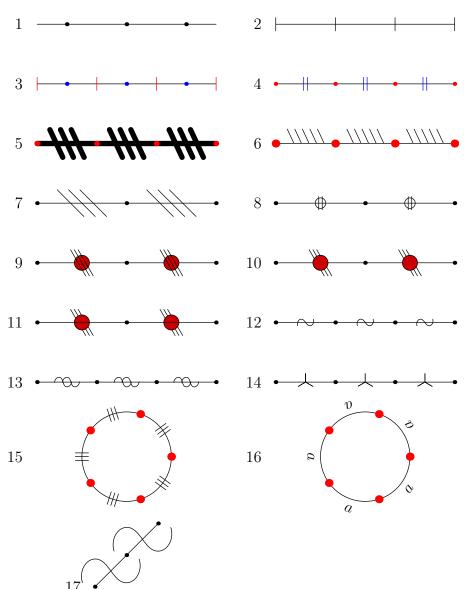
This package implements specialized routines for marking paths and angles. The principal mark routine provided by this package is

```
markroutine markinterval(int n=1, frame f, bool rotated=false);
```

which centers n copies of frame f within uniformly space intervals in arclength along the path, optionally rotated by the angle of the local tangent.

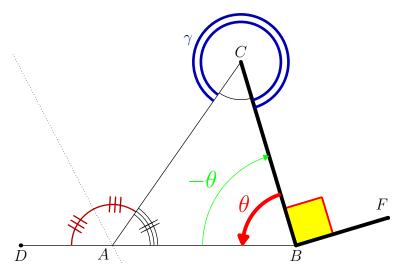
The marker (see [marker], page 105) routine can be used to construct new markers from these predefined frames:

For convenience, this module also constructs the markers StickIntervalMarker, CrossIntervalMarker, CircleBarIntervalMarker, and TildeIntervalMarker from the above frames. The example markers1.asy illustrates the use of these markers:



This package also provides a routine for marking an angle AOB:

as illustrated in the example markers2.asy.



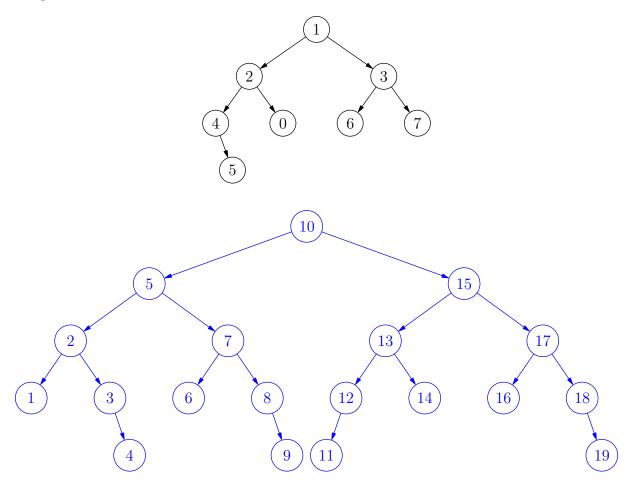
8.10 tree

This package implements an example of a dynamic binary search tree.

8.11 binarytree

This module can be used to draw an arbitrary binary tree and includes an input routine for the special case of a binary search tree, as illustrated in the example binarytreetest.asy:

```
import binarytree;
picture pic,pic2;
binarytree bt=binarytree(1,2,4,nil,5,nil,nil,0,nil,nil,3,6,nil,nil,7);
draw(pic,bt);
binarytree st=searchtree(10,5,2,1,3,4,7,6,8,9,15,13,12,11,14,17,16,18,19);
draw(pic2,st,blue);
add(pic.fit(),(0,0),10N);
add(pic2.fit(),(0,0),10S);
```



8.12 drawtree

This is a simple tree drawing module used by the example treetest.asy.

8.13 syzygy

This module automates the drawing of braids, relations, and syzygies, along with the corresponding equations, as illustrated in the example knots.asy.

8.14 feynman

This package, contributed by Martin Wiebusch, is useful for drawing Feynman diagrams, as illustrated by the examples eetomumu.asy and fermi.asy.

8.15 roundedpath

This package, contributed by Stefan Knorr, is useful for rounding the sharp corners of paths, as illustrated in the example file roundpath.asy.

8.16 animation

This module allows one to generate animations, as illustrated by the files wheel.asy, wavelet.asy, and cube.asy in the animations subdirectory of the examples directory.

These animations use the ImageMagick convert program to merge multiple images into a GIF or MPEG movie.

The related animate module, derived from the animation module, generates higher-quality portable clickable PDF movies, with optional controls. This requires installing the package

http://www.ctan.org/tex-archive/macros/latex/contrib/animate/animate.sty (version 2007/11/30 or later) in a new directory animate in the local LaTeX directory (for example, in /usr/local/share/texmf/tex/latex/animate). On UNIX systems, one must then execute the command texhash.

The example pdfmovie.asy in the animations directory, along with the slide presentations slidemovies.asy and intro.asy, illustrate the use of embedded PDF movies. The examples inlinemovie.tex and inlinemovie3.tex show how to generate and embed PDF movies directly within a LaTeX file (see Chapter 7 [LaTeX usage], page 82). The member function

of the animate structure accepts any of the animate.sty options, as described here:

http://www.ctan.org/tex-archive/macros/latex/contrib/animate/doc/animate.pdf

8.17 embed

This module provides an interface to the LaTeX package (included with MikTeX)

```
http://www.ctan.org/tex-archive/macros/latex/contrib/movie15
```

for embedding movies, sounds, and 3D objects into a PDF document. However, XeLaTeX users need to rename the modified version movie15_dvipdfmx.sty from

http://asymptote.svn.sourceforge.net/viewvc/asymptote/trunk/asymptote/patches/to movie15.sty and place it in their LaTeX path.

The latest version of the movie15 package requires both pdflatex version 1.20 or later and the file

http://www.ctan.org/tex-archive/macros/latex/contrib/oberdiek/ifdraft.dtx which can be installed by placing it in a directory ifdraft in the local LaTeX directory (e.g. /usr/local/share/texmf/tex/latex/ifdraft) and executing in that directory the commands:

```
tex ifdraft.dtx texhash
```

A more portable method for embedding movie files, which should work on any platform and does not require the movie15 or ifdraft packages, is provided by using the external module instead of embed. An example of these interfaces is provided in the file embeddedmovie.asy and externalmovie.asy in the animations subdirectory of the examples directory. For a higher quality movie generated directly by Asymptote, use the animate module along with the animate.sty package to embed a portable PDF animation (see [animate], page 93).

An example of embedding U3D code is provided in the file embeddedu3d.asy.

8.18 slide

This package provides a simple yet high-quality facility for making presentation slides, including portable embedded PDF animations (see the file slidemovies.asy). A simple example is provided in the file slidedemo.asy.

8.19 MetaPost

This package provides some useful routines to help MetaPost users migrate old MetaPost code to Asymptote. Further contributions here are welcome.

Unlike MetaPost, Asymptote does not implicitly solve linear equations and therefore does not have the notion of a whatever unknown. The routine extension (see [extension], page 34) provides a useful replacement for a common use of whatever: finding the intersection point of the lines through P, Q and p, q. For less common occurrences of whatever, one can use the built-in explicit linear equation solver solve instead.

8.20 unicode

Import this package at the beginning of the file to instruct LaTeX to accept unicode (UTF-8) standardized international characters. To use Cyrillic fonts, you will need to change the font encoding:

```
import unicode;
texpreamble("\usepackage{mathtext}\usepackage[russian]{babel}");
defaultpen(font("T2A","cmr","m","n"));
```

Support for Chinese, Japanese, and Korean fonts is provided by the CJK package:

```
http://www.ctan.org/tex-archive/languages/chinese/CJK/
```

The following commands enable the CJK song family (within a label, you can also temporarily switch to another family, say kai, by prepending "\CJKfamily{kai}" to the label string):

```
texpreamble("\usepackage{CJK}
\AtBeginDocument{\begin{CJK*}{GBK}{song}}
\AtEndDocument{\clearpage\end{CJK*}}");
```

8.21 latin1

If you don't have LaTeX support for unicode installed, you can enable support for Western European languages (ISO 8859-1) by importing the module latin1. This module can be used as a template for providing support for other ISO 8859 alphabets.

8.22 babel

This module implements the LaTeX babel package in Asymptote. For example:

```
import babel;
babel("german");
```

8.23 labelpath

This module uses the PSTricks pstextpath macro to fit labels along a path (properly kerned, as illustrated in the example file curvedlabel.asy), using the command

Here justify is one of LeftJustified, Centered, or RightJustified. The x component of a shift transform applied to the Label is interpreted as a shift along the curve, whereas the y component is interpreted as a shift away from the curve. All other Label transforms are ignored. This package requires the latex tex engine and inherits the limitations of the PSTricks \pstextpath macro.

8.24 labelpath3

This module, contributed by Jens Schwaiger, implements a 3D version of labelpath that does not require the PSTricks package. An example is provided in curvedlabel3.asy.

8.25 annotate

This module supports PDF annotations for viewing with Adobe Reader, via the function void annotate(picture pic=currentpicture, string title, string text, pair position);

Annotations are illustrated in the example file annotation.asy. Currently, annotations are only implemented for the latex (default) and tex TeX engines.

8.26 CAD

This package, contributed by Mark Henning, provides basic pen definitions and measurement functions for simple 2D CAD drawings according to DIN 15. It is documented separately, in the file CAD.pdf.

8.27 graph

This package implements two-dimensional linear and logarithmic graphs, including automatic scale and tick selection (with the ability to override manually). A graph is a guide (that can be drawn with the draw command, with an optional legend) constructed with one of the following routines:

Returns a graph using the scaling information for picture pic (see [automatic scaling], page 111) of the function f on the interval [T(a),T(b)], sampling at n points evenly spaced in [a,b], optionally restricted by the bool3 function cond on [a,b]. If cond is:

- true, the point is added to the existing guide;
- default, the point is added to a new guide;
- false, the point is omitted and a new guide is begun.

The points are connected using the interpolation specified by join:

- operator -- (linear interpolation; the abbreviation Straight is also accepted);
- operator .. (piecewise Bezier cubic spline interpolation; the abbreviation Spline is also accepted);
- Hermite (standard cubic spline interpolation using boundary condition notaknot, natural, periodic, clamped(real slopea, real slopeb)), or monotonic. The abbreviation Hermite is equivalent to Hermite(notaknot) for nonperiodic data and Hermite(periodic) for periodic data).

Returns a graph using the scaling information for picture pic of the parametrized function (x(t),y(t)) for t in the interval [T(a),T(b)], sampling at n points evenly spaced in [a,b], optionally restricted by the bool3 function cond on [a,b], using the given interpolation type.

Returns a graph using the scaling information for picture pic of the parametrized function z(t) for t in the interval [T(a),T(b)], sampling at n points evenly spaced in [a,b], optionally restricted by the bool3 function cond on [a,b], using the given interpolation type.

Returns a graph using the scaling information for picture pic of the elements of the array z, optionally restricted to those indices for which the elements of the boolean array cond are true, using the given interpolation type.

```
guide graph(picture pic=currentpicture, real[] x, real[] y,
```

Returns a graph using the scaling information for picture pic of the elements of the arrays (x,y), optionally restricted to those indices for which the elements of the boolean array cond are true, using the given interpolation type.

Returns a polar-coordinate graph using the scaling information for picture pic of the function f on the interval [a,b], sampling at n evenly spaced points, with the given interpolation type.

Returns a polar-coordinate graph using the scaling information for picture pic of the elements of the arrays (r,theta), using the given interpolation type.

An axis can be drawn on a picture with one of the following commands:

Draw an x axis on picture pic from x=xmin to x=xmax using pen p, optionally labelling it with Label L. The relative label location along the axis (a real number from [0,1]) defaults to 1 (see [Label], page 18), so that the label is drawn at the end of the axis. An infinite value of xmin or xmax specifies that the corresponding axis limit will be automatically determined from the picture limits. The optional arrow argument takes the same values as in the draw command (see [arrows], page 14). The axis is drawn before any existing objects in pic unless above=true. The axis placement is determined by one of the following axis types:

YZero(bool extend=true)

Request an x axis at y=0 (or y=1 on a logarithmic axis) extending to the full dimensions of the picture, unless extend=false.

YEquals(real Y, bool extend=true)

Request an x axis at y=Y extending to the full dimensions of the picture, unless extend=false.

Bottom(bool extend=false)

Request a bottom axis.

Top(bool extend=false)

Request a top axis.

```
BottomTop(bool extend=false)
```

Request a bottom and top axis.

Custom axis types can be created by following the examples in graph.asy. One can easily override the default values for the standard axis types:

```
import graph;
```

```
YZero=new axis(bool extend=true) {
  return new void(picture pic, axisT axis) {
    real y=pic.scale.x.scale.logarithmic ? 1 : 0;
    axis.value=I*pic.scale.y.T(y);
    axis.position=1;
    axis.side=right;
    axis.align=2.5E;
    axis.value2=Infinity;
    axis.extend=extend;
  };
};
YZero=YZero();
```

The default tick option is NoTicks. The options LeftTicks, RightTicks, or Ticks can be used to draw ticks on the left, right, or both sides of the path, relative to the direction in which the path is drawn. These tick routines accept a number of optional arguments:

If any of these parameters are omitted, reasonable defaults will be chosen:

Label format

override the default tick label format (defaultformat, initially "\$\%.4g\$"), rotation, pen, and alignment (for example, LeftSide, Center, or RightSide) relative to the axis. To enable LaTeX math mode fonts, the format string should begin and end with \$ see [format], page 28. If the format string is trailingzero, trailing zeros will be added to the tick labels; if the format string is "\%", the tick label will be suppressed;

ticklabel

is a function string(real x) returning the label (by default, format(format.s,x)) for each major tick value x;

bool beginlabel

include the first label;

bool endlabel

include the last label;

when automatic scaling is enabled (the default; see [automatic scaling], page 111), divide a linear axis evenly into this many intervals, separated by major ticks; for a logarithmic axis, this is the number of decades between labelled ticks;

int n divide each interval into this many subintervals, separated by minor ticks;

real Step the tick value spacing between major ticks (if N=0);

real step the tick value spacing between minor ticks (if n=0);

bool begin

include the first major tick;

bool end include the last major tick;

tickmodifier modify;

an optional function that takes and returns a tickvalue structure having real[] members major and minor consisting of the tick values (to allow modification of the automatically generated tick values);

real Size the size of the major ticks (in PostScript coordinates);

real size the size of the minor ticks (in PostScript coordinates);

bool extend;

extend the ticks between two axes (useful for drawing a grid on the graph);

pen pTick an optional pen used to draw the major ticks;

pen ptick an optional pen used to draw the minor ticks.

For convenience, the predefined tickmodifiers OmitTick(...real[] x), OmitTickInterval(real a, real b), and OmitTickIntervals(real[] a, real[] b) can be used to remove specific auto-generated ticks and their labels. The OmitFormat(string s=defaultformat ... real[] x) ticklabel can be used to remove specific tick labels but not the corresponding ticks. The tickmodifier NoZero is an abbreviation for OmitTick(O) and the ticklabel NoZeroFormat is an abbrevation for OmitFormat(O).

It is also possible to specify custom tick locations with LeftTicks, RightTicks, and Ticks by passing explicit real arrays Ticks and (optionally) ticks containing the locations of the major and minor ticks, respectively:

```
ticks LeftTicks(Label format="", ticklabel ticklabel=null, bool beginlabel=true, bool endlabel=true, real[] Ticks, real[] ticks=new real[], real Size=0, real size=0, bool extend=false, pen pTick=nullpen, pen ptick=nullpen)
```

Draw a y axis on picture pic from $y=y\min$ to $y=y\max$ using pen p, optionally labelling it with a Label L that is autorotated unless autorotate=false. The relative location of the label (a real number from [0,1]) defaults to 1 (see [Label], page 18). An infinite value of ymin or ymax specifies that the corresponding axis limit will be automatically determined from the picture limits. The optional arrow argument takes the same values as in the draw command (see [arrows], page 14). The axis is drawn before any existing objects in pic unless above=true. The tick type is specified by ticks and the axis placement is determined by one of the following axis types:

XZero(bool extend=true)

Request a y axis at x=0 (or x=1 on a logarithmic axis) extending to the full dimensions of the picture, unless extend=false.

XEquals(real X, bool extend=true)

Request a y axis at x=X extending to the full dimensions of the picture, unless extend=false.

Left(bool extend=false)

Request a left axis.

Right(bool extend=false)

Request a right axis.

LeftRight(bool extend=false)

Request a left and right axis.

• For convenience, the functions

and

can be respectively used to call yaxis and xaxis with the appropriate axis types XEquals(x,extend) and YEquals(y,extend). This is the recommended way of drawing vertical or horizontal lines and axes at arbitrary locations.

This convenience routine draws both x and y axes on picture pic from min to max, with optional labels xlabel and ylabel and any arrows specified by arrow. The axes are drawn on top of existing objects in pic only if above=true.

```
void axis(picture pic=currentpicture, Label L="", path g,
```

```
pen p=currentpen, ticks ticks, ticklocate locate,
arrowbar arrow=None, int[] divisor=new int[],
bool above=false, bool opposite=false);
```

This routine can be used to draw on picture pic a general axis based on an arbitrary path g, using pen p. One can optionally label the axis with Label L and add an arrow arrow. The tick type is given by ticks. The optional integer array divisor specifies what tick divisors to try in the attempt to produce uncrowded tick labels. A true value for the flag opposite identifies an unlabelled secondary axis (typically drawn opposite a primary axis). The axis is drawn before any existing objects in pic unless above=true. The tick locator ticklocate is constructed by the routine

where a and b specify the respective tick values at point(g,0) and point(g,length(g)), S specifies the autoscaling transformation, the function real time(real v) returns the time corresponding to the value v, and pair dir(real t) returns the absolute tick direction as a function of t (zero means draw the tick perpendicular to the axis).

• These routines are useful for manually putting ticks and labels on axes (if the variable Label is given as the Label argument, the format argument will be used to format a string based on the tick location):

```
void xtick(picture pic=currentpicture, Label L="", explicit pair z,
           pair dir=N, string format="",
           real size=Ticksize, pen p=currentpen);
void xtick(picture pic=currentpicture, Label L="", real x,
           pair dir=N, string format="",
           real size=Ticksize, pen p=currentpen);
void ytick(picture pic=currentpicture, Label L="", explicit pair z,
           pair dir=E, string format="",
           real size=Ticksize, pen p=currentpen);
void ytick(picture pic=currentpicture, Label L="", real y,
           pair dir=E, string format="",
           real size=Ticksize, pen p=currentpen);
void tick(picture pic=currentpicture, pair z,
          pair dir, real size=Ticksize, pen p=currentpen);
void labelx(picture pic=currentpicture, Label L="", explicit pair z,
            align align=S, string format="", pen p=nullpen);
void labelx(picture pic=currentpicture, Label L="", real x,
            align align=S, string format="", pen p=nullpen);
void labelx(picture pic=currentpicture, Label L,
            string format="", explicit pen p=currentpen);
void labely(picture pic=currentpicture, Label L="", explicit pair z,
            align align=W, string format="", pen p=nullpen);
void labely(picture pic=currentpicture, Label L="", real y,
            align align=W, string format="", pen p=nullpen);
void labely(picture pic=currentpicture, Label L,
```

```
string format="", explicit pen p=nullpen);
```

Here are some simple examples of two-dimensional graphs:

1. This example draws a textbook-style graph of $y = \exp(x)$, with the y axis starting at y = 0:

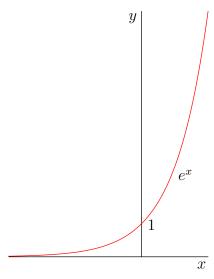
```
import graph;
size(150,0);

real f(real x) {return exp(x);}
pair F(real x) {return (x,f(x));}

xaxis("$x$");
yaxis("$y$",0);

draw(graph(f,-4,2,operator ..),red);

labely(1,E);
label("$e^x$",F(1),SE);
```



2. The next example draws a scientific-style graph with a legend. The position of the legend can be adjusted either explicitly or by using the graphical user interface xasy (see Chapter 11 [GUI], page 163). If an UnFill(real xmargin=0, real ymargin=xmargin) or Fill(pen) option is specified to add, the legend will obscure any underlying objects. Here we illustrate how to clip the portion of the picture covered by a label:

```
size(400,200,IgnoreAspect);
real Sin(real t) {return sin(2pi*t);}
```

real Cos(real t) {return cos(2pi*t);}

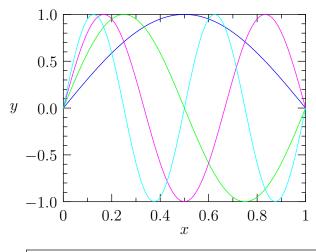
import graph;

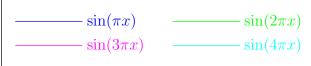
```
draw(graph(Sin,0,1),red,"$\sin(2\pi x)$");
draw(graph(Cos,0,1),blue,"$\cos(2\pi x)$");
xaxis("$x$",BottomTop,LeftTicks);
yaxis("$y$",LeftRight,RightTicks(trailingzero));
label("LABEL",point(0),UnFill(1mm));
add(legend(),point(E),20E,UnFill);
     1.0
     0.5
                                                                 \sin(2\pi x)
                         LABEL
 y
     0.0
                                                                 \cos(2\pi x)
   -0.5
   -1.0
               0.2
                       0.4
                                0.6
                                        0.8
                            x
```

To specify a fixed size for the graph proper, use attach: import graph;

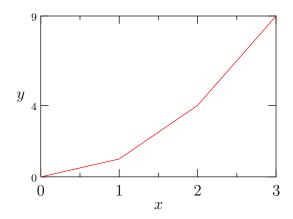
```
size(250,200,IgnoreAspect);
real Sin(real t) {return sin(2pi*t);}
real Cos(real t) {return cos(2pi*t);}
draw(graph(Sin,0,1),red,"$\sin(2\pi x)$");
draw(graph(Cos,0,1),blue,"$\cos(2\pi x)$");

xaxis("$x$",BottomTop,LeftTicks);
yaxis("$y$",LeftRight,RightTicks(trailingzero));
label("LABEL",point(0),UnFill(1mm));
attach(legend(),truepoint(E),20E,UnFill);
A legend can have multiple entries per line:
import graph;
size(8cm,6cm,IgnoreAspect);
```





3. This example draws a graph of one array versus another (both of the same size) using custom tick locations and a smaller font size for the tick labels on the y axis.



4. This example shows how to graph columns of data read from a file.

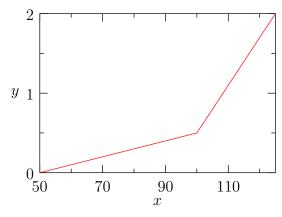
import graph;

```
size(200,150,IgnoreAspect);
file in=input("filegraph.dat").line();
real[][] a=in.dimension(0,0);
a=transpose(a);

real[] x=a[0];
real[] y=a[1];

draw(graph(x,y),red);

xaxis("$x$",BottomTop,LeftTicks);
yaxis("$y$",LeftRight,RightTicks);
```



5. The next example draws two graphs of an array of coordinate pairs, using frame alignment and data markers. In the left-hand graph, the markers, constructed with

using the path unitcircle (see [filltype], page 48), are drawn below each node. Any frame can be converted to a marker, using

In the right-hand graph, the unit n-sided regular polygon polygon(int n) and the unit n-point cyclic cross cross(int n, bool round=true, real r=0) (where r is an optional "inner" radius) are used to build a custom marker frame. Here markuniform(bool centered=false, int n, bool rotated=false) adds this frame at n uniformly spaced points along the arclength of the path, optionally rotated by the angle of the local tangent to the path (if centered is true, the frames will be centered within n evenly spaced arclength intervals). Alternatively, one can use markroutine marknodes to request that the marks be placed at each Bezier node of the path, or markroutine markuniform(pair z(real t), real a, real b, int n) to place marks at points z(t) for n evenly spaced values of t in [a,b].

These markers are predefined:

```
marker[] Mark={
  marker(scale(circlescale)*unitcircle),
  marker(polygon(3)),marker(polygon(4)),
  marker(polygon(5)),marker(invert*polygon(3)),
  marker(cross(4)),marker(cross(6))
};
marker[] MarkFill={
  marker(scale(circlescale)*unitcircle,Fill),marker(polygon(3),Fill),
  marker(polygon(4),Fill),marker(polygon(5),Fill),
  marker(invert*polygon(3),Fill)
};
The example also illustrates the errorbar routines:
void errorbars(picture pic=currentpicture, pair[] z, pair[] dp,
               pair[] dm={}, bool[] cond={}, pen p=currentpen,
               real size=0);
void errorbars(picture pic=currentpicture, real[] x, real[] y,
               real[] dpx, real[] dpy, real[] dmx={}, real[] dmy={},
               bool[] cond={}, pen p=currentpen, real size=0);
```

Here, the positive and negative extents of the error are given by the absolute values of the elements of the pair array dp and the optional pair array dm. If dm is not specified, the positive and negative extents of the error are assumed to be equal.

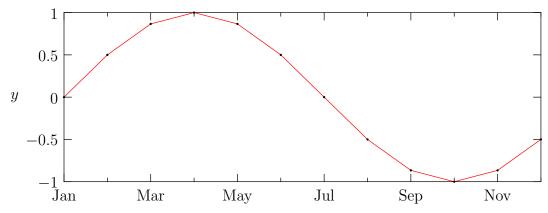
```
import graph;
picture pic;
real xsize=200, ysize=140;
size(pic,xsize,ysize,IgnoreAspect);
pair[] f={(5,5),(50,20),(90,90)};
pair[] df={(0,0),(5,7),(0,5)};
```

```
errorbars(pic,f,df,red);
draw(pic,graph(pic,f),"legend",
     marker(scale(0.8mm)*unitcircle,red,FillDraw(blue),above=false));
scale(pic,true);
xaxis(pic,"$x$",BottomTop,LeftTicks);
yaxis(pic,"$y$",LeftRight,RightTicks);
add(pic,legend(pic),point(pic,NW),20SE,UnFill);
picture pic2;
size(pic2,xsize,ysize,IgnoreAspect);
frame mark;
filldraw(mark,scale(0.8mm)*polygon(6),green,green);
draw(mark,scale(0.8mm)*cross(6),blue);
draw(pic2,graph(pic2,f),marker(mark,markuniform(5)));
scale(pic2,true);
xaxis(pic2,"$x$",BottomTop,LeftTicks);
yaxis(pic2,"$y$",LeftRight,RightTicks);
yequals(pic2,55.0,red+Dotted);
xequals(pic2,70.0,red+Dotted);
// Fit pic to W of origin:
add(pic.fit(),(0,0),W);
// Fit pic2 to E of (5mm, 0):
add(pic2.fit(),(5mm,0),E);
  100
                                          100
   80
                                           80
                    legend
   60
                                           60
y
                                        y
   40
                                           40
   20
                                           20
    0
                                            0
                                                   20
                                                         40
                                                               60
                                                                    80
     0
           20
                 40
                      60
                            80
                                 100
                                              0
                                                                          100
                                                            x
                    \boldsymbol{x}
```

6. A custom mark routine can be also be specified:

```
import graph;
size(200,100,IgnoreAspect);
markroutine marks() {
  return new void(picture pic=currentpicture, frame f, path g) {
    path p=scale(1mm)*unitcircle;
    for(int i=0; i <= length(g); ++i) {</pre>
      pair z=point(g,i);
      frame f;
      if(i % 4 == 0) {
        fill(f,p);
        add(pic,f,z);
      } else {
        if(z.y > 50) {
          pic.add(new void(frame F, transform t) {
              path q=shift(t*z)*p;
              unfill(F,q);
              draw(F,q);
            });
        } else {
          draw(f,p);
          add(pic,f,z);
      }
    }
  };
}
pair[] f=\{(5,5),(40,20),(55,51),(90,30)\};
draw(graph(f),marker(marks()));
scale(true);
xaxis("$x$",BottomTop,LeftTicks);
yaxis("$y$",LeftRight,RightTicks);
                     60
                     40
                   y
                     20
                      0
                             20
                                   40
                                               80
                                                     100
                                         60
```

7. This example shows how to label an axis with arbitrary strings. import graph;



8. The next example draws a graph of a parametrized curve. The calls to xlimits(picture pic=currentpicture, real min=-infinity, real max=infinity, bool crop=NoCrop);

and the analogous function ylimits can be uncommented to set the respective axes limits for picture pic to the specified min and max values. Alternatively, the function void limits(picture pic=currentpicture, pair min, pair max, bool crop=NoCrop); can be used to limit the axes to the box having opposite vertices at the given pairs). Existing objects in picture pic will be cropped to lie within the given limits if crop=Crop. The function crop(picture pic) can be used to crop a graph to the current graph limits.

```
import graph;
size(0,200);
real x(real t) {return cos(2pi*t);}
```

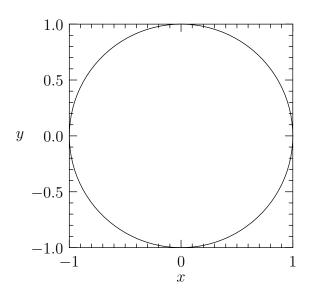
```
real y(real t) {return sin(2pi*t);}

draw(graph(x,y,0,1));

//xlimits(0,1,Crop);

//ylimits(-1,0,Crop);

xaxis("$x$",BottomTop,LeftTicks);
yaxis("$y$",LeftRight,RightTicks(trailingzero));
```



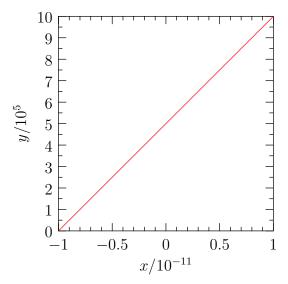
The next example illustrates how one can extract a common axis scaling factor.

```
import graph;
axiscoverage=0.9;
size(200,IgnoreAspect);
real[] x={-1e-11,1e-11};
real[] y={0,1e6};

real xscale=round(log10(max(x)));
real yscale=round(log10(max(y)))-1;

draw(graph(x*10^(-xscale),y*10^(-yscale)),red);

xaxis("$x/10^{"+(string)} xscale+"}$",BottomTop,LeftTicks);
yaxis("$y/10^{"+(string)} yscale+"}$",LeftRight,RightTicks(trailingzero));
```



Axis scaling can be requested and/or automatic selection of the axis limits can be inhibited with one of these scale routines:

```
void scale(picture pic=currentpicture, scaleT x, scaleT y);
```

This sets the scalings for picture pic. The graph routines accept an optional picture argument for determining the appropriate scalings to use; if none is given, it uses those set for currentpicture.

Two frequently used scaling routines Linear and Log are predefined in graph.

All picture coordinates (including those in paths and those given to the label and limits functions) are always treated as linear (post-scaled) coordinates. Use

```
pair Scale(picture pic=currentpicture, pair z);
```

to convert a graph coordinate into a scaled picture coordinate.

The x and y components can be individually scaled using the analogous routines

```
real ScaleX(picture pic=currentpicture, real x);
real ScaleY(picture pic=currentpicture, real y);
```

The predefined scaling routines can be given two optional boolean arguments: automin=false and automax=automin. These default to false but can be respectively set to true to enable automatic selection of "nice" axis minimum and maximum values. The Linear scaling can also take as optional final arguments a multiplicative scaling factor and intercept (e.g. for a depth axis, Linear(-1) requests axis reversal).

```
For example, to draw a log/log graph of a function, use scale(Log,Log): import graph;
```

```
size(200,200,IgnoreAspect);
real f(real t) {return 1/t;}
scale(Log,Log);
```

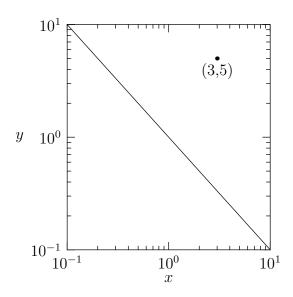
```
draw(graph(f,0.1,10));

//xlimits(1,10,Crop);

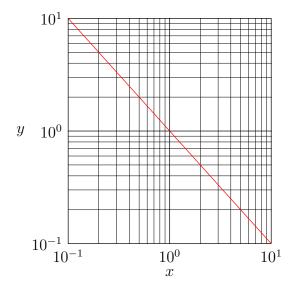
//ylimits(0.1,1,Crop);

dot(Label("(3,5)",align=S),Scale((3,5)));

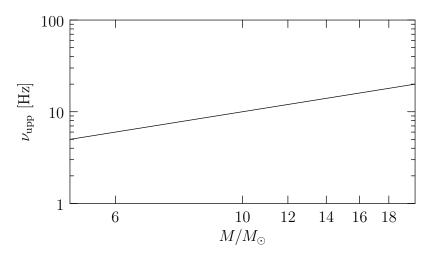
xaxis("$x$",BottomTop,LeftTicks);
yaxis("$y$",LeftRight,RightTicks);
```



By extending the ticks, one can easily produce a logarithmic grid:



One can also specify custom tick locations and formats for logarithmic axes: import graph;



It is easy to draw logarithmic graphs with respect to other bases:

```
import graph;
size(200,IgnoreAspect);
// Base-2 logarithmic scale on y-axis:
```

import graph;

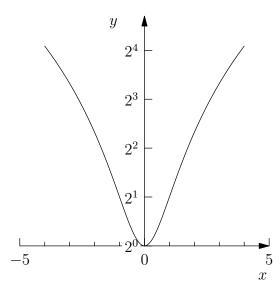
```
real log2(real x) {static real log2=log(2); return log(x)/log2;}
real pow2(real x) {return 2^x;}

scaleT yscale=scaleT(log2,pow2,logarithmic=true);
scale(Linear,yscale);

real f(real x) {return 1+x^2;}

draw(graph(f,-4,4));

yaxis("$y$",ymin=1,ymax=f(5),RightTicks(Label(Fill(white))),EndArrow);
xaxis("$x$",xmin=-5,xmax=5,LeftTicks,EndArrow);
```



Here is an example of "broken" linear x and logarithmic y axes that omit the segments [3,8] and [100,1000], respectively. In the case of a logarithmic axis, the break endpoints are automatically rounded to the nearest integral power of the base.

```
size(200,150,IgnoreAspect);

// Break the x axis at 3; restart at 8:
real a=3, b=8;

// Break the y axis at 100; restart at 1000:
real c=100, d=1000;

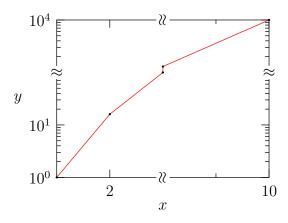
scale(Broken(a,b),BrokenLog(c,d));

real[] x={1,2,4,6,10};
real[] y=x^4;
```

```
draw(graph(x,y),red,MarkFill[0]);

xaxis("$x$",BottomTop,LeftTicks(Break(a,b)));
yaxis("$y$",LeftRight,RightTicks(Break(c,d)));

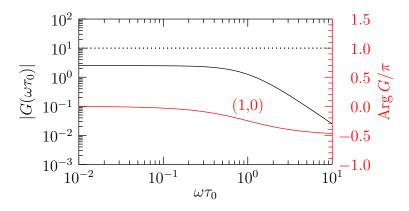
label(rotate(90)*Break,(a,point(S).y));
label(rotate(90)*Break,(a,point(N).y));
label(Break,(point(W).x,ScaleY(c)));
label(Break,(point(E).x,ScaleY(c)));
```



9. Asymptote can draw secondary axes with the routines

```
picture secondaryX(picture primary=currentpicture, void f(picture)); picture secondaryY(picture primary=currentpicture, void f(picture)); In this example, secondaryY is used to draw a secondary linear y axis against a primary logarithmic y axis:
```

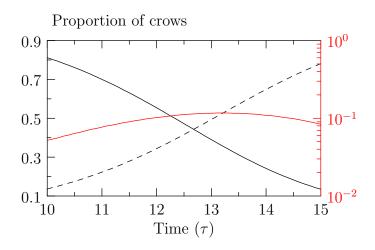
```
import graph;
texpreamble("\def\Arg{\mathop {\rm Arg}\nolimits}");
size(10cm,5cm,IgnoreAspect);
real ampl(real x) {return 2.5/(1+x^2);}
real phas(real x) {return -atan(x)/pi;}
scale(Log,Log);
draw(graph(ampl,0.01,10));
ylimits(0.001,100);
xaxis("$\omega\tau_0$",BottomTop,LeftTicks);
yaxis("$|G(\omega\tau_0)|$",Left,RightTicks);
picture q=secondaryY(new void(picture pic) {
    scale(pic,Log,Linear);
    draw(pic,graph(pic,phas,0.01,10),red);
```



A secondary logarithmic y axis can be drawn like this:

```
import graph;
size(9cm,6cm,IgnoreAspect);
string data="secondaryaxis.csv";
file in=input(data).line().csv();
string[] titlelabel=in;
string[] columnlabel=in;
real[][] a=in.dimension(0,0);
a=transpose(a);
real[] t=a[0], susceptible=a[1], infectious=a[2], dead=a[3], larvae=a[4];
real[] susceptibleM=a[5], exposed=a[6],infectiousM=a[7];
scale(true);
draw(graph(t,susceptible,t >= 10 & t <= 15));</pre>
draw(graph(t,dead,t >= 10 \& t <= 15),dashed);
xaxis("Time ($\tau$)",BottomTop,LeftTicks);
yaxis(Left,RightTicks);
picture secondary=secondaryY(new void(picture pic) {
```

```
scale(pic,Linear(true),Log(true));
draw(pic,graph(pic,t,infectious,t >= 10 & t <= 15),red);
yaxis(pic,Right,red,LeftTicks(begin=false,end=false));
});
add(secondary);
label(shift(5mm*N)*"Proportion of crows",point(NW),E);</pre>
```



10. Here is a histogram example, which uses the stats module.

```
import graph;
import stats;

size(400,200,IgnoreAspect);

int n=10000;

real[] a=new real[n];

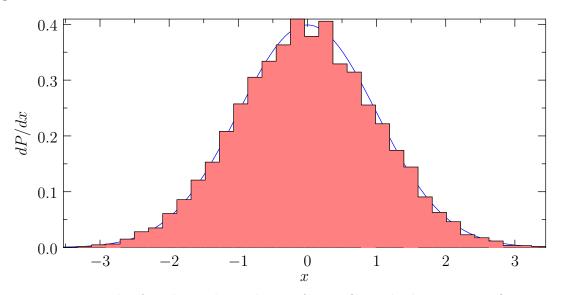
for(int i=0; i < n; ++i) a[i]=Gaussrand();

draw(graph(Gaussian,min(a),max(a)),blue);

// Optionally calculate "optimal" number of bins a la Shimazaki and Shinomoto.
int N=bins(a);

histogram(a,min(a),max(a),N,normalize=true,low=0,lightred,black,bars=false);

xaxis("$x$",BottomTop,LeftTicks);
yaxis("$dP/dx$",LeftRight,RightTicks(trailingzero));</pre>
```



11. Here is an example of reading column data in from a file and a least-squares fit, using the stats module.

```
size(400,200,IgnoreAspect);
import graph;
import stats;
file fin=input("leastsquares.dat").line();
real[][] a=fin.dimension(0,0);
a=transpose(a);
real[] t=a[0], rho=a[1];
// Read in parameters from the keyboard:
//real first=getreal("first");
//real step=getreal("step");
//real last=getreal("last");
real first=100;
real step=50;
real last=700;
// Remove negative or zero values of rho:
t=rho > 0 ? t : null;
rho=rho > 0 ? rho : null;
scale(Log(true),Linear(true));
int n=step > 0 ? ceil((last-first)/step) : 0;
real[] T,xi,dxi;
```

 10^{3}

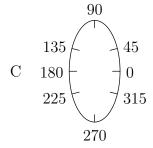
```
for(int i=0; i <= n; ++i) {
  real first=first+i*step;
  real[] logrho=(t >= first & t <= last) ? log(rho) : null;</pre>
  real[] logt=(t >= first & t <= last) ? -log(t) : null;
  if(logt.length < 2) break;</pre>
  // Fit to the line logt=L.m*logrho+L.b:
  linefit L=leastsquares(logt,logrho);
  T.push(first);
  xi.push(L.m);
  dxi.push(L.dm);
}
draw(graph(T,xi),blue);
errorbars(T,xi,dxi,red);
crop();
ylimits(0);
xaxis("$T$",BottomTop,LeftTicks);
yaxis("$\xi$",LeftRight,RightTicks);
   3
   2
 ξ
   1
   0^{\ }10^{2}
```

T

12. Here is an example that illustrates the general axis routine.

```
import graph;
size(0,100);
path g=ellipse((0,0),1,2);
```

```
scale(true);
axis(Label("C",align=10W),g,LeftTicks(endlabel=false,8,end=false),
    ticklocate(0,360,new real(real v) {
      path h=(0,0)--max(abs(max(g)),abs(min(g)))*dir(v);
      return intersect(g,h)[0];}));
```



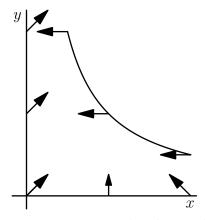
13. To draw a vector field of **n** arrows evenly spaced along the arclength of a path, use the routine

```
picture vectorfield(path vector(real), path g, int n, bool truesize=false,
                    pen p=currentpen, arrowbar arrow=Arrow);
as illustrated in this simple example of a flow field:
import graph;
defaultpen(1.0);
size(0,150,IgnoreAspect);
real arrowsize=4mm;
real arrowlength=2arrowsize;
typedef path vector(real);
// Return a vector interpolated linearly between a and b.
vector vector(pair a, pair b) {
  return new path(real x) {
    return (0,0)--arrowlength*interp(a,b,x);
  };
}
real f(real x) {return 1/x;}
real epsilon=0.5;
path g=graph(f,epsilon,1/epsilon);
int n=3;
draw(g);
xaxis("$x$");
```

pair b=(2pi,2pi);

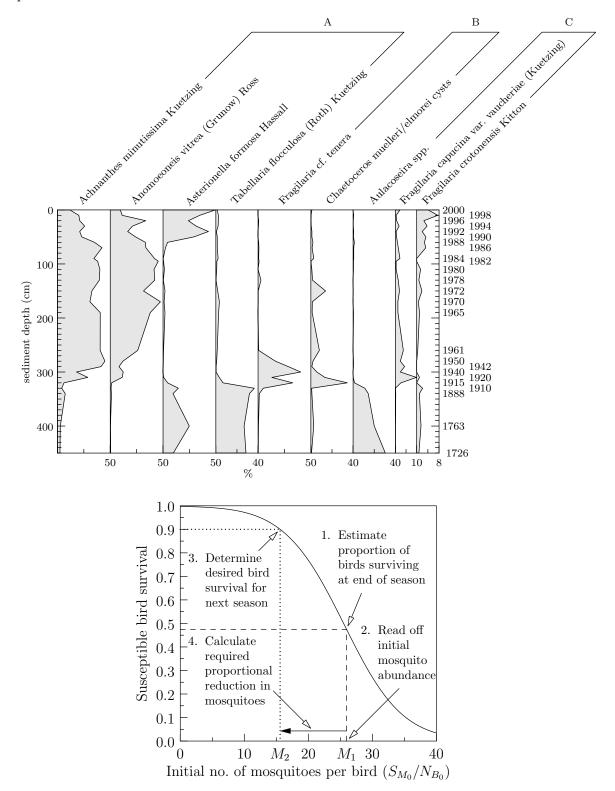
add(vectorfield(vector,a,b));

```
yaxis("$y$");
add(vectorfield(vector(W,W),g,n,true));
add(vectorfield(vector(NE,NW),(0,0)--(point(E).x,0),n,true));
add(vectorfield(vector(NE,NE),(0,0)--(0,point(N).y),n,true));
```



path vector(pair z) {return (0,0)--(sin(z.x),cos(z.y));}

15. The following scientific graphs, which illustrate many features of Asymptote's graphics routines, were generated from the examples diatom.asy and westnile.asy, using the comma-separated data in diatom.csv and westnile.csv.



8.28 palette

Asymptote can also generate color density images and palettes. The following palettes are predefined in palette.asy:

The function <code>cmyk(pen[] Palette)</code> may be used to convert any of these palettes to the CMYK colorspace.

A color density plot using palette palette can be generated from a function f(x,y) and added to a picture pic:

The function f will be sampled at nx and ny evenly spaced points over a rectangle defined by the points initial and final, respecting the current graphical scaling of pic. The color space is scaled according to the z axis scaling (see [automatic scaling], page 111). A bounds structure for the function values is returned:

```
struct bounds {
  real min;
  real max;
  // Possible tick intervals:
  int[] divisor;
}
```

This information can be used for generating an optional palette bar. The palette color space corresponds to a range of values specified by the argument range, which can be Full, Automatic, or an explicit range Range(real min, real max). Here Full specifies a range varying from the minimum to maximum values of the function over the sampling interval, while Automatic selects "nice" limits. The example imagecontour.asy illustrates how level sets (contour lines) can be drawn on a color density plot (see Section 8.36 [contour], page 149).

A color density plot can also be generated from an explicit real [[]] array data:

If the initial point is to the left and below the final point, by default the array indices are interpreted according to the Cartesian convention (first index: x, second index: y) rather than the usual matrix convention (first index: -y, second index: x).

To construct an image from an array of irregularly spaced points and an array of values f at these points, use one of the routines

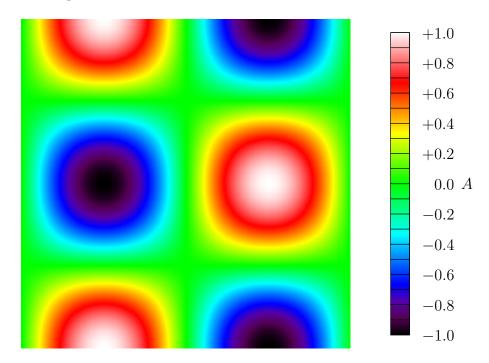
The color space of palette is taken to be over bounds bounds with scaling given by the z scaling of pic. The palette orientation is specified by axis, which may be one of Right, Left, Top, or Bottom. The bar is drawn over the rectangle from initial to final. The argument paletteticks is a special tick type (see [ticks], page 98) that takes the following arguments:

The image and palette bar can be fit to a frame and added and optionally aligned to a picture at the desired location:

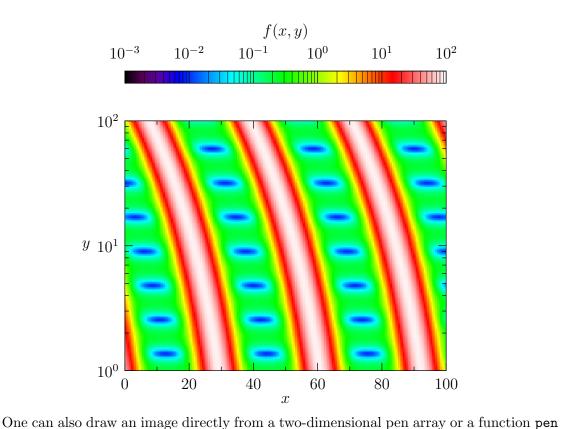
```
size(12cm,12cm);
import graph;
import palette;
int n=256;
real ninv=2pi/n;
real[][] v=new real[n][n];

for(int i=0; i < n; ++i)
   for(int j=0; j < n; ++j)
    v[i][j]=sin(i*ninv)*cos(j*ninv);

pen[] Palette=BWRainbow();</pre>
```



Here is an example that uses logarithmic scaling of the function values:



as illustrated in the following examples:

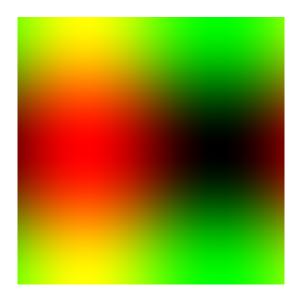
size(200);

```
import palette;
```

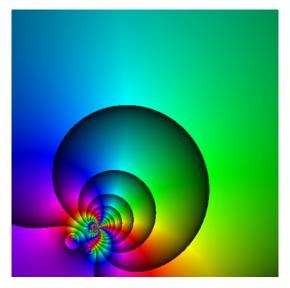
```
int n=256;
real ninv=2pi/n;
pen[][] v=new pen[n][n];

for(int i=0; i < n; ++i)
   for(int j=0; j < n; ++j)
    v[i][j]=rgb(0.5*(1+sin(i*ninv)),0.5*(1+cos(j*ninv)),0);</pre>
```

```
image(v,(0,0),(1,1));
```



```
import palette;
size(200);
real fracpart(real x) {return (x-floor(x));}
pair pws(pair z) {
 pair w=(z+exp(pi*I/5)/0.9)/(1+z/0.9*exp(-pi*I/5));
 return exp(w)*(w^3-0.5*I);
}
int N=512;
pair a=(-1,-1);
pair b=(0.5,0.5);
real dx=(b-a).x/N;
real dy=(b-a).y/N;
pen f(int u, int v) {
 pair z=a+(u*dx,v*dy);
 pair w=pws(z);
 real phase=degrees(w,warn=false);
 real modulus=w == 0 ? 0: fracpart(log(abs(w)));
  return hsv(phase,1,sqrt(modulus));
}
image(f,N,N,(0,0),(300,300),antialias=true);
```



For convenience, the module palette also defines functions that may be used to construct a pen array from a given function and palette:

```
pen[] palette(real[] f, pen[] palette);
pen[][] palette(real[][] f, pen[] palette);
```

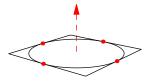
8.29 three

This module fully extends the notion of guides and paths in Asymptote to three dimensions. It introduces the new types guide3, path3, and surface. Guides in three dimensions are specified with the same syntax as in two dimensions except that triples (x,y,z) are used in place of pairs (x,y) for the nodes and direction specifiers. This generalization of John Hobby's spline algorithm is shape-invariant under three-dimensional rotation, scaling, and shifting, and reduces in the planar case to the two-dimensional algorithm used in Asymptote, MetaPost, and MetaFont [cf. J. C. Bowman, Proceedings in Applied Mathematics and Mechanics, 7:1, 2010021-2010022 (2007)].

For example, a unit circle in the XY plane may be filled and drawn like this: import three;

```
size(100);

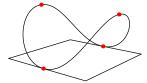
path3 g=(1,0,0)..(0,1,0)..(-1,0,0)..(0,-1,0)..cycle;
draw(g);
draw(0--Z,red+dashed,Arrow3);
draw(((-1,-1,0)--(1,-1,0)--(-1,1,0)--cycle));
dot(g,red);
```



and then distorted into a saddle:

```
import three;
```

```
size(100,0);
path3 g=(1,0,0)..(0,1,1)..(-1,0,0)..(0,-1,1)..cycle;
draw(g);
draw(((-1,-1,0)--(1,-1,0)--(1,1,0)--(-1,1,0)--cycle));
dot(g,red);
```



Module three provides constructors for converting two-dimensional paths to three-dimensional ones, and vice-versa:

```
path3 path3(path p, triple plane(pair)=XYplane);
path path(path3 p, pair P(triple)=xypart);
```

A Bezier surface, the natural two-dimensional generalization of Bezier curves, is defined in three_surface.asy as a structure containing an array of Bezier patches. Surfaces may drawn with one of the routines

The parameters nu and nv specify the number of subdivisions for drawing optional mesh lines for each Bezier patch. The optional name parameter is used as a prefix for naming the surface patches in the PRC model tree. Here material is a structure defined in three_light.asy:

```
struct material {
  pen[] p; // diffusepen,ambientpen,emissivepen,specularpen
  real opacity;
  real shininess;
...
}
```

These material properties are used to implement OpenGL-style lighting, based on the Phong-Blinn specular model. Sample Bezier surfaces are contained in the example files BezierSurface.asy, teapot.asy, and parametricsurface.asy. The structure render contains specialized rendering options documented at the beginning of module three.asy.

the constructor

The examples elevation.asy and sphericalharmonic.asy illustrate how to draw a surface with patch-dependent colors. The examples vertexshading and smoothelevation illustrate vertex-dependent colors, which is supported for both Asymptote's native OpenGL renderer and two-dimensional projections. Since the PRC output format does not currently support vertex shading of Bezier surfaces, PRC patches are shaded with the mean of the four vertex colors.

```
surface surface(path p, triple plane(pair)=XYplane);
and then filled:
```

```
draw(surface((0,0)--E+2N--2E--E+N..0.2E..cycle),red);
```

Planar Bezier surfaces patches are constructed using Orest Shardt's bezulate routine, which decomposes (possibly nonsimply connected) regions bounded (according to the zerowinding fill rule) by simple cyclic paths (intersecting only at the endpoints) into subregions bounded by cyclic paths of length 4 or less.

Arbitrary thick three-dimensional curves and line caps (which the OpenGL standard does not require implementations to provide) are constructed with

```
tube tube(path3 p, real width, render render=defaultrender);
```

this returns a tube structure representing a tube of diameter width centered approximately on g. The tube structure consists of a surface s and the actual tube center, path3 center. Drawing thick lines as tubes can be slow to render, especially with the Adobe Reader renderer. The setting thick=false can be used to disable this feature and force all lines to be drawn with linewidth(0) (one pixel wide, regardless of the resolution). By default, mesh and contour lines in three-dimensions are always drawn thin, unless an explicit line width is given in the pen parameter or the setting thin is set to false. The pens thin() and thick() defined in plain_pens.asy can also be used to override these defaults for specific draw commands.

There are four choices for viewing 3D Asymptote output:

1. Use the native Asymptote adaptive OpenGL-based renderer (with the command-line option -V and the default settings outformat="" and render=-1). If you encounter warnings from your graphics card driver, try specifying -glOptions=-indirect on the command line. On UNIX systems with graphics support for multisampling, we recommend installing the latest SVN (antialiased) version of the freeglut library (see [multisampling], page 6); the sample width can be controlled with the setting multisample. An initial screen position can be specified with the pair setting

position, where negative values are interpreted as relative to the corresponding maximum screen dimension. The default settings

```
import settings;
leftbutton=new string[] {"rotate","zoom","shift","pan"};
middlebutton=new string[] {"menu"};
rightbutton=new string[] {"zoom/menu","rotateX","rotateY","rotateZ"};
wheelup=new string[] {"zoomin"};
wheeldown=new string[] {"zoomout"};
```

bind the mouse buttons as follows:

- Left: rotate
- Shift Left: zoom
- Ctrl Left: shift viewport
- Alt Left: pan
- Middle: menu (must be unmodified; ignores Shift, Ctrl, and Alt)
- Wheel Up: zoom in
- Wheel Down: zoom out
- Right: zoom/menu (must be unmodified)
- Right double click: menu
- Shift Right: rotate about the X axis
- Ctrl Right: rotate about the Y axis
- Alt Right: rotate about the Z axis

The keyboard shortcuts are:

- h: home
- f: toggle fitscreen
- x: spin about the X axis
- y: spin about the Y axis
- z: spin about the Z axis
- s: stop spinning
- m: rendering mode (solid/mesh/patch)
- e: export
- c: show camera parameters
- p: play animation
- r: reverse animation
- : step animation
- +: expand
- =: expand
- >: expand
- -: shrink
- _: shrink

- <: shrink
- q: exit
- Ctrl-q: exit
- 2. Render the scene to a specified rasterized format outformat at the resolution of n pixels per bp, as specified by the setting render=n. A negative value of n is interpreted as |2n| for EPS and PDF formats and |n| for other formats. The default value of render is -1. By default, the scene is internally rendered at twice the specified resolution; this can be disabled by setting antialias=1. High resolution rendering is done by tiling the image. If your graphics card allows it, the rendering can be made more efficient by increasing the maximum tile size maxtile beyond the screen dimensions (indicated by maxtile=(0,0). If your video card generates unwanted black stripes in the output, try setting the horizontal and vertical components of maxtiles to something less than your screen dimensions. The tile size is also limited by the setting maxviewport, which restricts the maximum width and height of the viewport. On UNIX systems some graphics drivers support batch mode (-noV) rendering in an iconified window; this can be enabled with the setting iconify=true. Some (broken) UNIX graphics drivers may require the command line setting -glOptions=-indirect, which requests (slower) indirect rendering.
- 3. Embed the 3D PRC format in a PDF file and view the resulting PDF file with version 9.0 or later of Adobe Reader. In addition to the default settings.prc=true, this requires settings.outformat="pdf", which can be specified by the command line option -f pdf, put in the Asymptote configuration file (see [configuration file], page 159), or specified in the script before three.asy (or graph3.asy) is imported. Version 2008/10/08 or later of the movie15 package is also required (see Section 8.17 [embed], page 93). The example pdb.asy illustrates how one can generate a list of predefined views (see 100d.views). A stationary preview image with a resolution of n pixels per bp can be embedded with the setting render=n; this allows the file to be viewed with other PDF viewers. Alternatively, the file externalprc.tex illustrates how the resulting PRC and rendered image files can be extracted and processed in a separate LaTeX file. However, see Chapter 7 [LaTeX usage], page 82 for an easier way to embed three-dimensional Asymptote pictures within LaTeX. The open-source PRC specification is available from http://livedocs.adobe.com/acrobat_sdk/9/Acrobat9_HTMLHelp/API_References/PRCReference/PRC_Format_Specification/.
- 4. Project the scene to a two-dimensional vector (EPS or PDF) format with render=0. Only limited hidden surface removal facilities are currently available with this approach (see [PostScript3D], page 138).

Automatic picture sizing in three dimensions is accomplished with double deferred drawing. The maximal desired dimensions of the scene in each of the three dimensions can optionally be specified with the routine

The resulting simplex linear programming problem is then solved to produce a 3D version of a frame (actually implemented as a 3D picture). The result is then fit with another application of deferred drawing to the viewport dimensions corresponding to the usual two-dimensional picture size parameters. The global pair viewportmargin may be used to

add horizontal and vertical margins to the viewport dimensions. Alternatively, a minimum viewportsize may be specified. A 3D picture pic can be explicitly fit to a 3D frame by calling

```
frame pic.fit3(projection P=currentprojection);
```

and then added to picture dest about position with

```
void add(picture dest=currentpicture, frame src, triple position=(0,0,0));
```

For convenience, the three module defines 0=(0,0,0), X=(1,0,0), Y=(0,1,0), and Z=(0,0,1), along with a unitcircle in the XY plane:

```
path3 unitcircle3=X..Y..-X..-Y..cycle;
```

A general (approximate) circle can be drawn perpendicular to the direction normal with the routine

```
path3 circle(triple c, real r, triple normal=Z);
```

A circular arc centered at c with radius r from c+r*dir(theta1,phi1) to c+r*dir(theta2,phi2), drawing counterclockwise relative to the normal vector cross(dir(theta1,phi1),dir(theta2,phi2)) if theta2 > theta1 or if theta2 == theta1 and phi2 >= phi1, can be constructed with

The normal must be explicitly specified if c and the endpoints are colinear. If r < 0, the complementary arc of radius |r| is constructed. For convenience, an arc centered at c from triple v1 to v2 (assuming |v2-c|=|v1-c|) in the direction CCW (counter-clockwise) or CW (clockwise) may also be constructed with

When high accuracy is needed, the routines Circle and Arc defined in graph3 may be used instead. See [GaussianSurface], page 143 for an example of a three-dimensional circular arc.

The representation 0--0+u--0+u+v--0+v--cycle of the plane passing through point 0 with normal cross(u,v) is returned by

```
path3 plane(triple u, triple v, triple 0=0);
```

A three-dimensional box with opposite vertices at triples v1 and v2 may be drawn with the function

```
path3[] box(triple v1, triple v2);
```

For example, a unit box is predefined as

```
path3[] unitbox=box(0,(1,1,1));
```

Asymptote also provides optimized definitions for the three-dimensional paths unitsquare3 and unitcircle3, along with the surfaces unitdisk, unitplane, unitcube, unitcylinder, unitcone, unitsolidcone, unitfrustum(real t1, real t2), unitsphere, and unithemisphere.

These projections to two dimensions are predefined:

```
oblique
```

oblique(real angle)

The point (x,y,z) is projected to (x-0.5z,y-0.5z). If an optional real argument is given, the negative z axis is drawn at this angle in degrees. The projection obliqueZ is a synonym for oblique.

obliqueX

obliqueX(real angle)

The point (x,y,z) is projected to (y-0.5x,z-0.5x). If an optional real argument is given, the negative x axis is drawn at this angle in degrees.

obliqueY

obliqueY(real angle)

The point (x,y,z) is projected to (x+0.5y,z+0.5y). If an optional real argument is given, the positive y axis is drawn at this angle in degrees.

orthographic(triple camera, triple up=Z, triple target=O,

real zoom=1, pair viewportshift=0, bool showtarget=true, bool center=false)

This projects from three to two dimensions using the view as seen at a point infinitely far away in the direction unit(camera), orienting the camera so that, if possible, the vector up points upwards. Parallel lines are projected to parallel lines. The bounding volume is expanded to include target if showtarget=true. If center=true, the target will be adjusted to the center of the bounding volume.

orthographic(real x, real y, real z, triple up=Z, triple target=O,

real zoom=1, pair viewportshift=0, bool showtarget=true,

bool center=false)

This is equivalent to

orthographic((x,y,z),up,target,zoom,viewportshift,showtarget,center)

perspective(triple camera, triple up=Z, triple target=O,

real zoom=1, real angle=0, pair viewportshift=0,

bool showtarget=true, bool autoadjust=true,

bool center=autoadjust)

This projects from three to two dimensions, taking account of perspective, as seen from the location camera looking at target, orienting the camera so that, if possible, the vector up points upwards. If render=0, projection of three-dimensional cubic Bezier splines is implemented by approximating a two-dimensional nonuniform rational B-spline (NURBS) with a two-dimensional Bezier curve containing additional nodes and control points. If autoadjust=true, the camera will automatically be adjusted to lie outside the bounding volume for all possible interactive rotations about target. If center=true, the target will be adjusted to the center of the bounding volume.

```
bool showtarget=true, bool autoadjust=true,
             bool center=autoadjust)
           This is equivalent to
           perspective((x,y,z),up,target,zoom,angle,viewportshift,showtarget,
                       autoadjust, center)
The default projection, currentprojection, is initially set to perspective (5,4,2).
   We also define standard orthographic views used in technical drawing:
projection LeftView=orthographic(-X,showtarget=true);
projection RightView=orthographic(X,showtarget=true);
projection FrontView=orthographic(-Y,showtarget=true);
projection BackView=orthographic(Y,showtarget=true);
projection BottomView=orthographic(-Z,showtarget=true);
projection TopView=orthographic(Z,showtarget=true);
The function
void addViews(picture dest=currentpicture, picture src,
               projection[][] views=SixViewsUS,
               bool group=true, filltype filltype=NoFill);
adds to picture dest an array of views of picture src using the layout projection [[] views.
The default layout SixViewsUS aligns the projection FrontView below TopView and above
BottomView, to the right of LeftView and left of RightView and BackView. The predefined
layouts are:
projection[][] ThreeViewsUS={{TopView},
                               {FrontView, RightView}};
projection[][] SixViewsUS={{null,TopView},
                             {LeftView,FrontView,RightView,BackView},
                             {null,BottomView}};
projection[][] ThreeViewsFR={{RightView,FrontView},
                               {null,TopView}};
projection[][] SixViewsFR={{null,BottomView},
                             {RightView, FrontView, LeftView, BackView},
                             {null,TopView}};
projection[][] ThreeViews={{FrontView,TopView,RightView}};
projection[][] SixViews={{FrontView, TopView, RightView},
                           {BackView, BottomView, LeftView}};
   A triple or path3 can be projected to a pair or path, with project(triple, projection
```

P=currentprojection) or project(path3, projection P=currentprojection).

It is occasionally useful to be able to invert a projection, sending a pair z onto the plane.

It is occasionally useful to be able to invert a projection, sending a pair **z** onto the plane perpendicular to **normal** and passing through **point**:

triple invert(pair z, triple normal, triple point,

projection P=currentprojection);

```
A pair z on the projection plane can be inverted to a triple with the routine
triple invert(pair z, projection P=currentprojection);
A pair direction dir on the projection plane can be inverted to a triple direction relative
to a point v with the routine
triple invert(pair dir, triple v, projection P=currentprojection).
   Three-dimensional objects may be transformed with one of the following built-in trans-
form3 types (the identity transformation is identity4):
shift(triple v)
           translates by the triple v;
xscale3(real x)
           scales by x in the x direction;
yscale3(real y)
           scales by y in the y direction;
zscale3(real z)
           scales by z in the z direction;
scale3(real s)
           scales by s in the x, y, and z directions;
scale(real x, real y, real z)
           scales by x in the x direction, by y in the y direction, and by z in the z direction;
rotate(real angle, triple v)
           rotates by angle in degrees about an axis v through the origin;
rotate(real angle, triple u, triple v)
           rotates by angle in degrees about the axis u--v;
reflect(triple u, triple v, triple w)
           reflects about the plane through u, v, and w.
   When not multiplied on the left by a transform3, three-dimensional T<sub>E</sub>X Labels are
drawn as Bezier surfaces directly on the projection plane:
void label(picture pic=currentpicture, Label L, triple position,
            align align=NoAlign, pen p=currentpen,
            light light=nolight, string name="",
            render render=defaultrender, interaction interaction=
            settings.autobillboard ? Billboard : Embedded)
The optional name parameter is used as a prefix for naming the label patches in the PRC
model tree. The default interaction is Billboard, which means that labels are rotated
interactively so that they always face the camera. The interaction Embedded means that
```

the label interacts as a normal 3D surface, as illustrated in the example billboard.asy. Alternatively, a label can be transformed from the XY plane by an explicit transform3 or mapped to a specified two-dimensional plane with the predefined transform3 types XY, YZ,

ZX, YX, ZY, ZX. There are also modified versions of these transforms that take an optional argument projection P=currentprojection that rotate and/or flip the label so that it is more readable from the initial viewpoint.

A transform3 that projects in the direction dir onto the plane with normal n through point O is returned by

```
transform3 planeproject(triple n, triple 0=0, triple dir=n);
```

One can use

```
triple normal(path3 p);
```

to find the unit normal vector to a planar three-dimensional path p. As illustrated in the example planeproject.asy, a transform3 that projects in the direction dir onto the plane defined by a planar path p is returned by

```
transform3 planeproject(path3 p, triple dir=normal(p));
```

The functions

```
surface extrude(path p, triple axis=Z);
surface extrude(Label L, triple axis=Z);
```

return the surface obtained by extruding path p or Label L along axis.

Three-dimensional versions of the path functions length, size, point, dir, accel, radius, precontrol, postcontrol, arclength, arctime, reverse, subpath, intersect, intersections, intersectionpoint, intersectionpoints, min, max, cyclic, and straight are also defined.

The routine

```
real[][] intersections(path3 p, surface s, real fuzz=-1);
```

returns the intersection times of a path p with a surface s as a sorted array of real arrays of length 2, and

```
triple[] intersectionpoints(path3 p, surface s, real fuzz=-1);
```

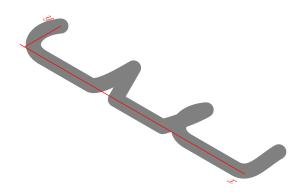
returns the corresponding intersection points. Here, the computations are performed to the absolute error specified by fuzz, or if fuzz < 0, to machine precision.

Here is an example showing all five guide3 connectors:

```
import graph3;
size(200);
currentprojection=orthographic(500,-500,500);
triple[] z=new triple[10];
z[0]=(0,100,0); z[1]=(50,0,0); z[2]=(180,0,0);
for(int n=3; n <= 9; ++n)
   z[n]=z[n-3]+(200,0,0);
path3 p=z[0]..z[1]---z[2]::{Y}z[3]
&z[3]..z[4]--z[5]::{Y}z[6]</pre>
```

```
&z[6]::z[7]---z[8]..{Y}z[9];
draw(p,grey+linewidth(4mm),currentlight);
xaxis3(Label(XY()*"$x$",align=-3Y),red,above=true);
```

yaxis3(Label(XY()*"\$y\$",align=-3X),red,above=true);



Three-dimensional versions of bars or arrows can be drawn with one of the specifiers None, Blank, BeginBar3, EndBar3 (or equivalently Bar3), Bars3, BeginArrow3, MidArrow3, EndArrow3 (or equivalently Arrow3), Arrows3, BeginArcArrow3, EndArcArrow3 (or equivalently ArcArrow3), MidArcArrow3, and ArcArrows3. Three-dimensional bars accept the optional arguments (real size=0, triple dir=0). If size=0, the default bar length is used; if dir=0, the bar is drawn perpendicular to the path and the initial viewing direction. The predefined three-dimensional arrowhead styles are DefaultHead3, HookHead3, TeXHead3. Versions of the two-dimensional arrowheads lifted to three-dimensional space and aligned according to the initial viewpoint (or an optionally specified normal vector) are also defined: DefaultHead2(triple normal=0), HookHead2(triple normal=0), TeXHead2(triple normal=0). These are illustrated in the example arrows3.asy.

Module three also defines the three-dimensional margins NoMargin3, BeginMargin3, EndMargin3, Margin3, Margins3, BeginPenMargin2, EndPenMargin2, PenMargin2, PenMargins2, BeginPenMargin3, EndPenMargin3, PenMargin3, PenMargin3, PenMargin3, BeginDotMargin3, EndDotMargin3, DotMargin3, DotMargins3, Margin3, and TrueMargin3.

The routine

can be used to draw on picture pic a pixel of width width at position v using pen p.

Further three-dimensional examples are provided in the files near_earth.asy, conicurv.asy, and (in the animations subdirectory) cube.asy.

Limited support for projected vector graphics (effectively three-dimensional nonrendered PostScript) is available with the setting render=0. This currently only works for piecewise planar surfaces, such as those produced by the parametric surface routines in the graph3 module. Surfaces produced by the solids package will also be properly rendered if the parameter nslices is sufficiently large.

In the module bsp, hidden surface removal of planar pictures is implemented using a binary space partition and picture clipping. A planar path is first converted to a structure face derived from picture. A face may be given to a two-dimensional drawing routine in place of any picture argument. An array of such faces may then be drawn, removing hidden surfaces:

Labels may be projected to two dimensions, using projection P, onto the plane passing through point O with normal cross(u,v) by multiplying it on the left by the transform

Here is an example that shows how a binary space partition may be used to draw a two-dimensional vector graphics projection of three orthogonal intersecting planes:

```
size(6cm,0);
import bsp;

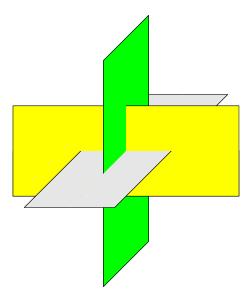
real u=2.5;
real v=1;

currentprojection=oblique;

path3 y=plane((2u,0,0),(0,2v,0),(-u,-v,0));
path3 l=rotate(90,Z)*rotate(90,Y)*y;
path3 g=rotate(90,X)*rotate(90,Y)*y;

face[] faces;
filldraw(faces.push(y),project(y),yellow);
filldraw(faces.push(l),project(l),lightgrey);
filldraw(faces.push(g),project(g),green);

add(faces);
```



8.30 obj

This module allows one to construct surfaces from simple obj files, as illustrated in the example files galleon.asy and triceratops.asy.

8.31 graph3

This module implements three-dimensional versions of the functions in graph.asy. To draw an x axis in three dimensions, use the routine

Analogous routines yaxis and zaxis can be used to draw y and z axes in three dimensions. There is also a routine for drawing all three axis:

The predefined three-dimensional axis types are

```
axis YZEquals(real y, real z, triple align=0, bool extend=false);
axis XZEquals(real x, real z, triple align=0, bool extend=false);
axis XYEquals(real x, real y, triple align=0, bool extend=false);
axis YZZero(triple align=0, bool extend=false);
axis XZZero(triple align=0, bool extend=false);
axis XYZero(triple align=0, bool extend=false);
axis Bounds(int type=Both, int type2=Both, triple align=0, bool extend=false);
```

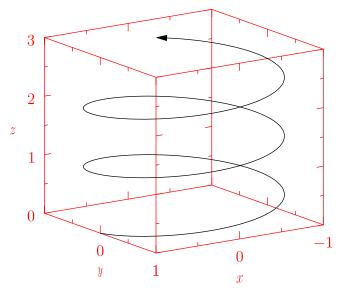
The optional align parameter to these routines can be used to specify the default axis and tick label alignments. The Bounds axis accepts two type parameters, each of which must be one of Min, Max, or Both. These parameters specify which of the four possible three-dimensional bounding box edges should be drawn.

The three-dimensional tick options are NoTicks3, InTicks, OutTicks, and InOutTicks. These specify the tick directions for the Bounds axis type; other axis types inherit the direction that would be used for the Bounds(Min, Min) axis.

Here is an example of a helix and bounding box axes with ticks and axis labels, using orthographic projection:

```
import graph3;
size(0,200);
size3(200,IgnoreAspect);
currentprojection=orthographic(4,6,3);
real x(real t) {return cos(2pi*t);}
real y(real t) {return sin(2pi*t);}
real z(real t) {return t;}

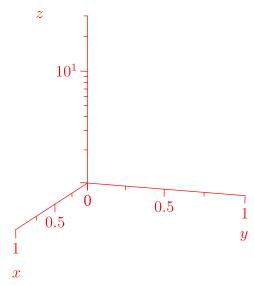
path3 p=graph(x,y,z,0,2.7,operator ..);
draw(p,Arrow3);
scale(true);
xaxis3(XZ()*"$x$",Bounds,red,InTicks(Label,2,2));
yaxis3(YZ()*"$y$",Bounds,red,InTicks(beginlabel=false,Label,2,2));
zaxis3(XZ()*"$z$",Bounds,red,InTicks);
```



The next example illustrates three-dimensional x, y, and z axes, without autoscaling of the axis limits:

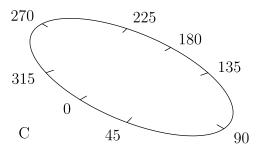
```
import graph3;
```

```
size(0,200);
size3(200,IgnoreAspect);
currentprojection=perspective(5,2,2);
scale(Linear,Linear,Log);
xaxis3("$x$",0,1,red,OutTicks(2,2));
yaxis3("$y$",0,1,red,OutTicks(2,2));
zaxis3("$z$",1,30,red,OutTicks(beginlabel=false));
```



One can also place ticks along a general three-dimensional axis:

```
import graph3;
size(0,100);
path3 g=yscale3(2)*unitcircle3;
currentprojection=perspective(10,10,10);
axis(Label("C",position=0,align=15X),g,InTicks(endlabel=false,8,end=false),
    ticklocate(0,360,new real(real v) {
        path3 h=0--max(abs(max(g)),abs(min(g)))*dir(90,v);
        return intersect(g,h)[0];},
        new triple(real t) {return cross(dir(g,t),Z);}));
```



Surface plots of matrices and functions over the region box(a,b) in the XY plane are also implemented:

```
surface surface(real[][] f, pair a, pair b, bool[][] cond={});
surface surface(real[][] f, pair a, pair b, splinetype xsplinetype,
                splinetype ysplinetype=xsplinetype, bool[][] cond={});
surface surface(real[][] f, real[] x, real[] y,
                splinetype xsplinetype=null, splinetype ysplinetype=xsplinetype,
                bool[][] cond={})
surface surface(triple[][] f, bool[][] cond={});
surface surface(real f(pair z), pair a, pair b, int nx=nmesh, int ny=nx,
                bool cond(pair z)=null);
surface surface(real f(pair z), pair a, pair b, int nx=nmesh, int ny=nx,
                splinetype xsplinetype, splinetype ysplinetype=xsplinetype,
                bool cond(pair z)=null);
surface surface(triple f(pair z), real[] u, real[] v,
                splinetype[] usplinetype, splinetype[] vsplinetype=Spline,
                bool cond(pair z)=null);
surface surface(triple f(pair z), pair a, pair b, int nu=nmesh, int nv=nu,
                bool cond(pair z)=null);
surface surface(triple f(pair z), pair a, pair b, int nu=nmesh, int nv=nu,
                splinetype[] usplinetype, splinetype[] vsplinetype=Spline,
                bool cond(pair z)=null);
```

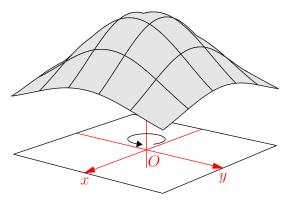
The final two versions draw parametric surfaces for a function f(u, v) over the parameter space box(a,b), as illustrated in the example parametric surface asy. An optional spline-type Spline may be specified. The boolean array or function cond can be used to control which surface mesh cells are actually drawn (by default all mesh cells over box(a,b) are drawn). Surface lighting is illustrated in the example files parametric surface asy and sinc.asy. Lighting can be disabled by setting light=nolight, as in this example of a Gaussian surface:

```
import graph3;
size(200,0);
currentprojection=perspective(10,8,4);
real f(pair z) {return 0.5+exp(-abs(z)^2);}
draw((-1,-1,0)--(1,-1,0)--(1,1,0)--(-1,1,0)--cycle);
draw(arc(0.12Z,0.2,90,60,90,25),ArcArrow3);
```

```
surface s=surface(f,(-1,-1),(1,1),nx=5,Spline);

xaxis3(Label("$x$"),red,Arrow3);
yaxis3(Label("$y$"),red,Arrow3);
zaxis3(XYZero(extend=true),red,Arrow3);

draw(s,lightgray,meshpen=black+thick(),nolight,render(merge=true));
label("$0$",0,-Z+Y,red);
```



A mesh can be drawn without surface filling by specifying nullpen for the surfacepen.

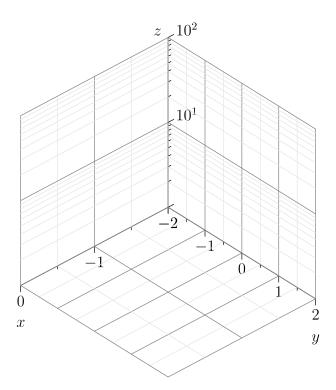
A vector field of $\mathtt{nu} \times \mathtt{nv}$ arrows on a parametric surface \mathtt{f} over $\mathtt{box(a,b)}$ can be drawn with the routine

as illustrated in the examples vectorfield3.asy and vectorfieldsphere.asy.

8.32 grid3

This module, contributed by Philippe Ivaldi, can be used for drawing 3D grids. Here is an example (further examples can be found in grid3.asy and at http://www.piprime.fr/files/asymptote/grid3/):

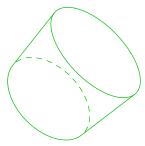
```
import grid3;
size(8cm,0,IgnoreAspect);
currentprojection=orthographic(0.5,1,0.5);
scale(Linear, Linear, Log);
limits((-2,-2,1),(0,2,100));
```



8.33 solids

This solid geometry package defines a structure revolution that can be used to fill and draw surfaces of revolution. The following example uses it to display the outline of a circular cylinder of radius 1 with axis O--1.5unit(Y+Z) with perspective projection:

```
import solids;
size(0,100);
revolution r=cylinder(0,1,1.5,Y+Z);
draw(r,heavygreen);
```



Further illustrations are provided in the example files cylinder.asy, cones.asy, hyperboloid.asy, and torus.asy.

The structure **skeleton** contains the three-dimensional wireframe used to visualize a volume of revolution:

```
struct skeleton {
   struct curve {
     path3[] front;
     path3[] back;
   }
   // transverse skeleton (perpendicular to axis of revolution)
   curve transverse;
   // longitudinal skeleton (parallel to axis of revolution)
   curve longitudinal;
}
```

8.34 tube

This package extends the tube surfaces constructed in three_arrows.asy to arbitrary cross sections, colors, and spine transformations. The routine

draws a tube along g with cross section section, after applying the transformation T(t) at relpoint(g,t). The parameter corner controls the number of elementary tubes at the angular points of g. A nonzero value of relstep specifies a fixed relative time step (in the sense of relpoint(g,t)) to use in constructing elementary tubes along g. The type coloredpath is a generalization of path to which a path can be cast:

```
struct coloredpath
{
  path p;
  pen[] pens(real);
  int colortype=coloredSegments;
}
```

Here p defines the cross section and the method pens(real t) returns an array of pens (interpreted as a cyclic array) used for shading the tube patches at relpoint(g,t). If colortype=coloredSegments, the tube patches are filled as if each segment of the section was colored with the pen returned by pens(t), whereas if colortype=coloredNodes, the tube components are vertex shaded as if the nodes of the section were colored.

A coloredpath can be constructed with one of the routines:

In the second case, the pens are independent of the relative time. In the third case, the array of pens contains only one pen, which depends of the relative time.

The casting of path to coloredpath allows the use of a path instead of a coloredpath; in this case the shading behaviour is the default shading behavior for a surface.

An example of tube is provided in the file trefoilknot.asy. Further examples can be found at http://www.piprime.fr/files/asymptote/tube/.

8.35 flowchart

This package provides routines for drawing flowcharts. The primary structure is a block, which represents a single block on the flowchart. The following eight functions return a position on the appropriate edge of the block, given picture transform t:

```
pair block.top(transform t=identity());
pair block.left(transform t=identity());
pair block.right(transform t=identity());
pair block.bottom(transform t=identity());
pair block.topleft(transform t=identity());
pair block.topright(transform t=identity());
pair block.bottomleft(transform t=identity());
pair block.bottomright(transform t=identity());
To obtain an arbitrary position along the boundary of the block in user coordinates, use:
pair block.position(real x, transform t=identity());
```

The center of the block in user coordinates is stored in block.center and the block size in PostScript coordinates is given by block.size.

A frame containing the block is returned by

```
frame block.draw(pen p=currentpen);
```

The following block generation routines accept a Label, string, or frame for their object argument:

rectangular block with an optional header (and padding dx around header and body):

block rectangle(object header, object body, pair center=(0,0),

pen headerpen=mediumgray, pen bodypen=invisible,

pen drawpen=currentpen,

real dx=3, real minheaderwidth=minblockwidth,

real minheaderheight=minblockwidth,

real minbodywidth=minblockheight,

real minbodyheight=minblockheight);

block rectangle(object body, pair center=(0,0),

pen fillpen=invisible, pen drawpen=currentpen,

real dx=3, real minwidth=minblockwidth,

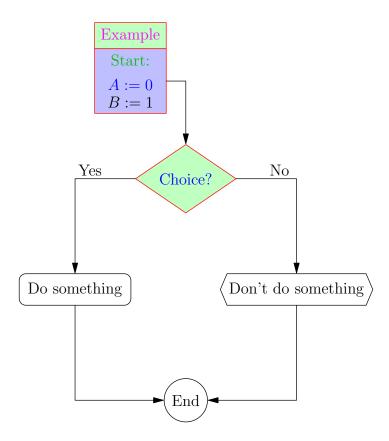
real minheight=minblockheight);

draw(block2);

```
parallelogram block:
          block parallelogram(object body, pair center=(0,0),
                               pen fillpen=invisible, pen drawpen=currentpen,
                               real dx=3, real slope=2,
                               real minwidth=minblockwidth,
                               real minheight=minblockheight);
diamond-shaped block:
          block diamond(object body, pair center=(0,0),
                         pen fillpen=invisible, pen drawpen=currentpen,
                         real ds=5, real dw=1,
                         real height=20, real minwidth=minblockwidth,
                         real minheight=minblockheight);
circular block:
          block circle(object body, pair center=(0,0), pen fillpen=invisible,
                        pen drawpen=currentpen, real dr=3,
                        real mindiameter=mincirclediameter);
rectangular block with rounded corners:
          block roundrectangle(object body, pair center=(0,0),
                                pen fillpen=invisible, pen drawpen=currentpen,
                                real ds=5, real dw=0, real minwidth=minblockwidth,
                                real minheight=minblockheight);
rectangular block with beveled edges:
          block bevel(object body, pair center=(0,0), pen fillpen=invisible,
                       pen drawpen=currentpen, real dh=5, real dw=5,
                       real minwidth=minblockwidth, real minheight=minblockheight);
  To draw paths joining the pairs in point with right-angled lines, use the routine:
path path(pair point[] ... flowdir dir[]);
The entries in dir identify whether successive segments between the pairs specified by point
should be drawn in the Horizontal or Vertical direction.
  Here is a simple flowchart example (see also the example controlsystem.asy):
size(0,300);
import flowchart;
block block1=rectangle(Label("Example", magenta),
                        pack(Label("Start:",heavygreen),"",Label("$A:=0$",blue),
                             "$B:=1$"),(-0.5,3),palegreen,paleblue,red);
block block2=diamond(Label("Choice?",blue),(0,2),palegreen,red);
block block3=roundrectangle("Do something",(-1,1));
block block4=bevel("Don't do something",(1,1));
block block5=circle("End",(0,0));
draw(block1);
```

```
draw(block3);
draw(block4);
draw(block5);

add(new void(picture pic, transform t) {
    blockconnector operator --=blockconnector(pic,t);
    // draw(pic,block1.right(t)--block2.top(t));
    block1--Right--Down--Arrow--block2;
    block2--Label("Yes",0.5,NW)--Left--Down--Arrow--block3;
    block2--Right--Label("No",0.5,NE)--Down--Arrow--block4;
    block4--Down--Left--Arrow--block5;
    block3--Down--Right--Arrow--block5;
});
```



8.36 contour

This package draws contour lines. To construct contours corresponding to the values in a real array c for a function f on box(a,b), use the routine

The integers nx and ny define the resolution. The default resolution, ngraph x ngraph (here ngraph defaults to 100) can be increased for greater accuracy. The default interpo-

lation operator is operator -- (linear). Spline interpolation (operator ..) may produce smoother contours but it can also lead to overshooting. The subsample parameter indicates the number of interior points that should be used to sample contours within each 1×1 box; the default value of 1 is usually sufficient.

To construct contours for an array of data values on a uniform two-dimensional lattice on box(a,b), use

To construct contours for an array of data values on a nonoverlapping regular mesh specified by the two-dimensional array z,

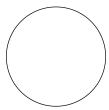
To construct contours for an array of values f specified at irregularly positioned points z, use the routine

The contours themselves can be drawn with one of the routines

The following simple example draws the contour at value 1 for the function $z = x^2 + y^2$, which is a unit circle:

```
import contour;
size(75);
```

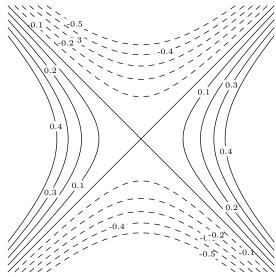
```
real f(real a, real b) {return a^2+b^2;}
draw(contour(f,(-1,-1),(1,1),new real[] {1}));
```



The next example draws and labels multiple contours for the function $z = x^2 - y^2$ with the resolution 100 x 100, using a dashed pen for negative contours and a solid pen for positive (and zero) contours:

```
import contour;
```

```
size(200);
```



The next example illustrates how contour lines can be drawn on color density images:

```
import graph;
import palette;
import contour;

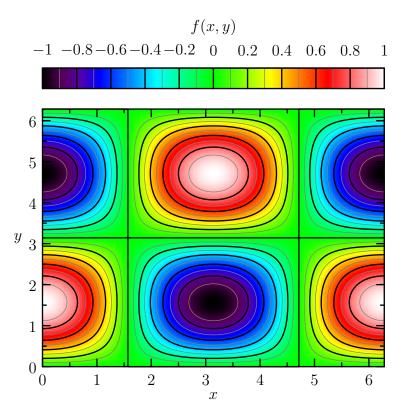
size(10cm,10cm,IgnoreAspect);

pair a=(0,0);
pair b=(2pi,2pi);

real f(real x, real y) {return cos(x)*sin(y);}

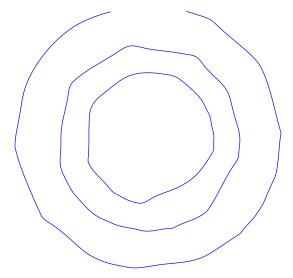
int N=200;
int Divs=10;
int divs=2;
```

```
defaultpen(1bp);
pen Tickpen=black;
pen tickpen=gray+0.5*linewidth(currentpen);
pen[] Palette=BWRainbow();
bounds range=image(f,Automatic,a,b,N,Palette);
// Major contours
real[] Cvals=uniform(range.min,range.max,Divs);
draw(contour(f,a,b,Cvals,N,operator --),Tickpen);
// Minor contours
real[] cvals;
for(int i=0; i < Cvals.length-1; ++i)</pre>
  cvals.append(uniform(Cvals[i],Cvals[i+1],divs)[1:divs]);
draw(contour(f,a,b,cvals,N,operator --),tickpen);
xaxis("$x$",BottomTop,LeftTicks,above=true);
yaxis("$y$",LeftRight,RightTicks,above=true);
palette("f(x,y), range, point(NW) + (0,0.5), point(NE) + (0,1), Top, Palette,
        PaletteTicks(N=Divs,n=divs,Tickpen,tickpen));
```



Finally, here is an example that illustrates the construction of contours from irregularly spaced data:

```
import contour;
size(200);
int n=100;
real f(real a, real b) {return a^2+b^2;}
srand(1);
real r() {return 1.1*(rand()/randMax*2-1);}
pair[] points=new pair[n];
real[] values=new real[n];
for(int i=0; i < n; ++i) {
   points[i]=(r(),r());
   values[i]=f(points[i].x,points[i].y);
}
draw(contour(points,values,new real[]{0.25,0.5,1},operator ..),blue);</pre>
```



In the above example, the contours of irregularly spaced data are constructed by first creating a triangular mesh from an array z of pairs:

```
int[][] triangulate(pair[] z);

size(200);
int np=100;
pair[] points;

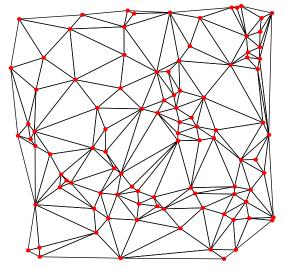
real r() {return 1.2*(rand()/randMax*2-1);}

for(int i=0; i < np; ++i)
    points.push((r(),r()));

int[][] trn=triangulate(points);

for(int i=0; i < trn.length; ++i) {
    draw(points[trn[i][0]]--points[trn[i][1]]);
    draw(points[trn[i][1]]--points[trn[i][2]]);
    draw(points[trn[i][2]]--points[trn[i][0]]);
}

for(int i=0; i < np; ++i)
    dot(points[i],red);</pre>
```



The example Gouraudcontour illustrates how to produce color density images over such irregular triangular meshes. Asymptote uses a robust version of Paul Bourke's Delaunay triangulation algorithm based on the public-domain exact arithmetic predicates written by Jonathan Shewchuk.

8.37 contour3

This package draws surfaces described as the null space of real-valued functions of (x, y, z) or real[][][] matrices. Its usage is illustrated in the example file magnetic.asy.

8.38 slopefield

```
To draw a slope field for the differential equation dy/dx = f(x,y) (or dy/dx = f(x)), use: picture slopefield(real f(real,real), pair a, pair b, int nx=nmesh, int ny=nx, real tickfactor=0.5, pen p=currentpen, arrowbar arrow=None);
```

Here, the points a and b are the lower left and upper right corners of the rectangle in which the slope field is to be drawn, nx and ny are the respective number of ticks in the x and y directions, tickfactor is the fraction of the minimum cell dimension to use for drawing ticks, and p is the pen to use for drawing the slope fields. The return value is a picture that can be added to currentpicture via the add(picture) command.

The function

```
path curve(pair c, real f(real, real), pair a, pair b);
```

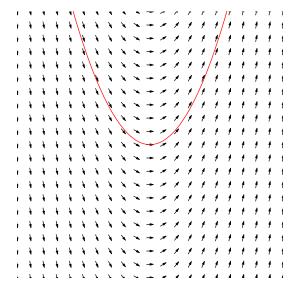
takes a point (c) and a slope field-defining function f and returns, as a path, the curve passing through that point. The points a and b represent the rectangular boundaries over which the curve is interpolated.

Both slopefield and curve alternatively accept a function real f (real) that depends on x only, as seen in this example:

```
import slopefield;
```

```
size(200);
```

```
real func(real x) {return 2x;}
add(slopefield(func,(-3,-3),(3,3),20,Arrow));
draw(curve((0,0),func,(-3,-3),(3,3)),red);
```



8.39 ode

The ode module, illustrated in the example odetest.asy, implements a number of explicit numerical integration schemes for ordinary differential equations.

9 Command-line options

Type asy -h to see the full list of command-line options supported by Asymptote:

Usage: ../asy [options] [file ...]

Options (negate by replacing - with -no):

-V,-View View output; command-line only

-a,-align C|B|T|Z Center, Bottom, Top, or Zero page alignment [C] -antialias n Antialiasing width for rasterized output [2]

-arcballradius pixels Arcball radius [750]

-auto3D Automatically activate 3D scene [true]

-autobillboard 3D labels always face viewer by default [true]

-autoimport string Module to automatically import

-autoplain Enable automatic importing of plain [true]

-autoplay Autoplay 3D animations [false]

-autorotate Enable automatic PDF page rotation [false]
-batchMask Mask fpu exceptions in batch mode [false]

-batchView View output in batch mode [false]

-bw Convert all colors to black and white [false]
-cd directory Set current directory; command-line only

-cmyk Convert rgb colors to cmyk [false]

-c,-command string Command to autoexecute

-compact Conserve memory at the expense of speed [false]

-d,-debug Enable debugging messages [false]

-divisor n Garbage collect using purge(divisor=n) [2]

-doubleclick ms Emulated double-click timeout [200]
-embed Embed rendered preview image [true]
-exitonEOF Exit interactive mode on EOF [true]
-fitscreen Fit rendered image to screen [true]

-framedelay ms Additional frame delay [0]
-framerate frames/s Animation speed [30]

-globalwrite Allow write to other directory [false]
-gray Convert all colors to grayscale [false]
-h,-help Show summary of options; command-line only

-historylines n Retain n lines of history [1000]
-iconify Iconify rendering window [false]
-inlineimage Generate inline embedded image [false]
-inlinetex Generate inline TeX code [false]

-interactiveMask Mask fpu exceptions in interactive mode [true]

-interactiveView View output in interactive mode [true]

-interactiveWrite Write expressions entered at the prompt to stdout [true]

-k,-keep Keep intermediate files [false]

-keepaux Keep intermediate LaTeX .aux files [false]

-level n Postscript level [3]

-1,-listvariables List available global functions and variables [false]

-localhistory Use a local interactive history file [false]

-loop Loop 3D animations [false]

-m,-mask Mask fpu exceptions; command-line only
-maxtile pair Maximum rendering tile size [(0,0)]
-maxviewport pair Maximum viewport size [(2048,2048)]

-multiline Input code over multiple lines at the prompt [false]
-multipleView View output from multiple batch-mode files [false]

-multisample n Multisampling width for screen images [4]

-O,-offset pair PostScript offset [(0,0)]

-f,-outformat format Convert each output file to specified format

-o,-outname name Alternative output directory/filename

-p,-parseonly Parse file [false]

-pdfreload Automatically reload document in pdfviewer [false]
-pdfreloaddelay usec Delay before attempting initial pdf reload [750000]

-position pair Initial 3D rendering screen position [(0,0)]
-prc Embed 3D PRC graphics in PDF output [true]

-prompt string Prompt [>]

-prompt2 string Continuation prompt for multiline input [..]

-q,-quiet Suppress welcome message [false]

-render n Render 3D graphics using n pixels per bp (-1=auto) [-1]

-resizestep step Resize step [1.2]

-reverse reverse 3D animations [false]
-rgb Convert cmyk colors to rgb [false]

-safe Disable system call [true]

-scroll n Scroll standard output n lines at a time [0]

-spinstep deg/s Spin speed [60]

-svgemulation Emulate unimplemented SVG shading [false]
-tabcompletion Interactive prompt auto-completion [true]

-tex engine latex|pdflatex|xelatex|tex|pdftex|context|none [latex]

-thick Render thick 3D lines [true]
-thin Render thin 3D lines [true]

-threads Use POSIX threads for 3D rendering [true]
-toolbar Show 3D toolbar in PDF output [true]

-s,-translate Show translated virtual machine code [false]
-twice Run LaTeX twice (to resolve references) [false]
-twosided Use two-sided 3D lighting model for rendering [true]

-u,-user string General purpose user string

-v,-verbose Increase verbosity level (can specify multiple times) [0]

-version Show version; command-line only

-wait Wait for child processes to finish before exiting [false]

-warn string Enable warning; command-line only

-where Show where listed variables are declared [false]

-zoomfactor factor Zoom step factor [1.05]
-zoomstep step Mouse motion zoom step [0.1]

All boolean options can be negated by prepending no to the option name.

If no arguments are given, Asymptote runs in interactive mode (see Chapter 10 [Interactive mode], page 161). In this case, the default output file is out.eps.

If - is given as the file argument, Asymptote reads from standard input.

If multiple files are specified, they are treated as separate Asymptote runs.

If the string autoimport is nonempty, a module with this name is automatically imported for each run as the final step in loading module plain.

Default option values may be entered as Asymptote code in a configuration file named config.asy (or the file specified by the environment variable ASYMPTOTE_CONFIG or -config option). Asymptote will look for this file in its usual search path (see Section 2.5 [Search paths], page 6). Typically the configuration file is placed in the .asy directory in the user's home directory (%USERPROFILE%\.asy under MSDOS). Configuration variables are accessed using the long form of the option names:

```
import settings;
outformat="pdf";
batchView=false;
interactiveView=true;
batchMask=false;
interactiveMask=true;
```

Command-line options override these defaults. Most configuration variables may also be changed at runtime. The advanced configuration variables dvipsOptions, hyperrefOptions, convertOptions, gsOptions, psviewerOptions, pdfviewerOptions, and glOptions allow specialized options to be passed as a string to the respective applications or libraries. The default value of hyperrefOptions is setpagesize=false,unicode,pdfborder=0 0 0.

```
If you insert import plain; settings.autoplain=true;
```

at the beginning of the configuration file, it can contain arbitrary Asymptote code.

The default output format is EPS for the (default) latex and tex tex engine and PDF for the pdflatex, xelatex, and context tex engines. Alternative output formats may be produced using the -f option (or outformat setting). To produce SVG output, first install dvisvgm (version 0.8.7 or later) from http://dvisvgm.sourceforge.net/down.html and be sure to use the latex or tex tex engine.

Asymptote can also produce any output format supported by the ImageMagick convert program (version 6.3.5 or later recommended; an Invalid Parameter error message indicates that the MSDOS utility convert is being used instead of the one that comes with ImageMagick). The optional setting -render n requests an output resolution of n pixels per bp. Antialiasing is controlled by the parameter antialias, which by default specifies a sampling width of 2 pixels. To give other options to convert, use the convertOptions setting or call convert manually. This example emulates how Asymptote produces antialiased tiff output at one pixel per bp:

```
asy -o - venn | convert -alpha Off -density 144x144 -geometry 50%x eps:- venn.tiff
If the option -nosafe is given, Asymptote runs in unsafe mode. This enables the int
system(string s) and int system(string[] s) calls, allowing one to execute arbitrary
shell commands. The default mode, -safe, disables this call.
```

A PostScript offset may be specified as a pair (in bp units) with the -0 option: asy -0 0,0 file

The default offset is zero. The default value of the page alignment setting align is Center.

The -c (command) option may be used to execute arbitrary Asymptote code on the command line as a string. It is not necessary to terminate the string with a semicolon. Multiple -c options are executed in the order they are given. For example

```
asy -c 2+2 -c "sin(1)" -c "size(100); draw(unitsquare)" produces the output 4 0.841470984807897
```

and draws a unitsquare of size 100.

The -u (user) option may be used to specify arbitrary Asymptote settings on the command line as a string. It is not necessary to terminate the string with a semicolon. Multiple -u options are executed in the order they are given. Command-line code like -u x=sqrt(2) can be executed within a module like this:

```
real x;
usersetting();
write(x);
```

When the -1 (listvariables) option is used with file arguments, only global functions and variables defined in the specified file(s) are listed.

Additional debugging output is produced with each additional -v option:

-v Display top-level module and final output file names.

-vv Also display imported and included module names and final LaTeX and dvips processing information.

-vvv Also output LaTeX bidirectional pipe diagnostics.

-vvvv Also output knot guide solver diagnostics.

-vvvvv Also output Asymptote traceback diagnostics.

10 Interactive mode

Interactive mode is entered by executing the command asy with no file arguments. When the -multiline option is disabled (the default), each line must be a complete Asymptote statement (unless explicitly continued by a final backslash character \); it is not necessary to terminate input lines with a semicolon. If one assigns settings.multiline=true, interactive code can be entered over multiple lines; in this mode, the automatic termination of interactive input lines by a semicolon is inhibited. Multiline mode is useful for cutting and pasting Asymptote code directly into the interactive input buffer.

Interactive mode can be conveniently used as a calculator: expressions entered at the interactive prompt (for which a corresponding write function exists) are automatically evaluated and written to stdout. If the expression is non-writable, its type signature will be printed out instead. In either case, the expression can be referred to using the symbol % in the next line input at the prompt. For example:

```
> 2+3
5
> %*4
20
> 1/%
0.05
> sin(%)
0.0499791692706783
> currentpicture
<picture currentpicture>
> %.size(200,0)
>
```

The % symbol, when used as a variable, is shorthand for the identifier operator answer, which is set by the prompt after each written expression evaluation.

The following special commands are supported only in interactive mode and must be entered immediately after the prompt:

```
help view the manual;
erase erase currentpicture;
reset reset the Asymptote environment to its initial state, except for changes to the settings module (see [settings], page 159), the current directory (see [cd], page 52), and breakpoints (see Chapter 14 [Debugger], page 166);
```

input FILE

does an interactive reset, followed by the command include FILE. If the file name FILE contains nonalphanumeric characters, enclose it with quotation marks. A trailing semi-colon followed by optional Asymptote commands may be entered on the same line.

quit exit interactive mode (exit is a synonym; the abbreviation q is also accepted unless there exists a top-level variable named q). A history of the most recent 1000 (this number can be changed with the historylines configuration variable) previous commands will be retained in the file .asy/history in the user's

home directory (unless the command-line option -localhistory was specified, in which case the history will be stored in the file .asy_history in the current directory).

Typing ctrl-C interrupts the execution of Asymptote code and returns control to the interactive prompt.

Interactive mode is implemented with the GNU readline library, with command history and auto-completion. To customize the key bindings, see: http://cnswww.cns.cwru.edu/php/chet/readline/readline.html

The file asymptote.py in the Asymptote system directory provides an alternative way of entering Asymptote commands interactively, coupled with the full power of Python. Copy this file to your Python path and then execute from within Python the commands

```
from asymptote import *
g=asy()
g.size(200)
g.draw("unitcircle")
g.send("draw(unitsquare)")
g.fill("unitsquare, blue")
g.clip("unitcircle")
g.label("\"$0$\", (0,0), SW")
```

11 Graphical User Interface

In the event that adjustments to the final figure are required, the preliminary Graphical User Interface (GUI) xasy included with Asymptote allows you to move graphical objects and draw new ones. The modified figure can then be saved as a normal Asymptote file.

11.1 GUI installation

As xasy is written in the interactive scripting language Python/TK, it requires Python (http://www.python.org), the Python Imaging Library (http://www.pythonware.com/products/pil/), and the tkinter package (included with Python under Microsoft Windows) be installed. Fedora Linux users can either install tkinter with the commands

```
yum install tkinter
yum install tk-devel
or manually install the tkinter, tix, tk, and tk-devel packages.
```

Pictures are deconstructed into the PNG image format, which supports full alpha channel transparency. Under Microsoft Windows, this requires Python 2.6.2 and the Python Imaging Library:

```
http://www.python.org/ftp/python/2.6.2/python-2.6.2.msi
http://effbot.org/downloads/PIL-1.1.7.win32-py2.6.exe.

On UNIX systems, place http://effbot.org/downloads/Imaging-1.1.7.tar.gz in the Asymptote source directory, and type (as the root user):

tar -zxf Imaging-1.1.7.tar.gz
cd Imaging-1.1.7
```

11.2 GUI usage

python setup.py install

A wheel mouse is convenient for raising and lowering objects within xasy, to expose the object to be moved. If a wheel mouse is not available, mouse Button-2 can be used to repeatedly lower an object instead. When run from the command line, xasy accepts a command line option -x n, which sets the initial magnification to n.

Deconstruction of compound objects (such as arrows) can be prevented by enclosing them within the commands

```
void begingroup(picture pic=currentpicture);
void endgroup(picture pic=currentpicture);
```

By default, the elements of a picture or frame will be grouped together on adding them to a picture. However, the elements of a frame added to another frame are not grouped together by default: their elements will be individually deconstructed (see [add], page 49).

12 PostScript to Asymptote

The excellent PostScript editor pstoedit (version 3.50 or later; available from http://sourceforge.net/projects/pstoedit/) includes an Asymptote backend. Unlike virtually all other pstoedit backends, this driver includes native clipping, even-odd fill rule, PostScript subpath, and full image support. Here is an example: asy -V /usr/local/share/doc/asymptote/examples/venn.asy

```
pstoedit -f asy venn.eps test.asy
asy -V test
```

If the line widths aren't quite correct, try giving pstoedit the -dis option. If the fonts aren't typeset correctly, try giving pstoedit the -dt option.

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13 Help

A list of frequently asked questions (FAQ) is maintained at

```
http://asymptote.sourceforge.net/FAQ
```

Questions on installing and using Asymptote that are not addressed in the FAQ should be sent to the Asymptote forum:

```
http://sourceforge.net/projects/asymptote/forums/forum/409349
```

Including an example that illustrates what you are trying to do will help you get useful feedback. LaTeX problems can often be diagnosed with the -vv or -vvv command-line options. Contributions in the form of patches or Asymptote modules can be posted here:

```
http://sourceforge.net/tracker/?atid=685685&group_id=120000
```

To receive announcements of upcoming releases, please subscribe to Asymptote at

```
http://freshmeat.net/projects/asy
```

If you find a bug in Asymptote, please check (if possible) whether the bug is still present in the latest Subversion developmental code (see Section 2.8 [Subversion], page 8) before submitting a bug report. New bugs can be submitted using the Bug Tracking System at

```
http://sourceforge.net/projects/asymptote
```

To see if the bug has already been fixed, check bugs with Status Closed and recent lines in http://asymptote.sourceforge.net/ChangeLog

Asymptote can be configured with the optional GNU library libsigsegv, available from http://libsigsegv.sourceforge.net, which allows one to distinguish user-generated Asymptote stack overflows (see [stack overflow], page 64) from true segmentation faults (due to internal C++ programming errors; please submit the Asymptote code that generates such segmentation faults along with your bug report).

14 Debugger

Asymptote now includes a line-based (as opposed to code-based) debugger that can assist the user in following flow control. To set a break point in file file at line line, use the command

```
The optional argument s may be used to conditionally set the variable ignore in plain_debugger.asy to true. For example, the first 10 instances of this breakpoint will be ignored (the variable int count=0 is defined in plain_debugger.asy): stop("test",2,quote{ignore=(++count <= 10);});
```

To set a break point in file file at the first line containing the string text, use void stop(string file, string text, code s=quote{});

To list all breakpoints, use:

void breakpoints();

To clear a breakpoint, use:

void clear(string file, int line);

To clear all breakpoints, use:

void clear();

The following commands may be entered at the debugging prompt:

void stop(string file, int line, code s=quote{});

```
h
            help;
            continue execution;
С
            step to the next instruction;
i
            step to the next executable line;
s
            step to the next executable line in the current file;
n
f
            step to the next file;
            return to the file associated with the most recent breakpoint;
r
t
            toggle tracing (-vvvvv) mode;
            quit debugging and end execution;
q
            exit the debugger and run to completion.
```

Arbitrary Asymptote code may also be entered at the debugging prompt; however, since the debugger is implemented with eval, currently only top-level (global) variables can be displayed or modified.

The debugging prompt may be entered manually with the call void breakpoint(code s=quote{});

15 Acknowledgments

Financial support for the development of Asymptote was generously provided by the Natural Sciences and Engineering Research Council of Canada, the Pacific Institute for Mathematical Sciences, and the University of Alberta Faculty of Science.

We also would like to acknowledge the previous work of John D. Hobby, author of the program MetaPost that inspired the development of Asymptote, and Donald E. Knuth, author of TEX and MetaFont (on which MetaPost is based).

The authors of Asymptote are Andy Hammerlindl, John Bowman, and Tom Prince. Sean Healy designed the Asymptote logo. Other contributors include Michail Vidiassov, Radoslav Marinov, Orest Shardt, Chris Savage, Philippe Ivaldi, Olivier Guibé, Jacques Pienaar, Mark Henning, Steve Melenchuk, Martin Wiebusch, and Stefan Knorr.

!	<
!	<
%	=
%	==
&	> 60
&	>=60
&&60	?
*	?60
*	^
*=61	^- 60 ^= 61 ^^ 12
+	
+	
-	2
	2D graphs
-=61	3
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	n1

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Airy	autoimport	
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alias 71	axialshade 1	6
align	axis	2
Align	azimuth	6
all		
Allow	To the state of th	
and	В	
AND	babel9	4
angle	background color	
animate	BackView	
animation		
-	Bar	
annotate	Bar3	
antialias	Bars 1	
antialiasing 6	Bars3	
append	barsize 1	
append 69	base modules	7
arc 30, 133	basealign 4	1
Arc	baseline	9
ArcArrow	batch mode	9
ArcArrow3	beep 3	0
ArcArrows	BeginArcArrow	
ArcArrows3	BeginArcArrow3	
arclength	BeginArrow	
arctime	BeginArrow3	
arguments	BeginBar1	
arithmetic operators	BeginBar3	
array	BeginDotMargin	
array iteration	BeginDotMargin3	
arrays	BeginMargin 1	
arrow	BeginMargin3	
Arrow	BeginPenMargin	
arrow keys9	BeginPenMargin2	
Arrow3	BeginPenMargin3	
arrows	BeginPoint	9
Arrows	Bessel 6	7
Arrows3	bevel	8
as	beveljoin	0
asin 67	Bezier curves	2
aSin 67	bezulate	0
asinh	Bi	
Aspect	Bi_deriv	
assert	Billboard	
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