

UNIVERSITY OF BRISTOL

JANUARY 2018 Examination Period

FACULTY OF ENGINEERING

**Examination for the Degree of
Bachelor and Master of Engineering and Bachelor and Master of Science**

**COMS-30002(J)
CRYPTOGRAPHY A**

**TIME ALLOWED:
2 Hours**

This paper contains *three* questions.
All answers will be used for assessment.
The maximum for this paper is *50 marks*.

Other Instructions:

- 1. Calculators must have the Faculty of Engineering Seal of Approval.**

TURN OVER ONLY WHEN TOLD TO START WRITING

Q1. For each of the questions below, four possible answers are presented. Select *all* the answers that you believe apply, or write “none” if you believe none apply. You do not need to justify your answer.

For each question, you can receive up to 3 points, with 3 points only for the perfect answer and one point deducted per incorrect classification, to a minimum of 0 points per question (e.g. if the correct answer is “A and B” then answering “B” leads to 2 points, whereas answering “B and C” only leads to 1 point).

[15 marks]

- (a) Which of the following modes most closely mirrors the one-time pad?
 - A. CTR
 - B. CBC
 - C. CFB
 - D. OFB
- (b) Which of the following statements is accurate?
 - A. AES is an SP Network
 - B. AES is a Feistel cipher
 - C. AES is an iterated cipher
 - D. AES uses key-whitening
- (c) In the sentences below, “harder than” should be interpreted as “known to be equally hard as or strictly harder than”.
 - A. Solving the DDH problem is harder than solving DLP
 - B. Solving the DDH problem is harder than solving the CDH problem
 - C. Solving the CDH problem is harder than solving DLP
 - D. Solving DLP is harder than solving the DDH problem.
- (d) Which of the following schemes are homomorphic?
 - A. Vanilla ElGamal
 - B. Vanilla RSA Encryption
 - C. RSA-OAEP
 - D. Hybrid ElGamal
- (e) The Chinese Remainder Theorem is commonly used to speed up
 - A. RSA encryption
 - B. RSA decryption
 - C. ElGamal encryption
 - D. ElGamal decryption

Q2. The one-time pad can be proven to be perfectly secret.

(a) Describe the three algorithms Kg, Enc, and Dec of the one-time pad.

[3 marks]

(b) Give the definition of perfect secrecy as a formal, probabilistic statement and describe in words what that statement intuitively captures.

[3 marks]

(c) There is an equivalent formalisation of perfect secrecy. Provide that statement and its intuitive meaning.

[2 marks]

(d) The one-time pad is seldom used directly and on its own in practice, say for secure e-mail. Why is this?

[5 marks]

(e) Imagine that one would create OTP-MAC in a similar way to CBC-MAC, by encrypting a message of arbitrary length and outputting the final 128 bits (padded with zeroes if needed) as the tag. Why is this OTP-MAC a bad idea?

[2 marks]

Q3. Schnorr signatures are a way of creating signature scheme based on the discrete logarithm problem in Schnorr subgroups of \mathbb{Z}_p^* . Key generation and signing work as follows.

Key generation Kg Selects random 2048-bit p and 256-bit q prime numbers such that q divides $p - 1$. It selects a random element $g \in \mathbb{Z}_p^*$ of order q . Let $G_q \subseteq \mathbb{Z}_p^*$ be the group of order q generated by g and let $H : G_q \times \{0, 1\}^* \rightarrow \mathbb{Z}_q$ be a hash function. Finally, it selects a random exponent $x \in \mathbb{Z}_q$ and sets $h \leftarrow g^x \bmod p$. Publish (p, q, g, h, H) as the verification key **vk** and keep (p, q, g, x, H) as the private signing key **sk**.

Signing Sign Takes as input the private signing key **sk** $= (p, q, g, x, H)$ and a message $m \in \{0, 1\}^*$. It selects a random element $w \in \mathbb{Z}_q$ and sets $a \leftarrow g^w \bmod p$ followed by $c \leftarrow H(a, m)$. Set $r \leftarrow w - cx \bmod q$. Return (c, r) as the signature on m .

With a suitable verification algorithm, Schnorr signatures can be proven secure—for some relevant notion of security—in the random oracle model based on the discrete logarithm problem.

(a) State the discrete logarithm problem.

[2 marks]

(b) Describe and motivate a relevant security notion for signature schemes.

[6 marks]

(c) In the security reduction, what component of the signature scheme would be modelled by the random oracle?

[1 mark]

(d) Describe a suitable verification algorithm (hint: recompute a).

[3 marks]

For a chosen-prefix preimage attack against the hash function H , an adversary is given a target digest $z \in \mathbb{Z}_q$ and target prefix $a \in G_q$, and has to find an m such that $z = H(a, m)$.

(e) Prove that if H is collision resistant, then it is also resistant against chosen-prefix preimage attacks.

[4 marks]

(f) Show how susceptibility of H against chosen-prefix preimage attacks leads to a vulnerability against the signature scheme; name the attack against the signature scheme as precisely as possible.

[4 marks]

END OF PAPER