

## Problem A. Max or Min

Input file:            **standard input**  
 Output file:        **standard output**  
 Time limit:         1 second  
 Memory limit:      256 megabytes

Kevin has  $n$  integers  $a_1, a_2, \dots, a_n$  arranged in a **circle**. That is, the numbers  $a_i$  and  $a_{i+1}$  ( $1 \leq i < n$ ) are neighbors. The numbers  $a_1$  and  $a_n$  are neighbors as well. Therefore, each number has exactly two neighbors.

In one minute, Kevin can set  $a_i$  to the minimum among three numbers:  $a_i$  and its two neighbors. Alternatively, Kevin can set  $a_i$  to the maximum among the same numbers. For example, if  $a_i = 5$  and  $a_i$  has two neighbors 3 and 2, and Kevin performs the minimum operation,  $a_i$  will be equal to 2. However, if he performs the maximum operation,  $a_i$  will remain 5.

For each  $x$  from 1 to  $m$ , find the minimum number of minutes to make all numbers equal  $x$ , or determine that it is impossible to do so.

### Input

The first line contains two integers  $n$  and  $m$  ( $3 \leq n \leq 2 \cdot 10^5$ ,  $1 \leq m \leq 2 \cdot 10^5$ ) — the number of integers in the circle, and the number of integers you need to find answers for.

The second line contains  $n$  integers  $a_1, a_2, \dots, a_n$  ( $1 \leq a_i \leq m$ ) — the integers in the circle.

### Output

Print  $m$  integers. The  $i$ -th integer should be equal to the minimum number of minutes that are needed to make all numbers equal  $i$  or  $-1$  if it's impossible.

### Example

standard input	standard output
7 5 2 5 1 1 2 3 2	5 5 7 -1 6

### Note

To make all numbers equal 2 Kevin needs at least 5 minutes. One of the possible sequence of operations:

1. Apply min operation to  $a_6$ , it will be equal to 2.
2. Apply max operation to  $a_4$ , it will be equal to 2.
3. Apply max operation to  $a_3$ , it will be equal to 5.
4. Apply min operation to  $a_2$ , it will be equal to 2.
5. Apply min operation to  $a_3$ , it will be equal to 2.

## Problem B. Level Up

Input file:            `standard input`  
Output file:         `standard output`  
Time limit:          2 seconds  
Memory limit:       256 megabytes

Being a fan of MMORPGs, Steve was really excited when World of Warcraft Classic was announced. He started playing from the first day and now has only two levels to go till the maximum level. Of course, he doesn't have as much time as he used to have when the game first appeared, so he really wants to finish these two levels as fast as possible.

In order to pass the first level, Steve needs  $s_1$  experience. Only after he gains it, he can move to the second level, in which he needs to get other  $s_2$  experience to pass it.

Steve has the list of  $n$  available quests. He knows that he may finish two levels with these quests. In order to pass the  $i$ -th quest, Steve needs  $t_i$  minutes. As a result, he will get  $x_i$  experience for this quest.

When Steve finishes a quest that takes him to the next level, the extra experience overflow is subtracted from the next level's required experience. Once he levels up, all the quests that are left will offer less experience  $y_i$ , but they will also require less time  $r_i$ .

Note that if Steve finishes a quest, he **can't** repeat this quest anymore (even in another level).

Given the list of quests, help Steve choose the order in which he will be doing the quests in order to finish the last two levels as fast as possible.

### Input

The first line contains three integers  $n, s_1, s_2$  ( $1 \leq n, s_1, s_2 \leq 500$ ) — the number of quests, experience required for the first level and experience required for the second level.

Each of the next  $n$  lines contains four integers  $x_i, t_i, y_i, r_i$  ( $1 \leq y_i < x_i \leq 500, 1 \leq r_i < t_i \leq 10^9$ ) where  $x_i$  and  $y_i$  are the experience you will gain from the  $i$ -th quest on the 1-st and 2-nd level respectively and  $t_i$  and  $r_i$  are the number of minutes you need to spend in order to pass this quest on the 1-st and 2-nd level respectively,

### Output

Print one number, representing the minimum number of minutes that Steve needs to finish two levels, or  $-1$  if there is no way to complete quests to finish both levels.

## Examples

standard input	standard output
2 100 100 100 100 10 10 101 11 100 10	110
4 20 20 40 1000 20 20 6 6 5 5 10 10 1 1 10 10 1 1	40
2 20 5 10 10 5 5 10 10 5 5	-1

## Problem C. Find the Array

Input file:           standard input  
Output file:         standard output  
Time limit:          2 seconds  
Memory limit:       256 megabytes

*This is an interactive problem.*

There is an array  $a$  of length  $n$ , consisting of **distinct** integers. It is guaranteed that every element of the array is a positive integer less than or equal to  $10^9$ . You have to find out the values of all the elements of it.

To do so, you can make up to 30 queries of the following two types:

- “1  $i$ ” ( $1 \leq i \leq n$ ) — ask the value of  $a_i$
- “2  $k\ i_1, i_2, \dots, i_k$ ” ( $2 \leq k \leq n$ ,  $1 \leq i_j \leq n$ , all  $i_j$  must be distinct) — the number  $k$  and  $k$  positions in the array. As the answer to this query you will receive  $\frac{k \cdot (k-1)}{2}$  integers —  $|a_{i_c} - a_{i_d}|$  for every  $c < d$ . In other words, you will receive  $\frac{k \cdot (k-1)}{2}$  absolute values of differences between all pairs of elements that are on positions  $i_1, i_2, \dots, i_k$ . Note that the answer on query 2 is randomly shuffled.

Once you know the answer, print it using the following query:

- “3  $a_1, a_2, \dots, a_n$ ” ( $1 \leq a_i \leq 10^9$ ) — the elements of the array  $a$ . After this query, your program must terminate. This query doesn’t count (i.e. you can make up to 30 queries of either of the first two types plus 1 query of the third type).

## Interaction Protocol

At the beginning, your program should read one integer  $n$  ( $1 \leq n \leq 250$ ) — the number of elements.

In order to make a query of the first type, print “1  $i$ ” ( $1 \leq i \leq n$ ). After this query, read one integer — the value of  $a_i$ .

In order to make a query of the second type, print “2  $k$ ” ( $2 \leq k \leq n$ ). Then in the same line print  $k$  space-separated distinct integers —  $i_1, i_2, \dots, i_k$ , ( $1 \leq i_j \leq n$ ). After this query read  $\frac{k \cdot (k-1)}{2}$  integers —  $|a_{i_c} - a_{i_d}|$  for every  $c < d$ . These values will be given in random order.

Once you know the answer, print “3”. Then in the same line, print  $n$  space-separated integers —  $a_1, a_2, \dots, a_n$  ( $1 \leq a_i \leq 10^9$ ). Your program must terminate after this query.

If for either of two first queries you get one number  $-1$  as the answer, then it means that you made more queries than allowed, or made an invalid query. Your program should immediately terminate (for example, by calling `exit(0)`). You will receive “**Wrong Answer**”. If you ignore this, you can get other verdicts since your program will continue to read from a closed stream.

After printing a query do not forget to output end of line and flush the output. Otherwise, you will get `Wall time-limit exceeded`. To do this, use:

- `fflush(stdout)` or `cout.flush()` in C++;
- `System.out.flush()` in Java;

- `stdout.flush()` in Python.

Precisely follow this format of interaction.

## Example

standard input	standard output
3	1 1
1	1 2
2	2 3 1 2 3
4 3 1	3 1 2 5

## Note

In the first query of type 1, we ask the value of  $a_1$  and receive 1 as the answer.

In the second query of type 2, we ask the value of  $a_2$  and receive 2 as the answer.

In the query of type 2, we ask all the possible differences between the elements of array with indexes 1, 2 and 3. And we get array 4, 3, 1 as the result. We know that the array contains values  $|a_1 - a_2|$ ,  $|a_1 - a_3|$ ,  $|a_2 - a_3|$ . Since we already know that  $|a_2 - a_1| = 1$ , one of the following is true:  $|a_1 - a_3| = 3$  and  $|a_2 - a_3| = 4$  or  $|a_2 - a_3| = 3$  and  $|a_1 - a_3| = 4$ . The only case that is possible, taking into account the constraints of the problem, is when  $|a_1 - a_3| = 4$  and  $|a_2 - a_3| = 3$  with  $a_3 = 5$ .

Since we know the values of all the elements of the array, we print them in the last query.

## Problem D. Cycle String?

Input file:           standard input  
 Output file:        standard output  
 Time limit:         1 second  
 Memory limit:      256 megabytes

Great wizard gave Alice and Bob a cycle string of length  $2 \cdot n$ , which has no repeated substrings of length  $n$ . In a cycle string, character  $s_{i+1}$  comes after  $s_i$ . Also,  $s_1$  comes after  $s_{2n}$ .

Unfortunately, evil gin shuffled all the symbols of the string. Help Alice and Bob restore the original string so that the above condition is satisfied.

### Input

The first line contains one string  $s$  of length  $2 \cdot n$  ( $2 \leq 2 \cdot n \leq 1\,000\,000$ ) which consists only of the lowercase Latin letters.

### Output

Print “NO” (without quotes) to the first line if it is impossible to restore the string so that the condition is satisfied. Otherwise, in the first line print “YES” (without quotes).

In the second line print one string — the restored string.

If there are multiple answers, print any.

### Examples

standard input	standard output
cbbabcbacbb	YES abbabcbccb
aa	NO
afedbc	YES afedbc

### Note

In the first example, substrings of the restored string are: “abbab”, “bbabc”, “babcb”, “abcbc”, “bcbcc”, “cbccb”, “bccba”, “ccbab”, “cbabb”, “babba”.

Note that the first example does not contain repetitions, however it can be rewritten as another cycle with no repetitions. Thus, the solution is not unique — the given example is also a correct solution.

In the second example, it is impossible to restore the string so that no repetition exists.

In the third example, there is no need to change anything.

## Problem E. Life Transfer

Input file:            `standard input`  
Output file:         `standard output`  
Time limit:          1 second  
Memory limit:       256 megabytes

*Note: “feli” is the local currency.*

In the great city of Nekoresti, there are  $n$  people for which we know their ages:  $a_i$  is the age of the  $i$ -th person. Currently, they are on vacation, so they decided to go on a trip to Pisiev to visit a Koshkseum, a famous museum. They can go either by car or by motorcycle:

- a **car** can transport  $k$  people (one driver which has to be at least  $l_c$  years old and  $k - 1$  passengers). The cost to rent a car is  $p_c$  feli.
- a **motorcycle** can transport only one person (which has to be at least  $l_m$  years old). The cost to rent a motorcycle is  $p_m$  feli.

Unfortunately, people have money issues, so they decided to consult Mewlin, the great local magician from the city. Using a formidable spell called Mucadabra, Mewlin can transfer age from one person to another. Formally, he can reduce the age  $x$  of a person and increase the age  $y$  of another person by the same amount (so the sum of ages is constant). The cost to transfer 1 unit of age is  $t$  feli. For magic medical reasons, the age of a person cannot be changed by more than  $d$  years (if the initial age is  $x$ , his age must be at least  $x - d$  and at most  $x + d$  at all times). Also, the age cannot go below 1 year old.

Help the people from Nekoresti to spend as little money as possible, so they can arrive in Pisiev.

### Input

The first line contains two integers  $n$  and  $k$  ( $1 \leq n, k \leq 10^5$ ) — the number of people and the maximum number of people that can be in one car.

The second line contains four integers  $l_c$ ,  $p_c$ ,  $l_m$  and  $p_m$  ( $1 \leq l_m < l_c \leq 10^5$ ,  $1 \leq p_m < p_c \leq 10^5$ ) — the minimum needed age to drive a car; the price of renting one car; the minimum needed age to drive a motorcycle and the price of renting one motorcycle.

The third line contains two integers  $t$  and  $d$  ( $0 \leq t, d \leq 10^5$ ) — the price of transferring one year and the maximum number of times the spells can be applied per each person.

The second line contains  $n$  integers  $a_1, a_2, \dots, a_n$  ( $1 \leq a_i \leq 10^5$ ) — the age of the  $i$ -th person.

### Output

Print one number, the smallest amount of feli the people need to spend in order for them to reach their destination. If there is no such solution, print  $-1$ .

## Examples

standard input	standard output
<pre>2 2 18 1000 16 1 5 3 16 15</pre>	<pre>1010</pre>
<pre>2 2 23 10 15 5 2 2 9 20</pre>	<pre>-1</pre>



## Problem F. Game on a Tree

Input file:            **standard input**  
 Output file:         **standard output**  
 Time limit:          1 second  
 Memory limit:       256 megabytes

Alice and Bob play a game on a tree. Initially, all nodes are white.

Alice is the first to move. She chooses any node and put a chip on it. The node becomes black. After that players take turns. In each turn, a player moves the chip from the current position to an ancestor or descendant node, as long as the node is not black. This node also becomes black. The player who cannot move the chip loses.

Who wins the game?

An *ancestor* of a node  $v$  in a rooted tree is any node on the path between  $v$  and the root of the tree.

A *descendant* of a node  $v$  in a rooted tree is any node  $w$  such that node  $v$  is located on the path between  $w$  and the root of the tree.

We consider that the root of the tree is 1.

### Input

The first line contains one integer  $n$  ( $1 \leq n \leq 100\,000$ ) — the number of nodes.

Each of the next  $n - 1$  lines contains two integers  $u$  and  $v$  ( $1 \leq u, v \leq n$ ) — the edges of the tree. It is guaranteed that they form a tree.

### Output

In a single line, print “Alice” (without quotes), if Alice wins. Otherwise, print “Bob”.

### Examples

standard input	standard output
4 1 2 2 3 3 4	Bob
7 2 1 2 6 1 3 2 5 7 2 2 4	Alice

### Note

In the first test case, the tree is a straight line and has 4 nodes, so Bob always can choose the last white node.

In the second test case, the optimal strategy for Alice is to place the chip on 3. This node will become black. Bob has to choose the node 1. Alice can choose any of 4, 5, 6, or 7. Bob can only choose 2. Alice chooses any of the white sons of 2, and Bob cannot make a move.

## Problem G. Projection

Input file:           standard input  
 Output file:        standard output  
 Time limit:         1 second  
 Memory limit:      512 megabytes



Everybody knows that you are a **TensorFlow** fan. Therefore, you've been challenged to recreate the **TensorFlow** logo from two projections.

Consider that you have a 3D volume,  $n \times m \times h$ , and two projections (two matrices with dimensions  $n \times m$  and  $n \times h$  with elements 0 and 1). You are asked to compute a possible sets of cubes that must be placed inside the 3D volume such that the 3D object created with the cubes throws the shadows specified by the projection-matrices, when the light comes from left and front. If it is not possible, just print  $-1$ . If it is possible you must find exactly two sets, one with the **maximum** amount of cubes and one with the **minimum** amount. You can assume there is no gravitation (the cubes are located inside the 3D volume exactly where they are placed, without requiring any support). We assume that 1 represents shadow and 0 represents light.

If there are multiple such solutions, you must output the minimum lexicographic one. One solution  $A$  is lexicographically smaller than another solution  $b$  if the first number that differs between the two solutions is smaller in  $a$  than in  $b$ .

For example, solution  $[(0, 0, 0), (1, 1, 1)]$  is smaller than  $[(1, 1, 1), (0, 0, 0)]$ .

### Input

The first line contains three integers separated by a single space  $n, m, h$  ( $1 \leq n, m, h \leq 100$ ) — the volume dimensions.

Each of the next  $n$  lines contains  $m$  characters, each being either 1 or 0 representing either a shadow area (1) or a light area (0), describing the projection from the light in the front.

Each of the next  $n$  lines contains  $h$  characters, with the same format as above, describing the projection from the light on the left.

### Output

The output should contain on the first line one number, either  $-1$  if there is no solution or  $k_{max}$  representing the maximum number of cubes we can assign in the volume that will generate the two projections given in the input.

The next  $k_{max}$  lines should contain triplets of numbers  $x, y, z$  ( $0 \leq x < n, 0 \leq y < m, 0 \leq z < h$ ) representing the cubes chosen in the lexicographically smallest solution with maximum number of cubes.

Then, only if there is a solution, one more line follows containing  $k_{min}$ , the minimum number of cubes we can assign in the volume that will generate the two projections given in the input.

After that, the next  $k_{min}$  lines should contain triplets of numbers  $x, y, z$  ( $0 \leq x < n$ ,  $0 \leq y < m$ ,  $0 \leq z < h$ ) representing the cubes in the lexicographically smallest solution with minimum number of cubes.

## Examples

standard input	standard output
5 3 3 111 010 010 010 010 111 100 110 100 100	14 0 0 0 0 0 1 0 0 2 0 1 0 0 1 1 0 1 2 0 2 0 0 2 1 0 2 2 1 1 0 2 1 0 2 1 1 3 1 0 4 1 0 8 0 0 0 0 1 1 0 2 2 1 1 0 2 1 0 2 1 1 3 1 0 4 1 0
2 2 2 00 00 11 11	-1
2 3 2 101 011 10 11	6 0 0 0 0 2 0 1 1 0 1 1 1 1 2 0 1 2 1 4 0 0 0 0 2 0 1 1 0 1 2 1

## Note

A cube at coordinates  $(x, y, z)$  will generate a shadow at line  $x$  and column  $y$  in the  $n \times m$  projection and line  $x$  and column  $z$  in the  $n \times h$  projection (indexed from 0).

## Problem H. Tree Permutations

Input file: standard input  
Output file: standard output  
Time limit: 1 second  
Memory limit: 256 megabytes

Once upon a time, Mr. Cool created a tree (an undirected graph without cycles) of  $n$  vertices, by assigning to each vertex  $i > 1$  two numbers:  $p_i < i$  — the direct ancestor of vertex  $i$  and  $w_i$  — the weight of the edge between vertex  $i$  and  $p_i$ . Vertex 1 is the root, so it does not have any ancestors.

You wanted to know what tree did Mr. Cool build, but Mr. Cool refused to tell this, but he gave you a tip:

He wrote all these numbers in one line. That's how he got array  $b$  of length  $2 \cdot n - 2$ .

$$b = [p_2, w_2, p_3, w_3, \dots, p_{n-1}, w_{n-1}, p_n, w_n]$$

Then he randomly shuffled it. That's how he got array  $a$ , and Mr. Cool presented you with it.

Since it is impossible to restore the tree knowing only values of array  $a$ , you decided to solve a different problem.

Let's call a tree  **$k$ -long**, if there are exactly  $k$  edges on the path between vertex 1 and  $n$ .

Let's call a tree  **$k$ -perfect**, if it is  $k$ -long and the sum of the weights of the edges on the path between vertex 1 and vertex  $n$  is maximal among all possible  $k$ -long trees that Mr. Cool could build.

Your task is to print the sum of the weights of the edges on the path between vertex 1 and vertex  $n$  for all possible  $k$ -perfect trees or print  $-1$  if a certain  $k$ -long tree could not be built by Mr. Cool.

### Input

The first line contains one integer  $n$  ( $2 \leq n \leq 10^5$ ) — the number of the vertices in the tree.

The second line contains  $2 \cdot n - 2$  integers  $a_1, a_2, \dots, a_{2n-2}$  ( $1 \leq a_i \leq n - 1$ ) — the elements of array  $a$ .

### Output

In one line, print  $n - 1$  space-separated integers  $w_1, w_2, w_3, \dots, w_{n-1}$ , where  $w_k$  — the sum of the weights of the edges on the path between vertex 1 and vertex  $n$  in a  $k$ -perfect tree. If there is no  $i$ -long tree, then  $w_i$  should be equal to  $-1$ .

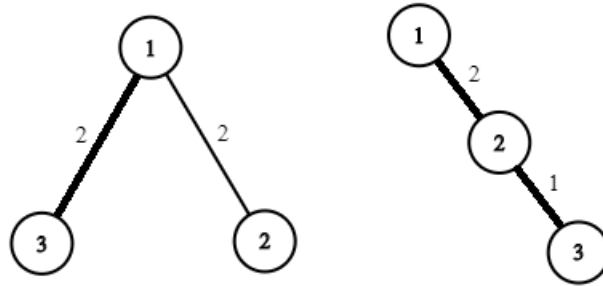
### Examples

standard input	standard output
3 1 1 2 2	2 3
3 2 2 2 2	-1 -1
6 1 4 5 4 4 4 3 4 4 2	-1 -1 -1 17 20

### Note

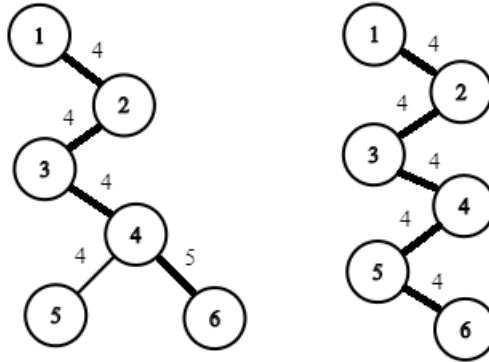
In the first example, the 1-perfect tree is defined by array  $[1, 2, 1, 2]$  (i.e.  $p_2 = 1, w_2 = 2, p_3 = 1, w_3 = 2$ ). The 2-perfect tree is defined by array  $[1, 2, 2, 1]$  (i.e.  $p_2 = 1, w_2 = 2, p_3 = 2, w_3 = 1$ ). Here are illustrations

of the 1-perfect tree and the 2-perfect tree respectively (path from vertex 1 to vertex  $n$  is drawn with bold lines):



In the second example, there are no  $k$ -perfect trees, that can be obtained by permuting array  $a$ .

In the third example, only 4-perfect tree and 5-perfect tree can be obtained. These are defined by arrays  $[1, 4, 2, 4, 3, 4, 4, 4, 4, 5]$  and  $[1, 4, 2, 4, 3, 4, 4, 4, 5, 4]$  respectively. Here are illustrations of them:



## Problem I. Absolute Game

Input file:            **standard input**  
 Output file:        **standard output**  
 Time limit:         1 second  
 Memory limit:      256 megabytes

Alice and Bob are playing a game. Alice has an array  $a$  of  $n$  integers, Bob has an array  $b$  of  $n$  integers. In each turn, a player removes one element of his array. Players take turns alternately. Alice goes first.

The game ends when both arrays contain exactly one element. Let  $x$  be the last element in Alice's array and  $y$  be the last element in Bob's array. Alice wants to maximize the absolute difference between  $x$  and  $y$  while Bob wants to minimize this value. Both players are playing optimally.

Find what will be the final value of the game.

### Input

The first line contains a single integer  $n$  ( $1 \leq n \leq 1\,000$ ) — the number of values in each array.

The second line contains  $n$  space-separated integers  $a_1, a_2, \dots, a_n$  ( $1 \leq a_i \leq 10^9$ ) — the numbers in Alice's array.

The third line contains  $n$  space-separated integers  $b_1, b_2, \dots, b_n$  ( $1 \leq b_i \leq 10^9$ ) — the numbers in Bob's array.

### Output

Print the absolute difference between  $x$  and  $y$  if both players are playing optimally.

### Examples

standard input	standard output
4 2 14 7 14 5 10 9 22	4
1 14 42	28

### Note

In the first example, the  $x = 14$  and  $y = 10$ . Therefore, the difference between these two values is 4.

In the second example, the size of the arrays is already 1. Therefore,  $x = 14$  and  $y = 42$ .

## Problem J. Graph and Cycles

Input file:           standard input  
Output file:         standard output  
Time limit:          2 seconds  
Memory limit:       256 megabytes

There is an undirected weighted complete graph of  $n$  vertices where  $n$  is odd.

Let's define a *cycle-array* of size  $k$  as an array of edges  $[e_1, e_2, \dots, e_k]$  that has the following properties:

- $k$  is greater than 1.
- For any  $i$  from 1 to  $k$ , an edge  $e_i$  has exactly one common vertex with edge  $e_{i-1}$  and exactly one common vertex with edge  $e_{i+1}$  and these vertices are distinct (consider  $e_0 = e_k$ ,  $e_{k+1} = e_1$ ).

It is obvious that edges in a cycle-array form a cycle.

Let's define  $f(e_1, e_2)$  as a function that takes edges  $e_1$  and  $e_2$  as parameters and returns the maximum between the weights of  $e_1$  and  $e_2$ .

Let's say that we have a cycle-array  $C = [e_1, e_2, \dots, e_k]$ . Let's define the *price of a cycle-array* as the sum of  $f(e_i, e_{i+1})$  for all  $i$  from 1 to  $k$  (consider  $e_{k+1} = e_1$ ).

Let's define a *cycle-split* of a graph as a set of non-intersecting cycle-arrays, such that the union of them contains all of the edges of the graph. Let's define the *price of a cycle-split* as the sum of prices of the arrays that belong to it.

There might be many possible cycle-splits of a graph. Given a graph, your task is to find the cycle-split with the minimum price and print the price of it.

### Input

The first line contains one integer  $n$  ( $3 \leq n \leq 999$ ,  $n$  is odd) — the number of nodes in the graph.

Each of the following  $\frac{n \cdot (n-1)}{2}$  lines contain three space-separated integers  $u$ ,  $v$  and  $w$  ( $1 \leq u, v \leq n, u \neq v, 1 \leq w \leq 10^9$ ), meaning that there is an edge between the nodes  $u$  and  $v$  that has weight  $w$ .

### Output

Print one integer — the minimum possible price of a cycle-split of the graph.



## Examples

standard input	standard output
3 1 2 1 2 3 1 3 1 1	3
5 4 5 4 1 3 4 1 2 4 3 2 3 3 5 2 1 4 3 4 2 2 1 5 4 5 2 4 3 4 2	35

## Note

Let's enumerate each edge in the same way as they appear in the input. I will use  $e_i$  to represent the edge that appears  $i$ -th in the input.

The only possible cycle-split in the first sample is  $S = \{[e_1, e_2, e_3]\}$ .  $f(e_1, e_2) + f(e_2, e_3) + f(e_3, e_1) = 1 + 1 + 1 = 3$ .

The optimal cycle-split in the second sample is  $S = \{[e_3, e_8, e_9], [e_2, e_4, e_7, e_{10}, e_5, e_1, e_6]\}$ . The price of  $[e_3, e_8, e_9]$  is equal to 12, the price of  $[e_2, e_4, e_7, e_{10}, e_5, e_1, e_6]$  is equal to 23, thus the price of the split is equal to 35.

## Problem K. Stranded Robot

Input file:            **standard input**  
 Output file:        **standard output**  
 Time limit:         4 seconds  
 Memory limit:      256 megabytes

A robot is stranded in the wreckage of an interstellar spaceship. There is a teleporter somewhere in the wreckage that can take the poor robot to safety.

The spaceship is spinning out of control along all axes. A nearby sun is shining light on the wreckage. The ship is also equipped with an artificial gravity generator. The artificial gravity always pulls the robot away from the sun, irrespective of the ship's orientation.

The robot is equipped with solar panels and must rely on solar power to move through the wreckage. When parts of the wreckage block out the sun, the robot is immobile. However, the robot always anchors itself after every move and does not risk thrashing about or falling into the void of space.

The wreckage and the area around it is represented by a three-dimensional grid  $a$  of size  $m \times n \times p$ . Each individual block can either be taken up by part of the ship or by vacuum. The ship blocks can be disconnected.

The robot starts off anchored to a piece of the ship. The robot chooses on its own when to move and when to wait for the sun to shine from a convenient direction.

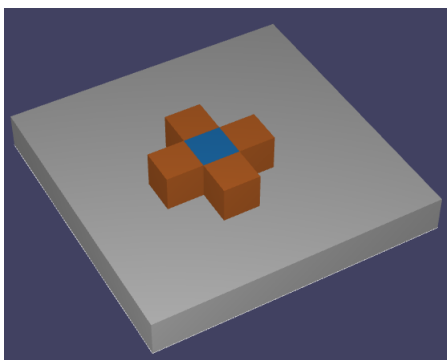
More formally, gravity can pull the robot in one of 6 directions, 2 directions along each of the 3 axes. A cell is lit by the sun if there is no wreckage from said cell in the direction opposite to gravity. Before making each move, the robot can effectively choose the direction in which it is pulled by the gravity. When making a move, both the original and the destination position must simultaneously be lit by the sun.

The following moves are allowed: (the light always shines from above in the following images; the blue block (or darker block in black-and-white printings) represents the robot, and the orange blocks (or lighter blocks) are possible destinations)

### 1. Moving across the floor

If the robot is resting on top of a block, it can move to an adjacent position, granted that it is at the same height.

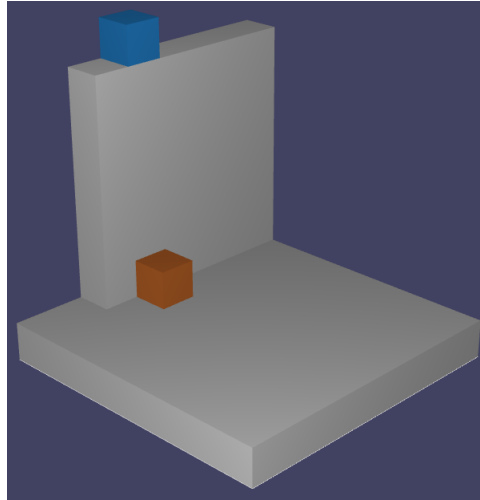
The robot can't move diagonally. The destination must also be lit by the sun.



### 2. Jumping off a cliff

The robot can take a step off a heightened position and subsequently drop down. There is no restriction on how long the drop is.

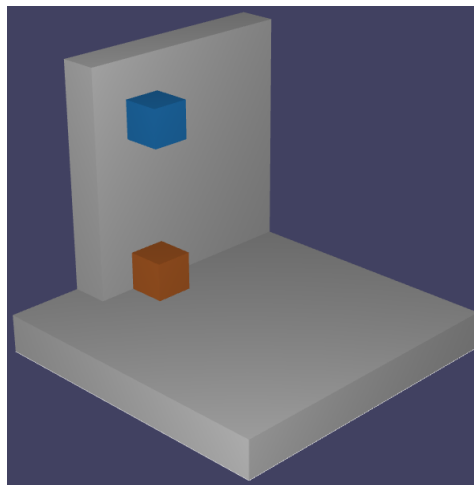
The robot can't drop down into the void of space, or to an unlit position.



### 3. Dropping down

If the robot is lit by the sun and it finds itself hanging, it can drop down. This can happen if the gravity's direction has changed.

The robot can't drop down into the void of space.



The goal is to reach the teleporter, if at all possible, using the smallest number of moves. The robot must be firmly anchored to the ship for the teleporter to work. In other words, the robot must find itself at the teleporter at the end of a valid move, and dropping down through it does not work. The teleporter doesn't block the sun or the robot's movement.

### Input

The first line contains three integers  $m, n, p$  ( $1 \leq m, n, p \leq 500$ ).

The spaceship and the area around the spaceship is described in  $p$  blocks.

The  $k$ -th block describes the block located on the  $k$ -th height. Each block consist of  $n$  lines.

The  $i$ -th line of the  $k$ -th block consists of  $m$  symbols. The  $j$ -th symbol is called  $a_{ijk}$ .

- If  $a_{ijk}$  is “\*”, then it is a full block.
- If  $a_{ijk}$  is “-”, then it is an empty block.
- If  $a_{ijk}$  is “R”, then this block contains the robot. It is guaranteed that there is only one such block. It is guaranteed that the robot is connected to a full block.
- If  $a_{ijk}$  is “T”, then this block contains the teleporter. It is guaranteed that there is only one such block.

## Output

Print the minimal number of moves required to reach the teleporter, or  $-1$  if the teleporter is unreachable.

## Examples

standard input	standard output
2 5 1 R- *- *- *T **	1
3 2 1 R-T ***	2
3 3 1 -R- -*- -T-	-1
5 4 2 -R--- -***** -***** -***** ----- ----- *T--- -----*	5

# The 2019 Southeastern Europe Regional Contest Editorial

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# Problem Editorial: “Max or Min”

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**Editorialist:** Stanislav Bezkorovainyi

Before solving the problem for all  $k$  from 1 to  $m$ , let's try to solve a problem for a fixed number  $k$ .

It's obvious, that if there is no element of array  $a$  that is equal to  $k$ , then the answer is  $-1$ .

Otherwise, the answer always exists. Let's prove it.

At first, let's replace all the elements in the array by the following rule:

- If  $a_i < k$ , then  $a_i := -1$  (replace  $a_i$  with  $-1$ )
- If  $a_i = k$ , then  $a_i := 0$  (replace  $a_i$  with  $0$ ).
- If  $a_i > k$ , then  $a_i := 1$  (replace  $a_i$  with  $1$ )

Now, the problem is equivalent to the previous one, we just have to make elements of the array equal to  $0$ .

Let's call *zebra* a subarray of the array  $a$  that consists of alternating numbers  $-1$  and  $1$ . Examples of zebras are:  $[-1]$ ,  $[1]$ ,  $[-1, 1, -1]$ ,  $[1, -1]$ ,  $[1, -1, 1, -1, 1, -1, 1]$ ,  $[-1, 1, -1, 1, -1, 1, -1, 1, -1]$ . Note, that since the array is arranged in a circle, arrays like, for example,  $[1, -1, -1, -1]$  have zebra  $[-1, 1, -1]$  that starts from the last element.

Since the elements are arranged in a circle, then cyclical shifts do not change it. Let's make necessary number of cyclical shifts to left, so that the first element of the array is equal to  $0$ . Now we know for sure, that there are no zebras that come through index  $1$ . It helps, because it ensures that finding all zebras in our cyclical string is now equivalent to finding all zebras in the string after the shifts. Why do we even need to find these zebras? We'll come to that later.

Let's define  $Z(a)$  as a multiset of lengths of the longest sequential zebras. How we build it:

At first we stick the pointer  $p$  at position  $1$ . Then the following algorithm works until  $p \leq n$ :

- If  $a_p = 0$ , then increase  $p$  by  $1$ . Repeat the algorithm.
- Otherwise, let  $len$  be the length of the longest possible zebra that starts from position  $p$ . Add  $len$  to  $Z(a)$  and increase  $p$  by  $len$ . Repeat the algorithm.

For example, if  $a$  was  $[k, -1, 1, -1, -1, 1, k, -1, -1]$ , then  $Z(a) = \{3, 2, 1, 1\}$  (the longest zebra that starts on position  $2$  has length  $3$ , the longest zebra that starts on position  $5$  has length  $2$ , the longest zebras that start on positions  $8$  and  $9$  have length  $1$ ).

Let's define function  $cnt(a, c)$  that returns number of occurrences of  $c$  in the array  $a$ . Let's define  $f(a)$  as a function that takes the array  $a$  as an argument and returns the following value:

$$f(a) = cnt(a, -1) + cnt(a, 1) + \sum_{z \in Z(a)} \left\lfloor \frac{z}{2} \right\rfloor$$

In other words,  $f(a)$  equals to number of elements that are not equal to  $0$  plus the sum of floored lengths of zebras divided by two.

**Claim.** The answer is equal to  $f(a)$ .

Those, who don't want to read lengthy proofs can skip to **How to implement it?** section. For those, who want to read the proof, here it is:

**Proof.**

- **Claim.**  $f(a) = 0$  if and only if all the elements of the array are equal to  $k$ .

**Proof.** If all the elements are equal to 0, then  $\text{cnt}(a, -1) = \text{cnt}(a, 1) = 0$  and there are definitely no zebras in the array, so  $f(a) = 0$ . It works in the other direction too: if  $f(a) = 0$ , then  $\text{cnt}(a, -1) + \text{cnt}(a, 1) = 0$ , so there are no elements that are not equal to 0, i.e. all of the elements of the array are equal to 0.

**What it gives us?** It means that our problem is equivalent to problem of reducing value of  $f(a)$  of our initial array to 0 in as few operations as possible.

- **Claim.** If we can transform our array  $a$  into array  $b$  in one operation, then  $f(b) \geq f(a) - 1$ . In other words, we can not reduce the value of the function more than by one in an operation.

**Proof.** At first, we should notice, that an operation that changes value of the element that is equal to 0 only increases value of  $\text{cnt}(a, -1) + \text{cnt}(a, 1)$ , thus  $f(b) > f(a)$  if  $b$  was obtained by changing value of an element in the array that is equal to 0.

Now it is left to prove that changing value of an element that is equal to either  $-1$  or  $1$  can not reduce the value of the function more than by one. I'll show you why changing an element that is equal to  $1$  can not do this. The proof for  $-1$  is very similar.

Let's suppose that  $a_i = 1$  for some  $i$ . Now let's look at all possible triples  $[a_{i-1}, a_i, a_{i+1}]$  (since the array is cyclical  $a_0 = a_n, a_{n+1} = a_1$ ). There are 9 possible triples:

- $[1, 1, 1]$ . We can not change  $a_i$  at all.
- $[-1, 1, 1]$ . We can make  $a_i$  equal to  $-1$ . Even though  $a_i$  might belong to a zebra, it is easy to proof that changing it to  $-1$  won't affect  $f(a)$  by more, than 1.
- $[1, 1, -1]$ . The same as the previous one.
- $[-1, 1, -1]$ . We can make  $a_i$  equal to  $-1$ . It doesn't change  $\text{cnt}(a, -1) + \text{cnt}(a, 1)$ , but it is obvious that  $a_i$  belongs to some zebra that starts on position  $p < i$ . By setting value of  $a_i$  as  $-1$  we "split" this zebra into two zebras which start at positions  $p$  and  $i + 1$ . Let's say that their lengths are  $\text{len}_1$  and  $\text{len}_2$  respectively. The length of the zebra before the split is  $\text{len}_1 + \text{len}_2 + 1$  (it included  $a_i$ ). Now,  $f(b) = f(a) - \lfloor \frac{\text{len}_1 + \text{len}_2 + 1}{2} \rfloor + \lfloor \frac{\text{len}_1}{2} \rfloor + \lfloor \frac{\text{len}_2}{2} \rfloor$ . It is easy to prove, that  $\lfloor \frac{\text{len}_1 + \text{len}_2 + 1}{2} \rfloor - \lfloor \frac{\text{len}_1}{2} \rfloor - \lfloor \frac{\text{len}_2}{2} \rfloor$  is either equal to 0 or 1. In order to prove this, you should test how expanding flooring brackets will work for cases, when:  $\text{len}_1$  is even and  $\text{len}_2$  is even,  $\text{len}_1$  is even and  $\text{len}_2$  is odd,  $\text{len}_1$  is odd and  $\text{len}_2$  is even, and both  $\text{len}_1$  and  $\text{len}_2$  are odd.
- $[0, 1, 1]$ . We can make  $a_i$  equal to 0. It will only affect  $\text{cnt}(a, 1)$ , because of that  $f(b) = f(a) - 1$ .
- $[1, 1, 0]$ . The same as the previous one.
- $[0, 1, 0]$ . We can make  $a_i$  equal to 0.  $f(b) = f(a) - 1$  as well.
- $[-1, 1, 0]$ . We can make  $a_i$  equal to  $-1$ . It will not affect  $\text{cnt}(a, -1) + \text{cnt}(a, 1)$ . It is also obvious that  $a_i$  belongs to some zebra. Let's suppose that it's length is  $\text{len}$ .  $f(b) = f(a) - \lfloor \frac{\text{len}}{2} \rfloor + \lfloor \frac{\text{len}-1}{2} \rfloor$ . It is obvious, that  $\lfloor \frac{\text{len}}{2} \rfloor - \lfloor \frac{\text{len}-1}{2} \rfloor$  is equal to either 0 or 1.
- $[0, 1, -1]$ . The same as the previous one.

**What it gives us?** Now we know that  $f(a)$  is the lower bound of number of operations needed to make all the elements of the array equal to 0.

- **Claim.** It is always possible to solve the problem in  $f(a)$  operations.

**Proof.** We do it using the following algorithm:

1. Look through all positions  $i$  such that  $a_i = 0$  and  $a_{i-1} \neq 0$ . If there are no such positions, then all the elements of the array are equal to 0 and thus the algorithm should stop. If such position  $i$  exists, then proceed to step 2.
2. If  $[a_{i-2}, a_{i-1}, a_i]$  is equal to either  $[0, -1, 0]$ ,  $[0, 1, 0]$ ,  $[-1, -1, 0]$  or  $[1, 1, 0]$ , then we set value  $a_{i-1}$  as  $k$ , which will obviously decrease the value of  $f(a)$ . After that, return to step 1. If the array is not equal to neither of the mentioned above, then don't do anything and go to step 3.

3. Now we know, that  $[a_{i-2}, a_{i-1}, a_i]$  is equal to either  $[1, -1, 0]$  or  $[-1, 1, 0]$ . It is obvious, that  $a_{i-2}$  belongs to some zebra of length greater than 1. By applying *min* or *max* operation respectively we can make  $a_{i-2}$  have same value as  $a_{i-1}$ .

If the zebra's length is greater, then 2, then after making  $a_{i-1}$  and  $a_{i-2}$  equal,  $a_{i-3}, a_{i-2}, a_{i-1}$  will have same values, thus, the zebra will no longer contain elements  $a_{i-2}$  and  $a_{i-1}$ , so it's length will be decreased by 2, so it's length divided by two will be decreased by one. This is why  $f(a)$  will be decreased by 1.

If the zebra's length is equal to 2, then, obviously, the zebra contains only elements  $a_{i-1}$  and  $a_{i-2}$ . If they are equal, then the zebra no longer exists at all. Thus,  $f(a)$  will be decreased by 1.

After making  $a_{i-2}$  equal to  $a_{i-1}$ , return to step 1.

**What it gives us?** Now we know, that  $f(a)$  is the lower bound of our answer and it is always achievable. Thus,  $f(a)$  is the answer.

**How to implement it?** It is obvious how to find  $f(a)$  in  $O(n)$  for a fixed  $k$ . But we need to solve the problem for all  $k$  from 1 to  $m$ .

Let's remember for each value from 1 to  $m$  on which positions they occur. And set all  $a_i := 1$ .

Let's define arrays  $p$ ,  $len$  and value  $sumz$ :

- Let's say that an element on a position  $i$  belongs to some zebra, then the value of  $p_i$  is the position from which the zebra starts. If  $a_i = 0$ , then  $p_i = i$ .
- If there is a zebra that starts from position  $i$ , then the value of  $len_i$  is equal to length of that zebra. If no zebra starts from position  $i$ , then  $len_i = 0$ .
- $sumz$  is equal to  $\sum_{z \in Z(a)} \lfloor \frac{z}{2} \rfloor$ , assuming that our array is not cyclical (i.e we do not count zebras that contain both positions 1 and  $n$ ).

Let's also define procedure  $upd(pos, val)$ .  $upd(pos, val)$  changes value of  $a_{pos}$  to  $val$  in a way that sustains all the arrays  $p$  and  $len$  and value  $sumz$  valid.

Now, let's make all  $p_i := 1$  and thus all  $len_i := 1$ ,  $sumz := 0$ . Now, we iterate  $k$  from 1 to  $m$ , solving the problem for every possible  $k$  in the following way:

- If the value  $k$  has no occurrences, then the answer for this  $k$  is  $-1$ .
- Otherwise, proceed:
  - Now, we should iterate over all positions  $i$ , where  $k$  occurred and call  $upd(i, 0)$ .
  - Now, answer  $ans_k$  for  $k$  is equal to number of positions, where  $k$  doesn't occur plus  $sumz$ . But we need to make sure the zebra that contains both elements on positions  $n$  and 1 (if it exists) is counted too. If  $a_1 = 0$  or  $a_n = 0$  or  $a_n = a_1$ , then the zebra doesn't exist at all, so we don't need to modify  $ans_k$ . Otherwise, the zebra exists and we need to modify  $ans_k$  in the following way:  $ans_k := ans_k - \lfloor \frac{len_1}{2} \rfloor - \lfloor \frac{len_{p_n}}{2} \rfloor + \lfloor \frac{len_1 + len_{p_n}}{2} \rfloor$ . This modifying is equivalent to deleting zebras that start on positions 1 and  $p_n$  (this is the zebra that contains  $n$ ) and adding zebra of length  $len_1 + len_{p_n}$ .
  - Now, we should iterate over all positions  $i$ , where  $k$  occurred and call  $upd(i, -1)$ .

This algorithm will always work, because, since we iterate  $k$  in ascending order, once position is assigned  $-1$  we will never have to change it's value again.

The only question is how to implement  $upd(pos, val)$ . In order to simplify it's implementation, I will use the fact if  $val = 0$ , then  $a_{pos}$  previously was 1, and if  $val = -1$ , then  $a_{pos}$  previously was 0.



- If  $val = 0$ , then do the following:
  - If  $p_{pos} + len_{p_{pos}} - 1 = pos$  (i.e the zebra, that contains position  $pos$  ends on it), then we should decrease  $sumz$  by  $\lfloor \frac{len_{p_{pos}}}{2} \rfloor - \lfloor \frac{len_{p_{pos}}-1}{2} \rfloor$  and decrease  $len_{p_{pos}}$  by one.
  - Otherwise, the zebra doesn't end on position  $pos$ , so transforming  $a_{pos}$  into  $k$  will split it. Decrease  $sumz$  by  $\lfloor \frac{len_{p_{pos}}}{2} \rfloor - \lfloor \frac{pos-p_{pos}}{2} \rfloor - \lfloor \frac{p_{pos}+len_{p_{pos}}-1-pos}{2} \rfloor$  (basicaly, we just split the zebra). Assign  $p_i := pos + 1$  for all  $i$  from  $pos + 1$  to  $p_{pos} + len_{p_{pos}} - 1$ . So now, we set  $len_{pos+1} := p_{pos} + len_{p_{pos}} - 1 - pos, len_{p_{pos}} := pos - p_{pos}$ .
  - In the end, in both cases, set  $p_{pos} := pos, len_{pos} := 0, a_{pos} := 0$ .
- If  $val = -1$ , then do the following:
  - At first, set  $a_{pos} := -1, len_{pos} := 1, p_{pos} := pos$ .
  - If  $pos < n$  and  $a_{pos+1} = 1$ , then there is a zebra that starts form position  $pos$  that contains zebra that starts from position  $pos + 1$  as a suffix. Increase  $sumz$  by  $\lfloor \frac{len_{pos+1}+1}{2} \rfloor - \lfloor \frac{len_{pos+1}}{2} \rfloor$ . Set  $p_i := pos$  for all  $i$  from  $pos + 1$  to  $pos + len_{pos+1}$ . Set  $len_{pos} := len_{pos+1} + 1, len_{pos+1} := 0$ .
  - If  $pos > 1$  and  $a_{pos-1} = 1$ , then there is a zebra that starts from position  $p_{pos-1}$  that has zebra that starts from position  $pos$  as a suffix. Increase  $sumz$  by  $\lfloor \frac{len_{p_{pos-1}}+len_{pos}}{2} \rfloor - \lfloor \frac{len_{p_{pos-1}}}{2} \rfloor - \lfloor \frac{len_{pos}}{2} \rfloor$ . Set  $p_i := p_{pos-1}$  for all  $i$  from  $pos$  to  $pos + len_{pos} - 1$ . Set  $len_{p_{pos-1}} := len_{p_{pos-1}} + len_{pos}, len_{pos} := 0$ .

In order to assign some value on a segment in  $O(\log n)$  you can use Segment Tree (you can read more about this data structure here [https://cp-algorithms.com/data\\_structures/segment\\_tree.html](https://cp-algorithms.com/data_structures/segment_tree.html)). Since  $upd(pos, val)$  is called exactly 2 times for each  $pos$ , then the procedure is called  $O(n)$  times. Every call is done in  $O(\log n)$ , thus, the final time complexity is  $O(n \log n)$ .

## Problem Editorial: “Level Up”

**Author:** Stefan Ruseti  
**Developer:** Adrian Budau  
**Editorialist:** Adrian Budau

The first idea would be to go through the quests in order and pick whether or not we solve that quest at the first level or at the second level.

We can use dynamic programming for this:  $dp[i][j][k]$  to be at quest  $i$  and have accumulated  $j$  experience at the first level and  $k$  experience at the second level.

When we are at a state  $dp[i][j][k]$  and are deciding what to do with quest  $i + 1$  we might run into this special case: we try to solve quest  $i + 1$  in the first level but we overflow the experience into the second level ( $j + x_i > S1$ ). We must remember to use the overflow experience at the second level. (so  $dp[i][j][k] \rightarrow dp[i + 1][S1][k + (j + x_i - S1)]$ ).

Unfortunately, we don't exactly try all possible orders to do the quests, and especially the quest that triggers the overflow. We solve quests independently at each level in the order we process the quests. But, there is a simple observation to be made: if in a correct solution we solve the quests  $(i_1, i_2, i_3, \dots, i_K)$  at the first level in this particular order, we could reorder quests  $i_1, i_2, \dots, i_{K-1}$  and get a different solution. We can not exactly pick any quest to be the last at level 1.

For example if we have 4 quests we want to do at the first level, 3 giving 30 experience each and 1 giving 90 experience and  $S1 = 100$  the only way to solve them at the first level is to first solve the 3 quests giving 30 each (therefore accumulating 90 experience) and then solving the 90 experience quest. We can see that if we can overflow using a quest giving  $U$  experience from the list  $i_1, i_2, \dots, i_K$  than we can overflow with any quest giving  $V \geq U$  experience.

So that means we could set the order of solving the quests: ascendant order by their first level experience ( $x_i$ ). Now we can use the dynamic programming solution describe above with no other modification.

Total complexity:  $\mathcal{O}(n \cdot s1 \cdot s2)$  time and  $\mathcal{O}(n \cdot s1 \cdot s2)$  or  $\mathcal{O}(s1 \cdot s2)$  (depending on implementation) memory.

# Problem Editorial: “Find the Array”

**Author:** Anton Trygub  
**Developer:** Stanislav Bezkorovainyi  
**Editorialist:** Stanislav Bezkorovainyi

If  $n \leq 30$ , that we can simply find the array using 30 queries of type 1. Otherwise we should do the following:

At first, we should find the position of maximum or minimum element of the array. Let's define those positions as  $w$  and  $u$  respectively. Now, use query of type 2 on the whole array. Let's denote the maximum element of the resulting array as  $M$ . It is obvious that  $M = a_w - a_u$ .

What does it mean if the maximum element of the resulting array on query of type 2 for some subset of indexes  $S$  is equal to  $M$ ? It means that both  $w$  and  $u$  belong to  $S$ . Now, we can using binary search find the leftmost position  $pos$  such that the maximum element of the resulting array on the query of type 2 for set of indexes  $\{1, 2, \dots, pos\}$  is equal to  $M$ .

Since both  $u$  and  $w$  belong to  $\{1, 2, \dots, pos\}$  and one of them does not belong to  $\{1, 2, \dots, pos - 1\}$ ,  $u$  or  $w$  is equal to  $pos$ . For now, it does not really matter whether  $u = pos$  or  $w = pos$ . What matters is that, since all  $a_i$  are distinct and that the value of  $a_{pos}$  is whether smaller or greater than value of any  $a_i$ , there is no such pair of indexes  $(j, i), i \neq j$  that  $|a_{pos} - a_i| = |a_{pos} - a_j|$ .

Let's denote  $b_i = |a_{pos} - a_i|$ . It's obvious, that  $b_{pos} = 0$ . Now we should find  $b_i$  for every  $1 \leq i \leq n, i \neq pos$ . All  $b_i$  will be distinct.

At first, let's define function  $F(Q) = B$  that takes some set of indexes  $Q$ , such that  $pos \notin Q$  as an argument and returns set  $B$  that contains all  $b_i$  for any  $i \in Q$ . How to implement such function in your program? That's quite simple. At first, let's denote set  $A$  as the result of query 2 on set of indexes  $Q$  and set  $A'$  as the result of query 2 on set  $Q \cup \{pos\}$ . Then,  $F(Q) = A' \setminus A$  (All elements, that belong to  $A'$ , but do not belong to  $A$ ). Since all  $b_i$  are different, size of set  $Q$  will always be equal to size of  $F(Q)$ .

Now, let's use divide and conquer approach.

Let's say that we have some set of positions  $Q$  and set  $B = F(Q)$ . We want to know  $b_i$  for every  $i \in Q$ . There are two possible scenarios:

- Size of  $Q$  is equal to 1. Then  $b_i$  is the only element of set  $B$ , while  $i$  is the only element of set  $Q$ . Thus, we know  $b_i$  without any additional queries.
- We split  $Q$  into two sets  $S_1, S_2$  such that first  $\lfloor \frac{|Q|}{2} \rfloor$  elements of set  $Q$  go into set  $S_1$  and the rest of them go into  $S_2$ . We can find  $B_1 = F(S_1), B_2 = F(S_2)$  in one call of function  $F$ :  $B_1 = F(S_1), B_2 = B \setminus B_1$ . Now, we should solve the problem independently for sets  $S_1, B_1$  and  $S_2, B_2$ .

At the beginning of the algorithm,  $Q$  contains all indexes except  $pos$  and  $B$  is calculated in a straightforward way  $B := F(Q)$ .

A split of a set takes 1 call of  $F$ , but wait a second... There are  $\approx n$  splits. It is no better than simply asking value for every index with query 1! Yeah, I know. But we can optimize it easily into  $O(\log_2 n)$  calls of  $F$ .

It is easy to notice that splits on the same depth of our “divide and conquer binary tree” can be done simultaneously with only one call of function  $F$ . Let's say that there are  $k$  sets  $Q_1, Q_2, \dots, Q_k$  about to be split on the same depth. Their first halves are  $S_{1_1}, S_{2_1}, \dots, S_{k_1}$ . Let's define  $A = F(S_{1_1} \cup S_{2_1} \cup \dots \cup S_{k_1})$  (Function of union of all  $S_{i_1}$ ). Now, for every  $1 \leq i \leq k$   $B_{i_1} = B_i \cap A, B_{i_2} = B_i \setminus B_{i_1}$ .

Depth of the tree is at most  $\lceil \log_2 n \rceil$ . So that we will make at most  $\lceil \log_2 n \rceil$  calls of function  $F$  + one call in order to get  $F(Q)$  for the initial set. Every call of  $F$  takes exactly 2 queries, so we will spend  $2 \cdot \lceil \log_2 n + 1 \rceil$  queries on this part of the program.

Now, we know  $b_i$  for every  $1 \leq i \leq n$ . Let's define  $pos_2$  as such index that  $b_{pos_2} = \max(b_1, b_2, \dots, b_n)$ . Since the minimal element has the greatest difference with the maximal element and vice versa, if  $a_{pos}$  is the minimal element, then  $a_{pos_2}$  is the maximal one and vice versa. Let's use query 1 to ask the value of  $a_{pos}$  and the value of  $a_{pos_2}$ . There are two possible scenarios:

- $a_{pos} > a_{pos_2}$ . That means  $a_{pos}$  is the maximal element of the array  $a$  and thus  $a_i = a_{pos} - b_i$  for every  $1 \leq i \leq n$ .
- $a_{pos} < a_{pos_2}$ . That means  $a_{pos}$  is the minimal element of the array  $a$  and thus  $a_i = a_{pos} + b_i$  for every  $1 \leq i \leq n$ .

Overall, we used  $1 + \lceil \log_2 n \rceil$  queries to find  $pos$ ,  $2 \cdot \lceil \log_2 n \rceil + 2$  queries to find  $b_i$  for every  $i$  and 2 queries to find values of  $a_{pos}$  and  $a_{pos_2}$ .

$$1 + \lceil \log_2 n \rceil + 2 \cdot \lceil \log_2 n \rceil + 2 + 2 = 5 + 3 \cdot \lceil \log_2 n \rceil = 5 + 3 \cdot \lceil \log_2 250 \rceil = 29$$

Note. Be careful while implementing binary search and function  $F$ . Otherwise, you may find yourself getting **Wrong Answer** due to very strict constraints.

## Problem Editorial: “Cycle String?”

**Author:** Anton Trygub  
**Developer:** Nazarii Denha  
**Editorialist:** Stanislav Bezkorovainyi

Let's say, that  $cnt_{ch}$  is the number of occurrences of character  $ch$  in the string. Let's say that  $cm$  is such a character, that  $cnt_{cm}$  is maximum among all  $cnt_{ch}$ .

Now, there are several possible scenarios.

**Scenario 1.**  $cnt_{cm} = 2 \cdot n$ . In other words, the string consists of the same characters. Obviously, the answer in that case is “NO”.

**Scenario 2.**  $cnt_{cm} = 2 \cdot n - 1$  and  $n \geq 2$ . In other words, the string consists of the same characters, except for one position. The answer in that case is “NO”.

**Proof.** Let's say that  $cm = “a”$  and the only symbol not equal to  $cm$  is equal to “c”. Our string will look like this:

aaaaaaaaaa . . . aaaaacaaaaa . . . aaaaaaaaaa

Since the string is cyclical, then cyclical shiftings of the string do not change it. Let's shift it the way that the only character that differs from the rest is placed on the right side of the string. Now the string looks like this:

$$\overbrace{a \text{ aaaaa } \dots \text{ aaaaaaa}}^{n \text{ characters}} aaaa \dots aaaaac$$

$$n \text{ characters}$$

**Scenario 3.**  $cnt_{cm} = 2 \cdot n - 2$ ,  $n \geq 3$  and the characters besides  $cm$  are the same. The answer in that case is “NO”.

**Proof.** Let's say that  $cm = “a”$  and the rest of the characters are equal to “c”. Now we shift the string with cyclic shifts in such a way that the first character of the string is “c”. Let's denote the other position on the string, where character “c” is placed as  $p$ . Now, our string looks like this:

$$\overbrace{caaaaa \dots aaac}^{p \text{ characters}} aaa \dots aaaaaa$$

It is obvious that  $p - 2 < n + 1$ . Otherwise, there would have been at least two strings of length  $n$  containing only characters “a” between 1 and  $p$ . For similar reason  $2 \cdot n - p < n + 1$ .

$$p - 2 < n + 1 \iff p < n + 3$$

$$2 \cdot n - p < n + 1 \iff n - p < 1 \iff p > n - 1$$

Now we know that  $p$  is either equal to  $n$  or  $n + 1$  or  $n + 2$ .

Why not  $n$ ? The substring  $t$  is equal to concatenation of string  $g_1$  and  $g_2$  (remember, our string is cyclical, so everything is fine):

$$\underbrace{ca}_{g_2} \overbrace{aaaa \dots aaac}^t \underbrace{aaaaaa \dots aaaaaa}_{g_1 = n - 2 \text{ same symbols}}$$

Why not  $n + 1$ ? The substring  $t$  is equal to substring  $g$ :

$$\overbrace{caaaaa \dots aaa}^t \overbrace{caaaaa \dots aaa}^g$$

Why not  $n + 2$ ? We can make  $n + 1$  cyclic shifts to left and after that we'll get the same situation as when  $p = n$ .

So there is no valid way to place  $p$ , so the answer is "NO".

**Scenario 4.**  $cnt_{cm} = 2 \cdot n - 2, n \geq 3$  and the characters besides  $cm$  are distinct. In this case the answer is "YES".

**Proof + the way to build such string.** Let's say that  $cm = "a"$  and the other two characters are equal to "c" and "d". We can build the answer this way:

$$\overbrace{caaa \dots aaa}^{n \text{ symbols}} \overbrace{daaa \dots aaa}^{n \text{ symbols}}$$

It is obvious that every substring will contain either "c" or "d" but not both of them. Since there is only one character "c", any two different substrings that contain "c" will have it on different positions and thus will be distinct. The same goes for any two substrings that contain "d".

**Scenario 5.**  $cnt_{cm} \leq n$ . In this case the answer is "YES".

**Proof + the way to build such string.** We can simply sort the string.

Let's suppose that it does not work and there are two identical substrings  $s_i$  and  $s_j$  of length  $n$  that start on positions  $i$  and  $j$  respectively, where  $i < j$ . Let's denote the first character of  $s_j$  as  $c$ . Let's say that at the beginning of the string  $s_j$  there are  $p \leq n$  characters  $c$ . Since  $s_i = s_j$  same should go for  $s_i$ . If character at position  $j + p - 1$  is equal to  $c$  too, then, since the string is sorted, there are at least  $j + p - i > p$  characters  $c$  at the beginning of the string  $s_i$ . We face contradiction and thus, the statement is false, i.e there are indeed no same substrings that start on different positions.

**Scenario 6.**  $n < cnt_{cm} \leq 2 \cdot n - 3$ . In this case the answer is "YES".

**Proof + the way to build such string.** Let's say that  $cm = "a"$  and the rest of the symbols are "b", "c", "d", ..., "z". We can put an arbitrary symbol besides  $cm$  on the first position, then  $n$  symbols  $cm$  (let's call it  $block_1$ ), then all the remaining symbols besides  $cm$  in any order (let's call it  $block_2$ ), then  $cnt_{cm} - n$  symbols  $cm$  (let's call it  $block_3$ ):

$$\overbrace{baaaaaaaaaaaaa \dots aaaaaaaaaaaaaa}^{block_1 - n \text{ symbols } cm} \overbrace{zbtbcydccceecgy \dots vwqxtheyqyui}^{block_2 - \text{the rest of the symbols}} \overbrace{aaaaaaaaaaaa \dots aaaaaaaaaaaaaa}^{block_3 - cnt_{cm} - n \text{ symbols } cm}$$

We should notice, that since  $cnt_{cm} \leq 2 \cdot n - 3 \iff cnt_{cm} - n \leq n - 3$ . And thus length of  $block_3$  is smaller than  $n - 2$ .

Why all of the substrings of length  $n$  will be unique?

- Substring that starts on position 1 is the only substring that starts from a symbol that is not equal to  $cm$  and is followed by  $n - 1$  characters  $cm$ .
- Substring that starts on position 2 is the only substring that consists only of characters  $cm$ .
- Substring that starts on position 3 is the only substring that has  $n - 1$  symbols  $cm$  at the beginning and one symbol that is not equal to  $cm$  in the end.
- Substrings that start on positions from 4 to  $n + 1$  are the only substrings that start from  $cm$  and they have a substring of length  $\geq 2$  that consists of characters that are not equal to  $cm$ . They will differ among themselves because they all have different number of  $cm$ -s at their beginning.

- Substrings that start on positions in  $block_2$  are the only substrings that start from symbol that is not equal to  $cm$  and have  $\leq n - 2$  characters  $cm$  in them. They will differ among themselves because lengths of their prefixes that do not contain symbol  $cm$  are distinct.
- Substrings that start from positions in  $block_3$  are the only substrings that start with symbol  $cm$  and have a symbol surrounded by  $cm$ -s that is not equal to  $cm$  (I'm talking about the first character of the string). They will differ among themselves by positions of the character that is not equal to  $cm$ .

## Problem Editorial: “Life Transfer”

**Author:** Eugenie Daniel Posdărascu  
**Developer:** Radu Vişan  
**Editorialist:** Adrian Budău

The easy approach to this problem is to fix the number of motorcycles to rent. The number of cars needed will be uniquely determined by this.

Now let's say we want  $C$  cars and  $M$  motorcycles. We need the oldest  $C$  people to drive the cars, and the next oldest  $M$  people to drive the motorcycles. For both of these groups we can divide the people into 4 categories:

- People that have at least  $d$  years more than the minimum age required to drive the vehicle. From these people, we can afford to take  $d$  years from each of them to give to someone else.
- People that have the minimum age required to drive the vehicle, but they are less than  $d$  years older than the minimum age required. From these people we can take years equal to their age out of which we subtract  $l_c$  or  $l_m$  respectively. This can be rewritten as the sum of their ages -  $l_c$  or  $l_m$  multiplied by their number.
- People that do not have the minimum age required to drive the vehicle, but need less than or equal to  $d$  years to reach it. This is symmetrical to the case above: they need years equal to  $l_p$  or  $l_m$  out of which we subtract the sum of their ages.
- People that are more than  $d$  years younger than the minimum required age to drive their vehicle. In this case, it means  $C$  and  $M$  can not represent the number of cars and motorcycles.

We can also divide the rest of the people in two categories:

- People with age at least  $d + 1$  out of which we can take  $d$  years to give to someone else (treated as the first category for cars/motorcycles)
- People with age less than or equal to  $d$ , out of which we can take years equal to their age minus 1 (treated as the second category for cars/motorcycles).

It can be seen that if we order all people by age all these categories of people become intervals which we can precompute for the whole array and then restrict depending on  $C$  and  $M$ . And the number of years we can take / we need to give become partial sums on this ordered array of years.

Total complexity:  $\mathcal{O}(n \log n)$  time and  $\mathcal{O}(n)$  space.



## Problem Editorial: “Game on a Tree”

Author: Anton Trygub  
Developer: Maksym Zub  
Editorialist: Stanislav Bezkorovainyi

It is easy to notice that if a vertex  $u$  is an ancestor of vertex  $v$ , then vertex  $v$  is a descendant of vertex  $u$ . And thus if we can move the chip from  $u$  to vertex  $v$ , then we can also move the chip from vertex  $v$  to vertex  $u$ .

Now let's build a new undirected graph  $G$ , that consists of  $n$  vertices. In graph  $G$  two vertices  $v$  and  $u$  are connected with an edge if and only if  $v$  is an ancestor of  $u$  or  $u$  is an ancestor of  $v$  in the original tree.

Obviously, playing the game on the tree is equivalent to playing the game on this graph, just with a small change. In the tree we could move the chip from vertex  $u$  to vertex  $v$  if and only if  $u$  was either ancestor of  $v$  or descendant of  $v$ . In graph  $G$  we can move the chip from vertex  $u$  to vertex  $v$  iff  $u$  and  $v$  are connected with an edge.

Now let's find a *maximum matching* of the graph. More information about matchings of graphs and their properties can be read on page <http://bit.ly/seerc-2019-matching>. But I'll provide a few definitions here:

A *matching* in a graph is a set of edges without common vertices.

A *maximum matching* is a matching that contains the largest possible number of edges.

A *perfect matching* is a matching which matches all vertices of the graph.

**Claim.** If a maximum matching of graph  $G$  is a perfect matching, then the second player wins. Otherwise, the first one wins.

**Proof.**

- At first, let's prove that if the matching is perfect, then the second player wins. Let's suppose that set of edges  $E$  is a perfect matching of graph  $G$ . As we all know, every edge in an undirected graph can be defined by a pair of numbers — pair of vertices that the edge connects. So, actually, by building a perfect matching we divide  $n$  vertices into  $\frac{n}{2}$  pairs of vertices  $(v_1, u_1), (v_2, u_2), \dots, (v_{\frac{n}{2}}, u_{\frac{n}{2}})$ , such that every vertex from graph  $G$  belongs to exactly one pair.

Let's see how Bob has to play in order to win this game. Let's say that on the first turn Alice chooses to put the chip on vertex  $a$ . Since the matching is perfect, vertex  $a$  belongs to some pair. Let it be  $(a, b)$ . Then Bob chooses to move the chip to vertex  $b$ . So now, all the vertices from the pair are used, so the players won't be able to move the chip on neither vertex  $a$  nor on vertex  $b$ . Now Alice has to put the chip in a vertex that belongs to another pair. And so on.

In general, if on turn  $k$  Alice decides to put the chip on vertex  $a_k$ , Bob finds the pair  $(a_k, b_k)$  to which  $a_k$  belongs and moves the chip to vertex  $b_k$ . Since we know that wherever Alice decides to put the chip Bob will still have the next vertex to go, it is obvious that Alice will lose, and thus Bob will win.

- Now let's prove that if the maximum matching is not perfect, then Alice can always win.

Let  $E$  be a maximum matching of graph  $G$ .  $E$  is a set of edges and thus, it can be also represented as a set of pairs of vertices  $(v_1, u_1), (v_2, u_2), \dots, (v_m, u_m)$ , where  $m$  is the size of  $E$ . Since  $E$  is not a perfect matching there is at least one vertex  $t$  that does not belong to any of the pairs.

At the first turn, Alice should choose any vertex that does not belong to any of the pairs. At the second turn, Bob will either have no vertices to go (and thus lose the game) or the only vertex he can go will belong to one of the pairs in matching  $E$ . Now the game is just the same as in the previous case, but now Bob is the first one to put the chip on a vertex that belongs to one of the pairs of the matching and because of that Bob will definitely lose.

But wait a second... In the previous case every vertex belonged to exactly one pair. Since  $E$  in this case is not perfect, there can be lots of vertices that do not belong to any of the pairs. Why can't Bob move to a vertex that does not belong to any of the pairs, just like Alice did in the first turn?

Let's suppose that Bob can do so. There are two possible cases:

- Bob decides to move the chip on a vertex that does not belong to any of the pairs on the second turn. If he can do so, the previous vertex and the new one can be matched, and thus  $E$  is not a maximum matching. Contradiction.
- Bob decides to move the chip on a vertex that does not belong to any of the pair on the  $k$ -th turn. Let vertices  $v_1, v_2, v_3, \dots, v_{k-1}, v_k$  be vertices on which the chip was placed on turns 1, 2, 3,  $\dots$ ,  $k-1$  and  $k$  respectively. Since Alice plays the game just like Bob in the case when  $E$  was perfect,  $k-1$  unordered pairs  $(v_2, v_3), (v_4, v_5), \dots, (v_{k-2}, v_{k-1})$  belong to  $E$ . Whats more, there are edges between  $k$  pairs of vertices  $(v_1, v_2), (v_3, v_4), \dots, (v_{k-1}, v_k)$ . We can delete the  $k-1$  pairs from  $E$  mentioned earlier and insert  $k$  pairs  $(v_1, v_2), (v_3, v_4), \dots, (v_{k-1}, v_k)$ . By doing that we create a new matching  $E_2$  that has size greater than that of matching  $E$ . That means that  $E$  is not a maximum matching. Contradiction.

So now we know how to solve the problem if know the size of maximum matching of the tree. But how to find it? You may try to use standard approaches for finding maximum matching, but these are very likely to get “Time limit exceeded” verdict. Here is author's approach that works in  $O(n)$  that capitalizes on the way how graph  $G$  was built.

We will use recursion + dynamic programming on the initial tree. Let's define  $dp_v$  as the minimal number of vertices that are left unmatched if we try to match all the vertices in the subtree of vertex  $v$ .

Let's say that we want to calculate  $dp_v$  for some vertex  $v$ . Now there are two possible senarios:

- Vertex  $v$  is a leaf. Obviously,  $dp_v = 1$ .
- Vertex  $v$  is not a leaf, so it has one or more direct descendants. Let's denote the direct descendants of vertex  $v$  as  $d_1, d_2, \dots, d_k$ . At first, let's calculate values of  $dp_{d_1}, dp_{d_2}, \dots, dp_{d_k}$ . Let's define  $D = \sum_{i=1}^k dp_{d_i}$ . If  $D$  is greater than 0, then we can match vertex  $v$  with one of it's unmatched descendants and get  $dp_v = D - 1$ . Otherwise, all of the descendants of vertex  $v$  were matched, so obviously vertex  $v$  has no vertex to match with, so  $dp_v = 1$ . We get the formula:

$$dp_v = \begin{cases} D - 1, & \text{if } D > 0 \\ 1, & \text{if } D = 0 \end{cases}, \text{ where } D = \sum_{i=1}^k dp_{d_i}$$

If  $dp_1 = 0$ , then we can match all of the vertices and thus the maximum matching is perfect, which means Bob will win. Otherwise, Alice will win.

## Problem Editorial: “Projection”

**Author:** Virgil Palanciuc  
**Developer:** Adrian Budău  
**Editorialist:** Adrian Budău

Let's call the 2 projections the  $NM$  and  $NH$  projections.

First of all, we can notice that a shadow on row  $X$  in one of the projections can only be generated from a cube that is at coordinates  $(X, \text{some}Y, \text{some}Z)$ . This means that the problem can be solved independently for each  $X$  from 1 to  $N$ .

First let's discuss the case of no solution: If at some row  $X$  one of the projections has at least one shadow (implying there is a cube at this row), but the other one doesn't (implying that there isn't any cube at this row) then obviously we can not construct a solution. If this doesn't happen there is always a solution and it will be obvious from the construction.

For the maximum number of cubes, the solution is unique. If the  $NM$  projection has a shadow  $(X, Y)$  and the  $NH$  solution has a shadow at  $(X, Z)$  then we can put a cube at  $(X, Y, Z)$ . There is no other spot we can place a cube, thus the solution is unique. Since it's unique we just have to print these cubes ordered by  $X$ , then by  $Y$ , then by  $Z$  to get the minimal lexicographically solution.

For the minimum number of cubes let's look at some row  $X$ . If the  $NM$  projection has  $P$  shadows it means there are at least  $P$  cubes at this row. Similarly, if the  $NH$  projection has  $Q$  shadows there are at least  $Q$  cubes at this row.

If  $P = Q$  and we have shadows in  $NM$  at  $(x, y_1), (x, y_2), \dots, (x, y_P)$  and shadows in  $NH$  at  $(x, z_1), (x, z_2), \dots, (x, z_Q)$  with  $y_1 < y_2 < \dots < y_P$  and  $z_1 < z_2 < \dots < z_Q$ . then we can place the  $P$  cubes at  $(x, y_1, z_1), (x, y_2, z_2), \dots, (x, y_P, z_P)$  and there is no solution using less cubes at this row (since we need at least  $P = Q$ ) and no smaller lexicographically one using only  $P$  cubes.

If  $P < Q$  we obviously need to use the same  $y$  multiple times (since we need at least  $Q$  cubes at this row). Because we require the minimum lexicographically solution we should use the smallest  $y$  multiple times. We can duplicate it until we have as many  $y$  as  $z$  and then apply the same idea as in the  $P = Q$  case.

If  $P > Q$  we do as in  $P < Q$  except we duplicate the smallest  $z$  and then apply the same idea as in the  $P = Q$  case.

Total complexity:  $(N \cdot M \cdot H)$  time and space.

# Problem Editorial: “Tree Permutations”

Author: Anton Trygub  
Developer: Stanislav Bezkorovainyi  
Editorialists: Anton Trygub and Stanislav Bezkorovainyi

Sort the array first. Say that  $a_1 \leq a_2 \leq \dots \leq a_{2n-2}$ .

- **Claim 1:** If  $a_i > i$  for some  $i$ , there is no valid tree at all.

**Proof:** Suppose that  $a_i > i$ . Obviously, this means that at most  $i - 1$  numbers don't exceed  $i$ , but, from the other side, parents of nodes  $2, 3, 4, 5, \dots, i + 1$  all have to be at most  $i$ . Contradiction.

- **Claim 2:** If  $a_i = i$  for some  $i$ , the path from  $n$  to  $1$  has to include  $i$ .

**Proof:** Suppose that  $a_i = i$ . Obviously, this means that at most  $i - 1$  numbers don't exceed  $i - 1$ . From the other side, parents of nodes  $2, 3, 4, 5, \dots, i$  all have to be at most  $i - 1$ . Therefore, all these  $i - 1$  numbers will be used as parents for  $2, 3, 4, 5, \dots, i$ , and therefore parents of all numbers from  $i + 1$  to  $n$  will be at least  $i$ . This means, that we can't get from  $n$  to  $1$  without coming through  $i$  (suppose that  $j$  is the last vertex on the path from  $n$  to  $1$  which is larger than  $i$ , then it's parent has to be  $i$ ).

- Let  $t_1, t_2, \dots, t_m$  be the set of indexes for which  $a_x = x$ , with  $t_1 = 1$ . Let  $S$  be the set of all numbers which appear among  $a_i$ , excluding  $t_j$  for  $1 \leq j \leq m$  — those are potential other vertices on the path. Obviously, there is no  $k$ -path for  $k < m$  (as every path from  $n$  to  $1$  has to include at least  $m + 1$  nodes  $t_1, \dots, t_m, n$ ). Also, there is no  $k$ -path for  $k > m + |S|$ , as the total possible number of different vertices in the path doesn't exceed  $m + |S| + 1$  (corresponding to  $n$ ).
- **Claim 3:** For  $m \leq k \leq m + |S|$ , the  $k$ -long trees exist. Sum of weights for  $k$ -perfect tree can then be computed as following: mark  $t_1, t_2, \dots, t_m$  as used, and  $k - m$  smallest elements from  $S$  as used. Answer is the sum of  $k$  largest of  $2n - 2 - k$  not marked numbers.

**Proof:** Obviously, this is the upper bound, it's left to prove that it's achievable. Denote  $k - m$  smallest elements of  $S$  as  $s_1 < s_2 < \dots < s_{k-m}$ . So our path will contain vertices  $path_1 < path_2 < \dots < path_k < path_{k+1}$ , where  $path_1 = 1$ ,  $path_{k+1} = n$ , and  $path_1, path_2, \dots, path_k$  is a sorted union of  $t_1, t_2, \dots, t_m, s_1, \dots, s_{k-m}$ . We now set parent of  $path_{i+1}$  to be  $path_i$  for each  $i$ .

Mark first occurrences of  $t_1, t_2, \dots, t_m, s_1, \dots, s_{k-m}$  in  $a$  as used. Also mark all the vertices that we've given a parent this way. There are  $2n - 2 - k$  indexes which are yet not used, let them be  $x_1 \leq x_2 \leq \dots \leq x_{2n-2-k}$ . Now look at all the  $n - k - 1$  vertices (except for  $1$ ) which aren't assigned a parent yet, assign  $a_{x_i}$  to the  $i$ -th of them (in increasing order). After that, distribute all  $n - 1$  numbers which are left can be used as weights, use  $k$  largest of them on the  $path$ , and others arbitrarily. This gives the answer from the claim, we just have to prove that we have actually produced a tree.

Let  $v_1, v_2, \dots, v_{n-k-1}$  be a sorted array of vertices which were not assigned a parent yet. Note, that vertex  $1$  can not have a parent, so  $v_1 \neq 1$ . Let's iterate over array  $v$ . Let's say that we want to assign a parent for vertex  $v_i$ . As said before, we want to assign parent of vertex  $v_i$  as  $a_{x_i}$ . This is valid if and only if  $a_{x_i} < v_i$ . Now we have to prove that  $a_{x_i} < v_i$  for any  $i$  from  $1$  to  $n - k - 1$ .

Let's suppose that it is not true and for some  $i$ ,  $a_{x_i} \geq v_i$ . Without loss of generality,  $path_j \leq a_{x_i} < path_{j+1}$  for some  $j$ . It means, that  $a_{x_i}$  is greater than or equal to the numbers of first  $j$  vertices of the  $path$  (i.e marked vertices) and  $a_{x_i} \geq v_i > v_{i-1} > \dots > v_1$  (i.e  $i$  unmarked vertices). Since  $a_{x_i}$  is greater than or equal to the numbers of the first  $i + j$  vertices it means, that  $a_{x_i} \geq i + j$ .

Now let's look at value  $x_i$ . What does it mean that an element has position  $x_i$  in array  $a$ ? It means that there are exactly  $x_i - 1$  elements before it in the array.

How many unmarked positions are there before  $x_i$ ? These are  $x_1, x_2, \dots, x_{i-1}$ . And thus, there are exactly  $i - 1$  unmarked positions before  $x_i$ .

How many marked positions are there before  $x_i$ ? As we already said,  $path_j \leq a_{x_i} < path_{j+1}$ . Since we marked the first occurrence of  $path_j$  in the array, so  $path_j > path_{j-1} > \dots > path_1$  these are all before of the position  $x_i$ .  $x_i < path_{j+1} < \dots < path_k$  these all, obviously go after position  $x_i$  since array  $a$  is sorted. So there are exactly  $j$  marked positions before  $x_i$ .

Since every position is either marked or not marked, there are exactly  $i - 1 + j$  positions before  $x_i$ . So,  $x_i = i - 1 + j + 1 = i + j$ .

Quick observation. From first two claims we know that if for some  $i$   $a_i > i$ , then there is no answer and if for some  $i$   $a_i = i$ , then we have already added  $i$  to our path. So, now we know that on every position  $i$  that has not been marked yet  $a_i < i$ . Thus,  $a_{x_i} < x_i$ . But we have already discovered, that  $a_{x_i} \geq i + j$  and that  $x_i = i + j$ , so we get  $a_{x_i} \geq x_i$ . Contradiction.

- Now let's move to the algorithmic side. We have to be able to calculate the sum of  $k$  largest numbers among  $x_1, x_2, \dots, x_{2n-k}$  where  $k - m$  smallest elements of  $S$  are discarded. It's the easy part: for  $k = m$ , the set of  $m$  largest numbers will be just  $m$  largest numbers in  $x$ . Now, maintain the set of values of  $S$  that are prohibited, and we haven't discarded yet. At every step, add next element of  $S$  to this set, and the next (in decreasing order)  $x_i$  in the set of the largest numbers. Now, while set of largest numbers contains some  $c$  that is in set **to be discarded**, delete one instance of  $c$  from both **to be discarded** and largest numbers, and add a next number into the largest instead. Sum recalculation is obvious.

## Problem Editorial: “Absolute Game”

Author: Roman Bilyi  
Developer: Roman Bilyi  
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**Claim.** This game is equivalent to the game where Alice has to choose  $x$  from her array and then Bob has to choose  $y$  from his array.

**Proof.** Since Alice can finish the game with whatever  $x$  she wants, the result of the game for Bob can't be better than in the game mentioned above. Now let's prove that it is always possible for Bob to achieve this.

At first, let's define an array  $c$  of  $n$  elements. Each  $c_i$  has such value, that the value of  $|a_i - b_{c_i}|$  is minimal possible. If for some position  $i$  there are more than one possible  $c_i$ , then choose the smallest one. In other words, we build an array such that if Alice finishes the game with  $x = a_i$ , then the best outcome for Bob is if  $y = b_{c_i}$ .

Let's say that in the first turn, Alice removes some element  $a_i$ . Then we should also remove the element  $c_i$  from array  $c$ . Now, since there are  $n - 1$  elements of the array  $c$ , there is at least one such position  $1 \leq p \leq n$ , that  $c_i \neq p$  for every  $i$ . Bob should remove the element  $b_p$ . It is optimal because whatever  $x$  we end up with, he will still be able to choose the best possible  $y$  from the remaining elements of array  $b$ .

In general, if after some turns there are  $k$  elements in Alice's array and Bob's array, now it's Alice's turn and if she removes an element  $a_i$  from her array, then Bob should remove an element  $b_p$  on such position  $p$  that there are no  $c_i = p$  among all possible  $i$  (i.e. among all such  $i$  that  $a_i$  have not been removed yet).

If Bob plays the way mentioned above, then after each his turn, for every  $x$  from the remaining array  $a$ , he will be able to choose the best possible  $y$  from the remaining array  $b$ .

**Ok, I got it. But how to implement it?** Implementation of the “light version” of the game is rather straightforward. Let's define an array  $ans$  such that  $ans_i$  is the answer if Alice decides to choose  $x$  to be equal to  $a_i$ . If  $x$  is equal to  $a_i$ , then Bob will obviously choose such  $y$  that  $|x - y|$  is minimal possible. Thus, we get the formula:

$$ans_i = \min_{j=1}^n |a_i - b_j|$$

Since Alice wants to maximize  $|x - y|$ , the final answer is equal to  $\max_{i=1}^n ans_i$ .

## Problem Editorial: “Graph and Cycles”

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Since  $n$  is always odd, then the degree of each vertex (i.e number of vertices adjacent to it) is always even, so there are even number of edges adjacent to each vertex.

Let's look at an edge  $e$  that connects two vertices  $(v_1, v_2)$ . In a cycle-array to which  $e$  will belong, this edge will be adjacent (i.e compared when calculating the price of the cycle-array) with exactly one edge, that is connected to  $v_1$  and exactly one edge that is connected to  $v_2$ .

So, if we look at a vertex  $v$ , then all the edges that are connected to it can be divided into  $\frac{n-1}{2}$  pairs of edges  $(e_1, e_2)$ , where  $e_1$  and  $e_2$  belong to same cycle-array and  $e_1$  is adjacent with  $e_2$ . So now, we can define the *price of a vertex* as the sum of maximums of weights in such pairs. In other words, if all the edges that are adjacent to a vertex  $v$  are arranged in pairs like that  $(e_1, e_2), (e_3, e_4), \dots, (e_{n-2}, e_{n-1})$ , then:

$$\text{Price of } v = \sum_{i=1}^{\frac{n-1}{2}} \max(\text{weight of } e_{2i-1}, \text{weight of } e_{2i})$$

It is easy to see that sum of prices for all of the vertices is equal to price of the cycle-split.

Let's say that edges  $e_1, e_2, \dots, e_{n-1}$  with weights  $w_1, w_2, \dots, w_{n-1}$  are connected to  $v$ . Let's say, that  $w'$  is a sorted array  $w$ . It is easy to prove that the smallest possible price of a vertex can be obtained if we split the weights into pairs in the following way:  $(w'_1, w'_2), (w'_3, w'_4), \dots, (w'_{n-2}, w'_{n-1})$ .

What is left to prove is that it is achievable. Let's say that for each vertex  $v$  we arranged edges adjacent to it into pairs, such that the price of the vertex is smallest possible. Let's denote this arrangement as  $(e'_1, e'_2), (e'_3, e'_4), \dots, (e'_{n-2}, e'_{n-1})$ . Let's call a pair  $(e'_i, e'_{i+1})$  **unfulfilled** if  $e'_i$  was used and  $e'_{i+1}$  weren't or vice versa.

Now, we can build the necessary cycle-split the following way:

- If all of the edges were used, then stop the algorithm.
- Otherwise, we should start forming a new cycle-array that will be added to the split. Find any vertex  $v$  that has an unused edge adjacent to it. We add this edge to the current cycle-array and mark it as used. Let's say that it is edge that connects vertex  $v$  with vertex  $u$ . Let's set our current vertex as  $u$  and repeat the following algorithm:
  - Let's suppose that our current vertex is some vertex  $u$ . If there are no unfulfilled pairs of it, then the cycle-array has been finally formed, thus we stop the algorithm.
  - Otherwise, there is exactly one unfulfilled pair. Wlog that it is pair  $(e'_i, e'_{i+1})$ .  
 If  $e'_i$  has not been used yet, then add it to our cycle-array and set the current vertex as a vertex that the edge  $e'_i$  connects  $u$  with. Now there are no unfulfilled pairs of the vertex  $u$ . It can be seen that  $e'_{i+1}$  was added just before we set  $u$  as our current vertex. Thus,  $e'_i$  and  $e'_{i+1}$  will be compared, when calculating price of the array.  
 The similar works for the case when  $e'_{i+1}$  has not been fulfilled.

The cycle array that is obtained this way is always valid, because it is an Euler's tour of the edges that belong to it.

So now, we have proven the solution. How to find the answer?

For any vertex  $v$  create an array  $w'$  that has all the weights of the edges that are connected to it in an ascending order. The price of this vertex is the sum of the values on even positions in the array  $w'$ . The price of the split is the sum of prices of all the vertices.

Total time complexity  $O(n^2 \log n)$ .



## Problem Editorial: “Stranded Robot”

**Author:** Vladimir Olteanu  
**Developer:** Vladimir Olteanu  
**Editorialists:** Vladimir Olteanu and Roman Bilyi

For the sake of simplicity, we will be assuming that  $M = N = P$ .

The key insight is that the robot can only attach itself to blocks that can be lit. For each of the 6 directions from which the sun can shine, there are at most  $N^2$  blocks that can be lit. The rest of the blocks make no difference whatsoever, and can be ignored.

Let us consider what happens when the sun is shining along the positive direction of the  $z$  axis. In order to determine which blocks are lit, we will need to compute the depth buffer  $zMin$ , where  $zMin[x, y]$  is the minimal  $z$  coordinate of all the blocks that share the same  $x$  and  $y$ , or  $-\infty$  if no such block exists.

From this perspective, the moves available to a robot located at  $(x, y, z)$  are:

- If  $z + 1 = zMin[x, y]$ , the robot is resting on top of a block, and can move to  $(x \pm 1, y, zMin[x \pm 1, y] - 1)$  or  $(x, y \pm 1, zMin[x, y \pm 1] - 1)$  if the  $zMin$  at the destination is greater or equal to  $z + 1$ .
- If  $z + 1 < zMin[x, y]$ , the robot can fall to  $(x, y, zMin[x, y] - 1)$ .

When the sun is shining along the negative direction of the  $z$  axis, we compute the depth buffer  $zMax$  instead, where  $zMax[x, y]$  is the maximal  $z$  coordinate of all the blocks that share the same  $x$  and  $y$ , or  $\infty$  if no such block exists. The moves available are:

- If  $z - 1 = zMax[x, y]$ , the robot can move to  $(x \pm 1, y, zMax[x \pm 1, y] + 1)$  or  $(x, y \pm 1, zMax[x, y \pm 1] + 1)$  if the  $zMax$  at the destination is less or equal to  $z - 1$ .
- If  $z - 1 > zMax[x, y]$ , the robot can fall to  $(x, y, zMax[x, y] + 1)$ .

These statements can be generalized for the other two axes.

There are at most  $6N^2$  cells that the robot can visit, and the shortest path to the teleporter can be found using BFS. This solution works in  $O(N^2)$  after reading the input. The total complexity is  $O(N^3)$  and the memory usage is  $O(N^2)$ .