

# International Collegiate Programming Contest The 2021 ACPC Girls Virtual July 10<sup>th</sup> 2021



The International Collegiate Programming Contest Sponsored by ICPC Foundation



### The 2021 ACPC Girls

(Contest Problems)



Virtual July 2021

### Problem A. Phantom of the Opera

Input file: phantom.in
Output file: standard output

Balloon Color: Pink

There's a friendly phantom living at the opera house. He likes to throw parties every weekend. His parties are very popular and lots of people line up at the door to get in.

To make the parties more fun, the phantom buys some masks and hands one mask to each guest entering the party. When the masks run out, the phantom shuts the door and refuses to let anyone else enter. To cheer up the people that could not enter the party, the phantom gives each of them a rose and sends them home. The phantom asked you to help him hand out the roses.

Given the number of people waiting outside the opera, and the number of masks the phantom has available, calculate the number roses you need to hand out.

#### Input

The first line contains an integer t  $(1 \le t \le 10^4)$  — the number of test cases.

Each test case is represented as one line containing 2 integers n ( $0 \le n \le 1000$ ) — the number of people waiting outside the opera, and m ( $0 \le m \le 1000$ ) — the number of masks the phantom has available.

#### Output

For each test case, output one integer — the number of roses you have to hand out.

phantom.in	standard output
4	3
10 7	0
10 100	0
123 123	30
30 0	

#### Problem B. Sambosa

Input file: sambosa.in
Output file: standard output

Balloon Color: Brown

Sambosa is a popular snack in the Arab region known for its spicy flavor. It can be stuffed with a variety of ingredients. In the Arabian Coastal Pastry Cafe ACPC, there are two variants, one with meat and another with cheese.

You and your friends sat on a circular table of N chairs. The waitress has M meat Sambosas and C cheese Sambosas. The waitress forgot who ordered what, so she is going to loop around the table starting from chair 1 and will give each person one Sambosa uniformally at random (i.e. if she has i meat Sambosa and j cheese Sambosas, she will give a meat one with probability  $\frac{i}{i+j}$  and a cheese one with probability  $\frac{j}{i+j}$ ). She will continue until she finish all Sambosas she has.

You like cheese Sambosa, given the number of chairs N, your position on the circle idx, the number of meat Sambosas M, and the number of meat Sambosas C, compute the expected number of cheese Sambosas you will get.

#### Input

The first number contains the number of test cases T ( $1 \le T \le 100$ ).

Each test case consists of one line of four integers N, idx, M, and C  $(1 \le idx \le N \le 10^3, 1 \le C, M \le 500)$ .

#### Output

For each test case, output one number: the expected number of cheese Sambosas you will get.

Your answer is considered correct if its absolute or relative error does not exceed  $10^{-4}$ .

Formally, let your answer be a, and the correct answer be b. Your answer is accepted if and only if  $\frac{|a-b|}{\max(1,|b|)} \le 10^{-4}$ .

#### Example

sambosa.in	standard output
2	0.5
2 1 1 1	1.2
5 3 4 6	

#### Note

Expected value is the probability-weighted average of a random variable. For example, if the random variable X can be 2 with probability  $\frac{2}{3}$  and 7 with probability  $\frac{1}{3}$  then  $Ex(X) = \frac{2}{3}.2 + \frac{1}{3}.7 = 3\frac{2}{3}$ . More generally: if variable X can be  $v_1, v_2, \ldots, v_n$  with probabilities  $p_1, p_2, \ldots, p_n$  then  $Ex(X) = \sum_{i=1}^n p_i.v_i$ .

### Problem C. The First University

Input file: fatimah.in
Output file: standard output

Balloon Color: Orange

Al-Qarawiyyin Mosque in Morocco is the first university in the world. It was founded by Fatimah Al-Fihri. One of the subjects that Fatimah liked was Mathematics. In particular, she introduced the Arabic numerals. Many of the mathematicians at the time were playing with prime numbers exploring the wonders of the patterns in them (long before their usage in encryption).

Fatimah found a page of a list of N numbers. She found that all numbers are products of unique primes (i.e. each number cannot be divided twice by a prime number). She also found that all prime divisors of the numbers are less than 42.

Fatimah wanted to cut the list into K consecutive sub-lists such that the sum of the Least Common Multiple (LCM) of each sub-list is maximized.

#### Input

The first line contains the number of test cases T ( $1 \le T \le 10$ ).

The first line of each test case consists of two integers N and K  $(1 \le K \le N \le 3000)$ .

The second line of each test case consists of the list A of N integers  $(1 \le A_i \le 10^8)$ .

#### Output

For each test case, output the maximum sum of the LCM of each sub-list after cutting the given list into K consecutive sub-lists.

#### Example

fatimah.in	standard output
2	12
4 2	217
4 2 6 2 3 2	
4 2	
30 5 7 7	

#### Note

In the first testcase, we can split the array into (6,2)(3,2) and get 6+6=12.

In the second testcase, we can split the array into (30, 5, 7)(7) and get 210 + 7 = 217.

### Problem D. A Nuclear Scientist

Input file: scientist.in
Output file: standard output

Balloon Color: Yellow

After the Foreign invasion on Egypt, most of the community schools were closed and replaced with western-style schools. Nabawiyya Musa was one of the activists for education in Egypt. She became a school director and always looked for talented students. One of them was Sameera Moussa who would later become a well-known nuclear scientist.

Sameera excelled at Algebra to the extent that in Grade 10 (1st secondary), she wrote a new textbook for Grade 10 Algebra under the supervision of Nabawiyya. The textbook was printed and used in most of the schools at the time.

Let's take an algebraic problem: given an array A, we want to add a number X such that the geometric mean of the array is an integer. The geometric mean of an array of length N is the Nth root of the product of the array  $\sqrt[N]{\prod A_i}$ .

#### Input

The first line contains the number of test cases T ( $1 \le T \le 20$ ).

The first line of each test case consists of one integer N ( $1 \le N \le 10^4$ ).

The second line of each test case consists of an array A of N integers  $(1 \le A_i \le 10^7)$ .

#### Output

For each test case, output one line contains one number X such that when X is added to A, the geometric mean is an integer. If there are multiple solutions, output the minimum one. It is guaranteed that X can be less than  $10^{18}$  for all test cases.

scientist.in	standard output
2	9
2	147
6 4	
3	
7 9 21	

### Problem E. Learning Languages

Input file: poet.in

Output file: standard output

Balloon Color: Gold

Nana Asmaa Uthman was a known poet in the Sokoto Caliphate in nowadays Nigeria. She taught many women reading, writing, religion, poetry, mathematics, and more from her father's large library. It's said she wrote poems in at least 4 languages and more than 60 books on different subjects.

For this problem, let's assume she wanted to learn English. English belongs to the Germanic languages family, which is known for very long words, like Hippopotomonstrosesquippedaliophobia which ironically means the fear of long words. Currently, Asmaa can pronounce correctly at least two syllables P1 and P2. Given a possibly long string S, Asmaa wants to find the largest substring that starts with P1 and ends with P2.

#### Input

The first number contains the number of test cases T ( $1 \le T \le 100$ ).

Each test case consists of 2 lines. The first line contains one string S ( $1 \le |S| \le 10^3$ ). The second line contains two space-separated strings P1 and P2 ( $1 \le |P1|, |P2| \le |S|$ ).

#### Output

For each test case, output one integer, the length of the longest substring of S that starts with P1 and ends with P2. If there is no such substring output 0.

poet.in	standard output
3	34
hippopotomonstrosesquippedaliophobia	0
pp phobia	0
arabcollegiateprogrammingcontenst	
pro col	
abbaaba	
abba bb	

#### Problem F. Hand-Copying

Input file: copy.in

Output file: standard output Balloon Color: Light Green

Aisha al-Qurtubiyya was a calligrapher and poet from Al-Andalus. She copied many manuscripts, books, and encyclopedias of her time.

Back then, books were priced based on the number of pages in them, and not their content or fame, as pages were copied by hand. Aisha went to a small library. All books are put on a shelf in a line. Each book has weight  $A_i$  and number of pages  $B_i$ . Aisha has a bag that can carry no more than W weight. She isn't greedy and only wants the number of pages to be not less than X. However, she doesn't want to walk a lot in the library. She wants to find the nearest book (smallest index) i such that she can choose some books up to and including book i with total weight no more than W and total number of pages not less than X.

#### Input

The first line contains the number of test cases t ( $1 \le t \le 20$ ).

The first line of each test case consists of three integers N, X and W ( $1 \le N \le 10^4, 1 \le X, W \le 10^4$ ), the number of books, the required number of pages, and the maximum weight Aisha can carry.

The second line of each test case consists of N integers  $A_i$  ( $1 \le A_i \le 10^4$ ).

The third line of each test case consists of N integers  $B_i$  ( $1 \le B_i \le 10^4$ ).

#### Output

For each test case, output the minimum index i such that Aisha can choose some books up to and including book i with total weight no more than W and total number of pages not less than X.

#### **Example**

copy.in	standard output
1	3
4 5 6	
3 4 1 4	
4 3 3 1	

#### Note

In the sample, if Aisha took the first book, she will only have 2 more weight units remaining. She can take the last book gathering 7 pages, which is enough, but this will make the final output (the last chosen book) 4. If instead she skipped the first book and took the second and third book, she will get 5 pages and a total weight of 6, which is enough, and the final optimal output is 3.

#### Problem G. D-Primes

Input file: dprime.in

Output file: standard output

Balloon Color: Blue

D-prime is the number that can be divided only by 1, D, and itself. It cannot be equal to 1 or D. Given a range L and R, find the number of D-primes in the range.

#### Input

The first number contains the number of test cases T ( $1 \le T \le 100$ ).

Each test case consists of one line of three integers D, L, and R  $(1 \le D \le 10^6, 1 \le L \le R \le 10^9)$ .

#### Output

For each test case, output one integer, the number of D-primes between L and R.

dprime.in	standard output
3	1
2 1 5	0
4 13 15	1
3 7 10	

#### Problem H. Hamza

Input file: arab.in

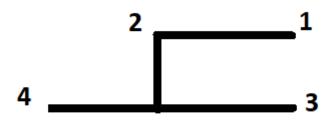
Output file: standard output

Balloon Color: Navy

The Arabic language is know for calligraphy, with different ways that characters can be connected and drawn. Although these letter connections made amazing artwork, they made printing more difficult as many letters can have different forms based on their position in the word. One of these characters is the letter Hamza which can be written in many ways:

### ءأئذإآؤ

While Allaa was investigating this, she wrote a program that can output the 2D coordinates of Hamza on a page. She assumes the Hamza is written with sharp right angles. More specifically, it consists of 4 distinct points, two on the top horizontal line and two on the bottom horizontal line. The two points on the right are vertically aligned, and the leftmost point of the top line is strictly to the right of the leftmost point of the bottom line as shown here:



There is a bug in Allaa's program that causes the output (8 integers representing the coordinates of 4 points) to be shuffled. Can you reconstruct the Hamza shape in the correct order?

#### Input

The first number contains the number of test cases T ( $1 \le T \le 100$ ).

Each test case consists of one line of 8 space-separated integers  $C_i$  ( $0 \le C_i \le 10^9$ ).

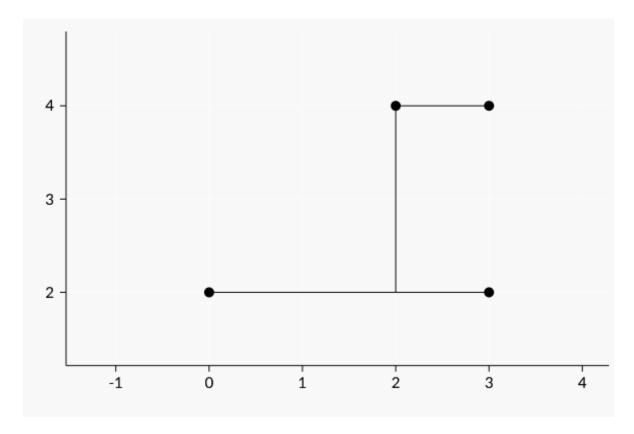
#### Output

For each test case, output one line with the correct order of the eight coordinates. The first two integers represent the X and Y coordinates of the first point in the image above, and so on. It is guaranteed that a correct answer always exists. If there are multiple correct answers, output any of them.

arab.in	standard output
1	3 4 2 4 3 2 0 2
3 3 2 2 4 0 4 2	

### Note

The sample case looks like this:



#### Problem I. Letter Dal

Input file: dal.in

Output file: standard output

Balloon Color: Violet

Daledah likes the first letter of her name, the letter Dal. She found a page with 2D points plotted on it. She wants to count how many Dals can be drawn out of these points. The same point can be used multiple times. The letter Dal consists of 3 points, two aligned horizontally on the bottom side and two aligned vertically on the right side. The two sides are equal. Help Daledah count the number of Dals she can draw.

#### Input

The first line contains the number of test cases T ( $1 \le T \le 6$ ).

The first line of each test case consists of one integer N ( $1 \le N \le 10^4$ ): the number of points.

N lines follow: each line contains two integers  $X_i$  and  $Y_i$  ( $0 \le X_i, Y_i \le 10^8$ ) the coordinates of the  $i^{th}$  point.

#### Output

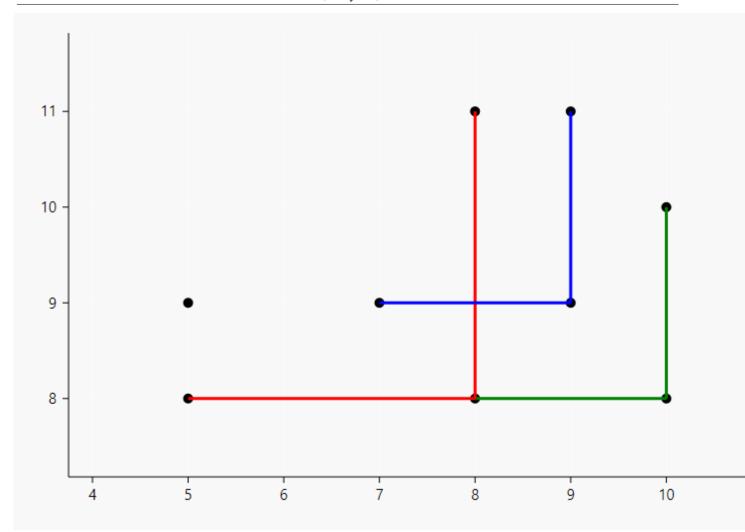
For each test case, output one integer: the number of (not necessary disjoint) letter Dal that can be formed from the given points.

#### Example

dal.in	standard output
1	3
9	
10 10	
10 8	
8 8	
9 11	
9 9	
7 9	
8 11	
5 8	
5 9	

#### Note

The sample case looks like this:



#### Problem J. XOR-K

Input file: xor.in

Output file: standard output

Balloon Color: White

Karima Al-Marwaziyya is well-known scholar of the 11th century AD (5th AH). She travelled between Turkmenistan, Baghdad, Quds, and Mecca. Throughout her travel, she needed strong horses. Each horse has a set of features represented as a binary string, if the  $i^{th}$  feature exists, then the  $i^{th}$  character will be 1. The strings are then encoded as integers.

Strangely enough, the combined features of a herd of horses is the XOR of their features. Karima can remove any number of features from any horse (i.e. make the bit that represents a feature 0 if it was 1 but not the other way around). Help her find the maximum value of the combined features of the herd.

#### Input

You will be given in the first line an integer N  $(1 \le N \le 10^6)$  the number of horses.

In the next line you will be given N integers  $k_i$  separated by spaces  $(0 \le k_i \le 10^6)$  the encoded features of the  $i^{th}$  horse.

#### Output

A single integer: the maximum XOR that can be obtained.

#### Example

xor.in	standard output
4	7
2 3 4 5	

#### Note

The XOR operation means the bitwise exclusive OR operation, which is 1 when bits differ and 0 otherwise.

#### Problem K. Elevator Game

Input file: elevator.in
Output file: standard output

Balloon Color: Silver

A new building built for the ACPC hotel consists of 9 floors numbered from 1 to 9. The building has an elevator of strange behavior. The elevator's controller pad is like the num pad on computer keyboards. The pad is described in the image below.



One has to enter instructions left L, right R, down D, or up U in order to reach the desired floor from the current floor. For example, if someone is on floor 2 and wants to go to floor 4, one way to build the instructions string is RRU to reach floor 4. Note that the instructions are cyclic (for example, if you are at floor number 8 and typed U the elevator will go to floor number 2).

You will be given the floor you are currently at and an experimental string. You are required to output the floor the elevator will end at.

#### Input

First line will be a string S ( $1 \le |S| \le 200000$ ), the instructions string. Second line will be the current floor X ( $1 \le X \le 9$ ).

#### Output

Output the number of the floor that will be reached after performing the instructions string.

elevator.in	standard output
RRU	4
2	

### Problem L. Growing World

Input file: phone.in

Output file: standard output

Balloon Color: Black

It's 7991 AD, and the world population has exploded. The phone number length has grown to cover this large population. Some numbers even reached a length of 100000 digits!

Mariam Mirza Khani, the famous mathematician, saved all her friends' numbers on her phone, but her phone got hacked and some digits of her best friend's number got replaced by ?. Mariam remembers the number of occurrences of each digit in her friend's number. She will try to restore the number by herself if only she knew in how many ways can she reconstruct her friend's number.

As the number of possible ways to reconstruct the number can be too large, she wants to know the number of ways modulo 1000000007.

#### Input

The first line of input contains a string S ( $1 \le |S| \le 100000$ ), representing the corrupted number. Followed by 10 lines each line has a digit and the number of occurrences of this digit that was changed to ?.

#### Output

Print the number of ways to reconstruct the number modulo  $1000000007(10^9 + 7)$ .

phone.in	standard output
0??52?08?15	6
0 0	
1 2	
2 0	
3 0	
4 0	
5 0	
6 2	
7 0	
8 0	
9 0	

### Problem M. Longest Statement?

Input file: graph.in

Output file: standard output Balloon Color: Dark Green

Given an undirected connected graph with no self loops or multiple edges, find the number of pairs of nodes such that the simple path between these two nodes is unique (there is no more than one simple path between these two nodes).

A simple path is a path that doesn't visit vertex or edge more than once.

#### Input

The first line contains two integers n, m  $(1 \le n \le 2 \cdot 10^5, n-1 \le m \le min(2 \cdot 10^5, \frac{n(n-1)}{2}))$  the number of nodes and edges respectively.

Each of the next m lines contains two integers x and y representing an edge between node x and node y  $(1 \le x < y \le n)$ .

#### Output

Print the number of such pairs.

graph.in	standard output
7 7	6
1 2	
1 3	
2 4	
2 5	
4 6	
4 7	
4 5	
12 14	10
1 2	
2 3	
3 4	
3 5	
4 5	
4 11	
1 7	
7 8	
8 9	
8 10	
9 10	
7 10	
9 12	
6 12	

#### Problem N. Cover science branches

Input file: branches.in
Output file: standard output

Balloon Color: Cyan

Fatimah Khatun the sister of the well-known Salah al-Din ibn Ayyub (aka Sitt al-Sham) focused on recovering schools and restoring knowledge lost during the European crusades. She financed scientists to write encyclopedias on different subjects.

To restore sciences systematically, she organized them into a tree. Each node represents a branch. However, it's not easy to make clear distinctions between subjects, so an encyclopedia in one subject may cover neighboring subjects but would require larger encyclopedia.

The subject (node) u can be covered with an encyclopedia of size  $A_u$ . The subject u can be covered and all of its neighbors (including its parent) with an encyclopedia of size  $B_u$ . Some subjects are special, they can cover themselves and all their neighbors and the neighbors' neighbors (distance from node  $u \leq 2$ ), with an encyclopedia of size  $C_u$ .

Given the tree of subjects and the three arrays A, B, and C, find the minimum sum of encyclopedias size (cost) to cover the whole tree. It is possible to cover a subject multiple times.

#### Input

The first line of the input contains T the number of testcases  $(1 \le T \le 10)$ .

The first line of each testcase contains N the size of the tree  $(1 \le N \le 10^5)$ .

The second line of each testcase contains N-1 integers  $P_i$  the parent of the i+1th node  $(1 \le P_i < i+1)$ .

The third line of each testcases contains N integers  $A_i$  ( $1 \le A_i \le 10^4$ ).

The forth line of each testcases contains N integers  $B_i$  ( $1 \le B_i \le 10^4$ ).

The fifth line of each testcases contains N integers  $C_i$ . If the ith node isn't special then  $C_i$  equals -1 otherwise  $(1 \le C_i \le 10^4)$ .

#### Output

For each line, print one integer minimum sum of encyclopedias size (cost) to cover the whole knowledge tree

branches.in	standard output
1	28
7	
1 1 2 2 4 4	
2 5 6 17 12 3 1	
8 42 10 21 15 7 30	
74 -1 -1 -1 9 -1	



If m >= n, output 0, else, output n-m.

## Problem Tutorial: "Sambosa"

DP[i][j] where i is the current number of meat sambosas and j is the current number of cheese sambosas. We can deduce the current position from the two parameter (i.e. (M-i+C-j))

if pos equals idx

$$DP[i][j] = j/(i+j) * (1 + DP[i][j-1]) + i/(i+j) * DP[i-1][j]$$

else

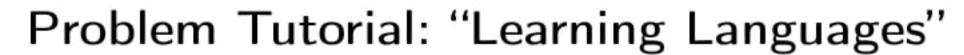
$$DP[i][j] = j/(i+j) * DP[i][j-1] + i/(i+j) * DP[i-1][j]$$

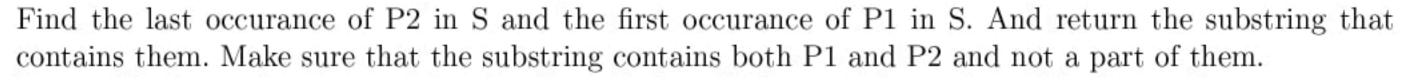
## Problem Tutorial: "The First University"

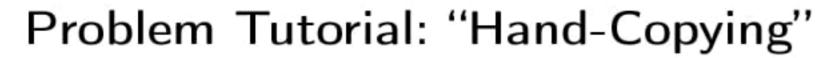
The simple DP solution is to compute DP[k][i] the optimal answer for cutting the prefix of size i into k sub-arrays. And we can loop over j >= i the end of the current subarray. This solution has complexity  $O(N^2K)$ . But we find that the LCM only changes in 13 position (the number of primes less than 42 # P). So we can compute the at most 13 valid j for each i in  $O(N^2)$  with total complexity  $O(N^2 + NK \cdot \# P)$ .

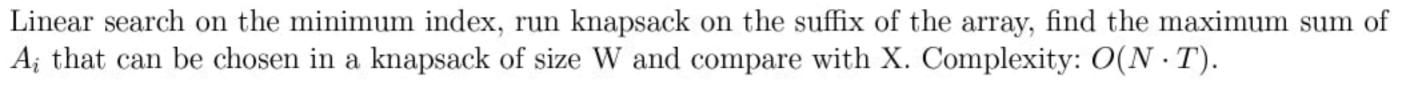
## Problem Tutorial: "A Nuclear Scientist"

Compute the prime factorization of the given array. We need to make all primes have power divisible by N+1. Multiply all primes that currently have power not divisible by N+1 till all of them have power divisible by N+1, or state that it's not possible if the result is larger than the limit.

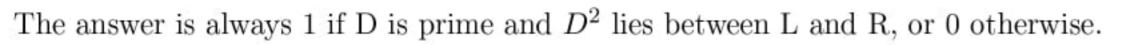








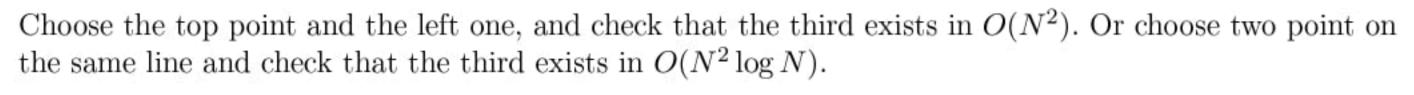




## Problem Tutorial: "Hamza"

Try all permutations of the coordinates and check that the current permutation represents a Hamza. The two top points align horizontally, and the two rightmost points align vertically. The two bottom points align horizontally. The two leftmost point, the top one is to the strich right of the bottom one.









### Problem Tutorial: "Elevator Game"

Simply follow the instructions. If the instruction is U add 3 unless the current button after performing the instruction is above 9 then minus 9. If the instruction is D minus three unless the current button after performing the instruction is below 1 then add 9. If the instruction is L minus one unless the current button is 1, 4 or 7 then add 2. If the instruction is R add one unless the current button is 3, 6 or 9 then minus 2.

Total Complexity O(|S|)

## Problem Tutorial: "Growing World"

Using combinatorics. Let N be the number of unknown digits,  $x_1$  be the number of occurrences of the first unknown digit,  $x_2$  the number of occurrences of the second unknown digit,  $x_3$  the number of third unknown digit, ...  $X_{10}$  the number of occurrences of the tenth unknown digit. Then the answer will be using the multinomial  $\binom{n}{x_1, x_2, x_3, ..., x_{10}}$ 

Total Complexity O(|S|)

## Problem Tutorial: "Longest Statement?"

nodes u and v have one valid path if the two nodes are articulation points and the edges connecting them on the path are bridges, so we construct another graph where two nodes are connected if they are connected in the original graph with a bridge and the two nodes are articulation points, and to count the number of valid pairs of nodes we count the number of pairs of nodes in the same connected component in the resulted graph.

## Problem Tutorial: "Cover science branches"

Using DP on trees you have to try all possibilities so every node will have 5 states:

- 1. need two level cover from the parent or parent neighbors
- 2. need one level cover from the parent
- 3. need zero level cover
- 4. will provide one level to neighbors distance <=1
- 5. will provide two level to neighbors distance <=2

Every node will calculate its states using children states and we can select any node as root so the answer will be the min between the third and fourth and the fifth state of the root.

explanation for states calculation:-

 $dp[1][u] = \sum_{v} \min(dp[s >= 2][v])$ : sum of all children need one level of cover or none  $\min(\text{states}(2,3,4,5))$ .

 $dp[2][u] = \sum_{v} \min(dp[s >= 3][v])$ : sum of all children that don't need any cover  $\min(\text{states}(3,4,5))$ .

 $dp[3][u] = min(\sum_{v} min(dp[s >= 3][v]) + a[u], min_{w}(\sum_{v \neq w} min(dp[s >= 3][v]) + min(dp[s >= 4][w])):$  minimum between (sum of all children need zero or less levels of cover min(states(3,4,5)) + a[u], or take cover from one child min(states (4,5)) and sum of the rest need zero or less levels of cover min(states(3,4,5))).

 $dp[4][u] = min(\sum_{v} min(dp[s >= 2][v]) + b[u], min_w \sum_{v \neq w} min(dp[s >= 2][v]) + dp[5][w])$ : minimum between (sum of all children need one or less levels of cover min(states(2,3,4,5)) + b[u], or take cover from one child state(5) and sum of the rest need one or less levels of cover min(states(2,3,4,5))).

 $dp[5][u]=C[u] + min(\sum_{v} min(dp[s >= 1][v])$ : C+sum of all children need two or less levels of cover min(states(1,2,3,4,5)) // only for the nodes from the given set.