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## SWERC'2016

Southwestern Europe Regional Contest

<http://swerc.up.pt/2016/>

**U.PORTO**  
FC  
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ACM International Collegiate Programming Contest 2016  
SWERC'2016: Southwestern Europe Regional Contest  
<http://swerc.up.pt/2016/>

## Contents

<b>Problem A:</b> Within Arm's Reach . . . . .	3
<b>Problem B:</b> Bribing Eve . . . . .	5
<b>Problem C:</b> Candle Box . . . . .	7
<b>Problem D:</b> Dinner Bet . . . . .	9
<b>Problem E:</b> Passwords . . . . .	11
<b>Problem F:</b> Performance Review . . . . .	13
<b>Problem G:</b> Cairo Corridor . . . . .	15
<b>Problem H:</b> Pascal's Hyper-Pyramids . . . . .	17
<b>Problem I:</b> The White Rabbit Pocket Watch . . . . .	19
<b>Problem J:</b> Risky Lottery . . . . .	21
<b>Problem K:</b> Balls and Needles . . . . .	23

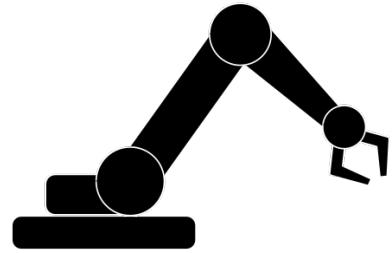
## Judges and Problem Setters

- Ana Paula Tomás (FCUP, University of Porto)
- André Susano Pinto (Google)
- Eduardo Marques (FCUP, University of Porto)
- Margarida Mamede (Nova University of Lisbon)
- Miguel Araújo (FCUP, University of Porto & CMU, US)
- Miguel Oliveira (FEUP, University of Porto)
- Pedro Guerreiro (University of Algarve)
- Pedro Pereira (INESC-TEC)
- Pedro Ribeiro (FCUP, University of Porto)
- Pedro Vasconcelos (FCUP, University of Porto)
- Sérgio Crisóstomo (FCUP, University of Porto)

# Within Arm's Reach

João wants to join the robotic football team of his university. However, since he knows little about robotics and mathematics, he decided to build a 2-dimensional robotic arm to bootstrap his knowledge.

The robotic arm is composed of  $N$  segments of various lengths. The segments can form any angle between them, including configurations that make it appear to self-intersect when viewed from above. The robotic arm works great, but it is not trivial to position the arm's tip as close as possible to given  $x, y$  target coordinates with so many joints to control. Can you help João?



## Task

Given the robotic arm description and target coordinates relative to the arm's origin, calculate a configuration that places the arm's tip as close as possible to the target.

## Input

The first line contains  $N$ , the number of segments composing the robotic arm.  $N$  lines follow, each with an integer  $L_i$  describing the length of the  $i$ th segment from the fixed point until the arm's tip. There is one more line with 2 integers: the  $x, y$  coordinates of the target point to reach.

## Constraints

$1 \leq N \leq 20$  Number of segments in the robotic arm

$1 \leq L_i \leq 1\,000$  Length of the  $i$ th segment

$-20\,000 \leq x, y \leq 20\,000$  Target coordinates to attempt to reach

## Output

The output should contain  $N$  lines, each containing two real numbers  $x_i, y_i$  indicating the coordinates of the tip of the  $i$ th segment.

The length of the  $i$ th segment computed from the solution and input  $L_i$  may not differ by more than 0.01. Similarly, the absolute error between the solution's distance to the target and the minimum possible distance to the target cannot exceed 0.01.

Note that, in general, there are many solutions. Your program may output any of them.

## Problem A

---

### Sample Input 1

```
3  
5  
3  
4  
5 3
```

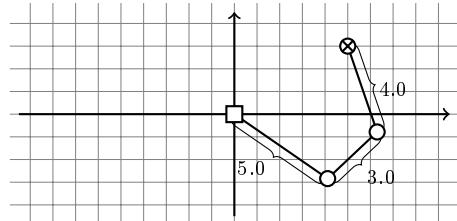
### Sample Output 1a

```
4.114 -2.842  
6.297 -0.784  
5.000 3.000
```

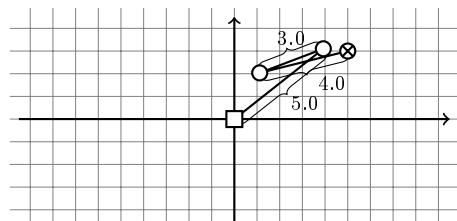
### Sample Output 1b

```
3.923 3.100  
1.118 2.037  
5.000 3.000
```

### Sample 1 Explanation



Sample Output 1a



Sample Output 1b

## Problem A

---

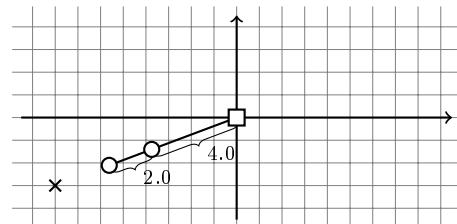
### Sample Input 2

```
2  
4  
2  
-8 -3
```

### Sample Output 2

```
-3.745 -1.404  
-5.618 -2.107
```

### Sample 2 Explanation



Sample Output 2

# Bribing Eve

Eve works at a magazine that does product reviews and publishes recommendations to consumers. They are working on a new mobile phones review and have decided on two reproducible tests that score each device's battery lifetime and performance using an integer between 1 and 1000.

These two scores,  $x_1$  and  $x_2$ , are then combined with a weights vector  $w = [w_1, w_2]$  to produce an overall score:

$$s = w_1x_1 + w_2x_2.$$

The final review ranking is then obtained by sorting the products by decreasing order of  $s$ . Additionally, when multiple products get exactly the same score, Eve decides how to order them.

Maria (a fake name to mask her identity) tried to bribe Eve to tweak the results to get her product higher on the list. Eve argued that she was not able to tamper the evaluation of each test, but Maria suggested to tweak the weights  $w$  used when computing the overall score. The weights  $w$  must be non-negative and at least one of them must be positive, but the values are decided by Eve.

Eve is thinking whether to modify the weights in Maria's benefit or not, and asked you to determine what are the best and worst possible ranking positions for Maria's product.



## Task

Given a list of all products scores in battery and performance  $[x_1, x_2]$  tests, find out what are the best and worst positions in the ranking that can be given to Maria's product when the weights  $[w_1, w_2]$  and the order for tied products are left for Eve to decide.

## Input

The first line has the number  $N$  of products being compared.  $N$  lines follow, each containing two integers  $x_1$  and  $x_2$  indicating a product's score in the battery and performance tests. Maria's product is the first on the list.

## Constraints

$$1 \leq N \leq 100\,000 \quad \text{Number of products}$$

$$1 \leq x_1, x_2 \leq 1\,000 \quad \text{Performance of a product in the tests}$$

## **Problem B**

---

## **Problem B**

---

### **Output**

The output consists of two numbers  $A$  and  $B$ , indicating the best and worst possible positions that Maria's product can get on the ranking given Eve's ability to modify the weights and ordering in case of a tie.

### **Sample Input**

```
5
7 7
11 10
8 5
1 1
12 12
```

### **Sample Output**

```
3 4
```

# Candle Box

Rita loves her Birthday parties. She is really happy when blowing the candles at the *Happy Birthday*'s clap melody. Every year since the age of four she adds her birthday candles (one for every year of age) to a candle box. Her younger daydreaming brother Theo started doing the same at the age of three. Rita and Theo boxes look the same, and so do the candles.

One day Rita decided to count how many candles she had in her box:

— *No, no, no! I'm younger than that!*

She just realized Theo had thrown some of his birthday candles in her box all these years. Can you help Rita fix the number of candles in her candle box?



## Task

Given the difference between the ages of Rita and Theo, the number of candles in Rita's box, and the number of candles in Theo's box, find out how many candles Rita needs to remove from her box such that it contains the right number of candles.

## Input

The first line of the input has one integer  $D$ , corresponding to the difference between the ages of Rita and Theo.

The second line has one integer  $R$ , corresponding to the number of candles in Rita's box.

The third line has one integer  $T$ , corresponding to the number of candles in Theo's box.

## Constraints

$1 \leq D \leq 20$  Difference between the ages of Rita and Theo

$4 \leq R < 1\,000$  Number of candles in Rita's box

$0 \leq T < 1\,000$  Number of candles in Theo's box

## Output

An integer representing the number of candles Rita must remove from her box such that it contains the right number of candles.

## **Problem C**

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## **Problem C**

---

### **Sample Input**

2  
26  
8

### **Sample Output**

4

# Dinner Bet

Cesar and Raul like betting and good food, in no particular order. They want to try out a new fancy restaurant and they decided to make a bet – they are going to play a game and the loser pays for dinner.

They have a box with  $N$  balls. Each ball contains a distinct number between 1 and  $N$ . Then, the game proceeds with these steps:

- Initially, each person picks  $C$  distinct numbers between 1 and  $N$  and writes them down on a separate card.
- In each round,  $D$  balls are drawn from the box uniformly at random. Cesar and Raul mark down the ball numbers that appear in their respective card. The  $D$  balls are then returned to the box.
- The game stops when a player is able to mark on the card all the numbers he chose. That player is the winner. If both players finish at the same time, it is a draw and they will split the dinner.



They are quite eager to try out this new restaurant and they're now wondering: how many rounds will the game last?

## Task

Given the number  $N$  of balls, the number  $D$  of balls they draw from the box in each round, the amount  $C$  of numbers in their cards and the numbers they wrote down, find the expected number of rounds the game will last.

## Input

The first line of the input consists of three space separated integers:  $N$ ,  $D$ , and  $C$ .  $N$  is the number of balls,  $D$  is the number of balls drawn in each round, and  $C$  is the cards' size. Each of the following two lines contains  $C$  space separated integers: the numbers Cesar and Raul wrote down, respectively.

## Constraints

$1 \leq N \leq 50$  Number of balls in the box

$1 \leq D \leq \min(10, N)$  Number of balls drawn in each round

$1 \leq C \leq \min(10, N)$  Cards' size

## **Problem D**

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## **Problem D**

---

### **Output**

The output is the expected number of rounds of the game.

The result will be considered correct as long as the absolute error does not exceed  $10^{-3}$ .

### **Sample Input 1**

```
2 1 1
1
2
```

### **Sample Output 1**

```
1.00000
```

### **Sample 1 Explanation**

There are 2 balls. Cesar picked number 1 and Raul picked number 2. In the first round, either number 1 or 2 will be drawn and so one of them wins right away.

### **Sample Input 2**

```
30 5 10
2 3 5 7 11 13 17 19 23 29
20 18 16 14 12 10 8 6 4 2
```

### **Sample Output 2**

```
13.30378
```

# Passwords

It's that time of the year again when you go back to work and need to choose new passwords for all your services. The rules enforced by the system administrators are very strict, and the password you choose must obey the following restrictions:

- It must contain only letters and digits.
- It must have between  $A$  and  $B$  characters (inclusive).
- It must have at least one lowercase letter, one uppercase letter and one digit.
- It cannot contain any word of a collection of forbidden words (the blacklist).



A word of the blacklist is considered to be contained in the password if it appears as a substring, that is, if it occurs as a consecutive sequence of characters (regardless of it being upper or lower case). For instance, `swerc` is a substring of `SwErC`, `2016swerc2016` or `SWERC16`, but it is not a substring of `ICPC` or `sw16erc`.

Additionally, for the purposes of avoiding the blacklist, you cannot use `l33t`. Specifically, some digits can be interpreted as letters, namely the 0 ('o'), 1 ('i'), 3 ('e'), 5 ('s') and 7 ('t'). This implies that for example `5w3rC` would be an occurrence of `swerc`, and that `abcL337def` contains the word `leet`.

You cannot stop thinking about all these rules and you wonder how many different valid passwords there are... Can you calculate how many passwords would obey these rules?

## Task

Given a blacklist with  $N$  words and two integers  $A$  and  $B$ , your task is to compute the number of different valid passwords that exist following the given constraints: made up of only letters and digits; length between  $A$  and  $B$  (inclusive); at least one lowercase letter, one uppercase letter, and one digit; no blacklisted substring. Since this number can be very big, compute it modulo 1 000 003.

## Input

The first line contains two integers,  $A$  and  $B$ , specifying respectively the minimum and maximum length of the password. The second line contains an integer  $N$ , the number of words of the blacklist. The following  $N$  lines each contains a string  $W_i$  indicating a word in the blacklist. These words are formed only by lowercase letters.

## Problem E

---

## Problem E

### Constraints

$3 \leq A \leq B \leq 20$  Size of the password  
 $0 \leq N \leq 50$  Number of words in the blacklist  
 $1 \leq \text{length}(W_i) \leq 20$  Size of each blacklist word

### Output

The output should contain a single line with an integer indicating the number of valid passwords modulo 1 000 003. A valid password is one that respects all of the given constraints.

### Sample Input

```
3 5
9
swerc
icpc
fbi
cia
bio
z
hi
no
yes
```

### Sample Output

607886

### Sample Explanation

In this case there are exactly 378 609 020 valid passwords and

$$378\,609\,020 \bmod 1\,000\,003 = 607\,886.$$

Some examples of valid passwords are: aA1, B23tT, 1g9K or B2j.

Some examples of invalid passwords are: aaA (it does not contain digits), 12a (it does not contain upper case letters), a12A34 (length > 5) or bB10 (contains bio as substring).

# Performance Review

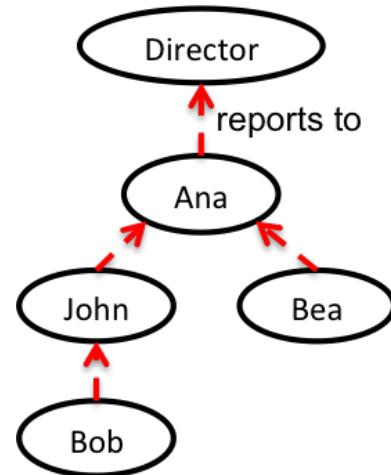
Employee performance reviews are a necessary evil in any company. In a performance review, employees give written feedback about each other on the work done recently. This feedback is passed up to their managers which then decide promotions based on the feedback received.

Maria is in charge of the performance review system in the engineering division of a famous company. The division follows a typical structure. Each employee (except the engineering director) reports to one manager and every employee reports directly or indirectly to the director.

Having the managers assessing the performance of their direct reports has not worked very well. After thorough research, Maria came up with a new performance review system. The main idea is to complement the existing corporate structure with a technical rank for each employee. An employee should give feedback only about subordinates with lower technical level.

Hence, the performance review will work as follows. Employees prepare a summary of their work, estimate how much time it takes to review it, and then request their superiors with higher technical rank to review their work.

Maria is very proud of this new system, but she is unsure if it will be feasible in practice. She wonders how much time each employee will waste writing reviews. Can you help her out?



## Task

Given the corporate structure of the engineering division, determine how much time each employee will spend writing performance reviews.

## Input

The first line of input has one integer  $E$ , the number of employees, who are conveniently numbered between 1 and  $E$ . The next  $E$  lines describe all the employees, starting at employee 1 until employee  $E$ . Each line contains three space-separated integers  $m_i \ r_i \ t_i$ , the manager, the technical rank and the expected time to perform the review of employee  $i$ . The engineering director has no manager, represented with  $m_i = -1$ . The other employees have  $m_i$  between 1 and  $E$ .

## Problem F

---

## Problem F

### Constraints

- $1 \leq E \leq 100\,000$  Number of employees  
 $1 \leq r_i \leq 100\,000$  Technical rank of each employee  
 $1 \leq t_i \leq 100\,000$  Expected time to perform each review

### Output

The output contains  $E$  lines. Line  $i$  has the time employee  $i$  will spend writing reviews.

### Sample Input

```
5
4 4 80
1 1 40
-1 10 60
3 5 50
4 8 70
```

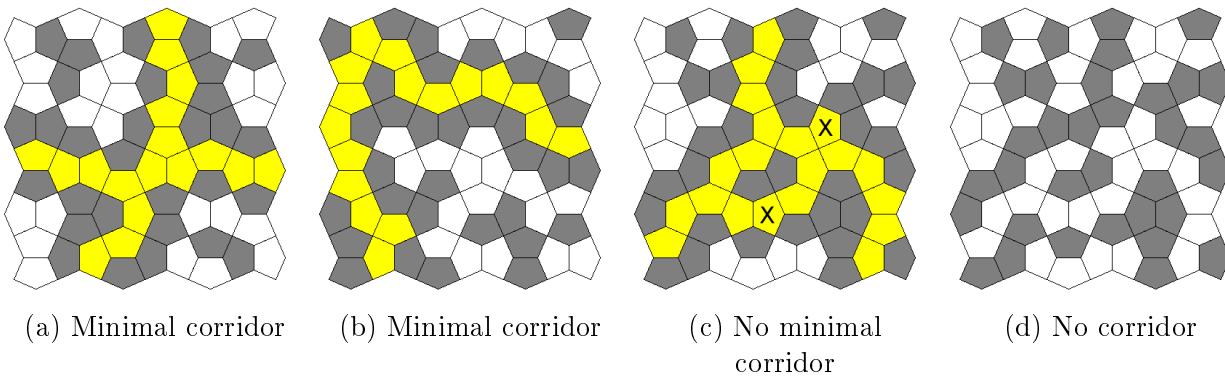
### Sample Output

```
40
0
240
120
0
```

# Cairo Corridor

The *Cairo pentagonal tiling* is a decomposition of the plane using semiregular pentagons. Its name is given because several streets in Cairo are paved using variations of this design.

Consider a bounded tiling where each pentagon is either *clear* (white) or *filled in* (grey). A *corridor* is a maximal set of clear adjacent pentagons that connect the four borders of the tiling. Pentagons are considered adjacent if they share an edge, not just a corner. It is easy to verify that there can be at most one corridor in each tiling. A corridor is said to be *minimal* if it has no superfluous pentagon, that is, if any pentagon of the corridor was filled in, the set of remaining pentagons would not be a corridor.



The figure above depicts four example tilings. In the first three cases, there is a corridor which is highlighted in yellow. Besides, the corridors of figures (a) and (b) are minimal, but the one in figure (c) is not: for example, the tiles marked ‘X’ (among others) could be filled in and a corridor would still exist. In the rightmost tiling there is no corridor.

The tilings shown in figures (a) and (c) correspond to sample input 1.

## Task

Write a program that reads textual descriptions of Cairo tilings, and for each one determines if a corridor exists and is minimal. In the latter case, the program should compute the *size of the corridor*, i.e., the number of clear pentagonal tiles of the corridor.

## Problem G

## Problem G

### Input

The first line of input is a positive decimal integer  $T$  of tilings to be processed. Each tiling description  $k$  has a first line with two positive decimal integers,  $N_k$  and  $M_k$ , separated by a space. The following  $N_k$  lines contain  $2 \times M_k$  binary digits representing pairs  $a_{ij}, b_{ij}$  of tiles (0 is clear and 1 is full) in alternating horizontal/vertical adjacency following a “checkerboard” pattern, as is illustrated in the figure below.

### Constraints

- $1 \leq T \leq 10$  Number of tilings
- $1 \leq \sum_{k=1}^T N_k \leq 250$  Total number of lines
- $1 \leq \sum_{k=1}^T M_k \leq 250$  Total number of tile pairs

### Output

The output consists of  $T$  lines; the  $k$ -th line should be the size of the corridor of the  $k$ -th tiling, if there exists a minimal corridor, and NO MINIMAL CORRIDOR, otherwise.

### Sample Input 1

```

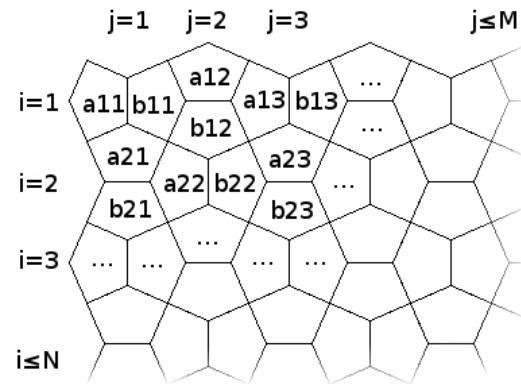
2
6 6
010101001001
001000101100
110101001101
010010000100
001110110010
001001101010
6 6
010010110010
001100100111
000110100101
011001100101
100100011100
011010001101

```

### Sample Output 1

17

NO MINIMAL CORRIDOR



### Sample Input 2

```

1
3 4
11110111
01000000
11110111

```

### Sample Output 2

9

## Pascal's Hyper-Pyramids

We programmers know and love Pascal's triangle: an array of numbers with 1 at the top and whose entries are the sum of the two numbers directly above (except numbers at both ends, which are always 1). For programming this generation rule, the triangle is best represented left-aligned; then the numbers on the left column and on the top row equal 1 and every other is the sum of the numbers immediately above and to its left. The numbers highlighted in bold correspond to the base of Pascal's triangle of height 5:



1	1	1	1	1
1	2	3	4	
1	3	6		
1	4			
1				

*Pascal's hyper-pyramids* generalize the triangle to higher dimensions. In 3 dimensions, the value at position  $(x, y, z)$  is the sum of up to three other values:

- $(x, y, z - 1)$ , the value immediately below it if we are not on the bottom face ( $z = 0$ );
  - $(x, y - 1, z)$ , the value immediately behind it if we are not on the back face ( $y = 0$ );
  - $(x - 1, y, z)$ , the value immediately to the left if we are not on the leftmost face ( $x = 0$ ).

The following figure depicts Pascal's 3D-pyramid of height 5 as a series of plane cuts obtained by fixing the value of the  $z$  coordinate.

$$\begin{array}{c|ccccc} z=0 & z=1 & z=2 & z=3 & z=4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & \\ 1 & 3 & 6 & & \\ 1 & 4 & & & \\ \hline 1 & & 2 & 3 & 4 \\ & 2 & 6 & \mathbf{12} & \\ & 3 & \mathbf{12} & & \\ & 4 & & & \\ \hline & & 1 & 3 & 6 \\ & & 3 & \mathbf{12} & \\ & & 6 & & \\ & & 4 & & \\ \hline & & & 1 & \end{array}$$

For example, the number at position  $x = 1, y = 2, z = 1$  is the sum of the values at  $(0, 2, 1)$ ,  $(1, 1, 1)$  and  $(1, 2, 0)$ , namely,  $6 + 3 + 3 = 12$ . The base of the pyramid corresponds to a plane of positions such that  $x + y + z = 4$  (highlighted in bold above).

The size of each layer grows quadratically with the height of the pyramid, but there are many repeated values due to symmetries: numbers at positions that are permutations of one another must be equal. For example, the numbers at positions  $(0, 1, 2)$ ,  $(1, 2, 0)$  and  $(2, 1, 0)$  above are all equal to 3.

**Task**

Write a program that, given the number of dimensions  $D$  of the hyper-space and the height  $H$  of a hyper-pyramid, computes the set of numbers at the base.

**Input**

A single line with two positive integers: the number of dimensions,  $D$ , and the height of the hyper-pyramid,  $H$ .

**Constraints**

$2 \leq D < 32$  Number of dimensions

$1 \leq H < 32$  Height

$D$  and  $H$  are such that all numbers in the hyper-pyramid are less than  $2^{63}$ .

**Output**

The set of numbers at the base of the hyper-pyramid, with no repetitions, one number per line, and in ascending order.

**Sample Input 1**

2 5

**Sample Output 1**

1  
4  
6

**Sample Input 2**

3 5

**Sample Output 2**

1  
4  
6  
12

# The White Rabbit Pocket Watch

*Alice: How can it be?*

*Rabbit: Trust me Alice. It always takes the same time. When I go from my home up the road to Queen of Hearts' Castle, my watch counts nine hours. However, if I continue down to Mad Hatter's House, my watch counts just two hours in total. Isn't that great?*

*Alice: How can it be Rabbit? The path is longer and you take a shorter time to do it? How can it be?*

*Rabbit: Trust me Alice! It is all recorded in my logbook. You can check it. All my trips are there...*

*Alice: Rabbit, I do not think it can help me...*

*Rabbit: Alice, no matter where you are, or where you want to go, or the track you choose, you'll be able to find how long it takes you.*

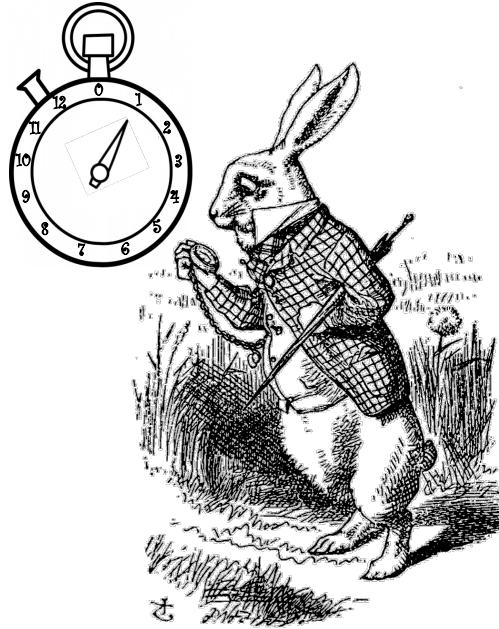
*Alice: Really?*

*Rabbit: For sure!*

Poor Rabbit, poor Alice.

White Rabbit is helping Alice finding a quick way home through the Rabbit's hole with his holy logbook of trips. The problem lies in the chronometer of its bizarre pocket watch (it displays the hours from zero to 12), and the way the Rabbit counts the time with it: If a journey takes 14 hours (real time), seeing the pointer resting above number one, he assumes it took one hour long.

Given that the White Rabbit is telling the truth, can you help Alice finding how long the shortest path home takes, using the Rabbit's logbook of trips?



## Task

Your task is to find the shortest real time it takes for Alice to go from her present location to the Rabbit's hole. For each trip, the White Rabbit wrote down the trip time, the number of visited locations (not necessarily distinct) and the sequence in which they were visited. That sequence defines the trip because there is at most one direct track between any two locations in the Wonderland and it takes the same time both ways.

## Problem I

---

## Problem I

### Input

The first line contains four integers  $N$ ,  $A$ ,  $R$  and  $T$ , where:  $N$  is the number of distinct locations;  $A$  identifies the place where Alice is located;  $R$  corresponds to the Rabbit's hole location; and  $T$  is the number of trips recorded in White Rabbit's logbook. All locations are identified by numbers from 1 to  $N$ . Each of the next  $T$  lines describes a trip logged with format  $d\ p\ a_1\ a_2\ \dots\ a_p$ , where  $d$  is the trip duration (according to White Rabbit),  $p$  is the number of locations and  $a_1\ a_2\ \dots\ a_p$  is the sequence of visited locations.

### Constraints

$2 \leq N \leq 200$  Number of locations

$1 \leq T \leq 500$  Number of trips in the logbook

$2 \leq p \leq 800$  Number of (possibly repeated) locations in a trip

$1 \leq d_{ij} \leq 12$  Real time of the direct track between  $a_i$  and  $a_j$  (if it exists)

There are at most 200 direct tracks

### Output

An integer representing the shortest (real) time it takes for Alice to get home.

### Sample Input 1

```
3 1 3 3
3 4 1 2 3 2
4 3 1 2 1
1 4 1 2 1 3
```

### Sample Output 1

```
9
```

### Sample Input 2

```
5 5 1 9
0 3 1 2 3
1 4 1 4 2 3
6 4 3 4 1 3
11 5 1 3 4 2 1
4 4 1 2 4 1
6 6 1 2 3 1 4 3
7 4 2 3 4 1
11 3 4 3 5
12 5 5 2 4 2 5
```

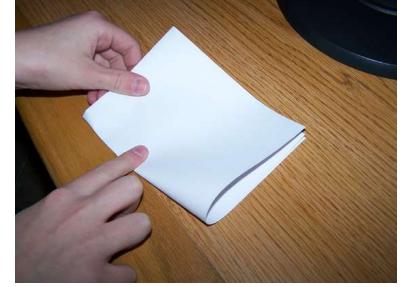
### Sample Output 2

```
6
```

# Risky Lottery

Prof. Peter decided to surprise his class by organizing a new lottery with a very peculiar system.

He starts by announcing a small positive number  $M$ . Afterwards, each student is going to secretly write an integer from the set  $\{1, \dots, M\}$  on a slip of paper that is then folded. After everyone has selected a number, they reveal all choices and whoever wrote down the lowest unique number is the winner! For instance, if there are only three students in the class, two of them select number 2 and the other selects number 5, then the student who chose number 5 is the winner.



The lottery was a success, but Prof. Peter is now wondering what strategy his students should have used. If everyone follows the same optimal randomized strategy, with which probability should each number be chosen so that they maximize their chances of winning? A strategy is optimal if, when everyone is following it, then no individual student can improve his winning probability by selecting a different strategy. Can you help Prof. Peter?

## Task

Given  $N$ , the number of students in the class, and  $M$ , the largest number they can write down, determine the optimal randomized strategy (followed by all students). That is, determine the probability of selecting each number between 1 and  $M$ .

## Input

There is one line with two integers:  $N$ , which is the number of students in the class, and  $M$ , which is the largest integer each student can choose.

## Constraints

$$3 \leq N \leq 7 \quad \text{Number of students in the class}$$

$$1 \leq M \leq N + 1 \quad \text{Maximum number that can be selected}$$

## Output

The output consists of  $M$  lines. Line  $k$  should have the probability of selecting number  $k$ . The result will be considered correct as long as the absolute error does not exceed  $10^{-3}$ .

## **Problem J**

---

## **Problem J**

---

### **Sample Input 1**

3 3

### **Sample Output 1**

0.46410  
0.26795  
0.26795

### **Sample Input 2**

7 1

### **Sample Output 2**

1.00000

### **Sample Input 3**

5 6

### **Sample Output 3**

0.35785  
0.31502  
0.19107  
0.09512  
0.03515  
0.00580

# Balls and Needles

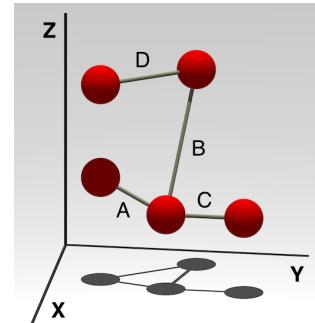
Joana Vasconcelos is a Portuguese artist who uses everyday objects in her creations, like electric irons or plastic cutlery. She is an inspiration to Ana, who wants to make ceiling hanging sculptures with straight knitting needles and balls of wool. For safety reasons, there will be a ball at each end of each needle. Knitting needles vary in colour, length and thickness (to allow intersections of needles).



Sculptures are to be exhibited in room corners, which provide a 3D Cartesian coordinate system, with many lamps on the ceiling. Sculpture designs are made with the coordinates of the centres of the balls of wool in which knitting needles are stuck. That is, each needle  $N$  is represented by a set of two different triples:  $N = \{(x, y, z), (x', y', z')\}$ .

Ana dislikes *closed chains*. A *true closed chain* is a sequence of  $k$  distinct needles,  $N_1, N_2, \dots, N_k$  (for some  $k \geq 3$ ), such that:

- $N_1 = \{(x_1, y_1, z_1), (x_2, y_2, z_2)\}, N_2 = \{(x_2, y_2, z_2), (x_3, y_3, z_3)\}, \dots,$
- $N_k = \{(x_k, y_k, z_k), (x_{k+1}, y_{k+1}, z_{k+1})\}, \text{ and } (x_{k+1}, y_{k+1}, z_{k+1}) = (x_1, y_1, z_1).$



But her dislike of closed chains is so extreme that the shadow of the sculpture on the floor has to be free of “floor closed chains”. Given any needle  $N = \{(x, y, z), (x', y', z')\}$ , let  $N^\downarrow = \{(x, y), (x', y')\}$  denote the shadow of needle  $N$  on the floor. For Ana (who is an artist), a *floor closed chain* is also a sequence of  $k$  distinct needles,  $N_1, N_2, \dots, N_k$  (for some  $k \geq 3$ ), such that:

- $N_i^\downarrow \neq N_j^\downarrow$ , for every  $1 \leq i < j \leq k$  (the  $k$  needle shadows are all distinct);
- $N_1^\downarrow = \{(x_1, y_1), (x_2, y_2)\}, N_2^\downarrow = \{(x_2, y_2), (x_3, y_3)\}, \dots,$
- $N_k^\downarrow = \{(x_k, y_k), (x_{k+1}, y_{k+1})\}, \text{ and } (x_{k+1}, y_{k+1}) = (x_1, y_1).$

Consider the sculpture depicted in the figure, which has the following four knitting needles:

$$\begin{aligned} A &= \{(12, 12, 8), (10, 5, 11)\}, & B &= \{(12, 12, 8), (4, 14, 21)\}, \\ C &= \{(12, 12, 8), (12, 20, 8)\}, & D &= \{(4, 14, 21), (10, 5, 21)\}. \end{aligned}$$

This structure is not free of closed chains because, although there is no true closed chain, the sequence of needles  $A, B, D$  is a floor closed chain.

## Task

Write a program that, given the knitting needles of a sculpture, determines whether there is a true or a floor closed chain in the sculpture.

## Problem K

---

## Problem K

### Input

The first line of the input has one integer,  $K$ , which is the number of knitting needles in the sculpture. Each of the following  $K$  lines contains six integers,  $x_1$ ,  $y_1$ ,  $z_1$ ,  $x_2$ ,  $y_2$ , and  $z_2$ , which indicate that  $\{(x_1, y_1, z_1), (x_2, y_2, z_2)\}$  is the set of triples of a needle. Any two distinct needles are represented by different sets of triples.

### Constraints

$1 \leq K \leq 50\,000$  Number of knitting needles  
 $1 \leq x_i, y_i, z_i < 1\,000$  Coordinates of each triple

### Output

The output has two lines, each one with a string. The string in the first line is: `True closed chains`, if there is some true closed chain in the sculpture; `No true closed chains`, otherwise. The string in the second line is: `Floor closed chains`, if there is some floor closed chain in the sculpture; `No floor closed chains`, otherwise.

### Sample Input 1

```
4
12 12 8 10 5 11
12 12 8 4 14 21
12 12 8 12 20 8
4 14 21 10 5 21
```

### Sample Output 1

```
No true closed chains
Floor closed chains
```

### Sample Input 3

```
3
50 50 50 100 100 100
100 100 100 50 50 90
50 50 90 50 50 50
```

### Sample Output 3

```
True closed chains
No floor closed chains
```

### Sample Input 2

```
4
1 1 1 2 2 2
2 2 2 1 5 5
9 4 4 9 4 2
9 4 4 9 9 4
```

### Sample Output 2

```
No true closed chains
No floor closed chains
```

### Sample Input 4

```
3
1 1 5 1 3 7
1 3 7 4 4 5
4 4 5 1 1 5
```

### Sample Output 4

```
True closed chains
Floor closed chains
```

# Problem Analysis Session

SWERC Judges

SWERC 2016



# C - Candle Box

## Problem

Consider two finite arithmetic progressions:

- $R = 4 + 5 + \dots$  number of Rita's candles
- $T = 3 + 4 + \dots$  number of Theo's candles

Theo placed  $X$  of its candles in Rita's box.

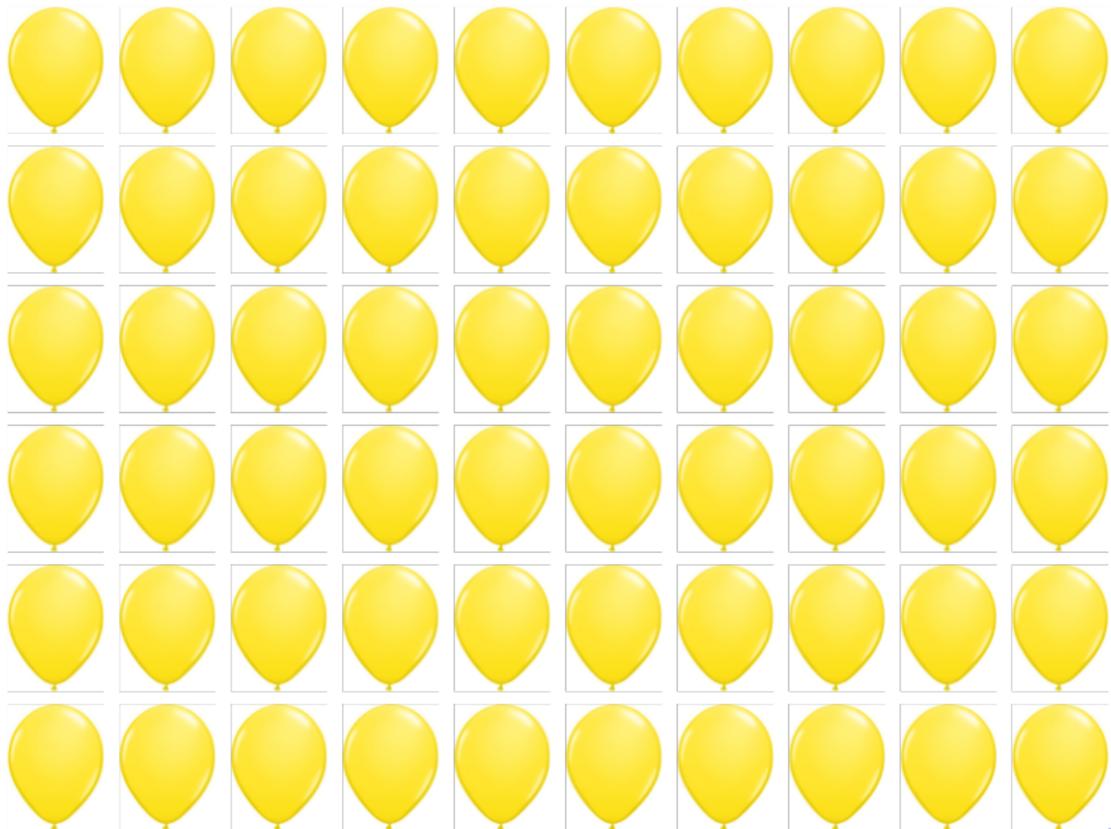
Given  $R + X$ ,  $T - X$ , and  $D$  (difference between the ages of Rita and Theo), find out  $X$ .

## Categories

- Math, arithmetic progression



# C - Candle Box



# C - Candle Box

## Sample Solution

Given  $(R + X)$ ,  $(T - X)$ , and  $D$ :

- $R = \sum_{k=0}^n (4 + k)$  and  $T = \sum_{k=0}^{n-D+1} (3 + k)$
- To compute  $R$ , find out  $n$  such that  $R + T = (R + X) + (T - X)$ 
  - either analytically
  - or iteratively computing the sum of arithmetic progressions  $R$ ,  $T$ , until the condition holds
- The answer is  $(R + X) - R$

# K - Balls and Needles

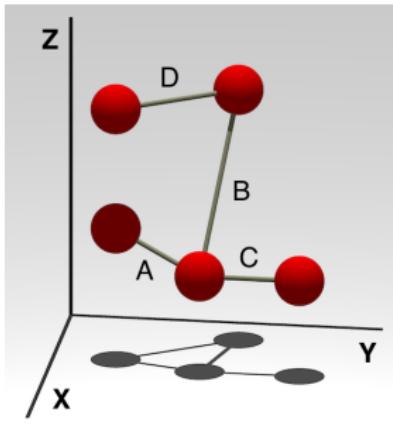
## Problem

Given a set of needles / line segments  $((x_1, y_1, z_1), (x_2, y_2, z_2))$  in 3D:

- Is there a true closed chain, i.e., a **cycle** in the (simple) graph whose edges are the line segments?
- Is there a floor closed chain, i.e., a **cycle** in the (simple) graph whose edges are the line segment projections  $((x_1, y_1), (x_2, y_2))$ ?

## Categories

Graphs – Cycle detection



# K - Balls and Needles

## Sample Solution

- Sculpture with  $K$  needles / line segments
- **Graph for 3D:**
  - $|E| = K$ ,  $|V| \leq 2K$
  - Map each point  $(x, y, z)$  to an integer  $0, 1, 2, \dots$
- **Graph for 2D:**
  - $|E| \leq K$ ,  $|V| \leq 2K$
  - Map each point  $(x, y)$  to an integer  $0, 1, 2, \dots$
  - Leave **self-loops** out
  - Do not add **repeated edges**
- **Cycle detection:** DFS or Union Find
- Time and space complexity:  $\mathcal{O}(K)$

# H - Hyper-Pyramids

## Problem

Calculate the numbers at the base of Pascal's "triangle" generalized to higher-dimensions.

## Categories

Math, dynamic programming



# H - Hyper-Pyramids

General recurrence for dimension  $D$ :

$$\begin{aligned} C(x_1, \dots, x_D) &= C(x_1 - 1, x_2, \dots, x_D) \\ &\quad + C(x_1, x_2 - 1, \dots, x_D) \\ &\quad + \dots \\ &\quad + C(x_1, x_2, \dots, x_D - 1), && \text{if all } x_i > 0 \\ C(x_1, \dots, x_D) &= 1 && \text{otherwise.} \end{aligned}$$

Base of the hyper-pyramid of height  $H$ :

$$\{C(x_1, \dots, x_D) \mid 0 \leq x_i < H \wedge \sum_i x_i = H - 1\}$$

# H - Hyper-Pyramids

Implementation:

- Use 64-bit integer arithmetic
- Recursively generate vectors of size  $D$  that sum  $H - 1$
- Calculate recurrence using **memoization**
- Collect results into a set
- Choice of memo and set data structures is not critical

However, the above is not enough, we need to **exploit symmetry**:

- $C(x_1, \dots, x_D) = C(\pi \circ (x_1, \dots, x_D))$  for any permutation  $\pi$
- Normalize **memo keys**, e.g. ascending order  
(sort vectors before lookup/insert)
- Generate only **ascending vectors**  $x_1 \leq \dots \leq x_D$  that sum  $H - 1$

# H - Hyper-Pyramids

Alternative approach (for the more math-savvy):

- Numbers in Pascal's hyper-pyramids are the **multinomial coefficients**
- Compute directly using the **closed factorials formula**:

$$C(x_1, x_2, \dots, x_D) = \binom{H-1}{x_1, x_2, \dots, x_D} = \frac{(H-1)!}{x_1! x_2! \cdots x_D!}$$

- No recurrence, no memoization
- Caveat: requires larger precision (BigIntegers) or careful implementation to avoid overflow with 64-bits

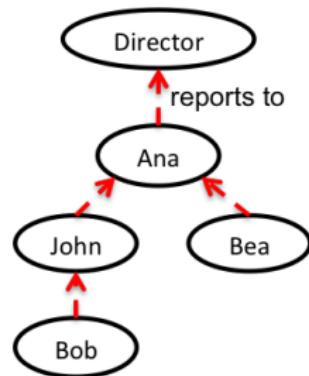
# F - Performance Review

## Problem

Given a rooted tree with  $N$  nodes, where each node has a (key, value) pair, find for each node  $x$  the sum of values in its children whose key is smaller than  $x$ 's key.

## Categories

- Graph Theory - Trees
- Data Structures



# F - Performance Review

## Naive approach

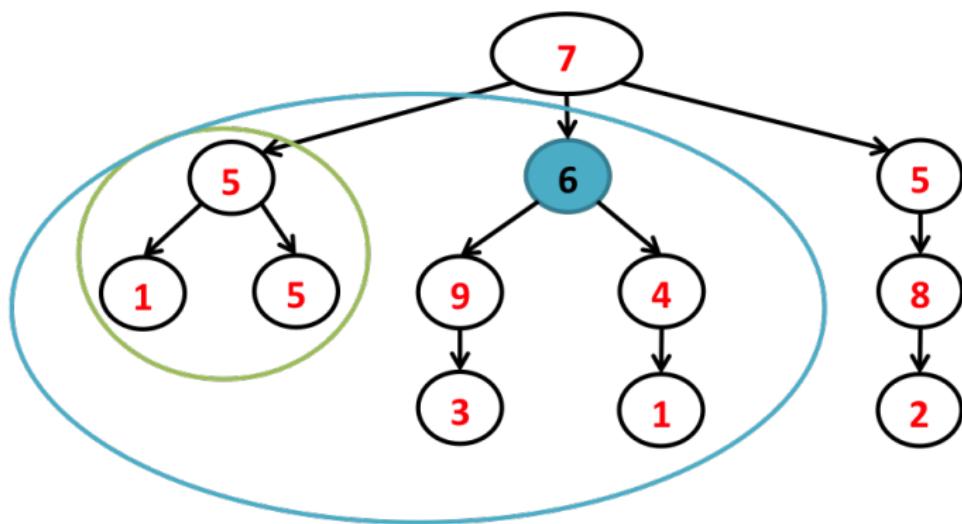
- Process each node's children and sum the values
- Complexity is  $\mathcal{O}(N^2)$  — too slow

## Several valid approaches

- Post-order traversal of the tree + Fenwick Tree
- Pre-order traversal + Segment Tree
- Process nodes from the deepest leaves to the root and merge (key,value) pairs

# F - Performance Review

## Sample Solution



# F - Performance Review

## Sample Solution

- Use a data structure  $\mathcal{T}$  that supports:
  - $\text{Add}(k, v)$ : add the (key, value) pair to  $\mathcal{T}$
  - $\text{Sum}(\text{key})$ : return the sum of values in  $\mathcal{T}$  with keys less than  $\text{key}$
- Post-order traversal of the given tree

$\text{Dfs}(\text{node}, \mathcal{T})$

$\text{prev} = \mathcal{T}.\text{Sum}(\text{node.key})$

for each child of node

$\text{Dfs}(\text{child}, \mathcal{T})$

$\text{node.review\_time} = \mathcal{T}.\text{sum}(\text{node.key}) - \text{prev}$

$\mathcal{T}.\text{Add}(\text{node.key}, \text{node.value})$

- Using an efficient data structure like a Fenwick Tree, this approach takes  $\mathcal{O}(N \log N)$  time and  $\mathcal{O}(N)$  space

## B - Bribing Eve

### Problem

Find the best and worst positions that a product  $m$  from a list can have when sorted by a score:

$$s = w_1 x_1 + w_2 x_2$$

with  $w_1, w_2 \in \mathbb{R}_0^+$   
and  $w_1 + w_2 > 0$ .

### Categories

- Sweep
- Geometry



# B - Bribing Eve

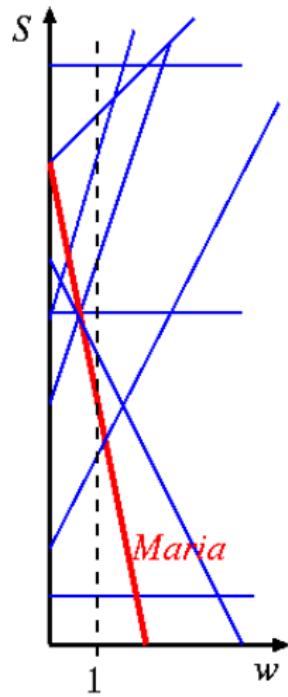
## Naive approach

- For every product  $i$  find the weights  $s_m = s_i$ .
- Use every valid weight found to sort the products and find the best and worst ranking position.
- Complexity is  $\mathcal{O}(N^2 \log N)$  — too slow

## Sweep

- Find a way to sweep through all valid weights while keeping a set of best/worst products.
- Complexity is  $\mathcal{O}(N \log N)$

## B - Bribing Eve



$$S = w_1x_1 + w_2x_2$$
$$\frac{1}{w_1 + w_2}S = \frac{w_1}{w_1 + w_2}x_1 + \frac{w_2}{w_1 + w_2}x_2$$

Therefore, we can reduce the analysis to:

$$S = w_1x_1 + w_2x_2, \text{ with } w_1 + w_2 = 1$$

That is,  $S = x_2 + (x_1 - x_2)w$ , with  $0 \leq w \leq 1$ .

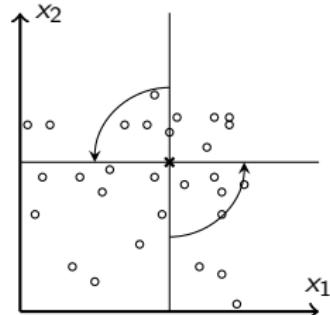
### Solution approach:

Find the intersection points in  $O(n)$ . Sort them.

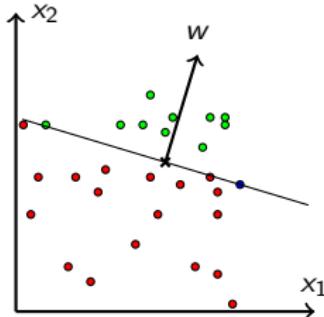
Linear Sweep to update counters.

## B - Bribing Eve

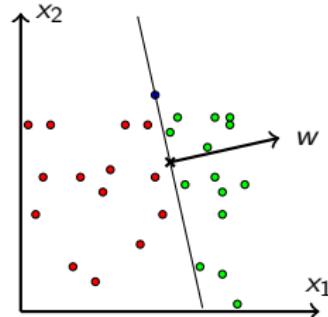
### Radial sweep



### Best ranking



### Worst ranking



### Caveats

- Watch for boundary conditions - **degenerate cases**.
- Be careful of products with same test scores.
- Test with epsilon if solving with floats.

# D - Dinner Bet

## Problem

Given the number  $N$  of balls, the number  $D$  of balls drawn each round and the  $C$  values chosen by each of the **two players**, find the expected number of rounds their game will last.

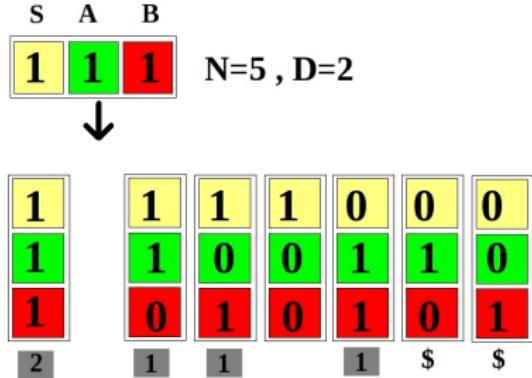
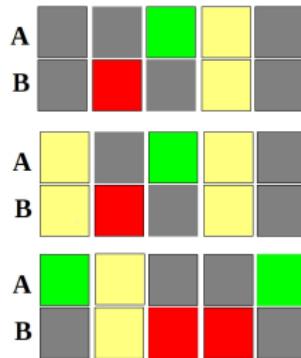
## Categories

- Dynamic Programming
- Probabilities, Combinatorics
- (Absorbing Markov Chains)

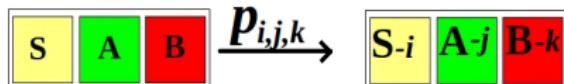


# D - Dinner Bet

State abstraction Shared, Player A, Player B



State Transition (with probability  $p_{i,j,k}$ )



$$D - (N - S - A - B) \leq i + j + k \leq D$$

$$p_{i,j,k} = \frac{\binom{S}{i} \binom{A}{j} \binom{B}{k} \binom{N-A-B-S}{D-i-j-k}}{\binom{N}{D}}$$

# D - Dinner Bet

## Sample Solution

- The game ends when  $S = 0 \wedge (A = 0 \vee B = 0)$
- The expected number of rounds satisfies the recurrence

$$\mathcal{E}_{R,S,A,B} = \sum_{i+j+k \leq D} p_{i,j,k} \mathcal{E}_{R-1,S-i,A-j,B-k}$$

$$\mathcal{E}_{R,0,A,0} = \mathcal{E}_{R,0,0,B} = R$$

- Compute  $\mathcal{E}$  using Dynamic Programming:  $\mathcal{O}(C^3 D^3)$  time

## Another formula...

$$\begin{aligned}\mathbb{E}_{S,A,B} &= \frac{1}{1 - p_{0,0,0}} \left( 1 + \sum_{i+j+k \neq 0} p_{i,j,k} \mathbb{E}_{S-i,A-j,B-k} \right) \\ \mathbb{E}_{0,A,0} &= \mathbb{E}_{0,0,B} = 0\end{aligned}$$

## Sample Solution

- The game lasts the most when  $N = 50$  and  $D = 1$  and the players have the same key
- With 1 number left, the probability of continuing playing after  $\mathcal{R}$  rounds is  $(\frac{49}{50})^{\mathcal{R}} = 0.98^{\mathcal{R}}$
- The contribution to the result is  $\mathcal{R} \times 0.98^{\mathcal{R}}$ , so an estimate for the minimum number of iterations needed is finding  $\mathcal{R} \times 0.98^{\mathcal{R}} < EPS$ .
- Considering 1000 rounds was more than enough for a  $10^{-3}$  epsilon

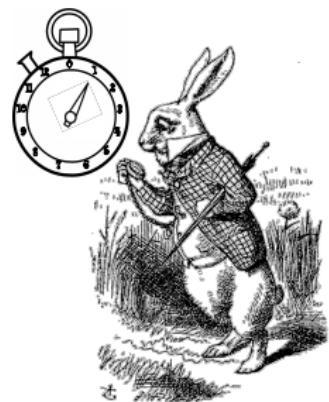
# I - The White Rabbit Pocket Watch

## Problem

Given a set of **tracks** over an undirected weighted graph with **unknown edge weights**, along with the sum **modulo 13** of its edge weights, determine the weight of the **shortest path** between two given nodes.

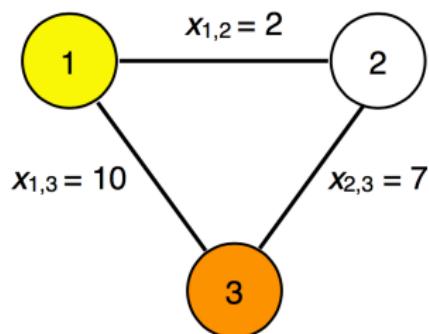
## Categories

- Graph theory / shortest path
- System of linear equations in a prime field



# I - The White Rabbit Pocket Watch

Tracks	Duration in $\mathbb{Z}_{13}$
1 → 2 → 3 → 2	3
1 → 2 → 1	4
1 → 2 → 1 → 3	1



$$\begin{cases} x_{1,2} + 2x_{2,3} \equiv 3 \pmod{13} \\ 2x_{1,2} \equiv 4 \pmod{13} \\ 2x_{1,2} + x_{1,3} \equiv 1 \pmod{13} \end{cases}$$

The system has a unique solution.

# I - The White Rabbit Pocket Watch

## Solution Overview:

- Build a system of **linear equations** from the track information
- Solve the system by **Gaussian elimination** in  $\mathbb{Z}_{13}$  to find the edge weights (the problem statement guarantees a unique solution)
- Compute the **shortest path** between the target nodes (e.g. Dijkstra)
- Time complexity is  $O(|E|^3)$

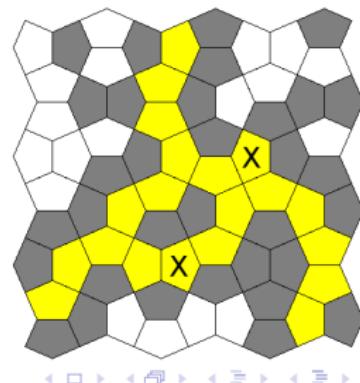
## Problem

Given a pentagonal tiling:

- Is there a **corridor**, i.e., a maximal set of clear adjacent pentagons that connect the 4 borders?
- (If yes) Is the corridor **minimal**, i.e., does it have no superfluous pentagon?
- (If yes) How many pentagons has the minimal corridor?

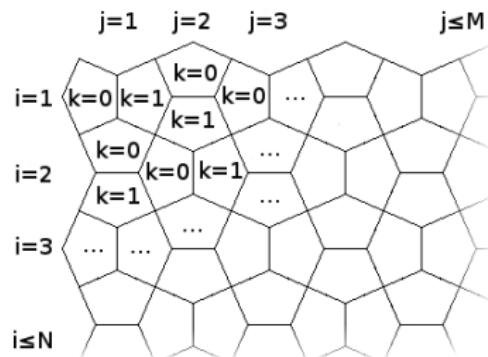
## Categories

Graphs – Connectivity



## Sample Solution

- Model empty tiles and their adjacencies as a **graph**
  - Each tile identified by  $(i, j, k)$
  - Adjacency and border checks by case analysis, e.g. **left border**:  
$$(i + j \text{ is even} \wedge j = 1 \wedge k = 0) \vee (i + j \text{ is odd} \wedge j = 1)$$
  - $|V| \leq 2 \times N \times M \approx 125\,000$ ,  
 $|E| \leq 5 \times 125\,000$
- Search for a corridor (**connected component** that touches all borders): DFS, BFS or Union Find
- Check the corridor for minimality



How to **check minimality**?

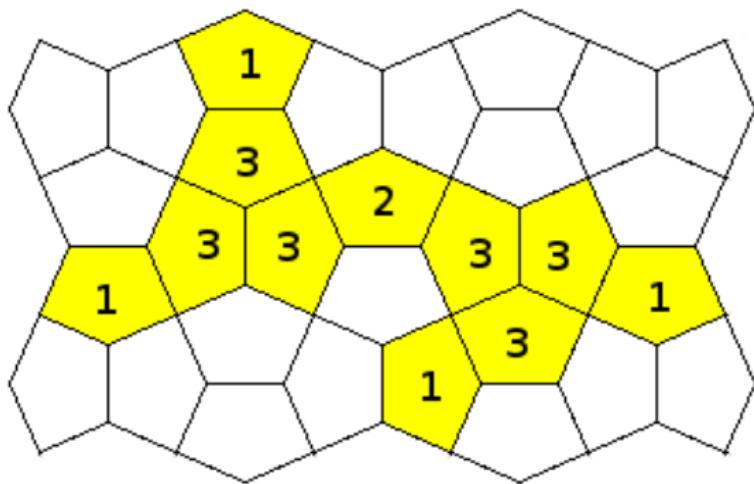
## Naive check

- Delete each node and check if the remaining nodes are a corridor
- Time complexity:  $\mathcal{O}((N \times M)^2)$  — **too high**

## Better approach

- Check only **critical nodes**, i.e.
  - with degree 1
  - with degree  $\geq 3$  and their adjacents
- Bounded number:
  - at most **4** nodes with degree 1
  - at most **6** nodes with degree  $\geq 3$
- Time and space complexity:  $\mathcal{O}(N \times M)$

# G - Cairo Corridor



Sample corridor with 6 nodes of degree 3

# E - Passwords

## Problem

Given a blacklist with  $N$  words and two integers  $A$  and  $B$ , the task was to compute the number of different valid passwords obeying:

- length is between  $A$  and  $B$  (inclusive)
- at least one lowercase letter, one uppercase letter, and one digit
- no blacklisted substring.

## Categories

- Dynamic Programming
- Strings, Aho-Corasick

Password:

\*\*\*\*\*|

Password strength: Weak

## Overview

- Generating all passwords (obviously) would exceed time limit
- We can count... using **dynamic programming** (DP)
- Our DP state needs to have into account all restrictions
  - Size of the string (**easy**) - *integer*
  - Having > 1 upper/lower/digit (**easy**) - *3 booleans*
  - Avoiding blacklisted substrings (**not so easy...**)

## Main Difficulty

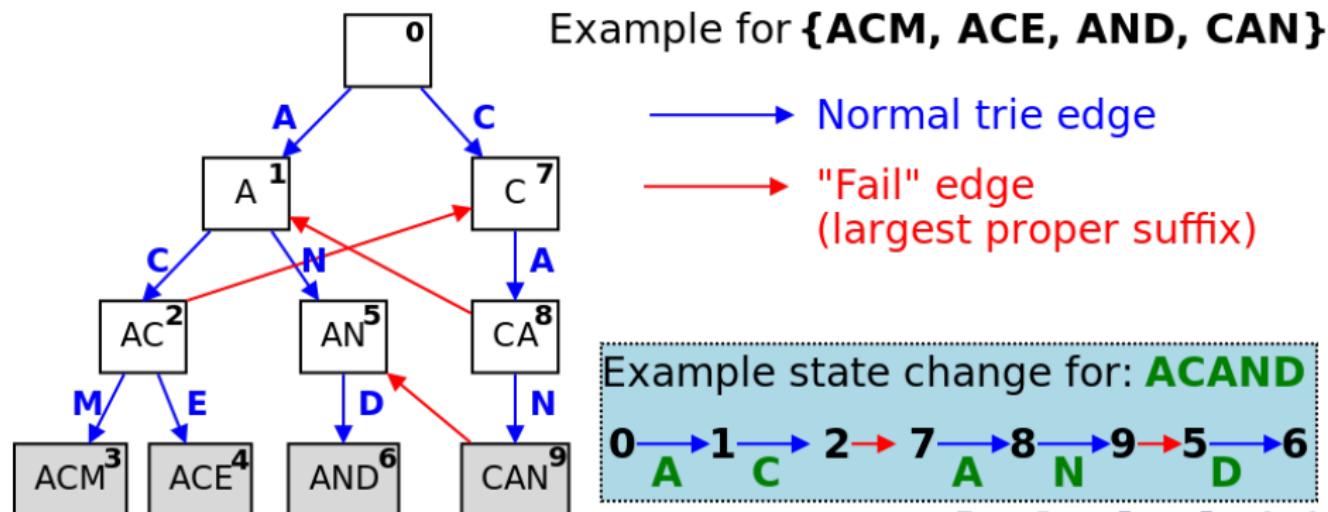
- How can we incorporate in the DP state how to avoid all blacklisted words at the same time?

# E - Passwords

Idea - Use an automaton

Capture at which “position” we are at all blacklisted words

- We could use something like Aho-Corasick
- A “position” is a node in the automaton



# E - Passwords

## Sample Solution

- Use a five dimensional DP state:  $(S, L, U, D, N)$ 
  - $S$  - Size of current password
  - $L, U, D$  - Yes/No we have at least one upper/lower/digit
  - $N$  - Position/Node in the automaton
- Let  $\delta(N, c)$  be the transition from node  $N$  using character  $c$
- $count(S, L, U, D, N) = 0$  if  $N$  is a word; otherwise:
$$\sum_{c \in \{a..z\}} count(S + 1, true, U, D, \delta(N, c)) +$$
$$\sum_{c \in \{A..Z\}} count(S + 1, L, true, D, \delta(N, lower(c))) +$$
$$\sum_{c \in \{0..9\}} count(S + 1, L, U, true, \delta(N, leet(c)))$$
- Solution in `count(0,false,false,false," ")`
- Construction of automaton could be “inefficient” /not optimal

## Caveat

Care should be taken with words that contain other smaller words

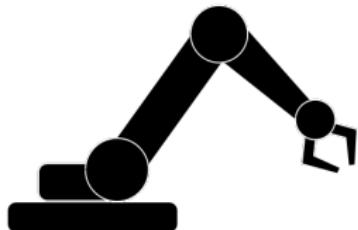
# A - Within Arm's Reach

## Problem

Given a robotic arm description and target coordinates, calculate a configuration that places the arm's tip as close as possible to the target.

## Categories

- Geometry



# A - Within Arm's Reach

## Difficulties

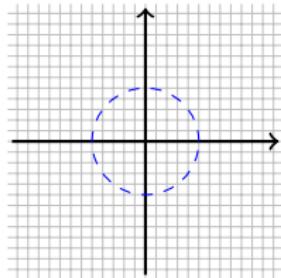
- Geometric
- Multiple output - need to construct any valid solution. Too much freedom. How to handle it?

Possible approach: understanding what are the reachable areas and how to step between them allows to construct a solution from target to origin.

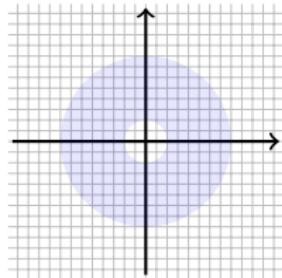
## Useful geometric knowledge

- A polygon has every side smaller or equal to the sum of all other sides.
- Re-ordering segments does not affects reachable positions.
- Problem can be rotated to place target on an axis.

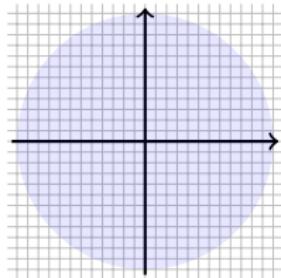
# A - Within Arm's Reach



$$L = \{5\}$$
$$r \in [5, 5]$$



$$L = \{5, 3\}$$
$$r \in [2, 8]$$



$$L = \{5, 3, 4\}$$
$$r \in [0, 12]$$

## Reachable area

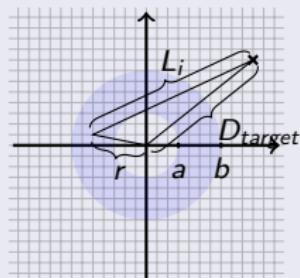
- The set of reachable points given a set  $L$  of segments is a single disc.
- One can also see  $r$  as a segment that can be added to  $L$  such that it is possible to construct a polygon.

# A - Within Arm's Reach

## Step function

To find a point in a disc  $r \in [a, b]$  that is at distance  $L_i$  of a target:

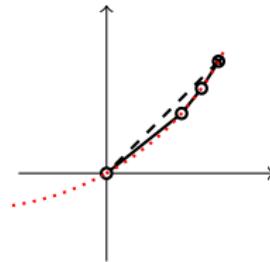
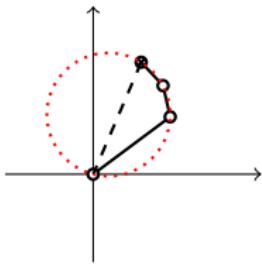
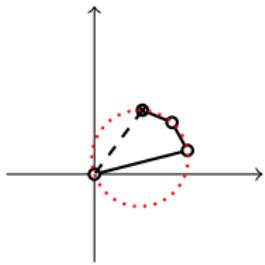
- Pick any  $r$  such that  $\{r, L_i, D_{target}\}$  are the sides of a valid triangle
- From the triangle sides, calculate the angles and then the point.



## Solution overview

- Calculate reachable disc when using every prefix of  $n$  segments.
- Construct solution from tip to origin.
- Print from origin to tip.

# A - Within Arm's Reach



## Other possible solution

- Interpret the problem as building a polygon with  $N + 2$  sides.
- The extra sides are the distance from the origin to target and the min side to make it possible to construct a polygon.
- Limit the solution space by trying to inscribe the polygon sides as cords on a circumference.
- Binary search on the circumference radius.

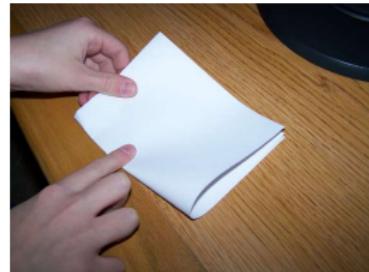
# J - Risky Lottery

## Problem

Approximate numerically the symmetric equilibrium strategy of the **lowest unique bet** game with  $N$  players choosing numbers from 1 to  $M$ .

## Categories

Game Theory, Numerical Methods



## J - Risky Lottery

### Key Insight

In an optimal strategy, the **probability of winning is the same** regardless of the number we select.

(Otherwise, we should be picking numbers with higher probability of winning more frequently than we currently are.)

Example (3 players, 3 numbers):

$$\begin{cases} PWin_1(P) = \lambda \\ PWin_2(P) = \lambda \\ PWin_3(P) = \lambda \\ P_1 + P_2 + P_3 = 1 \end{cases} \Leftrightarrow \begin{cases} (P_2 + P_3)^2 = \lambda \\ P_1^2 + P_3^2 = \lambda \\ P_1^2 + P_2^2 = \lambda \\ P_1 + P_2 + P_3 = 1 \end{cases}$$

(Note:  $\lambda$  is not trivial because of ties.)

## Solution sketch

Any iterative method that finds a solution should be accepted.

Even something *basic*:

- we only need  $10^{-3}$  precision, so let's pick  $\text{EPS} = 10^{-5}$ ;
- calculate  $\text{avg} = \sum_i P\text{Win}_i(P)/M$ ;
- if  $(P\text{Win}_i(P) > \text{avg})$  then  $P_i += \text{EPS}$  else  $P_i -= \text{EPS}$ ;
- iterate until converged.

# Questions?