

The German Collegiate Programming Contest 2017

GCPC 2017



Problems

- A Drawing Borders
- B Buildings
- C Joyride
- D Pants On Fire
- E Perpetuum Mobile
- F Plug It In!
- G Water Testing
- H Ratatöskr
- I Überwatch
- J Word Clock
- K You Are Fired!

Do not open before the contest has started.

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Problem A

Drawing Borders

Somewhere in the great North American plains live the tribes of chiefs *Blue Eagle*, *Red Beaver*, and *Green Serpent*. Their population is scattered over numerous villages all over the land and conflict arises whenever members of different tribes meet while traveling across the plains.

To put an end to the constant animosities the chiefs have decided that the land should be divided between the tribes so that they can avoid each other when moving between villages belonging to the same tribe. More precisely, they want to construct two border fences – thus dividing the land into three regions – such that two villages lie in the same region precisely when they belong to the same tribe.

The villages are represented by points in the Euclidean plane that are colored blue, red or green, depending on the tribe, and the fences should be drawn in the form of two polygons. The polygons may not touch or intersect themselves or each other and none of the points may lie on their boundary. (Make sure to read the constraints in the Output section!)

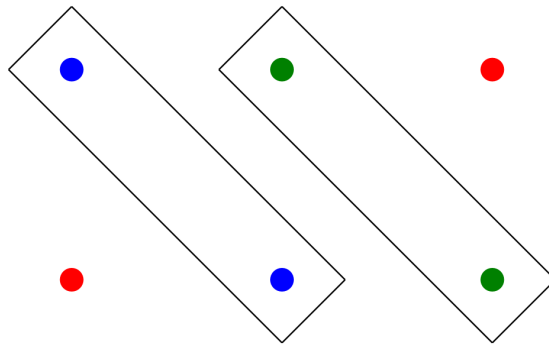


Figure A.1: Illustration of the sample.

Input

The input consists of:

- one line with an integer n ($3 \leq n \leq 100$), the number of villages.
- n lines, each with three integers x, y, c ($-1\,000 \leq x, y \leq 1\,000$, $1 \leq c \leq 3$), representing a village at coordinates (x, y) of color c ($1 = \text{blue}$, $2 = \text{red}$, $3 = \text{green}$). All positions are unique. There is at least one village of each color.

Output

If there is no solution, print `impossible`. Otherwise, print the two polygons in the following format:

- one line with an integer m ($3 \leq m \leq 1\,000$), the number of vertices of the polygon.
- m lines, each with two real numbers x, y ($-3\,000 \leq x, y \leq 3\,000$), the vertices of the polygon in either clockwise or counter-clockwise order. The numbers may be given with up to five decimal places (additional places will be rounded off).

Sample Input 1

```
6
0 0 2
0 1 1
1 0 1
1 1 3
2 0 3
2 1 2
```

Sample Output 1

```
4
-0.3 1.0
1.0 -0.3
1.3 0.0
0.0 1.3
4
0.7 1.0
2.0 -0.3
2.3 0.0
1.0 1.3
```

Problem B

Buildings

As a traveling salesman in a globalized world, Alan has always moved a lot. He almost never lived in the same town for more than a few years until his heart yearned for a different place. However, this newest town is his favorite yet - it is just so colorful. Alan has recently moved to Colorville, a smallish city in between some really nice mountains. Here, Alan has finally decided to settle down and build himself a home - a nice big house to call his own.

In Colorville, many people have their own houses - each painted with a distinct pattern of colors such that no two houses look the same. Every wall consists of exactly $n \times n$ squares, each painted with a given color (windows and doors are also seen as unique “colors”). The walls of the houses are arranged in the shape of a regular m -gon, with a roof on top. According to the deep traditions of Colorville, the roofs should show the unity among Colorvillians, so all roofs in Colorville have the same color.

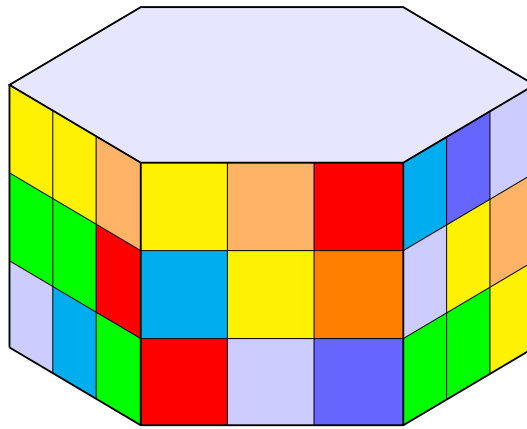


Figure B.1: Example house design for $n = 3, m = 6$.

Of course, Alan wants to follow this custom to make sure he fits right in. However, there are so many possible designs to choose from. Can you tell Alan how many possible house designs there are? (Two house designs are obviously the same if they can be translated into each other just by rotation.)

Input

The input consists of:

- one line with three integers n , m , and c , where
 - n ($1 \leq n \leq 500$) is the side length of every wall, i.e. every wall consists of $n \times n$ squares;
 - m ($3 \leq m \leq 500$) is the number of corners of the regular polygon;
 - c ($1 \leq c \leq 500$) the number of different colors.

Output

Output s where s is the number of possible different house designs. Since s can be very large, output $s \bmod (10^9 + 7)$.

Sample Input 1

1 3 1

Sample Output 1

1

Sample Input 2

2 5 2

Sample Output 2

209728

Problem C

Joyride

It is another wonderful sunny day in July – and you decided to spend your day together with your little daughter Joy. Since she really likes the fairy-park in the next town, you decided to go there for the day. Your wife (unfortunately she has to work) agreed to drive you to the park and pick you up again. Alas, she is very picky about being on time, so she told you exactly when she will be at the park's front entrance to pick you up and you have to be there at exactly that time. You clearly also don't want to wait outside, since this would make your little daughter sad – she could have spent more time in the park!

Now you have to plan your stay at the park. You know when you will arrive and when you will have to depart. The park consists of several rides, interconnected by small pavements. The entry into the park is free, but you have to pay for every use of every ride in the park. Since it is Joy's favorite park, you already know how long using each ride takes and how much each ride costs. When walking through the park, you obviously must not skip a ride when walking along it (even if Joy has already used it), or else Joy would be very sad. Since Joy likes the park very much, she will gladly use rides more than once. Walking between two rides takes a given amount of time.

Since you are a provident parent you want to spend as little as possible when being at the park. Can you compute how much is absolutely necessary?

Input

The input consists of:

- one line with an integer x ($1 \leq x \leq 1\,000$) denoting the time between your arrival and the time you will be picked up (in minutes);
- one line with three integers n , m , and t , where
 - n ($1 \leq n \leq 1\,000$) is the number of rides in the park;
 - m ($1 \leq m \leq 1\,000$) is the number of pavements;
 - t ($1 \leq t \leq 1\,000$) is the number of minutes needed to pass over a pavement from one ride to another.
- m lines each containing two integers a and b ($1 \leq a, b \leq n$) stating that there is a pavement between the rides a and b .
- n lines each containing two integers t and p ($1 \leq t, p \leq 10^6$) stating that the corresponding ride takes t minutes and costs p Euro.

You always start at ride 1 and have to return to ride 1 at the end of your stay, since the entry is located there. This means that you have to use the ride 1 at least twice (once upon entry and once upon exit). You can take a ride more than once, if you have arrived at it.

Output

Output one line containing either a single integer, the minimum amount necessary to stay x minutes in the park, or `It is a trap.` (including the period) if it is not possible to stay exactly x minutes.

Sample Input 1

```
4
4 4 1
1 2
2 3
3 4
4 1
1 2
2 1
5 4
3 3
```

Sample Output 1

```
8
```

Sample Input 2

```
6
4 4 1
1 2
2 3
3 4
4 1
1 2
2 1
5 4
3 3
```

Sample Output 2

```
5
```


Problem D

Pants On Fire

Donald and Mike are the leaders of the free world and haven't yet (after half a year) managed to start a nuclear war. It is so great! It is so tremendous!

Despite the great and best success of Donald's Administration, there are still a few things he likes to complain about.

The Mexican government is much smarter, much sharper, and much more cunning.
And they send all these bad hombres over because they don't want to pay for them.
They don't want to take care of them.

Donald J. Trump, First Republican Presidential Debate, August 6, 2015

He also frequently compares `Mexicans` to other bad people (like `Germans`, since they are exporting so many expensive cars to the US). Due to the tremendous amount of statements he has made (mostly containing less than 140 characters ...) the "Fake-News" New York Telegraph (NYT) has to put in a lot of effort to clarify and comment on all the statements of Donald. To check a statement, they have a list of facts they deem to be true and classify Donald's statements into three groups: real facts (which are logical conclusions from their list of true facts), exaggerations (which do not follow, but are still consistent with the papers list of facts), and alternative facts (which contradict the knowledge of the newspaper).

They have asked you to write a program helping them to classify all of Donald's statements – after all it is hard for a journalist to go through them all and check them all, right?

Input

The input consists of:

- one line containing two integers n and m , where
 - n ($1 \leq n \leq 200$) is the number of facts deemed true by the NYT;
 - m ($1 \leq m \leq 200$) is the number of statements uttered by the Donald.
- n lines each containing a statement deemed true by the NYT.
- m lines each containing a statement uttered by the Donald.

All statements are of the form `a are worse than b`, for some strings `a` and `b`, stating that `a` is (strictly) worse than `b`. The strings `a` and `b` are never identical. Both `a` and `b` are of length between 1 and 30 characters and contain only lowercase and uppercase letters of the English alphabet.

Note that Donald's statements may contain countries that the NYT does not know about. You may assume that worseness is transitive and that the first n lines do not contain any contradictory statement. Interestingly, Donald's press secretary (Grumpy Sean) has managed to convince him not to make up countries when tweeting, thus the input mentions at most 193 different countries.

Output

For every of the m statements of Donald output one line containing

- `Fact` if the statement is true given the n facts of the NYT
- `Alternative Fact` if the inversion of the statement is true given the n facts of the NYT
- `Pants on Fire` if the statement does not follow, but neither does its inverse.

Sample Input 1

```
4 5
Mexicans are worse than Americans
Russians are worse than Mexicans
NorthKoreans are worse than Germans
Canadians are worse than Americans
Russians are worse than Americans
Germans are worse than NorthKoreans
NorthKoreans are worse than Mexicans
NorthKoreans are worse than French
Mexicans are worse than Canadians
```

Sample Output 1

```
Fact
Alternative Fact
Pants on Fire
Pants on Fire
Pants on Fire
```

Problem E

Perpetuum Mobile

The year is 1902. Albert Einstein is working in the patent office in Bern. Many patent proposals contain egregious errors; some even violate the law of conservation of energy. To make matters worse, the majority of proposals make use of non-standard physical units that are not part of the metric system (or not even documented). All proposals are of the following form:

- Every patent proposal contains n energy converters.
- Every converter has an unknown input energy unit associated with it.
- Some energy converters can be connected: If converter a can be connected to converter b such that one energy unit associated with a is turned into c input units for b , then this is indicated by an arc $a \xrightarrow{c} b$ in the proposal. The output of a can be used as input for b if and only if such an arc from a to b exists.

Einstein would like to dismiss all those proposals out of hand where the energy converters can be chained up in a cycle such that more energy is fed back to a converter than is given to it as input, thereby violating the law of conservation of energy.

Einstein's assistants know that he is born for higher things than weeding out faulty patent proposals. Hence, they take care of the most difficult cases, while the proposals given to Einstein are of a rather restricted form: Every *admissible* patent proposal given to Einstein does not allow for a cycle where the total product of arc weights exceeds 0.9. By contrast, every *inadmissible* patent proposal given to Einstein contains a cycle where the the number of arcs constituting the cycle does not exceed the number of converters defined in the proposal, and the total product of arc weights is greater or equal to 1.1.

Could you help Einstein identify the inadmissible proposals?

Input

The input consists of:

- one line with two integers n and m , where
 - n ($2 \leq n \leq 800$) is the number of energy converters;
 - m ($0 \leq m \leq 4000$) is the number of arcs.
- m lines each containing three numbers a_i , b_i , and c_i , where
 - a_i and b_i ($1 \leq a_i, b_i \leq n$) are integers identifying energy converters;
 - c_i ($0 < c_i \leq 5.0$) is a decimal number

indicating that the converter a_i can be connected to the converter b_i such that one input unit associated with a_i is converted to c_i units associated with b_i . The number c_i may have up to 4 decimal places.

Output

Output a single line containing `inadmissible` if the proposal given to Einstein is inadmissible, `admissible` otherwise.

Sample Input 1

```
2 2
1 2 0.5
2 1 2.3
```

Sample Output 1

```
inadmissible
```

Sample Input 2

```
2 2
1 2 0.5
2 1 0.7
```

Sample Output 2

```
admissible
```

Problem F

Plug It In!

Adam just moved into his new apartment and simply placed everything into it at random. This means in particular that he did not put any effort into placing his electronics in a way that each one can have its own electric socket.

Since the cables of his devices have limited reach, not every device can be plugged into every socket without moving it first. As he wants to use as many electronic devices as possible right away without moving stuff around, he now tries to figure out which device to plug into which socket. Luckily the previous owner left behind a plugbar which turns one electric socket into 3.

Can you help Adam figure out how many devices he can power in total?

Input

The input consists of:

- one line containing three integers m , n and k , where
 - m ($1 \leq m \leq 1\,500$) is the number of sockets;
 - n ($1 \leq n \leq 1\,500$) is the number of electronic devices;
 - k ($0 \leq k \leq 75\,000$) is the number of possible connections from devices to sockets.
- k lines each containing two integers x_i and y_i indicating that socket x_i can be used to power device y_i .

Sockets as well as electronic devices are numbered starting from 1.

The plugbar has no cable, i.e. if it is plugged into a socket it simply triples it.

Output

Output one line containing the total number of electrical devices Adam can power.

Sample Input 1

```
3 6 8
1 1
1 2
1 3
2 3
2 4
3 4
3 5
3 6
```

Sample Output 1

```
5
```

Sample Input 2

```
4 5 11
1 1
1 2
1 3
2 1
2 2
2 3
3 1
3 2
3 3
4 4
4 5
```

Sample Output 2

```
5
```

Sample Input 3

```
3 5 7
1 1
1 2
2 2
2 3
2 4
3 4
3 5
```

Sample Output 3

```
5
```

Problem G

Water Testing

You just bought a large piece of agricultural land, but you noticed that – according to regulations – you have to test the ground water at specific points on your property once a year. Luckily the description of these points is rather simple. The whole country has been mapped using a Cartesian Coordinate System (where $(0, 0)$ is the location of the Greenwich Observatory). The corners of all land properties are located at integer coordinates according to this coordinate system. Test points for ground water have to be erected on every point inside a property whose coordinates are integers.

Input

The input consists of:

- one line with a single integer n ($3 \leq n \leq 100\,000$), the number of corner points of your property;
- n lines each containing two integers x and y ($-10^6 \leq x, y \leq 10^6$), the coordinates of each corner.

The corners are ordered as they appear on the border of your property and the polygon described by the points does not intersect itself.

Output

The number of points with integer coordinates that are strictly inside your property.

Sample Input 1

```
4
0 0
0 10
10 10
10 0
```

Sample Output 1

```
81
```

This page is intentionally left (almost) blank.

Problem H

Ratatöskr

Ratatöskr is a squirrel that lives in a giant (but finite) mythical tree called Yggdrasil. He likes to gossip, which sets the other inhabitants of the tree against each other. Ratatöskr is thus hunted by the two ravens of Odin, which are called Hugin and Munin, to bring him to justice.

The tree is made up of nodes connected by branches. Initially, the ravens and the squirrel sit on three different nodes. Now the following happens repeatedly:

- On Odin's signal, one of the ravens launches into the air and flies to another node of the tree (or possibly back to its previous position), while the other stays where it is.
- During this maneuver, Ratatöskr can travel along the branches to reach another node, but may not pass through a node where a raven sits. He is much quicker than the ravens and will reach his destination before the flying raven lands.

Ratatöskr gets captured if one of the ravens flies to his position and there is no other node he can escape to.

Help Odin determine an optimal strategy for capture, i.e. the minimum number of signals he has to give until Ratatöskr is guaranteed to be captured by a raven.

Input

The input consists of:

- one line with a single integer n ($3 \leq n \leq 80$), the number of nodes in the tree. The nodes are labeled $1, \dots, n$.
- one line with a single integer r ($1 \leq r \leq n$), the initial position of the squirrel Ratatöskr.
- one line with a single integer h ($1 \leq h \leq n$), the initial position of the raven Hugin.
- one line with a single integer m ($1 \leq m \leq n$), the initial position of the raven Munin.
- $n - 1$ lines each containing two integers i and j ($1 \leq i < j \leq n$), indicating a branch between nodes i and j .

The positions r , h and m are distinct. There is at most one branch between any two nodes and every node is reachable from every other node by a sequence of branches.

Output

One line containing an integer s , the number of signals that the ravens need to capture Ratatöskr in an optimal strategy. If Ratatöskr can escape them indefinitely, output `impossible`.

Sample Input 1

```
4
1
2
4
1 4
2 4
3 4
```

Sample Output 1

```
1
```

Sample Input 2

4
1
2
3
1 4
2 4
3 4

Sample Output 2

2

Problem I

Überwatch

The lectures are over, the assignments complete and even those pesky teaching assistants have nothing left to criticize about your coding project. Time to play some video games! As always, your procrastinating self has perfect timing: Cold Weather Entertainment just released *Überwatch*, a competitive first person video game!

Sadly, you aren't very good at these kind of games. However, *Überwatch* offers more than just skill based gameplay. In *Überwatch* you can defeat all opponents in view with a single button press using your ultimate attack. The drawback of this attack is that it has to charge over time before it is ready to use. When it is fully charged you can use it at any time of your choosing. After its use it immediately begins to charge again.

With this knowledge you quickly decide on a strategy:

- Hide from your opponents and wait for your ultimate attack to charge.
- Wait for the right moment.
- Defeat all opponents in view with your ultimate attack.
- Repeat.

After the game your teammates congratulate you on your substantial contribution. But you wonder: How many opponents could you have defeated with optimal timing?

The game is observed over n time slices. The ultimate attack is initially not charged and requires m time slices to charge. This first possible use of the ultimate attack is therefore in the $(m+1)$ -th time slice. If the ultimate attack is used in the i -th time slice, it immediately begins charging again and is ready to be fired in the $(i+m)$ -th time slice.

Input

The input consists of:

- one line with two integers n and m , where
 - n ($1 \leq n \leq 300\,000$) is the game duration;
 - m ($1 \leq m \leq 10$) is the time needed to charge the ultimate attack in time slices.
- one line with n integers x_i ($0 \leq x_i \leq 32$) describing the number of opponents in view during a time slice in order.

Output

Output the maximum number of opponents you can defeat.

Sample Input 1

```
4 2
1 1 1 1
```

Sample Output 1

```
1
```

Sample Input 2

```
9 3
1 1 2 2 3 2 3 2 1
```

Sample Output 2

```
5
```

Problem J

Word Clock

You work at a company producing *word clocks*. These are clocks which, instead of an ordinary clock face, have a grid of letters that is used to display time. Inside the clock are LEDs, one behind each letter, which are lit up in such a way that the illuminated letters spell out the current time as a sentence. See the example below:

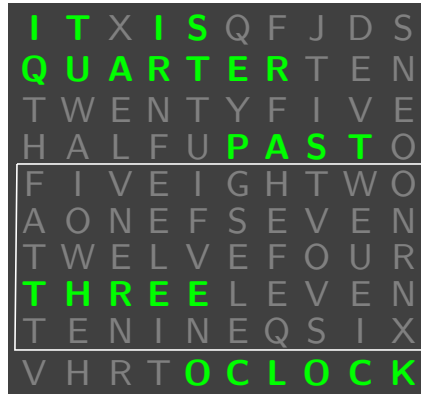


Figure J.1: Example word clock displaying the time 3:15. Note that for efficiency reasons, words are allowed to overlap, e.g. FIVE and EIGHT.

Recently your company decided to go international, and you have been tasked with designing the clock faces for various languages. For this purpose, the team of translators has already compiled a list of all the words needed to tell the time and arranged them into groups according to their position in the sentence (e.g. in the above example the numbers from ONE to TWELVE form such a group, as do the words PAST and TO). This means that you won't have to worry about grammar, as you will only consider a single group at a time.

Given such a group of words and the size of a subgrid, find a way to place all words in the grid or determine that this is impossible. Words have to be written from left to right and are not allowed to wrap from one line to another.

Input

The input consists of:

- one line with three integers h , w , n , where
 - h ($1 \leq h \leq 18$) is the height of the grid;
 - w ($1 \leq w \leq 18$) is the width of the grid;
 - n ($1 \leq n \leq 18$) is the number of words.
- one line with n words, the words to fit inside the grid. Each word consists of at least 1 and at most 18 uppercase letters of the English alphabet. The words are distinct.

Output

If there is no solution, print `impossible`. Otherwise print h lines, each with w uppercase letters, the grid of the word clock. If there is more than one solution, any will be accepted.

Sample Input 1

5 10 12
ONE TWO THREE FOUR FIVE SIX SEVEN EIGHT NINE TEN ELEVEN TWELVE

Sample Output 1

FIVEIGHTWO
AONEFSEVEN
TWELVEFOUR
THREELEVEN
TENINEQSIX

Sample Input 2

5 10 12
EIN ZWEI DREI VIER FUENF SECHS SIEBEN ACHT NEUN ZEHN ELF ZWOELF

Sample Output 2

ZWOELFUENF
SECHSIEBEN
JZEHNEUNYP
DREINSZWEI
VIERQACHTC

Sample Input 3

5 10 12
UNO DUE TRE QUATTRO CINQUE SEI SETTE OTTO NOVE DIECI UNDICI DODICI

Sample Output 3

impossible

Problem K

You Are Fired!

The latest quarterly report of the Pump Organization does not look promising at all: the costs exploded while the revenue decreased by 42%. Taking a closer look at the numbers, CEO Dagobert J. Pump comes to the conclusion that the only way to save the company is to fire some of the employees.

The problem is that CEO Pump just recently claimed in an interview that "he is the greatest job creator that God ever created". So if he fired too many people, the dishonest corrupted left-wing media would certainly report about it, which would make him look extremely unreliable. The PR experts of the Pump Organization already estimated a number of employees that CEO Pump may fire without raising any attention in the media. To be on the safe side, CEO Pump further decides to fire as few employees as possible.

Taking a list of all employees of his company and their respective salary, CEO Pump has to save at least d Dollars by firing not more than k employees. If this is possible, CEO Pump wants to personally tell each and every one of them: **YOU ARE FIRED!**

Input

The input consists of:

- one line with three integers n , d , and k , where
 - n ($1 \leq n \leq 10\,000$) is the number of employees of the Pump Organization;
 - d ($1 \leq d \leq 10^9$) is the amount of money that CEO Pump needs to save at least;
 - k ($1 \leq k \leq n$) is the maximum number of employees that CEO Pump can fire without raising attention in the media.
- n lines each containing a string s_i and an integer c_i , where
 - s_i ($1 \leq |s_i| \leq 7$) is the name of the i -th employee;
 - c_i ($1 \leq c_i \leq 100\,000$) is the salary of the i -th employee.

All s_i are distinct and only contain lowercase and uppercase letters of the English alphabet.

Output

If there is no way to save at least d Dollars by firing not more than k employees, output *impossible*.

If there is a way to save at least d Dollars by firing not more than k employees, then the output consists of:

- one line containing a single integer x ($1 \leq x \leq k$), where x is the number of people to be fired.
- x lines each containing the string s_i , **YOU ARE FIRED!**, where s_i is the name of the i -th employee to be fired.

If there are multiple solutions, any of them will be accepted.

Sample Input 1

```
5 2000 3
John 999
Lyndon 450
Richard 1234
Gerald 1001
Jimmy 300
```

Sample Input 2

```
3 5555 2
Ronald 1000
George 2000
Bill 3000
```

Sample Output 1

```
2
Richard, YOU ARE FIRED!
Gerald, YOU ARE FIRED!
```

Sample Output 2

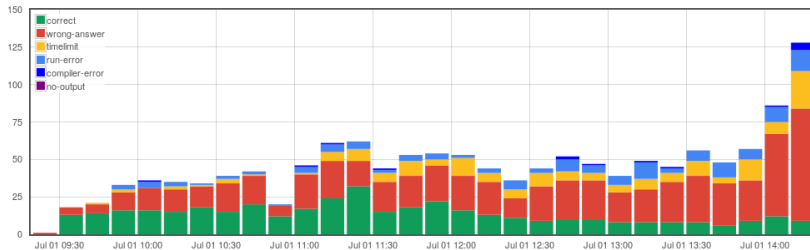
```
impossible
```


German Collegiate Programming Contest 2017

The GCPC 2017 Jury

01.07.2016

Statistics



Statistics

Problem	Min LOC	Max LOC
Borders	48	337
Buildings	26	90
Joyride	46	84
Pants on Fire	30	97
Plug It In	51	206
Perpetuum Mobile	30	142
Water Testing	17	269
Ratatöskr	56	131
Überwatch	14	72
Word Clock	79	168
You are fired	21	91
total	418	1687

K: You Are Fired! - Sample Solution

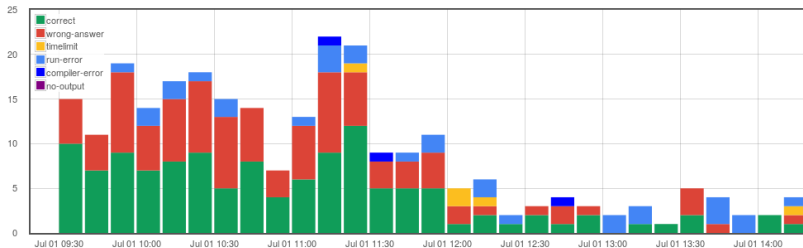
Problem

Given a list of employees with their respective salary, decide if there is a way to save d dollars by firing at most k of the employees.

Solution

- ▶ Sort all employees by their salary.
- ▶ If the accumulated salary of the k best earning employees is smaller than d , print out "impossible".
- ▶ Otherwise: Fire employees in descending order of their salary. Stop if their accumulated weight is $\geq d$.

K: You Are Fired! - Statistics



D: Pants on Fire - Sample Solution

Problem

You are given N statements of the form A are worse than B and one further such statement $S = X$ are worse than Y .

Decide whether

- ▶ S logically follows
- ▶ $\neg S$ logically follows
- ▶ none of the two

Solution

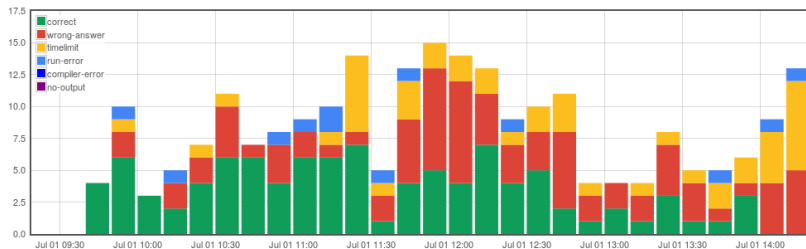
- ▶ are worse than induces an ordering relation \prec
- ▶ Use Floyd-Warshall to compute the transitive hull of the N given statements, which is \prec

D: Pants on Fire - Sample Solution

Solution

- ▶ Check whether the order S or its inverse (Y are worse than X , i.e. $Y \prec X$) are in \prec .
 - ▶ If $S \in \prec$: Fact
 - ▶ Else If $\bar{S} \in \prec$: Alternative Fact
 - ▶ Else: Pants on Fire
-
- ▶ Using DFS was also possible

D: Pants on Fire - Statistics



I: Uberwatch - Sample Solution

Problem

A game of Überwatch is played over n time slices. The player can defeat all currently visible opponents. The attack must be charged for m time slices after every use and at the beginning of the game. What is the maximum number of opponents that can be defeated?

Observations

The greedy strategy of firing as fast as possible might skip a time slice needed for the optimal solution. → **WA**

Only two choices in every time slice:

- ▶ Fire
Must not have fired for atleast m time slices. Score current number of opponents.
- ▶ Wait
Score does not change.

I: Uberwatch - Sample Solution

Problem

A game of Überwatch is played over n time slices. The player can defeat all currently visible opponents. The attack must be charged for m time slices after every use and at the beginning of the game. What is the maximum number of opponents that can be defeated?

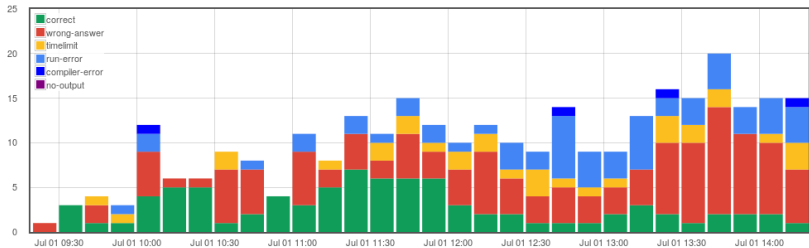
Solution

Solution for time slice t :

$$solution(t) = \max(\underbrace{opponents(t) + solution(t - m)}_{\text{Fire}}, \underbrace{solution(t - 1)}_{\text{Wait}})$$

One dimensional dynamic programming to get $O(n)$ run time.
Special case the initial charging.

I: Uberwatch - Statistics



C: Joyride - Sample Solution

Problem

Given a graph $G = (V, E)$, per node a time t_v and a cost c_v , and a time t^* for all edges. You are traversing the graph starting at 1 and at every node you pass you have stay kt_v time and spend kc_v money with $k \geq 1$

What is the least amount of money you need to spend s.t. you arrive at 1 after X time?

Solution

The problem is a variant of Knapsack.

The time is the “weight”, X is the size of the bucket, and c_v is the “gain” of an edge.

Extend standard Knapsack-DP to handle the graph structure.

C: Joyride - Sample Solution

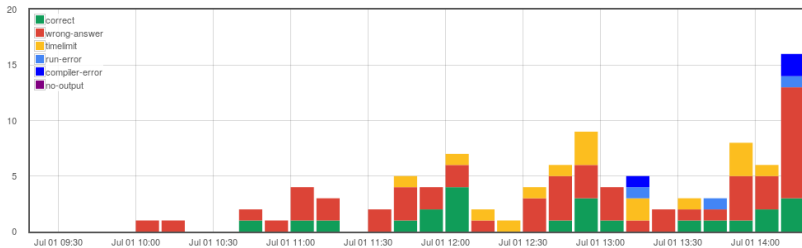
Use DP over the current node v and the time t you can move freely from e .

$$dp(v, t) = \begin{cases} \infty & t < 0 \\ c_v + \min\left(\{dp(v, t - t_v)\} \cup \{dp(v', t - t^* - t_v) \mid (v', v) \in E\}\right) & \text{if } t \geq 0 \end{cases}$$

Additional special treatment for entering the park:

The answer is $dp(1, X - t_1) + c_v$

C: Joyride - Statistics



G: Water Testing - Sample Solution

Problem

Given a polygon $P = (p_1, \dots, p_n)$ s.t. all p_i are on integer coordinates.

How many integer points are strictly inside P ?

Solution

Use Pick's theorem.

$$A = I + \frac{R}{2} - 1$$

where

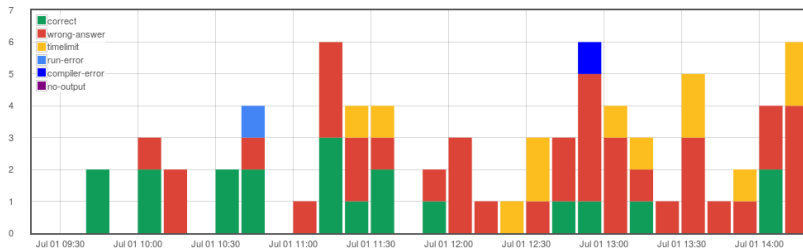
- ▶ A - Area of P
- ▶ I - Integer points strictly inside P
- ▶ R - Integer points in the border of P

G: Water Testing - Sample Solution

$$A = I + \frac{R}{2} - 1$$

- ▶ Compute A with any interger-safe method
- ▶ R can be computed using gcd
 - ▶ each edge of the polygon is defined by a $(\Delta_x, \Delta_y) = (|p_1^x - p_2^x|, |p_1^y - p_2^y|)$ pair
 - ▶ we have to determine how many $k \in \mathbb{N}^+$ there are s.t. $\frac{\Delta_x}{k}$ and $\frac{\Delta_y}{k}$ are integer.
 - ▶ but this is just $\gcd(\Delta_x, \Delta_y)$

G: Water Testing - Statistics



E: Perpetuum Mobile - Sample Solution

Problem

Given a directed graph with arc weights in $[0.0001, 5]$, decide whether the graph contains a cycle with total product > 1 .

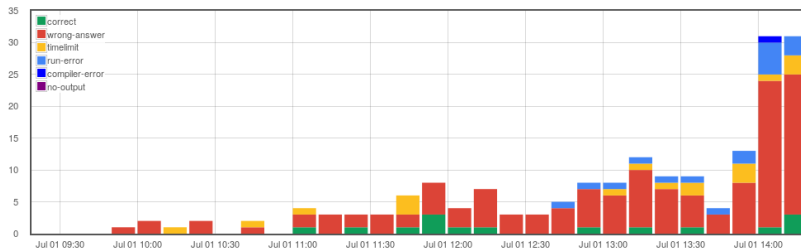
Solution

- ▶ Given a cycle in the graph, let c_i be the arc weight of the i th arc in the cycle. We know: $c_i > 0$ for all arcs i . Then

$$\prod_i c_i > 1 \iff -\sum_i \log(c_i) < 0$$

- ▶ Convert arc weights c_i to $-\log(c_i)$ and use Bellman-Ford or Floyd-Warshall to decide whether a negative cycle exists in the transformed graph. This is precisely the case if there is a cycle with product > 1 in the original graph.

E: Perpetuum Mobile - Statistics



F: Plug it in - Sample Solution

Problem

- ▶ Adam placed his electronics randomly into his room.
- ▶ Not every device can be plugged into every socket.
- ▶ How many devices can Adam power using 1 powerbar which triples 1 socket?

Solution

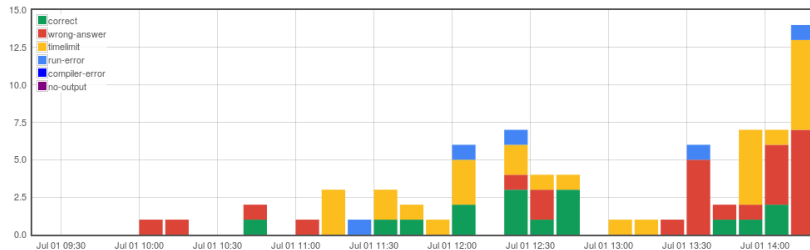
- ▶ Naive idea: Compute a maximum bipartite matching (possibly using a maximum flow algorithm) for each of the n possible locations of the powerbar
- ▶ This is, however, too time-consuming!

F: Plug it in - Sample Solution

Solution

- ▶ Create a bipartite graph with sockets on the left, devices on the right and a connection between a socket and a device if the device can be plugged into the socket.
- ▶ Compute a maximum bipartite matching without the powerbar
- ▶ For each matched socket with more than 1 neighbor:
 - ▶ Triple the socket (i.e. add 2 unoccupied copies of the socket / raise the capacity in the flow network).
 - ▶ Compute the maximum bipartite matching / maximum flow in the resulting graph.
 - ▶ Use the bipartite matching of the original graph as a starting point for the matching of the new graph!
- ▶ Take the maximum across all computed matchings.

F: Plug it in - Statistics



B: Buildings - Sample Solution

Problem

How many possible houses are there (up to rotation) with m walls consisting of $n \times n$ bricks, each colored in one of c colors?

Solution

For each wall, there are $c^{(n \times n)}$ possible designs.
But how to deal with rotation?

B: Buildings - Sample Solution

Burnside's Lemma

The number of orbits (*distinct results*) for a group of operations (*possible rotations*) acting on a set X (*the set of walls*) is exactly the average number of elements (*wall designs*) fixed by each group element (*rotation by a fixed n*).

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

B: Buildings - Sample Solution

Application

Consider all rotations rot_i rotating the house by i .

1. Since the house has m walls, all designs have to have periodicity m (i.e. repeat after m walls).
2. Rotation by i fixes all wall designs every i walls, i.e. it fixes all designs with periodicity i .
3. \Rightarrow Rotation by i fixes a design iff the design has periodicity $\gcd(i, m)$.
4. There are exactly $\{\text{number of wall designs}\}^{\gcd(i, m)}$ possible designs with periodicity $\gcd(i, m)$.

B: Buildings - Sample Solution

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Solution

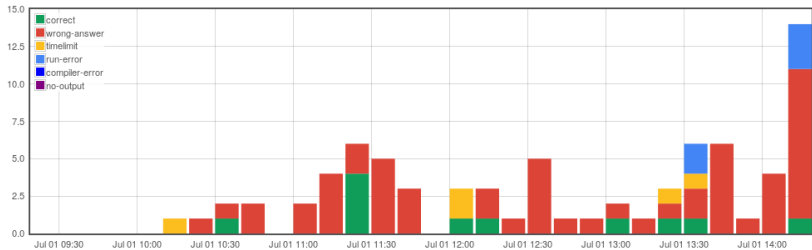
- ▶ Let $N = c^{(n \times n)}$ be the number of wall designs.
- ▶ There are exactly

$$\frac{1}{m} \sum_{0 \leq i < m} N^{\gcd(i, m)}$$

possible house designs up to rotation.

- ▶ Compute this number modulo 1000000007 so it does not overflow long.

B: Buildings - Statistics



H: Ratatoeskr - Sample Solution

Problem

A tree of n nodes and starting positions (nodes) of a squirrel and two ravens are given. On a signal of Odin, one raven flies into the air and lands again. Meanwhile, the squirrel can travel in the tree, but may not pass over a node occupied by the other raven.

Output the minimum number of signals after which the ravens are guaranteed to have captured the squirrel.

Possible Approaches

1. Dynamic programming on possible states
2. Force the squirrel into the shallowest subtree
3. Find the 'highest' node the squirrel can reach

H: Ratatoeskr - Sample Solution

Approach 1: Dynamic Programming

- ▶ Idea: Dynamic programming by backwards breadth-first-search on the states
- ▶ The final states are when the squirrel is at the same position as a raven
- ▶ To move between states, consider the possible ways for the ravens and the squirrel to travel (bearing in mind the squirrel cannot move past a raven)
- ▶ Already computed states must be skipped to avoid an infinite loop
- ▶ Runtime $O(n^4)$

H: Ratatoeskr - Sample Solution

Approach 2: DFSs on the tree

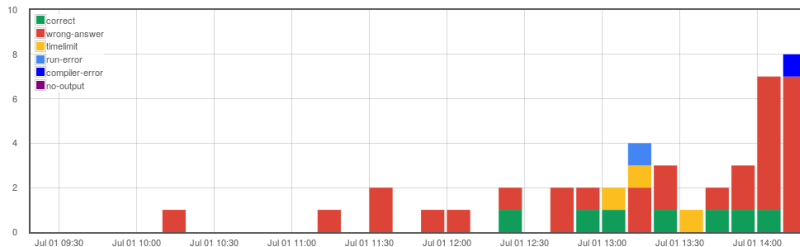
- ▶ Idea: Drive the squirrel into the shallowest subtree
- ▶ For every node, root the tree at this node and compute its depth
- ▶ The minimum depth is the number of signals required
- ▶ Runtime: $O(n^2)$

H: Ratatoeskr - Sample Solution

Approach 3: Assign heights to nodes

- ▶ Idea: Node heights correspond to the number of signals required (minus 1)
- ▶ Leaves are assigned a height of 0
- ▶ A node has a height of k if it becomes a leaf after all nodes of height $< k$ have been removed
- ▶ The squirrel should travel to the highest node (a 'center' of the tree) it can reach
- ▶ Compute the highest position the squirrel can reach from its starting position
- ▶ If there are two centers, add 1 for an extra signal
- ▶ Runtime: $O(n)$

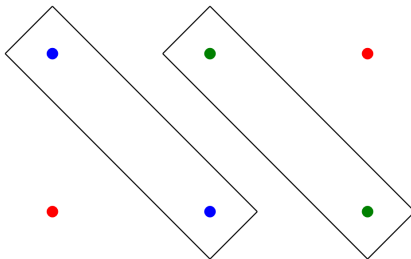
H: Ratatoeskr - Statistics



A: Borders - Sample Solution

Problem

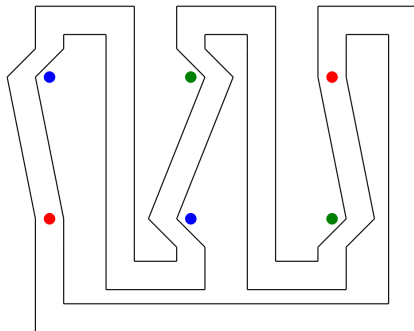
Given a set of blue, red and green points in the plane, draw two polygons so that each color has its own region.



A: Borders - Sample Solution

Solution 1

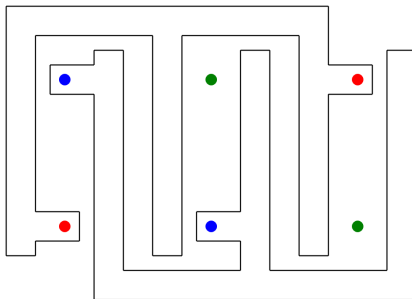
- ▶ Sort all points lexicographically, then go column by column.
- ▶ Draw the polygons in parallel, shifting left or right depending on color.



A: Borders - Sample Solution

Solution 2

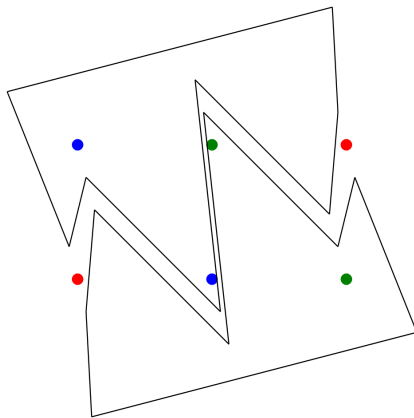
- ▶ Sort all points lexicographically, then go column by column.
- ▶ Draw two comb-shaped polygons from top and bottom, adding protrusions to “capture” the appropriate color.



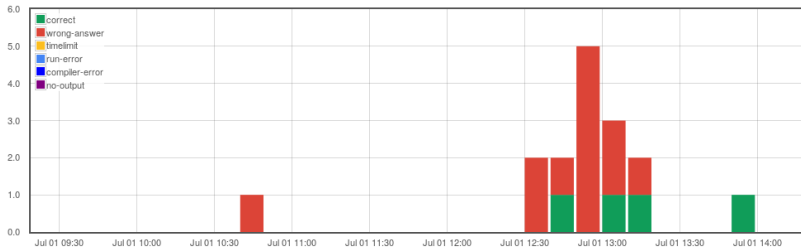
A: Borders - Sample Solution

Solution 3

- Pick a direction at random and do a zigzag in that direction.



A: Borders - Statistics



J: Word Clock - Sample Solution

Problem

Given a set S of at most 18 words and a rectangular grid, fill the grid with letters such that every word occurs in the grid.

Solution

Use dynamic programming: for $S' \subseteq S$ and $s \in S'$ let

$f(S', s)$ = the earliest possible end position (row, column) of a text containing all the words from S' and ending in s .

- ▶ A solution exists if and only if one of the positions $f(S, s)$ still lies in the grid.
- ▶ To reconstruct the solution one needs to keep track of the best predecessor states while building the DP table (see next slide).

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J: Word Clock - Sample Solution

Solution

$f(S', s)$ = the earliest possible end position (row, column) of a text containing all the words from S' and ending in s .

The DP table can be constructed as follows:

- ▶ From the current state (S', s) try to append every $t \notin S'$ to the text, maximizing overlap between s and t .
If t doesn't fit in the current line, start a new line.
The new state is $(S' \cup \{t\}, t)$.
- ▶ The maximal overlaps between words can be precalculated.
Possible pitfall: some words may be substrings of other words.
- ▶ Time complexity: $\mathcal{O}(n^2 2^n)$.

J: Word Clock - Statistics

