# 习题答案

## 习题 1.2

1. （1） (1) $\frac{\pi}{3}(l^2-h^2)h$;  (2) $xy\sqrt{4-x^2-y^2}$;  （3）$8xy\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}$;

   （4）$\left|\sqrt{2}-\sqrt{(x-1)^2+y^2+(z+1)^2}\right|$

2. (1) $\left\{(x,y)\middle|4x^2+y^2\geq 1\right\}$;  (2) $\left\{(x,y)\middle|xy>0\right\}$;  (3) $\left\{(x,y)\middle|y^2\geq 1, x^2+y^2+z^2<4\right\}$;

   (4) $\mathbb{R}^2$

3. $f(x,y)=\begin{cases}\frac{1-y}{1+y}x^2, & y\neq -1;\\ 0, & y=-1\end{cases}$.

4. (1) $k=0$;  （2）不是;  (3) $k=2$.

5. 记 $d=[(x-a)^2+(y-b)^2+(z-c)^2]^{\frac{1}{2}}$,

   $F_x=-G\frac{Mm_0}{d^3}(x-a), F_y=-G\frac{Mm_0}{d^3}(y-b), F_z=-G\frac{Mm_0}{d^3}(z-c)$.

6. $[1,4]\times[1,2]$,  $u=a^2-\left(\frac{v}{a}\right)^2$.

7. (1) $u^2+v^2=\frac{1}{R^2}$;  (2)$u=v, v\geq\frac{1}{2}$.

## 习题 1.3

1. (1) 1;  (2) 0;  (3) 0;  (4),(5),(6) 不存在；（7）0；（8）0；（9）不存在;

   (10) $\frac{2}{3}$;  (11) 不存在；（12）$-\frac{1}{3}$.

2. (1) $\frac{\ln 3}{3}$;  (2) 0;  (3) 0;  (4) 0;  (5) 0;  (6) 0.

3. (1) 0，, 1, 不存在;  (2) $\frac{1}{2}$, 1, 不存在;  (3) 不存在，不存在， 0.

6. (1) 连续;  (2) 不连续;  (3) 连续;  (4) 不连续.

7. (1) $\left\{(x,y)\middle|x+y\neq 0\right\}$;  (2) $\left\{(x,y)\middle|x^2+y^2\neq 0\right\}$;

(3) $\{(x,y) | x+y \notin \mathbb{Z}\}$; (3) $\{(x,y) | x+y=0\}$.

11. (1) 2;  (2) 1;  (3) 1;  (4) 无;  (5) 无.

## 习题 1.4

1. (1) $z'_x = 2axy + by^2$,  $z'_y = ax^2 + 2bxy$;

   (2) $z'_x = 4x\tan(x^2+y^2)\sec^2(x^2+y^2)$,  $z'_y = 4y\tan(x^2+y^2)\sec^2(x^2+y^2)$;

   (3) $z'_x = \dfrac{1}{y} - \dfrac{y}{x^2}$,  $z'_y = -\dfrac{x}{y^2} + \dfrac{1}{x}$;

   (4) $z'_x = -\dfrac{2xy}{x^4+y^2}$,  $z'_y = \dfrac{x^2}{x^4+y^2}$;

   (5) $z'_x = \dfrac{1}{\sqrt{x^2-y^2}}$,  $z'_y = -\dfrac{y}{x\sqrt{x^2-y^2}+(x^2-y^2)}$;

   (6) $z'_x = e^{-y} - ye^{-x}$,  $z'_y = -xe^{-y} + e^{-x}$;

   (7) $z'_x = -\ln 2 \cdot 2^{xy} y\sin(1+2^{xy})$,  $z'_y = -\ln 2 \cdot 2^{xy} x\sin(1+2^{xy})$;

   (8) $u'_x = -\dfrac{y}{x^2}\operatorname{ch}\dfrac{y}{x}$,  $u'_y = \dfrac{1}{x}\operatorname{ch}\dfrac{y}{x} + z\operatorname{sh} yz$,  $u'_z = y\operatorname{sh} yz$;

2. （1）不可微；　（2）不可微；　（3）不可微；　（4）可微。

4. (1) $-\cos 1\left(\dfrac{\sqrt{2}}{2}dx + \dfrac{1}{2}dy - \dfrac{1}{2}dz\right)$;  (2) $e^{x+y+z}(dx+dy+dz)$;

   (3) $2(x+y)(dx+dz)$; (4) $\dfrac{1}{\sqrt{1+x^2+y^2+z^2}}(xdx+ydy$

   (5) $\dfrac{2}{(x+y)^2}(ydx - xdy)$;  (6) $-\dfrac{e^{xy}}{\sqrt{1-e^{2xy}}}(ydx + xdy)$;

   (7) $\dfrac{xdx+ydy+zdz}{1+x^2+y^2+z^2}$;  (8) $2\sum_{i=1}^{n}\sum_{j=1}^{n} x_i dx_j$.

5. $2576cm^3$.  6. $-0.17cm$.  10.任意方向.

11. (1) $\dfrac{1}{5}$;  (2) $\dfrac{4}{81}$;  (3) $-2n\sqrt{n}$;  (4) $\sqrt{n}$.

12. (1) $\dfrac{1}{\sqrt{x^2+y^2}}(x,y)$; (2) $\dfrac{1}{(x+y+z)^2}(y^2z+yz^2, z^2x+zx^2, x^2y+xy^2)$;

(3) $(1\cdot;\cdot 1, \quad , 1)$; $\sum_{i=1}^{n}(x_i 4)\ 2\cdots$ (

13. $2\sqrt{2}$, $(0,1,1)$, $\mathbf{l}:\mathbf{l}\perp(0,1,1)$.

14. (1) $u''_{xx}=-2a^2\cos 2(ax-by)$, $u''_{xy}=2ab\cos 2(ax-by)$, $u''_{yy}=-2b^2\cos 2(ax-by)$;

(2) $u''_{xx}=\alpha^2 e^{-\alpha x}\sin\beta y$, $u''_{xy}=-\alpha\beta e^{-\alpha x}\cos\beta y$, $u''_{yy}=-\beta^2 e^{-\alpha x}\sin\beta y$;

(3) $u''_{xx}=(-2y+xy^2)e^{-xy}$, $u''_{xy}=(-2x+x^2y)e^{-xy}$, $u''_{yy}=x^3e^{-xy}$;

(4) $u''_{xx}=-\dfrac{1}{\left(x+\sqrt{1-y^2}\right)^2}$, $u''_{xy}=\dfrac{y}{\sqrt{1-y^2}\left(x+\sqrt{1-y^2}\right)^2}$,

$$u''_{yy}=\frac{x+(1+y^2)\sqrt{1-y^2}}{\sqrt{1-y^2}\left(x\sqrt{1-y^2}+\left(1-y^2\right)\right)^2}.$$

## 习题 1.5

1. (1) $J=\begin{pmatrix}\dfrac{x}{\sqrt{x^2+y^2}} & \dfrac{y}{\sqrt{x^2+y^2}}\\ -\dfrac{y}{x^2+y^2} & \dfrac{x}{x^2+y^2}\end{pmatrix}$, $\mathbb{R}^2\setminus\{(0,0)\}$;

(2) $J=\begin{pmatrix}e^x\cos y & -e^x\sin y\\ e^x\sin y & e^x\cos y\end{pmatrix}$, $\mathbb{R}^2$;

(3) $J=\begin{pmatrix}\dfrac{y^2-x^2}{\left(x^2+y^2\right)^2} & -\dfrac{2xy}{\left(x^2+y^2\right)^2}\\ -\dfrac{2xy}{\left(x^2+y^2\right)^2} & \dfrac{x^2-y^2}{\left(x^2+y^2\right)^2}\end{pmatrix}$, $\mathbb{R}^2\setminus\{(0,0)\}$;

(4) $J=\begin{pmatrix}\dfrac{x}{x^2+y^2} & \dfrac{y}{x^2+y^2}\\ -\dfrac{y}{x^2+y^2} & \dfrac{x}{x^2+y^2}\end{pmatrix}$, $\mathbb{R}^2\setminus\{(0,0)\}$.

2. $\begin{pmatrix} \sin\theta\cos\varphi & -r\sin\theta\sin\varphi & r\cos\theta\cos\varphi \\ \sin\theta\sin\varphi & r\sin\theta\cos\varphi & r\cos\theta\sin\varphi \\ \cos\theta & 0 & -r\sin\theta \end{pmatrix}.$

3. (1) $z'_x = \dfrac{x^2y - y^3}{x^4 + 3x^2y^2 + y^4}, \quad z'_y = \dfrac{xy^2 - x^3}{x^4 + 3x^2y^2 + y^4};$

(2) $z'_x = 3f'_1 + 4f'_2, \quad z'_y = 2(f'_1 - f'_2);$

(3) $z'_x = 2xf'_1 + ye^{xy}f'_2, \quad z'_y = -2yf'_1 + xe^{xy}f'_2;$

(4) $z'_x = f'_1 + f'_2 + f'_3, \quad z'_y = f'_2 - f'_3;$

(5) $z'_x = y - \dfrac{y}{x^2}f(xy) + \dfrac{y^2}{x}f'(xy), \quad z'_y = x + \dfrac{1}{x}f(xy) + yf'(xy);$

(6) $z'_x = (1 + \ln x)f'_1 + 2f'_2, \quad z'_y = -f'_2.$

4. $-e^{-x}\ln(e^{-x} - \ln x) - \dfrac{e^{-x}}{e^{-x} - \ln x}\left(e^{-x} + \dfrac{1}{x}\right).$

8. $\dfrac{\partial^2\omega}{\partial u^2} + \dfrac{\partial\omega}{\partial u} = 0.$

9. (1) $J = \left(\begin{array}{cc|} 1 & 1 \\ \dfrac{x}{x^2+y^2} & \dfrac{y}{x^2+y^2} \\ -\dfrac{y(x^2+y^2)}{x^2} & \dfrac{x^2+y^2}{x} \end{array}\right)\begin{pmatrix} \dfrac{y^2-x^2}{(x^2+y^2)^2} & \dfrac{-2xy}{(x^2+y^2)^2} \\ \dfrac{-2xy}{(x^2+y^2)^2} & \dfrac{x^2-y^2}{(x^2+y^2)^2} \end{pmatrix}, \quad dY = J(dx, dy)^T;$

(2) $J = \dfrac{1}{x^2+y^2}\begin{pmatrix} \ln(x^2+y^2) & 2\arctan\dfrac{y}{x} \\ \ln(x^2+y^2) & -2\arctan\dfrac{y}{x} \end{pmatrix}\begin{pmatrix} x & y \\ -y & x \end{pmatrix}, \quad dY = J(dx, dy)^T;$

(3) $J = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad dY = (dx, dy)^T.$