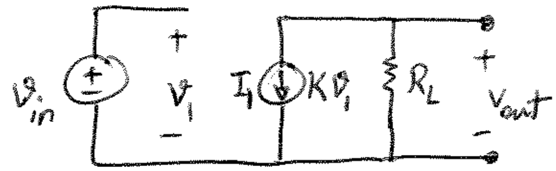
Chapter 4

(4.1)



$K = 20\,mA/V$

$\left|\frac{v_{out}}{v_{in}}\right| = 15 \qquad v_{in} = v_1$

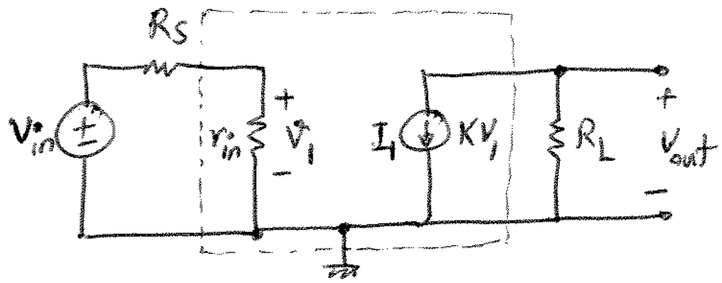
$$v_{out} = -I_1 R_L = -K R_L v_{in}$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = -K R_L \Rightarrow \left|\frac{v_{out}}{v_{in}}\right| = K R_L$$

$$\Rightarrow K R_L = 15 \Rightarrow R_L = \frac{15}{20\,mA/V} = 750\,\Omega$$

$$\boxed{R_L = 750\,\Omega}$$

4.2



$$\frac{V_{out}}{V_{in}} = ?$$

$$V_1 = \frac{r_{in}}{r_{in}+R_S} V_{in}$$

$$I_1 = K V_1$$

$$V_{out} = -R_L I_1$$

$$\Rightarrow V_{out} = -K R_L V_1$$

$$\Rightarrow V_{out} = -K R_L \frac{r_{in}}{r_{in}+R_S} V_{in}$$

$$\Rightarrow \boxed{A_V = \frac{V_{out}}{V_{in}} = -K R_L \frac{r_{in}}{r_{in}+R_S}}$$

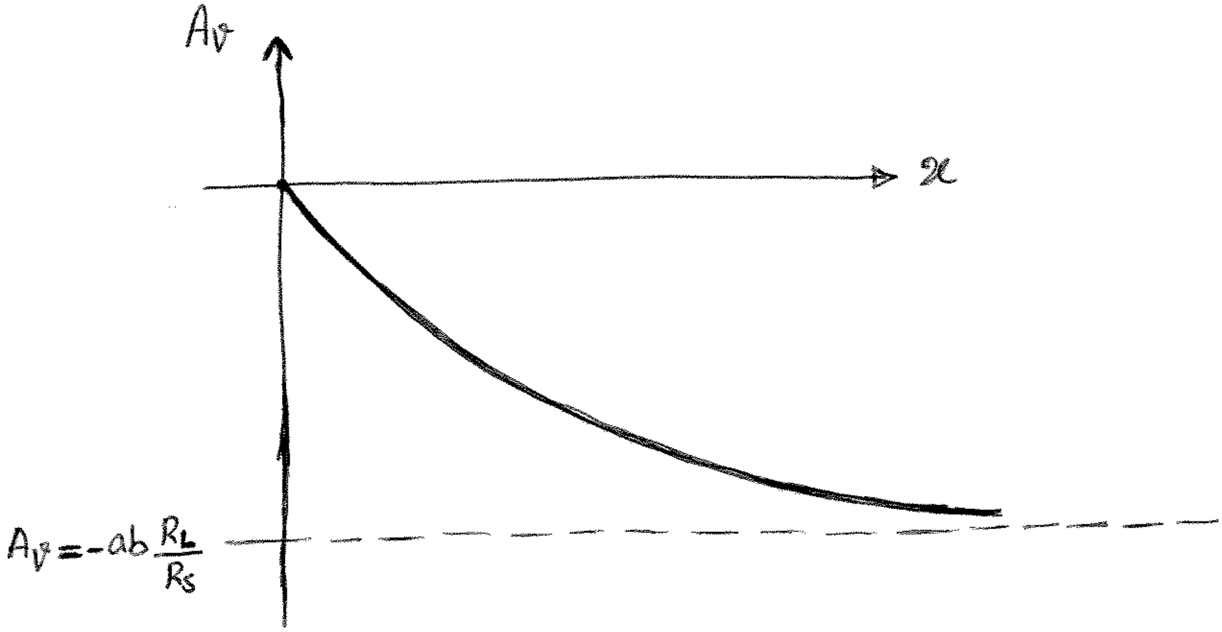
4.3 From Solution for problem 4.2,

$a > 0$
$b > 0$
$x \geq 0$

$$A_v = -K R_L \frac{r_{in}}{r_{in} + R_S}$$

$$\xrightarrow[K = bx]{r_{in} = a/x} A_v = -bx R_L \frac{a/x}{a/x + R_S} = -bR_L \frac{a}{\frac{a}{x} + R_S}$$

$$\Longrightarrow \boxed{A_v = -bR_L \left(\frac{x}{1 + \frac{R_S}{a} x} \right)}$$



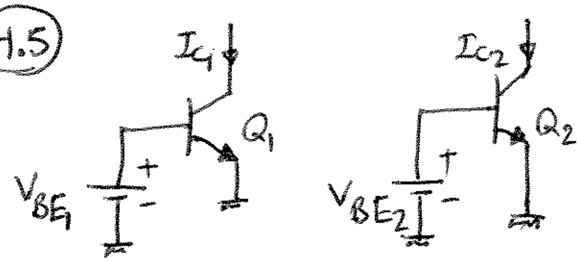
4.4 From equation (4.8) page 136,

$$I_C = \frac{A_E q D_n n_i^2}{N_E W_B} e^{\frac{V_{BE}}{V_T}} \qquad W_B \equiv \text{width of the Base}$$

if $W_B \uparrow 2 \Rightarrow I_C \downarrow 2$

$\boxed{\text{Collector current decreases by a factor of two}}$

(4.5)



$V_T = 26\,mV$

$I_{C_1} = I_{C_2}$

$V_{BE_1} - V_{BE_2} = 20\,mV$

$$I_C = \frac{A_E q D_n n_i^2}{N_E W_B}\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$$ equation (4.8) page 136

$$\Rightarrow I_C \simeq \frac{A_E q D_n n_i^2}{N_E W_B} e^{\frac{V_{BE}}{V_T}}$$ $A_E \equiv$ Cross Section

if $I_{C_1} = I_{C_2}$

$$\Rightarrow A_{E_1} \frac{q \cancel{D_n n_i^2}}{\cancel{N_E} W_B} e^{\frac{V_{BE_1}}{V_T}} = A_{E_2} \frac{q \cancel{D_n n_i^2}}{\cancel{N_E} W_B} e^{\frac{V_{BE_2}}{V_T}}$$

$$\Rightarrow \frac{A_{E_2}}{A_{E_1}} = \frac{e^{V_{BE_1}/V_T}}{e^{V_{BE_2}/V_T}}$$

$$\Rightarrow \frac{A_{E_2}}{A_{E_1}} = e^{(V_{BE_1} - V_{BE_2})/V_T} = e^{20\,mV/26\,mV}$$

$$\Rightarrow \boxed{\frac{A_{E_2}}{A_{E_1}} = e^{\frac{20}{26}} \simeq 2.16}$$

(6a) $I_X = 1^{mA} \Rightarrow I_{Q_1} = I_{Q_2} = 0.5^{mA}$

$$I_{Q_1} = I_{S_1} e^{\frac{V_{BE_1}}{V_T}} \Rightarrow 5\times10^{-4} = 3\times10^{-16} e^{\frac{V_B}{26^{mV}}}$$

$$\Rightarrow V_B = 26^{mV} \ln\left(\tfrac{5}{3}\times10^{12}\right) \Rightarrow \boxed{V_B \simeq 731.7^{mV}}$$

(6b) $I_Y = I_{S_3} e^{\frac{V_B}{V_T}}$

$$\Rightarrow I_{S_3} = I_Y e^{-\frac{V_B}{V_T}} = 2.5\times10^{-3} \times e^{-\frac{V_B}{26^{mV}}} = 2.5\times10^{-3} \times \frac{1}{\frac{5}{3}\times10^{12}}$$

$$\Rightarrow \boxed{I_{S_3} = 1.5\times10^{-15} A}$$

7a) $I_X = I_1 + I_2$

$$\Rightarrow I_X = I_{S_1} e^{\frac{V_B}{V_T}} + I_{S_2} e^{\frac{V_B}{V_T}} \Rightarrow I_X = (I_{S_1} + I_{S_2}) e^{\frac{V_B}{V_T}}$$

$$\Rightarrow V_B = V_T \ln\left(\frac{I_X}{I_{S_1} + I_{S_2}}\right) \xrightarrow{I_{S_1} = 2I_{S_2}} \boxed{V_B = V_T \ln\left(\frac{I_X}{\frac{3}{2} I_{S_1}}\right)}$$

$$V_B = 26\times10^{-3} \ln\left(\frac{1.2\times10^{-3}}{\frac{3}{2}\times5\times10^{-16}}\right) \Longrightarrow \boxed{V_B \simeq 730.6 \text{ mV}}$$

7b) Transistors at the edge of the active mode $\Rightarrow V_C = V_B$

applying KVL, we have:

$$V_{CC} = R_C I_X + V_B \Rightarrow \boxed{R_C = \frac{V_{CC} - V_B}{I_X}}$$

$$\Rightarrow R_C = \frac{2.5 - 0.73}{1.2\times10^{-3}}$$

$$\Rightarrow \boxed{R_C \simeq 1475\ \Omega}$$

8a Same as 7a,

$$\boxed{V_B \simeq 730.6\,mV}$$

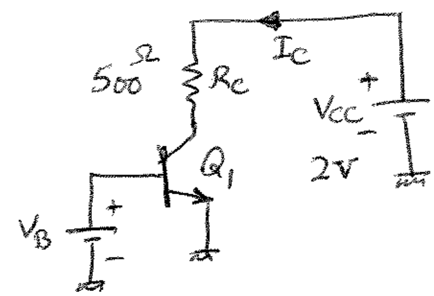
8b According to 7b,

$$R_C = \frac{V_{CC} - V_B}{I_X} = \frac{1.5 - 0.73}{1.2 \times 10^{-3}}$$

$$\Rightarrow \boxed{R_C \simeq 642\ \Omega}$$

9 Q_1 is at the edge of the active region $\Rightarrow V_C = V_B$

applying KVL, we have:



$$V_{CC} = R_C I_C + V_C$$

$$\xRightarrow{V_C = V_B} V_{CC} = R_C I_C + V_B$$

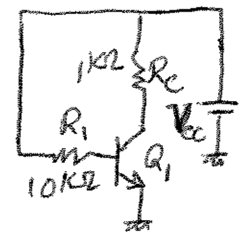
$$\Rightarrow V_{CC} = R_C I_S e^{V_B/V_T} + V_B$$

$$\Rightarrow 500\,\Omega \times 5\times10^{-16}\, e^{\frac{V_B}{26\,mV}} + V_B = 2V$$

Using numerical methods or simply, trial & error:

$$\boxed{V_B \simeq 760\ mV}$$

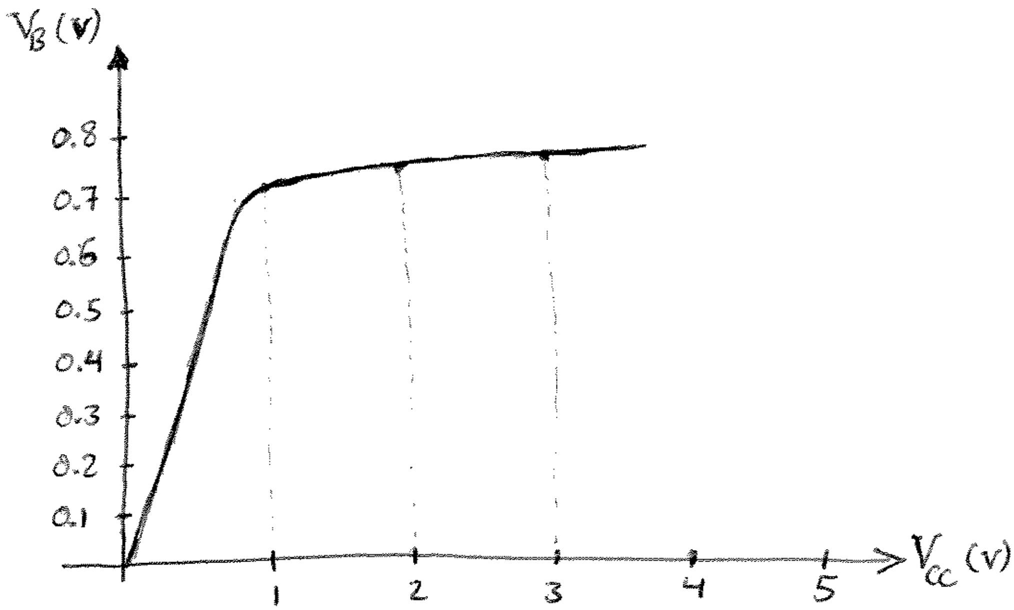
(10) Q_1 at the edge of saturation $\Rightarrow V_C = V_B$



Hence: $V_{CC} = R_C I_C + V_B$

$$\Rightarrow V_{CC} = R_C I_S e^{V_B/V_T} + V_B$$

$$\xrightarrow{I_S = 3\times10^{-16}A} V_{CC} = 3\times10^{-13} e^{V_B/V_T} + V_B \xrightarrow[V_{CC}=2^V]{\text{with}} \boxed{V_B \simeq 755^{mV}}$$



(11) Assuming $I_E \simeq I_C$, we can write:

Applying KVL: $1.5^{V} = V_{BE} + V_X$ where $V_X = 1^{k\Omega} \times I_E$

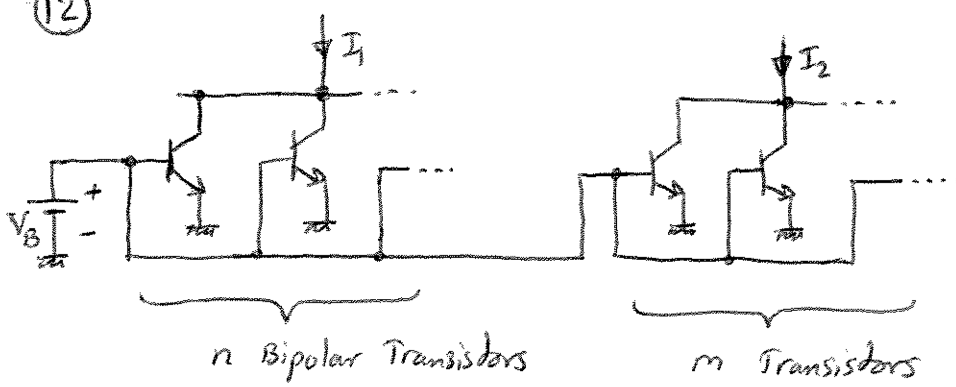
Hence: $1.5 = V_{BE} + 1^{k\Omega} \times I_C$

$$\Rightarrow 1.5 = V_{BE} + 1^{k\Omega} \times I_S e^{\frac{V_{BE}}{V_T}}$$

$$\xrightarrow[V_T = 26mV]{I_S = 6\times10^{-16}A} 1.5 = V_{BE} + 6\times10^{-13} e^{V_{BE}/26^{mV}} \Rightarrow \boxed{V_{BE} \simeq 724.5^{mV}}$$

$$V_X = 1.5 - V_{BE} \Rightarrow \boxed{V_X \simeq 775.5 \text{ mV}}$$

(12)



$$\left.\begin{aligned} I_1 &= n\,I_C = n\,I_S\,e^{V_B/V_T} \\ I_2 &= m\,I_C = m\,I_S\,e^{V_B/V_T} \end{aligned}\right\} \Rightarrow \frac{I_1}{I_2} = \frac{n}{m}$$

$$\Rightarrow \boxed{\frac{n}{m} = \frac{1^{mA}}{1.5^{mA}} = \frac{2}{3}} \xrightarrow{\text{choose}} \boxed{\begin{aligned} n &= 2 \\ m &= 3 \end{aligned}}$$

$$I_1 = n\,I_C = n\,I_S\,e^{\frac{V_B}{V_T}}$$

$$\Rightarrow I_1 = n \times 3 \times 10^{-16}\,e^{\frac{V_B}{26^m}} = 1^{mA} \overset{n=2}{\Longrightarrow} \boxed{V_B \simeq 750^{mV}}$$

(13) Using the same technique as in problem 12, we have:

$$\frac{n_1}{I_1} = \frac{n_2}{I_2} = \frac{n_3}{I_3}$$

$$\Rightarrow \frac{n_1}{0.2} = \frac{n_2}{0.3} = \frac{n_3}{0.45} \Rightarrow \boxed{\frac{n_1}{4} = \frac{n_2}{6} = \frac{n_3}{9}}$$

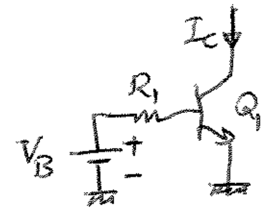
So lets choose $\boxed{\begin{cases} n_1 = 4 \\ n_2 = 6 \\ n_3 = 9 \end{cases}}$

Hence,

$$I_1 = n_1 I_S e^{V_B/V_T} \Rightarrow 0.2 \times 10^{-3} = 4 \times 3 \times 10^{-16} e^{\frac{V_B}{26mV}}$$

$$\Rightarrow \boxed{V_B \simeq 672\ mV}$$

(14) From KVL,



$$V_B = R_1 I_B + V_{BE_{Q_1}}$$

$$I_B = \frac{I_C}{\beta} = \frac{1^{mA}}{100} \Rightarrow \boxed{I_B = 10^{-5} A}$$

$$V_{BE_{Q_1}} = V_T \ln\left(\frac{I_C}{I_S}\right) = 26\times10^{-3} \ln\left(\frac{10^{-3}}{7\times10^{-16}}\right)$$

$$\Rightarrow \boxed{V_{BE_{Q_1}} \simeq 727.7\ mV}$$

Therefore,

$$V_B = R_1 I_B + V_{BE_{Q_1}}$$

$$\simeq 10^{K\Omega} \times 10^{-5} A + 728\times10^{-3}$$

$$\Rightarrow V_B \simeq 0.1 + 0.728 \Rightarrow \boxed{V_B \simeq 0.828\ V}$$

(15) According to the solution for problem 14, we have:

Applying KVL: $V_B = R_B I_B + V_{BE}$

$$\Rightarrow V_B = R_B \frac{I_C}{\beta} + V_T \ln\left(\frac{I_C}{I_S}\right)$$

$$\Rightarrow 0.8 = 10^4 \times \frac{I_C}{100} + 26\times10^{-3} \ln\left(\frac{I_C}{7\times10^{-16}}\right)$$

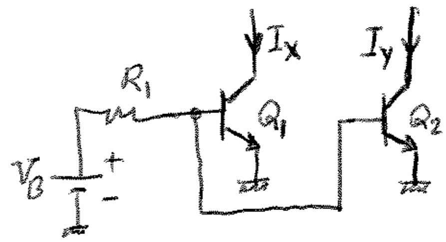
$$\Rightarrow 0.8 = 100\, I_C + 26\times10^{-3} \ln\left(\frac{I_C}{7\times10^{-16}}\right)$$

using trial & error or numerical methods,

$$\boxed{I_C \simeq 7.85\times10^{-4}\,A = 785\,\mu A}$$

16

$$\begin{cases} I_X = I_{S_1} \exp\left(\frac{V_{BE_1}}{V_T}\right) \\ I_Y = I_{S_2} \exp\left(\frac{V_{BE_2}}{V_T}\right) \\ V_{BE_1} = V_{BE_2} = V_{BE} \end{cases}$$



$$\Rightarrow \frac{I_X}{I_Y} = \frac{I_{S_1}}{I_{S_2}} = \frac{2\cancel{I_{S_2}}}{\cancel{I_{S_2}}} \Rightarrow \boxed{\frac{I_X}{I_Y} = 2}$$

$$\begin{cases} I_X = \beta_1 I_{B_1} \\ I_Y = \beta_2 I_{B_2} \\ \beta_1 = \beta_2 \end{cases}$$

$$\Rightarrow \boxed{\frac{I_{B_1}}{I_{B_2}} = \frac{I_X}{I_Y} = 2}$$

Applying KVL:

$$V_B = R_1 \left(I_{B_1} + I_{B_2}\right) + V_{BE}$$

$$V_{BE} = V_{BE_1} = V_T \ln\left(\frac{I_X}{I_{S_1}}\right) = 26^{mV} \ln\left(\frac{1^{mA}}{4\times10^{-16}}\right) \simeq 742\,mV$$

$$I_{B_1} = \frac{I_X}{\beta} \xrightarrow{\beta=100} I_{B_1} = \frac{1^{mA}}{100} = 10\mu A$$

$$\frac{I_{B_1}}{I_{B_2}} = 2 \longrightarrow I_{B_2} = \frac{I_{B_1}}{2} = \frac{10^{\mu A}}{2} \Rightarrow I_{B_2} = 5\mu A$$

Hence: $V_B = 5\times10^3\,\Omega \times \left(10\mu A + 5\mu A\right) + 0.742^V$

$$= 0.075 + 0.742 \Rightarrow \boxed{V_B \simeq 0.817^V}$$

(17) Applying KVL:

$$V_B = R_1 (I_{B_1} + I_{B_2}) + V_{BE} \xlongequal{\beta_1=\beta_2=\beta} \frac{R_1}{\beta}(I_{C_1} + I_{C_2}) + V_{BE}$$

$$\Rightarrow V_B = \frac{R_1}{\beta}(I_{S_1} + I_{S_2}) \exp\left(\frac{V_{BE}}{V_T}\right) + V_{BE}$$

$$\xRightarrow{\beta=100} 0.8^V = \frac{5000^\Omega}{100}(3\times10^{-16} + 5\times10^{-16}) \exp\left(\frac{V_{BE}}{26^{mV}}\right) + V_{BE}$$

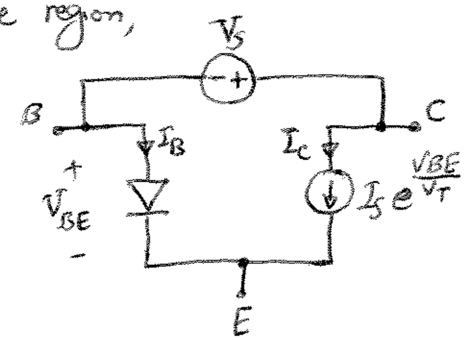
$$\Rightarrow 0.8^V = 4\times10^{-14} \cdot \exp\left(\frac{V_{BE}}{26^{mV}}\right) + V_{BE}$$

Numerical methods or Trial & error $\Rightarrow \boxed{V_{BE} \simeq 732\ mV}$

$$I_X = I_{S_1} \exp\left(\frac{V_{BE}}{V_T}\right) = 3\times10^{-16}\left[\exp\left(\frac{732}{26}\right)\right] \Rightarrow \boxed{I_X \simeq 506\ \mu A}$$

$$I_Y = I_{S_2} \exp\left(\frac{V_{BE}}{V_T}\right) = 5\times10^{-16} \exp\left(\frac{732}{26}\right) \Rightarrow \boxed{I_Y \simeq 843\ \mu A}$$

(18) Since Transistor is in Forward active region,



No change across V_{BE}

$\Downarrow$

No change in I_B

$\Downarrow$

No change in I_C

(19) $g_m = \dfrac{I_C}{V_T}$

$$\Rightarrow g_m = \frac{I_S \exp\left(\frac{V_{BE}}{V_T}\right)}{V_T} \Rightarrow \boxed{V_{BE} = V_T \ln\left(\frac{g_m V_T}{I_S}\right)}$$

$$\xrightarrow[g_m = \frac{1}{13\Omega}]{I_S = 6\times10^{-16} A} V_{BE} = 26^{mV} \cdot \ln\left(\frac{\frac{1}{13\Omega} \times 26\times10^{-3}}{6\times10^{-16}}\right)$$

$$\Rightarrow \boxed{V_{BE} \simeq 750\ mV}$$

20 $g_m = \frac{I_C}{V_T}$

$$\Delta g_m = \frac{\Delta I_C}{V_T} = \frac{1}{V_T}\Delta\left(I_S e^{V_{BE}/V_T}\right) \simeq \frac{I_S}{V_T^2} e^{\frac{V_{BE}}{V_T}} \Delta V_{BE}$$

$$\Rightarrow \boxed{\Delta g_m \simeq \frac{I_C}{V_T^2} \Delta V_{BE}}$$

$$\Rightarrow \Delta g_m \simeq \frac{g_m}{V_T} \Delta V_{BE}$$

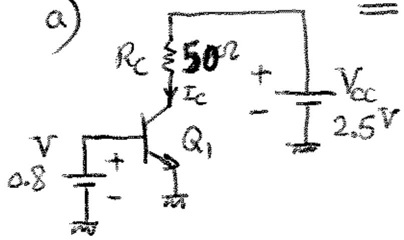
$$\Rightarrow \boxed{\frac{\Delta g_m}{g_m} \simeq \frac{1}{V_T} \Delta V_{BE}}$$

$$\left.\frac{\Delta g_m}{g_m}\right|_{I_C = 1\,mA} \overset{max}{=\!=} 0.1 \quad \Rightarrow \Delta V_{BE_{max}} = 0.1\, V_T$$

$$\Rightarrow \boxed{\Delta V_{BE} \leqslant 2.6\ mV}$$

(21) $V_A = \infty \Rightarrow r_o = \infty$, $\quad I_S = 8\times 10^{-16} A$, $\quad \beta = 100$

a)



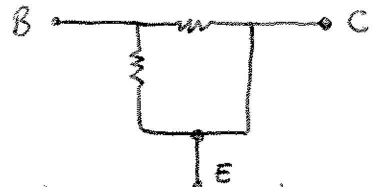
$\underline{\underline{\text{If}}}$ in Forward active region,

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$= 8\times 10^{-16} \exp\left(\frac{800}{26}\right)$$

$$\Rightarrow \boxed{I_C \simeq 18.5 mA}$$

Small signal model



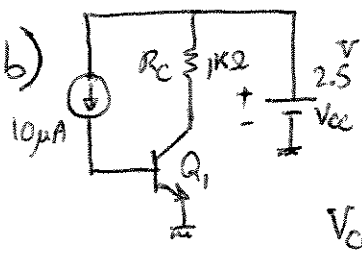
$$\Rightarrow \boxed{V_{CE} \simeq 1.575 \text{ V}}$$

Hence Transistor should be in Forward Active

$$I_{C_Q} = \frac{V_{CC} - V_{CE}}{R_C} = \frac{2.5 - 1.575}{50\Omega} = \frac{0.925}{50} \Rightarrow I_{C_Q} \simeq 18.5 mA$$

which matches with I_C

b)



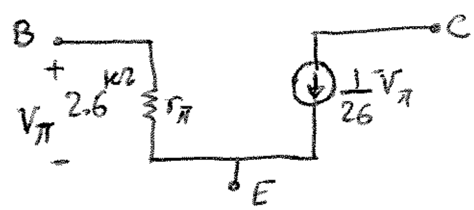
Assuming in Forward active region,

$$I_C = \beta I_B = 100 \times 10^{\mu A} \Rightarrow \boxed{I_C = 1^{mA}}$$

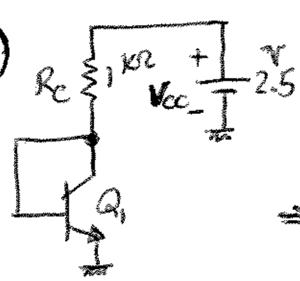
$$V_{CE} = V_{CC} - R_C I_C = 2.5 - 1^{K\Omega} \times 1^{mA} \Rightarrow \boxed{V_{CE} = 1.5^V}$$

$$g_m = \frac{I_C}{V_T} = \frac{1^{mA}}{26^{mV}} \Rightarrow \boxed{g_m = \frac{1}{26}} \Omega^{-1}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{1/26} \Rightarrow \boxed{r_\pi \simeq 2600^{\Omega}}$$

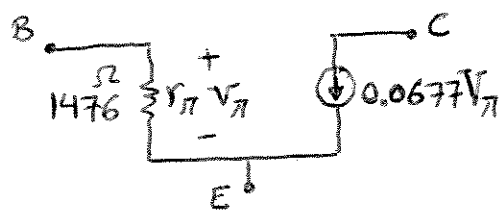


c)



Applying KVL,

$$V_{CC} \simeq R_C I_C + V_{BE}$$



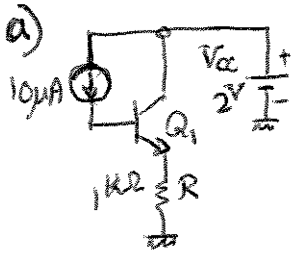
$$\Rightarrow 2.5^V \simeq 1^{K\Omega} \times I_C + V_{BE}$$

$$\Rightarrow 2.5^V \simeq 8\times 10^{-13} \cdot \exp\left(\frac{V_{BE}}{V_T}\right) + V_{BE} \Rightarrow \boxed{V_{BE} \simeq 739^{mV}}$$

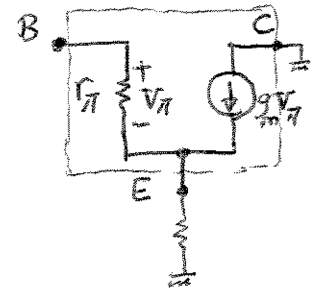
$$g_m = \frac{I_C}{V_T} = \frac{\frac{V_{CC} - V_{BE}}{R_C}}{V_T} \Rightarrow g_m = \frac{2.5 - 0.739}{1^{K\Omega} \times 0.026} \Rightarrow \boxed{g_m \simeq 67.7 \text{ mS}}, \boxed{I_C = 1.76^{mA}}$$

(22) $V_A = \infty \Rightarrow r_o = \infty$, $I_S = 8\times10^{-16}A$, $\beta = 100$

a)



$I_C = \beta I_B = 100\times10^{\mu A} \Rightarrow \boxed{I_C = 1^{mA}}$



$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 26^{mV} \times \ln\left(\frac{10^{-3}}{8\times10^{-16}}\right)$

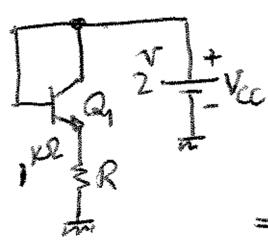
$\Rightarrow \boxed{V_{BE} \simeq 724\,mV}$

$V_{CE} = V_{CC} - RI_E \simeq V_{CC} - RI_C = 2 - 1^{k\Omega}\times 1^{mA} \Rightarrow \boxed{V_{CE} = 1^V}$

$g_m = \frac{I_C}{V_T} = \frac{1^{mA}}{26^{mV}} \Rightarrow \boxed{g_m = \frac{1}{26\Omega}}$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{1/26} \Rightarrow \boxed{r_\pi = 2.6^{k\Omega}}$

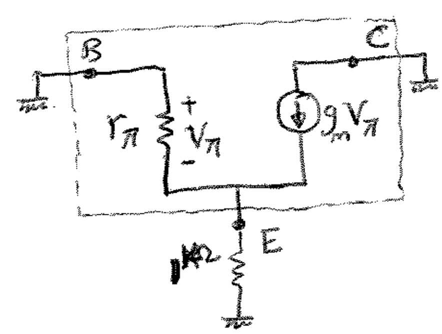
b)



Applying KVL,

$V_{CC} = V_{BE} + RI_E$

$\Rightarrow V_{CC} \simeq V_{BE} + RI_C$



$\Rightarrow V_{CC} \simeq V_{BE} + RI_S \exp\left(\frac{V_{BE}}{V_T}\right)$

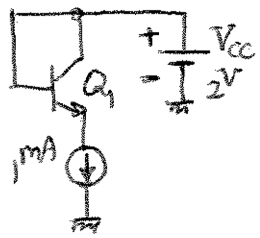
$\Rightarrow 2^V \simeq V_{BE} + 8\times10^{-13}\exp\left(\frac{V_{BE}}{26^{mV}}\right) \Rightarrow \boxed{V_{BE} \simeq 730\,mV}$

$\boxed{V_{CE} = V_{BE} = 730^{mV}}$ $I_C = 8\times10^{-16}\exp\left(\frac{730}{26}\right) \Rightarrow \boxed{I_C \simeq 1.2mA}$

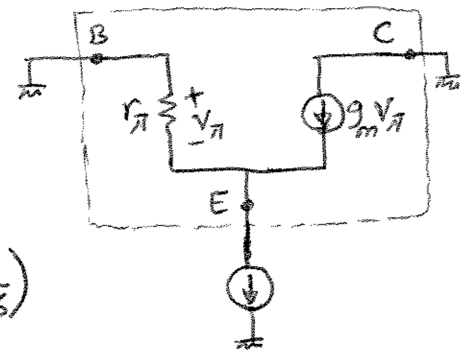
$g_m = \frac{I_C}{V_T} = \frac{1.2^{mA}}{26^{mV}} \Rightarrow \boxed{g_m \simeq 46\,mS}$ $r_\pi = \frac{\beta}{g_m} \xrightarrow{\beta=100} \boxed{r_\pi \simeq 2167^{\Omega}}$

(22) Continued ...

c)



$\boxed{I_C \simeq I_E = 1^{mA}}$



$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right)$$

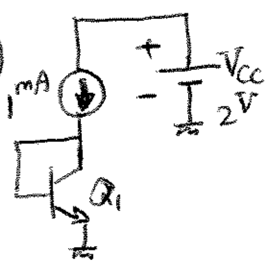
$$= 26^{mV} \ln\left(\frac{1^{mA}}{8 \times 10^{-16}}\right)$$

$$\Rightarrow \boxed{V_{BE} \simeq 724 \text{ mV}}$$

$$\boxed{V_{CE} = V_{BE} = 724 \text{ mV}} \qquad g_m = \frac{I_C}{V_T} = \frac{1^{mA}}{26^{mV}} \Rightarrow \boxed{g_m = \frac{1}{26\Omega}}$$

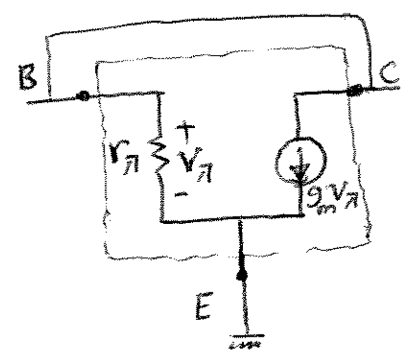
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{1/26} \Rightarrow \boxed{r_\pi \simeq 2.6 \text{ k}\Omega}$$

d)



As in part c, we have,

$\boxed{I_C \simeq 1 \text{mA}}$



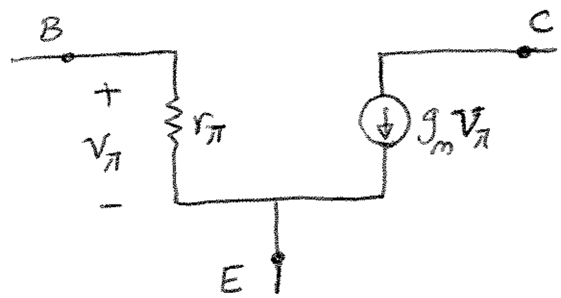
$\boxed{V_{CE} \simeq 724 \text{ mV}}$

$\boxed{g_m = \frac{1}{26\Omega}}$ $\qquad$ $\boxed{r_\pi = 2.6 \text{ k}\Omega}$

(23) $I_C = I_S \exp\left(\frac{V_{BE}}{nV_T}\right)$ $\qquad I_C = \beta I_B$

$$g_m = \frac{\partial I_C}{\partial V_{BE}} = \frac{1}{nV_T} I_S \exp\left(\frac{V_{BE}}{nV_T}\right) = \frac{I_C}{nV_T}$$

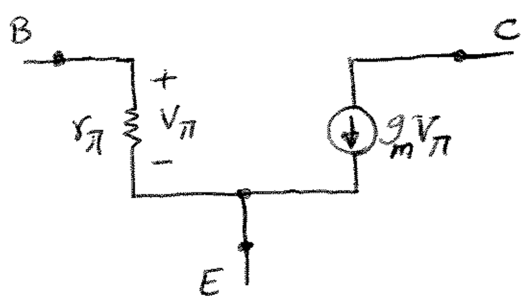
$$r_\pi = \frac{\partial V_{BE}}{\partial I_B} = \frac{\partial V_{BE}}{\frac{1}{\beta}\partial I_C} = \frac{\beta}{g_m} = \frac{n\beta V_T}{I_C}$$



(24) $I_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)$, $I_C = \alpha I_B^2 \Rightarrow \frac{\partial I_B}{\partial I_C} = \frac{1}{2\sqrt{\alpha I_C}}$

$$g_m = \frac{\partial I_C}{\partial v_{BE}} = \frac{I_S}{V_T}\exp\left(\frac{v_{BE}}{V_T}\right) = \boxed{\frac{I_C}{V_T}}$$

$$r_\pi = \frac{\partial V_{BE}}{\partial I_B} = \frac{\partial V_{BE}}{\frac{1}{2\sqrt{\alpha I_C}}\,\partial I_C} = \frac{2\sqrt{\alpha I_C}}{g_m} = \frac{2\sqrt{\alpha I_C}}{I_C/V_T} = \boxed{2V_T\sqrt{\frac{\alpha}{I_C}}}$$



(25) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$ $\quad$ V_{BE} is Constant

$$\Delta I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\frac{1}{V_A}\,\Delta V_{CE}$$

$$\Rightarrow \frac{\Delta I_C}{I_C} = \frac{\cancel{I_S \exp\left(\frac{V_{BE}}{V_T}\right)}\frac{1}{V_A}\cdot \Delta V_{CE}}{\cancel{I_S \exp\left(\frac{V_{BE}}{V_T}\right)}\left[1 + \frac{V_{CE}}{V_A}\right]} = \frac{\Delta V_{CE}}{V_A + V_{CE}}$$

$$\frac{\Delta I_C}{I_{C_{min}}} < 0.05 \Rightarrow \frac{\Delta V_{CE}}{V_A + V_{CE_{min}}} < 0.05$$

$$\Rightarrow 20\,\Delta V_{CE} < V_A + V_{CE_{min}}$$

$$\left.\begin{array}{l}\Delta V_{CE} = 2^V \\ V_{CE_{min}} = 1^V\end{array}\right\} \Rightarrow 40 < V_A + 1 \Rightarrow \boxed{V_A > 39^V}$$

26

a) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) = 5\times10^{-17} \exp\left(\frac{800^{mV}}{26^{mV}}\right) \simeq \boxed{1.15 mA}$

$V_X = V_{CC} - R_C I_C = 2.5^V - 1^{k\Omega} \times 1.15^{mA}$

$\boxed{V_X = 1.35 V}$ Transistor is in Forward Active Region

b) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$

$\Rightarrow I_C = 5\times10^{-17} \exp\left(\frac{800}{26}\right)\left[1 + \frac{V_X}{5^V}\right]$ equation $\underline{1}$

Also we know: $V_X = V_{CC} - R_C I_C \Rightarrow I_C = \frac{V_{CC} - V_X}{R_C}$ equation $\underline{2}$

equations 1, 2 $\Rightarrow \frac{V_{CC} - V_X}{R_C} = 5\times10^{-17} \exp\left(\frac{800}{26}\right)\left[1 + \frac{V_X}{5}\right]$

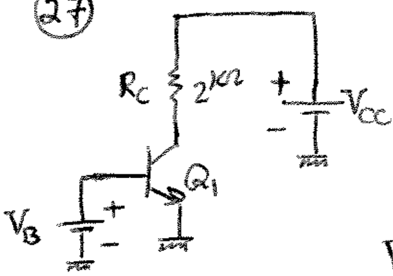
$\Rightarrow V_X + 5\times10^{-14} \exp\left(\frac{800}{26}\right)\left[1 + \frac{V_X}{5}\right] = 2.5$

$\Rightarrow 1.2306\, V_X \simeq 1.347$

$\Rightarrow \boxed{V_X \simeq 1.095 \; V} \xrightarrow{\text{equation } \underline{1}} \boxed{I_C \simeq 1.406 mA}$

Transistor is in Forward Active Region

(27)



$I_S = [illegible] \times 10^{-17} A \qquad V_A = 5^V$

Applying KVL:

$$V_{CC} = R_C I_C + V_{CE}$$

$$\Rightarrow V_{CC} = R_C I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right] + V_{CE}$$

$$\xRightarrow{V_{BE} \text{ Constant}} \boxed{\Delta V_{CC} = \left[R_C I_S \exp\left(\frac{V_{BE}}{V_T}\right)\frac{1}{V_A} + 1\right] \cdot \Delta V_{CE}} \quad \text{equation } 1$$

$$I_C = I_S e^{\frac{V_{BE}}{V_T}}\left[1 + \frac{V_{CE}}{V_A}\right] \Rightarrow \Delta I_C = I_S e^{V_{BE}/V_T} \times \frac{1}{V_A} \Delta V_{CE}$$

$$\Rightarrow \boxed{\Delta V_{CE} = \frac{1}{I_S e^{V_{BE}/V_T} \times \frac{1}{V_A}} \cdot \Delta I_C} \quad \text{equation } 2$$

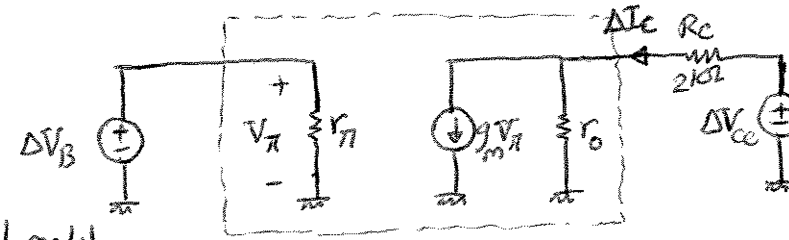
$$\text{equations } 1, 2 \Rightarrow \Delta I_C = \frac{I_S e^{\frac{V_{BE}}{V_T}} \times \frac{1}{V_A}}{1 + R_C I_S e^{\frac{V_{BE}}{V_T}} \times \frac{1}{V_A}} \cdot \Delta V_{CC}$$

$$\Rightarrow \boxed{\Delta I_C = \frac{I_S \exp\left(\frac{V_{BE}}{V_T}\right)}{V_A + R_C I_S \exp\left(\frac{V_{BE}}{V_T}\right)} \cdot \Delta V_{CC}} = \frac{1}{r_o + R_C} \cdot \Delta V_{CC}$$

Could also be obtained using small signal model

$$\Rightarrow \Delta I_C = \frac{2.31 \times 10^{-4}}{5 + 0.4613} \times 0.5 \Rightarrow \Delta I_C \simeq 0.021 mA$$

28



We use small signal model,

Assuming that the required ΔV_B is small enough.

Applying superposition,

$$\Delta I_C = \left(\frac{1}{r_o + R_C}\right)\Delta V_{CC} + \left(\frac{g_m r_o}{r_o + R_C}\right)\Delta V_B$$

$$\Delta I_C = 0 \implies \boxed{\Delta V_B = -\frac{1}{g_m r_o}\Delta V_{CC}}$$

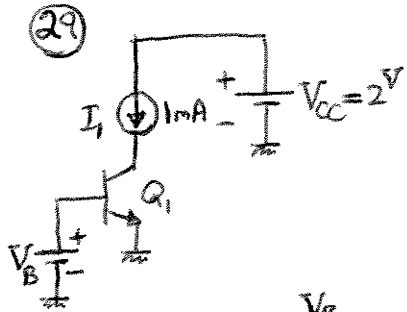
$$\Delta V_B = -\frac{1}{\frac{\cancel{I_C}}{V_T}\cdot\frac{V_A}{\cancel{I_C}}}\Delta V_{CC} \implies \Delta V_B = -\frac{V_T}{V_A}\Delta V_{CC}$$

$$\implies \Delta V_B = -\frac{26\times10^{-3}}{5}\times(3-2.5)$$

$$\implies \boxed{\Delta V_B = -2.6\ mV}$$

which is small enough for small signal model

29



$I_S = 3\times10^{-17}A$

a) $I_C = I_S e^{\frac{V_B}{V_T}} \Longrightarrow V_B = V_T \ln\left(\frac{I_C}{I_S}\right) = 26^{mV} \ln\left(\frac{10^{-3}}{3\times10^{-17}}\right)$

$$\Longrightarrow \boxed{V_B \simeq 809.6 \text{ mV}}$$

b) $I_C = I_S\, e^{\frac{V_B}{V_T}}\left(1 + \frac{V_{CE}}{V_A}\right)$

$10^{-3} = 3\times10^{-17}\, e^{V_B/V_T}\left(1 + \frac{1.5}{5}\right) \Rightarrow e^{V_B/V_T} = \frac{10^{14}}{3.9}$

$\Longrightarrow V_B = 26^{mV} \ln\left(\frac{10^{14}}{3.9}\right) \Longrightarrow \boxed{V_B \simeq 802.8 \text{ mV}}$

30 $I_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$

$$r_o^{-1} = \frac{dI_C}{dV_{CE}} = I_S \exp\left(\frac{v_{BE}}{V_T}\right) \cdot \frac{1}{V_A} \simeq \frac{I_C}{V_A} \Rightarrow r_o \simeq \frac{V_A}{I_C}$$

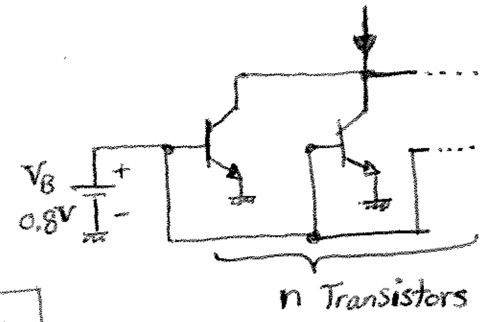
$$r_o > 10^{k\Omega} \Rightarrow \frac{V_A}{I_C} > 10^{k\Omega}$$

$$\Rightarrow V_A > 10^{k\Omega} \times 2^{mA}$$

$$\Rightarrow \boxed{V_A > 20\,V}$$

(31) $I_S = 5\times10^{-16}\,A$, $V_A = 8\,V$

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1+\frac{V_{CE}}{V_A}\right]$$

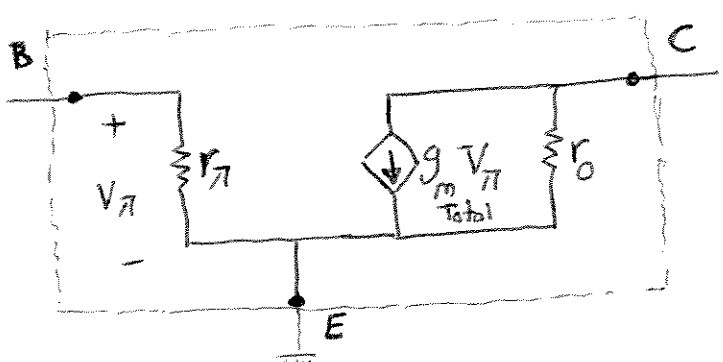


$$\boxed{g_{m_{Total}} = \frac{I_{C_{Total}}}{V_T} \simeq \frac{n I_S \exp\left(\frac{V_{BE}}{V_T}\right)}{V_T}}$$

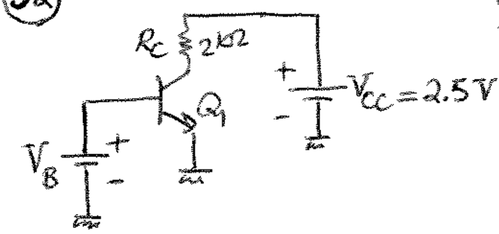
$$\Rightarrow g_{m_{Total}} \simeq \frac{n\times5\times10^{-16}\exp\left(\frac{800}{26}\right)}{26\,mV} \Rightarrow \boxed{g_{m_{Total}} \simeq 0.4435\,n}$$

$$r_o^{-1} = \frac{\partial I_{C_{Total}}}{\partial V_{CE}} = \frac{\partial}{\partial V_{CE}}\left[n I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1+\frac{V_{CE}}{V_A}\right]\right]$$

$$\Rightarrow \boxed{r_o = \frac{V_A}{n I_S \exp\left(\frac{V_{BE}}{V_T}\right)}} \qquad \boxed{r_\pi = \frac{\beta}{g_{m_{Total}}} \overset{\beta=100}{\simeq} \frac{225.5}{n}}$$



(32)



$I_S = 6 \times 10^{-16} A$, $V_A = \infty$

a) Q_1 at the edge of the active region $\Rightarrow V_{CE} = V_{BE}$

applying KVL, $V_{CC} = R_C I_C + V_{CE}$

$$\xrightarrow[\text{edge}]{\text{at the}} V_{CC} = R_C I_C + V_{BE} \Rightarrow R_C I_S e^{\frac{V_{BE}}{V_T}} + V_{BE} = V_{CC}$$

$$\Rightarrow 2^{k\Omega} \times 6 \times 10^{-16} A \; e^{V_B/26mV} + V_B = 2.5 \Rightarrow \boxed{V_B \simeq 728.5^{mV}}$$

b) Applying KVL, $V_{CC} = R_C I_C + V_{CE}$

Soft Saturation $\Rightarrow V_{CE} = V_{BE} - 0.2^V$

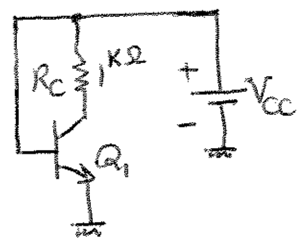
$$\Rightarrow V_{CC} = R_C I_C + V_{BE} - 0.2^V$$

$$\Rightarrow 2^{k\Omega} \times 6 \times 10^{-16} e^{V_B/26mV} + V_B = 2.7V$$

$$\Rightarrow \boxed{V_B \simeq 731.5 mV}$$

So V_B Can increase by 3mV

(33)



$I_S = 7\times10^{-16} A$, $V_A = \infty$

$\Downarrow$

$r_o = \infty$

Applying KVL,

$$V_{CC} = R_C I_C + V_{CE} \xrightarrow{V_{CE} = V_{BE} - 0.2^V} R_C I_C + V_{BE} - 0.2^V = V_{CC}$$

$$\Rightarrow R_C I_S e^{\frac{V_{BE}}{V_T}} + V_{BE} - 0.2^V = V_{CC}$$

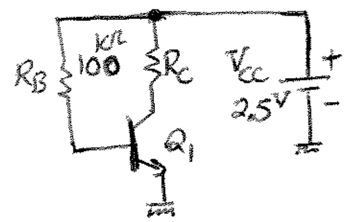
$$\xRightarrow{V_{BE} = V_{CC}} R_C I_S e^{\frac{V_{CC}}{V_T}} + \cancel{V_{CC}} - 0.2 = \cancel{V_{CC}}$$

$$\Rightarrow R_C I_S e^{V_{CC}/V_T} = 0.2^V$$

$$\Rightarrow 1^{K\Omega} \times 7\times10^{-16} e^{V_{CC}/26^{mV}} = 0.2^V$$

$$\Rightarrow \boxed{V_{CC} \simeq 686 \text{ mV}}$$

(34) $I_S = 2\times10^{-17}A$, $V_A = \infty$ $\beta = 100$



$$\begin{cases} V_{CC} = R_C I_C + V_{CE}\ , \quad V_{CE} = V_{BE} - 0.2^V \\ V_{CC} = R_B I_B + V_{BE} \Rightarrow V_{CC} = R_B \frac{I_C}{\beta} + V_{BE} \end{cases}$$

$$R_B \frac{I_C}{\beta} + V_{BE} = V_{CC} \Rightarrow \frac{R_B}{\beta} I_S e^{\frac{V_{BE}}{V_T}} + V_{BE} = V_{CC}$$

$$\Rightarrow \frac{100^{k\Omega}}{100} \times 2\times10^{-17} e^{\frac{V_{BE}}{26^{mV}}} + V_{BE} = 2.5^V$$

$$\Rightarrow \boxed{V_{BE} \simeq 833.5\ mV}$$

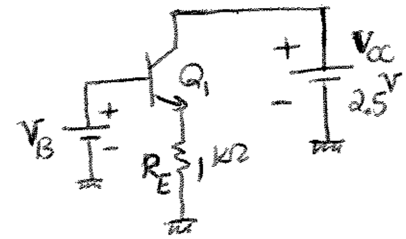
Soft Saturation $\Rightarrow V_{CE} = V_{BE} - 0.2^V \Rightarrow \boxed{V_{CE} = 692.5^{mV}}$

$$V_{CC} = R_C I_C + V_{CE} \Rightarrow R_C = \frac{V_{CC} - V_{CE}}{I_C}$$

$$\Rightarrow R_C = \frac{V_{CC} - V_{CE}}{I_S \exp\left(\frac{V_{BE}}{V_T}\right)} = \frac{2.5 - 0.6925}{2\times10^{-17} \exp\left(\frac{892.5}{26}\right)}$$

$$\Rightarrow \boxed{R_C \simeq 112\ \Omega}$$

(35) $I_S = 5\times10^{-16}A$, $V_A = \infty \Rightarrow r_o = \infty$



Soft Saturation $\Rightarrow V_{BC} = 200\,mV$

$$\Rightarrow V_B = V_C + 0.2^V \Rightarrow \boxed{V_B = 2.7\,V}$$

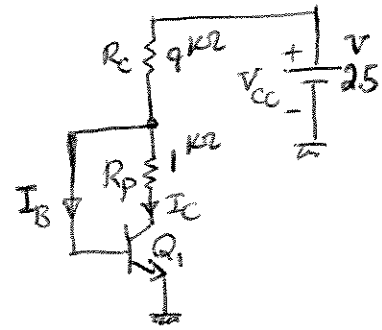
Applying KVL $\Rightarrow V_B = V_{BE} + R_E I_E \xrightarrow{I_E \simeq I_C} V_B = V_{BE} + R_E I_C$

$$\Rightarrow V_{BE} + 1^{k\Omega} \times I_S\, e^{\frac{V_{BE}}{V_T}} = 2.7^V$$

$$\Rightarrow V_{BE} + 5\times10^{-13}\, e^{\frac{V_{BE}}{V_T}} = 2.7^V \Rightarrow \boxed{V_{BE} \simeq 754\,mV}$$

$$I_C = I_S\, e^{V_{BE}/V_T} = 5\times10^{-16}\, e^{\frac{0.754}{0.026}} \Rightarrow \boxed{I_C \simeq 2\,mA}$$

(36) $\beta = 100, \quad V_A = \infty \Rightarrow r_o = \infty$



$$V_{BC} = 0.2\,V \Rightarrow R_P I_C = 0.2\,V$$

$$\Rightarrow \boxed{I_C = \frac{0.2\,V}{R_P}}$$

$$V_{BE} = V_{CC} - R_C(I_B + I_C)$$

$$\xrightarrow{\beta=100} V_{BE} = V_{CC} - \frac{\beta+1}{\beta} R_C I_C \Rightarrow \boxed{V_{BE} = V_{CC} - \frac{\beta+1}{\beta}\frac{R_C}{R_P} \times 0.2}$$

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \Rightarrow I_S = I_C \exp\left(-\frac{V_{BE}}{V_T}\right)$$

$$\Rightarrow \boxed{I_S = \frac{0.2}{R_P} \exp\left[\frac{0.2}{V_T} \cdot \frac{\beta+1}{\beta} \cdot \frac{R_C}{R_P} - \frac{V_{CC}}{V_T}\right]}$$

$$\xrightarrow{\beta=100} \boxed{I_S \simeq \frac{0.2}{R_P} \exp\left[\frac{0.2}{V_T}\frac{R_C}{R_P} - \frac{V_{CC}}{V_T}\right]}$$

$$\Rightarrow \boxed{I_S \simeq 4.06 \times 10^{-16}\,A}$$

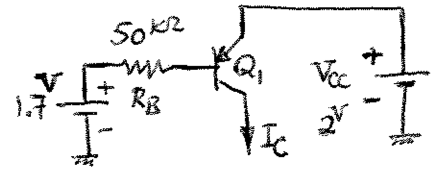
37 $I_{S_1} = 3I_{S_2} = 6\times10^{-16}A$

$$I_1 = I_{S_1}\exp\left(\frac{V_{EB_1}}{V_T}\right) = 6\times10^{-16}\exp\left(\frac{300}{26}\right) \Rightarrow I_1 \simeq 6.155\times10^{-11}\,A$$

$$I_2 = I_{S_2}\exp\left(\frac{V_{EB_2}}{V_T}\right) = 2\times10^{-16}\exp\left(\frac{820}{26}\right) \Rightarrow I_2 \simeq 10\,mA$$

$$I_X = I_1 + I_2 \Rightarrow \boxed{I_X \simeq 10\,mA}$$

(38) $I_S = 2\times10^{-17}A$ $\qquad \beta = 100$



Applying KVL,

$$V_{CC} = V_{EB} + R_B I_B + 1.7^V$$

$$\Rightarrow 2^V = V_{EB} + R_B \frac{I_C}{\beta} + 1.7^V$$

$$\Rightarrow 0.3^V = V_{EB} + \frac{50^{k\Omega}}{100} I_C$$

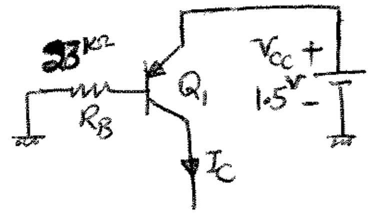
$$\Rightarrow 0.3^V = V_{EB} + 500 \times I_S \, e^{\frac{V_{EB}}{V_T}}$$

$$\Rightarrow 0.3^V = V_{EB} + 10^{-14} e^{V_{EB}/26mV} \Rightarrow \boxed{V_{EB} \simeq 0.3\ V}$$

$$I_C = I_S \, e^{\frac{V_{EB}}{V_T}} \Rightarrow I_C = 2\times10^{-17} e^{\frac{300}{26}}$$

$$\Rightarrow \boxed{I_C \simeq 2.05 \times 10^{-12}\ A}$$

(39) $I_C = 3\,mA$, $\beta = 100$, $R_B = 23\,k\Omega$



Applying KVL,

$$V_{CC} = V_{EB} + R_B I_B \implies V_{CC} = V_{EB} + R_B \frac{I_C}{\beta}$$

$$\implies -I_C \frac{R_B}{\beta} + V_{CC} = V_{EB}$$

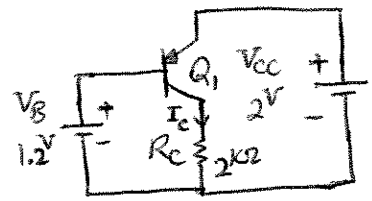
$$I_C = I_S e^{V_{EB}/V_T}$$

$$\implies I_S = I_C e^{-\frac{V_{EB}}{V_T}}$$

$$\implies \boxed{I_S = I_C e^{\frac{1}{V_T}\left(\frac{R_B I_C}{\beta} - V_{CC}\right)}}$$

$$\implies \boxed{I_S \simeq 8.85 \times 10^{-17} A}$$

(40) At the edge of active $\Rightarrow V_{BC} = 0$



$$I_C = \frac{V_B - V_{BC}}{R_C} = \frac{V_B}{R_C}$$

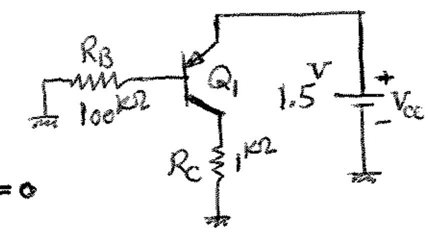
$$\Rightarrow I_C = \frac{1.2^V}{2^{k\Omega}} \Rightarrow \boxed{I_C \simeq 0.6\ mA}$$

$$I_C = I_S \exp\left(\frac{V_{EB}}{V_T}\right) \Rightarrow I_S = I_C \exp\left(-\frac{V_{EB}}{V_T}\right)$$

$$\Rightarrow I_S = 0.6 \times 10^{-3} \exp\left(-\frac{800}{26}\right)$$

$$\Rightarrow \boxed{I_S \simeq 2.6 \times 10^{-17}\ A}$$

41 $I_S = 8\times10^{-16}A$



At the edge of the active mode $\Rightarrow V_{BC}=0$

$$\Rightarrow V_{EB} = V_{EC}$$

Applying KVL,

$$V_{CC} = V_{EC} + R_C I_C \xrightarrow{V_{EB}=V_{EC}} V_{CC} = V_{EB} + R_C I_C$$

$$\Rightarrow V_{EB} + R_C I_S e^{\frac{V_{EB}}{V_T}} = V_{CC}$$

$$\Rightarrow V_{EB} + 8\times10^{-13} e^{\frac{V_{EB}}{26^{mV}}} = 1.5 \Rightarrow \boxed{V_{EB} \simeq 718\,mV}$$

$$I_C = I_S e^{V_{EB}/V_T} \Rightarrow \boxed{I_C \simeq 0.788\,mA}$$

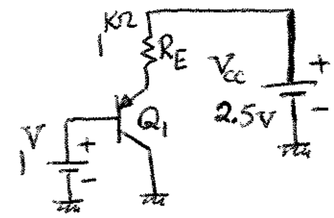
Applying KVL,

$$V_{BC}=0 \Rightarrow V_B = V_C \Rightarrow R_B I_B = R_C I_C$$

$$\Rightarrow R_B \frac{\cancel{I_C}}{\beta} = R_C \cancel{I_C} \Rightarrow \boxed{\beta = \frac{R_B}{R_C}}$$

$$\Rightarrow \beta = \frac{100^{k\Omega}}{1^{k\Omega}} \Rightarrow \boxed{\beta = 100}$$

(42) $I_S = 3\times10^{-17}A$



Applying KVL,

$$V_{CC} = R_E I_E + V_{EB} + 1^V \xrightarrow{I_E \simeq I_C} V_{CC} = R_E I_C + V_{EB} + 1^V$$

$$\Rightarrow 2.5 = 1^{K\Omega} \times 3\times10^{-17} e^{V_{EB}/26^{mV}} + V_{EB} + 1^V$$

$$\Rightarrow V_{EB} + 3\times10^{-14} e^{V_{EB}/26^{mV}} = 1.5^V$$

$$\Rightarrow \boxed{V_{EB} \simeq 800.5 \text{ mV}}$$

$$I_C = I_S e^{\frac{V_{EB}}{V_T}} = 3\times10^{-17} e^{\frac{800.5}{26}} \Rightarrow \boxed{I_C \simeq 0.705 \text{ mA}}$$

(43) $I_S = 3\times10^{-17}A$, $\beta = 100$, $V_A = \infty \Rightarrow \boxed{r_o = \infty}$

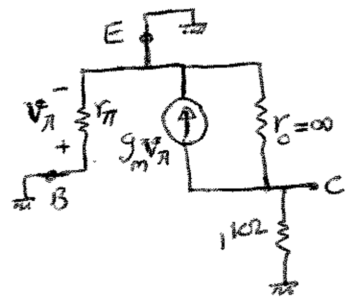
a) $V_{EB} = 2.5 - 1.7 = 0.8\ V$

$$I_C = I_S\, exp\left(\frac{V_{EB}}{V_T}\right) = 3\times10^{-17} exp\left(\frac{800}{26}\right) \Rightarrow \boxed{I_C \simeq 0.692^{mA}}$$

$$V_{EC} = V_{CC} - R_C I_C = 2.5 - 1^{k\Omega} \times 0.692^{mA} \Rightarrow \boxed{V_{EC} \simeq 1.808^{V}}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.692^{mA}}{26^{mV}} \Rightarrow \boxed{g_m \simeq 26.6\,mS}$$

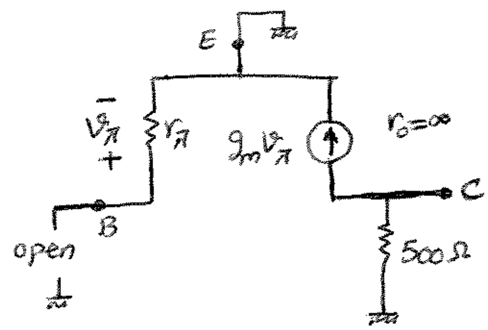
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{26.6\times10^{-3}} \Rightarrow \boxed{r_\pi \simeq 3.76\ k\Omega}$$



b) $V_{EB} = V_T \ln\left(\frac{I_C}{I_S}\right) \Rightarrow V_{EB} = V_T \ln\left(\frac{\beta I_B}{I_S}\right)$

$$\Rightarrow V_{EB} = 26^{mV} \times \ln\left(\frac{100\times20\times10^{-6}}{3\times10^{-17}}\right)$$

$$\Rightarrow \boxed{V_{EB} \simeq 827.6\,mV}$$



$$I_C = \beta I_B \Rightarrow \boxed{I_C = 2mA}$$

$$V_{EC} = V_{CC} - R_C I_C = 2.5 - 0.5^{k\Omega} \times 2^{mA} \Rightarrow \boxed{V_{EC} = 1.5\ V}$$

$$g_m = \frac{I_C}{V_T} = \frac{2^{mA}}{26^{mV}} \Rightarrow \boxed{g_m \simeq 77\,mS} \qquad r_\pi = \frac{\beta}{g_m} \Rightarrow \boxed{r_\pi \simeq 1.3\,k\Omega}$$

(43) Continued .....

c) Applying KVL,

$$V_{cc} = V_{EB} + (I_C + I_B) \times 2^{k\Omega} \simeq V_{EB} + 2^{k\Omega} \times I_C$$

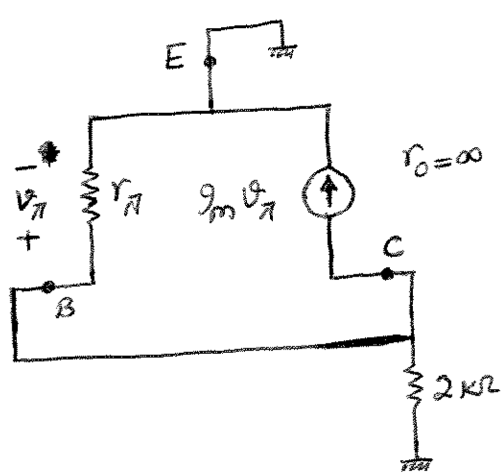
$$\Rightarrow \quad V_{EB} + 2^{k\Omega} \times I_S e^{\frac{V_{EB}}{V_T}} = V_{cc}$$

$$\Rightarrow \quad V_{EB} + 6 \times 10^{-14} e^{V_{EB}/26^{mV}} = 2.5^{V} \quad \Rightarrow \quad \boxed{V_{EB} \simeq 805 mV}$$

$$I_C \simeq \frac{V_{cc} - V_{EB}}{R} = \frac{2.5 - 0.805}{2^{k\Omega}} \quad \Rightarrow \quad \boxed{I_C \simeq 847.5 \mu A}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.8475 \times 10^{-3}}{0.026} \quad \Rightarrow \quad \boxed{g_m \simeq 32.6 \text{ mS}}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{32.6 \times 10^{-3}} \quad \Rightarrow \quad \boxed{r_\pi \simeq 3068 \ \Omega}$$

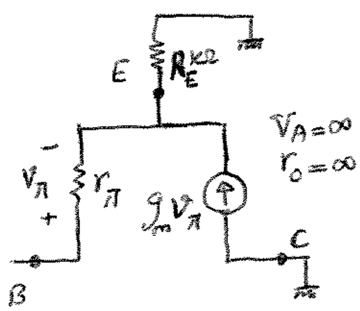


(44) $I_S = 3\times10^{-17} A$, $\beta = 100$, $V_A = \infty \Rightarrow \boxed{r_o = \infty}$

a) Applying KVL,

$$V_{CC} = R_E I_E + V_{EC} \xrightarrow{I_E \simeq I_C} V_{EC} = V_{CC} - R_E I_C \qquad \boxed{I_C = \beta I_B = 0.2^{mA}}$$

$$\Rightarrow \boxed{V_{EC} = V_{CC} - \beta R_E I_B} \xrightarrow[\substack{R_E = 2^{k\Omega} \\ V_{CC} = 2.5^{V}}]{I_B = 2^{\mu A}} \boxed{V_{EC} = 2.1\ V}$$



Transistor is in Forward Active Region

b) Applying KVL,

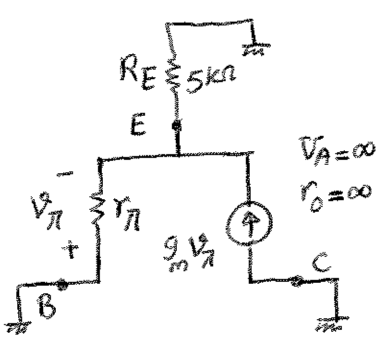
Forward Active Region

$$V_{CC} = R_E I_E + V_{EB} \Rightarrow V_{CC} \simeq R_E I_C + V_{EB}$$

$$\Rightarrow 2.5 = 5^{k\Omega} \times 3\times10^{-17} e^{V_{EB}/V_T} + V_{EB}$$

$$\Rightarrow \boxed{V_{EB} \simeq 781.9\ mV}$$

From Circuit $V_{EC} = V_{EB}$



$$g_m = \frac{I_C}{V_T} = \frac{3\times10^{-17} e^{\frac{781.9}{26}}}{0.026} \Rightarrow \boxed{g_m \simeq 0.0133\ S}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.0133} \Rightarrow \boxed{r_\pi \simeq 7538\ \Omega}$$

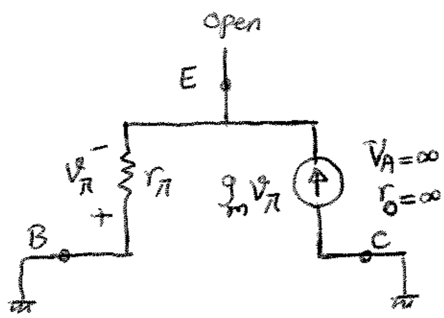
(44) Continued .....

c) $I_E = 0.5mA \Longrightarrow \boxed{I_C \simeq 0.5mA}$

$$I_C = I_S e^{V_{EB}/V_T} \Longrightarrow 0.5mA = 3\times10^{-17} e^{\frac{V_{EB}}{26mV}} \Longrightarrow \boxed{V_{EB} \simeq 791.6^{mV}}$$

In the given Circuit : $V_{EC} = V_{EB}$

$$g_m = \frac{I_C}{V_T} = \frac{0.5^{mA}}{26^{mV}} \Rightarrow \boxed{g_m \simeq 19.2mS}$$

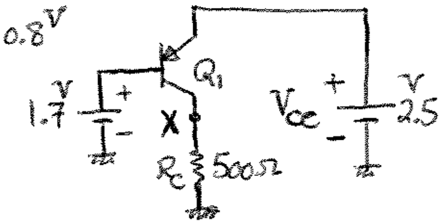


$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.0192} \Rightarrow \boxed{r_\pi \simeq 5.2k\Omega}$$

Forward Active Region

(45) $I_S = 5 \times 10^{-17} A$

$V_{EB} = 0.8^V$



a) $V_A = \infty \Rightarrow r_o = \infty$

$$I_C = I_S e^{V_{EB}/V_T} \Rightarrow I_C = 5\times10^{-17} e^{\frac{800}{26}} \Rightarrow \boxed{I_C \simeq 1.15 mA}$$

$$V_X = R_C I_C = 0.5^{k\Omega} \times 1.15^{mA} \Rightarrow \boxed{V_X \simeq 0.58^V}$$

b) $V_A = 6V$

$$I_C = I_S e^{\frac{V_{EB}}{V_T}}\left(1 + \frac{V_{EC}}{V_A}\right) \quad , \quad V_{EC} = V_{CC} - R_C I_C$$

$$\Rightarrow I_C = I_S e^{V_{EB}/V_T}\left(1 + \frac{V_{CC} - R_C I_C}{V_A}\right)$$

$$\Rightarrow I_C = I_S e^{V_{EB}/V_T}\left(1 + \frac{V_{CC}}{V_A}\right) - \frac{I_S R_C}{V_A} e^{\frac{V_{EB}}{V_T}} I_C$$

$$\Rightarrow I_C = \frac{I_S e^{V_{EB}/V_T}\left(1 + \frac{V_{CC}}{V_A}\right)}{1 + \frac{I_S R_C}{V_A} e^{V_{EB}/V_T}} = \frac{5\times10^{-17} e^{\frac{800}{26}}\left(1 + \frac{2.5}{6}\right)}{1 + \frac{5\times10^{-17} \times 0.5^{k\Omega}}{6} e^{800/26}}$$

0.745 V

$$\Rightarrow \boxed{I_C = 1.49 mA} \qquad V_X = R_C I_C = 500 \times 1.49 \times 10^{-3} \Rightarrow \boxed{V_X = .745^V}$$

(46) $r_o = 60\,k\Omega$, $I_C = 2\,mA$

$$r_o = \frac{V_A}{I_C} \Rightarrow 60 \times 10^{3}\,\Omega = \frac{V_A}{2 \times 10^{-3}\,A} \Rightarrow \boxed{V_A = 120\,V}$$

(47) $r_o = 60\ k\Omega$, $\quad I_C = 1\ mA$

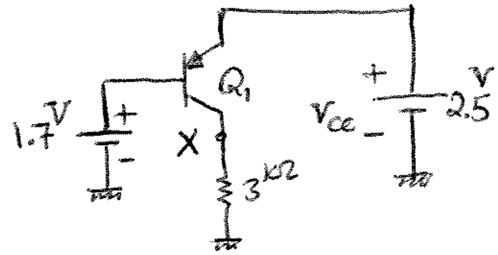
$$r_o = \frac{V_A}{I_C} \Rightarrow \boxed{V_A = r_o \cdot I_C} \Rightarrow \boxed{V_A \propto I_C}$$

$$\Rightarrow V_A = 60^{k\Omega} \times 1^{mA}$$

$$\Rightarrow \boxed{V_A = 60\ V}$$

V_A is half the value in problem 46 as V_A is proportional to I_C.

48 $V_A = 5\,v$



a) At the edge of active mode

$$\Rightarrow \boxed{V_X = V_B = 1.7\,v}$$

$$I_C = \frac{V_X}{R_C} = \frac{1.7\,v}{3\,k\Omega} \Rightarrow \boxed{I_C \simeq 0.567\,mA}$$

$$I_C = I_S\, e^{\frac{V_{EB}}{V_T}}\left(1 + \frac{V_{EC}}{V_A}\right) \Rightarrow \boxed{I_S = \frac{I_C\, e^{-\frac{V_{EB}}{V_T}}}{1 + \frac{V_{EC}}{V_A}}}$$

$$I_S = \frac{0.567 \times 10^{-3} \times e^{-\frac{800}{26}}}{1 + \frac{2.5 - 1.7}{5}} \Rightarrow \boxed{I_S \simeq 2.118 \times 10^{-17}\,A}$$

b) $V_A = \infty$

$$I_C = I_S\, e^{\frac{V_{EB}}{V_T}} \Rightarrow \boxed{I_S = I_C\, e^{-\frac{V_{EB}}{V_T}}}$$

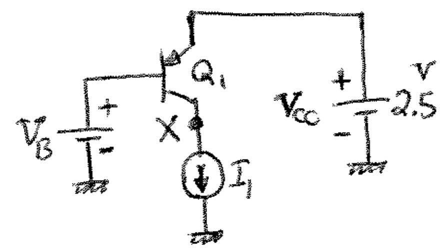
$$I_S = 0.567 \times 10^{-3}\, e^{-\frac{800}{26}} \Rightarrow \boxed{I_S \simeq 2.457 \times 10^{-17}\,A}$$

I_S increases

(49) The direction of Currents in large-signal model shows how Currents would flow when The pnp Transistor is properly DC biased.

The direction of currents in small-signal model shows how the ac Currents flow when ac voltage across Base-Emitter increases.

(50) $I_S = 6\times10^{-16}\,A$, $\quad V_A = 5\,V$, $\quad I_1 = 2\,mA$



a) $$I_C = I_S e^{V_{EB}/V_T}\left(1+\frac{V_{EC}}{V_A}\right)$$

$$\Rightarrow V_{EB} = V_T \ln\left(\frac{I_C}{I_S\left(1+\frac{V_{EC}}{V_A}\right)}\right)$$

$$\xrightarrow[V_{EB}=V_{CC}-V_B]{V_{EC}=V_{CC}-V_X} \boxed{V_B = V_{CC} - V_T \ln\left(\frac{I_C}{I_S\left(1+\frac{V_{CC}-V_X}{V_A}\right)}\right)}$$

$$\Rightarrow V_B = 2.5 - 0.026\ln\left(\frac{2\times10^{-3}}{6\times10^{-16}\left(1+\frac{2.5-1}{5}\right)}\right) \Rightarrow \boxed{V_B \simeq 1.757\,V}$$

b) $$I_C = I_S e^{V_{EB}/V_T}\left(1+\frac{V_{EC}}{V_A}\right) \Rightarrow 1+\frac{V_{EC}}{V_A} = \frac{I_C}{I_S} e^{-V_{EB}/V_T}$$

$$\xrightarrow[V_{EB}=V_{CC}-V_B]{V_{EC}=V_{CC}-V_X} \boxed{V_X = V_{CC} - V_A\left(\frac{I_C}{I_S}e^{-V_{EB}/V_T} - 1\right)}$$

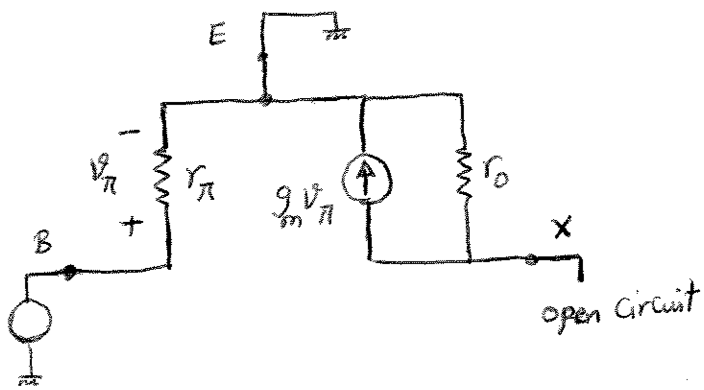
$$\Delta V_X \simeq \frac{dV_X}{dV_{EB}}\Delta V_{EB} \Rightarrow \Delta V_X \simeq \frac{V_A}{V_T}\cdot\frac{I_C}{I_S} e^{-V_{EB}/V_T}\Delta V_{EB}$$

$$\xrightarrow{\Delta V_{EB} = -\Delta V_B} \boxed{\Delta V_X \simeq -\frac{V_A}{V_T}\cdot\frac{I_C}{I_S} e^{-V_{EB}/V_T}\Delta V_B}$$

$$\Rightarrow \Delta V_X \simeq -\frac{5}{0.026}\times\frac{2\times10^{-3}}{6\times10^{-16}}\exp\left(-\frac{2.5-1.757}{0.026}\right)\times0.1\times10^{-3} \Rightarrow \boxed{\Delta V_X \simeq -24.9\,mV}$$

(50) Continued .....

c)



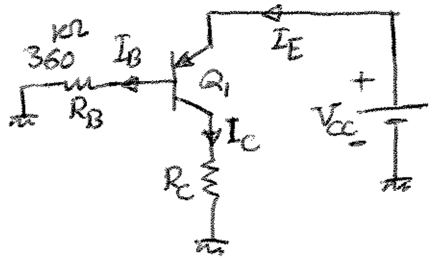
$$r_o = \frac{V_A}{I_C} = \frac{5^V}{2mA} \Rightarrow \boxed{r_o \simeq 2.5\ k\Omega}$$

$$g_m = \frac{I_C}{V_T} = \frac{2^{mA}}{0.026\ V} \Rightarrow \boxed{g_m \simeq 76.9\ mS}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{\frac{2}{26}} \Rightarrow \boxed{r_\pi = 1.3\ k\Omega}$$

(51) $\beta = 100,\quad V_A = \infty \Rightarrow r_o = \infty$

$R_B = 360\,k\Omega$



a) given: $V_C = V_B + 0.2^V$

$$\Rightarrow R_C I_C = R_B I_B + 0.2^V$$

$$\Rightarrow R_C I_C = R_B \frac{I_C}{\beta} + 0.2^V \Rightarrow \boxed{I_C = \frac{0.2^V}{R_C - \frac{R_B}{\beta}}} \Rightarrow \boxed{I_C = 0.5^{mA}}$$

$$I_C = I_S e^{+\frac{V_{EB}}{V_T}} \Rightarrow I_S = I_C e^{-\frac{V_{EB}}{V_T}} \Rightarrow I_S = I_C e^{-\left(\frac{V_{CC} - R_B I_B}{V_T}\right)}$$

$$\Rightarrow \boxed{I_S = \left(\frac{0.2}{R_C - \frac{R_B}{\beta}}\right) \exp\left[-\frac{1}{V_T}\left(V_{CC} - R_B \times \frac{0.2^V}{\beta\left(R_C - \frac{R_B}{\beta}\right)}\right)\right]}$$

$$\Rightarrow \boxed{I_S \simeq 10^{-15} A = 1\ fA}$$

b) $g_m = \frac{I_C}{V_T}$

$$\Rightarrow \boxed{g_m = \frac{0.2^V}{V_T\left(R_C - \frac{R_B}{\beta}\right)}} \Rightarrow \boxed{g_m \simeq 19.23\ mS}$$

(52) $I_S = 5\times10^{-16}A$, $\beta = 100$, $V_A = \infty \Rightarrow r_o = \infty$

a) $V_{EB} = 0 \Rightarrow Q_1$ is off $\quad I_C \cong 0$

b) $I_B = 0 \Rightarrow Q_1$ is off

c) Applying KVL : $V_{CC} \cong V_{EB} + 1^{k\Omega} \times I_C$

$$\Rightarrow V_{EB} + 1^{k\Omega} \times I_S e^{\frac{V_{EB}}{V_T}} \simeq V_{CC} \Rightarrow V_{EB} + 5\times10^{-13} e^{\frac{V_{EB}}{26^{mV}}} \simeq 2.5^V$$

$$\Rightarrow \boxed{V_{EB} \simeq 751\ mV} \qquad I_C = 5\times10^{-16} e^{\frac{0.751}{0.026}} \Rightarrow \boxed{I_C \simeq 1.8mA}$$

with this current, Transistor is Saturated. Note $V_B < V_C$ Always

(d) $V_{BC} = 0 \Rightarrow$ Transistor is at the edge of Saturation

e) $I_C \cong 0.5mA \Rightarrow V_{EB} = V_T \ln\left(\frac{I_C}{I_S}\right) = 26^{mV} \ln\left(\frac{0.5mA}{5\times10^{-16}}\right)$

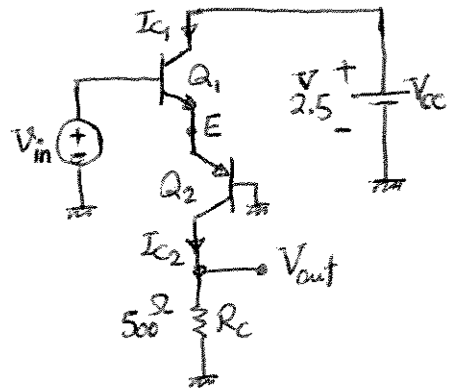
$$\Rightarrow \boxed{V_{EB} \simeq 718\ mV}$$

$V_{Collector} = 500^{\Omega} \times I_C \Rightarrow \boxed{V_C = 0.25^V}$

As $V_B = 0$, $V_C = 0.25^V \Rightarrow$ Transistor is soft saturated

(53) $I_{S_1} = 3I_{S_2} = 5\times10^{-16}A$, $\beta_1=100$, $\beta_2=50$, $V_A=\infty \Rightarrow r_o=\infty$

a) $V_{B_2}=0^V \xrightarrow[\text{Bias by } 200^{mV}_{max}]{BC\ Q_2\ \text{Forward}} V_{C_{2\,max}} = 0.2\ V$



$$\Rightarrow I_{C_{2\,max}} = \frac{V_{C2max}}{R_C} = \frac{0.2^V}{500^\Omega}$$

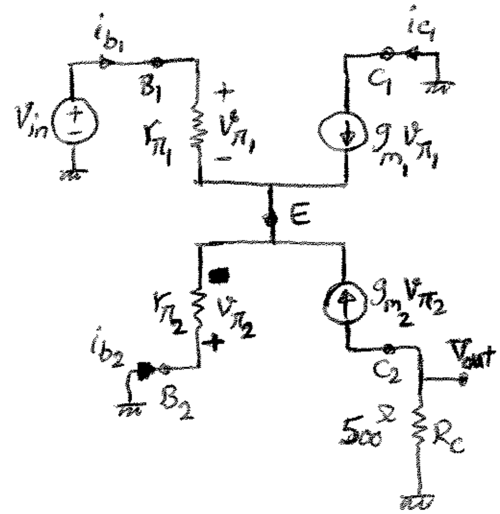
$$\Rightarrow \boxed{I_{C_{2\,max}} = 0.4\ mA}$$

As shown $I_{C_1} = I_{C_2}$

$$V_{in_{max}} = V_{BE_{1\,max}} + V_{EB_{2\,max}} = V_T \ln\frac{I_{C_1max}}{I_{S_1}} + V_T \ln\frac{I_{C_2max}}{I_{S_2}}$$

$$\Rightarrow V_{in_{max}} = 26^{mV}\cdot\left[\ln\frac{0.4\times10^{-3}}{5\times10^{-16}} + \ln\frac{0.4\times10^{-3}}{\frac{5}{3}\times10^{-16}}\right] \Rightarrow \boxed{V_{in_{max}} \simeq 1.454^V}$$

b) $g_{m_1} = \frac{I_{C_1}}{V_T} = \frac{0.4^{mA}}{26\ mV}$



$g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{0.4^{mA}}{26^{mV}}$

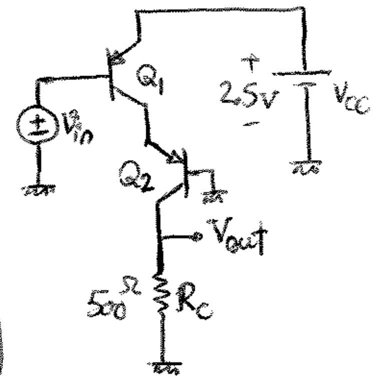
$$\Rightarrow \boxed{g_{m_1} = g_{m_2} \simeq 15.4\ mS}$$

$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = \frac{100}{\frac{0.4}{26}} \Rightarrow \boxed{r_{\pi_1} = 6.5\ k\Omega}$

$r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = \frac{50}{0.4/26} \Rightarrow \boxed{r_{\pi_2} = 3.25\ k\Omega}$

$V_A = \infty \Rightarrow \boxed{r_o = \infty}$

(54) $I_{S_1} = 3I_{S_2} = 5\times10^{-16}\,A$, $\beta_1 = 100$, $\beta_2 = 50$, $V_A = \infty$



a) $V_{B_2} = \phi \xrightarrow[\text{Forward biased by } 200mV]{Q_2 \text{ Base-Collector}} V_{C_2} = 0.2^V$

$$\Rightarrow I_{C_{2\,max}} = \frac{V_{C_2 max}}{R_C} = \frac{0.2^V}{500^\Omega} \Rightarrow \boxed{I_{C_{2\,max}} = 0.4mA}$$

As shown: $I_{C_1} \simeq I_{C_2}$ (Note: $I_{C_1} = I_{E_2} = \frac{\beta_2+1}{\beta_2} I_{C_2}$ precisely)

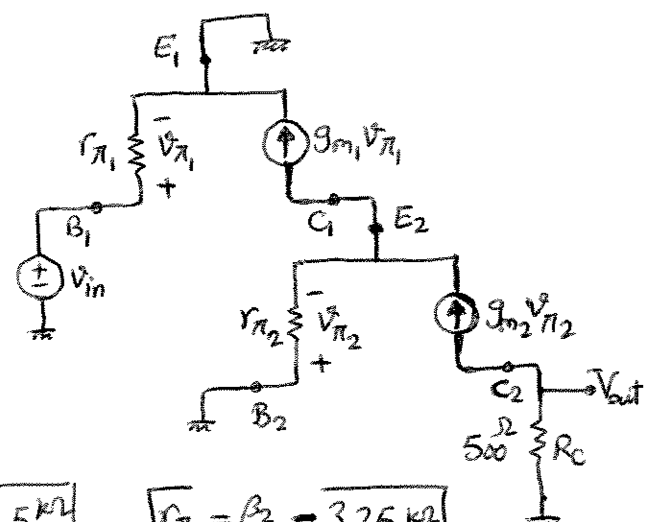
$$I_{C_1} \cong I_{S_1} e^{V_{EB_1}/V_T} \Rightarrow V_{EB_1} = V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right) \Rightarrow V_{CC} - V_{in} = V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right)$$

$$\Rightarrow \boxed{V_{in} = V_{CC} - V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right)} \Rightarrow V_{in} = 2.5 - 0.026 \ln\left(\frac{4\times10^{-4}}{5\times10^{-16}}\right)$$

$$\Rightarrow \boxed{V_{in} \simeq 1.787\ V}$$ This is minimum acceptable V_{in}

b) $g_{m_1} = \frac{I_{C_1}}{V_T} \simeq \frac{0.4^{mA}}{26^{mV}}$

$g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{0.4^{mA}}{26^{mV}}$



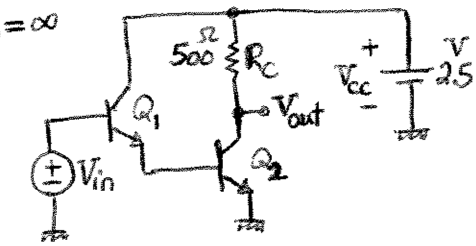
$$\Rightarrow \boxed{g_{m_1} = g_{m_2} \simeq 15.4\ mS}$$

$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = \frac{100}{\frac{0.4}{26}} \Rightarrow \boxed{r_{\pi_1} = 6.5^{k\Omega}}$ $\boxed{r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = 3.25\,k\Omega}$

Q1 in active mode

$$V_{EB_2} = V_T \ln\left(\frac{I_{C_2}}{I_{S_2}}\right) = 26^{mV} \ln\left(\frac{0.4\times10^{-3}}{\frac{5}{3}\times10^{-16}}\right) \Rightarrow \boxed{V_{EB_2} \simeq 741\,mV} \Rightarrow \boxed{V_{EC_1} \simeq 1.759\ V}$$

(55) $I_{S_1} = 3I_{S_2} = 5\times10^{-16}A$, $\beta_1 = 100$, $\beta_2 = 50$, $V_A = \infty$



a) Q_2 is softly saturated $\Rightarrow V_{BC_2} = 0.2^V$

$$V_{BC_2} = 0.2^V \Rightarrow V_{B_2} - V_{C_2} = 0.2^V \Rightarrow V_{BE_2} - (V_{CC} - R_C I_{C_2}) = 0.2^V$$

$$\Rightarrow V_{BE_2} + R_C I_{S_2} e^{\frac{V_{BE_2}}{V_T}} = V_{CC} + 0.2$$

$$\Rightarrow V_{BE_2} + 500 \times \frac{5}{3} \times 10^{-16} e^{\frac{V_{BE_2}}{V_T}} = 2.5 + 0.2 \Rightarrow \boxed{V_{BE_2} \simeq 800\,mV}$$

$$V_{BE_1} = V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right) \qquad I_{C_2} = I_{S_2} e^{\frac{V_{BE_2}}{V_T}} = \frac{5}{3}\times10^{-16} e^{\frac{800}{26}} \Rightarrow \boxed{I_{C_2} \simeq 3.8^{mA}}$$

$$V_{BE_1} = V_T \ln\left(\frac{I_{C_2}/\beta_2}{I_{S_1}}\right) = 26^{mV} \ln\left(\frac{\frac{3.8\times10^{-3}}{50}}{5\times10^{-16}}\right) \Rightarrow \boxed{V_{BE_1} \simeq 669.4\,mV}$$

$$V_{in} = V_{BE_1} + V_{BE_2} \Rightarrow \boxed{V_{in} \simeq 1.469\ V}$$ Maximum allowable value for V_{in}

b)

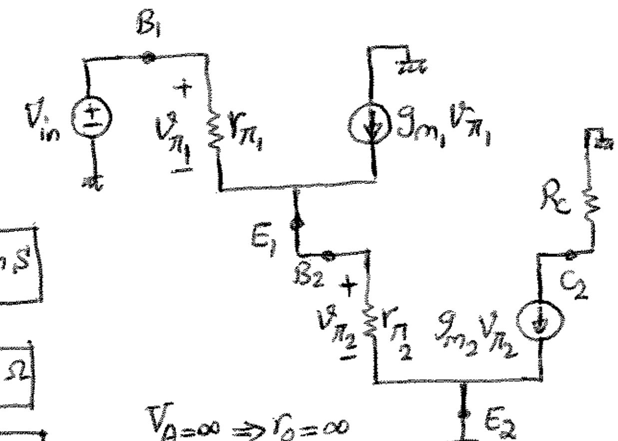
$$g_{m_1} = \frac{I_{C_1}}{V_T} = \frac{I_{C_2}/\beta_2}{V_T} = \frac{\frac{3.8^{mA}}{50}}{26^{mV}}$$

$$\Rightarrow \boxed{g_{m_1} \simeq 2.9\ mS}$$

$$g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{3.8^{mA}}{26^{mV}} \Rightarrow \boxed{g_{m_2} \simeq 146\ mS}$$

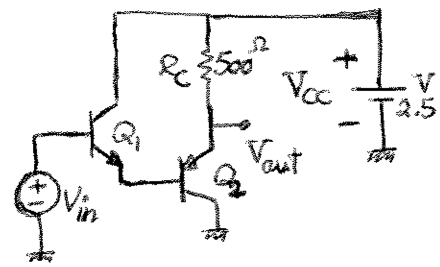
$$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = \frac{100}{2.9\times10^{-3}} \Rightarrow \boxed{r_{\pi_1} \simeq 34321\,\Omega}$$

$$r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = \frac{50}{146\times10^{-3}} \Rightarrow \boxed{r_{\pi_2} \simeq 342\,\Omega}$$



$V_A = \infty \Rightarrow r_o = \infty$

56 $I_{S_1} = 2I_{S_2} = 6\times10^{-17}A, \quad \beta_1 = 80, \quad \beta_2 = 100$



a) $I_{C_2} = 2^{mA}$

$$V_{EB_2} = V_T \ln\frac{I_{C_2}}{I_{S_2}} = 26^{mV}\ln\left(\frac{2\times10^{-3}}{3\times10^{-17}}\right) \simeq 827.6^{mV}$$

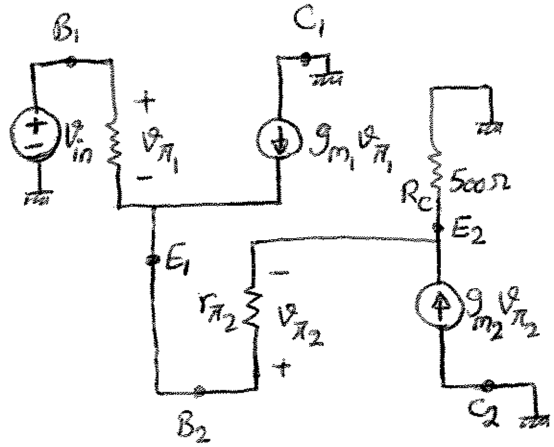
$$V_{BE_1} = V_T \ln\frac{I_{C_1}}{I_{S_1}} = 26^{mV}\ln\left(\frac{\frac{2\times10^{-3}}{100}}{6\times10^{-17}}\right) \simeq 689.9\ mV$$

$$\boxed{V_{in} = V_{CC} - R_C I_{C_2} - V_{EB_2} + V_{BE_1}} = 2.5 - 0.5^{k\Omega}\times2^{mA} - 0.8276 + 0.6899$$

$$\Rightarrow \boxed{V_{in} \simeq 1.362\ V}$$

b) $g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{2^{mA}}{26^{mV}} \Rightarrow \boxed{g_{m_2} \simeq 76.9\ mS}$

$g_{m_1} = \frac{I_{C_1}}{V_T} = \frac{2^{mA}/100}{26^{mV}} \Rightarrow \boxed{g_{m_1} \simeq 769\mu S}$



$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = \frac{80}{\frac{1}{1300}}$

$\Rightarrow \boxed{r_{\pi_1} = 104\ k\Omega}$

$r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = \frac{100}{2/26} \Rightarrow \boxed{r_{\pi_2} = 1300\Omega}$

$V_A = \infty \Rightarrow r_o = \infty$