

Problems

⊕ Section 2.1

1. In the two-category case, under the Bayes' decision rule the conditional error is given by Eq. 7. Even if the posterior densities are continuous, this form of the conditional error virtually always leads to a discontinuous integrand when calculating the full error by Eq. 5.

- (a) Show that for arbitrary densities, we can replace Eq. 7 by $P(\text{error}|x) = 2P(\omega_1|x)P(\omega_2|x)$ in the integral and get an upper bound on the full error.
- (b) Show that if we use $P(\text{error}|x) = \alpha P(\omega_1|x)P(\omega_2|x)$ for $\alpha < 2$, then we are not guaranteed that the integral gives an upper bound on the error.
- (c) Analogously, show that we can use instead $P(\text{error}|x) = P(\omega_1|x)P(\omega_2|x)$ and get a lower bound on the full error.
- (d) Show that if we use $P(\text{error}|x) = \beta P(\omega_1|x)P(\omega_2|x)$ for $\beta > 1$, then we are not guaranteed that the integral gives an lower bound on the error.

⊕ Section 2.2

4. Generalize the minimax decision rule in order to classify patterns from three categories having triangle densities as follows:

$$p(x|\omega_i) = T(\mu_i, \delta_i) \equiv \begin{cases} (\delta_i - |x - \mu_i|)/\delta_i^2 & \text{for } |x - \mu_i| < \delta_i \\ 0 & \text{otherwise,} \end{cases}$$

where $\delta_i > 0$ is the half-width of the distribution ($i = 1, 2, 3$). Assume for convenience that $\mu_1 < \mu_2 < \mu_3$, and make some minor simplifying assumptions about the δ_i 's as needed, to answer the following:

- (a) In terms of the priors $P(\omega_i)$, means and half-widths, find the optimal decision points x_1^* and x_2^* under a zero-one (categorization) loss.
- (b) Generalize the minimax decision rule to *two* decision points, x_1^* and x_2^* for such triangular distributions.
- (c) Let $\{\mu_i, \delta_i\} = \{0, 1\}, \{.5, .5\}$, and $\{1, 1\}$. Find the minimax decision rule (i.e., x_1^* and x_2^*) for this case.
- (d) What is the minimax risk?