信号处理作业及答案

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1 Homework 9

project 9.1. 用直接I型和直接II型(标准型)结构实现以下系统函数

$$H(z) = \frac{3 + 4.2z^{-1} + 0.8z^{-2}}{2 + 0.6z^{-1} - 0.4z^{-2}} \tag{1}$$

answer :

$$2y(n) + 0.6y(n-1) - 0.4y(n-2) = 3x(n) + 4.2x(n-1) + 0.8x(n-2) \tag{2}$$

或

$$y(n) + 0.3y(n-1) - 0.2y(n-2) = 1.5x(n) + 2.1x(n-1) + 0.4x(n-2) \tag{3}$$

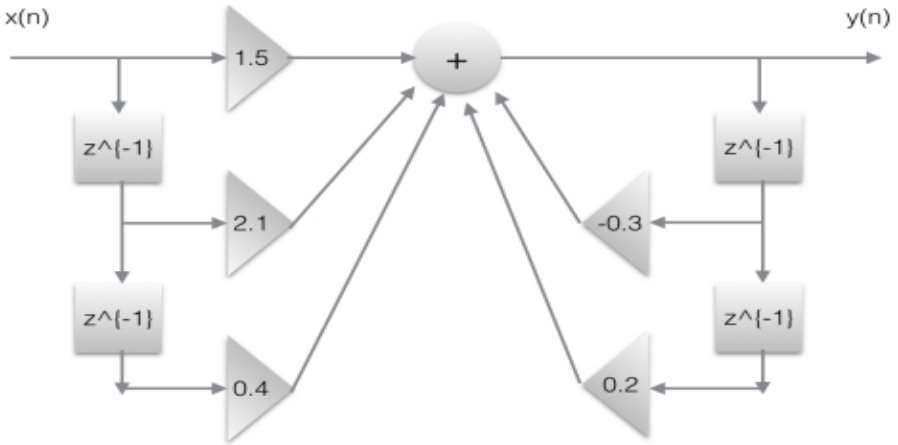

Figure 1:

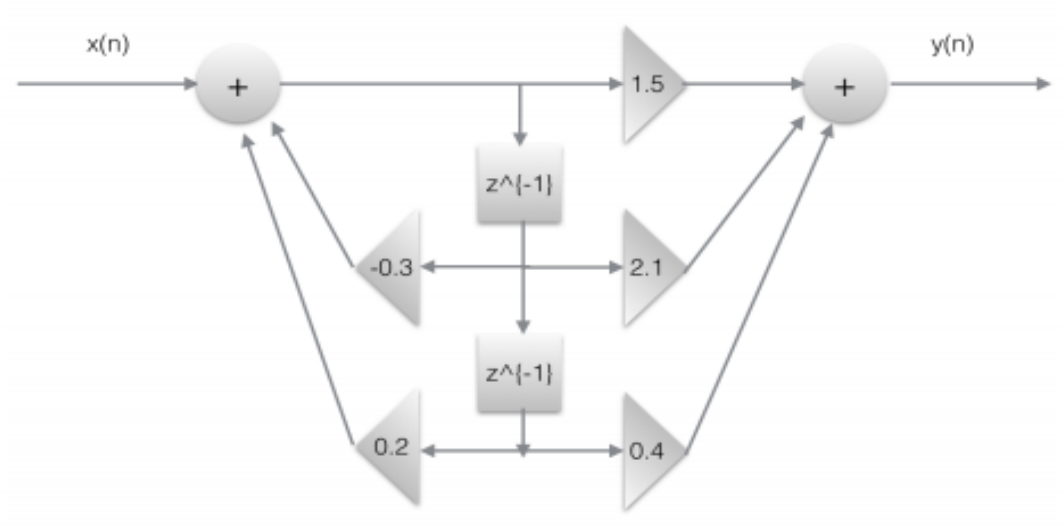

Figure 2:

project 9.2. 设滤波器差分方程为

$$y(n) = x(n) + x(n-1) + \frac{1}{3}y(n-1) + \frac{1}{4}y(n-2) \tag{4}$$

a : 用直接I型结构实现此差分方程;

b : 用直接II型(标准型)结构实现此差分方程;

c : 求系统的频率响应(幅度及相位)

answer :

a :

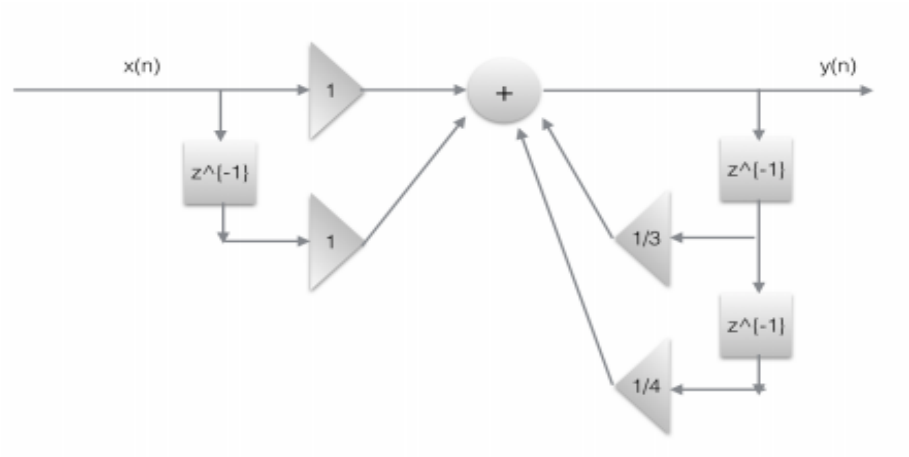

Figure 3:

b :

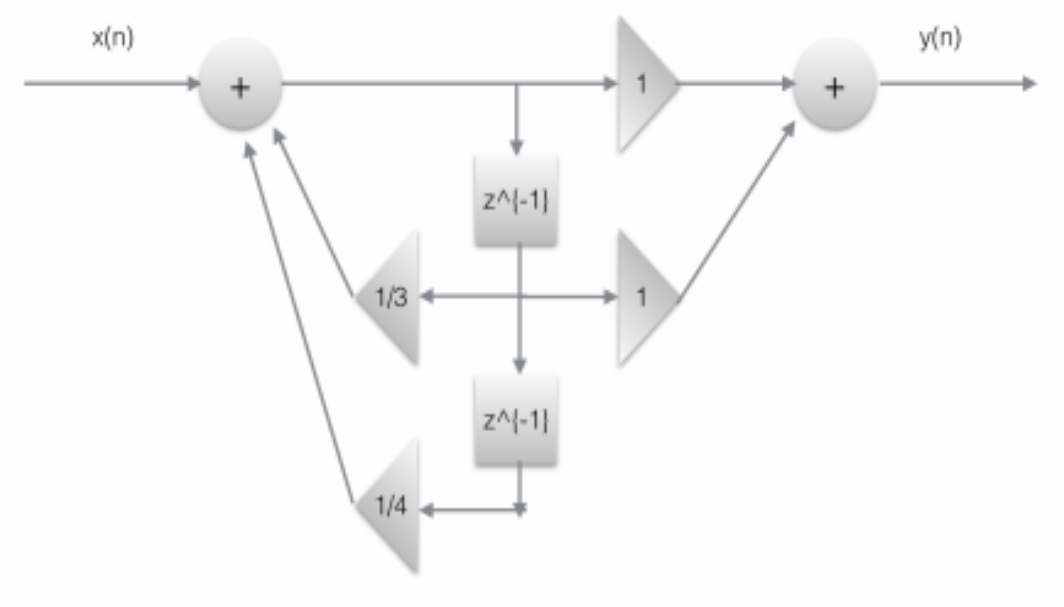

Figure 4:

c :

$$H(\omega) = \frac{\sum b(k)e^{-jk\omega}}{\sum a(k)e^{-jk\omega}} = \frac{1+e^{-j\omega}}{1-\frac{1}{3}e^{-j\omega}-\frac{1}{4}e^{-2j\omega}} \tag{5}$$

令

$$A = 1+cos\omega, B = sin\omega, C = 1-\frac{1}{3}cow\omega-\frac{1}{4}cos2\omega, D = \frac{1}{3}sin\omega+\frac{1}{4}sin2\omega \tag{6}$$

有

$$H(\omega) = \frac{A-Bj}{C+Dj} = \frac{(AC-BD)-j(BC+AD)}{C^2+D^2} \tag{7}$$

模长为

$$|H(\omega)| = \sqrt{\frac{A^2+B^2}{C^2+D^2}}H(\omega) = \sqrt{\frac{4(1+cos\omega)}{\frac{169}{72}-cos\omega-cos2\omega}} \tag{8}$$

相位记为

$$\theta(\omega) = arg(H(\omega)) \tag{9}$$

$$= arctan-\frac{BC+AD}{AC-BD} = arctan(-\frac{19sin\omega+3sin2\omega}{8+5cos\omega-3cos2\omega}) \tag{10}$$

project 9.3. 已知离散函数的差分方程为

$$y(n) = x(n) + 4x(n-1) + 0.7y(n-1) - 0.1y(n-2) \tag{11}$$

求

$a:$ 系统传递函数$H(z)$;

$b:$ 系统的单位冲激响应$h(n)$;

$c:$ 画出系统的零极点分布;

$d:$ 说明系统频响的高低通特性;

$e:$ 说明系统的稳定性;

answer :

$a:$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 4z^{-1}}{1 - 0.7z^{-1} + 0.1z^{-2}} \tag{12}$$

$$= \frac{z(z+4)}{(z-0.2)(z-0.5)} \tag{13}$$

$b:$

$$h(n) = \sum Res[H(z)z^{n-1}] = \sum Res[\frac{z^n(z+4)}{(z-0.2)(z-0.5)}], z \geq 0 \tag{14}$$

一阶极点$z = 0.2$和$z = 0.5$,两处留数为

$$Res[\frac{z^n(z+4)}{(z-0.2)(z-0.5)}]|_{z=0.2} = \frac{z^n(z+4)}{((z-0.2)(z-0.5))'}|_{z=0.2} = -14(0.2)^n \tag{15}$$

$$Res[\frac{z^n(z+4)}{(z-0.2)(z-0.5)}]|_{z=0.5} = \frac{z^n(z+4)}{((z-0.2)(z-0.5))'}|_{z=0.5} = -15(0.5)^n \tag{16}$$

$$\tag{17}$$

因而

$$h(n) = -14(0.2)^n u(n) + 15(0.5)^n u(n) \tag{18}$$

$c:$

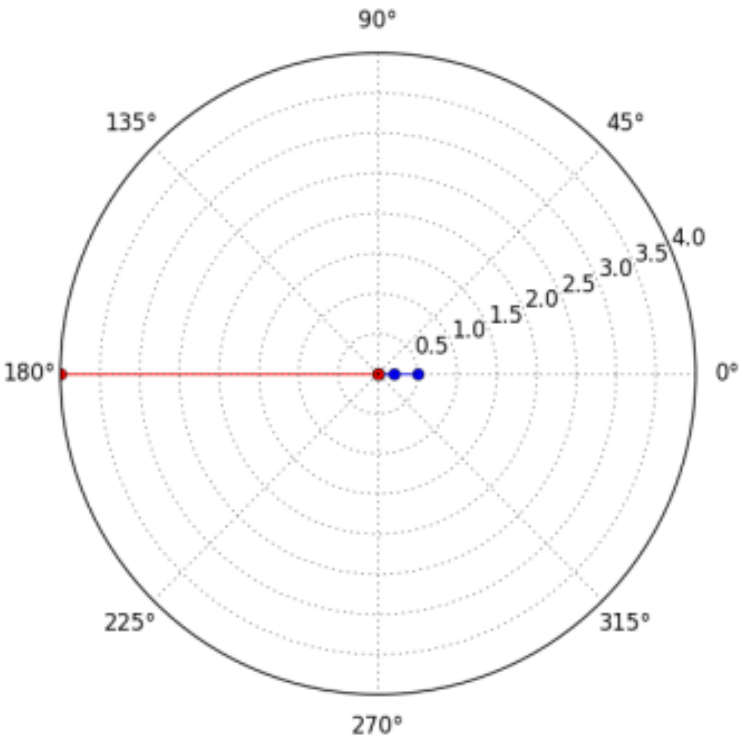

Figure 5:

其中,蓝点为极点，红点为零点

d :

$$H(\omega) = \frac{e^{j\omega}(e^{j\omega} + 4)}{(e^{j}\omega - 0.2)(e^{j\omega - 0.5})} \tag{19}$$

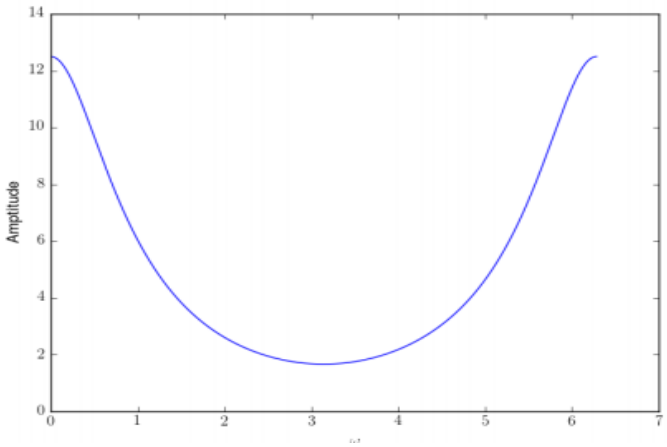

Figure 6:

如图所示，为低通的

e :

$$\sum |h(n)| = 14 \sum_{0}^{\infty} (0.2)^n + 15 \sum_{0}^{\infty} (0.5)^n \tag{20}$$

显然是(绝对)收敛的