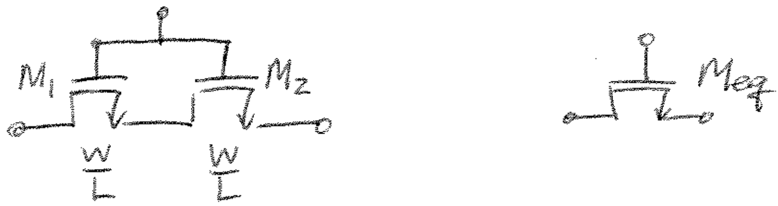
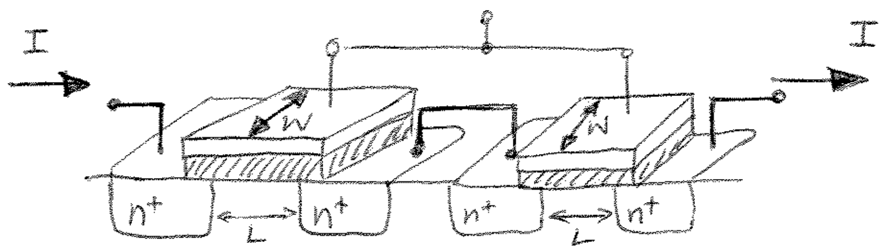
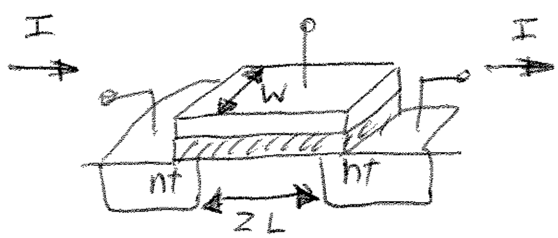
1.



Intuitively, this is similar to having twice of the original channel length:



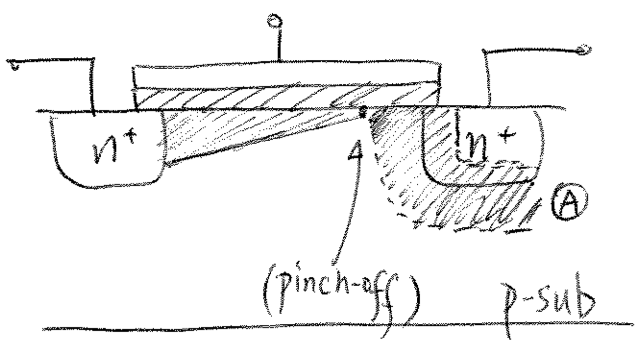
Since current flowing into either non-gate terminals must come out at the other terminal (KCL) and the intermediate node is equipotential, this is as if we have a Meq with width W & length 2L:



This approximation can simplify a lot of calculations.

2. A key point to remember: the charge density APPROACHES zero (not EQUALS) at pinch-off. In other words, Q is never exactly equal to zero (albeit very close.) Another way to view this phenomenon is by observing $I = Q \cdot v$: recognize that v is finite. Since we get some finite value of I at pinch-off, we expect $Q \neq 0$.

Consider the following:



The shaded region, Ⓐ, represents a reverse-biased pn junction. Just as a diode, there exist minority profiles on p & n sides, which $\neq 0$.

Pinch-off implies that the depletion region created no longer has free carriers. The depletion still sweeps all electrons from inversion channel to drain.

3\. Given: $C_{ox} = 10\ fF/\mu m^2$ $\quad W = 5\mu m$ $\quad L = 0.1\mu m$

$V_{GS} - V_{TH} = 1V$ $\quad V_{DS} = 0$

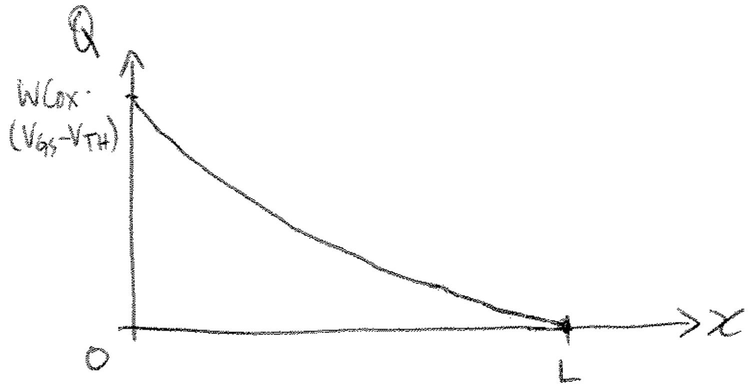
Find: total charge stored in channel, Q_{tot}

$$Q_{tot} = W C_{ox} (V_{GS} - V_{TH}) L$$

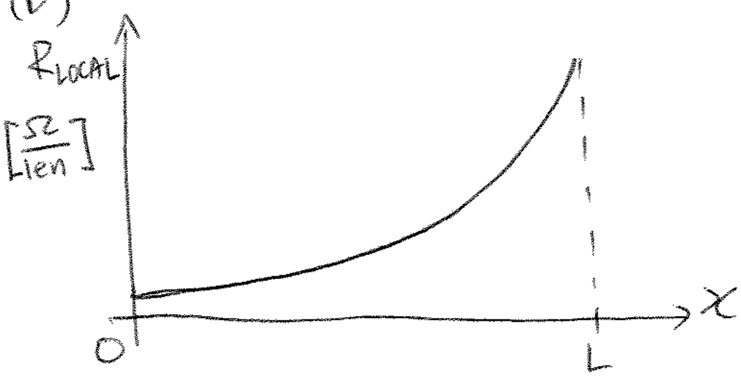
$$= (5\mu m)(10\ fF/\mu m^2)(1V)(0.1\mu m) = 5\ fC$$

4. (a) $Q = W C_{ox} (V_{GS} - V_{TH} - V(x))$

$= -W C_{ox} \cdot V(x) + W C_{ox} (V_{GS} - V_{TH})$



(b)



$R \propto \frac{1}{\mu Q}$

↑ mobility of charge.

5. $I_D = WC_{ox}\left[V_{GS} - V(x) - V_{TH}\right]\mu_n \dfrac{dV(x)}{dx}$

Define : $A = \dfrac{I_D}{WC_{ox}\mu_n}$, $B = V_{GS} - V_{TH}$

$$\Rightarrow \quad A = (B - V)\frac{dV}{dx} = \frac{d}{dx}\left(BV - \frac{V^2}{2}\right)$$

Integrating $A = \dfrac{d}{dx}(BV - V^2/2)$ gives:

$$Ax = BV - V^2/2 \Rightarrow V^2 - 2BV + 2Ax = 0$$

Using quadratic formula:

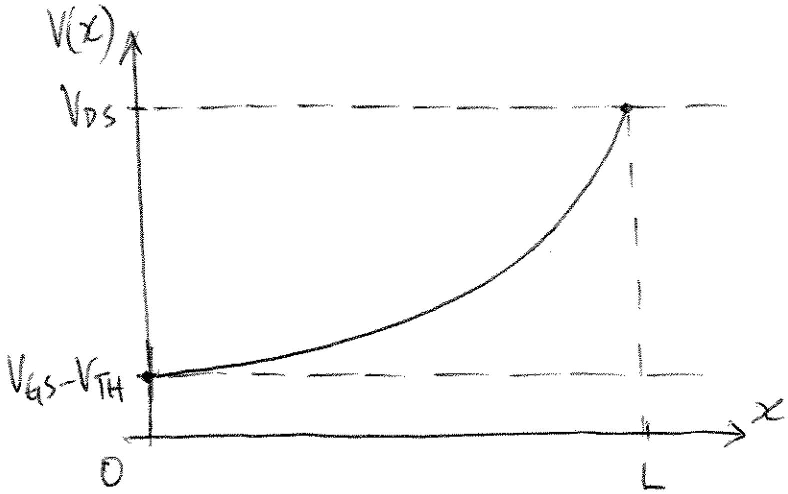
$$V_{+,-} = \frac{2B \pm \sqrt{4B^2 - 4 \cdot 2A}}{2} = B \pm \sqrt{B^2 - 2Ax}$$

$$= B\left(1 \pm \sqrt{1 - 2\left(\frac{A}{B^2}\right)x}\right)$$

$$= (V_{GS} - V_{TH})\left\{1 \pm \sqrt{1 - \left[2 \cdot \frac{I_D}{WC_{ox}\mu_n(V_{GS} - V_{TH})^2}\right]x}\right\}$$

We know that $0 \leq V(x) \leq V_{GS} - V_{TH}$ (pinch-off), and the term inside the square root is > 0. Therefore, we take V_- as the solution.

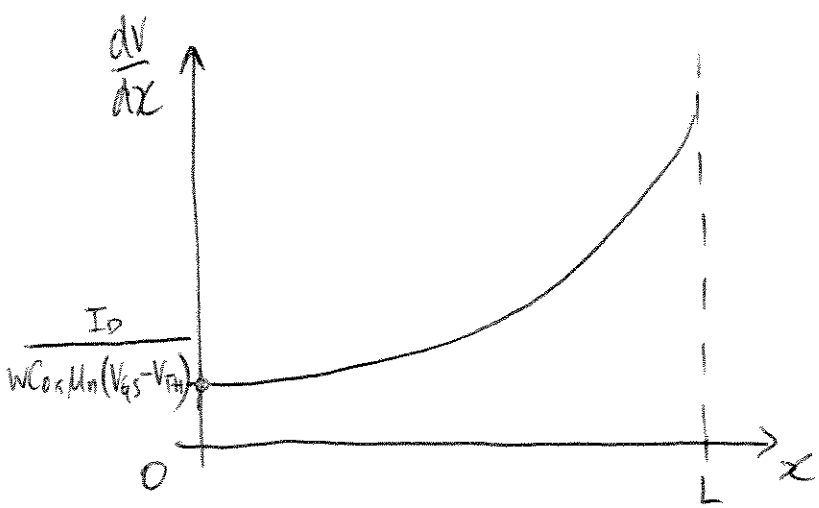
i.e. $V(x) = (V_{GS}-V_{TH})\left\{ 1 - \sqrt{1 - \left[\frac{2I_D}{WC_{ox}\mu_n (V_{GS}-V_{TH})^2} \right] x} \right\}$



$\because$ $I_D \propto W$

$\Rightarrow V(x)$ is independent of W.

$$\frac{dV}{dx} = \frac{I_D}{WC_{ox}\mu_n (V_{GS}-V_{TH})} \cdot \left[1 - \frac{2I_D \cdot x}{WC_{ox}\mu_n (V_{GS}-V_{TH})^2} \right]^{-\frac{1}{2}}$$



6. No.

By varying $V_{GS}-V_{TH}$ & V_{DS}, we can only obtain $\mu_n C_{ox} \frac{W}{L}$, but not $\mu_n C_{ox}$ & $\frac{W}{L}$ individually.

7. Given : NMOS $\quad I_D = 1mA \quad V_{GS} - V_{TH} = 0.6V$

$I_D = 1.6mA \quad V_{GS} - V_{TH} = 0.8V$

(triode region) $\quad \mu_n C_{ox} = 200 \frac{\mu A}{V^2}$

Find V_{DS} & W/L.

$$1\,mA = \mu_n C_{ox} \frac{W}{L} \left[(0.6)V_{DS} - V_{DS}^2/2\right] \quad \text{——— ①}$$

$$1.6\,mA = \mu_n C_{ox} \frac{W}{L} \left[(0.8)V_{DS} - V_{DS}^2/2\right] \quad \text{——— ②}$$

② ÷ ① : $1.6 = \dfrac{0.8 V_{DS} - V_{DS}^2/2}{0.6 V_{DS} - V_{DS}^2/2} = \dfrac{1.6 - V_{DS}}{1.2 - V_{DS}}$

$\Rightarrow V_{DS} = \dfrac{1.6(0.2)}{0.6} \approx 0.533\ V$

$$\Rightarrow \frac{W}{L} = \frac{I_D}{\mu_n C_{ox}\left[(V_{GS} - V_{TH})V_{DS} - V_{DS}^2/2\right]}$$

$$= \frac{1\,mA}{200\frac{\mu A}{V^2}\left[(0.6V)(0.533V) - (0.533V)^2/2\right]}$$

$\approx 28.$

8. $I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2\right]$

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = \frac{1}{2}\mu C_{ox}\frac{W}{L}\cdot 2V_{DS} = \mu C_{ox}\frac{W}{L}V_{DS}$$

$$g_m\big|_{V_{DS}=0} = 0.$$

Intuitively, when $V_{GS} > V_{TH}$, mobile charges (channel) become available. This determines the on-resistance. But since there is no I_D ($\because V_{DS}=0$), it does not matter if there is an incremental change in V_{GS} (i.e. ∂V_{GS}). Since varying V_{GS} gives no change in I_D, $g_m|_{V_{DS}=0} = 0$.

9. Given: $V_{DD} = 1.8\ V$ $\frac{W}{L} = 20$ $\mu_n C_{ox} = 200\ \frac{\mu A}{V^2}$

$V_{TH} = 0.4\ V$

Find minimum-on resistance.

$$R_{on} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{DD} - V_{TH})}$$

$$= \frac{1}{\left(200\ \frac{\mu A}{V^2}\right)(20)\ (1.8 - 0.4)V} = 179.\ \Omega$$

10. $$500 = \frac{1}{\mu_n C_{ox} \frac{W}{L} (1 - V_{TH})}$$

$$400 = \frac{1}{\mu_n C_{ox} \frac{W}{L} (1.5 - V_{TH})}$$

For the same NMOS, $\mu_n C_{ox}$ & $\frac{W}{L}$ are fixed

$$\Rightarrow \quad 500(1 - V_{TH}) \stackrel{?}{=} 400(1.5 - V_{TH})$$

$$500(0.6) \neq 400(1.1)$$

$\therefore$ This is not possible.

11. $I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2\right]$

$$r_{DS,tri} \triangleq \left(\frac{\partial I_D}{\partial V_{DS}}\right)^{-1} = \left[\frac{\partial}{\partial V_{DS}}\left\{\frac{1}{2}\mu C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2\right]\right\}\right]^{-1}$$

$$= \left[\mu C_{ox}\frac{W}{L}(V_{GS}-V_{TH}) - \mu C_{ox}\frac{W}{L}V_{DS}\right]^{-1}$$

$$= \frac{1}{\mu C_{ox}\frac{W}{L}(V_{GS}-V_{TH}-V_{DS})}$$

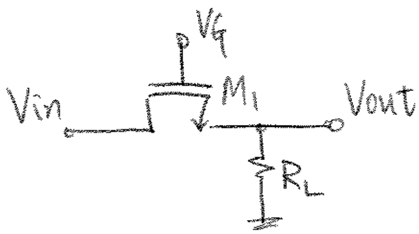
12. When MOS operates as a resistor,

$$R_{on} = \frac{1}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}$$

$$\Rightarrow \tau = R_{on} C_{GS} = \frac{W L C_{ox}}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})} = \frac{L^2}{\mu (V_{GS} - V_{TH})}$$

To minimize the time constant,
1) use minimum channel length, and
2) maximize overdrive voltage.

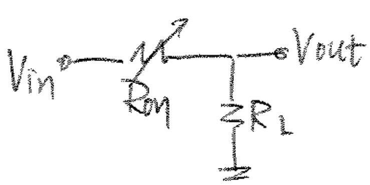
13.



Given $V_{in} \approx 0$
$V_G = 1.8\ V$
$R_L = 100\Omega$

Find $\frac{W}{L}$ such that signal output attenuates by only 5%.

$V_{in} \approx 0$ implies that we can approximate M_1 as a linear resistance controlled by V_G. Therefore, the equivalent circuit becomes a resistive divider:



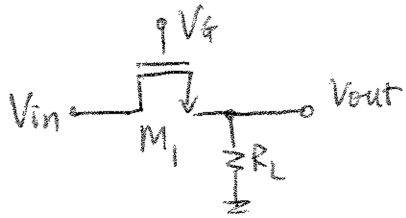
$$V_{out} = 0.95\, V_{in} = \frac{R_L}{R_{on} + R_L} V_{in}$$

$$\Rightarrow R_{on} \cong 5.3\,\Omega$$

$$\therefore \frac{W}{L} = \frac{1}{\mu C_{ox}(V_{GS} - V_{TH}) R_{on}} \approx \frac{1}{200\frac{\mu A}{V^2}(1.8 - 0.4)(5.3\Omega)}$$

$$= 674.$$

14.



$V_0 \sim$ few mV.

(a) $V_{in} = V_0 \cos\omega t$ $\qquad V_{out} = 0.95(V_0 \cos\omega t)$

$$V_{out} = \frac{R_L}{R_{ON}+R_L} V_{in} \quad \Rightarrow \frac{R_L}{R_{ON}+R_L} = 0.95V_0$$

$$R_{ON} = \frac{R_L}{\left(\frac{0.95V_0}{1-0.95V_0}\right)} = \frac{1}{\mu_n C_{ox}\frac{W}{L}(V_G - V_{TH})}$$

$$\therefore \frac{W}{L} = \frac{0.95V_0/(1-0.95V_0)}{\mu_n C_{ox} R_L (V_G - V_{TH})}$$

(b) $V_{out} = 0.95\,V_{in} = 0.95(V_0\cos\omega t + 0.5)$

$\approx 0.95 \times 0.5 = 0.475$

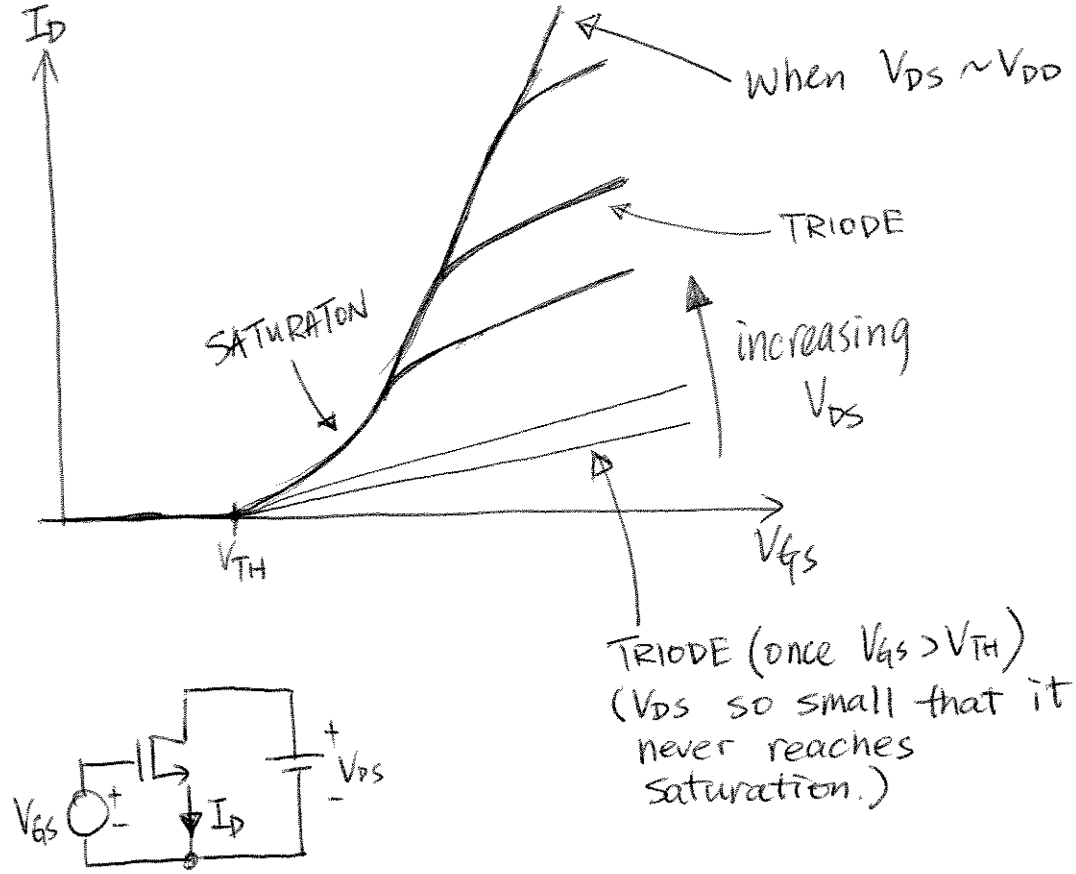
($\because$ V_0 is relatively small)

$$\therefore R_{ON} = \frac{R_L}{0.9} = \frac{1}{\mu_n C_{ox}\frac{W}{L}(V_G - V_{TH})}$$

$$\Rightarrow \frac{W}{L} = \frac{0.9}{\mu_n C_{ox} R_L (V_G - V_{TH})}$$

Results show that if there is no DC voltage as input, the R_{ON} varies with changing sinewave. With a DC bias voltage, R_{ON} becomes more stable (independent of V_o).

15.



I_D
When $V_{DS} \sim V_{DD}$
TRIODE
SATURATON
increasing V_{DS}
V_{TH}
V_{GS}
TRIODE (once $V_{GS} > V_{TH}$)
(V_{DS} so small that it never reaches saturation.)
+
V_{DS}
-
V_{GS}
+
-
I_D

16. The peak of the parabola signifies pinch-off (i.e. $V_{DS} = V_{GS} - V_{TH}$). This means that (with $\lambda = 0$) I_D cannot be increased further by increasing V_{DS}. Since this curve must be continuous, the peak I_D must originate from the peak of the parabola.

17. $I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^{\alpha}$, $\alpha < 3$

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \cdot \alpha (V_{GS} - V_{TH})^{\alpha - 1}$$

$$= \frac{\alpha}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^{\alpha - 1}$$

$$= \frac{\alpha I_D}{(V_{GS} - V_{TH})}$$

18. $$I_D = W C_{ox} (V_{GS} - V_{TH}) v_{SAT}$$

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = W C_{ox} v_{SAT}$$

19. (a) OFF $\because$ $V_{GS} = 0$

(b) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(c) TRIODE (LINEAR) $\because$ $V_{GS} > V_{TH}$ &
$V_{DS} \ll 2(V_{GS} - V_{TH})$

(d) TRIODE $\because$ $V_{GS} > V_{TH}$ & $V_{DS} < V_{GS} - V_{TH}$
(REMEMBER : MOSFET is symmetric)

(e) TRIODE $\because$ $V_{GS} > V_{TH}$ & $V_{DS} < V_{GS} - V_{TH}$

(f) OFF $\because$ $V_{GS} = 0$

(g) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(h) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(i) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

20. (a) OFF $\because$ $V_{GS} = 0$ $(V_{GS} < V_{TH})$

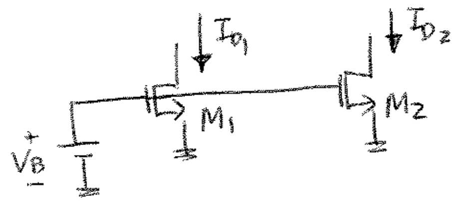
(b) OFF $\because$ $V_{GS} = 0$ $(V_{GS} < V_{TH})$

(c) TRIODE (LINEAR) $\because$ $V_{GS} > V_{TH}$ &

$V_{DS} \ll 2(V_{GS} - V_{TH})$

(d) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

21.



$$0.99 I_{D2} < I_{D1} < 1.01 I_{D2}$$

Since M_1 & M_2 are treated as current sources, they are assumed to be in saturation.

Evaluate λ at boundaries:

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_B - V_{TH})^2 (1 + \lambda V_{DS1}) \quad \text{——— ①}$$

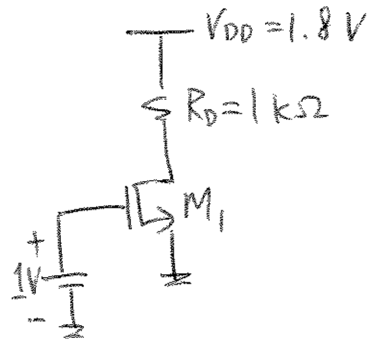
$$I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_B - V_{TH})^2 (1 + \lambda V_{DS2}) \quad \text{——— ②}$$

① ÷ ② : $$\frac{I_{D1}}{I_{D2}} = \frac{0.99 I_{D2}}{I_{D2}} = \frac{1 + \lambda V_{DS1}}{1 + \lambda V_{DS2}}$$

$$\therefore \lambda = \frac{0.01}{0.99 V_{DS2} - V_{DS1}} = \frac{0.01}{0.99(1V) - (0.5V)} = 0.02 V^{-1}$$

Maximum tolerable $\lambda = 0.02 V^{-1}$

22.



$\lambda = 0$, $V_{TH} = 0.4\,V$

$\mu_n C_{ox} = 200\,\frac{\mu A}{V^2}$

M_1 sits at the edge of saturation when $V_{DS} = V_{GS} - V_{TH}$.

$$\Rightarrow V_{DS,edge} = (1 - 0.4)\,V = 0.6\,V$$

By KCL, $I_{D_1} = I_{R_D} = \frac{V_{DD} - V_{DS}}{R_D} = \frac{1.2\,V}{1\,k\Omega} = 1.2\,mA$

$$\therefore I_{D_1} = 1.2\,mA = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$$

$$\Rightarrow \frac{W}{L} = \frac{2 I_{D_1}}{\mu_n C_{ox}(V_{GS} - V_{TH})^2} = \frac{2(1.2mA)}{\left(200\,\frac{\mu A}{V^2}\right)\left(1 - 0.4\right)^2 V^2}$$

$$\cong 33.$$

23. If gate oxide thickness, t_{ox}, doubles, the corresponding capacitance, $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$, is halved.

$\Rightarrow \mu_n C_{ox}$ is also halved

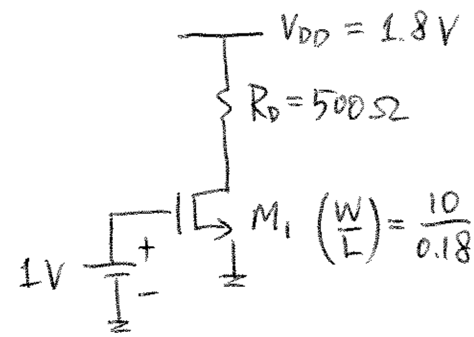
$\Rightarrow I_{D_1}$ is halved $\Rightarrow V_{DS}$ increases

$\Rightarrow M_1$ stays in saturation ($V_{DS} > V_{GS} - V_{TH}$)

$$I_{D_1} = \frac{1.2\,mA}{2} = 0.6\,mA$$

$$\Rightarrow V_{DS} = (1.8\,V) - (0.6\,mA)(1\,k\Omega) = 1.2\,V$$

24.



$\lambda = 0$

To avoid triode region, $V_{DS} \geq V_{GS} - V_{TH}$.

$\Rightarrow V_{DS} \geq 1 - 0.4 = 0.6\ V$

$$\Rightarrow I_{D_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2$$

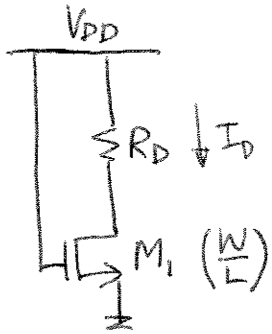
$$= \frac{1}{2}\left(200\ \frac{\mu A}{V^2}\right)\left(\frac{10}{0.18}\right)(0.6)^2 = 2\ mA$$

By KCL, $\dfrac{V_{DD} - V_{DS}}{R_D} = 2mA$

$\therefore\ V_{DD} = (2mA)(500\Omega) + 0.6V = 1.6\ V$

Minimum $V_{DD} = 1.6\ V$

25.



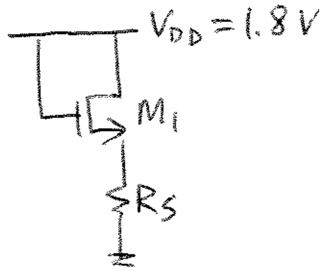
$\lambda = 0$

When M_1 operates at the edge of saturation, $V_{DS} = V_{GS} - V_{TH}$. Also, by KCL:

$$I_{R_D} = I_{D_1} \Rightarrow \frac{V_{DD} - (V_{DD} - V_{TH})}{R_D} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{DD} - V_{TH})^2$$

$$\therefore V_{TH} = R_D \cdot \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{DD} - V_{TH})^2}_{I_D}$$

26.



$\lambda = 0$

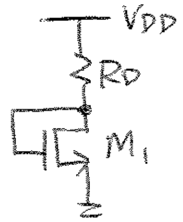
Find $\left(\frac{W}{L}\right)$ with bias current $= I_1$.

Since $V_{DS} = V_{GS}$ for M_1, this device always operates in saturation region (given $V_{GS} > V_{TH}$).

By KCL, $I_1 = I_{R_S}$; by Ohm's law, $V_S = I_1 R_S$

$$\Rightarrow \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{DD} - I_1 R_S - V_{TH})^2 = I_1$$

$$\therefore \frac{W}{L} = \frac{2 I_1}{\mu_n C_{ox} (V_{DD} - I_1 R_S - V_{TH})^2}$$

27. 

Calculate I_1 if $\lambda = 0$.
Assume $V_{GS} > V_{TH}$

By KCL, $I_{R_D} = I_1$

$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}_{\triangleq B}(V_{GS} - V_{TH})^2 = \frac{V_{DD} - V_{GS}}{R_D}$$

Re-arrange this to quadratic form:

$$V_{GS}^2(BR_D) - V_{GS}(2BR_DV_{TH} - 1) + (V_{TH}^2 \cdot BR_D - V_{DD}) = 0$$

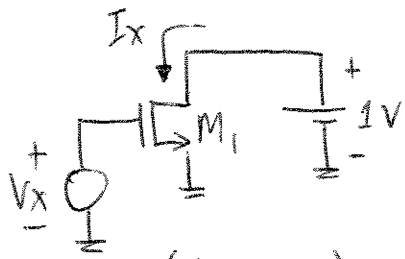
$$\Rightarrow V_{GS_{1,2}} = \frac{(BR_DV_{TH} - 1) \pm \sqrt{BR_D(V_{DD} - V_{TH}) + 1}}{BR_D}$$

$$= \frac{\left(\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right) \cdot R_D \cdot V_{TH} - 1\right) \pm \sqrt{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)R_D(V_{DD} - V_{TH}) + 1}}{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)}$$

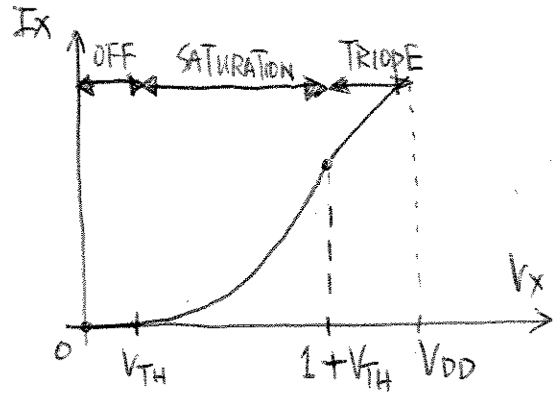
Whether the answer is V_{GS_1} or V_{GS_2} depends on other parameters. Also note that since M_1 is diode-connected, it never goes into triode (i.e either OFF or SATURATION.) This helps in eliminating one of the solutions.

After solving V_{GS}, $I_D = I_1 = \dfrac{V_{DD} - V_{GS}}{R_D}$

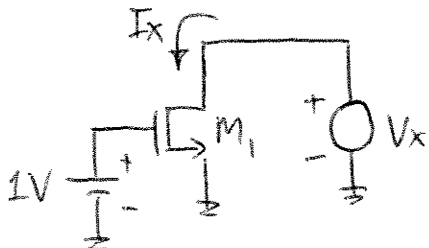
28. (a)

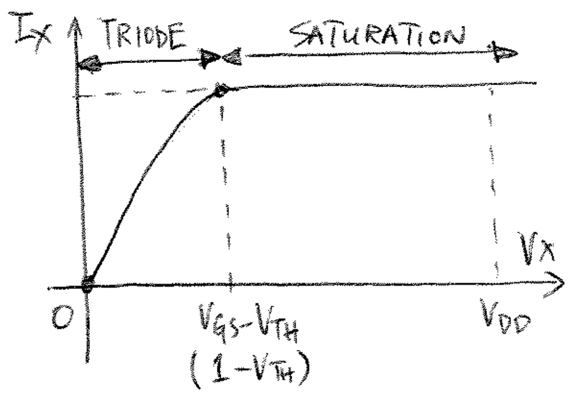


$(V_{TH} = 0.4)$

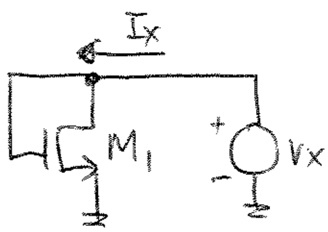


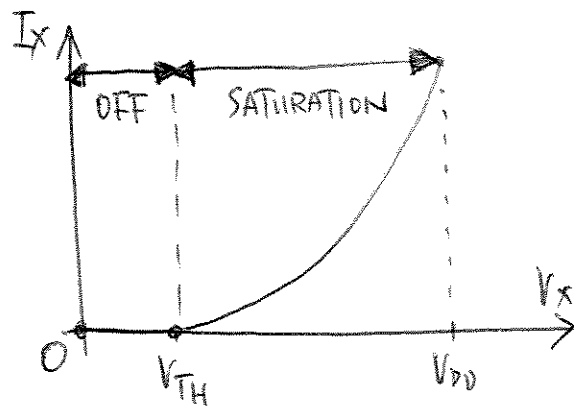
(b)



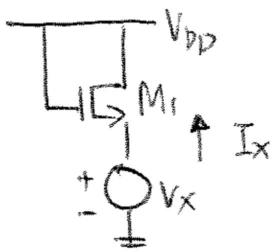


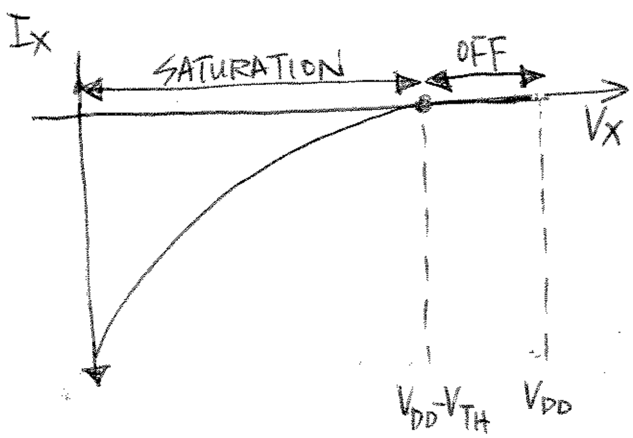
(c)



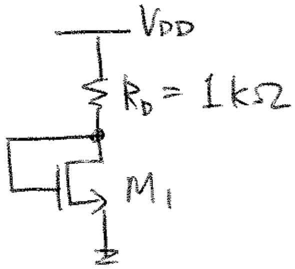


(d)





29.



$\left(\frac{W}{L}\right) = \frac{10}{0.18}$, $\lambda = 0.1\ V^{-1}$

Find I_{D_1}

Since M_1 is diode-connected, it operates in saturation.

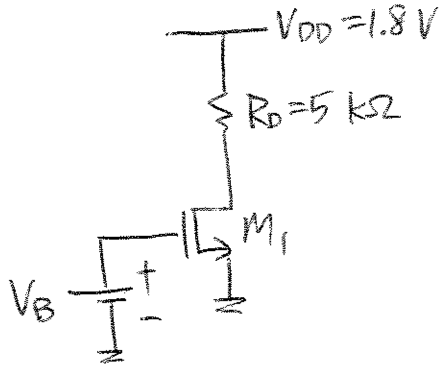
By KCL, $\frac{V_{DD} - V_G}{R_D} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_G - V_{TH})^2 (1 + \lambda V_G)$

One can solve this by (1) using a graphing calculator, (2) trial-and-error, (3) or iteratively finding V_G.

Using any method gives $V_G \approx 0.807$ V

$\Rightarrow I_D = \frac{V_{DD} - V_G}{R_D} \approx 1$ mA

30.



$\frac{W}{L} = \frac{20}{0.18}$, $\lambda = 0.1\ V^{-1}$

At the edge of saturation,

$$I_{D_1} = \frac{V_{DD} - (V_B - V_{TH})}{R_D} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_B - V_{TH})^2(1+\lambda(V_B - V_{TH}))$$

This equation can be solved by using a graphing calculator, special programs, or iteratively.

Using any method gives $V_B \approx 0.57\ V$
$(I_D \approx 0.33\ mA)$

31. An NMOS device with $\lambda = 0$ must provide a transconductance of $1/50 \ \frac{1}{\Omega}$.

(a) Given $I_D = 0.5\ mA$, find W/L.

$$g_m = \frac{1}{50} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$$\Rightarrow \frac{W}{L} = \frac{g_m^2}{2 \mu_n C_{ox} I_D} = \frac{\left(1/50 \ \frac{1}{\Omega}\right)^2}{2\left(200 \frac{\mu A}{V^2}\right)(0.5 mA)} \approx 2000$$

(b) Given $V_{GS} - V_{TH} = 0.5V$, find W/L.

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$\Rightarrow \frac{W}{L} = \frac{g_m}{\mu_n C_{ox}(V_{GS} - V_{TH})} = \frac{\left(1/50 \ \frac{1}{\Omega}\right)}{\left(200 \frac{\mu A}{V^2}\right)(0.5 V)} \approx 200$$

(c) Given $V_{GS} - V_{TH} = 0.5V$, find I_D.

$$\Rightarrow I_D = \frac{g_m (V_{GS} - V_{TH})}{2} = \frac{\left(1/50 \ \frac{1}{\Omega}\right)(0.5V)}{2} \approx 5 mA$$

32. (a) $g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}$ (I_D constant)

Doubling (W/L) implies a $\sqrt{2}$ times increase in g_m: $g_{m_{NEW}} = \sqrt{2\mu_n C_{ox}\left(2\frac{W}{L}\right) I_D} = \sqrt{2}\, g_m$.

(b) $g_m = \frac{2I_D}{V_{GS}-V_{TH}}$ (I_D constant)

Doubling $(V_{GS}-V_{TH})$ decreases g_m by half:

$g_{m_{NEW}} = \frac{2I_D}{2(V_{GS}-V_{TH})} = \frac{1}{2} g_m$

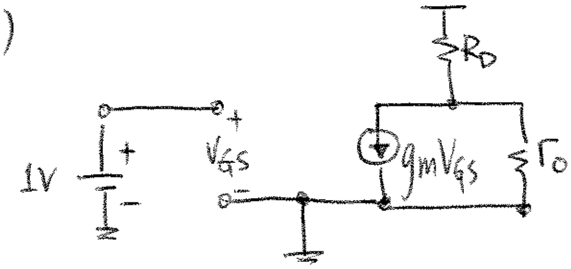
(c) $g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}$ (W/L constant)

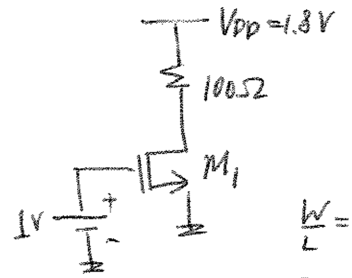
Doubling I_D increases g_m by $\sqrt{2}$ times.

(d) $g_m = \frac{2I_D}{V_{GS}-V_{TH}}$ ($V_{GS}-V_{TH}$ constant)

Doubling I_D increases g_m by 2 times.

33. (a)





$\frac{W}{L} = \frac{20}{0.18}$

$\lambda = 0.1V^{-1}$

First, verify M_1 is in saturation:

$$V_{DS} = V_{DD} - I_D R_D = V_{DD} - R_D \cdot \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2(1+\lambda V_{DS})$$

$$= 1.8 - 100 \cdot \frac{1}{2}\, 200\frac{\mu A}{V^2}\left(\frac{20}{0.18}\right)(1-0.4)^2(1+\lambda V_{DS})$$

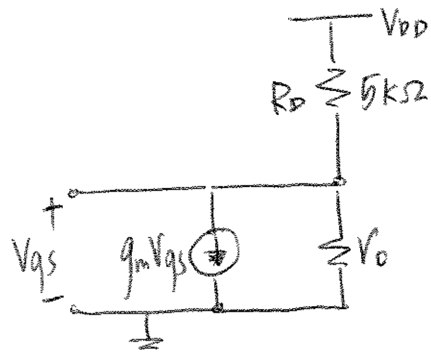
Solving this gives $V_{DS} \approx 1.35V > V_{GS}-V_{TH}$.

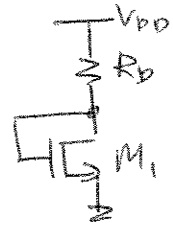
$$\therefore\ g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH}) = \left(200\frac{\mu A}{V^2}\right)\left(\frac{20}{0.18}\right)(1-0.4V)$$

$$\approx 0.013\ ^1/_\Omega$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{\lambda\left(\frac{V_{DD}-V_{DS}}{R_D}\right)} = \frac{1}{0.1V^{-1}\left(\frac{0.45V}{100\Omega}\right)} \approx 2222.\Omega$$

(b)





By KCL, $\dfrac{V_{DD}-V_{GS}}{R_D} = \dfrac{1}{2}\mu_n C_{ox}\dfrac{W}{L}(V_{GS}-V_{TH})^2(1+\lambda V_{GS})$

Solving this yields $V_{GS} \approx 0.546\,V > V_{TH}$

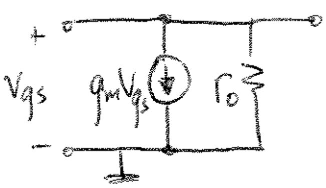
$$\Rightarrow g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH}) = \frac{200\mu A}{V^2}\left(\frac{20}{0.18}\right)(0.146V)$$

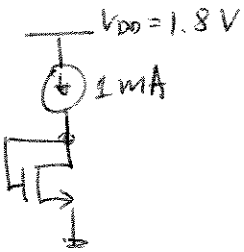
$$\approx 0.00324\ {}^1/_\Omega$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{\lambda\left(\frac{V_{DD}-V_{GS}}{R_D}\right)} = \frac{1}{0.1V^{-1}\left(\frac{1.8-0.546}{5k}\right)}$$

$$\approx 40.\ k\Omega$$

(c)





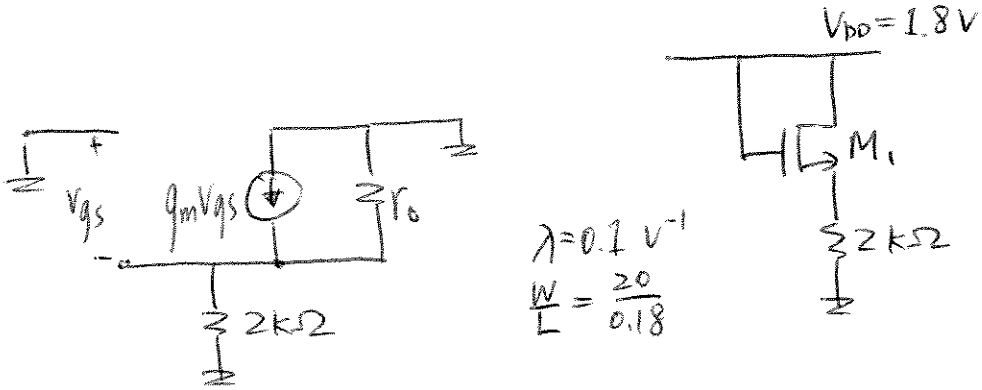
(Note: ideal current source is open in small signal)

$$g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} = \sqrt{2\left(200 \frac{\mu A}{V^2}\right)\left(\frac{20}{0.18}\right)(1\,mA)}$$

$$\approx 0.0067 \ {}^1/_\Omega$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.1\,V^{-1})(1\,mA)} = 10\,k\Omega$$

(d)



(Note: ideal voltage source is shorted to GND in this problem because V_{DD} is si -ended.)

By KCL, $I_D = I_R$

$$\Rightarrow \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[(V_{DD}-V_S)-V_{TH}\right]^2\left[1+\lambda(V_{DD}-V_S)\right] = V_S/2k\Omega.$$

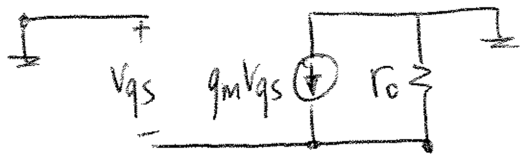
Solving this (analytically or numerically) gives $V_S \approx 1.18\ V$

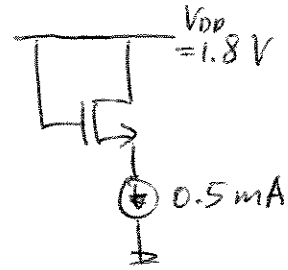
$$\Rightarrow I_D = V_S/2k\Omega \approx 0.59\ mA.$$

$$g_m = \frac{2I_D}{V_{GS}-V_{TH}} = \frac{2(0.59mA)}{(1.8-1.18-0.4)V} \approx 0.0054\ 1/\Omega$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.1\,V^{-1})(0.59mA)} \approx 16.9\ k\Omega$$

(e)





$$0.5\ mA = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[(V_{DD}-V_S)-V_{TH}\right]^2\left[1+\lambda(V_{DD}-V_S)\right]$$

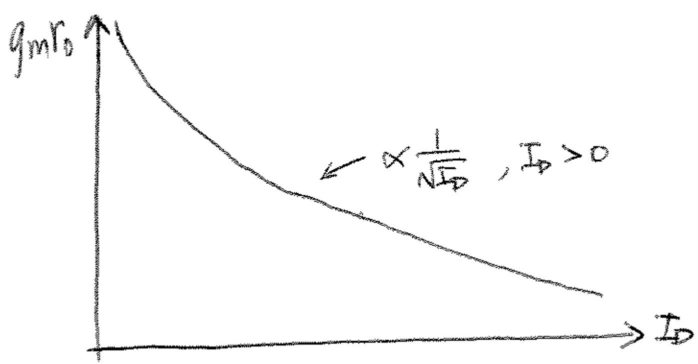
Solving this equation gives $V_S \approx 1.19\ V$

$$\Rightarrow\ g_m = \frac{2I_D}{V_{GS}-V_{TH}} = \frac{2(0.5mA)}{(1.8-1.19-0.4)V} \approx 0.0048\frac{1}{\Omega}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.1\ V^{-1})(0.5mA)} = 20\ K\Omega.$$

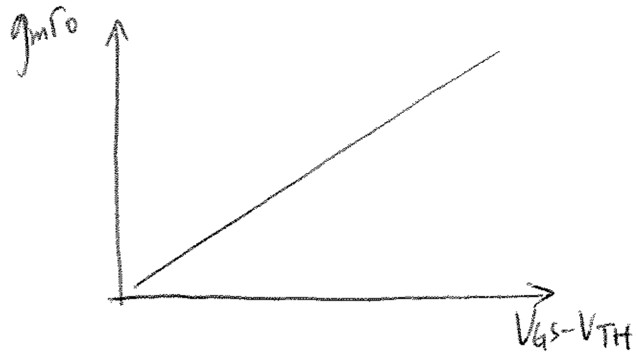
34. $g_m = \sqrt{2\mu C_{ox} \frac{W}{L} I_D}$ $\quad r_o = \left(\frac{\partial I_D}{\partial V_{DS}}\right)^{-1} = \frac{1}{\lambda I_D}$

$$g_m r_o = \frac{\sqrt{2\mu C_{ox}(W/L) I_D}}{\lambda I_D} = \frac{1}{\lambda} \sqrt{\frac{2\mu C_{ox}(W/L)}{I_D}}$$

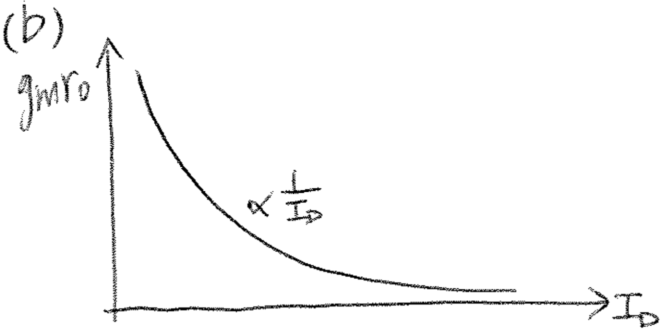


35 (a) $g_m = \mu C_{ox} \frac{W}{L}(V_{GS} - V_{TH})$ $\qquad r_o = \frac{1}{\lambda I_D}$

$$g_m r_o = \frac{\mu C_{ox}(W/L)(V_{GS} - V_{TH})}{\lambda I_D}$$



(b)



36. Given NMOS with $\lambda = 0.1\ V^{-1}$ $g_m r_o = 20$, $V_{DS} = 1.5\,V$

determine W/L if $I_D = 0.5\,mA$.

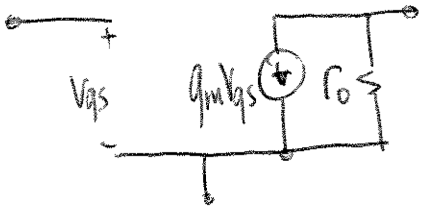
$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.1\,V^{-1})(0.5\,mA)} = 20\ k\Omega$$

$$\Rightarrow g_m = \frac{20}{20\,k\Omega} = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}$$

$$\therefore \frac{W}{L} = \left(\frac{20}{20\,k\Omega}\right)^2 \frac{1}{2\mu_n C_{ox} I_D}$$

$$= \left(\frac{1}{1\,k\Omega}\right)^2 \frac{1}{2\left(200\frac{\mu A}{V^2}\right)(0.5\,mA)} \approx 5.$$

37.



Given $\lambda = 0.2\ V^{-1}$

$g_m r_o = 20$

$V_{DS} = 1.5\ V$

$I_D = 0.5\ mA$

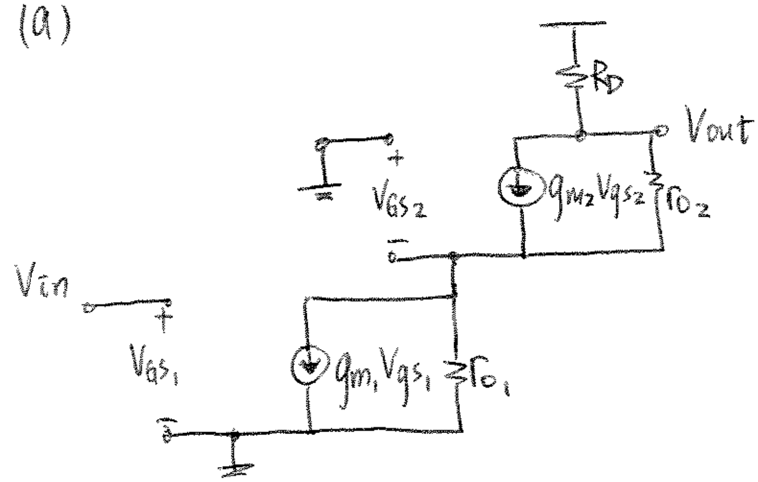
Calculate $\frac{W}{L}$.

$$g_m = \frac{20}{r_o} = 20 \cdot \lambda I_D = 20(0.2V^{-1})(0.5\ mA) = 0.002\ 1/\Omega$$

$$\Rightarrow g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

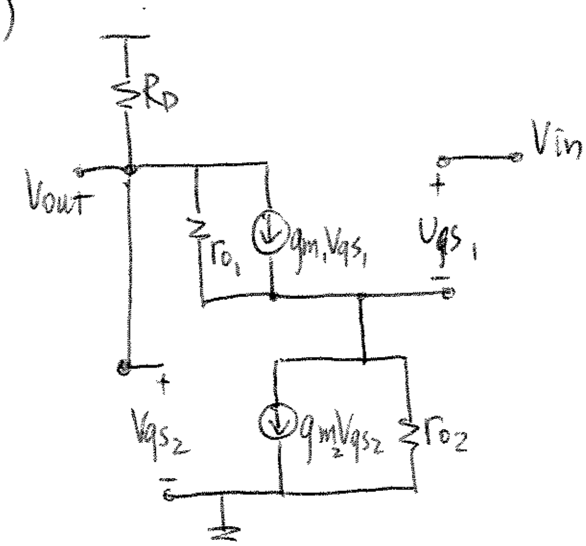
$$\therefore \frac{W}{L} = \frac{g_m^2}{2 \mu_n C_{ox} I_D} = \frac{(0.0002\ 1/\Omega)^2}{2\left(200 \frac{\mu A}{V^2}\right)(0.5 mA)} = 20$$

38. (a)



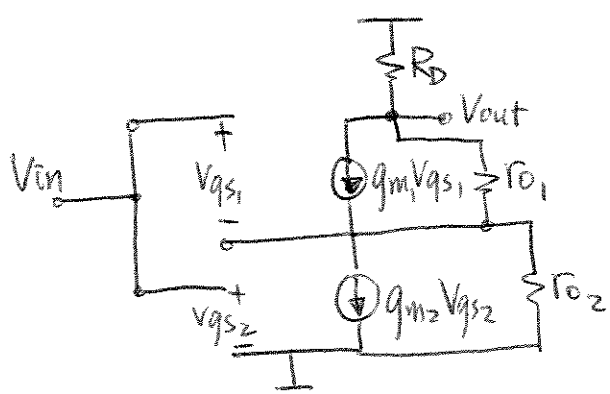
R_D
Vout
+
V_{GS2}
−
$g_{m2}V_{gs2}$
r_{o2}
Vin
+
V_{gs1}
−
$g_{m1}V_{gs1}$
r_{o1}

(b)



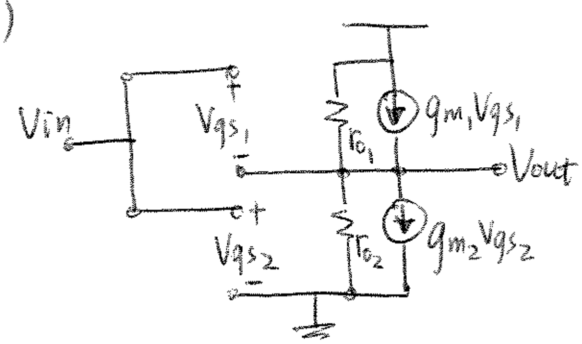
R_D
Vin
Vout
+
r_{o1}
$g_{m1}V_{gs1}$
V_{gs1}
−
+
V_{gs2}
−
$g_{m2}V_{gs2}$
r_{o2}

(c)

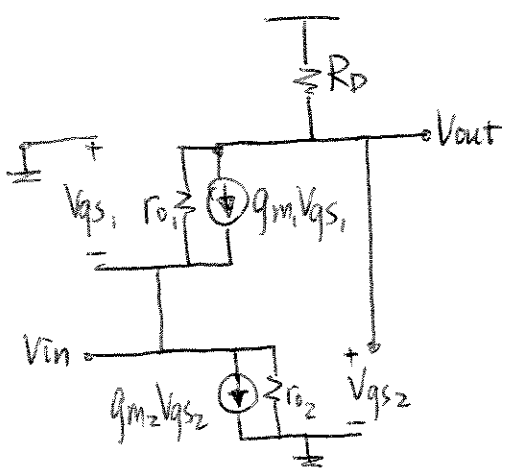


R_D
Vout
+
Vin
V_{gs1}
−
$g_{m1}V_{gs1}$
r_{o1}
+
V_{gs2}
−
$g_{m2}V_{gs2}$
r_{o2}

(d)


Vin
Vgs1
ro1
gm1Vgs1
Vout
Vgs2
ro2
gm2Vgs2

(e)


RD
Vout
Vgs1
ro1
gm1Vgs1
Vin
gm2Vgs2
ro2
Vgs2

39. (a) OFF $\because |V_{SG}| = 0$

(b) OFF $\because |V_{SG}| < |V_{TH}| = 0.4V$

(c) SATURATION $\because |V_{SD}| > |V_{SG}| - |V_{TH}|$

(d) OFF $\because V_{SG} < |V_{TH}|$

40. (a) SATURATION $\because$ $V_{SD} > V_{SG} - |V_{TH}|$

(b) LINEAR (RESISTIVE) $\because$ $V_{SG} > |V_{TH}|$

$V_{SD} \ll 2(V_{SG} - |V_{TH}|)$

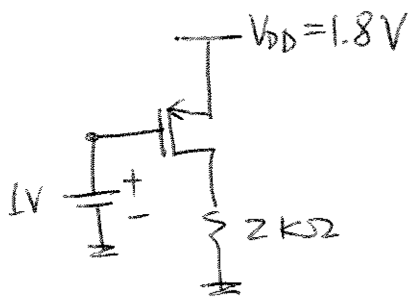
(c) (EDGE OF) SATURATION $\because$ $V_{SG} > |V_{TH}|$

$V_{SD} = V_{SG} - |V_{TH}|$

(d) TRIODE $\because$ $V_{SG} > |V_{TH}|$

$V_{SD} < V_{SG} - |V_{TH}|$

41.



$\lambda = 0$

At the edge of saturation, $V_{SD} = V_{SG} - |V_{TH}|$
$\Rightarrow V_D = 1.4\ V.$

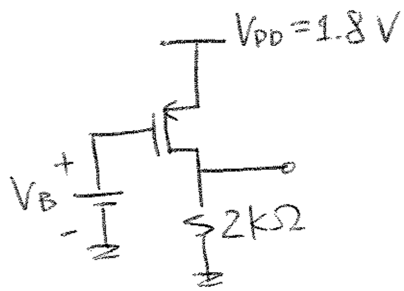
By KCL, $I_{D1} = I_R$

$$\Rightarrow \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{SG} - |V_{TH}|)^2 = \frac{V_D}{2\,k\Omega}$$

$$\therefore \frac{W}{L} = \frac{V_D}{2\,k\Omega} \cdot \frac{2}{\mu_p C_{ox}(V_{SG} - |V_{TH}|)^2}$$

$$= \frac{1.4\ V}{2\,k\Omega} \cdot \frac{2}{100\,\frac{\mu A}{V^2}\,(0.8\ V - 0.4\ V)^2} \approx 87.5$$

42.



$\lambda = 0$

When $V_B = 1V$, $W/L = 87.5$

When $V_B = 0.8V$,

$$I_D = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{SG} - |V_{TH}|)^2$$

$$= \frac{1}{2}\left(100\frac{\mu A}{V^2}\right)(87.5)(1-0.4)^2 V^2 \approx 1.6\,mA$$

$\Rightarrow V_D = I_D(2k\Omega) \approx 3.2V$, which exceeds the supply voltage!

$\therefore$ PMOS goes into triode:
($\because I_D$ is too large)

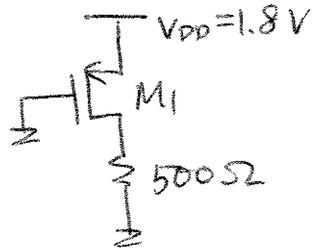
By KCL,

$$\frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[(V_{SG} - |V_{TH}|)\cdot 2V_{SD} - V_{SD}^2\right] = (V_{DD} - V_{SD})/2k\Omega$$

Solving this equation numerically (or trial-and-error) gives $V_{SD} \approx 0.18\ V$

$$\Rightarrow I_D = \frac{V_{DD} - V_{SD}}{2k\Omega} = \frac{(1.8 - 0.18)V}{2k\Omega} \approx 0.81\ mA$$

43 (a)



Assume M_1 in triode (since V_{SG} is large). Note: if assumption is incorrect, results will show that.

By KCL, $I_D = I_R$

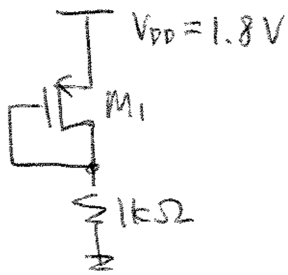
$$\Rightarrow \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[(V_{SG} - |V_{TH}|)\cdot 2V_{SD} - V_{SD}^2\right] = \frac{V_{DD} - V_{SD}}{500\Omega}$$

This is a quadratic relation on V_{SD}. Solving it yields $V_{SD} \approx 0.42\,V$

Verify assumption: $V_{SD} \overset{?}{<} V_{SG} - |V_{TH}|$
$0.42 < 1.8 - 0.4 = 1.4$ ($\checkmark$)

$$I_D = \frac{V_{DD} - V_{SD}}{500\Omega} = \frac{(1.8 - 0.42)V}{500\Omega} \approx 2.8\,mA$$

(b)



$\frac{W}{L} = \frac{10}{0.18}$ $\lambda = 0$

$\mu_p C_{ox} = 100 \frac{\mu A}{V^2}$

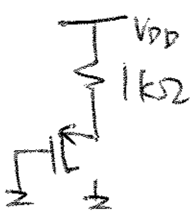
By KCL,

$$\frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left(V_{SG} - |V_{TH}|\right)^2 = \frac{(V_{DD} - V_{SG})}{1k\Omega}$$

Solving this quadratic equation gives
$V_{SG} \approx 0.61\ V$

$$\Rightarrow I_D = \frac{V_{DD} - V_{SG}}{1k\Omega} = \frac{(1.8 - 0.61)V}{1k\Omega} \approx 1.2\ mA$$

(c)



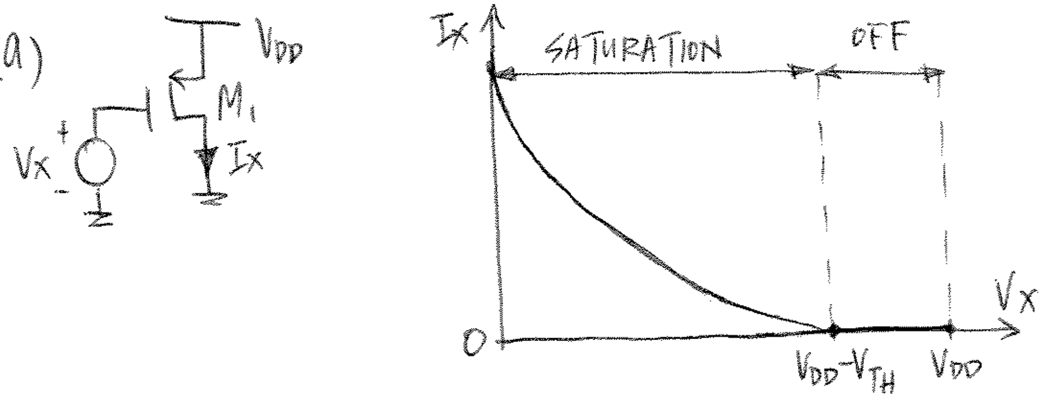
By KCL,

$$\frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left(V_{SG} - |V_{TH}|\right)^2 = \frac{V_{DD} - V_{SG}}{1k\Omega}$$

Solving this gives $V_{SG} \approx 0.61\ V$

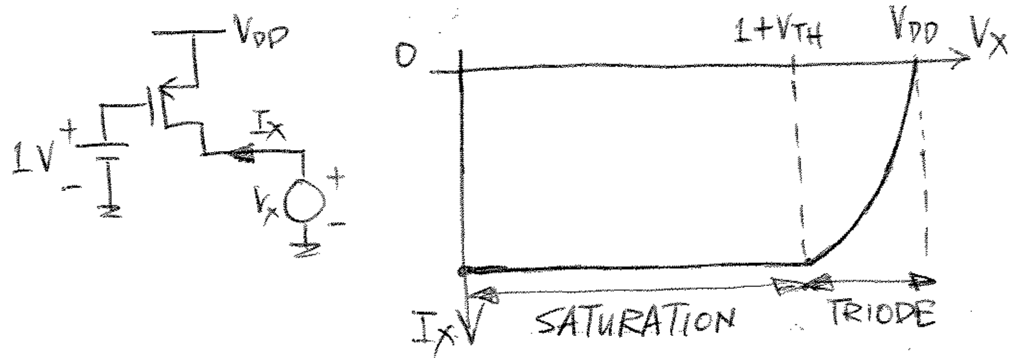
$$\Rightarrow I_D = \frac{V_{DD} - V_{SG}}{1k\Omega} \approx 1.2\ mA$$

44. (a)



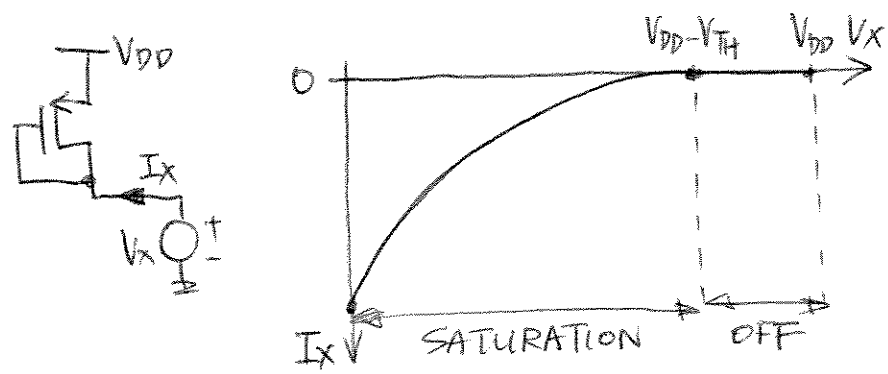
V_{DD}
M_1
V_X
I_X
SATURATION
OFF
0
$V_{DD}-V_{TH}$
V_{DD}

(b)



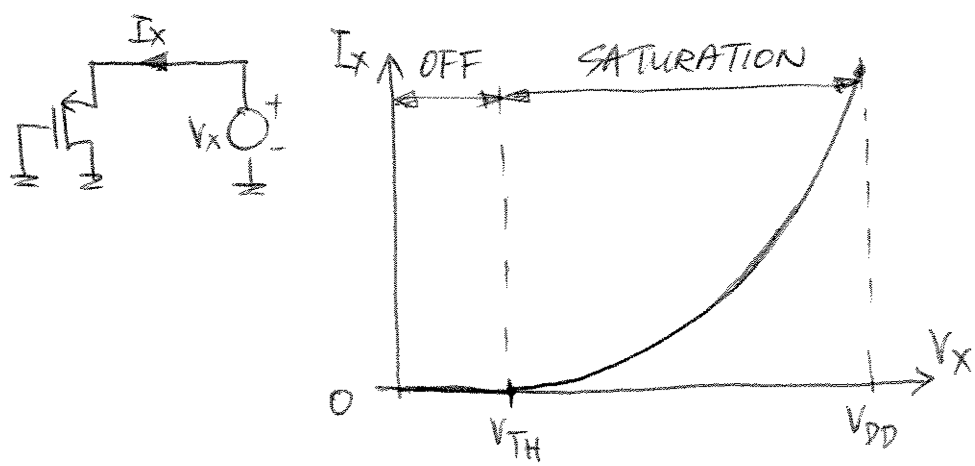
V_{DD}
1V
I_X
V_X
0
$1+V_{TH}$
SATURATION
TRIODE

(c)



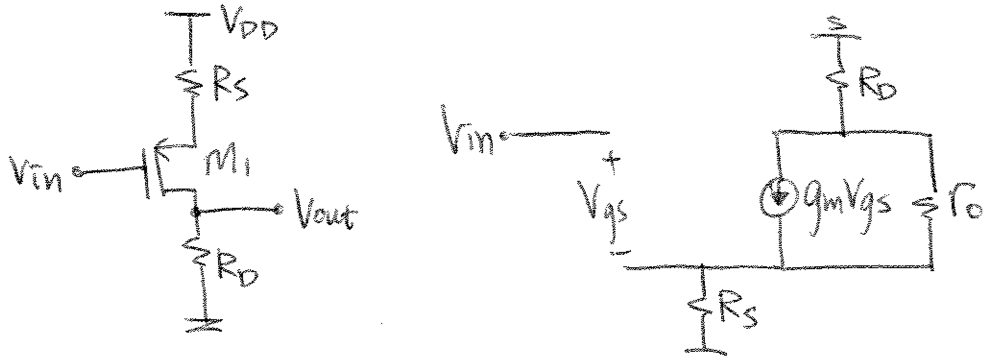
V_{DD}
I_X
V_X
0
$V_{DD}-V_{TH}$
SATURATION
OFF

(d)



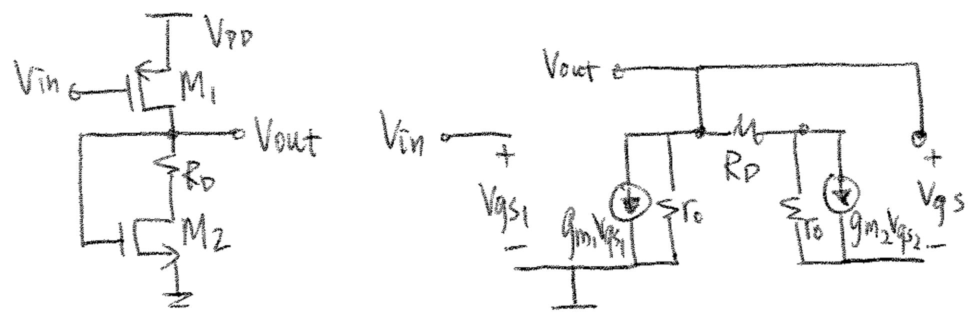
I_X
V_X
OFF
SATURATION
0
V_{TH}
V_{DD}

45. (a)



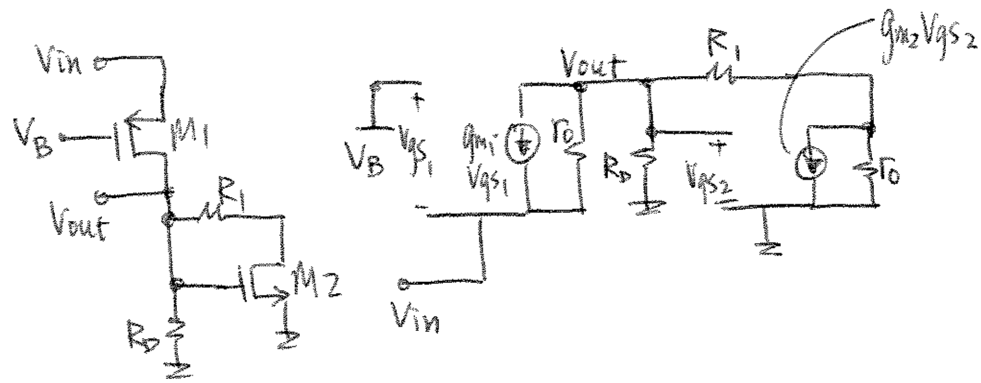
VDD
Rs
Vin
M1
Vout
RD
Vin
+
Vgs
−
RD
gmVgs
ro
Rs

(b)



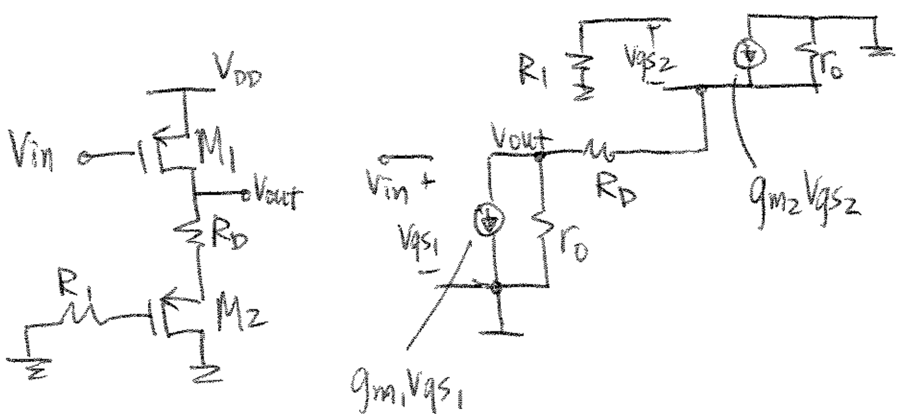
VDD
Vin
M1
Vout
RD
M2
Vout
Vin
+
Vgs1
−
gm1Vgs1
ro
RD
ro
gm2Vgs2
+
Vgs
−

(c)



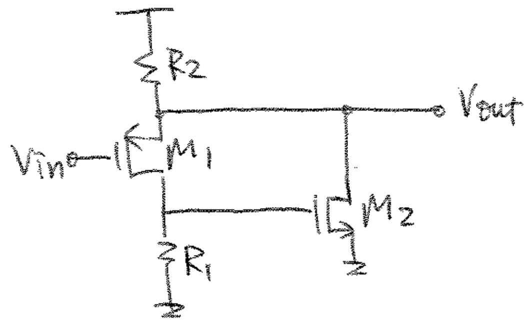
Vin
VB
M1
Vout
R1
M2
RD
Vout
R1
gm2Vgs2
+
VB
Vgs1
−
gm1Vgs1
ro
RD
+
Vgs2
ro
Vin

(d)

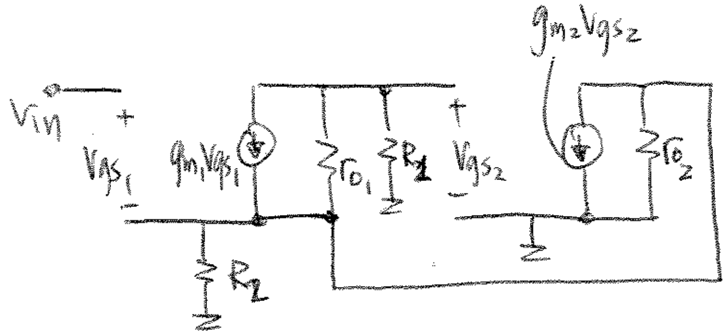


VDD
Vin
M1
Vout
RD
R1
M2
R1
+
Vgs2
−
ro
Vout
RD
gm2Vgs2
Vin +
Vgs1
−
ro
gm1Vgs1

(e)

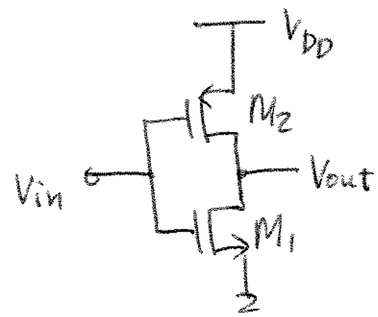


R2
Vout
Vin
M1
M2
R1



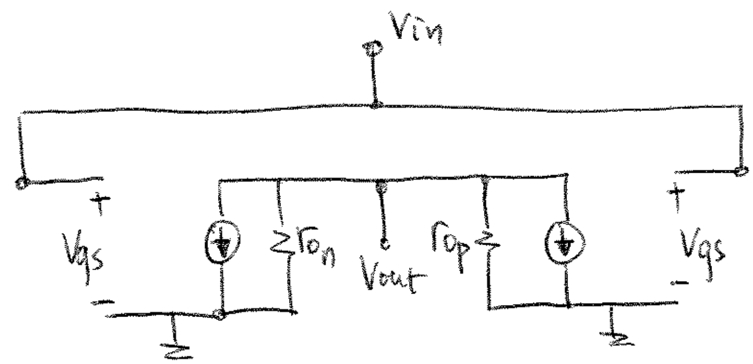
gm2Vgs2
Vin
+
Vgs1
-
gm1Vgs1
ro1
R1
+
Vgs2
-
ro2
R2

46.



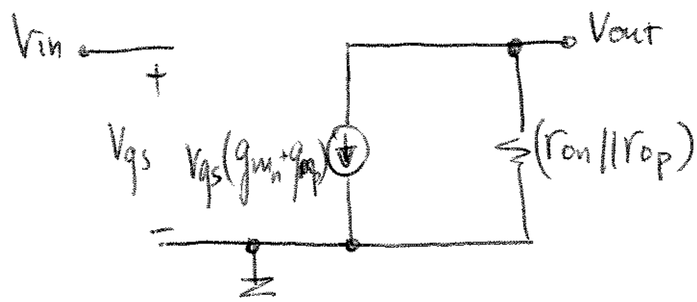
Assume λ_n & λ_p.

(a)



They are in "parallel" because from the small-signal model, both their respective SOURCE and DRAIN nodes are the same.

(b) Assuming both M_1 & M_2 are in saturation, we can combine r_o's & g_m's :



$$\therefore \quad \frac{V_{out}}{V_{in}} = -(g_{mn} + g_{mp})(r_{on} // r_{op})$$