1. Consider a two-class, two-dimensional classification task, where the feature vectors in each of the classes ω_1, ω_2 are distributed according to

$$p(\boldsymbol{x}|\omega_1) = \frac{1}{\left(\sqrt{2\pi\sigma_1^2}\right)^2} \exp\left(-\frac{1}{2\sigma_1^2}(\boldsymbol{x}-\boldsymbol{\mu}_1)^T(\boldsymbol{x}-\boldsymbol{\mu}_1)\right)$$

$$p(\boldsymbol{x}|\omega_2) = \frac{1}{\left(\sqrt{2\pi\sigma_2^2}\right)^2} \exp\left(-\frac{1}{2\sigma_2^2}(\boldsymbol{x}-\boldsymbol{\mu}_2)^T(\boldsymbol{x}-\boldsymbol{\mu}_2)\right)$$

with

$$\boldsymbol{\mu}_1 = [1, 1]^T, \quad \boldsymbol{\mu}_2 = [1.5, 1.5]^T, \quad \sigma_1^2 = \sigma_2^2 = 0.2$$

Assume that $P(\omega_1) = P(\omega_2)$ and design a Bayesian classifier

(a) that minimizes the error probability

(b) that minimizes the average risk with loss matrix

$$\Lambda = \begin{bmatrix} 0 & 1 \\ 0.5 & 0 \end{bmatrix}$$

Using a pseudorandom number generator, produce 100 feature vectors from each class, according to the preceding pdfs. Use the classifiers designed to classify the generated vectors. What is the percentage error for each case? Repeat the experiments for $\boldsymbol{\mu}_2 = [3.0, 3.0]^T$.

pdfs = probability density functions

2. In a two-class classification task, we constrain the error probability for one of the classes to be fixed, that is, $\epsilon_1 = \epsilon$. Then show that minimizing the error probability of the other class results in the likelihood test

$$\text{decide } \boldsymbol{x} \text{ in } \omega_1 \text{ if } \frac{P(\omega_1|\boldsymbol{x})}{P(\omega_2|\boldsymbol{x})} > \theta$$

where θ is chosen so that the constraint is fulfilled. This is known as the *Neyman–Pearson test*, and it is similar to the Bayesian minimum risk rule.

3. In a three-class, two-dimensional problem the feature vectors in each class are normally distributed with covariance matrix

$$\Sigma = \begin{bmatrix} 1.2 & 0.4 \\ 0.4 & 1.8 \end{bmatrix}$$

The mean vectors for each class are $[0.1, 0.1]^T$, $[2.1, 1.9]^T$, $[-1.5, 2.0]^T$. Assuming that the classes are equiprobable, (a) classify the feature vector $[1.6, 1.5]^T$ according to the Bayes minimum error probability classifier.