Homework Deadline May 24 (Thursday)

Problem 1 (score 4)

- For the linear SVM in the non-separable case

$$\begin{aligned} \min_{w,b,\varepsilon_i} \quad & \frac{1}{2}\|w\|_2^2 + C\sum_{i=1}^{N}\varepsilon_i \\ \text{s.t.} \quad & y_i\left(\langle w, x_i\rangle + b\right) \geq 1 - \varepsilon_i, \\ & \varepsilon_i \geq 0, \quad \forall i = 1, \ldots, N \end{aligned}$$

derive its dual problem and express the optimal hyperplane $f(x) = \langle w^*, x\rangle + b^*$ using the solution to the dual problem

Problem 2 (score 3)

- In clustering, we often need to measure the distance between two sets. A function $d: X \times X \to [0, \infty)$ is called a *distance* if for all $x, y, z \in X$ the following 4 conditions are satisfied:
  - Non-negativity: $d(x, y) \geq 0$
  - Identity: $d(x, y) = 0 \Leftrightarrow x = y$
  - Symmetry: $d(x, y) = d(y, x)$
  - Triangle inequality $d(x, z) \leq d(x, y) + d(y, z)$
- Is the following function a distance between two sample sets A and B?

$$H(A, B) = \max(h(A, B), h(B, A))$$

- where $h(A, B) = \max_{a \in A} \min_{b \in B} \left|\left|a - b\right|\right|_2$

Problem 3 (score 3)

- Consider the directed graph shown on the right in which none of the variables is observed.
  - Show that $a \perp\!\!\!\perp b | \emptyset$
  - Suppose we now observe the variable d. Show that in general $a \not\perp\!\!\!\perp b | d$