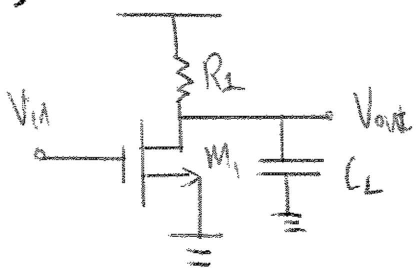
1)



$R_1 = 1\,k\Omega$

$C_L = 1pF$

$$V_{out} = -g_{m1} R_1 // \frac{1}{C_L S} V_{in} \Rightarrow \frac{V_{out}}{V_{in}} = -g_m R_1 // \frac{1}{C_L S}$$

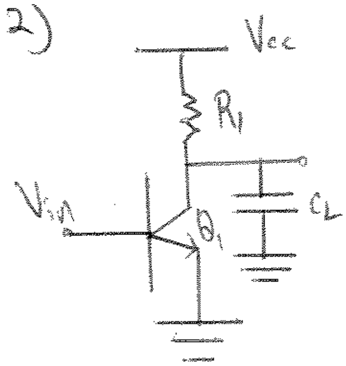
$$\frac{V_{out}}{V_{in}} = -g_{m1}\left(\frac{R_1}{R_1 C_L S + 1}\right), \quad S \to j\omega, \quad \frac{V_{out}}{V_{in}}(j\omega) = -g_{m1}\left(\frac{R_1}{R_1 C_L j\omega + 1}\right)$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_1}{\sqrt{1 + (\omega R_1 C_L)^2}}, \quad \text{Fall by } 10\% = \left|\frac{V_{out}}{V_{in}}\right| = g_m R_1\, 0.9$$

$$\Rightarrow \frac{g_m R_1}{\sqrt{1 + (\omega R_1 C_L)^2}} = g_m R_1\, 0.9 \Rightarrow \omega_{-10\%} = 4.84 \times 10^9 \text{ rad/s}$$

$$2\pi f = 4.84 \times 10^9 \Rightarrow f_{-10\%} = 7.708 \times 10^8 \text{ Hz}$$

2)



-3dB bandwidth = 1 GHz

$C_L = 2pF$

Power = 2mW

Low freq gain ?

Power = $2.5V\, I_c$, $I_c = 0.8\,mA$

Dominant Pole at the output = $\dfrac{1}{R_1 C_L} = 2\pi(1GHz)$

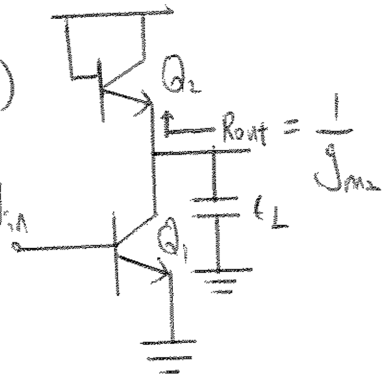
$R_1 = 79.58$ Ohm.

Low freq gain : $-g_m R_1 = \dfrac{-I_c R_1}{V_T} = \dfrac{(79.58)(0.8)}{26}$

$A_v\Big|_{low\text{-}freq} = -2.45$

3)

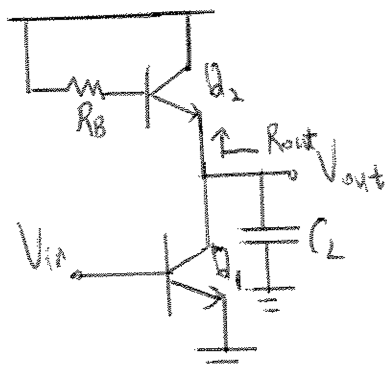
a)



Assume $\beta >> 1$

$$-3dB = \frac{g_{m2}}{C_L}$$

b)

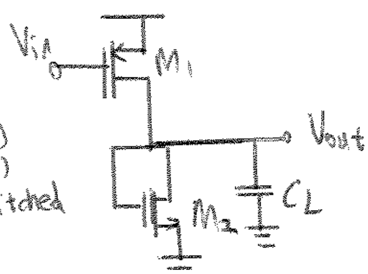


$$R_{out} = \frac{1}{g_{m2}} + \frac{R_B}{\beta+1}$$

$$-3dB = \frac{(\beta+1)\, g_{m2}}{C_L\,[\beta + R_B g_{m2}]}$$

d)

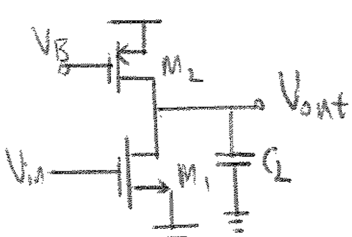
note, parts d) and c) are switched



$$R_{out} = \frac{1}{g_{m2}} \,//\, r_{o2} \,//\, r_{o1} \approx \frac{1}{g_{m2}}$$

$$-3dB = \frac{g_{m2}}{C_L}$$

c)

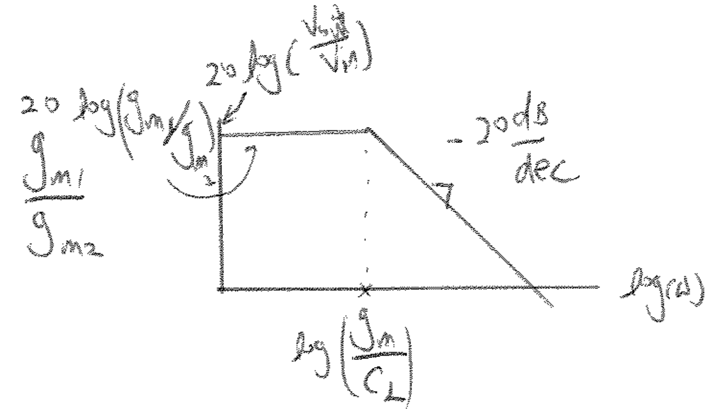


$$R_{out} = r_{o1} \,//\, r_{o2}$$

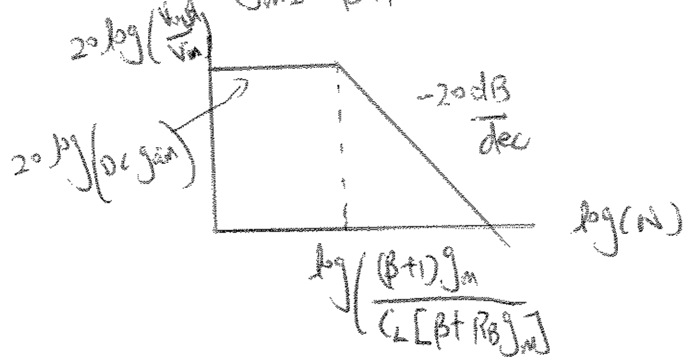
$$-3dB = \frac{1}{(r_{o1} \,//\, r_{o2})\, C_L}$$

4)

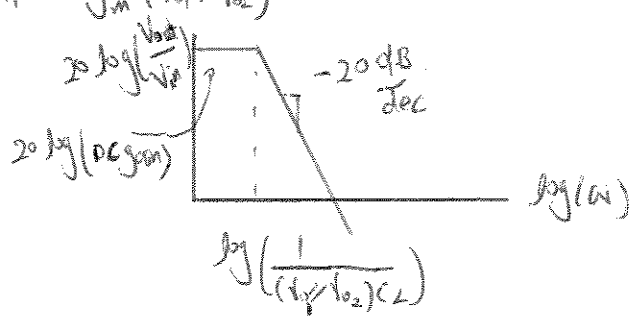
a) $|DC\ gain| = \dfrac{g_{m1}}{g_{m2}}$



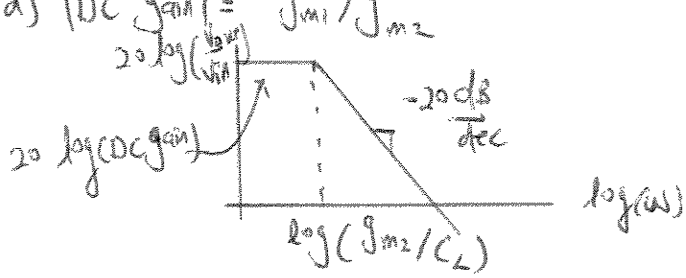
b) $|DC\ gain| = g_{m1}\left(\dfrac{1}{g_{m2}} + \dfrac{R_B}{\beta+1}\right)$



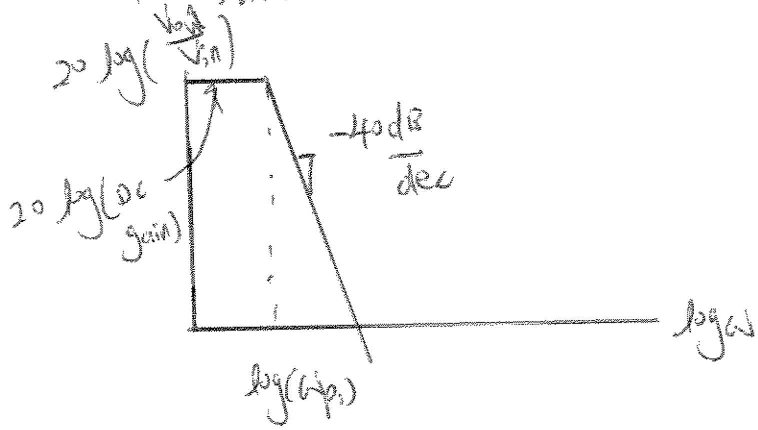
c) $|DC\ gain| = g_m(r_{o1} // r_{o2})$



d) $|DC\ gain| = g_{m1}/g_{m2}$



5) 2 poles at ω_{p1}, means slope is -40 dB/dec

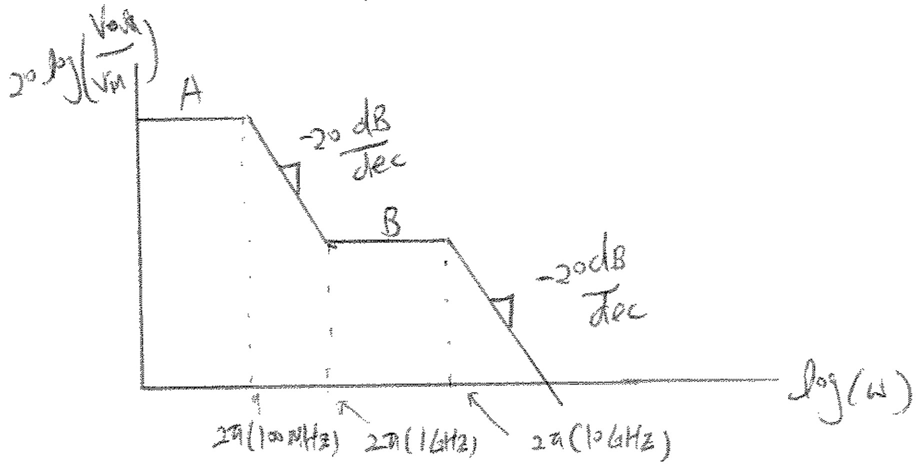


* Assuming transfer function is in the form of
$$\frac{A}{\left(\sqrt{\left(\frac{\omega}{\omega_{p_1}}\right)^2+1}\right)^2}$$

6)

Poles at 100 MHz, 10 GHz

Zero at 1 GHz.

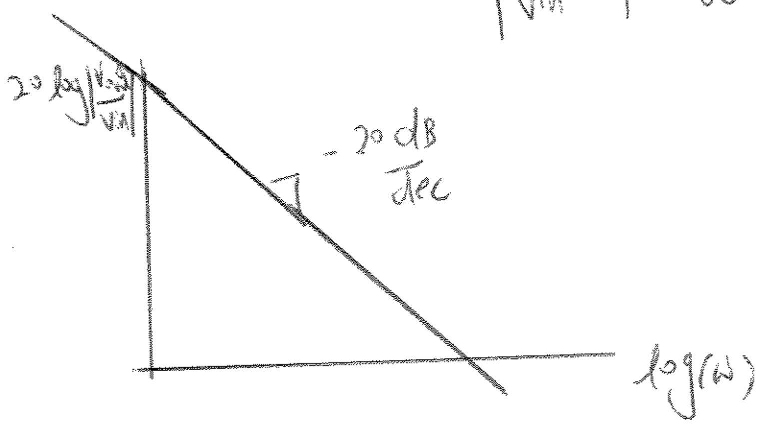


$A(100\text{ MHz}) = B(1\text{ GHz})$

$B = 0.1A$

7) Ideal Integrator : $\frac{V_{out}}{V_{in}}(s) = \frac{1}{s}$

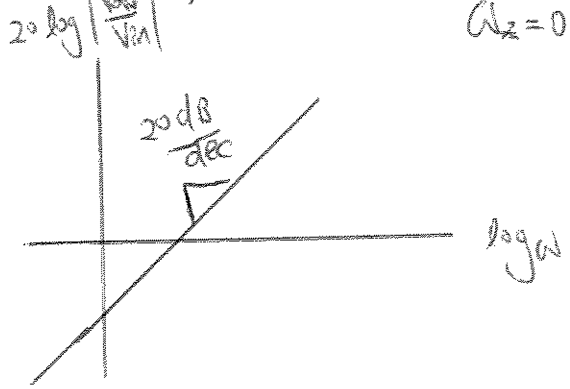
$$\left|\frac{V_{out}}{V_{in}}(j\omega)\right| = \frac{1}{\omega}$$



For an integrator, the gain at arbitrary low freq approaches infinity.

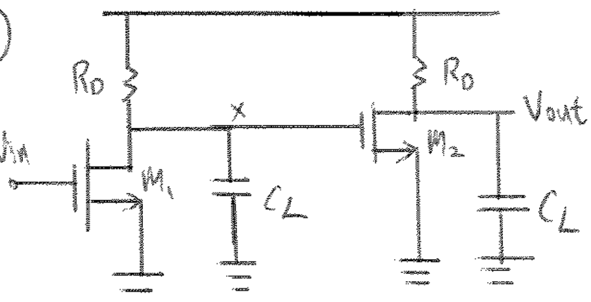
8) Ideal differentiator : $s = \frac{V_{out}}{V_{in}}$, $\left|\frac{V_{out}}{V_{in}}(j\omega)\right| = \omega$

$\omega_z = 0$



For an ideal differentiator, gain at arbitrary high freq approaches infinity.

9)



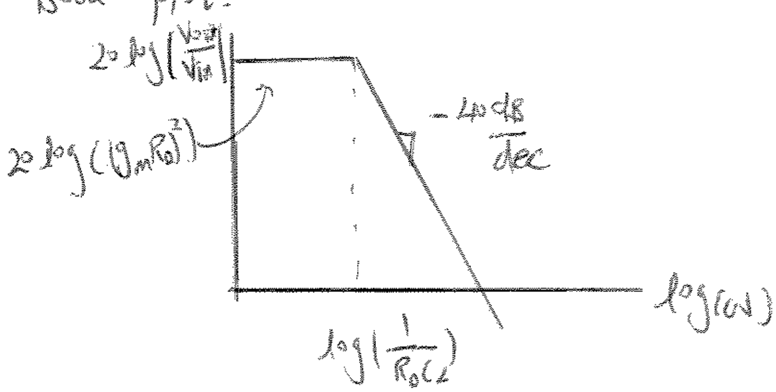
$\lambda = 0$, & neglect other caps.

DC gain: $\dfrac{V_x}{V_{in}} = -g_m R_D$, $\dfrac{V_{out}}{V_x} = -g_m R_D$

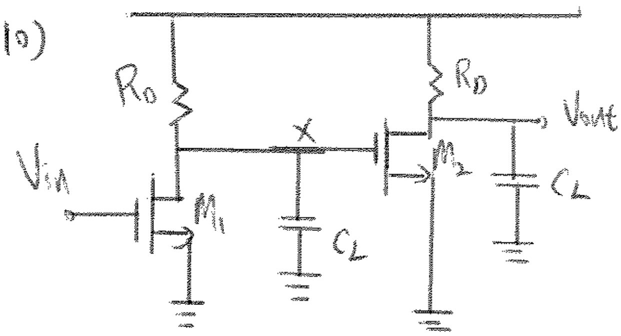
$$\frac{V_{out}}{V_{in}} = (g_m R_D)^2 \qquad \text{(At DC)}$$

2 poles at $\dfrac{1}{R_D C_L}$

Bode plot:



10)



$$\frac{V_x}{V_{in}}(s) = -g_m\left(R_D \,//\, \frac{1}{C_L s}\right), \quad \frac{V_{out}}{V_x}(s) = -g_m\left(\frac{R_D}{R_D C_L s + 1}\right)$$

$$= -g_m\left(\frac{R_D}{R_D C_L s + 1}\right)$$

$$H(s) = \frac{V_x}{V_{in}}(s)\,\frac{V_{out}}{V_x}(s) = \left(\frac{g_m R_D}{R_D C_L s + 1}\right)^2$$

$$s \to \omega j, \quad H(j\omega) = \left(\frac{g_m R_D}{1 + R_D C_L j\omega}\right)^2$$

$$|H(j\omega)| = \frac{(g_m R_D)^2}{1 + (R_D C_L \omega)^2}$$

-3dB Band Width:

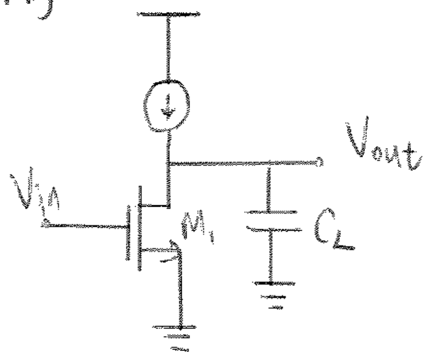
$$\frac{(g_m R_D)^2}{1 + (R_D C_L \omega)^2} = \frac{(g_m R_D)^2}{\sqrt{2}}$$

$$\Rightarrow (R_D C_L \omega)^2 + 1 = \sqrt{2}$$

$$\Rightarrow \omega = \frac{\sqrt{\sqrt{2} - 1}}{R_D C_L} = \frac{0.6436}{R_D C_L} \; (\text{rad}/s)$$

$$2\pi f = \frac{0.6436}{R_D C_L} \Rightarrow f = \frac{0.10243}{R_D C_L} \; (\text{Hz})$$

1 a)



$\lambda > 0$

Since $\lambda > 0$, and we have an ideal current source, the impedance looking from out to ground is $r_o // \frac{1}{C_L s}$

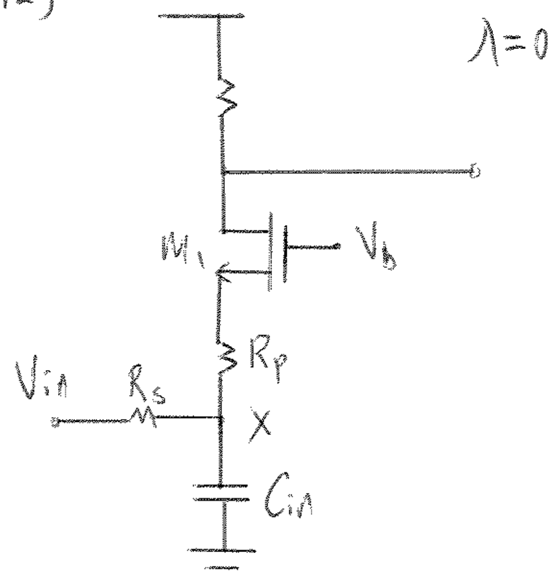
So, $V_{out} = -g_m V_{in}\left(r_o // \frac{1}{C_L s}\right)$

$H(s) = -g_m\left(\frac{r_o}{r_o C_L s+1}\right)$, $|H(j\omega)| = \frac{g_m r_o}{\sqrt{(r_o C_L \omega)^2+1}}$

For $\lambda \to 0$, $r_o \to \infty \Rightarrow H(s) \to \frac{-g_m r_o}{r_o C_L s}$

$H(s) = \frac{-g_m}{C_L s}$. A pole at origin, thus operating as an ideal integrator.

12)



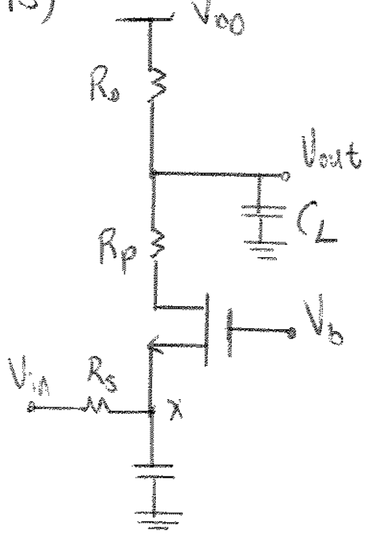
To find input pole, let $V_{in} = 0$ and find the equivalent resistance and capacitance from node X to ground.

$$R_X = R_S // \left(R_p + \frac{1}{g_{m_1}} \right), \quad C_X = C_{in}$$

$$\omega_{p,in} = \frac{1}{C_{in} \left[R_S // \left(R_p + \frac{1}{g_{m_1}} \right) \right]}$$

$$\omega_{p,out} = \frac{1}{R_D C_L}$$

13)



$\lambda = 0$, neglect all other caps.

$R_X = R_S // \frac{1}{g_m}$

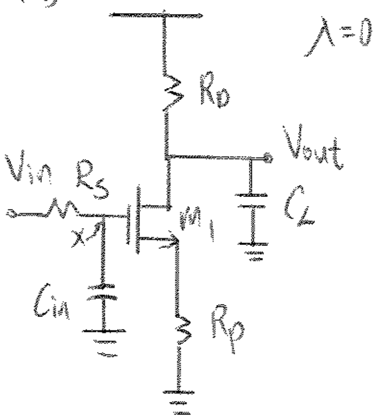
$C_X = C_{in}$

$R_{out} = R_D$ (since $r_o = \infty$)

$C_{out} = C_L$

$$\omega_{pin} = \frac{1}{(R_S // \frac{1}{g_m}) C_{in}}, \qquad \omega_{pout} = \frac{1}{R_D C_L}$$

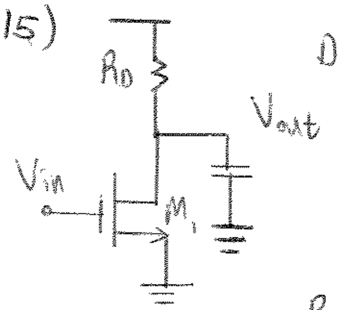
14)



$R_X = R_S$, $R_{out} \approx R_D$

$C_X = C_{in}$, $C_{out} \approx C_L$

$$\omega_{pin} = \frac{1}{R_S C_{in}} \quad , \quad \omega_{pout} = \frac{1}{R_D C_L}$$

15) 

DC Gain: $g_m R_D = \dfrac{2 I_D R_D}{V_{eff}}$

where $V_{eff} = V_{GS} - V_{th}$

Band Width: $\dfrac{1}{R_D C_L}$

Power Consumption: $V_{DD} I_D$

F.O.M. (11.5) $= \dfrac{\text{Gain} \times \text{Band Width}}{\text{Power Consumption}}$

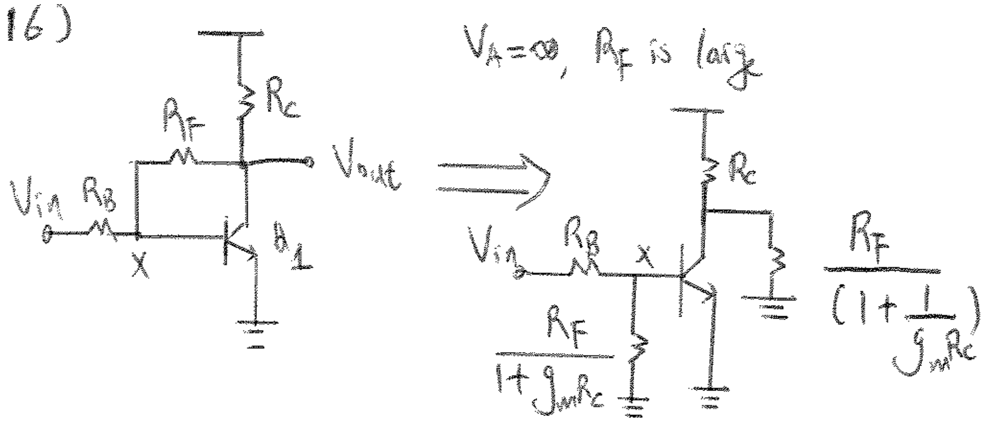
$$= \frac{\left(\dfrac{2 I_D R_D}{V_{eff}}\right)\left(\dfrac{1}{R_D C_L}\right)}{V_{DD} I_D}$$

$$= \frac{2}{V_{eff} V_{DD} C_L}$$

For practical design, $V_{eff} > V_t$, thus bipolar has a larger F.O.M. than MOS.

16)

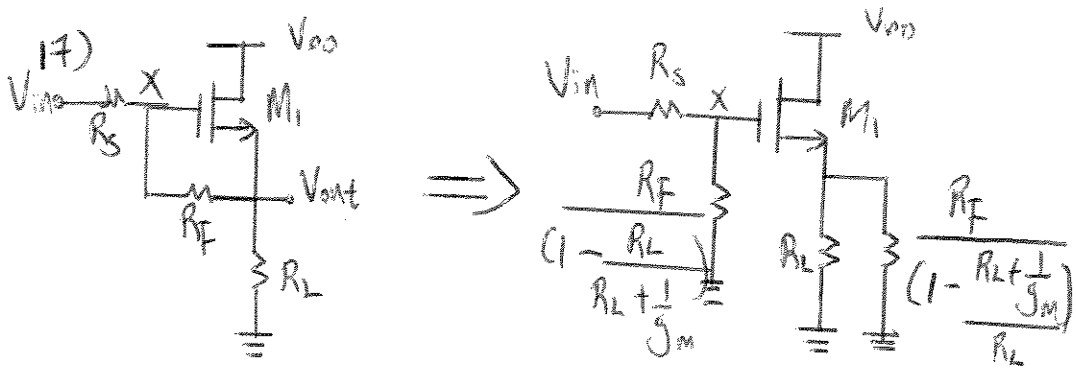
$V_A = \infty$, R_F is large



$$R_X = R_B \,//\, \left(\frac{R_F}{1+g_mR_C}\right), \quad R_{out} = R_C \,//\, \left(\frac{R_F}{1+\frac{1}{g_mR_C}}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{-R_{out}}{\frac{1}{g_m} + \frac{R_X}{\beta+1}} = \frac{-R_C \,//\, \left(\frac{R_F}{1+1/g_mR_C}\right)}{\frac{1}{g_m} + \frac{R_B \,//\, \left(R_F/(1+g_mR_C)\right)}{\beta+1}}$$

17)



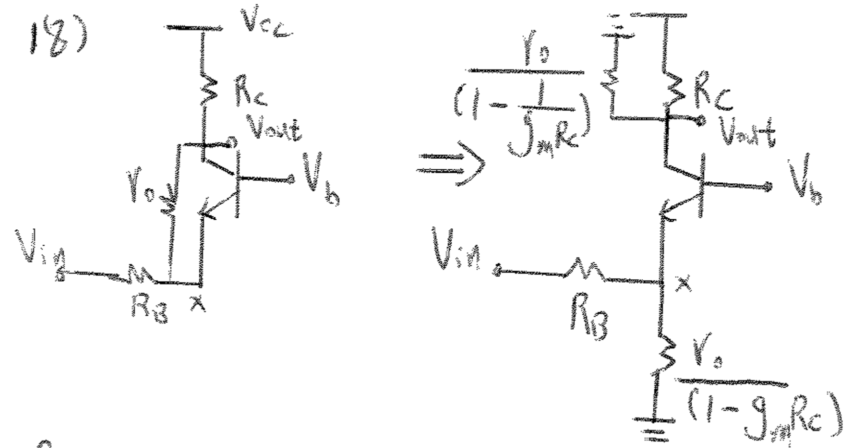
$$R_{out}: R_L \,//\, \frac{R_F}{1 - \frac{R_L + \frac{1}{g_m}}{R_L}} = R_L \,//\, \frac{R_F}{-\frac{1}{g_m R_L}}$$

$$R_{out}: R_L \,//\, -R_F g_m R_L$$

(note that R_{out} may be negative)

$$\frac{V_{out}}{V_{in}} = \frac{R_L \,//\, -R_F g_m R_L}{R_L \,//\, (-R_F R_L g_m) + \frac{1}{g_m}}$$

1(8)

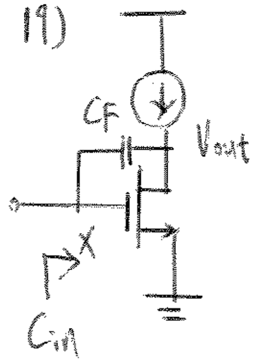


$$R_{out} = R_c \,//\left(\frac{r_o}{1-\frac{1}{g_m R_c}}\right)$$

$$R_x = R_B \,//\left(\frac{r_o}{1-g_m R_c}\right)$$

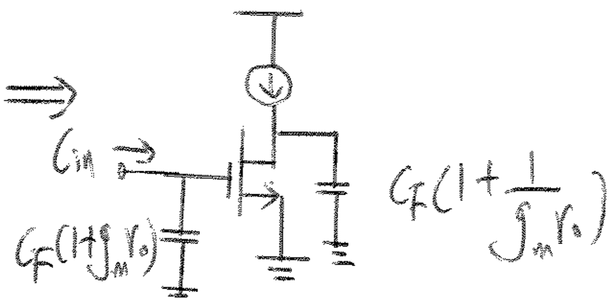
$$\frac{V_{out}}{V_{in}} = \frac{R_{out}}{R_x + \frac{1}{g_m}} = \frac{R_c \,//\left(\frac{r_o}{1-\frac{1}{g_m R_c}}\right)}{R_B \,//\left(\frac{r_o}{1-g_m R_c}\right) + \frac{1}{g_m}}$$

19)



$\lambda > 0$, DC gain $= -g_m r_o$

$\Longrightarrow$



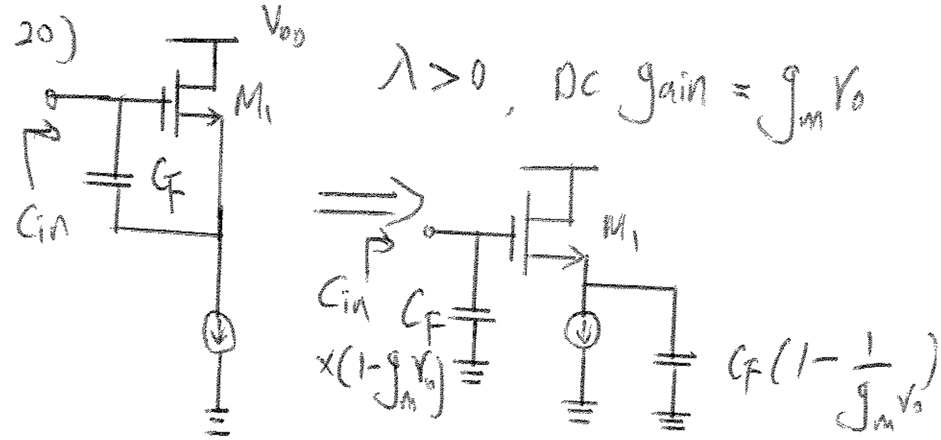
$C_{in} = C_F(1+g_m r_o)$, neglecting other caps.

As $\lambda \to 0$, $r_o \to \infty$, DC gain $\to \infty$,

$C_{in} \to \infty$, this bandwidth will $\longrightarrow 0$.

20)

$\lambda > 0$, DC gain $= g_m r_o$

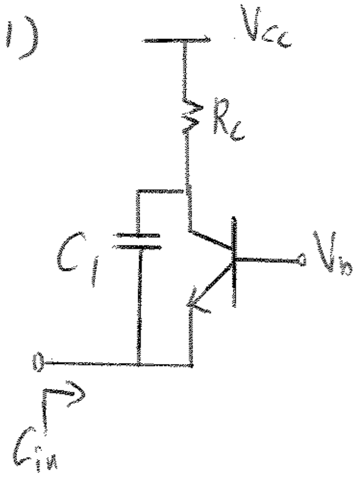


$$C_{in} = C_F(1-g_m r_o)$$

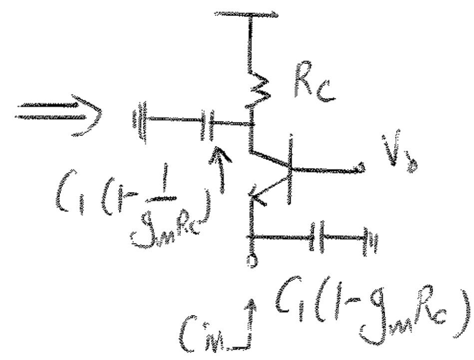
As $\lambda \to 0$, $r_o \to \infty$, $g_m r_o \to \infty$, $C_{in} = -\infty$

When $C \to$ negative in value, we have inductive activity. So right here, we have an effective infinite inductor.

21)



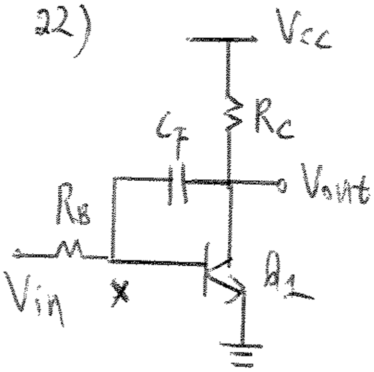
DC gain: $g_m R_c$



$$C_{in} = C_1 (1 - g_m R_c).$$

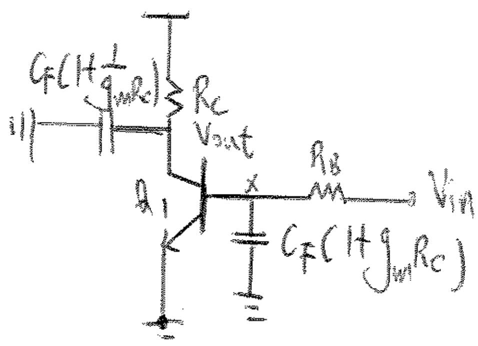
If $g_m R_c$ is designed to be larger than 1, as it normally would, we will have inductive action.

22)



DC gain (from x to out): $-g_m R_C$

$\Downarrow$



$C_{in} = C_F(1 + g_m R_C)$

$R_{in} = R_B // r_\pi$

$C_{out} = C_F\left(1 + \frac{1}{g_m R_C}\right)$

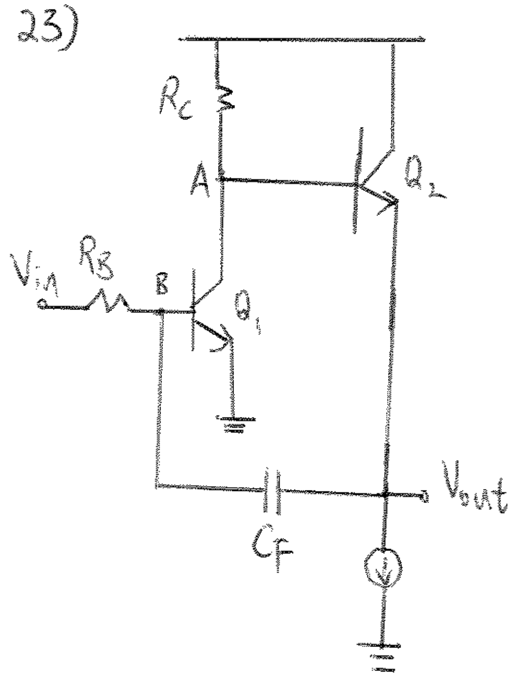
$R_{out} = R_C$

$$\omega_{pin} = \frac{1}{R_B // r_\pi \left[C_F(1 + g_m R_C)\right]}$$

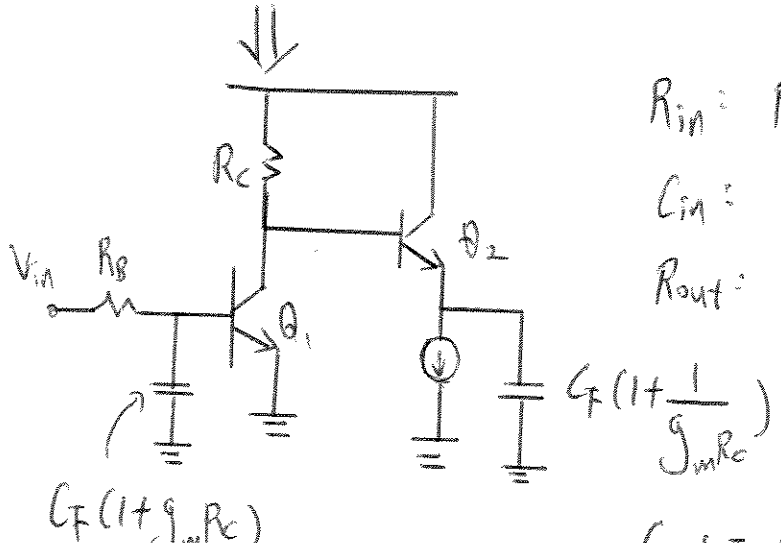
$$\omega_{pout} = \frac{1}{R_C C_F\left(1 + \frac{1}{g_m R_C}\right)} \approx \frac{1}{R_C C_F}$$

(If $g_m R_C \gg 1$)

23)



The gain from B to A is $-g_mR_c$, from A to out is 1 (since we have an ideal current source). So the gain from B to out is $-g_mR_c$.



R_{in}: $R_B // r_\pi$

C_{in}: $C_F(1+g_mR_c)$

R_{out}: $\frac{1}{g_m} + \frac{R_c}{\beta+1}$

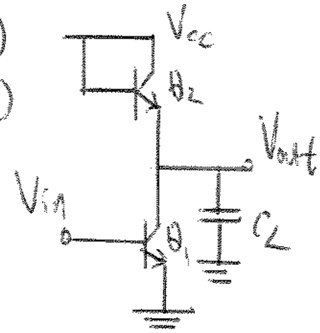
$C_{out} = C_F(1+\frac{1}{g_mR_c})$

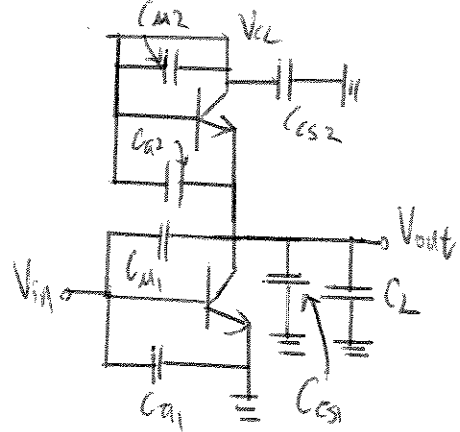
$$\omega_{pin} = \frac{1}{R_B//r_\pi[C_F(1+g_mR_c)]}$$

$$\omega_{pout} = \frac{1}{\left(\frac{1}{g_m}+\frac{R_c}{\beta+1}\right)C_F\left(1+\frac{1}{g_mR_c}\right)} \approx \frac{1}{\left(\frac{1}{g_m}+\frac{R_c}{\beta+1}\right)C_F}, \ (g_mR_c \gg 1)$$

24)

a)



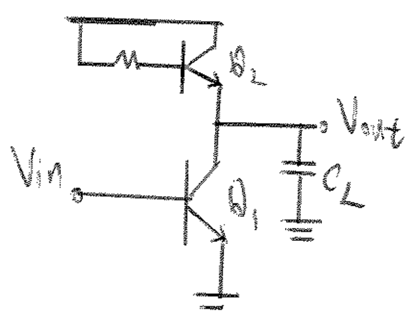


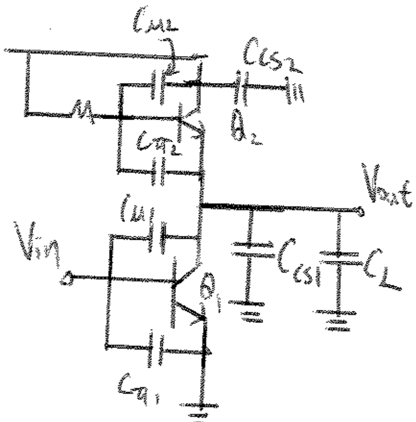
$C_{\pi 2}$, C_{CS1}, C_L are in parallel

$C_{\mu 2}$, C_{CS2} are grounded on both ends.

(and technically in parallel as well)

b)



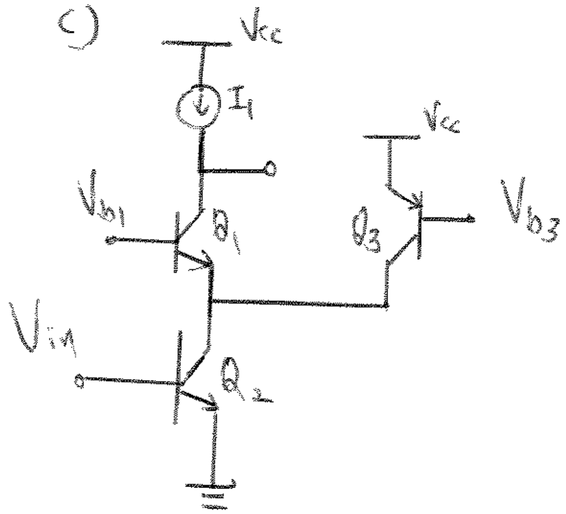


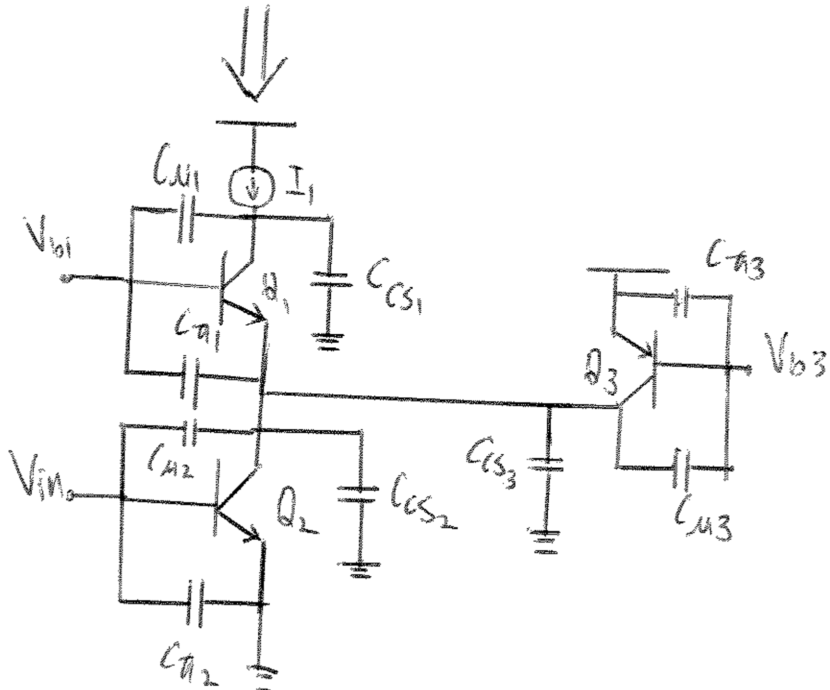
C_{CS1}, C_L are in parallel

C_{CS2} is grounded on both ends

24)

c)





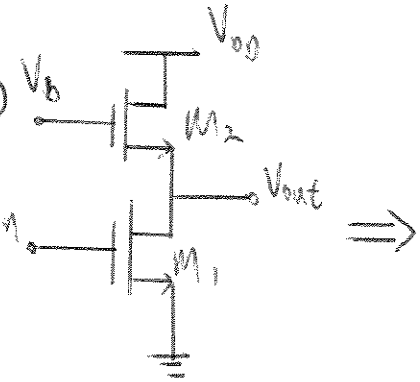
$C_{\pi 1}$, C_{CS_2}, C_{CS_3}, $C_{\mu 3}$ are in parallel

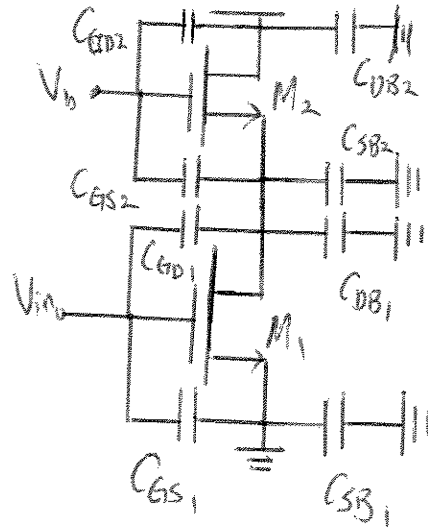
$C_{\mu 1}$, C_{CS_1} are also in parallel

$C_{\pi 3}$ is grounded on both ends

25)

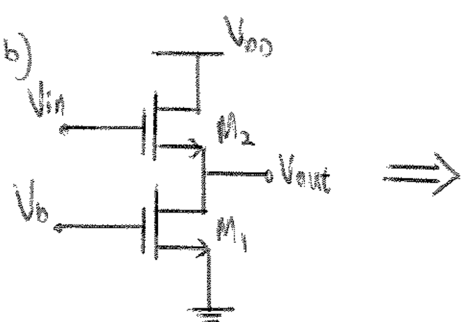
a)

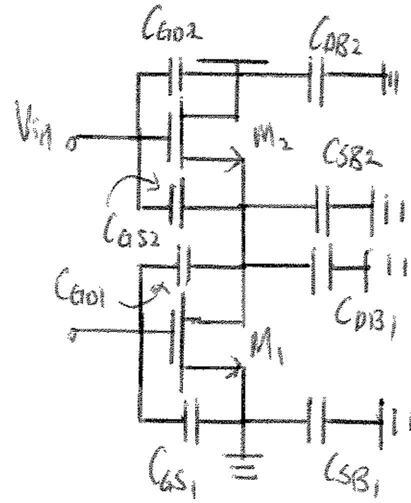




C_{GS2}, C_{SB2}, C_{DB1} are in parallel
C_{GD2}, C_{DB2} are in parallel and grounded on both ends
C_{SB1} is grounded on both ends.

b)





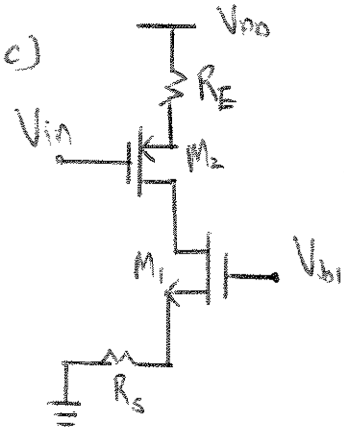
C_{GD1}, C_{DB1}, C_{SB2} are in parallel

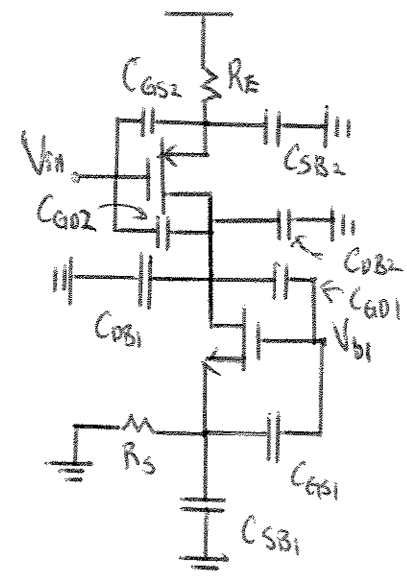
C_{GS1}, C_{SB1} are in parallel and grounded on both ends

C_{DB2} is grounded on both ends.

25)

c)



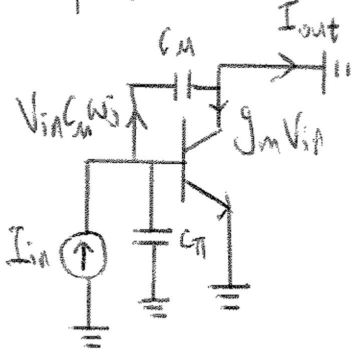


C_{DB2}, C_{GD1}, C_{DB1}, are in parallel

C_{SB1}, C_{GS1} are also in parallel.

26)

Bipolar



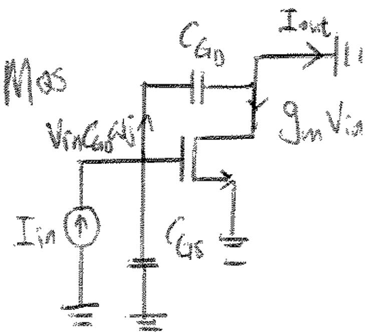
$$V_{in} = (I_{in})\left(\frac{1}{[C_\mu + C_\pi]\omega j}\right)$$ (Assuming we are at freq, and r_π can be neglected)

$$I_{out} = V_{in} C_\mu \omega j - g_m I_{in}\left(\frac{1}{[C_\mu + C_\pi]\omega j}\right)$$

$$\frac{I_{out}}{I_{in}} = \frac{C_\mu \omega j - g_m}{[C_\mu + C_\pi]\omega j} \Rightarrow \left|\frac{I_{out}}{I_{in}}\right| = \frac{\sqrt{(g_m)^2 + (C_\mu \omega_T)^2}}{[C_\mu + C_\pi]\omega_T} = 1$$

$$\omega_T^2 = \frac{g_m^2}{2C_\mu C_\pi + C_\pi^2} \Rightarrow \omega_T = 2\pi f_T = \frac{g_m}{\sqrt{2C_\mu C_\pi + C_\pi^2}}$$

MOS



Similarly for MOS, with C_μ and C_π replaced by C_{GD} and C_{GS} respectively.

$$\omega_T = 2\pi f_T = \frac{g_m}{\sqrt{2C_{GD}C_{GS} + C_{GS}^2}}$$

27)

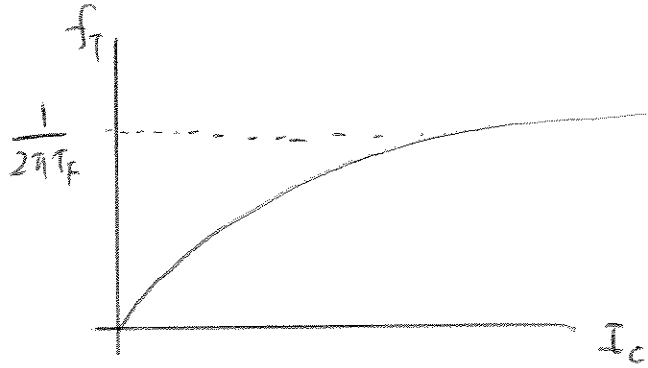
$$C_\pi = g_m \tau_F + C_{je}$$

$$2\pi f_T = \frac{g_m}{C_\pi} = \frac{g_m}{g_m \tau_F + C_{je}}$$

Assume C_{je} to be independent of I_C.

a) $$2\pi f_T = \frac{\frac{I_C}{V_T}}{\frac{I_C}{V_T}\tau_F + C_{je}} \Rightarrow f_T = \frac{I_C}{2\pi(I_C\tau_F + V_T C_{je})}$$

b)



As $I_C \to \infty$, $f_T \to \frac{1}{2\pi\tau_F}$

28)

$$C_{GS} \approx \left(\frac{2}{3}\right) W L C_{ox}$$

$$2\pi f_T = \frac{g_m}{C_{GS}} = \frac{\frac{W}{L}\mu_n C_{ox}(V_{GS}-V_{TH})}{\frac{2}{3} W L \; C_{ox}}$$

$$2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(\; V_{GS} - V_{TH})$$

29)

$$2\pi f_T = \frac{3}{2}\frac{2I_D}{WLC_{ox}}\frac{1}{(V_{GS}-V_{TH})}$$

Apparently, f_T decreases with the overdrive. However, when we look closely, I_D is actually proportional to $(V_{GS}-V_{TH})^2$ (In saturation), so f_T is proportional to $(V_{GS}-V_{TH})$.

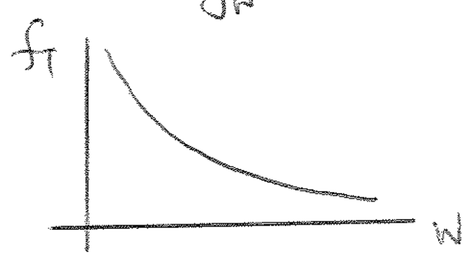
3c)

a) As $W \uparrow$, $(V_{GS} - V_{TH})$ has to $\downarrow$ by

$\frac{1}{\sqrt{W}}$ in order to maintain I_D constant

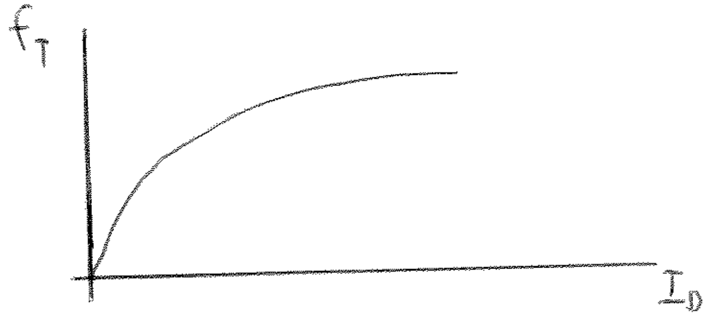
using equation $2\pi f_T = \frac{3}{2} \frac{\mu_n}{L^2} (V_{GS} - V_{TH})$

$2\pi f_T \propto \frac{1}{\sqrt{W}}$



b) $I_D \uparrow$, W constant it means $V_{GS} - V_{TH} \uparrow$ with $\sqrt{I_D}$. Using equation $2\pi f_T = \frac{3}{2} \frac{\mu_n}{L^2} (V_{GS} - V_{TH})$

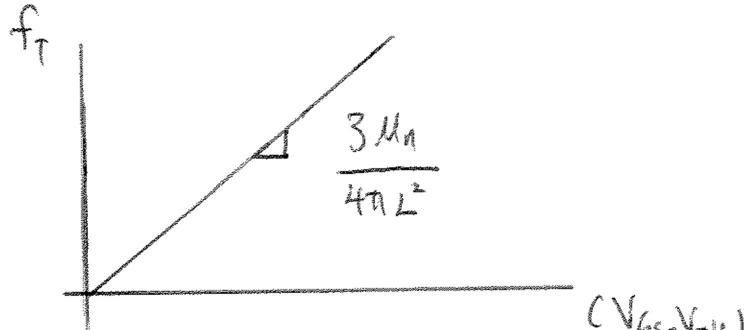
$2\pi f \propto \sqrt{I_D}$



31)

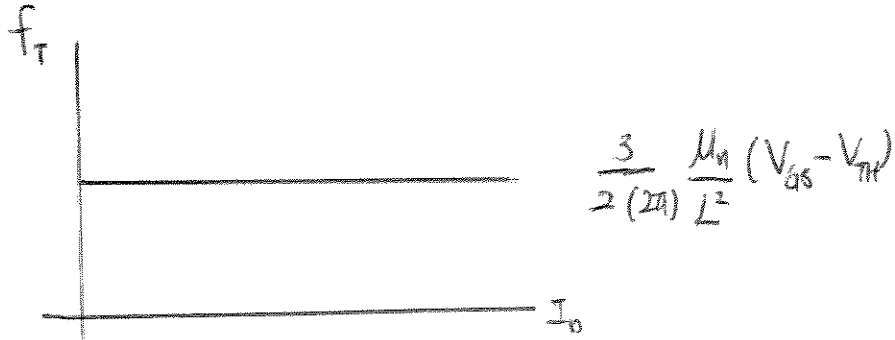
Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$

a) $2\pi f_T \propto (V_{GS}-V_{TH})$



b) Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$

$2\pi f$ = constant for all I_D.

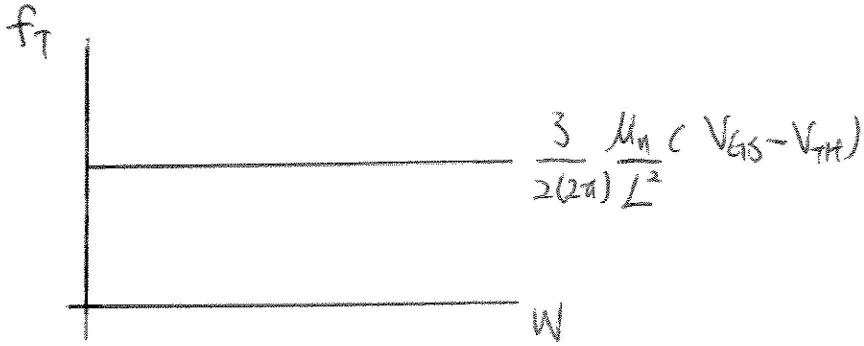


32)

a)

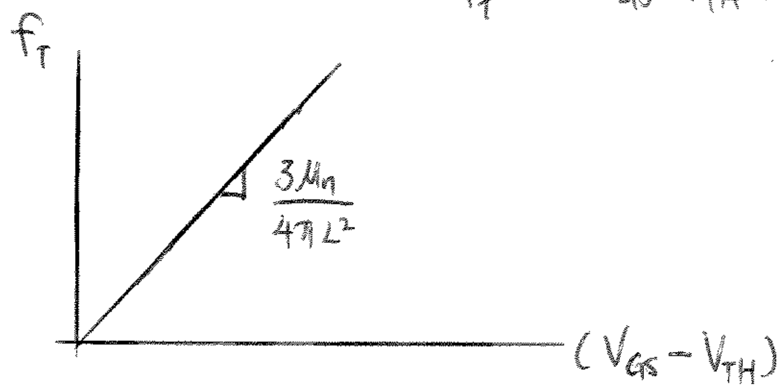
Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$

we Know that $2\pi f_T$ is constant for all W.



b) Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$.

we Know that $2\pi f_T \propto (V_{GS}-V_{TH})$.



33)

a) $I_D = \frac{1}{2}\frac{W}{L}\mu_n C_{ox}(V_{GS}-V_{TH})^2$

As $L\uparrow$, to maintain the same current and overdrive voltage, $W\uparrow$ as well.

So W also $2X$.

b) Since $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$, and

L $2X$ while $(V_{GS}-V_{TH})$ is constant,

$f_T \downarrow$ by $\frac{3}{4}$ or $f_{T_{new}} = \frac{1}{4} f_{T_{old}}$.

34)

a) $V_{GS} - V_{TH} \longrightarrow \frac{1}{2}(V_{GS} - V_{TH})$

Constant I_D and $W\uparrow$ (L constant)

$$2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$$

$$f_{T_{new}} = \frac{f_{T,old}}{2}$$

b) $V_{GS} - V_{TH} \longrightarrow \frac{1}{2}(V_{GS} - V_{TH})$

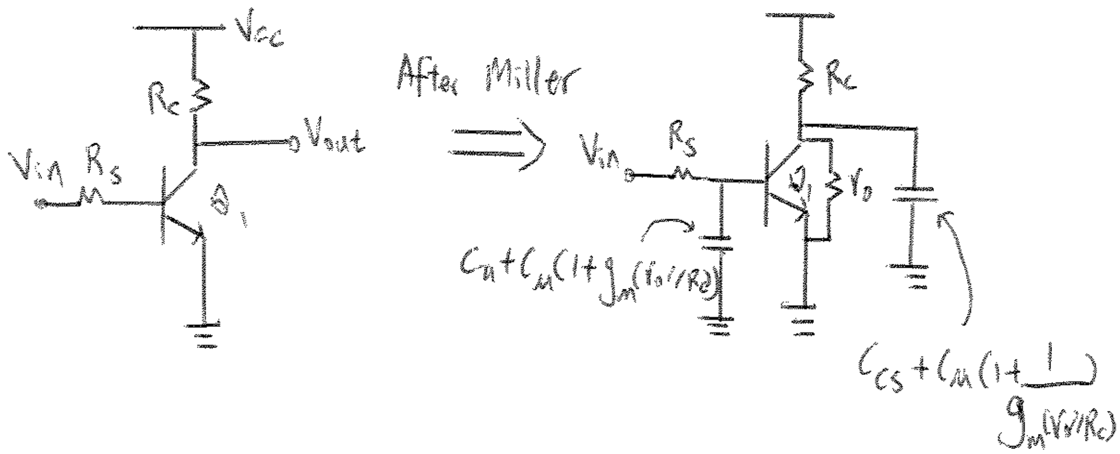
Constant W and $I_D \downarrow$ (L constant)

$$2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$$

$$f_{T_{new}} = \frac{f_{T,old}}{2}$$

35)

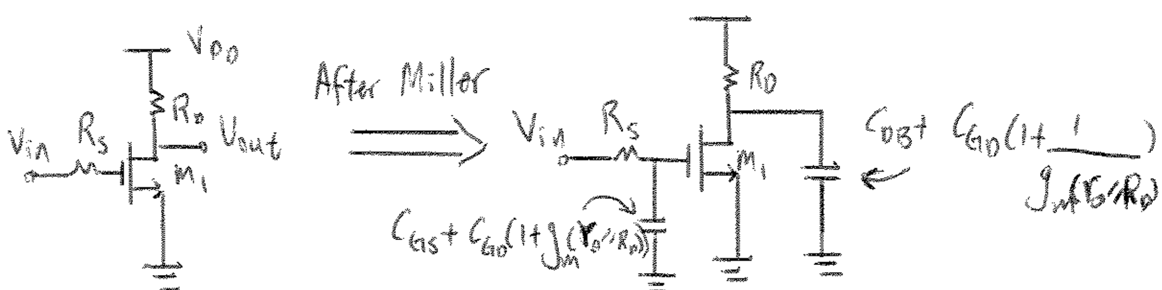
Bipolar CE Stage



$$\omega_{p_{in}} : \frac{1}{(R_S // r_\pi)[C_\pi + C_\mu(1+g_m(r_o // R_C))]}$$

$$\omega_{p_{out}} : \frac{1}{(R_C // r_o)[C_{CS} + C_\mu(1+1/g_m(r_o // R_C))]}$$

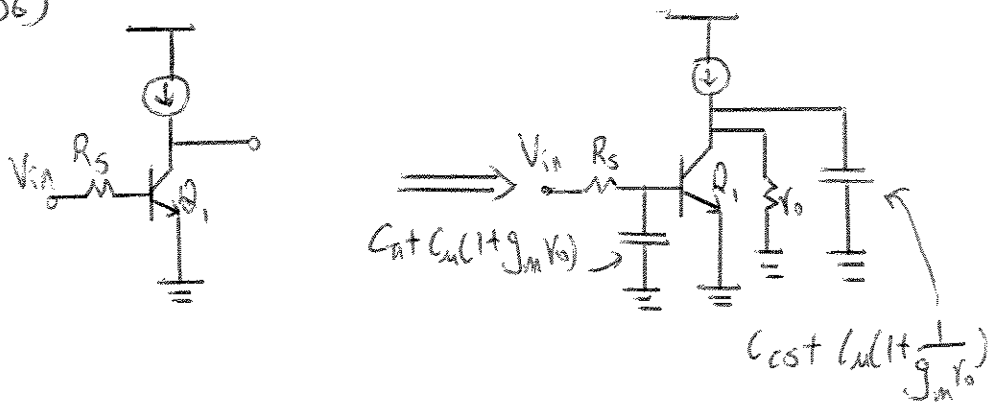
MOS CS Stage



$$\omega_{p_{in}} : \frac{1}{R_S[C_{GS} + C_{GD}(1+g_m(r_o // R_D))]}$$

$$\omega_{p_{out}} : \frac{1}{(R_D // r_o)[C_{DB} + C_{GD}(1+1/g_m(r_o // R_D))]}$$

36)



$$\omega_{pin} = \frac{1}{(R_s // r_\pi)[C_\pi + C_\mu(1+g_m r_o)]}$$

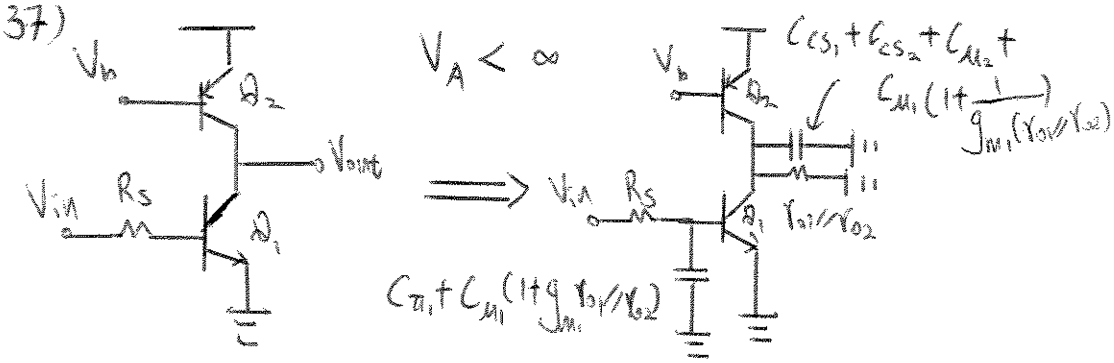
$$\omega_{pout} = \frac{1}{r_o[C_{cs} + C_\mu(1+1/g_m r_o)]}$$

$$H(s) = \frac{\text{DC gain}}{\left(1+\frac{s}{\omega_{pin}}\right)\left(1+\frac{s}{\omega_{pout}}\right)}$$

$$H(s) = \frac{g_m r_o \left(r_\pi / (r_\pi + R_s)\right)}{\left(1+\frac{s}{1/(R_s // r_\pi)[C_\pi + C_\mu(1+g_m r_o)]}\right)\left(1+\frac{s}{1/(r_o[C_{cs}+C_\mu(1+\frac{1}{g_m r_o})])}\right)}$$

37)

$V_A < \infty$



$$\omega_{pin} = \frac{1}{(R_S // r_{\pi_1})\left[C_{\pi_1} + C_{\mu}(1 + g_{m_1}(r_{o1} // r_{o2}))\right]}$$

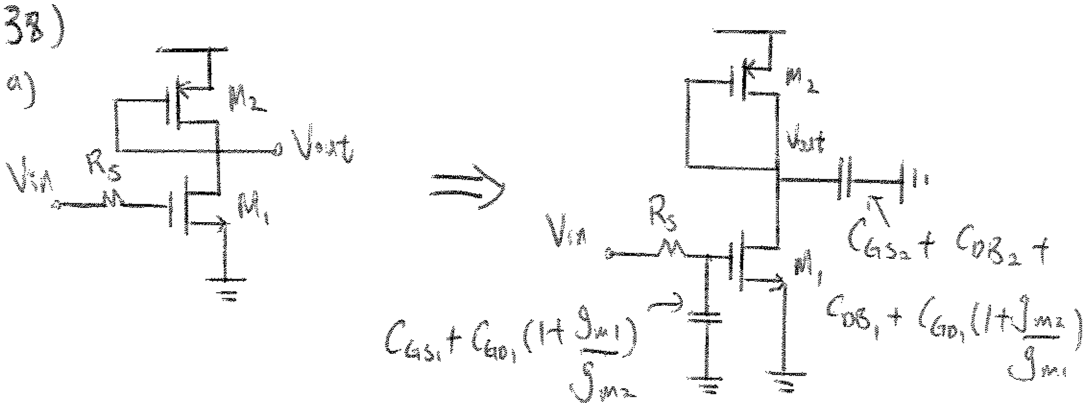
$$\omega_{pout} = \frac{1}{(r_{o1} // r_{o2})\left[C_{CS_1} + C_{CS_2} + C_{\mu_2} + C_{\mu_1}\left(1 + 1/(g_{m_1} r_{o1} // r_{o2})\right)\right]}$$

$$g_m r_o \gg 1$$

$$H(s) = \frac{g_m(r_{o1} // r_{o2})(r_\pi / r_\pi + R_S)}{\left(1 + R_S\left[C_{\pi_1} + C_{\mu}(g_m(r_{o1} // r_{o2}))\right]s\right)\left(1 + (r_{o1} // r_{o2})\left[C_{CS_1} + C_{CS_2} + C_{\mu_2} + C_{\mu_1}\right]s\right)}$$

38)

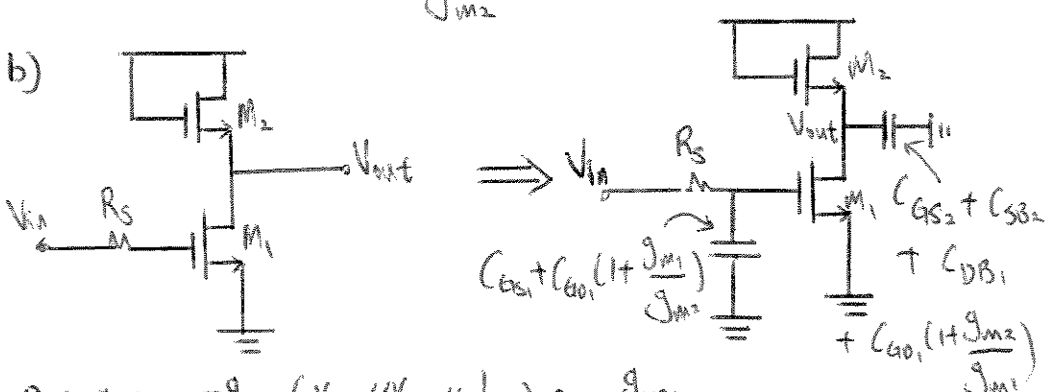
a)



$$\text{DC gain} = -g_{m_1}\left(r_{o_1} \,//\, r_{o2} \,//\, \frac{1}{g_{m2}}\right) \approx -\frac{g_{m_1}}{g_{m_2}}$$

$$\omega_{pin} = \frac{1}{R_S\left(C_{GS_1} + C_{GD_1}\left(1+\frac{g_{m1}}{g_{m2}}\right)\right)} \qquad \omega_{pout} = \frac{g_{m2}}{\left(C_{GS_2} + C_{DB_2} + C_{DB_1} + C_{GD_1}\left(1+\frac{g_{m2}}{g_{m1}}\right)\right)}$$

b)



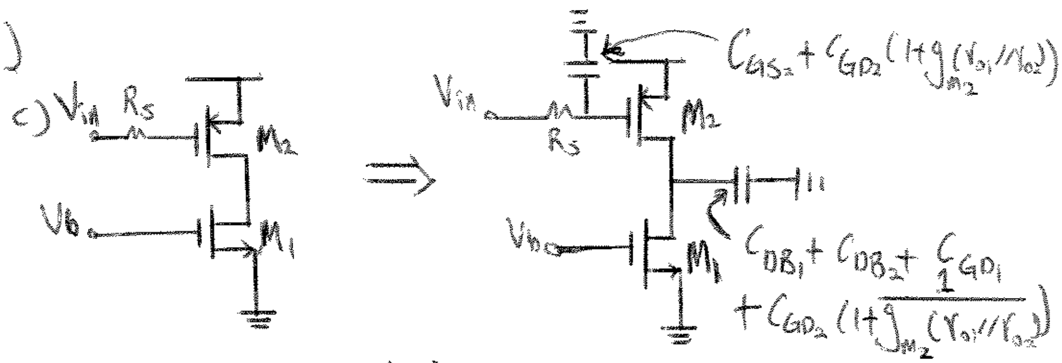
$$\text{DC gain} = -g_{m1}\left(r_{o_1} \,//\, r_{o2} \,//\, \frac{1}{g_{m2}}\right) \approx -\frac{g_{m1}}{g_{m2}}$$

$$\omega_{pin} = \frac{1}{R_s\left(C_{GS_1} + C_{GD_1}\left(1+\frac{g_{m1}}{g_{m2}}\right)\right)}$$

$$\omega_{pout} = \frac{g_{m2}}{C_{SB_2} + C_{GS_2} + C_{DB_1} + C_{GD_1}\left(1+\frac{g_{m2}}{g_{m1}}\right)}$$

38)

c)



DC gain: $-g_{m_2}(r_{o1}//r_{o2})$

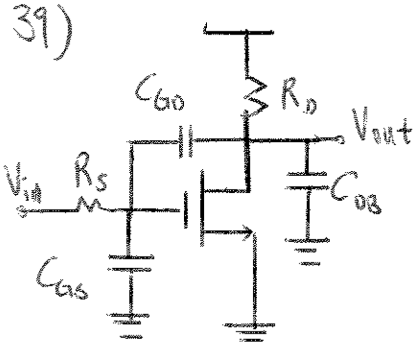
$$\omega_{pin} = \frac{1}{R_S\left(C_{GS_2} + C_{GD_2}(1+g_{m_2}(r_{o1}//r_{o2}))\right)}$$

$$\omega_{pout} = \frac{1}{(r_{o1}//r_{o2})\left[C_{DB_1} + C_{DB_2} + C_{GD_1} + C_{GD_2}\left(1+\frac{1}{g_{m_2}(r_{o1}//r_{o2})}\right)\right]}$$

$$\omega_{pout} \approx \frac{1}{(r_{o1}//r_{o2})\left[C_{DB_1} + C_{DB_2} + C_{GD_1} + C_{GD_2}\right]}$$

since $g_{m_2}(r_{o1}//r_{o2}) \gg 1$

39)



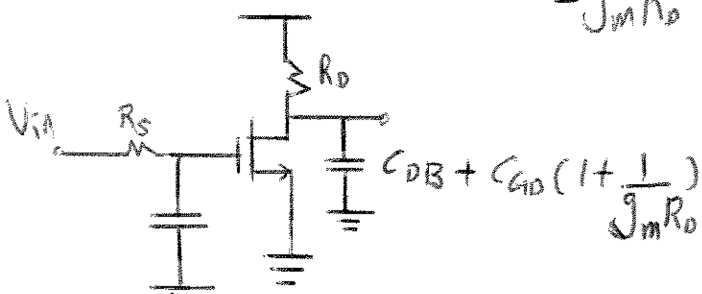
$R_S = 200\Omega \quad C_{GS} = 50fF$

$R_D = 1K\Omega \quad C_{GD} = 10fF$

$I_D = 1mA \quad C_{DB} = 15fF$

a) Miller's approximation:

$$-g_m R_D = \frac{-(1)(2)(1)}{(0.2)} = -10$$



$$\omega_{pin} = \frac{1}{R_S(C_{GS} + (1+g_m R_D)C_{GD})} = \frac{1}{200(50fF + (11)(10fF))} = 31.25GHz$$

$$\omega_{pout} = \frac{1}{R_D\left(C_{DB} + \left(1+\frac{1}{10}\right)C_{GD}\right)} = \frac{1}{1000(15fF + (1+0.1)10fF)} = 38.46GHz$$

3q)

b) Equation $\frac{V_{out}}{V_{Thev}}(s) = \frac{(C_{XY}s - g_m)R_L}{as^2 + bs + 1}$

$a = R_{Thev} R_L (C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})$

$R_{Thev} = R_S$, $C_{in} = C_{GS}$, $C_{out} = C_{DB}$, $R_L = R_D$, $C_{XY} = C_{GD}$

$a = (200)(1000)[(50)(10^{-15})(10)(10^{-15}) + (15)(10^{-15})(10)(10^{-15}) + (50)(10^{-15})(15)(10^{-15})] = 2.8 \times 10^{-22}$

$b = (1 + g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L (C_{XY} + C_{out})$

$b = (1+10)(10)(10^{-15})(200) + (200)(50 \times 10^{-15}) + (1000)(10)(10^{-15}) + 1000(15)(10^{-15})$

$b = 5.7 \times 10^{-11}$

So denominator $= (2.8 \times 10^{-22} s^2 + 5.7 \times 10^{-11} s + 1)$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$s = -1.93909 \times 10^{10}, \; -1.84181 \times 10^{11}$$

$\omega_{p_1} = 19.39\,GHz$, $\omega_{p_2} = 184.2\,GHz$

Which one is ω_{pin}, and which one is ω_{pout}? This can be seen from inspection, at output and high frequency C_{GD} starts to become a short and thus the output resistance collapses to $1/g_m$, and pushes the output pole out. Whereas at the input the pole location does not change too much because R_S is small and C_{GS} is large. Therefore, we conclude that when solving the transfer function directly, the ω_{pin} is 19.39 GHz (on the same order as

39)

b). that obtained from Miller's approximation), while ω_{pout} is pushed out significantly, 184.2 GHz (when compared to that obtained from Miller's approximation).

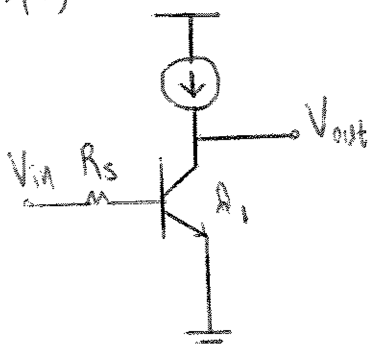
<u>Miller Approximation</u>

$\omega_{pin} = 31.25\,GHz$
$\omega_{pout} = 38.46\,GHz$

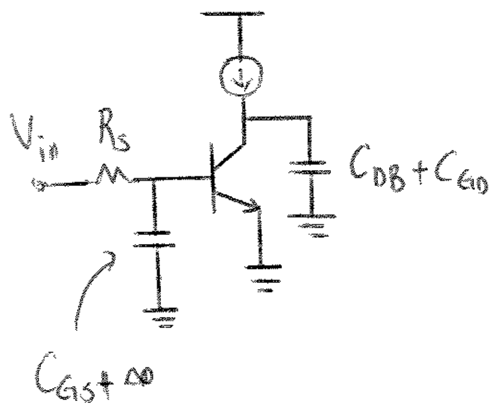
<u>Transfer Function</u>

$\omega_{pin} = 19.39\,GHz$
$\omega_{pout} = 184.2\,GHz$

40)



a) Miller's Approximation: DC gain: $-\infty$



$$\omega_{pin} = \frac{1}{R_S(\infty)} = 0, \quad \omega_{pout} = \frac{1}{\infty(C_{DB}+C_{GD})} = 0$$

b) Transfer Function:

$$\frac{V_{out}}{V_{thev}}(s) = \frac{(C_{XY}s - g_m)R_L}{as^2 + bs + 1}$$

$$a = R_{Thev} R_L (C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})$$

$$b = (1+g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L (C_{XY} + C_{out})$$

40)

b) $R_L \to \infty$

$$\frac{V_{out}}{V_{thev}} = \frac{C_{XY}S - g_m}{S[R_{thev}(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})S + g_m C_{XY} R_{thev} + (C_{XY} + C_{out})]}$$

So $\omega_{p_1} = 0$

$$\omega_{p_2} = \frac{(g_m C_{XY} R_{thev} + (C_{XY} + C_{out}))}{R_{thev}[C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out}]}$$

$$\omega_{p_2} = \frac{g_m C_\mu R_S /\!/ R_\pi + C_\mu + C_{CS}}{R_S /\!/ r_\pi [C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS}]}$$

$\omega_{p_1} = \omega_{pin}$, $\omega_{p_2} = \omega_{pout}$.

Miller:

$\omega_{pin} = 0$, $\omega_{pout} = 0$

Again, the output pole predicted by the transfer function is pushed out, and the input poles are similar. (In fact, they are the same this time.)
This shows one of the short-comings of Miller's approximation.

4i) <u>Dominant-Pole approximation:</u>

$$\omega_{p_1} = \frac{1}{(1+g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L (C_{XY} + C_{out})}$$

$$\omega_{p_1} = 0 \qquad (\text{Since } R_L = \infty)$$

$$\omega_{p_2} = \frac{(1+g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L (C_{XY} + C_{out})}{R_{Thev} R_L (C_{in} C_{XY} + C_{out} C_{XY} + C_{in} C_{out})}$$

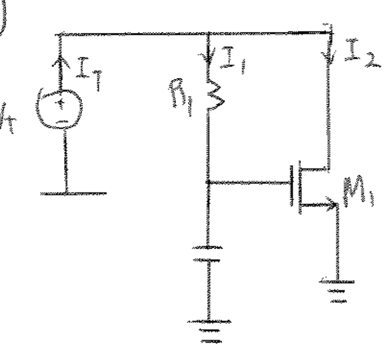
Since $R_L = \infty$

$$\omega_{p_2} = \frac{g_m C_{XY} R_{Thev} + C_{XY} + C_{out}}{R_{Thev} (C_{in} C_{XY} + C_{out} C_{XY} + C_{in} C_{out})}$$

$$\omega_{p_2} = \frac{g_m C_\mu R_S /\!/ R_\pi + C_\mu + C_{CS}}{R_S (C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS})}$$

Dominant-Pole approximation gives the same result as the transfer function method.

42)



$\lambda = 0$, and neglect other capacitances.

$I_T = I_1 + I_2$

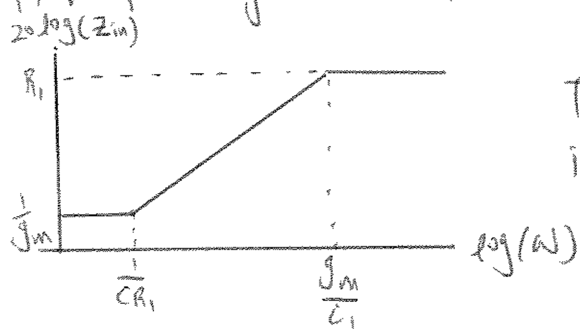
$$I_1 = \frac{V_T}{\left(R_1 + \frac{1}{C_1 S}\right)}, \quad I_2 = \frac{g_{m_1} V_T}{C_1 R_1 S + 1}$$

$$I_T = \frac{C_1 S V_T}{C_1 R_1 S + 1} + \frac{g_{m_1} V_T}{C_1 R_1 S + 1} \Rightarrow \frac{V_T}{I_T} = \frac{C_1 R_1 S + 1}{C_1 S + g_{m_1}}$$

$$S \to j\omega \Rightarrow \frac{C_1 R_1 (j\omega) + 1}{C_1 j\omega + g_{m_1}} = Z_T(j\omega)$$

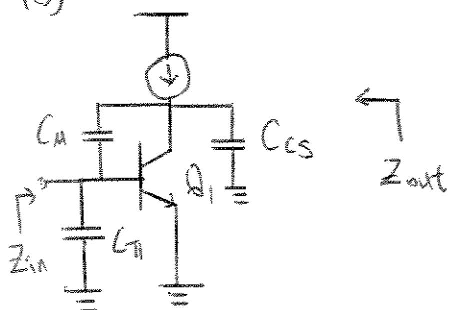
$$|Z_T| = |Z_{in}| = \frac{\sqrt{(C_1 R_1 \omega)^2 + 1}}{\sqrt{(C_1 \omega)^2 + g_{m_1}^2}} = \frac{\sqrt{(C_1 R_1 \omega)^2 + 1}}{g_m \sqrt{\left(\frac{C_1 \omega}{g_{m_1}}\right)^2 + 1}}$$

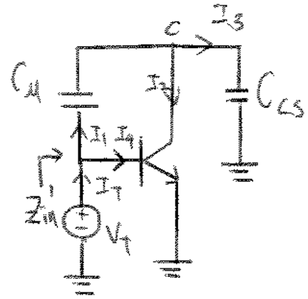
At $\omega = \frac{1}{C_1 R_1}$, we have a zero, at $\omega = \frac{g_{m_1}}{C_1}$, we have a pole. If $R_1 > \frac{1}{g_{m_1}}$, the zero is at a lower frequency than the pole, and the bode-plot for magnitude would look like the following.



The bode-plot shows an impedance that increases with frequency, an inductive behavior.

43)





$$Z_{in} = Z_{in}' \,//\, \frac{1}{C_\pi s} \;, \quad I_T = I_1 + I_4 = C_\mu s V_{bc} + \frac{g_m V_T}{\beta}$$

$$V_{bc} = V_T - V_c \;, \quad V_c = (I_1 - g_m V_T)\frac{1}{C_{cs} s}$$

$$I_1 = \left[V_T - (I_1 - g_m V_T)\frac{1}{C_{cs} s}\right] C_\mu s$$

$$I_1 = V_T\left[C_\mu s + \frac{g_m C_\mu}{C_{cs}}\right] \Big/ \left(1 + \frac{C_\mu}{C_{cs}}\right)$$

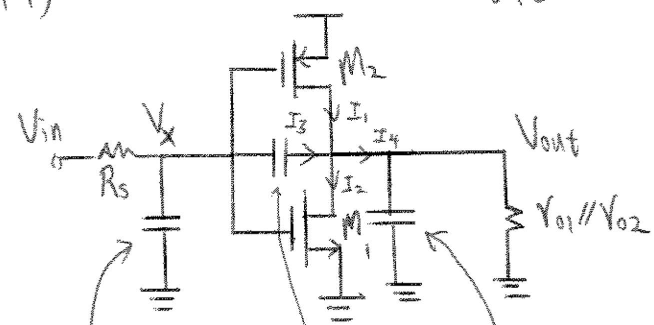
$$I_T = V_T\left[C_\mu s + \frac{g_m C_\mu}{C_{cs}}\right] \Big/ \left(1 + \frac{C_\mu}{C_{cs}}\right) + \frac{g_m V_T}{\beta}$$

$$Z_{in}' = \frac{V_T}{I_T} = \frac{1}{\dfrac{g_m}{\beta} + \dfrac{C_\mu s}{\left(1 + \frac{C_\mu}{C_{cs}}\right)} + \dfrac{g_m \frac{C_\mu}{C_{cs}}}{\left(1 + \frac{C_\mu}{C_{cs}}\right)}}$$

$$Z_{in} = Z_{in}' \,//\, \frac{1}{C_\pi s} = r_\pi \,//\, \frac{1}{\dfrac{C_{cs} C_\mu s}{C_{cs} + C_\mu}} \,//\, \frac{1}{C_\pi s} \,//\, \frac{C_{cs} + C_\mu}{g_m C_\mu}$$

$$Z_{out} = \frac{1}{(C_\mu + C_{cs}) s}$$

44) $\lambda > 0$



$C_{GS_2} + C_{GS_1} = C_C$

$C_{GD_1} + C_{GD_2} = C_B$

$C_{DB_2} + C_{DB_1} = C_A$

$$V_{out} = I_4 \Big(\underbrace{r_{o1} // r_{o2} // \frac{1}{[C_{DB_2} + C_{DB_1}]s}}_{Z_{out}} \Big)$$

$$I_4 = I_1 + I_3 - I_2$$

$$I_1 = (0 - V_x) g_{m_2}$$

$$I_2 = V_x g_{m_1}$$

$$I_3 = (V_x - V_{out}) \big((C_{GD_1} + C_{GD_2}) s$$

$$I_4 = -V_x g_{m_2} + (V_x - V_{out}) C_B s - V_x g_{m_1}$$

$$V_{out} = Z_{out} \left[-V_x (g_{m_2} + g_{m_1}) + (V_x - V_{out}) C_B s \right]$$

Writing a node equation at x.

$$\frac{V_x - V_{in}}{R_s} + V_x C_c s + (V_x - V_{out}) C_B s = 0$$

$$V_x = \frac{V_{out} C_B s + V_{in}/R_s}{(1/R_s + C_c s + C_B s)}$$

44)

Subsitute everything and we get

$$V_{out} = Z_{out}\left[-(g_{m1}+g_{m2})\left(\frac{V_{out}C_B s + V_{in}/R_s}{1/R_s + C_c s + C_B s}\right) + \left(\frac{V_{out}C_B s + V_{in}/R_s}{1/R_s + C_c s + C_B s} - V_{out}\right)C_B s\right]$$

collect all the V_{out}'s on one-side and likewise for V_{in}'s, we will get

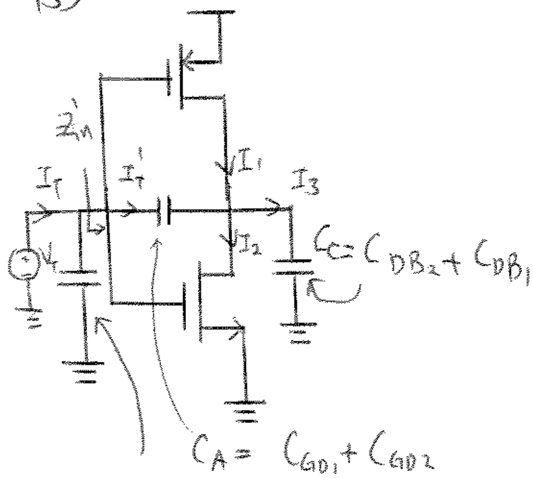
$$\frac{V_{out}}{V_{in}} = \frac{\frac{Z_{out}}{R_s}\left(C_B s - (g_{m1}+g_{m2})\right)}{1/R_s + (C_c+C_B)s + Z_{out}C_B s(g_{m1}+g_{m2}) + Z_{out}C_B s\left(\frac{1}{R_s}+(C_c+C_B)s\right) - Z_{out}C_B^2 s^2}$$

where $Z_{out} = r_{o1} // r_{o2} // \dfrac{1}{[C_{DB1}+C_{DB2}]s}$

$$C_B = C_{GD1} + C_{GD2}$$

$$C_c = C_{GS1} + C_{GS2}$$

45)



$$Z_{in} = \frac{V_T}{I_T} = \frac{1}{C_B} \,//\, Z_{in}'$$

$$Z_{in}' = \frac{V_T}{I_T'}$$

$$I_T' = \left[V_T - \left(I_3 \frac{1}{C_C s}\right)\right] C_A s$$

$$I_3 = I_T' - V_T g_{m2} - g_{m1} V_T$$

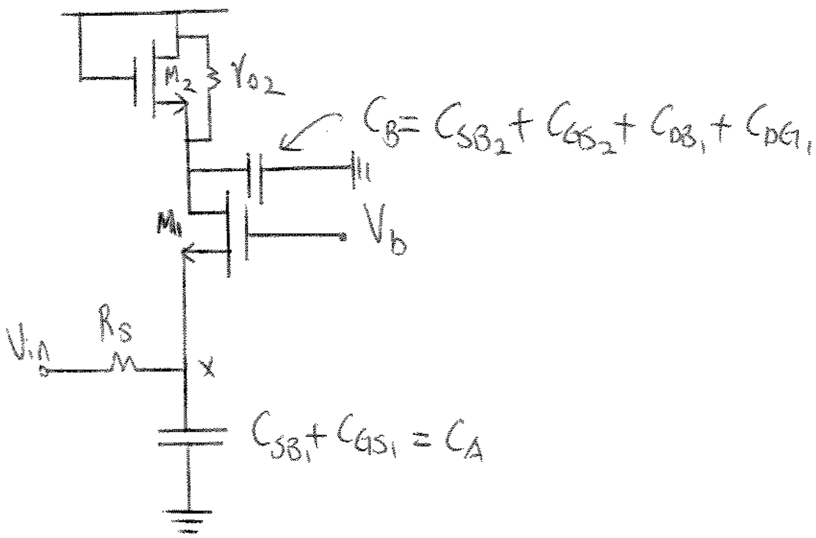
We get $\Rightarrow$ $I_T'\left(1 + \frac{C_A}{C_c}\right) = V_T\left[C_A s + (g_{m1} + g_{m2})\frac{C_A}{C_c}\right]$

$$Z_{in}' = \frac{V_T}{I_T'} = \frac{\left(1 + \frac{C_A}{C_c}\right)}{\left[C_A s + (g_{m1} + g_{m2})\frac{C_A}{C_c}\right]}$$

$$Z_{in} = \frac{1}{[C_{GS_1} + C_{GS_2}]s} \,//\, \frac{\left(1 + \frac{C_{GD_1} + C_{GD_2}}{C_{DB_1} + C_{DB_2}}\right)}{\left[(C_{GD_1} + C_{GD_2})s + (g_{m1} + g_{m2})\frac{C_{GD_1} + C_{GD_2}}{C_{DB_2} + C_{DB_1}}\right]}$$

4b)

a)



$$V_{out} = -(0 - V_x) g_{m_1} \left[\frac{1}{g_{m2}} \,//\, \frac{1}{C_B s} \right] = V_x g_{m_1} \left[\frac{1}{g_{m2}} \,//\, \frac{1}{C_B s} \right]$$

Node equation at X, $\dfrac{V_x - V_{in}}{R_s} + V_x C_A S - g_m (0 - V_x) = 0$

$$V_x \left(\frac{1}{R_s} + C_A S + g_m \right) = \frac{V_{in}}{R_s} \Rightarrow V_x = \frac{V_{in}}{(1 + R_s C_A S + R_s g_m)}$$

subsitute in V_x and solving for $V_{out}/V_{in} \Rightarrow$

$$\frac{V_{out}}{V_{in}} = \frac{g_{m_1} [\, 1/g_{m2} \,//\, \frac{1}{C_B s} \,]}{(1 + R_s C_A S + R_s g_m)}$$

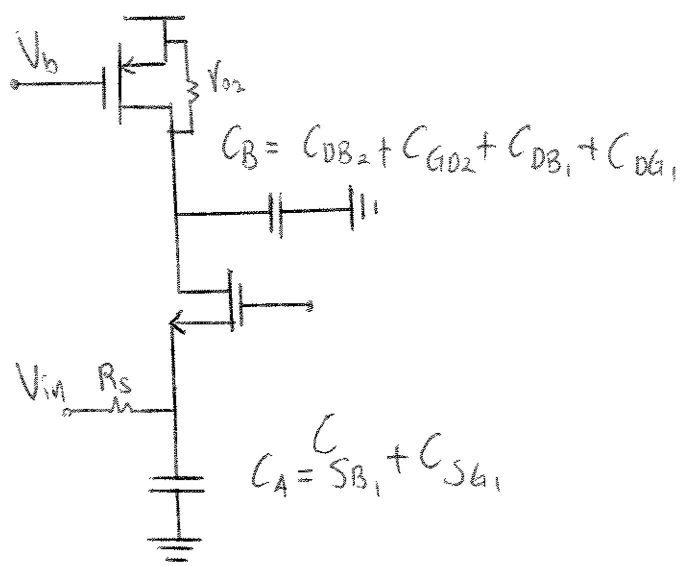
$$\frac{V_{out}}{V_{in}} = \frac{g_{m_1} (1/g_{m2})}{(C_B (1/g_{m2}) S + 1)(1 + R_s C_A S + R_s g_{m_1})}$$

Where $C_B = C_{SB_2} + C_{GS_2} + C_{DB_1} + C_{DG_1}$

$C_A = C_{SB_1} + C_{GS_1}$

4b)

b)



Similar to part a), with $\frac{1}{g_{m_2}}$ replaced by r_{o2},

and different C_B

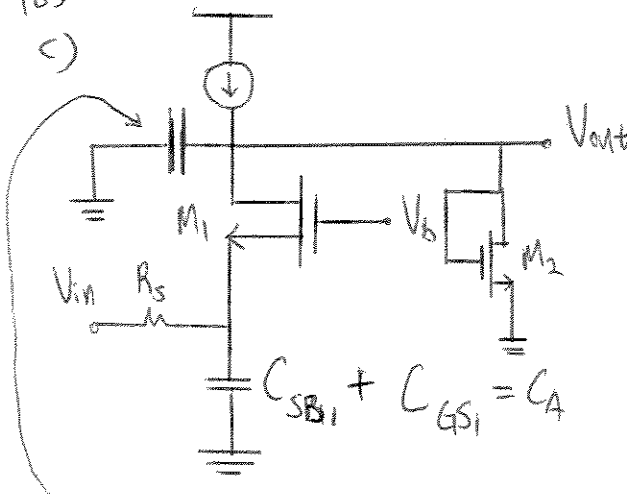
So $$\frac{V_{out}}{V_{in}} = \frac{g_{m_1} r_{o2}}{(C_B r_o S + 1)(1 + R_S C_A S + R_S g_{m_1})}$$

Where $C_B = C_{DB_2} + C_{GD2} + C_{DB_1} + C_{DG_1}$

$C_A = C_{SB_1} + C_{SG_1}$

4b)

c)



$C_B = C_{DB_1} + C_{GD_1} + C_{DB_2} + C_{GS_2}$

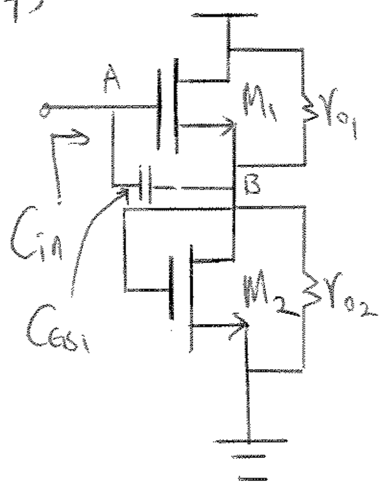
AC-wise, this circuit is very similar to part a). Its transfer function is the same as part a), except for C_B.

$$\frac{V_{out}}{V_{in}} = \frac{g_{m_1}(1/g_{m_2})}{\left(C_B(1/g_{m_2})S+1\right)\left(1+R_S C_A S + R_S g_{m_1}\right)}$$

where $C_B = C_{DB_1} + C_{GD_1} + C_{DB_2} + C_{GS_2}$

$C_A = C_{SB_1} + C_{GS_1}$

47)



DC gain from A to B:

$$A_V = \frac{\frac{1}{g_{m2}} // r_{o1} // r_{o2}}{\frac{1}{g_{m2}} // r_{o1} // r_{o2} + \frac{1}{g_{m1}}}$$

$$A_V \approx \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m2}} + \frac{1}{g_{m1}}} = \frac{g_{m1}}{g_{m1} + g_{m2}}$$

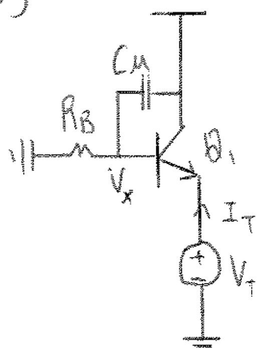
since $g_m r_o >> 1$

Using Miller's Capacitance:

$$C_{in} = C_{GS1}(1 - A_V) = C_{GS1}\left(1 - \frac{g_{m1}}{g_{m1} + g_{m2}}\right)$$

$$C_{in} = C_{GS1}\left(\frac{g_{m2}}{g_{m2} + g_{m1}}\right)$$

48)



$V_A = \infty$,

$\frac{\beta}{\beta+1} \approx 1$, if $\beta \gg 1$

$$I_T = -(V_x - V_T)\, g_m \alpha \approx -(V_x - V_T)\, g_m$$

$$V_x = \frac{I_T}{\beta}\left(R_B \parallel \frac{1}{C_\mu s}\right)$$

$$I_T = \left(V_T - \frac{I_T}{\beta}\left(R_B \parallel \frac{1}{C_\mu s}\right)\right) g_m$$

$$I_T = g_m V_T - \frac{g_m}{\beta}\left(R_B \parallel \frac{1}{C_\mu s}\right) I_T$$

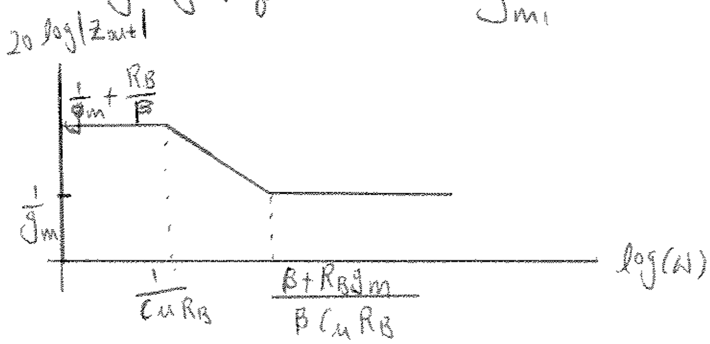
$$I_T\left(1 + \frac{g_m}{\beta}\left(R_B \parallel \frac{1}{C_\mu s}\right)\right) = g_m V_T$$

$$\frac{V_T}{I_T} = \frac{1}{g_m} + \frac{R_B \parallel \frac{1}{C_\mu s}}{\beta} = \frac{\beta C_\mu R_B\left(s + \frac{\beta + R_B g_m}{\beta C_\mu R_B}\right)}{g_m \beta (1 + C_\mu R_B s)}$$

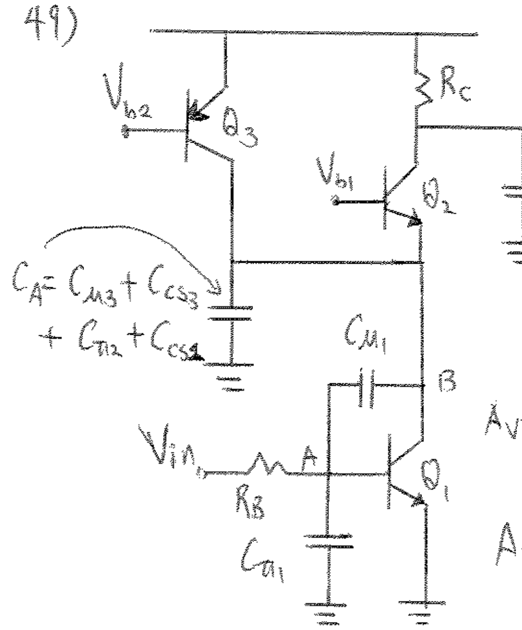
Zero: $\frac{\beta + R_B g_m}{\beta C_\mu R_B}$, Pole: $\frac{1}{C_\mu R_B}$

At DC, $|Z_{out}| = \frac{1}{g_{m1}} + \frac{R_B}{\beta}$

At very high freq: $|Z_{out}| = \frac{1}{g_{m1}}$



49)



DC gain from A to B:

$A_V = -\left(r_{\pi 2} // \frac{1}{g_{m2}} \right) g_{m1}$

$A_V \simeq -\frac{g_{m1}}{g_{m2}}$ (Since $\beta >> 1$)

We have $I_{C2} = 0.25 I_{C1}$, $g_m = \frac{I_C}{V_T} \Rightarrow g_{m2} = 0.25 g_{m1}$

$A_V = -\frac{g_{m1}}{g_{m2}} = -4$. (If $I_{C2} = I_{C1}$, $A_V = -1$)

Applying Miller's Thereom: $C_{in} = C_{\pi 1} + C_{\mu 1}(1+4) = C_{\pi 1} + 5C_{\mu 1}$

$C_B = C_A + C_\mu (1 + \frac{1}{4}) = C_A + C_\mu (\frac{5}{4})$

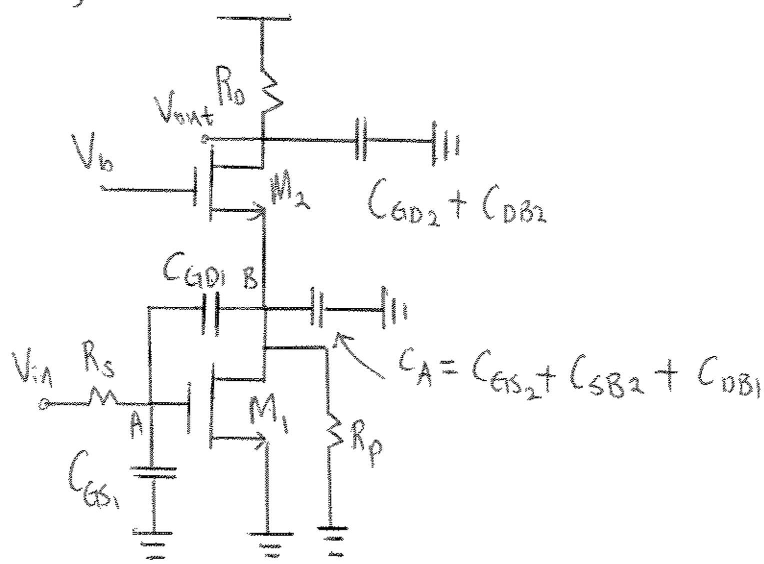
$\omega_{p in} (\omega_{pA}) = \frac{1}{(R_B // r_\pi)[C_{\pi 1} + 5C_\mu]}$, $\omega_{pB} = \frac{0.25 g_{m1}}{[C_A + \frac{5}{4} C_{\mu 1}]}$

$\omega_{pB} = \frac{g_{m2}}{[C_A + \frac{5}{4} C_{\mu 1}]}$, $\omega_{p out} = \frac{1}{R_C [C_{CS2} + C_{\mu 2}]}$

Where $C_A = C_{\mu 3} + C_{CS3} + C_{\pi 2} + C_{CS1}$.

Since the DC gain is increased, Miller effect is more significant.
(In magnitude)

50)



DC gain from A to B is $-g_{m1}\left(R_P \parallel \frac{1}{g_{m2}}\right)$

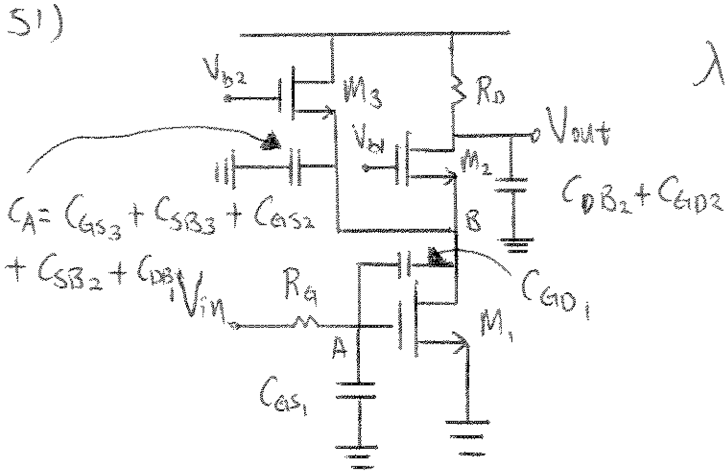
Applying Miller's Theorem:

$$\omega_{p_{in}} (\omega_{pA}) = \frac{1}{R_S\left(C_{GS1} + C_{GD1}\left(1 + g_{m1}\left(R_P \parallel \frac{1}{g_{m2}}\right)\right)\right)}$$

$$\omega_{pB} = \frac{1}{R_P \parallel \frac{1}{g_{m2}} \left[C_{GS2} + C_{SB2} + C_{DB1} + C_{GD1}\left(1 + 1/g_{m1}\left(R_P \parallel \frac{1}{g_{m2}}\right)\right)\right]}$$

$$\omega_{p_{out}} = \frac{1}{R_D\left(C_{GD2} + C_{DB2}\right)}$$

5')



$\lambda = 0$

DC gain from A to B: $-g_{m_1}\left(\frac{1}{g_{m3}} \parallel \frac{1}{g_{m2}}\right) = -g_{m_1}\left(\frac{1}{g_{m2}+g_{m3}}\right)$

Applying Miller's Theorem:

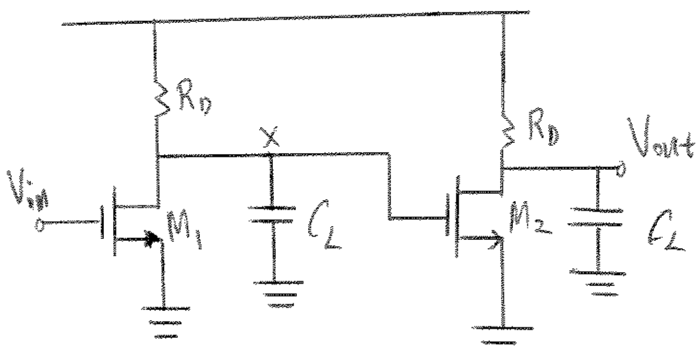
$$\omega_{pin}\ (\omega_{pa}) = \frac{1}{R_G\left(C_{GS_1} + C_{GD_1}\left(\frac{g_{m_1}+g_{m2}+g_{m3}}{g_{m2}+g_{m3}}\right)\right)}$$

$$\omega_{pB} = \frac{g_{m3}+g_{m2}}{\left(C_A + C_{GD_1}\left(\frac{g_{m_1}+g_{m2}+g_{m3}}{g_{m_1}}\right)\right)}$$

$$\omega_{pout} = \frac{1}{R_D\left(C_{DB2} + C_{GD2}\right)}$$

Where $C_A = C_{GS3} + C_{SB3} + C_{GS2} + C_{SB2} + C_{DB_1}$

52)



Bias Current = 1mA (each stage)

$C_L = 50fF$

$\mu_n C_{ox} = 100\,\mu A/V^2$, $A_V = 20$, $-3dB$: 1GHz

DC gain: $(g_m R_D)^2 = 20$

$-3dB$ bandwidth: $0.10243/(R_D C_L) = 1\,GHz$

Since $C_L = 50fF$, $R_D = 2048.6\,\Omega$

$(g_m R_D)^2 = 20 \Rightarrow g_m = 0.002183 = \frac{2I_D}{V_{eff}} \Rightarrow V_{eff} = 0.916V$

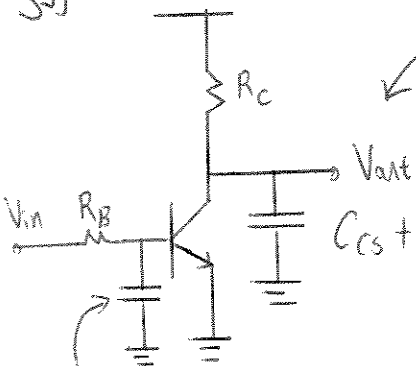
$V_{eff} = V_{GS} - V_{th} = 0.916V$

$g_m = \mu_n C_{ox} \frac{W}{L}(V_{eff}) \Rightarrow \frac{W}{L} = \frac{g_m}{\mu_n C_{ox}(V_{eff})} = 23.83$

So $R_D = 2.05K$, $C_L = 50fF$

$V_{GS} - V_{th} = 0.916V$, $W/L = 23.83$

53)



$C_{CS} + \left(1 + \frac{1}{g_m R_C}\right) C_\mu$

$C_\pi + C_\mu (1 + g_m R_C)$

After apply Miller's theorem

$\omega_{pin} = (2\pi)(500MHz)$

$\omega_{pout} = (2\pi)(2G)$

$I_C = 1mA$, $C_\pi = 20fF$,

$C_\mu = 5fF$, $C_{CS} = 10fF$

$V_A = \infty$.

Low frequency Voltage gain: $\frac{V_{out}}{V_{in}} = \frac{-R_C}{1/g_m + \frac{R_B}{\beta+1}}$

$$\omega_{pin} = \frac{1}{(R_B // r_\pi)(C_\pi + C_\mu(1 + g_m R_C))} = (2\pi)(500\,MHz)$$

$$\omega_{pout} = \frac{1}{R_C[C_{CS} + (1 + 1/(g_m R_C))C_\mu]} = (2\pi)(2G)$$

$$\Rightarrow g_m = 2\pi(2G)[g_m R_C C_{CS} + g_m R_C C_\mu + C_\mu]$$

$$\Rightarrow R_C = \left(\frac{g_m}{(2\pi)(2G)} - C_\mu\right) / \left(g_m(C_{CS} + C_\mu)\right)$$

$$g_m = \frac{I_C}{V_T} = 0.0386 \frac{1}{\Omega}, \quad R_C = 5296.53\,\Omega$$

53)

In order to maximize low frequency gain V_{out}/V_{in}, R_B should be as small as possible (restricted by the input pole location). So $R_B \parallel r_\pi \approx R_B$.

$$\omega_{pin} \approx \frac{1}{R_B\left(C_\pi + C_\mu(1+g_mR_C)\right)} = (2\pi)(500\times10^6)$$

$$g_mR_c = 204.446$$

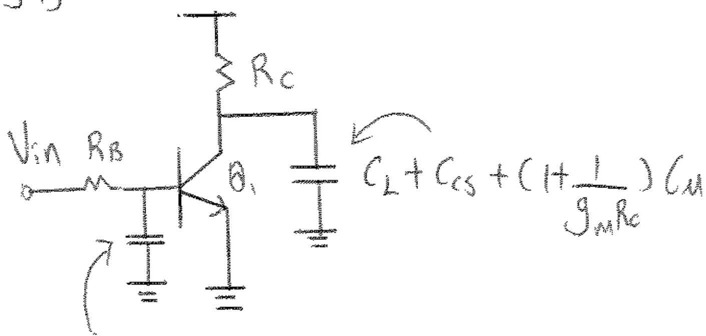
$$R_B = \frac{1}{\omega_{pin}\left(C_\pi + C_\mu(1+g_mR_c)\right)} \approx 303.95\Omega$$

So

$$R_B = 303.95\Omega$$

$$R_c = 5296.53\Omega$$

54)



$C_\pi + (1+g_m R_C)C_\mu$

Low freq Voltage gain: $\frac{V_{out}}{V_{in}} = \frac{-R_C}{1/g_m + \frac{R_B}{\beta+1}}$

$$\omega_{pout} = \frac{1}{R_C[C_L + C_{CS} + (1+\frac{1}{g_m R_C})C_\mu]} = (2\pi)(2\,GHz)$$

$$g_m = \frac{I_C}{V_T} = 0.0386\ \frac{1}{\Omega}$$

$$g_m = (2\pi)(2GHz)[g_m R_C[C_L + C_{CS}] + g_m R_C C_\mu + C_\mu]$$

$$R_C = \left[\frac{g_m}{(2\pi)(2G)} - C_\mu\right] / (g_m[C_L + C_{CS} + C_\mu])$$

$R_C = 2269.94\Omega \approx 2.27\,K\Omega$

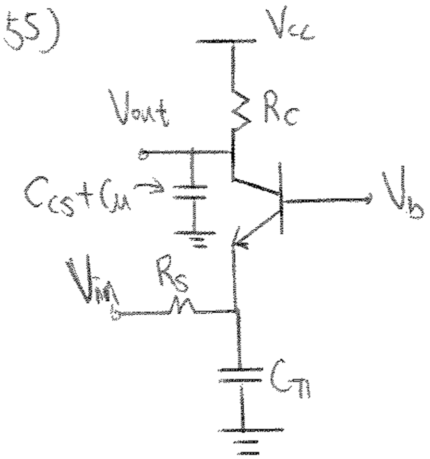
Again, to maximize low freq gain, R_B should be as small as possible, so $R_B // r_\pi \approx R_B$

$$\omega_{pin} \approx \frac{1}{R_B(C_\pi + C_\mu(1+g_m R_C))} = (2\pi)(50\times10^6),\ g_m R_C = 87.62$$

$R_B = 687.35\Omega$

So, $R_C = 2.27K\Omega$, $R_B = 687.35\Omega$

55)



$V_A = \infty$, $I_C = 1\,mA$, $R_S = 50\,\Omega$,
$C_\pi = 20\,fF$, $C_{cs} = 20\,fF$, $C_\mu = 5\,fF$

$-3dB$ bandwidth: $10\,GHz$

Since the output node sees a larger capacitance and resistance than the input, (R_C usually large for large gain), dominant pole and thus $-3dB$ bandwidth occurs at the output.

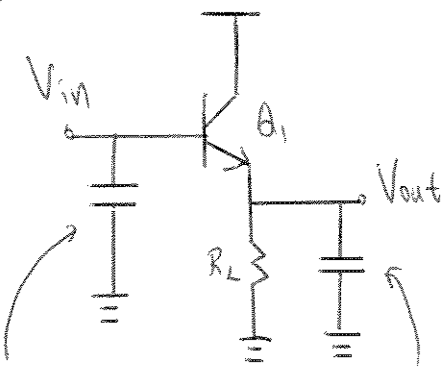
$$\omega_{pout} = \frac{1}{R_C\,[C_\mu + C_{cs}]} = (2\pi)(10\,GHz)$$

$$R_C = 636.62\,\Omega \quad , \quad \frac{1}{g_m} = \frac{25.9\,mV}{1\,mA}$$

Maximum achievable gain: $\dfrac{R_C}{R_S + \frac{1}{g_m}} = 8.4$

Here we have a tradeoff between gain and bandwidth.

56)



DC gain: $\dfrac{R_L}{R_L + 1/g_m}$

$V_A = \infty$

$C_\mu = 10fF,\ C_\pi = 100fF$

$C_\mu + C_\pi\left(1 - \dfrac{R_L}{R_L + 1/g_m}\right)$

$C_\pi\left(1 - \dfrac{R_L + 1/g_m}{R_L}\right)$

$$C_{in} < 50fF \Rightarrow C_\mu + C_\pi\left(1 - \frac{R_L}{R_L + 1/g_m}\right) < 50fF$$

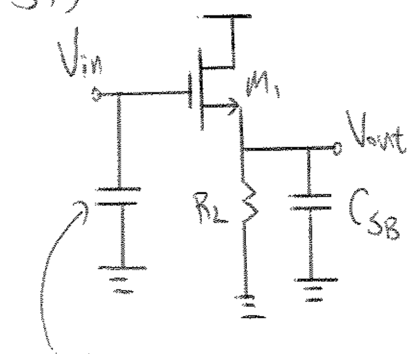
$$10fF + 100fF\left(1 - \frac{R_L}{R_L + 1/g_m}\right) < 50fF$$

$$100fF\left(1 - \frac{R_L}{R_L + 1/g_m}\right) < 40fF$$

$$\left(\frac{\frac{1}{g_m}}{R_L + 1/g_m}\right) < 0.4$$

$$R_L > \frac{3}{2g_m} = 38.85\Omega$$

57)



$R_L = 100\Omega$, $I_D = 1mA$

$A_V = \frac{V_{out}}{V_{in}} = 0.8$ $\mu_n C_{ox} = 100 \mu A/V^2$

$L = 0.18 \mu m$, $\lambda = 0$, $C_{GD} \approx 0$,

$C_{SB} \approx 0$, $C_{GS} = (2/3) WL C_{ox}$

$C_{ox} = 12 fF/(\mu m)^2$

$C_{GD} + C_{GS}(1 - 0.8)$

$C_{in} = C_{GD} + C_{GS}(0.2)$, $C_{in} = C_{GS}(0.2) = C_{in(min)}$

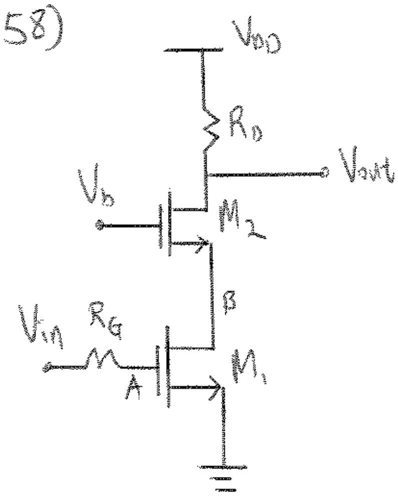
$$A_V = \frac{R_L}{R_L + 1/g_m} = 0.8, \quad \frac{1}{g_m} = 25 = \frac{V_{eff}}{2I_D}$$

$$V_{eff} = 50mV, \quad I_D = \frac{1}{2}\frac{W}{L}\mu_n C_{ox}(V_{eff})^2 \Rightarrow W = 1440$$

$$C_{in,min} = 0.2\, C_{GS} = 0.2\left(\frac{2}{3}\right) WL\, C_{ox} = 414.72 fF$$

or $C_{in,min} = 0.415 pF$

58)



$\omega_{pin} = 5\text{ GHz}$, $\omega_{pout} = 10\text{ GHz}$

$V_{eff} = 200\text{ mV}\ (V_{GS} - V_{th})$, $I_D = 0.5\text{mA}$

$\lambda = 0$, $C_{GS} = (2/3)\,WL\,C_{ox}$,

$L = 0.18\mu m$, $\mu_n C_{ox} = 100\mu A/V^2$

$C_{GD} = WC_o$, $C_o = 0.2\text{fF}/\mu m$

$C_{ox} = 12\text{fF}/\mu m^2$

DC gain from A to B: $\dfrac{-g_{m1}}{g_{m2}} = 1$

$C_{in} = C_{GS} + C_{GD}(1 + g_{m1}/g_{m2}) = C_{GS} + 2C_{GD}$

$I_D = \dfrac{1}{2}\dfrac{W}{L}\mu_n C_{ox}(V_{eff})^2 = 0.5\text{ mA} \Rightarrow \dfrac{W}{L} = 250$

$L = 0.18\mu$, $W = 45\mu$

$$\omega_{pin} = (2\pi)(5\times10^9) = \frac{1}{R_G\left[\frac{2}{3}(45)(0.18)(12\text{fF}/\mu m^2) + (0.2)(45)(2)\right]}$$

$R_G = 384.43\Omega$

$$\omega_{pout} = \frac{1}{R_D[0.2W]} = (10\times10^9)(2\pi), \quad W = 45\mu m$$

$\Rightarrow R_D = 1.8K\Omega$ (1768.4Ω) exact value

$$\text{Gain} = |g_m R_D| = \frac{2I_D}{V_{eff}} R_D = 8.842$$

59)

$W_2 = 4W_1$, $V_{eff_2} = \frac{V_{eff_1}}{2}$ (To maintain the current constant)

$V_{eff_1} = 200mV$, $V_{eff_2} = 100mV$ (Assume V_{eff_1} is not changed)

DC gain: $-\frac{g_{m_1}}{g_{m_2}} = \frac{-g_{m_1}}{2g_{m_1}} = -\frac{1}{2}$

$$\omega_{pin} = \frac{1}{R_G\left[\frac{2}{3}WLC_{ox} + (0.2)W\left(\frac{1}{2}\right)\right]} = (5\times10^9)(2\pi)$$

$W = 45\mu$

$\Rightarrow R_G = 459.32\,\Omega$

$$R_D = \frac{1}{(10\times10^9)(2\pi)(0.2)(4)(45)} = 442.097\,\Omega$$

DC gain: $|g_{m_1}R_D| = \frac{2I_D}{V_{eff_1}}R_D = 2.2105$