1. 解：$\min_{w,b} \frac{1}{2}\langle w, w\rangle + C\sum_i \varepsilon_i$

$$s.t.\quad y_i(\langle w, x_i\rangle + b) \geq 1-\varepsilon_i,\ \varepsilon_i \geq 0$$

首先构造拉格朗日函数：

$$L(w,b,\varepsilon,\alpha,\beta) = \frac{1}{2}\langle w,w\rangle + C\sum_i \varepsilon_i + \sum_i \alpha_i [1-y_i(\langle w, x_i\rangle + b) - \varepsilon_i] + \sum_i \beta_i(-\varepsilon_i)$$

求偏导可得：$0 = \frac{\partial}{\partial w} L(w,b,\alpha) = w + \sum_i \alpha_i(-y_i x_i) \Rightarrow w = \sum_i \alpha_i y_i x_i$

$$0 = \frac{\partial}{\partial b} L(w,b,\alpha) = -\sum_i \alpha_i y_i \Rightarrow \sum_i \alpha_i y_i = 0$$

$$0 = \frac{\partial}{\partial \varepsilon_i} = C - \alpha_i - \beta_i \Rightarrow \alpha_i = C - \beta_i \in C$$

代回原拉格朗日函数有：$L(w,b,\alpha) = -\frac{1}{2}\sum_{i,j}\alpha_i\alpha_j y_i y_j \langle x_i, x_j\rangle + \sum_i \alpha_i + C\sum_i \varepsilon_i - \sum_i \alpha_i \varepsilon_i - \sum_i \beta_i \varepsilon_i$

由 $C-\alpha_i-\beta_i=0$，有：$C\sum_i \varepsilon_i = \sum_i \alpha_i \varepsilon_i + \sum_i \beta_i \varepsilon_i$

$$\therefore g(\alpha,\beta) = \min_{w,b} L(w,b,\alpha) = -\frac{1}{2}\sum_{i,j}\alpha_i\alpha_j y_i y_j \langle x_i, x_j\rangle + \sum_i \alpha_i$$

其对偶形式为：

$$\min_{\alpha} \frac{1}{2}\sum_{i,j}\alpha_i\alpha_j y_i y_j \langle x_i, x_j\rangle - \sum_i \alpha_i$$

$$s.t.\quad \sum y_i \alpha_i = 0$$

$$0 \leq \alpha_i \leq C$$

即与可分情况较为相似，只是约束条件由 $\alpha_i \geq 0$ 变成了 $0\leq\alpha_i\leq C$

故 $w^* = \sum_i y_i \alpha_i^* x_i = \sum_{\alpha_i^* \neq 0} y_i \alpha_i^* x_i$

$$b^* = y_i - \langle w^*, x_i\rangle \qquad (0 \leq \alpha_i^* \leq C)$$

即超平面为 $f(x) = \langle w^*, x\rangle + b^*$

$$= \sum_{\alpha_i^* \neq 0} y_i \alpha_i^* \langle x_i, x\rangle + b^* \qquad (0 \leq \alpha_i^* \leq C)$$

2. 解：$p(a,b,c,d) = p(a)p(b)p(c|a,b)p(d|c)$

$$p(a,b) = \sum_c \sum_d p(a,b,c,d)$$

$$= p(a)p(b)\sum_c p(c|a,b)\sum_d p(d|c)$$

$$= p(a)p(b)$$

故 $a \perp\!\!\!\perp b \mid \emptyset$

当d被观测到后，$p(a,b|d) = \dfrac{\sum_c p(a,b,c,d)}{\sum_a \sum_b \sum_c p(a,b,c,d)}$

$$= \frac{p(a)p(b)p(d|a,b)}{p(d)}$$

在一般情况下，上式 $\neq p(a|d)p(b|d)$

$\therefore a \not\perp\!\!\!\perp b \mid d$