1. (a)

$$n_i(T=300K) = 1.66\cdot10^{15}(300)^{3/2}\cdot \exp\left[\frac{-(0.66eV)}{2(1.38\cdot10^{-23}J/K)(300K)}\right]$$

$$= 2.5\cdot10^{13}\ cm^{-3}$$

$$n_i(T=600K) = 1.66\cdot10^{15}(600)^{3/2}\cdot \exp\left[\frac{-(0.66eV)}{2(1.38\cdot10^{-23}J/K)(600K)}\right]$$

$$= 4.15\cdot10^{16}\ cm^{-3}$$

Comparing these results with those in Example:

$$\frac{n_i(Ge @ 300K)}{n_i(Si @ 300K)} \approx 2315. \qquad \frac{n_i(Ge @ 600K)}{n_i(Si @ 600K)} \approx 27.$$

At higher temperature, the exponential terms approaches one, which implies that $n_i \sim T^{3/2}$, independent of bandgap energy, Eg.

(b) For any doped material, $n\cdot p = n_i^2$. Assuming at $T=300K$,

$$p = 5\cdot10^{16}cm^{-3}$$

$$n = [n_i(T=300K)]^2/p = \frac{(2.5\cdot10^{13}cm^{-3})^2}{5\cdot10^{16}cm^{-3}} = 1.25\cdot10^{10}cm^{-3}$$

2. (a) Mobility of electrons in Si = 1350 $cm^2/V\text{-}s$
Mobility of holes in Si = 480 $cm^2/V\text{-}s$

$\Rightarrow$ velocity of electrons $= \mu_n E = \left(1350 \frac{cm^2}{V\text{-}s}\right)\left(0.1 \frac{V}{\mu m}\right)$

$= 1.35 \cdot 10^4 \ m/s$

velocity of holes $= \mu_p E = \left(480 \frac{cm^2}{V\text{-}s}\right)\left(0.1 \frac{V}{\mu m}\right)$

$= 4.8 \cdot 10^3 \ m/s$

(b) Given $E = 0.1 \ V/\mu m$ hole current negligible
$\mu_n = 1350 \ cm^2/V\text{-}s$ $\mu_p = 480 \ cm^2/V\text{-}s$

$$J_{tot} = 1 \ mA/\mu m^2 = q[\mu_n n E + \mu_p p E] \approx q \mu_n \cdot n E$$

$$\therefore n = \frac{J_{tot}}{q \mu_n E} = \frac{1 mA/\mu m^2}{(1.6 \cdot 10^{-19} C)(1350 \ cm^2/V\text{-}s)(0.1 \ V/\mu m)}$$

$$= 4.6 \cdot 10^{17} \ cm^{-3}$$

3. Given $L = 0.1\,um$ $\quad A = (0.05\,um)^2$ $\quad V = 1V$

$\mu_n = 1350\ cm^2/V\text{-}s$ $\quad \mu_p = 480\ cm^2/V\text{-}s$

$n = 10^{17}\ cm^{-3}$ (assuming n-type dopant)

(a) $$n_i(T=300K) = 5.2\cdot 10^{15}(300)^{3/2}\exp\left[\frac{-1.12\,eV}{2(1.38\cdot 10^{-23}\,J/k)(300k)}\right]$$

$$= 1.08\cdot 10^{10}\ cm^{-3}$$

$p = n_i^2/n = 1.17\cdot 10^3\ cm^{-3}$ $\qquad E = V/L = 10\ V/um$

$$\therefore\ I_{tot} = A\cdot J_{tot} = A\cdot q[\mu_n n + \mu_p p]E$$

$$= A\cdot q\,[\mu_n n + \mu_p (n_i^2/n)]E$$

$$= (0.05\,um)^2(1.6\cdot 10^{-19}C)\left[1350\frac{cm^2}{V\text{-}s}(10^{17}cm^{-3}) + 480\frac{cm^2}{V\text{-}s}(1.17\cdot 10^3\,cm^{-3})\right]$$

$$\cdot (10\ V/um)$$

$\Rightarrow I_{tot} \approx 0.054\ mA$

(b) @ 400K : $n_i = 3.7 \cdot 10^{12}\ cm^{-3}$

$\underline{p} = n_i^2/n = 1.4 \cdot 10^{8}\ cm^{-3}$

$\underline{E} = 10\ V/um$

$$\therefore I_{tot} = A \cdot q\left[\mu_n n + \mu_p (n_i^2/n)\right] E$$

$$= (0.05\,um)^2 (1.6 \cdot 10^{-19} C)\left[1350\ \frac{cm^2}{V\text{-}s}(10^{17} cm^{-3}) + 480\ \frac{cm^2}{V\text{-}s}(1.4 \cdot 10^{8} cm^{-3})\right] \cdot (10\ V/um)$$

$$\Rightarrow I_{tot} \approx 0.054\ mA.$$

4. Given $L = 0.1\ \mu m$ $\quad A = (0.05\ \mu m)^2$ $\quad V = 1V$

$\mu_n = 3900\ cm^2/V\text{-}s$ $\quad \mu_p = 1900\ cm^2/V\text{-}s$

$n = 10^{17} cm^{-3}$ (assuming n-type dopant)

(a) From previous problem,

@ 300k: $n_i = 2.5 \cdot 10^{13}\ cm^{-3}$ $\quad p = n_i^2/n = 6.3 \cdot 10^9 cm^{-3}$

$E = 10\ V/\mu m$

$$I_{tot} = A \cdot J_{tot} = A\, q \left[\mu_n n + \mu_p (n_i^2/n)\right] E$$

$$= (0.05\,\mu m)^2 (1.6 \cdot 10^{-19} C) \left[3900 \frac{cm^2}{V\text{-}s}(10^{17} cm^{-3}) + 1900 \frac{cm^2}{V\text{-}s}(6.3 \cdot 10^{9} cm^{-3})\right] \cdot (10\ V/\mu m)$$

$$\Rightarrow I_{tot} \approx 62.4\ mA$$

(b) @ 400 k: $n_i = 2.9 \cdot 10^{15} cm^{-3}$ $\quad p = 8.5 \cdot 10^{13}\ cm^{-3}$

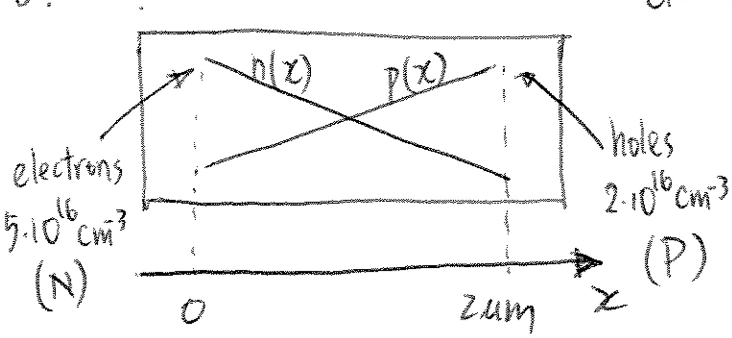
$E = 10\ V/\mu m$

$$I_{tot} = A\, q \left[\mu_n n + \mu_p (n_i^2/n)\right] E$$

$$= (0.05\,\mu m)^2 (1.6 \cdot 10^{-19} C) \left[3900 \frac{cm^2}{V\text{-}s}(10^{17} cm^{-3}) + 1900 \frac{cm^2}{V\text{-}s}(8.5 \cdot 10^{13} cm^{-3})\right] \cdot (10\ V/\mu m)$$

$$\Rightarrow I_{tot} \approx 62.4\ mA$$

5.



Given $D_n = 34\ cm^2/s$
$D_p = 12\ cm^2/s$
$L = 2\ \mu m$
$A = (1\ \mu m)^2$

The injected carriers diffuse from one end to the other.

$$I_{tot} = A \cdot J_{tot} = A \cdot q\left[\frac{dn}{dx}D_n - \frac{dp}{dx}D_p\right]$$

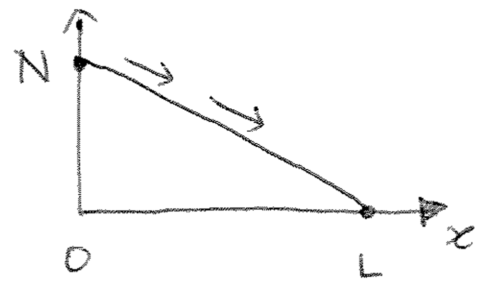
$$= A \cdot q\left[D_n\left(-\frac{N}{L}\right) - D_p\left(\frac{P}{L}\right)\right]$$

$$= (1\ \mu m)^2 (1.6 \cdot 10^{-19} C)\left[34\frac{cm^2}{s}\left(-\frac{5 \cdot 10^{16} cm^{-3}}{2\ \mu m}\right) - 12\frac{cm^2}{s}\left(\frac{2 \cdot 10^{16} cm^{-3}}{2\ \mu m}\right)\right]$$

$$= -15.5\ \mu A.$$

6. Given Area = a

find total e ectrons stored.



$$n(x) = -\frac{N}{L}x + N$$

$\therefore$ total electrons stored

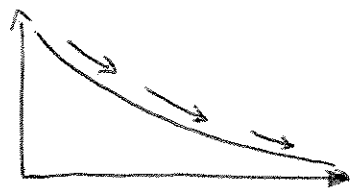
$$= \int a \cdot n(x)\, dx = \int_0^L a\left(-\frac{N}{L}x + N\right)dx$$

$$= aN\left(-\frac{x^2}{2L} + x\right)\Bigg|_0^L = \underline{\frac{aNL}{2}}$$

7. Given Area = a

find total electrons stored.

$$n(x) = N \cdot \exp\left(\frac{-x}{L_d}\right)$$



$\therefore$ total electrons stored

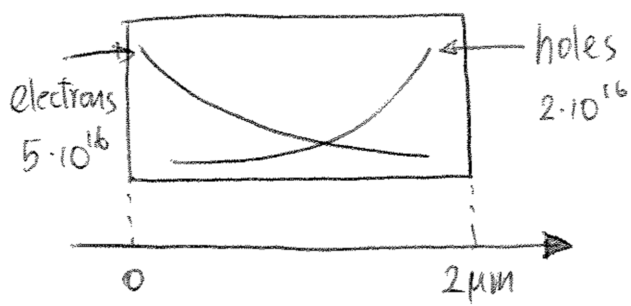
$$= \int a\, n(x)\, dx = \int_0^\infty a \cdot N \cdot \exp\left(\frac{-x}{L_d}\right) dx$$

$$= aN\left(-L_d \cdot \exp\frac{-x}{L_d}\right)\Bigg|_0^\infty = aNL_d.$$

For the linear profile, the result depends on the length, L.

For the exponential profile, the result is constant (since L_d is constant.)

8.



$$n(x) = N \exp(-x/L_d)$$

$$p(x) = P \exp\left(\frac{x-2}{L_d'}\right)$$

$N = 5 \cdot 10^{16}\,cm^{-3}$ $\quad$ $P = 2 \cdot 10^{16}\,cm^{-3}$

total number of electrons $= \int a \cdot n \, dx$

$$= \int_0^2 a \cdot n(x)\, dx = aN\left(-L_d \cdot \exp(-x/L_d)\right)\Big|_0^2$$

$$= aNL_d\left[1 - \exp(-2/L_d)\right]$$

total number of holes $= \int a \cdot p \, dx$

$$= \int_0^2 a \cdot p(x)\, dx = aP\left(L_d' \cdot \exp\left(\frac{x-2}{L_d'}\right)\right)\Big|_0^2$$

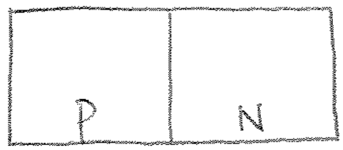
$$= aPL_d'\left[1 - \exp\left(-\frac{2}{L_d'}\right)\right]$$

9. Drift is analogous to water flow in a river.

Water flows from top of mountain to bottom because of gravitational field; electron flows from one terminal to the other because of electric field.

<u>DRIFT</u>		<u>WATER FLOW</u>
electrons	$\longleftrightarrow$	water
electric field	$\longleftrightarrow$	gravitational field.
drift/current	$\longleftrightarrow$	water flow

10. (a)



Assume Si.

$N_A = 4\cdot 10^{16}\,cm^{-3}$ $\quad N_D = 5\cdot 10^{17}\,cm^{-3}$

$$p_p \approx N_A = 4\cdot 10^{16}\,cm^{-3}$$
$$n_p = \frac{n_i^2}{p_p} = \frac{(1.08\cdot 10^{10}\,cm^{-3})^2}{4\cdot 10^{16}\,cm^{-3}} \approx 2.9\cdot 10^{3}\,cm^{-3}$$

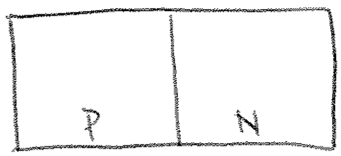
$$n_n \approx N_D = 5\cdot 10^{17}\,cm^{-3}$$
$$p_n = \frac{n_i^2}{n_n} = \frac{(1.08\cdot 10^{10}\,cm^{-3})^2}{5\cdot 10^{17}\,cm^{-3}} \approx 2.3\cdot 10^{2}\,cm^{-3}$$

(b) $V_0 = \frac{kT}{q}\ln\left(\frac{N_A\cdot N_D}{n_i^2}\right)$

@ 250K : $V_0 = 0.905\ V$
@ 300K : $V_0 = 0.848\ V$
@ 350K : $V_0 = 0.789\ V$

Towards higher temperatures, $V_0 \sim T\ln\left(\frac{1}{T^3}\right)$.
That is, overall, V_0 drops with higher T.

11. Given $N_D = 3 \cdot 10^{16}\ cm^{-3}$ $\quad n_i = 1.08 \cdot 10^{10}\ cm^{-3}$



find V_0.

$$V_0 = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right) = \frac{kT}{q} \ln\left(\frac{N_D}{n_i}\right)$$

$$= \frac{(1.38 \cdot 10^{-23}\ J/K)(300K)}{1.6 \cdot 10^{-19} C} \ln\left(\frac{3 \cdot 10^{16}\ cm^{-3}}{1.08 \cdot 10^{10}\ cm^{-3}}\right)$$

$$= 0.384\ V$$

12. Given $N_D = 3\cdot 10^{16}\ cm^{-3}$ $\quad N_A = 2\cdot 10^{15}\ cm^{-3}$

$V_R = 1.6\ V$ $\quad \epsilon_{si} = 11.7\times 8.85\cdot 10^{-14} \frac{F}{cm^2}$

(a) $n_i = 1.08\cdot 10^{10}\ cm^{-3}$

$$V_0 = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) \approx (26mV)\ln\left[\frac{3\cdot 10^{16}\times 2\cdot 10^{15}}{(1.08\cdot 10^{10})^2}\right]$$

$$= 0.698\ V$$

$$C_{j0} = \sqrt{\frac{\epsilon_{si}\cdot q}{2}\cdot\frac{N_A N_D}{N_A+N_D}\cdot\frac{1}{V_0}}$$

$$= \left[\frac{11.7\times 8.85\cdot 10^{-14}\times q}{2}\cdot\frac{3\times 10^{16}\times 2\times 10^{15}}{3\times 10^{16}+2\times 10^{15}}\cdot\frac{1}{V_0}\right]^{\frac{1}{2}}$$

$$= 0.149\ fF/\mu m^2$$

$$\therefore\ C_j(V_R) = \left[1 + 1.6/V_0\right]^{-\frac{1}{2}}\times C_{j0} = 0.082\ fF/\mu m^2$$

(b) Given $C_{j,new} = 2\cdot C_{j,old}$

$$\Rightarrow \sqrt{\frac{\frac{q\epsilon_{si}}{2}\cdot\frac{N_A N_D}{N_A+N_D}\cdot\frac{1}{V_0}}{1+\frac{V_R}{V_0}}} = \sqrt{\frac{\frac{q\epsilon_{si}}{2}\cdot\frac{N_A N_D'}{N_A+N_D'}\cdot\frac{1}{V_0'}}{1+\frac{V_R}{V_0'}}}\times 2$$

Squaring both sides & simplifying gives:

$$\frac{\left(\frac{N_D}{N_A+N_D}\right)}{V_0+V_R} = 4\cdot\frac{\left(\frac{N_D'}{N_A+N_D'}\right)}{V_0'+V_R}$$, where N_D' = old value.

Here, there is only one variable, N_D (new value). The solution can be found iteratively by solving this equation. But we can make an assumption that $V_0+V_R \approx V_0'+V_R$ since $V_R = 1.6\ V$, the dominant term. Then we verify V_0 & V_0' afterwards.

$$\Rightarrow \frac{N_D}{N_A+N_D} = 4\frac{N_D'}{N_A+N_D'}$$

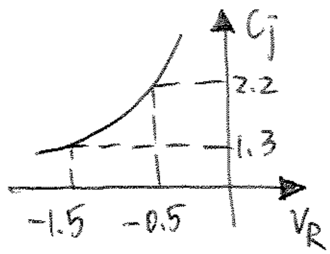
$$\Rightarrow N_D = \frac{4N_D'N_A}{N_A-3N_D'} = \frac{4(2\cdot10^{15})(3\cdot10^{16})}{(3\cdot10^{16})-3\cdot(2\cdot10^{15})} \approx 1.00\cdot10^{16}\ cm^{-3}$$

$$\Rightarrow \frac{N_D}{N_D'} = \frac{1\cdot10^{16}}{2\cdot10^{15}} \approx 5$$

Verify: $V_{0,old} = 0.698\ V \Rightarrow V_0+V_R \approx 2.3\ V$
$V_{0,new} = 0.740\ V \Rightarrow V_0+V_R \approx 2.3\ V$ (✓)

∴ Increase N_D by 5 times.

13.



$$\frac{C_{j0}}{\sqrt{1+\frac{0.5}{V_0}}} = 2.2 \quad \text{——} \ ①$$

$$\frac{C_{j0}}{\sqrt{1+\frac{1.5}{V_0}}} = 1.3 \quad \text{——} \ ②$$

① ÷ ② : $\dfrac{1+{1.5}/{V_0}}{1+{0.5}/{V_0}} = \left(\dfrac{2.2}{1.3}\right)^2 \Rightarrow V_0 = 0.0365\ V$

Substitute V_0 into ①:

$$C_{j0} = 2.2\sqrt{1+\frac{0.5}{V_0}} \approx 8.43\ fF/\mu m^2$$

$$\Rightarrow \frac{N_A N_D}{N_A+N_D} = (C_{j0})^2 \cdot V_0 \cdot \frac{2}{\varepsilon_{si} q}$$

$$= \left(8.43\ \frac{fF}{\mu m^2}\right)^2 \times (0.0365V) \cdot \frac{2}{\varepsilon_{si} q} \approx 3.13 \cdot 10^{17}\ cm^{-3}$$

Fix a value for $N_A > \frac{N_A N_D}{N_A + N_D} \triangleq y$

$$N_A = 2 \cdot 10^{18}\,cm^{-3} \Rightarrow N_D = \frac{y N_A}{N_A - y}$$

$$= \frac{(3.13 \cdot 10^{17}\,cm^{-3})(2 \cdot 10^{18}\,cm^{-3})}{(2 \cdot 10^{18} - 3.13 \cdot 10^{17})\,cm^{-3}}$$

$$\approx 3.71 \cdot 10^{17}\,cm^{-3}$$

14 (a) In forward bias, $I_D = 1mA$, $V_D = 750mV$

$$\therefore I_S \approx I_D e^{-\frac{V_D}{V_T}} = (1mA)\exp[-750mV/26mV]$$

$$= 2.97 \cdot 10^{-16} A$$

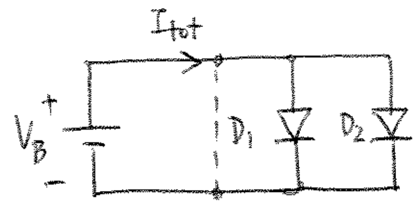
(b) Since $I_S \propto$ Area, doubling area implies doubling I_S. From (a),

$$I_D = 1\ mA = 2 \times I_S e^{V_D/V_T}$$

$$\therefore V_D = V_T \ln\left(\frac{I_D}{2I_S}\right) = (26mV)\ln\left(\frac{1mA}{2 \cdot 2.97 \times 10^{-16} A}\right)$$

$$= 0.732\ V$$

15 (a)



$$I_{tot} = I_{D_1} + I_{D_2} = I_{S_1}(e^{V_B/V_T} - 1) + I_{S_2}(e^{V_B/V_T} - 1)$$

$$= (I_{S_1} + I_{S_2})(e^{V_B/V_T} - 1)$$

Therefore, the parallel combination operates as an exponential device, with an equivalent saturation current of $I_{S_1} + I_{S_2}$.

(b) By KVL, $V_{D_1} = V_{D_2}$

$$\Rightarrow V_T \ln\left(\frac{I_{D_1}}{I_{S_1}}\right) = V_T \ln\left(\frac{I_{D_2}}{I_{S_2}}\right)$$

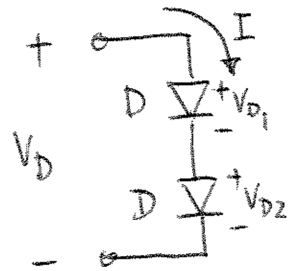
Also, $I_{tot} = I_{D_1} + I_{D_2} \Rightarrow I_{D_2} = I_{tot} - I_{D_1}$

$$\therefore V_T \ln\left(\frac{I_{D_1}}{I_{S_1}}\right) = V_T \ln\left(\frac{I_{tot} - I_{D_1}}{I_{S_2}}\right)$$

$$\Rightarrow I_{D_1} = I_{tot}\left(\frac{I_{S_1}}{I_{S_1} + I_{S_2}}\right)$$

$$\Rightarrow I_{D_2} = I_{tot}\left(\frac{I_{S_2}}{I_{S_1} + I_{S_2}}\right)$$

16. (a)



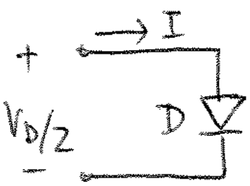
Suppose $I = I_s\left(e^{\frac{V_{D1}}{V_T}} - 1\right)$

$I_{D_2} = I_s\left(e^{\frac{V_{D2}}{V_T}} - 1\right)$

By KCL, $I_{D_1} = I_{D_2} = I$

$$\Rightarrow \left(e^{V_{D1}/V_T} - 1\right) = \left(e^{V_{D2}/V_T} - 1\right) \Rightarrow V_{D_1} = V_{D2} = \frac{V_D}{2}$$

$$\therefore\ I = I_s\left(e^{(V_D/2)/V_T} - 1\right)$$



Therefore, a series combination can be viewed as a single two-terminal device with exponential characteristics.

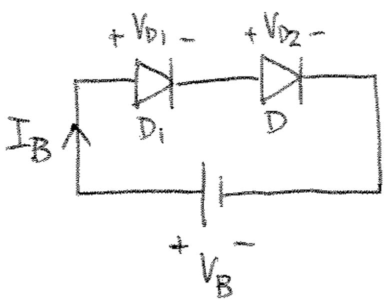
(b) Suppose V_i = initial V_D. Need 10x
V_f = final V_D increase in I.

$$\Rightarrow 10 = \frac{I_s\left(e^{V_f/V_T} - 1\right)}{I_s\left(e^{V_i/V_T} - 1\right)} \approx e^{\frac{V_f - V_i}{V_T}}$$

$$\therefore\ \Delta V = V_f - V_i = V_T \ln(10) = (26\,mV)\ln(10)$$

$$\approx 60.\ mV.$$

17.



Find I_B, V_{D1}, V_{D2} in terms of V_B, $I_{,}$, I_{S2}

By KVL, $V_B = V_{D1} + V_{D2} = V_T \ln\left(\dfrac{I_B}{I_{S1}}\right) + V_T \ln\left(\dfrac{I_B}{I_{S2}}\right)$

$$\Rightarrow \quad V_B = V_T \ln\left(\frac{I_B^2}{I_{S1} I_{S2}}\right)$$

$$\therefore \quad I_B = \sqrt{I_{S1} I_{S2} \cdot \exp V_B/V_T} = \sqrt{I_{S1} I_{S2}} \cdot \exp\left(\frac{V_B}{2V_T}\right).$$

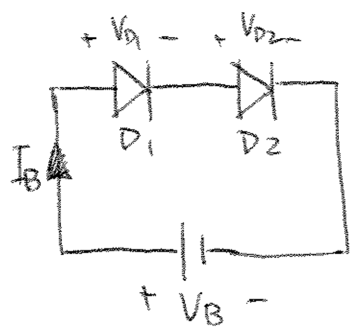
$$V_{D1} = V_T \ln\left(\frac{I_B}{I_{S1}}\right) = V_T \ln\left(\frac{\sqrt{I_{S1} I_{S2}}}{I_{S1}} \cdot \exp \frac{V_B}{2V_T}\right)$$

$$= V_T \ln \sqrt{\frac{I_{S2}}{I_{S1}}} + \frac{V_B}{2}$$

$$V_{D2} = V_T \ln\left(\frac{I_B}{I_{S2}}\right) = V_T \ln\left(\frac{\sqrt{I_{S1} I_{S2}}}{I_{S2}} \cdot \exp \frac{V_B}{2V_T}\right)$$

$$= V_T \ln \sqrt{\frac{I_{S1}}{I_{S2}}} + \frac{V_B}{2}$$

18.



$$V_B = V_T \ln \frac{I_B}{I_{S1}} + V_T \ln \frac{I_B}{I_{S2}} = V_T \ln\left(\frac{I_B^2}{I_{S1} I_{S2}}\right)$$

$$\Rightarrow I_B = \sqrt{I_{S1} I_{S2}} \cdot \exp \frac{V_B}{2V_T}$$

Increase I_B by 10 times:

$$I_{B,new} = 10 I_B$$

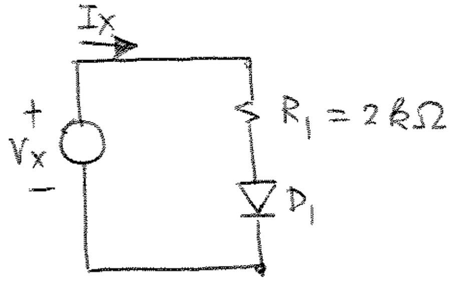
$$\Rightarrow V_{B,new} = V_T \ln\left(\frac{I_{B,new}^2}{I_{S1} I_{S2}}\right) = V_T \ln\left[\frac{(10 I_B)^2}{I_{S1} I_{S2}}\right]$$

$$= V_T \ln\left(\frac{I_B^2}{I_{S1} I_{S2}}\right) + V_T \ln 100$$

$$= V_B + V_T \ln 100 \approx V_B + 0.120\text{ V}$$

∴ V_B increases by 0.120 V.

19.



$$I_{D_1} = I_S\left(e^{\frac{V_{D1}}{V_T}} - 1\right)$$

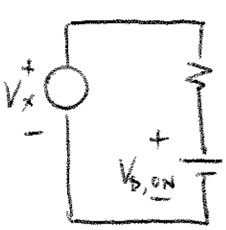
$$I_S = 2 \cdot 10^{-15} A$$

By KVL,
$$\begin{aligned} V_X &= I_X R_1 + V_{D1} \\ &= I_X R_1 + V_T \ln\left(I_{D_1}/I_S\right) \\ &= I_X R_1 + V_T \ln\left(I_X/I_S\right) \end{aligned}$$

This can be solved directly with special programs or graphing calculators. But this can be solved iteratively, by hand.

$\boxed{V_X = 0.5V}$

We suppose that D_1 is on.
$\Rightarrow$ current flows through D_1.
Assume a $V_{D,ON}$:
$\Rightarrow V_{D_1} = 0.4V$



$$\Rightarrow I_X = \frac{V_X - V_{D1}}{R_1} = \frac{(0.5-0.4)V}{2k\Omega} = 0.05\,mA$$

$$V_{D_1} = V_T \ln\left(\frac{I_X}{I_S}\right) = (0.026V)\ln\left(\frac{0.05\,mA}{2 \cdot 10^{-15} A}\right) \approx 0.62\,V$$

$\therefore$ Contradiction because V_{D_1} exceeds V_X !!
This means our assumption is incorrect
$\Rightarrow D_1$ is OFF $\Rightarrow V_{D_1} = V_X = 0.5V$ $\quad I_X = 0$

$\boxed{V_X = 0.8\text{ V}}$ Suppose D_1 is on. (This is a reasonable assumption since most diodes turn on at around $V_D = 0.7$ V.)

For startup, use $V_{D_1} = 0.7$ V.

$$V_{D_1} = 0.7\text{ V} \Rightarrow I_X = \frac{V_X - V_{D_1}}{R_1} = 0.05\text{ mA}$$

$$\Rightarrow V_{D_1} = V_T \ln(I_X/I_{S_1}) \approx 0.622\text{ V}$$

$$V_{D_1} = 0.622\text{ V} \Rightarrow I_X = \frac{(0.8 - 0.622)\text{V}}{2\text{ k}\Omega} = 0.089\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026\text{ V})\ln\left(\frac{0.089\text{ mA}}{2\cdot 10^{-15}\text{ A}}\right) \approx 0.637\text{ V}$$

$$V_{D_1} = 0.637\text{ V} \Rightarrow I_X = \frac{(0.8 - 0.637)\text{V}}{2\text{ k}\Omega} = 0.082\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026\text{V})\ln\left(\frac{0.082\text{ mA}}{2\cdot 10^{-15}\text{ A}}\right) \approx 0.635\text{V}$$

$$V_{D_1} = 0.635\text{V} \Rightarrow I_X = \frac{(0.8 - 0.635)\text{V}}{2\text{ k}\Omega} = 0.083\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026\text{V})\ln\left(\frac{0.083\text{ mA}}{2\cdot 10^{-15}\text{A}}\right) \approx 0.635\text{V}$$

$\therefore$ With an accuracy of three decimal points,

$V_{D_1} \approx 0.635$ V

$I_X \approx 0.082$ mA.

(Of course, more iterations give a more accurate result.)

$\boxed{V_X = 1\text{ V}}$ Suppose, again, that D_1 is on. Use V_{D_1} from previous calculations as starting point.

$$V_{D_1} = 0.635\text{ V} \Rightarrow I_X = \frac{(1-0.635)\text{V}}{2\text{ k}\Omega} = 0.18\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026\text{V})\ln\left(\frac{0.18\text{mA}}{2\cdot 10^{-15}\text{A}}\right) \approx 0.656\text{ V}$$

$$V_{D_1} = 0.656\text{V} \Rightarrow I_X = \frac{(1-0.656)\text{V}}{2\text{ k}\Omega} = 0.17\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026\text{V})\ln\left(\frac{0.17\text{mA}}{2\cdot 10^{-15}\text{A}}\right) \approx 0.655\text{ V}$$

$$V_{D_1} = 0.655\text{ V} \Rightarrow I_X = \frac{(1-0.655)\text{V}}{2\text{ k}\Omega} = 0.17\text{ mA}$$

$$\Rightarrow V_{D_1} = 0.655\text{ V}$$

$$\therefore V_{D_1} \approx 0.655\text{ V}$$
$$I_X \approx 0.17\text{ mA}.$$

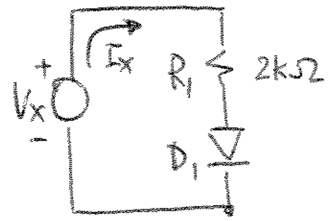
$\boxed{V_X = 1.2\text{ V}}$ Using similar assumptions as those in previous calculations,

$$V_{D_1} = 0.655\text{ V} \Rightarrow I_X = 0.27\text{ mA} \Rightarrow V_{D_1} \approx 0.667\text{V}$$
$$V_{D_1} = 0.667\text{ V} \Rightarrow I_X = 0.27\text{ mA} \Rightarrow V_{D_1} \approx 0.666\text{V}$$
$$V_{D_1} = 0.666\text{V} \Rightarrow I_X = 0.27\text{ mA} \Rightarrow V_{D_1} \approx 0.666\text{V}$$

$$\therefore I_X \approx 0.27\text{mA} \qquad V_{D_1} = 0.666\text{ V}$$

For more than 3x increase in I_x, V_{D_1} only increases by ~30 mV, which is less than 10% of the turn-on voltage of the diode. In other words, once the diode conducts current, its voltage varies marginally (expected due to its exponential characteristic). This also implies that the diode, once on, can allow any amount of current to flow through (until $V_{D_1} \times I_{D_1}$ becomes so large that the diode simply "breaks down".)

20.



Since $I_{S_1} \propto$ Area, I_{D_1} becomes:

$$I_{D_1} = \underbrace{10 \times (2 \cdot 10^{-15} A)}_{I_{S_1}'} \left(e^{\frac{V_{D1}}{V_T}} - 1\right)$$

$\boxed{V_X = 0.8\ V}$ Suppose D_1 is on. Assume $V_{D_1} = 0.7V$

$$V_{D_1} = 0.7\ V \Rightarrow I_X = \frac{V_X - V_{D_1}}{R_1} = \frac{0.1\ V}{2\ k\Omega} = 0.05\ mA$$

$$\Rightarrow V_{D_1} = V_T \ln(I_X / I_{S_1}') = (0.026V) \ln\left(\frac{0.05\ mA}{20 \cdot 10^{-15} A}\right)$$

$$= 0.563\ V$$

$$V_{D_1} = 0.563V \Rightarrow I_X = \frac{(0.8 - 0.563)\ V}{2\ k\Omega} = 0.12\ mA$$

$$\Rightarrow V_{D_1} = (0.026\ V) \ln\left(\frac{0.12\ mA}{20 \cdot 10^{-15} A}\right) \approx 0.585\ V$$

$$V_{D_1} = 0.585V \Rightarrow I_X = \frac{(0.8 - 0.585)V}{2\ k\Omega} = 0.11\ mA$$

$$\Rightarrow V_{D_1} = (0.026\ V) \ln\left(\frac{0.11\ mA}{20 \cdot 10^{-15} A}\right) \approx 0.583\ V$$

$$V_{D_1} = 0.583\ V \Rightarrow I_X = \frac{(0.8 - 0.583)V}{2\ k\Omega} = 0.11\ mA$$

$$\Rightarrow V_{D_1} = 0.583\ V$$

$\therefore$ $V_{D_1} \approx 0.583\ V$

$I_X \approx 0.11\ mA.$

$\boxed{V_X = 1.2\,V}$ Suppose D_1 is on. Use results from previous calculations as starting point.

$$V_{D_1} = 0.583\,V \Rightarrow I_X = \frac{(1.2-0.583)V}{2\,k\Omega} = 0.31\,mA$$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{0.31mA}{20\times10^{-15}A}\right) \approx 0.610\,V$$

$$V_{D_1} = 0.610\,V \Rightarrow I_X = \frac{(1.2-0.610)V}{2\,k\Omega} = 0.30\,mA$$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{0.30\,mA}{20\cdot10^{-15}A}\right) \approx 0.609\,V$$

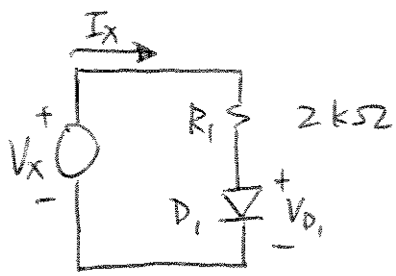
$$V_{D_1} = 0.609\,V \Rightarrow I_X = \frac{(1.2-0.609)V}{2\,k\Omega} = 0.30\,mA$$

$$\Rightarrow V_{D_1} = 0.609\,V$$

$\therefore\ V_{D_1} \approx 0.609\,V$

$I_X \approx 0.30\,mA.$

By increasing the cross-section area of D_1, intuitively this means D_1 can conduct same amount of current with less V_{D_1}. The results have shown that in this problem, V_{D_1} is less and I_X is more.

21. 

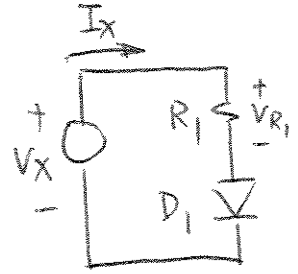
Given: @ $V_X = 2V$, $V_{D_1} = 850mV$

$$\Rightarrow I_X = \frac{V_X - V_{D_1}}{R_1} = 0.58\ mA$$

$$\therefore\ I_S = \frac{I_X}{(e^{V_D/V_T} - 1)} \approx I_X \exp[-V_{D}/V_T]$$

$$= (0.58\ mA) \exp[-0.85/0.026] \approx 3.64 \cdot 10^{-18} A$$

22.



Given $V_{R_1} = V_X/2$, find V_X.
$I_S = 2 \cdot 10^{-16} A$.

By KCL,

$$\frac{V_{R_1}}{R_1} = I_S\left(e^{V_{D1}/V_T} - 1\right)$$

Also, $V_{R_1} = V_{D_1} = V_X/2$ (KVL).

$$\therefore \frac{V_X/2}{R_1} = I_S \cdot \left(\exp\left[\frac{V_{D_1}/2}{V_T}\right] - 1\right)$$

This must be solved iteratively. From experience, suppose $V_X = 2V$.

$$V_X = 2V \Rightarrow I_X = \frac{V_X/2}{R_1} = \frac{1V}{2k\Omega} = 5mA$$

$$\Rightarrow V_X = 2 \cdot V_{D_1} = 2V_T \ln(I_X/I_S)$$

$$= 2(0.026V)\ln\left(\frac{5mA}{2 \cdot 10^{-16}A}\right) \approx 1.48\ V$$

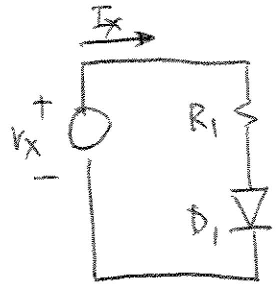
$$V_X = 1.48V \Rightarrow I_X = \frac{1.48/2\ V}{2k\Omega} = 0.37\ mA$$

$$\Rightarrow V_X = 2(0.026V)\ln\left(\frac{0.37mA}{2 \cdot 10^{-16}A}\right) \approx 1.47\ V$$

$$V_X = 1.47V \Rightarrow I_X = \frac{(1.47)/2\ V}{2k\Omega} = 0.37\ mA$$

$$\Rightarrow V_X = 1.47\ V$$

23.



Given $V_X = 1V \Rightarrow I_X = 0.2mA$

$V_X = 2V \Rightarrow I_X = 0.5mA$

Find R_1 and I_S.

By KVL, $V_{D_1} = V_X - I_X R_1 = V_T \ln\left(\frac{I_X}{I_S}\right)$

$$\Rightarrow \quad 1 - (0.2mA)R_1 = (0.026V)\ln\left(\frac{0.2mA}{I_S}\right) \quad \text{———} \; ①$$

$$2 - (0.5mA)R_1 = (0.026V)\ln\left(\frac{0.5mA}{I_S}\right) \quad \text{———} \; ②$$

$$② - ① : \quad 1 - (0.3mA)R_1 = (0.026V)\ln\left(\frac{0.5}{0.2}\right)$$

$$\Rightarrow R_1 = \frac{1-(0.026) \;\; V}{0.3m \;\; A} = 3.25 \; k\Omega$$

Substitute R_1 into ①:

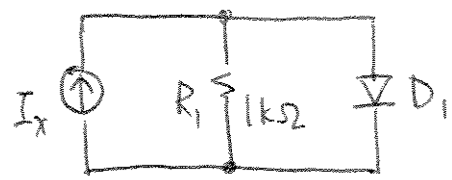
$$I_S = I_X \cdot \exp\left[-\frac{V_X - I_X R_1}{V_T}\right]$$

$$= (0.2mA)\exp\left[-\frac{1-(0.2m)(3.25k)}{0.026}\right] \approx 2.94 \cdot 10^{-10} A$$

$\therefore \; R_1 \approx 3.25 \; k\Omega$

$I_S \approx 2.94 \cdot 10^{-10} A.$

24.



Given $I_S = 3 \cdot 10^{-16} A$, find V_{D_1}.

By KCL, $I_x = \dfrac{V_{D_1}}{R_1} + I_{D_1} = \dfrac{V_T}{R_1} \ln\left(\dfrac{I_{D_1}}{I_S}\right) + I_{D_1}$

Since I_x, V_T, R_1 and I_S are known, this can be solved directly with special programs or graphing calculators. However, this can be also solved by iterations: Assume a V_{D_1}, calculate I_{D_1}, and re-iterate on V_{D_1}.

Assume $V_{D_1} = 0.7\,V$ as starting point.

$\boxed{I_x = 1\,mA}$

$V_{D_1} = 0.7\,V \Rightarrow I_{D_1} = I_x - V_{D_1}/R_1 = 1\,mA - \dfrac{0.7V}{1k\Omega} = 0.3\,mA$

$\Rightarrow V_{D_1} = V_T \ln\left(I_x/I_S\right)$

$= (0.026V) \ln\left(\dfrac{0.3mA}{3 \cdot 10^{-16}A}\right) \approx 0.718\,V$

$V_{D_1} = 0.718\,V \Rightarrow I_{D_1} = 1\,mA - \dfrac{0.718\,V}{1\,k\Omega} = 0.28\,mA$

$\Rightarrow V_{D_1} = (0.026\,V) \ln\left(\dfrac{0.28\,mA}{3 \cdot 10^{-16}A}\right) \approx 0.717\,V$

$$V_{D_1} = 0.717\,V \Rightarrow I_{D_1} = 1mA - \frac{0.717V}{1\,k\Omega} = 0.28\,mA$$

$$\Rightarrow V_{D_1} = 0.717\,V$$

$\therefore V_{D_1} \approx 0.717\,V.$

$\boxed{I_X = 2mA}$ Assume $V_{D_1} = 0.717\,V$ from previous result.

$$V_{D_1} = 0.717\,V \Rightarrow I_{D_1} = 2mA - \frac{0.717V}{1\,k\Omega} = 1.28\,mA$$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{1.28\,mA}{3\cdot 10^{-16}A}\right) \approx 0.756V$$

$$V_{D_1} = 0.756\,V \Rightarrow I_{D_1} = 2mA - \frac{0.756V}{1\,k\Omega} = 1.24\,mA$$

$$\Rightarrow V_{D_1} = (0.026\,V)\ln\left(\frac{1.24mA}{3\cdot 10^{-16}A}\right) \approx 0.755V$$

$$V_{D_1} = 0.755V \Rightarrow I_{D_1} = 2mA - \frac{0.755\,V}{1\,k\Omega} = 1.24\,mA$$

$$\Rightarrow V_{D_1} = 0.755V$$

$\therefore V_{D_1} = 0.755\,V$

$\boxed{I_X = 4\text{ mA}}$ Assume $V_{D_1} = 0.755\text{V}$ from previous result.

$$V_{D_1} = 0.755\text{V} \Rightarrow I_{D_1} = 4\text{mA} - \frac{0.755\text{V}}{1\text{k}\Omega} = 3.25\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026)\text{V} \ln\left(\frac{3.25\text{mA}}{3\cdot 10^{-16}\text{A}}\right) \approx 0.780\text{ V}$$

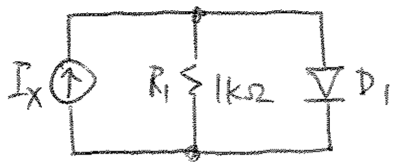
$$V_{D_1} = 0.780\text{ V} \Rightarrow I_{D_1} = 4\text{mA} - \frac{0.780\text{V}}{1\text{k}\Omega} = 3.22\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026\text{ V})\ln\left(\frac{3.22\text{mA}}{3\cdot 10^{-16}\text{A}}\right) \approx 0.780\text{V}$$

$\therefore V_{D_1} \approx 0.780\text{ V}.$

Note: As I_X increases, I_{D_1} increases, while (V_{D_1}/R_1) stays relatively the same. Because of the exponential characteristic, the diode, once on, will <u>absorb</u> as much current as necessary to satisfy KCL.

25.



Given $I_{D_1} = 0.5mA$ when $I_X = 1.3mA$, find I_S.

This means $V_{D_1} = (I_X - I_{D_1})R_1$

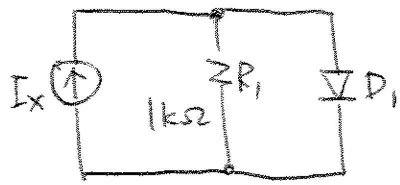
$= (0.8mA) 1k\Omega = 0.8V$

$\Rightarrow I_S = I_{D_1} \cdot exp\left[-V_{D_1}/V_T\right]$

$= (0.5mA)\ exp\left[-0.8V/0.026V\right]$

$\approx 2.17 \cdot 10^{-17} A$

2b



Given $I_{R_1} = I_x/2$

$I_S = 3 \cdot 10^{-16}\,A$

find I_x.

$$V_{D_1} = \frac{I_x}{2} \cdot R_1 = V_T \ln\left(\frac{I_x/2}{I_S}\right)$$

This can be solved directly with special programs or graphing calculators. Alternatively, one can solve this iteratively by hand.

Assume $V_D = 0.8\ V$.

$$V_D = 0.8\ V \Rightarrow (I_x/2) = \frac{V_D}{R_1} = \frac{0.8\ V}{1\ k\Omega} = 0.8\ mA$$

$$\Rightarrow V_D = V_T \ln\left(\frac{I_x/2}{I_S}\right) = (0.026V)\ln\left(\frac{0.8\,mA}{3 \cdot 10^{-16}A}\right)$$

$$\approx 0.744\ V$$

$$V_D = 0.744\ V \Rightarrow I_x/2 = \frac{0.744V}{1k\Omega} = 0.744\ mA$$

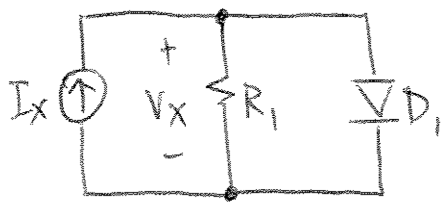
$$\Rightarrow V_D = (0.026V)\ln\left(\frac{0.744\,mA}{3 \cdot 10^{-16}A}\right) \approx 0.742\ V$$

$$V_D = 0.742V \Rightarrow I_x/2 = \frac{0.742V}{1k\Omega} = 0.742 \text{ mA}$$

$$\Rightarrow V_D = (0.026V) \ln\left(\frac{0.742mA}{3 \cdot 10^{-16}A}\right) \approx 0.742V$$

$$\therefore I_x = 2(0.742mA) = 1.48 \text{ mA}$$

27.



Given $I_X = 1mA \rightarrow V_X = 1.2V$
$I_X = 2mA \rightarrow V_X = 1.8V$

find R_1 and I_S.

$I_{D_1} = I_X - V_X/R_1$ (KCL)

By KVL, $V_X = V_T \ln\left(\frac{I_{D_1}}{I_S}\right) = V_T \ln\left(\frac{I_X - V_X/R_1}{I_S}\right)$

$$\Rightarrow (1.2\ V) = (0.026\ V) \ln\left[\frac{(1mA) - (1.2V)/R_1}{I_S}\right] \text{ ——— } ①$$

$$(1.8\ V) = (0.026\ V) \ln\left[\frac{(2mA) - (1.8\ V)/R_1}{I_S}\right] \text{ ——— } ②$$

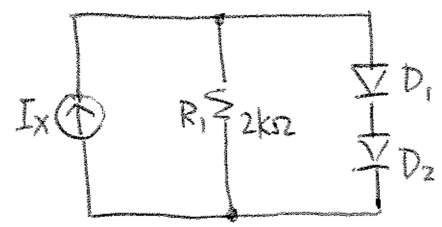
$$② - ①: \quad 0.6V = (0.026V) \ln\left(\frac{2mA - 1.8V/R_1}{1mA - 1.2V/R_1}\right)$$

$$\Rightarrow R_1 = \frac{1.2 \cdot \exp[0.6/0.026] - 1.8}{1\,mA \cdot \exp[0.6/0.026] - 2\,mA} \approx 1.2\ k\Omega$$

$$I_S = I_D \exp[-V_X/V_T] = \left(2mA - \frac{1.8V}{1.2k\Omega}\right) \exp\left[\frac{-1.8V}{0.026V}\right]$$

$$\approx 4.29 \cdot 10^{-34}\ A.$$

28.



Given $D_1 = D_2$ with
$I_S = 5 \cdot 10^{-16} A$

Find V_{R_1} for $I_X = 2mA$.

Current through the diodes $= I_D$
$= I_X - \dfrac{V_{R_1}}{R_1}$ where V_{R_1} = voltage across R_1

$$\Rightarrow V_{R_1} = 2 \cdot V_T \ln\left(\frac{I_D}{I_S}\right) = 2\left[V_T \ln\left(\frac{I_X}{I_S} - \frac{V_{R_1}}{I_S R_1}\right)\right]$$

This can be solved directly with special programs or graphing calculators or by hand iteratively.

Assume a V_{R_1}, calculate I_D, and re-iterate on new $V_{R_1} = (2 \times V_{D_1})$. From experience, most diodes conduct at $V_D \approx 0.7$ V. Assume $V_{R_1} = 1.4$ V.

$$V_{R_1} = 1.4 \text{ V} \Rightarrow I_D = I_X - \frac{V_{R_1}}{R_1} = 2mA - \frac{1.4V}{2k\Omega} = 1.3mA$$

$$\Rightarrow V_{R_1} = 2 V_T \ln\left(\frac{I_D}{I_S}\right)$$

$$= 2(0.026V)\ln\left(\frac{1.3mA}{5 \times 10^{-16}A}\right) \approx 1.49 \text{ V}$$

$V_{R_1} = 1.49V \Rightarrow I_D = 2mA - \frac{1.49}{2k\Omega} = 1.26\ mA$

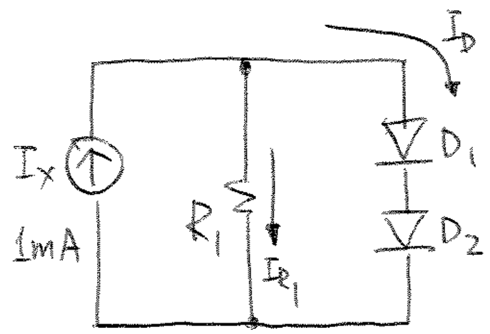
$\Rightarrow V_{R_1} = 2(0.026V)\ln\left(\frac{1.26\ mA}{5 \cdot 10^{-16}A}\right) \approx 1.48V$

$V_{R_1} = 1.48V \Rightarrow I_D = 2mA - \frac{1.48V}{2k\Omega} = 1.26\ mA$

$\Rightarrow V_{R_1} = 1.48V$

$\therefore$ voltage across $R_1 = 1.48\ V$

29.



Given $I_{R_1} = 0.5\,mA$,
$I_S = 5\cdot 10^{-16} A$ for each diode.

Find R_1.

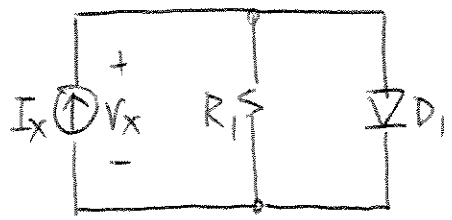
By KCL, $I_D = I_X - I_{R_1} = 0.5\,mA$

$$\Rightarrow V_{D_1} = V_{D_2} = V_T \ln\left(\frac{I_D}{I_S}\right) = 0.026 \ln\left(\frac{0.5mA}{5\cdot 10^{-16}A}\right)$$

$$\approx 0.718\ V$$

$$\therefore R_1 = \frac{V_{R_1}}{I_{R_1}} = \frac{2V_{D_1}}{I_{R_1}} = \frac{2(0.718V)}{0.5mA} = 2.87\,k\Omega$$

30.

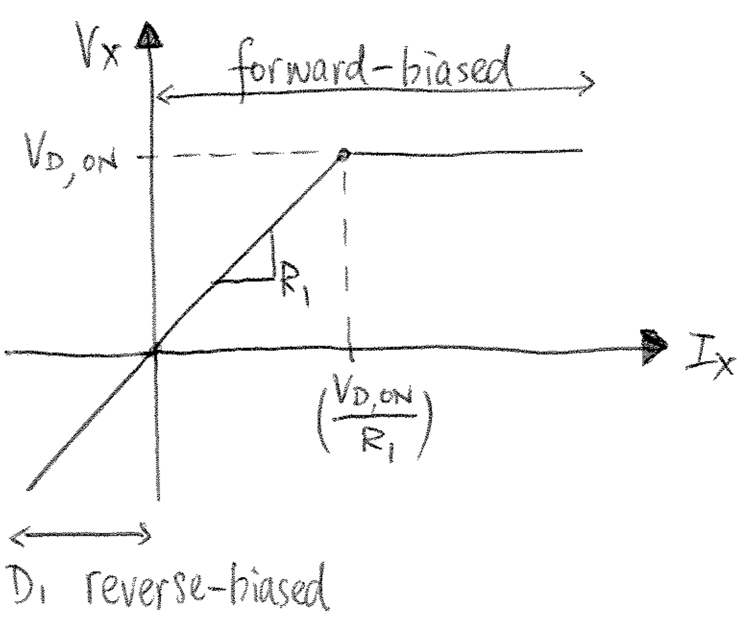


(a) Constant-voltage model:

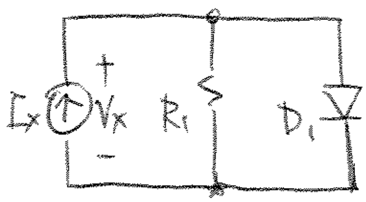
Consider, first, the extreme cases: when D_1 is off, we have the following:

This implies V_X is linearly proportional to I_X

When D_1 is on, V_X is fixed (by KVL) by D_1 ($= V_{D,ON}$). This implies that any additional current from I_X cannot flow through R_1, which means D_1 will absorb all the currents to satisfy KVL.



(b) exponential model :



Assume I_S negligible.

When D_1 is off, most of I_x flows through R_1. When D_1 is on, V_{D_1} $(= V_x)$ follows this relationship:

$$V_{D_1} = V_x = V_T \ln\left(\frac{I_{D_1}}{I_S}\right) = V_T \ln\left(\frac{I_x}{I_S} - \frac{V_x}{I_x R_1}\right)$$

$$\Rightarrow I_x = I_S \exp(V_x/V_T) + V_x/R_1$$

$$\approx I_S \exp(V_x/V_T) \quad \text{when } D_1 \text{ is forward-biased } (V_x > V_T)$$

i.e. $V_x \approx V_T \ln(I_x/I_S)$