信号处理作业及答案

Shi Zheng

December 15, 2016

1 Homework 2

project 2.1. 证明：一个函数与单位阶跃函数的卷积等于该函数的积分,即

$$f(t) * u(t) = \int_{-\infty}^{t} f(t)\, dt$$

answer ：

$$\begin{aligned}
f(t) * u(t) &= \int_{-\infty}^{+\infty} f(t-\tau)u(\tau)\, d\tau && (1)\\
&= \int_{-\infty}^{0} f(t-\tau)u(\tau)\, d\tau + \int_{0}^{+\infty} f(t-\tau)u(\tau)\, d\tau && (2)\\
&= \int_{-\infty}^{t} f(t)\, dt && (3)\\
& && (4)
\end{aligned}$$

project 2.2. 证明：已知某信号$f_0(t)$是一个关于纵轴对称的三角波，设它的底边长为2，高为1，试绘出信号$f(t)$的波形：

$$f(t) = \sum_{k=\infty}^{\infty} f_0(t) * \delta(t-2n)$$

并回答f(t)是否是周期信号?如是,其周期为多少?

answer ： 参考课本1.4节

$$f(t) = \sum_{n=-\infty}^{+\infty} f_0(t) * \delta(t-2n) \tag{5}$$
$$= \sum_{n=-\infty}^{+\infty} f_0(t-2n) \tag{6}$$
$$= f_0(t-2n) \tag{7}$$
$$\tag{8}$$

其周期为二，波形如图所示

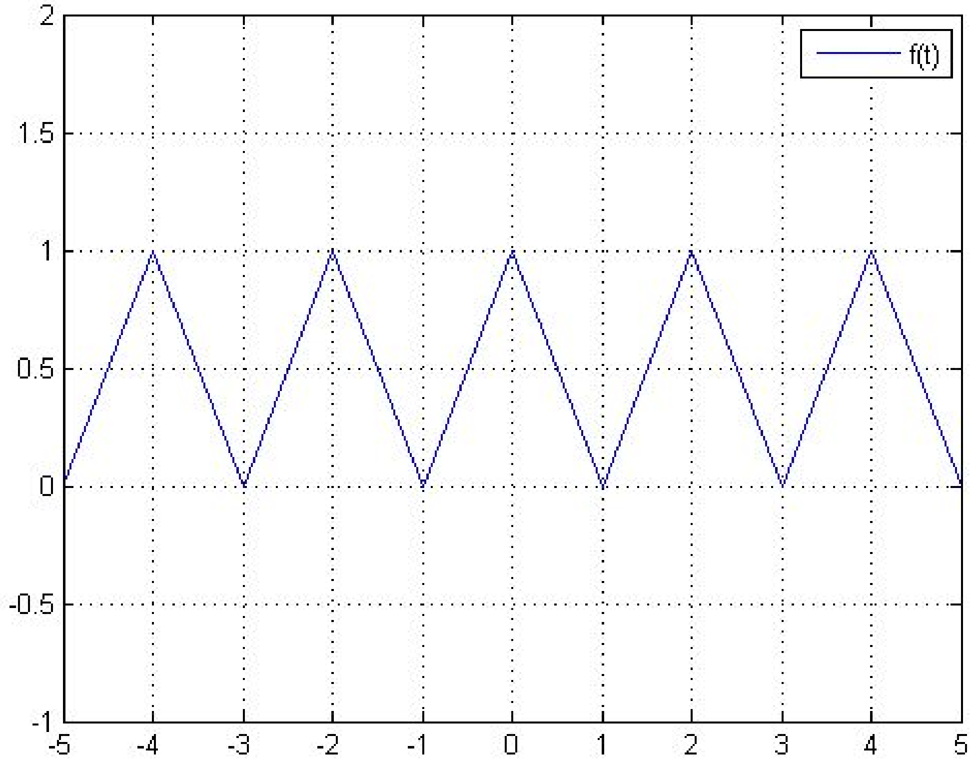

Figure 1:

2 Homework 3

project 3.1. 已知$f(t) = sin(t)cos(2t) + 5cos(3t)sin(4t)$，求该函数的傅里叶级数

answer ：

三角形式:

$$f(x) = sin(x)cos(2x) + 5cos(3x)sin(4x) \tag{9}$$
$$= \frac{sin(3x) - sin(x)}{2} + \frac{5sin(7x) + sin(x)}{2} \tag{10}$$
$$= 2sin(x) + \frac{3x}{2} + \frac{5sin(7x)}{2} \tag{11}$$
$$\tag{12}$$

指数形式:

$$f(x) = 2sin(x) + \frac{3x}{2} + \frac{5sin(7x)}{2} \tag{13}$$
$$= j(e^{-jx} - e^{jx}) + \frac{j}{4}(e^{-3jx} - e^{3jx}) + \frac{5j}{4}(e^{-7jx} - e^{7jx}) \tag{14}$$
$$= 2sin(x) + \frac{3x}{2} + \frac{5sin(7x)}{2} \tag{15}$$
$$\tag{16}$$

project 3.2.

$$\text{已知 } f(t) = \begin{cases} t, & 0 \le t < \tau \\ \tau, & \tau \le t < 2\tau \\ 0, & t < 0 \; or \; t \ge 2\tau \end{cases} \text{，求该函数的傅里叶级数}$$

answer :

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)\, dt \tag{17}$$
$$= \int_0^{\tau} te^{(-j\,\omega t)}\, dt + \int_{\tau}^{2\,\tau} \tau e^{(-j\,\omega t)}\, dt \tag{18}$$
$$= \frac{(j\,\omega\tau + 1)e^{(-j\,\omega\tau)}}{\omega^2} - \frac{1}{\omega^2} + \tau\left(-\frac{j\,e^{(-j\,\omega\tau)}}{\omega} + \frac{j\,e^{(-2j\,\omega\tau)}}{\omega}\right) \tag{19}$$
$$= \frac{j\,\omega\tau e^{-2j\,\omega\tau} + e^{-j\,\omega\tau} - 1}{\omega^2} \tag{20}$$
$$= \frac{j\,\tau}{\omega} e^{-2j\,\omega\tau} + \frac{1}{\omega^2} e^{-j\,\omega\tau} - \frac{1}{\omega^2} \tag{21}$$
$$= \frac{j\,\tau\cos(2\,\omega\tau)}{\omega} + \frac{\tau\sin(2\,\omega\tau)}{\omega} + \frac{\cos(\omega\tau)}{\omega^2} - \frac{j\,\sin(\omega\tau)}{\omega^2} - \frac{1}{\omega^2} \tag{22}$$

3 Homework 4

project 4.1. 已知$f(t) = e^{\frac{-t^2}{20}}$，为分析某时刻下的“局部频谱”,可选适合的窗函数$w(t,t_0)$,并截取$f(t)$在$t_0$ 附近的信号，即$f_w(t,t_0) = f(t)\cdot w(t,t_0)$

a： 求信号$f(t)$的FT

b： 现不妨取窗函数$w(t,t_0) = e^{\frac{-(t-t_0)^2}{2}}$，试分析$t_0 = 0$时刻下对应的“局部频谱”，即求$f_W(t,0)$的FT

c： 画出信号$f(t)$的频谱图与信号f(t)在$t_0 = 0$时刻下的”局部频谱”图，并进行对比

answer ：

a:

$$\begin{aligned} F(\omega) &= \int_{-\infty}^{+\infty} e^{\frac{-t^2}{20}} e^{-j\omega t}\,dt && (23)\\ &= e^{-\frac{100\omega^2}{20}} + \int_{-\infty}^{+\infty} e^{\frac{-t+10jw^2}{20}}\,dt && (24)\\ &= e^{-5\omega^2}\cdot\sqrt{20}\cdot\sqrt{\pi} && (25)\\ &= 2\sqrt{5}\cdot e^{-5\omega^2} && (26)\end{aligned}$$

b: 同理，$w(t,t_0) = e^{\frac{-t^2}{2}}$,由时移特性:

$$F_w(\omega,t_0) = \sqrt{2\pi}e^{-\frac{\omega^2}{2}}\cdot e^{-j\omega t_0} = \sqrt{2\pi}e^{-\frac{\omega^2}{2}-j\omega t_0} \tag{27}$$

由$f_w(t,t_0) = f(t)\cdot w(t,t_0)$：

$$\begin{aligned} F(\omega) &= \frac{1}{2\pi}F(\omega)*F_w(\omega,0) && (28)\\ &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} 2\sqrt{5\pi}\cdot e^{-5(\omega-s)^2}\sqrt{2\pi}\cdot e^{-\frac{s^2}{2}}\,ds && (29)\\ &= \sqrt{10}\int_{-\infty}^{+\infty} e^{-5(\omega-s)^2-0.5s^2}\,ds && (30)\\ &= \sqrt{10}e^{-\frac{5\omega^2}{11}}\int_{-\infty}^{+\infty} e^{-\frac{11}{2}(s-\frac{10}{11}\omega)^2}\,ds && (31)\\ &= \sqrt{10}\cdot e^{-\frac{5\omega^2}{11}}\cdot\frac{\sqrt{22}}{11}\cdot\sqrt{\pi} && (32)\\ &= \frac{2\sqrt{55\pi}}{11}\cdot e^{-\frac{5\omega^2}{11s}} && (33)\end{aligned}$$

c:　　如图所示:

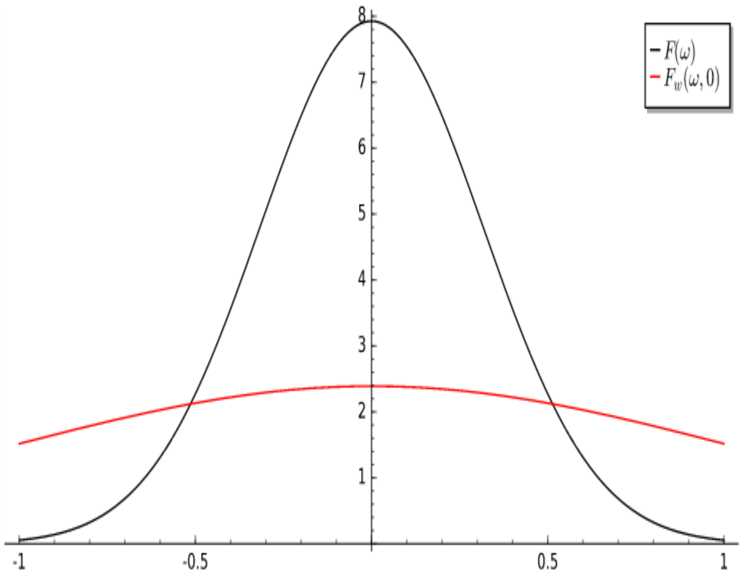

Figure 2:

project 4.2. 已知符号函数$sgn(t) = \begin{cases} 1, & t \geq 0 \\ -1, & t < 0 \end{cases}$不满足绝对可积条件，但却存在FT，请求出对应的FT。

answer ： 参考≪信号处理原理≫2.4.4符号函数

$$
\begin{aligned}
sgn(t) &= \lim_{a \to \infty} sgn(t) \cdot e^{-a|5|} && (34) \\
&= F(\omega) = \lim_{a \to 0} \frac{-2j\omega}{a^2 + \omega^2} && (35) \\
&= \frac{2}{j\omega} && (36)
\end{aligned}
$$