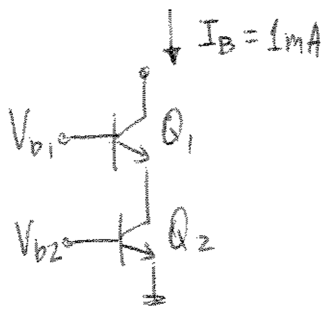
1.



$I_S = 6 \cdot 10^{-17} A$ $\quad \beta = 100$

$V_A \rightarrow \infty$ $\quad V_T = \frac{kT}{q}$

$\alpha = \frac{\beta}{\beta + 1}$

(a) $V_{b2} = V_T \ln \left(\frac{I_B / \alpha^2}{I_S} \right) = (0.026V) \ln \left(\frac{1.02mA}{6 \cdot 10^{-17} A} \right)$

$\approx 0.792 \ V$

(b) From the configuration,

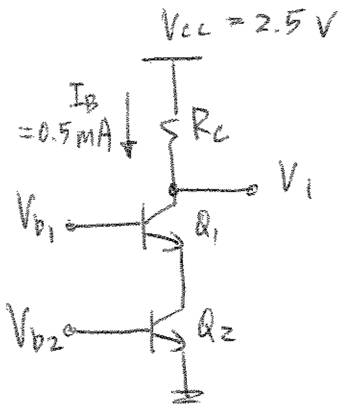
$V_{b1} = V_{CE_2} + V_{BE_1} = (V_{BE_2} - 300mV) + V_{BE_1}$

$V_{BE_1} = V_T \ln (I_B / I_S) = (0.026V) \ln \left(\frac{1mA}{6 \cdot 10^{-17} A} \right)$

$\approx 0.792 \ V$

$\therefore \ V_{b1} = (0.792 - 0.3) + 0.79 = 1.28 \ V$

2.



(a) $V_{b2} = V_{BE2} = V_T \ln\left(\frac{I_B/\alpha^2}{I_S}\right) = (0.026V)\ln\left(\frac{0.51\,mA}{6\cdot 10^{-17}A}\right)$

$\approx 0.774\ V$

$V_{BE_1} = V_{b_1} - V_{C2} = V_{b_1} - (V_{b2} - 300mV)$

$\Rightarrow V_{b_1} = V_{BE_1} + V_{b2} - 0.3V$

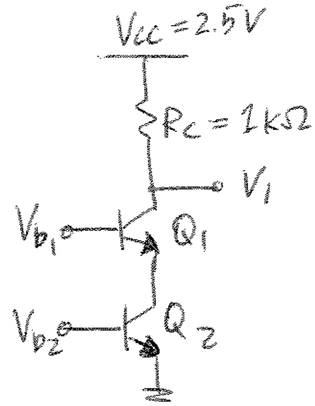
$= (0.026V)\ln\left(\frac{0.5mA}{6\cdot 10^{-17}A}\right) + (0.774V) - (0.3V)$

$\approx 1.25\ V$

(b) $V_1 = V_{b_1} - 0.3V = 0.95V$

$\therefore\ R_C = \frac{V_{CC} - V_1}{I_B} = \frac{(2.5 - 0.95)V}{0.5mA} \approx 3.1\ k\Omega$

3. From previous experience, assume both V_{BE1} & $V_{BE2} = 0.8\,V$



$$\Rightarrow V_1 = V_{CE_1} + V_{CE_2}$$

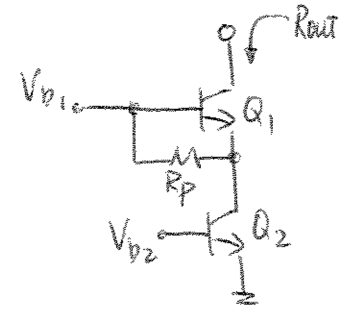
$$= (V_{BE_1} - 200mV) + (V_{b2} - 200mV)$$

$$= 1.2\ V$$

* By KCL, maximum bias current

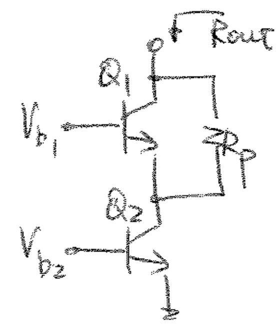
$$\approx \frac{V_{cc} - V_1}{R_c} = \frac{(2.5 - 1.2)V}{1k\Omega} = 1.3\ mA.$$

4. (a) R_p appears in parallel with r_{π_1}



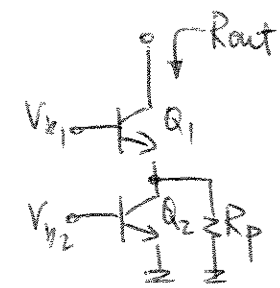
$$\therefore R_{out} = [1+g_{m_1}(r_{o_2} \| r_{\pi_1} \| R_p)]r_{o_1} + (r_{o_2} \| r_{\pi_1} \| R_p)$$

(b) R_p appears in parallel with r_{o_1}



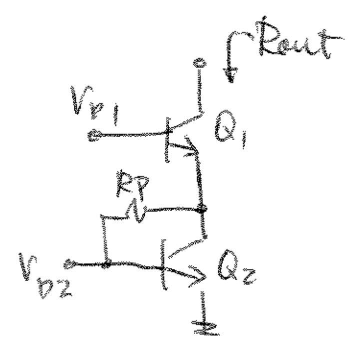
$$\therefore R_{out} = [1+g_{m_1}(r_{o_2} \| r_{\pi_1})](r_{o_1} \| R_p) + (r_{o_2} \| r_{\pi_1})$$

(c) R_p appears in parallel with r_{o_2}



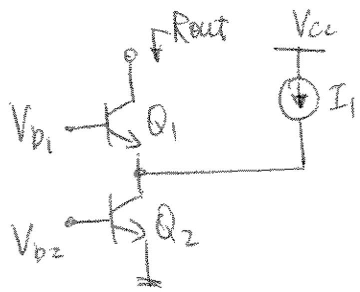
$$\therefore R_{out} = [1+g_{m_1}(r_{o_2} \| r_{\pi_1} \| R_p)]r_{o_1} + (r_{o_2} \| r_{\pi_1} \| R_p)$$

(d) R_p appears in parallel with r_{o_2} (in small-signal) $\because V_{b2}$ is AC GND.



$$\therefore R_{out} = [1+g_{m_1}(r_{o_2} \| r_{\pi_1} \| R_p)]r_{o_1} + (r_{o_2} \| r_{\pi_1} \| R_p)$$

5.



$I_1 = 0.5\,mA$

$I_{C_1} = 0.5\,mA \qquad I_{C_2} = 1\,mA.$

$= 2 I_{C_1}$

$\beta = 100 \qquad V_A = 5V$

$$R_{out} = g_{m_1} r_{o_1} (r_{o_2} \parallel r_{\pi_1})$$

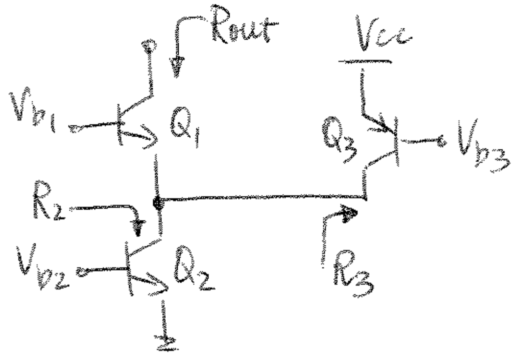
$$= \frac{I_{C1}}{V_T} \cdot \frac{V_A}{I_{C1}} \cdot \frac{V_{A2}/I_{C2} \cdot \beta V_T/I_{C1}}{V_{A2}/I_{C2} + \beta V_T/I_{C1}}$$

$$= \frac{V_A}{V_T} \cdot \frac{\frac{V_{A2}/2}{I_{C1}} \cdot \beta V_T/I_{C1}}{\frac{V_{A2}/2}{I_{C1}} + \beta V_T/I_{C1}} \approx \frac{1}{I_{C_1}} \cdot \frac{V_A}{V_T} \cdot \frac{\beta V_A V_T}{V_A + 2\beta V_T}$$

$$= \frac{1}{0.5mA} \cdot \frac{5V}{0.026V} \cdot \frac{100(5V)(0.026V)}{(5V) + 2(100)(0.026V)}$$

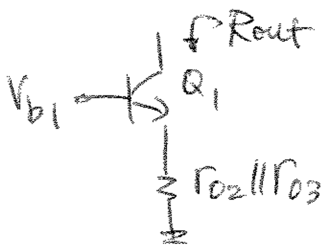
$\therefore\ R_{out} \approx 490.\ k\Omega.$

6.



$R_3 = r_{o3}$ (V_{cc} & V_{b3} are AC GND)

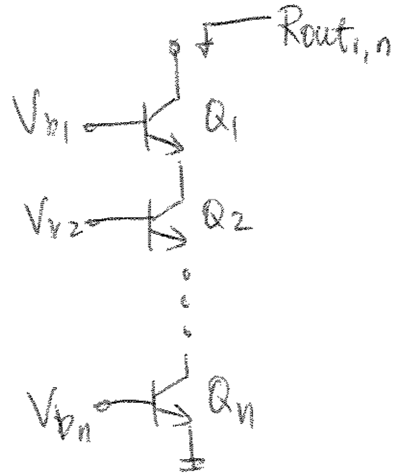
$R_2 = r_{o2}$ (V_{b2} is AC GND)



$$\therefore R_{out} = [(1+g_{m1}(r_{o2} \| r_{o3} \| r_{\pi 1})] r_{o1} + (r_{o2} \| r_{o3} \| r_{\pi 1})$$

$$\approx g_{m1} r_{o1} (r_{o2} \| r_{o3} \| r_{\pi 1})$$

7.



Suppose $R_{out_{i,j}}$ is the output impedance of the cascode circuit with BJTs $Q_i, Q_{i+1}, Q_{i+2}, \ldots Q_{j-1}, Q_j$.

$$R_{out_{n-1,n}} = [1 + g_{m_{n-1}}(r_{o_n} \| r_{\pi_{n-1}})] r_{o_{n-1}} + (r_{o_n} \| r_{\pi_{n-1}})$$

$$\approx g_{m_{n-1}}(r_{o_n} \| r_{\pi_{n-1}}) r_{o_{n-1}} \approx g_{m_{n-1}} r_{\pi_{n-1}} r_{o_{n-1}}$$

$$= \beta r_o \qquad (\text{usually, } r_\pi \ll r_o)$$

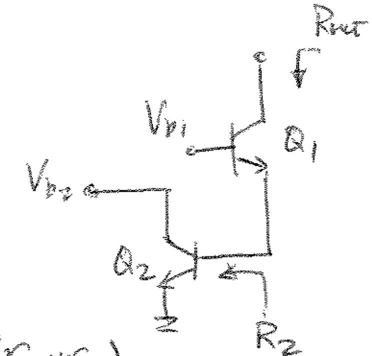
$$R_{out_{n-2,n}} = [1 + g_{m_{n-2}}(\beta r_o \| r_{\pi_{n-2}})] r_o + (\beta r_o \| r_{\pi_{n-2}})$$

$$\approx g_m r_\pi r_o + r_\pi \cong \beta r_o$$

This means that $R_{out} \approx \beta r_o$ even if an extra BJT is employed in the cascode configuration.

i.e. $R_{out_{MAX}} \approx \beta r_o$

8. (a) $R_2 = (r_{\pi_2} \parallel r_{\pi_1})$



$$\therefore R_{out} = [1 + g_{m1} R_2] r_{o1} + R_2$$

$$= [1 + g_{m1}(r_{\pi_1} \parallel r_{\pi_2})] r_{o1} + (r_{\pi_1} \parallel r_{\pi_2})$$

(b) In part (a), $I_{C2} = \beta I_{C1}$ $(= I_{B2})$

$$\therefore R_{out(a)} = \left[1 + g_{m1}\left(\frac{\beta V_T}{I_{C1}} \parallel \frac{V_T}{I_{C1}}\right)\right] r_{o1} + (r_{\pi_1} \parallel r_{\pi_2})$$

$$\approx \left(1 + g_{m1}\frac{V_T}{I_{C1}}\right) r_{o1} + \frac{V_T}{I_{C1}}$$

$$= 2 r_{o1} + V_T / I_{C1}$$

$$R_{out,cascode} = [1 + g_{m1}(r_{o2} \parallel r_{\pi_1})] r_{o1} + (r_{o2} \parallel r_{\pi_1})$$

$$\approx [1 + g_{m1} r_{\pi_1}] r_{o1} + r_{\pi_1}$$

$$\approx \beta r_{o1} + r_{\pi_1} = \beta r_{o1} + V_A / I_{C1}$$

Compare term-by-term:

$$\left.\begin{array}{l} 2 r_{o1} \ll \beta r_{o1} \\ V_T \ll V_A \end{array}\right\} \Rightarrow R_{out(a)} \ll R_{out,cascode}$$

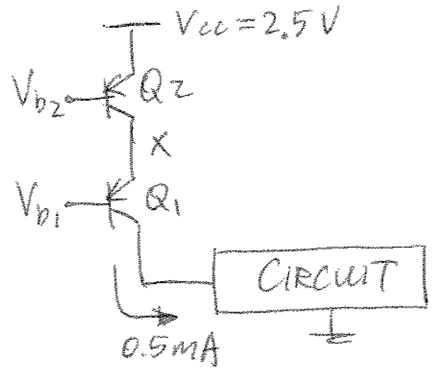
i.e. using (a) reduces the effect of having a cascode configuration.

9. $$R_{out} = \frac{1}{I_C} \cdot \frac{V_A}{V_T} \cdot \frac{\beta V_A V_T}{V_A + \beta V_T}$$

$$\approx \frac{1}{I_C} \cdot \frac{V_A}{\cancel{V_T}} \cdot \frac{\beta \cancel{V_A} \cancel{V_T}}{\cancel{V_A}} = \beta \frac{V_A}{I_C} = \beta r_o$$

$$= R_{out,max}$$

This means that $R_{out,max}$ is often achieved with 2-BJT cascode.

10. 

$I_S = 10^{-16}$ A $\quad \beta = 100$

$I_{BIAS} = 0.5$ mA

(a) $I_{BIAS} \approx I_{C2} = 0.5$ mA

$$\therefore V_{b2} = V_{cc} - |V_{be2}|$$
$$= V_{cc} - V_T \ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right)$$
$$= (2.5V) - (0.026V) \ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right) \approx 1.74V$$

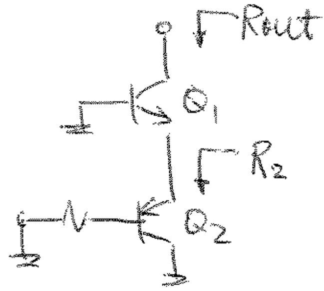
(b) $|V_{CB2}| = V_X - V_{b2} = 200\text{mV}$

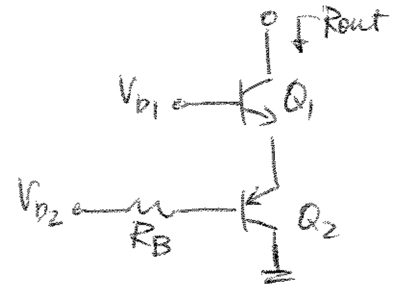
$\Rightarrow V_{c2} = V_{b2} + |V_{CB2}| = 1.94V$

$$\therefore V_{b1} = V_{c2} - |V_{BE1}| = V_{c2} - V_T \ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right)$$
$$= (1.94V) - (0.026V) \ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right) \approx 1.18V$$

$\Rightarrow$ Maximum allowable $V_{b1} = 1.18V$

11. (a)





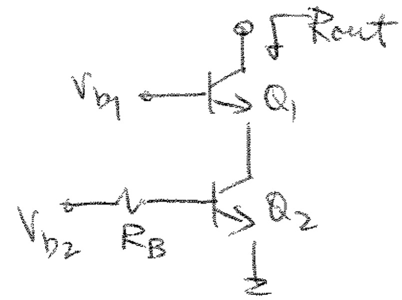
(AC-small signal)

Looking into emitter of Q2,

$$R_2 = \frac{1}{\left(\frac{\beta+1}{R_B + r_{\pi_2}} + \frac{1}{r_{o_2}}\right)}$$

$$\Rightarrow R_{out} = [1 + g_{m_1}(R_2 \parallel r_{\pi_1})] r_{o_1} + (R_2 \parallel r_{\pi_1})$$

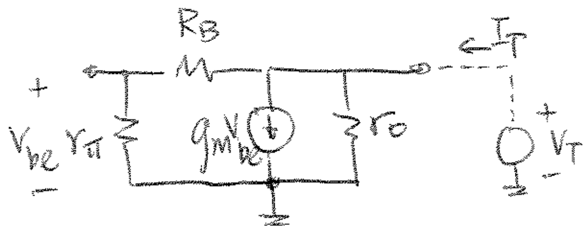
(b) R_B does not affect Q_2 in small-signal R_{out}:

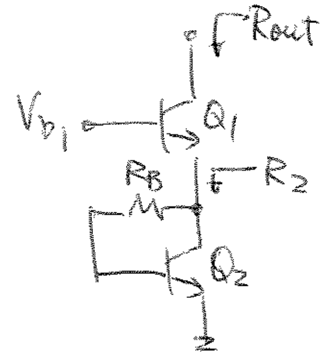


$$\therefore R_{out} = [1 + g_{m_1}(r_{o_2} \parallel r_{\pi_1})] r_{o_1} + (r_{o_2} \parallel r_{\pi_1})$$

This is a cascode stage.

(c) Use small-signal analysis:



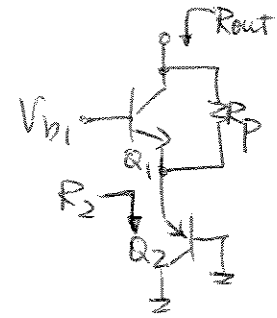


By KCL, $I_T = \dfrac{V_T}{R_B + r_{\pi_2}} + g_{m_2} \dfrac{V_T\, r_{\pi_2}}{r_{\pi_2} + R_B} + \dfrac{V_T}{r_{o_2}}$

$$\Rightarrow \quad R_2 = \frac{V_T}{I_T} = \frac{1}{\left(\dfrac{\beta+1}{R_B + r_{\pi_2}} + \dfrac{1}{r_o}\right)} \approx \frac{1}{\beta/(R_B + r_{\pi_2}) + 1/r_{o_2}}$$

$$\therefore R_{out} = \left[1 + g_{m_1}(R_2 \parallel r_{\pi_1})\right] r_{o_1} + (R_2 \parallel r_{\pi_1})$$
$$\approx g_{m_1} r_{o_1} (R_2 \parallel r_{\pi_1})$$

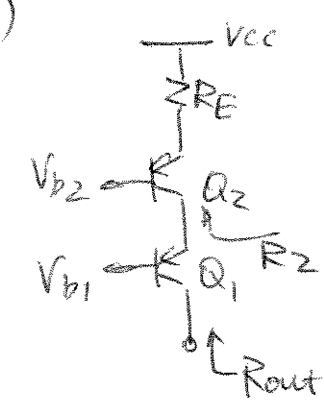
(d) R_p appears in parallel with r_{o1}.



$$R_2 = r_{o2} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}$$
$$\approx r_{o2} \parallel \frac{r_{\pi 2}}{\beta} \parallel r_{\pi 2}$$
$$\approx r_{o2} \parallel (r_{\pi 2}/\beta) \approx r_{\pi 2}/\beta$$

$$\therefore R_{out} = [1 + g_{m1}(R_2 \parallel r_{\pi 1})](r_{o1} \parallel R_p) + (R_2 \parallel r_{\pi 1})$$
$$\approx g_{m1}(r_{o1} \parallel R_p)(r_{\pi 1} \parallel R_2)$$

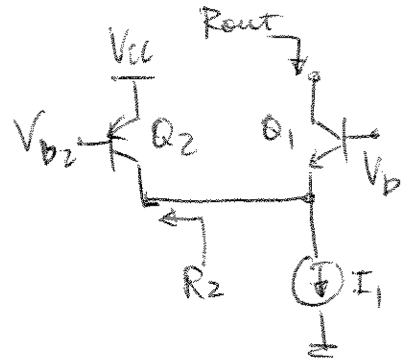
(e) $R_2 = [1 + g_{m2}(R_E \parallel r_{\pi 2})] r_{o2} + (R_E \parallel r_{\pi 2})$
$$\approx g_{m2}(R_E \parallel r_{\pi 2}) r_{o2}$$



$$\therefore R_{out} = [1 + g_{m1}(R_2 \parallel r_{\pi 1})] r_{o1} + (R_2 \parallel r_{\pi 1})$$
$$\approx g_{m1}(R_2 \parallel r_{\pi 1}) r_{o1}$$
$$= g_{m1} [g_{m2} r_{o2}(R_E \parallel r_{\pi 2}) \parallel r_{\pi 1}] r_{o1}$$

This is a cascode stage.

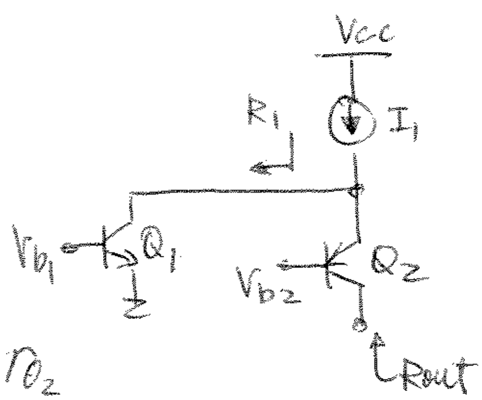
(f) $R_2 = r_{o2}$



$$\therefore R_{out} = [1 + g_{m1}(R_2 \| r_{\pi_1})] r_{o1} + (R_2 \| r_{\pi_1})$$

$$\approx g_{m1} r_{o1} (R_2 \| r_{\pi_1})$$
$$= g_{m1} r_{o1} (r_{\pi_1} \| r_{o2})$$

(g) $R_1 = r_{o1}$
(output impedance of a common-emitter)

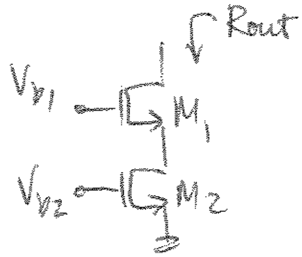


$$\therefore R_{out} = [1 + g_{m2}(R_1 \| r_{\pi 2})] r_{o2}$$

$$+ (R_1 \| r_{\pi 2})$$

$$\approx g_{m2} r_{o2} (r_{o1} \| r_{\pi 2})$$

12.



$I_D = 0.5\,mA$ $\quad R_{out} \geq 50\,k\Omega$

$\mu_n C_{ox} = 100 \frac{\mu A}{V^2}$ $\quad \frac{W}{L} = \frac{20}{0.18}$

Calculate max λ.

Assume M_1 & M_2 in saturation.

$$\Rightarrow R_{out} \approx g_{m1} r_{o1} r_{o2}$$
$$= \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} \times \frac{1}{\lambda I_D} \times \frac{1}{\lambda I_D} \geq 50\,k\Omega.$$

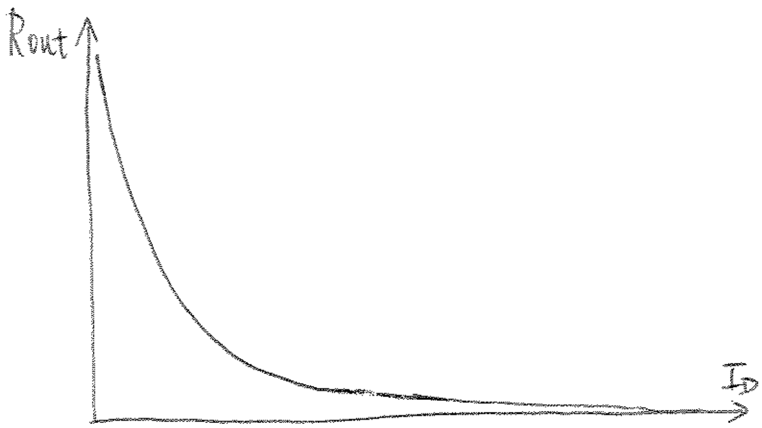
(All quantities are known).

Solve for λ:

$$\lambda_{max} \approx 0.51\ V^{-1}$$

13. (a) $R_{out} = g_{m2} r_{o1} r_{o2} = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} \cdot \frac{1}{\lambda I_D} \cdot \frac{1}{\lambda I_D}$

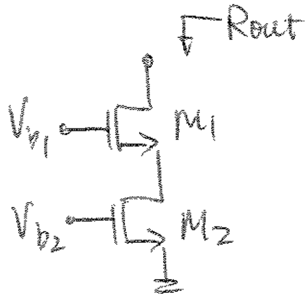
$$= 2\mu_n C_{ox} (W/L) \, (I_D)^{-3/2}$$



(b) R_{out} (BJT) $\propto I_B^{-1}$
R_{out} (MOS) $\propto I_B^{-3/2}$

$\therefore$ MOS cascode is a stronger function of I in terms of Rout.

14.



$\left(\frac{W}{L}\right)_1 = 30/0.18 \qquad \left(\frac{W}{L}\right)_2 = 20/0.18$

$I_{BIAS} = 0.5\,mA$

$\mu_n C_{ox} = 100\,\frac{\mu A}{V^2} \qquad V_{TH} = 0.4\,V$

(a) $I_{D2} = I_{BIAS} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH})^2$

$$\Rightarrow V_{b2} = \sqrt{\frac{2 I_{BIAS}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}} + V_{TH}$$

$$= \sqrt{\frac{2(0.5mA)}{(100\,\frac{\mu A}{V^2})(20/0.18)}} + 0.4V \approx 0.7\,V$$

M_2 operates in saturation as long as

$V_{GS_2} - V_{TH} \le V_{DS_2} \Rightarrow V_{DS_2} \ge 0.3\,V.$

Observe that $V_{GS_1} = V_{b_1} - V_{DS_2}$

$$I_{D_1} = I_{BIAS} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b_1} - V_{DS_2} - V_{TH})^2$$

$$\Rightarrow V_{b_1} \ge \sqrt{\frac{2 I_{BIAS}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}} + 0.4V + 0.3V$$

$$= \sqrt{\frac{2(0.5mA)}{(100\,\frac{\mu A}{V^2})(30/0.18)}} + 0.7V \approx 0.95\,V.$$

$\therefore$ Minimum $V_{b_1} = 0.95V.$

(b) $R_{out} = (1 + g_{m_1} r_{o2}) r_{o1} + r_{o2}$

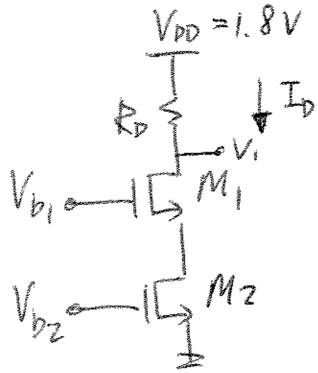
$$= \left(1 + \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{BIAS}} \cdot \frac{1}{\lambda I_{BIAS}}\right) \cdot \frac{1}{\lambda I_{BIAS}} + \frac{1}{\lambda I_{BIAS}}$$

$$= \left[1 + \sqrt{2\left(100\frac{\mu A}{V^2}\right)\left(\frac{30}{0.18}\right)(0.5mA)} \cdot \frac{1}{(0.1)(0.5m)}\right] \cdot \frac{1}{(0.1)(0.5m)}$$

$$+ \frac{1}{(0.1)(0.5mA)}$$

$$\approx 1.67 \ M\Omega$$

15.



$\left(\frac{W}{L}\right)_1 = \frac{20}{0.18}$ $\left(\frac{W}{L}\right)_2 = \frac{40}{0.18}$

$\mu_n C_{ox} = 100\ \frac{\mu A}{V^2}$ $V_{TH} = 0.4\text{ V}.$

$I_D = 1\text{ mA}$ $R_D = 500\,\Omega$

(a) Both M_1 & M_2 must stay in saturation.

$$\Rightarrow V_1 = 1.8 - I_D R_D = 1.8 - (1\text{mA})(500\,\Omega) = 1.3\text{ V}.$$

Want this value equal to that which makes M_1 operates at the edge of saturation.

$$\therefore V_{b_1} = V_1 + V_{TH} = 1.3 + 0.4 = 1.7\text{ V}.$$

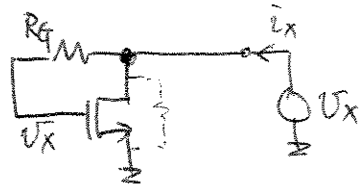
(b) $I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot \left[(V_{b_1} - V_X) - V_{TH}\right]^2 = 1\text{mA}.$

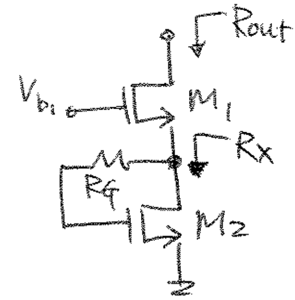
$$\Rightarrow V_X = V_{b_1} - V_{TH} - \sqrt{\frac{2 I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}}$$

$$= (1.7\text{ V}) - (0.4\text{ V}) - \sqrt{\frac{2(1\text{mA})}{(100\ \mu A/V^2)(20/0.18)}}$$

$$\approx 1.276\text{ V}.$$

1b. (a) First compute R_X:

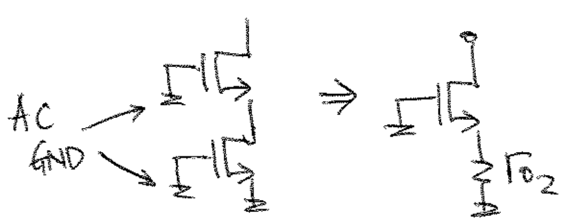


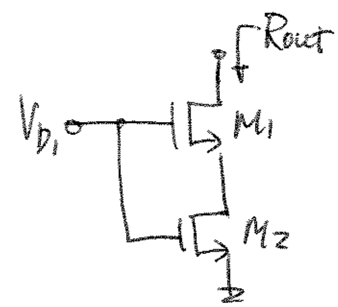


$$I_X = g_{m_2} v_X + v_X / r_{o2} \Rightarrow R_X = v_X / i_X = \frac{1}{g_{m2} + 1/r_{o2}}$$

$$\therefore R_{OUT} = g_{m1} r_{o1} R_X = \frac{g_{m1} r_{o1}}{g_{m2} + 1/r_{o2}}$$

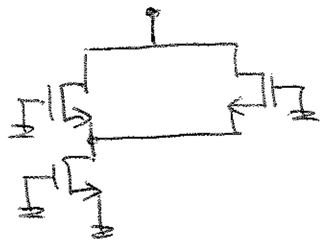
(b) Equivalent circuit:

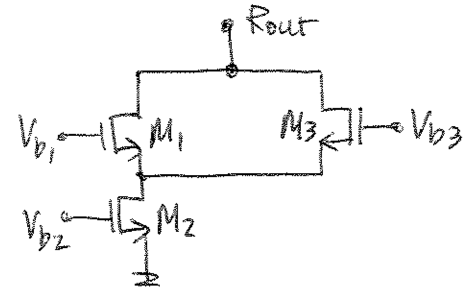


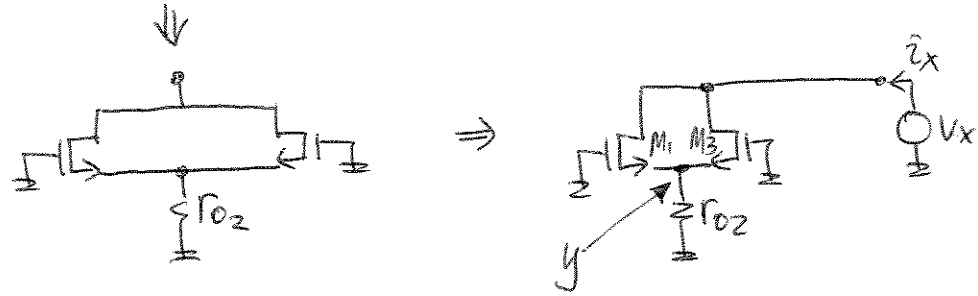


$$\therefore R_{OUT} = g_{m1} r_{o1} r_{o2}$$

(c) Equivalent Circuit:







By KCL, $V_y = i_x \cdot r_{o2}$ —— ①

$$i_x = g_{m1}(-V_y) + g_{m3}(-V_y) + (v_x - V_y)\left(\frac{1}{r_{o1}} + \frac{1}{r_{o3}}\right) \quad ②$$

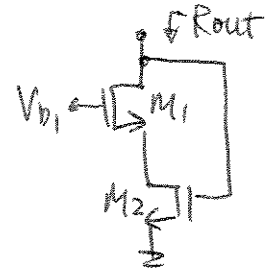
Substitute ① into ② and re-arrange:

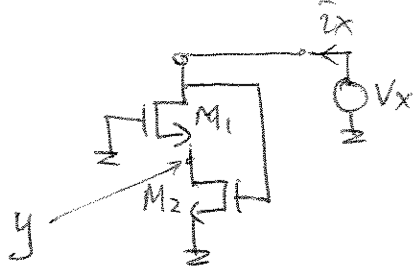
$$R_{out} = \frac{v_x}{i_x} = (r_{o1} \parallel r_{o3}) + r_{o2}(r_{o1} \parallel r_{o3})(g_{m1} + g_{m3}) + r_{o2}$$

$$\approx r_{o2}(r_{o1} \parallel r_{o3})(g_{m1} + g_{m3})$$

(Intuitively this makes sense because we have 2 NMOSs in parallel — $= g_m v_{gs}$ adds up, and r_o's are splitting total current, i_x. This is as if an equivalent NMOS replacing M_1 & M_3 with $g_m = (g_{m1} + g_{m3})$ & $r_o = (r_{o1} \parallel r_{o3})$.)

(d) Examine the equivalent circuit with a test voltage:





By observation, i_x must flow through both M_1 & M_2.

By KCL,

$$i_x = g_{m2} v_x + v_y / r_{o2}$$

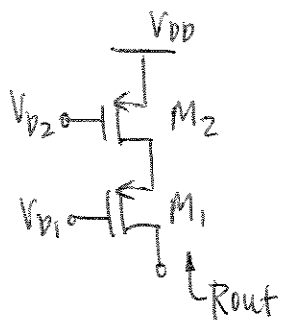
$$i_x = g_{m1}(-v_y) + (v_x - v_y)/r_{o1}$$

Substitute ① into ② and re-arrange:

$$R_{out} = \frac{v_x}{i_x} = \frac{g_{m1} r_{o2} + \frac{r_{o2}}{r_{o1}} + 1}{g_{m1} g_{m2} r_{o2} + (g_{m2} r_{o2} + 1)\left(\frac{1}{r_{o1}}\right)}$$

$$\approx \frac{r_{o2}\left(g_{m1} + \frac{1}{r_{o1}}\right)}{g_{m2} r_{o2}\left(g_{m1} + \frac{1}{r_{o1}}\right)} \approx \frac{1}{g_{m2}}$$

17.



$I_{BIAS} = 0.5\ mA$

$R_{out} = 40\ k\Omega$

$\mu_p C_{ox} = 50\ \mu A/V^2$

$\lambda = 0.2\ V^{-1}$

$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2$

$$R_{out} = 40\ k\Omega = (g_{m1} r_{o2} + 1) r_{o1} + r_{o2}$$

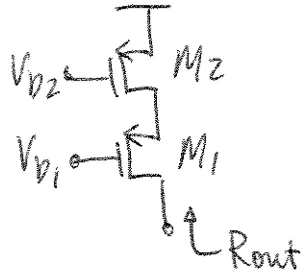
$$\Rightarrow g_{m1} = \sqrt{2 \mu_p C_{ox} \left(\frac{W}{L}\right)_1 I_{BIAS}} = \left(\frac{R_{out} - r_{o2}}{r_{o1}} - 1\right) \cdot \frac{1}{r_{o2}}$$

$$\therefore \left(\frac{W}{L}\right)_1 = \left[\left(\frac{R_{out} - r_{o2}}{r_{o1}} - 1\right) \frac{1}{r_{o2}}\right]^2 \cdot \frac{1}{2 \mu_p C_{ox} I_{BIAS}}$$

$$= \left\{\left[\frac{(40k\Omega) - [(0.2)(0.5m)]^{-1}}{[(0.2)(0.5m)]^{-1}} - 1\right] \cdot [0.2 \cdot 0.5m]\right\}^2 \cdot \frac{1}{2\left(50 \frac{\mu A}{V^2}\right)(0.5mA)}$$

$$\approx 0.8$$

18.



$$R_{out} = g_{m1} r_{o1} r_{o2} = \sqrt{2 \mu_p C_{ox} \left(\frac{W}{L}\right)_1 I_D} \cdot \frac{1}{\lambda I_D} \cdot \frac{1}{\lambda I_D}$$

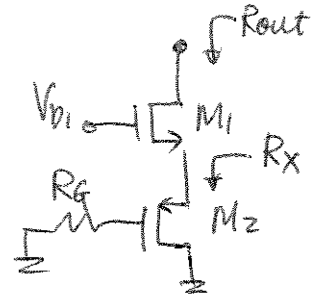
If W_1 & W_2 increase by N times and L_1, L_2, and I_D remain unchanged:

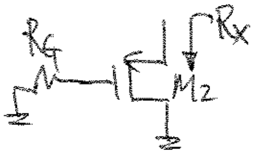
$$R_{out}(\text{New}) = \sqrt{2 \mu_p C_{ox} \left(\frac{NW}{L}\right) I_D} \cdot \left(\frac{1}{\lambda I_D}\right)^2$$

$$= \sqrt{N} \sqrt{2 \mu_p C_{ox} \frac{W}{L} I_D} \left(\frac{1}{\lambda I_D}\right)^2 = \sqrt{N}\, R_{out}$$

$\therefore$ R_{out} is increased by $\sqrt{N}$ times.

19. (a) R_X is the input impedence of a common-gate configuration:





"Looking into" the source of M_2,

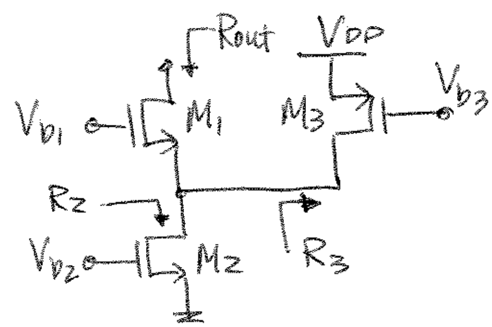
$$R_X = \frac{1}{g_{m2}} \parallel r_{o2}$$

$$\therefore R_{out} = g_{m1} r_{o1} R_X = g_{m1} r_{o1} \left(\frac{1}{g_{m2}} \parallel r_{o2}\right)$$

(b) From observation,

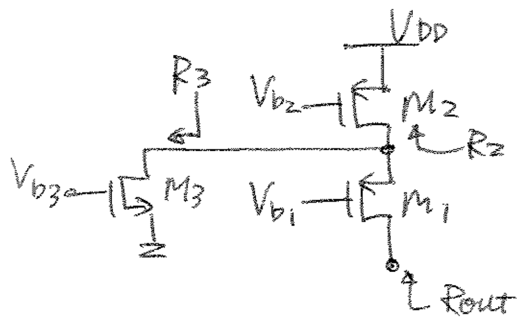
→ $R_3 = r_{o3}$ ($\because V_{SG} = 0$ in AC)

→ $R_2 = r_{o2}$ ($\because V_{SG} = 0$ in AC)



$$\therefore R_{out} = g_{m1} r_{o1} (R_2 \parallel R_3) = g_{m1} r_{o1} (r_{o2} \parallel r_{o3})$$

(c) By observation,

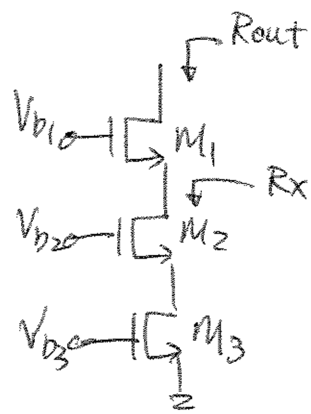


$R_2 = r_{o2}$ ($V_S = V_G = $ AC GND)

$R_3 = r_{o3}$ ($V_S = V_G = $ AC GND)

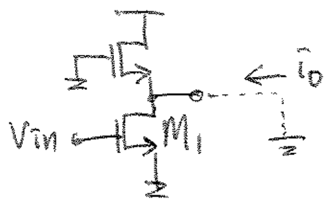
$\therefore R_{out} = g_{m1} r_{o1} (R_2 \| R_3) = g_{m1} r_{o1} (r_{o2} \| r_{o3})$

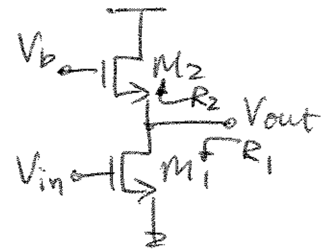
(d) $R_X = g_{m2} r_{o2} r_{o3}$



$$\Rightarrow R_{out} = g_{m1} r_{o1} R_X$$
$$= g_{m1} g_{m2} r_{o1} r_{o2} r_{o3}$$

20.(a) Equivalent circuit:



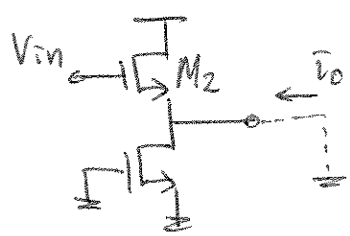


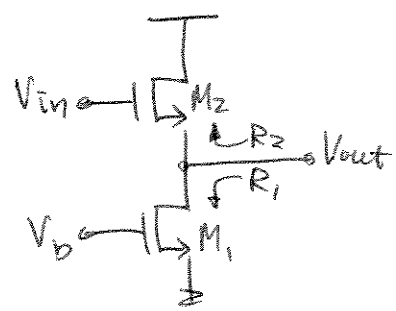
$$i_o = G_m v_{in} \Rightarrow G_m = \frac{i_o}{v_{in}} = g_{m1}$$

$$R_1 = r_{o1} \ ; \ R_2 = 1/g_{m2}$$

$$\therefore A_v = -G_m R_{out} = -g_{m1}(r_{o1} \,//\, 1/g_{m2})$$

(b) Equivalent circuit:



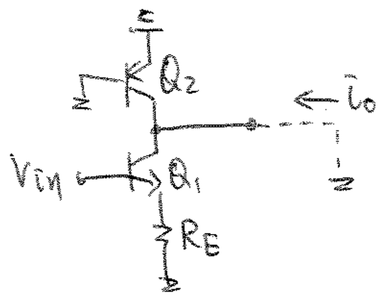


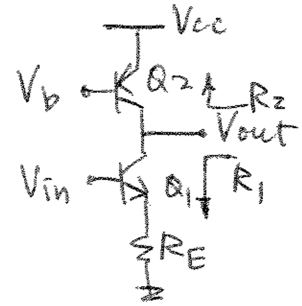
$$-i_o = g_{m2} V_{in} \Rightarrow G_m = \frac{i_o}{V_{in}} = -g_{m2}$$

$$R_1 = r_{o1} \ ; \ R_2 = 1/g_{m2}$$

$$\therefore A_v = -G_m R_{out} = g_{m2}(r_{o1} \,//\, 1/g_{m2})$$

(c) Equivalent circuit:





With output node shorted, this is a common-emitter stage with degeneration.

$$\Rightarrow G_m = \frac{g_{m1}}{g_{m1}(R_E \| r_{o1}) + 1}$$

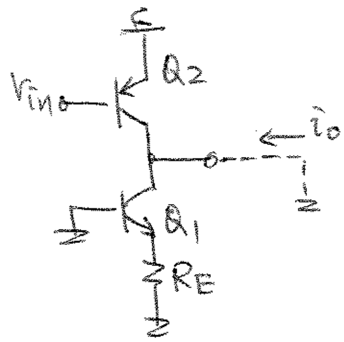
$$R_1 = [1 + g_{m1}(R_E \| r_{\pi 1})] r_{o1} + (R_E \| r_{\pi 1})$$

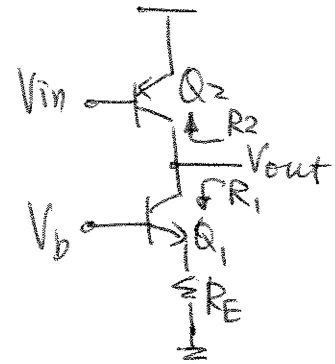
$$R_2 = r_{o2}$$

$$\Rightarrow R_{out} = R_1 \| R_2$$

$$\therefore A_v = -G_m R_{out} = -\frac{g_{m1}\left(\{[1 + g_{m1}(R_E \| r_{\pi 1})] r_{o1} + (R_E \| r_{\pi 1})\} \| r_{o2}\right)}{g_{m1}(R_E \| r_{o1}) + 1}$$

(d) Equivalent circuit:





With output shorted to AC GND, circuit becomes a simple common-emitter stage:

$$\Rightarrow G_m = g_{m2}$$

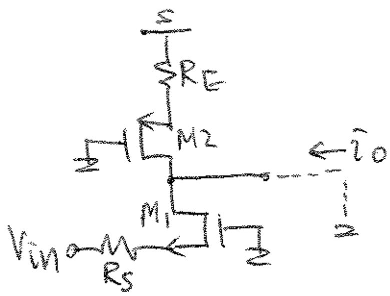
$$R_1 = [1 + g_{m1}(R_E \parallel r_{\pi 1})] r_{o1} + (R_E \parallel r_{\pi 1})$$

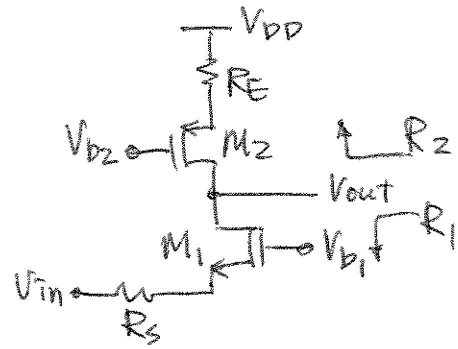
$$R_2 = r_{o2}$$

$$\Rightarrow R_{out} = R_1 \parallel R_2$$

$$\therefore A_v = -G_m R_{out} = -g_{m2}\left(\{[1 + g_{m1}(R_E \parallel r_{\pi 1})] r_{o1} + (R_E \parallel r_{\pi 1})\} \parallel r_{o2}\right)$$

(e) Equivalent circuit:





Observing that i_o must flow through M_1 only:

$$i_o = g_{m1}\left(-\left(\overbrace{v_{in} + i_o R_s}^{\text{gate voltage of } M_1}\right)\right)$$

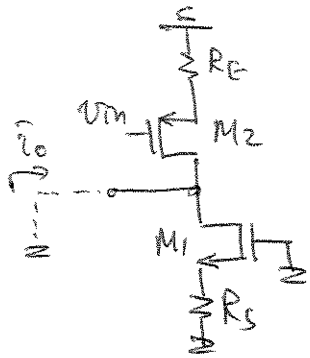
$$\Rightarrow G_m = \frac{i_o}{v_{in}} = \frac{-g_{m1}}{1+g_{m1}R_s}$$

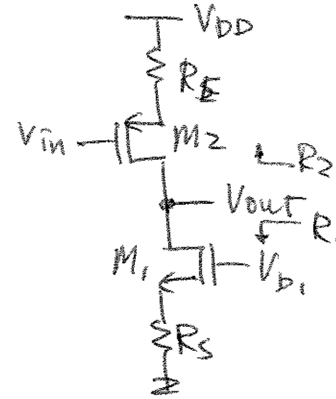
$$R_1 = (1+g_{m1}R_S)r_{o1} + R_S$$
$$R_2 = (1+g_{m2}R_E)r_{o2} + R_E$$
$$R_{out} = R_1 \parallel R_2$$

$$\therefore A_v = -G_m R_{out}$$

$$= \frac{g_{m1}}{1+g_{m1}R_s}\left\{\left[(1+g_{m1}R_s)r_{o1} + R_s\right] \parallel \left[(1+g_{m2}R_E)r_{o2} + R_E\right]\right\}$$

(f) Equivalent circuit:





This is a common-source stage with degeneration:

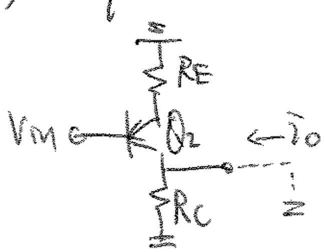
$$\Rightarrow G_m = \frac{g_{m2}}{1+g_{m2}R_E}$$

$$R_1 = (1+g_{m1}R_S)r_{o1} + R_S$$
$$R_2 = (1+g_{m2}R_E)r_{o2} + R_E$$

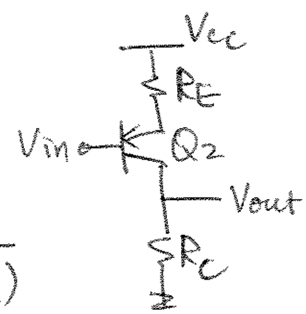
$$\therefore A_v = -G_m(R_1 \| R_2)$$

$$= \frac{g_{m2}}{1+g_{m2}R_E}\left\{\left[(1+g_{m1}R_S)r_{o1}+R_S\right] \| \left[(1+g_{m2}R_E)r_{o2}+R_E\right]\right\}$$

(g) Equivalent circuit:



$$\Rightarrow G_m = \frac{g_{m2}}{1+g_{m2}(R_E \| r_{\pi 2})}$$



$$R_{out} = \left\{\left[1+g_{m2}(R_E \| r_{\pi 2})\right]r_{o2} + (R_E \| r_{\pi 2})\right\} \| R_C$$

$$\Rightarrow A_v = -G_m R_{out} = \frac{g_{m2}R_{out}}{1+g_{m2}(R_E \| r_{\pi 2})}$$

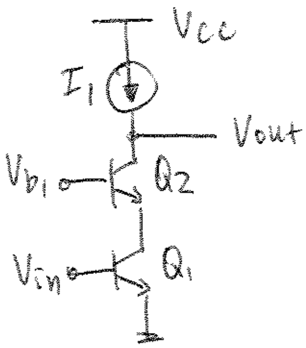
21. $A_v = -g_{m1} r_{o1} g_{m1} (r_{o1} \parallel r_{\pi 2})$

$$= -\frac{I_{C1}}{V_T} \cdot \frac{V_{A1}}{I_{C1}} \cdot \frac{I_{C1}}{V_T} \cdot \frac{1}{\frac{I_{C1}}{V_{A1}} + \frac{I_{C2}}{\beta V_T}}$$

Since $I_{C1} \approx I_{C2}$,

$$A_v \approx -\frac{V_{A1}/V_T^2}{\frac{1}{V_{A1}} + \frac{1}{\beta V_T}} = -\frac{\beta V_A^2}{V_T(V_A + \beta V_T)}$$

22.

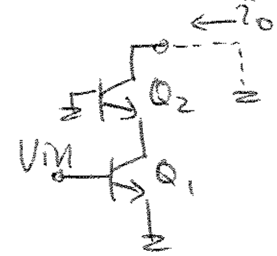


$A_v = 500$ $\quad I_1 = 1mA$

$\beta_1 = \beta_2 = 100$

Determine minimum $V_{A_1} = V_{A_2}$.

Using small-signal analysis,



$$G_M = \frac{i_o}{v_{in}} = g_{m1}\left(\frac{\beta+1}{\beta}\right) = \frac{I_1}{V_T}\left(\frac{\beta+1}{\beta}\right)$$

$$R_{out} = [1+g_{m2}(r_{o1}\|r_{\pi 2})]r_{o2} + (r_{o1}\|r_{\pi 2})$$

$$\approx g_{m2}(r_{o1}\|r_{\pi 2})r_{o2} = \frac{\beta V_A^2}{I_C(V_A+\beta V_T)}$$

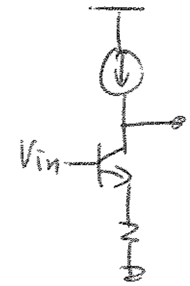
$$\Rightarrow A_v = -G_m R_{out}$$

$$= -\frac{I_1}{V_T}\left(\frac{\beta+1}{\beta}\right)\cdot\frac{\beta V_A^2}{I_C(V_A+\beta V_T)} = 500$$

$\Rightarrow$ All values are given. V_A is solved using the quadratic formula:

$\therefore V_A \approx 0.65\ V$

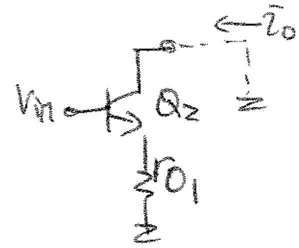
23. (a) Even though Rout is independent of where Vin is applied, Gm changes:



The circuit is a commom-emitter with degeneration, which always has Gm ≤ Gm of commom-emitter stage without degeneration.

Alternatively, this circuit has less gain because it only has one amplifier stage.

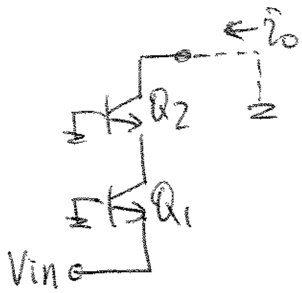
(b) $G_m = \dfrac{i_o}{v_{in}} = \dfrac{g_{m2}}{1+g_{m2}(r_{o1} \| r_{o2})}$



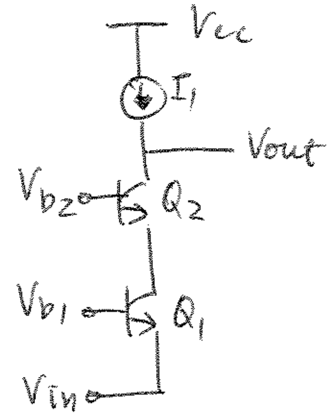
$R_{out} = [1+g_{m2}(r_{o1} \| r_{\pi 2})] r_{o2} + r_{o1}$

$\Rightarrow A_v = -G_m R_{out} = \dfrac{g_{m2}\{[1+g_{m2}(r_{o1} \| r_{\pi 2})] r_{o2} + r_{o1}\}}{1+g_{m2}(r_{o1} \| r_{o2})}$

24. Equivalent circuit:



$\beta \to \infty$.

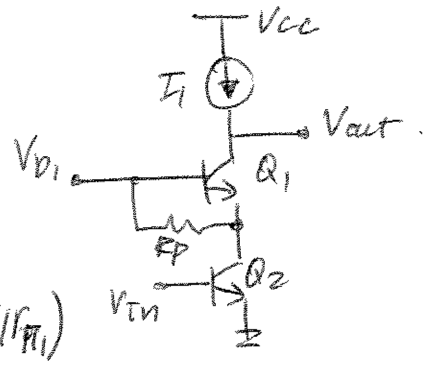


$$G_m = \frac{i_o}{V_{in}} \approx -g_{m1}$$

$$R_{out} = [1 + g_{m2}(r_{o1} \| r_{\pi 2})]\, r_o + (r_{o1} \| r_{\pi 2})$$

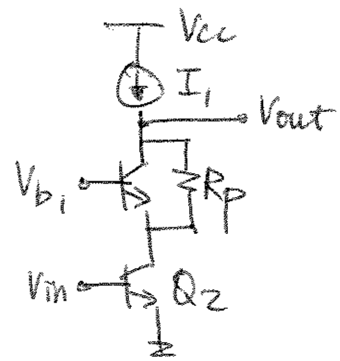
$$\Rightarrow A_v = -G_m R_{out} = g_{m1}\left[\{1 + g_{m2}(r_{o1} \| r_{\pi 2})\} r_{o2} + (r_{o1} \| r_{\pi 2})\right]$$

25. (a) From lecture, we know that the voltage gain of a BJT cascode circuit $\approx -g_{m2} r_{o2} g_{m2} (r_{o2} \| r_{\pi_1})$

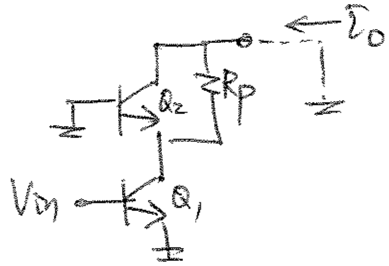


This circuit resembles such, and the only difference is that r_{π_1} now becomes $(r_{\pi_1} \| R_p)$

$$\therefore \; A_v \approx -g_{m2}^2 r_{o2} (r_{o2} \| r_{\pi_1} \| R_p)$$



(b) Equivalent circuit:



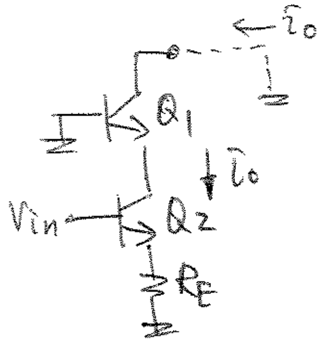
$$G_m = \frac{i_o}{V_{in}} = \frac{\beta+1}{\beta} g_{m1} \approx g_{m1}$$

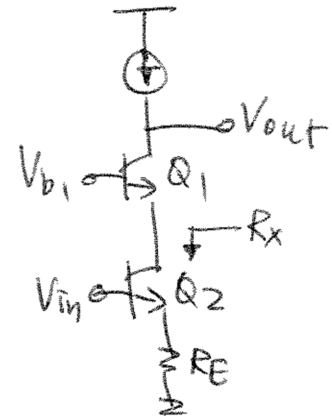
$$R_{out} = [1 + g_{m2}(r_{o1} \| r_{\pi 2})](r_{o2} \| R_p) + (r_{o1} \| r_{\pi 2})$$

$$\therefore \; A_v = -G_m R_{out}$$

$$= -g_{m1} \{[1 + g_{m2}(r_{o1} \| r_{\pi 2})](r_{o2} \| R_p) + (r_{o1} \| r_{\pi 2})\}$$

(c) Equivalent circuit:





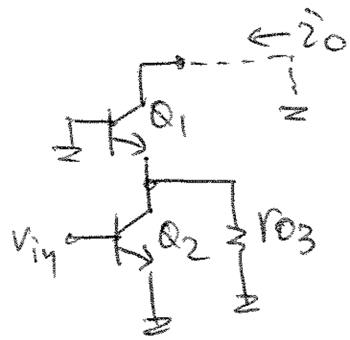
$$G_m = \frac{i_o}{v_{in}} = \frac{g_{m2}}{1 + g_{m2} R_E}$$ (small-signal analysis)

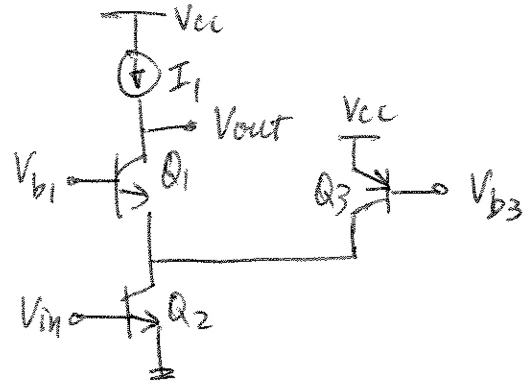
$$\begin{aligned} R_{out} &= (1 + g_{m1} R_X) r_{o1} + R_X \\ &= [1 + g_{m1} [(1 + g_{m2} R_E) r_{o2} + R_E]] r_{o1} \\ &\quad + [(1 + g_{m2} R_E) r_{o2} + R_E] \end{aligned}$$

$$\therefore A_v = -G_m R_{out}$$

$$= \frac{g_{m2}}{1 + g_{m2} R_E} \{ [1 + g_{m1} [(1 + g_{m2} R_E) r_{o2} + R_E]] r_{o1} + [(1 + g_{m2} R_E) r_{o2} + R_E] \}$$

(d) Equivalent circuit:

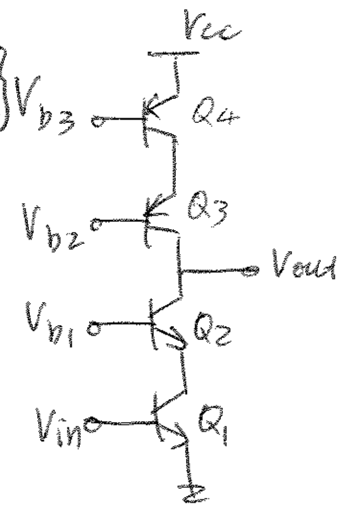




This resembles the BJT cascode topology, only now r_{o2} becomes $(r_{o2} \| r_{o3})$

$$\Rightarrow A_v \approx -g_{m2}^2 (r_{o2} \| r_{o3})(r_{o2} \| r_{o3} \| r_{\pi 1})$$

2b. $A_v = -g_{m1}\Big\{\big[\underbrace{g_{m2} r_{o2}(r_{o1} \| r_{\pi 2})}_{R_{on}}\big] \,\|\, \big[\underbrace{g_{m3} r_{o3}(r_{o4} \| r_{\pi 3})}_{R_{op}}\big]\Big\}$



$$R_{on} = \frac{(V_{An}/V_T)}{\left(\frac{1}{V_{An}} + \frac{1}{\beta_n V_T}\right) I_C}$$

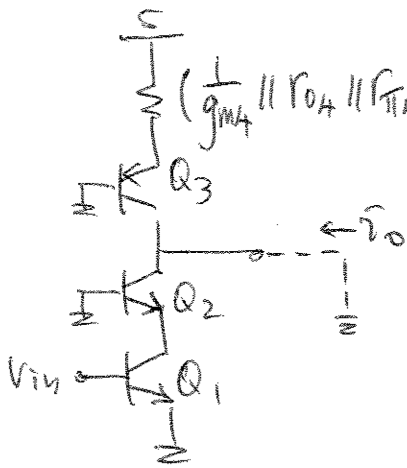
$$R_{op} = \frac{(V_{AP}/V_T)}{\left(\frac{1}{V_{Ap}} + \frac{1}{\beta_p V_T}\right) I_C} \qquad g_{m1} = \frac{I_C}{V_T}$$

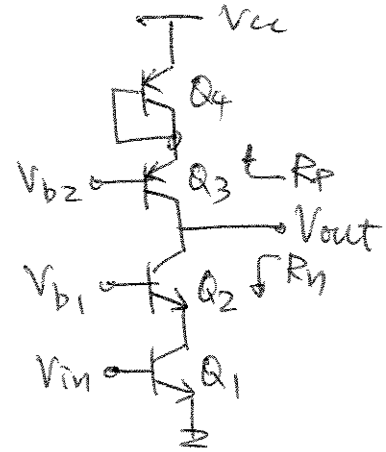
$$\therefore A_v = \frac{-\;(I_C/V_T)}{\dfrac{\left(\frac{1}{V_{An}} + \frac{1}{\beta_n V_T}\right) I_C}{V_{An}/V_T} + \dfrac{\left(\frac{1}{V_{Ap}} + \frac{1}{\beta_p V_T}\right) I_C}{V_{AP}/V_T}}$$

$$= \frac{V_{AN} \cdot V_{AP}}{V_T^2 \left(\frac{V_{AP}}{V_{AN}} + \frac{V_{AP}}{\beta_N V_T} + \frac{V_{AN}}{V_{AP}} + \frac{V_{AN}}{\beta_P V_T}\right)}$$

$\therefore A_v$ is independent of bias current, I_C.

27. Equivalent circuit.





$$G_m = g_{m1} = \frac{i_o}{v_{in}} = \frac{i_{c1}}{v_{in}}$$

$$R_{out} = R_p || R_n$$

$$R_p = \left[1 + g_{m3}\left(\frac{1}{g_{m4}} || r_{o4} || r_{\pi 4} || r_{\pi 3}\right)\right] r_{o3} + \left[\frac{1}{g_{m4}} || r_{o4} || r_{\pi 4} || r_{\pi 3}\right]$$

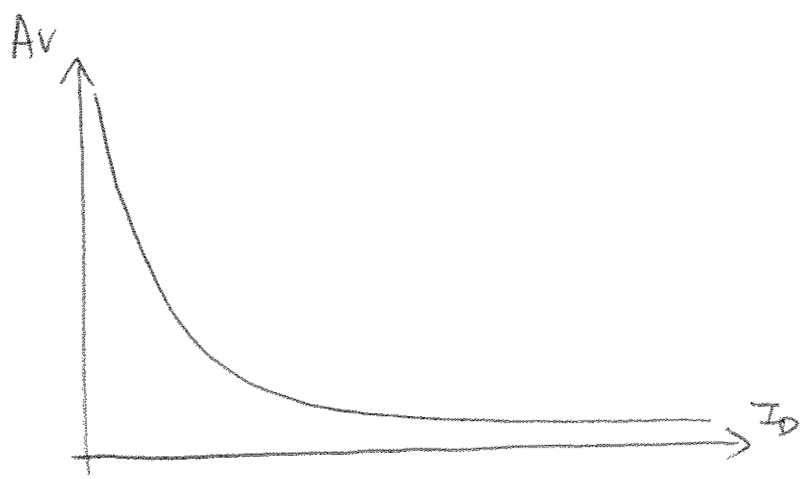
$$R_n = \left[1 + g_{m2}(r_{o1} || r_{\pi 2})\right] r_{o2} + (r_{o1} || r_{\pi 2})$$

$$\therefore A_v = -G_m R_{out} = -g_{m1}(R_p || R_n)$$

28. $|A_v| = g_{m1} r_{o1} g_{m2} r_{o2}$

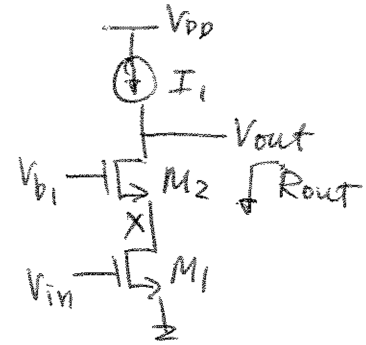
$$= \sqrt{2 \mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_D} \cdot \frac{1}{\lambda I_D} \cdot \sqrt{2 \mu_n C_{ox} \left(\frac{W}{L}\right)_2 I_D} \cdot \frac{1}{\lambda I_D}$$

$$= 2 \mu_n C_{ox} \sqrt{\left(\frac{W}{L}\right)_1 \left(\frac{W}{L}\right)_2} \cdot \frac{1}{\lambda^2 I_D}$$



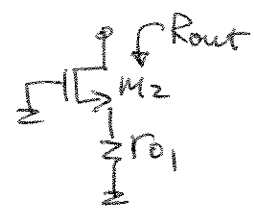
29. $|A_v| = 200$

$\mu_n C_{ox} = 100\ \frac{\mu A}{V^2}$ $\qquad \lambda = 0.1\ V^{-1}$



Determine $\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2$

$R_{out} = (1 + g_{m2} r_{o1}) r_{o2} + r_{o1}$



$G_m \cong g_{m1}$ (short-circuit current flows through both M_1 & M_2)

$$|A_v| = G_m R_{out} = g_{m1}\left[(1 + g_{m2} r_{o1}) r_{o2} + r_{o1}\right]$$

$$\approx g_{m1} g_{m2} r_{o1} r_{o2} = (g_m r_o)^2 = 200$$

$\left(\because \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 \text{ and } I_{D1} = I_{D2}\right)$

$$(g_m r_o)^2 = \left(\frac{2 I_D}{V_{GS} - V_{TH}} \cdot \frac{1}{\lambda I_D}\right)^2 = 200$$

$$\Rightarrow V_{GS} - V_{TH} = \left(\sqrt{200} \cdot \lambda/2\right)^{-1} = \left[\sqrt{200} \cdot (0.05\ V^{-1})\right]^{-1}$$

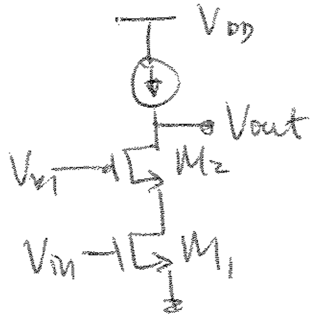
$$\approx 1.41\ V$$

$$\Rightarrow \quad I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH})^2$$

$$\therefore \left(\frac{W}{L}\right) = \frac{2 I_D}{\mu_n C_{ox} (V_{GS} - V_{TH})^2}$$

$$= \frac{2(1 \text{ mA})}{100 \frac{\mu A}{V^2} (1.41V)^2} \approx 10$$

30.



$$\left(\frac{W}{L}\right)_{1,new} = N\left(\frac{W}{L}\right)_1$$

$$\left(\frac{W}{L}\right)_{2,new} = N\left(\frac{W}{L}\right)_2$$

$$\lambda_{n,1} = \lambda_{n,2}$$

$$A_{v_{new}} \approx -g_{m1} g_{m2} r_{o1} r_{o2}$$

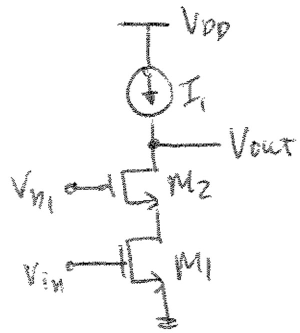
$$= -\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,new} I_D} \cdot \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_{2,new} I_D} \cdot \frac{1}{\lambda I_D} \cdot \frac{1}{\lambda I_D}$$

$$= -\sqrt{N}\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_D} \cdot \sqrt{N}\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_D} \cdot \frac{1}{\lambda I_D} \cdot \frac{1}{\lambda I_D}$$

$$= -N\left(g_{m1} g_{m2} r_{o1} r_{o2}\right) = -N \cdot A_{v,old}.$$

Gain is N times of original value:

31.



$$\left(\frac{W}{L}\right)_{1,new} = \frac{1}{N}\left(\frac{W}{L}\right)_1$$

$$\left(\frac{W}{L}\right)_{2,new} = \frac{1}{N}\left(\frac{W}{L}\right)_2$$

Assume $\lambda_{n,1} = \lambda_{n,2}$

$$A_{v_{new}} \approx -g_{m1}\left(g_{m2}\left(r_{o1} r_{o2}\right)\right)$$

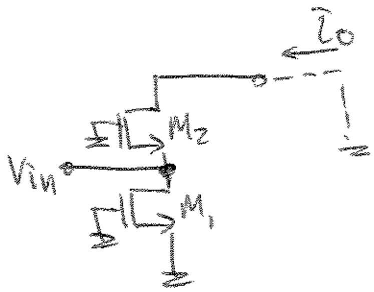
$$= -\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,new} \cdot I_{D1}} \cdot \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_{2,new} I_{D1}} \cdot \left(\frac{1}{\lambda I_D}\right)^2$$

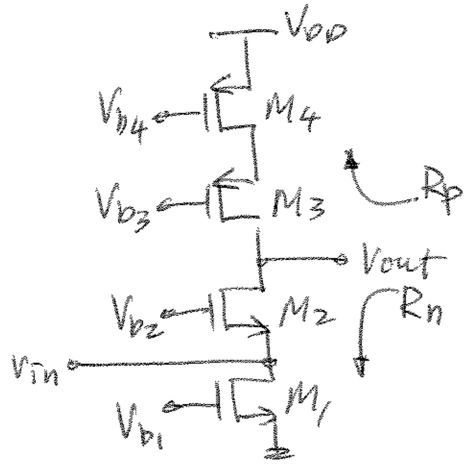
$$= -\sqrt{\frac{1}{N}}\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_D} \cdot \sqrt{\frac{1}{N}} \cdot \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_D} \cdot \left(\frac{1}{\lambda I_D}\right)^2$$

$$= -\frac{1}{N}\, g_{m1} g_{m2} r_{o1} r_{o2} = -\frac{1}{N}\left(A_{v,old}\right)$$

Gain is $1/N$ of original value.

32.





By KCL,

$$\frac{i_o}{V_{in}} = -\left(g_{m2} + \frac{1}{r_{o1} \| r_{o2}}\right) = G_m$$

$$R_n = r_{o2} \qquad R_p \approx g_{m3} r_{o3} r_{o4}$$

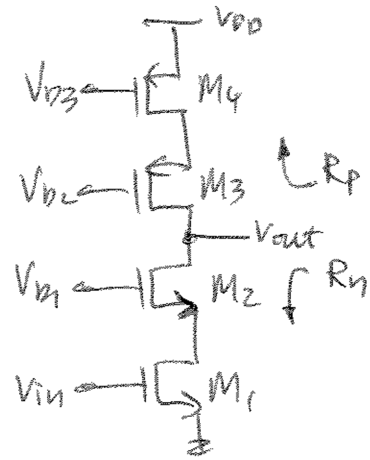
$$\therefore A_v = -G_m (R_n \| R_p) = \left(g_m + \frac{1}{r_{o1} \| r_{o2}}\right)(r_{o2} \| g_{m3} r_{o3} r_{o4})$$

33. $\left(\frac{W}{L}\right) = 20/0.18$

$\mu_n C_{ox} = 100\ \mu A/V^2$

$\mu_p C_{ox} = 50\ \mu A/V^2$

$\lambda_n = 0.1\ V^{-1}$ $\lambda_p = 0.15\ V^{-1}$



Calculate I_{BIAS} such as $A_v = 500$.

The cascode circuit has gain

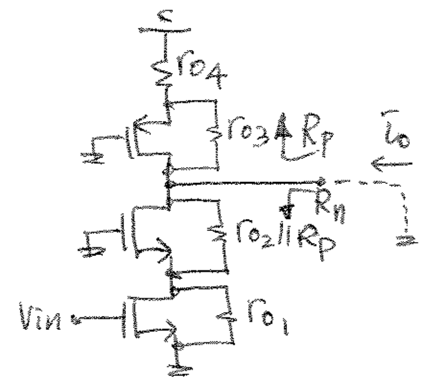
$$\approx -g_{m1} \cdot [g_{m2} r_{o1} r_{o2} \parallel g_{m3} r_{o3} r_{o4}]$$

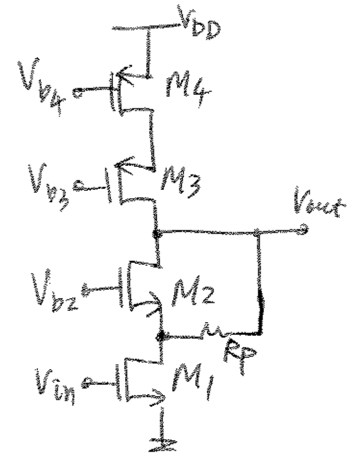
$$\Rightarrow 500 = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D} \left(\frac{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)}}{(\lambda_n)^2\, I_D^{3/2}} \parallel \frac{\sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)}}{(\lambda_p)^2\, I_D^{3/2}} \right)$$

All quantities are known. Solving I_D gives:

$I_D = I_{BIAS} \approx 1.06$ mA.

34 (a) Equivalent circuit:



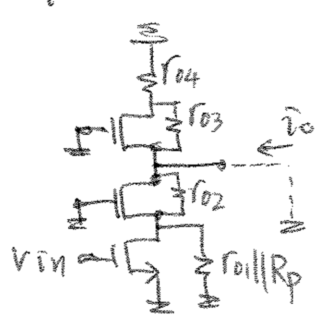


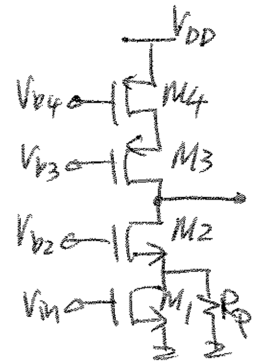
$$G_m = \frac{i_o}{v_{in}} \approx g_{m1} \quad (\because g_m r_o \gg 1)$$

$$R_p = g_{m3} r_{o3} r_{o4} \qquad R_n = g_{m2}(r_{o2} \| R_p) r_{o1}$$

$$\therefore A_v = -G_m R_{out} = -g_{m1}\left[g_{m3} r_{o3} r_{o4} \,\|\, g_{m2}(r_{o2} \| R_p) r_{o1}\right]$$

(b) Equivalent circuit



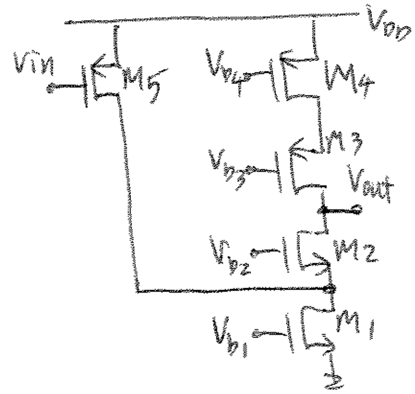


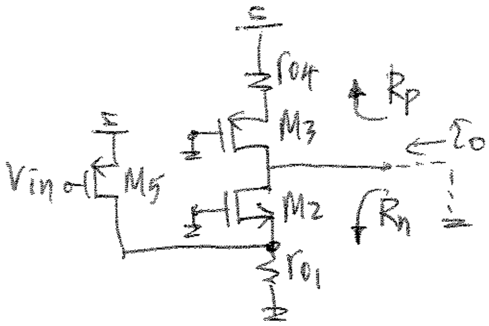
$$G_m = i_o / v_{in} \approx g_{m1} \quad (\because g_m r_o \gg 1)$$

$$R_p = g_{m3} r_{o3} r_{o4} \qquad R_n = g_{m2}(r_{o1} \| R_p) r_{o2}$$

$$\therefore A_v = -G_m R_{out} = -g_{m1}\left[g_{m3} r_{o3} r_{o4} \,\|\, g_{m2}(r_{o1} \| R_p) r_{o2}\right]$$

(c) Equivalent circuit:





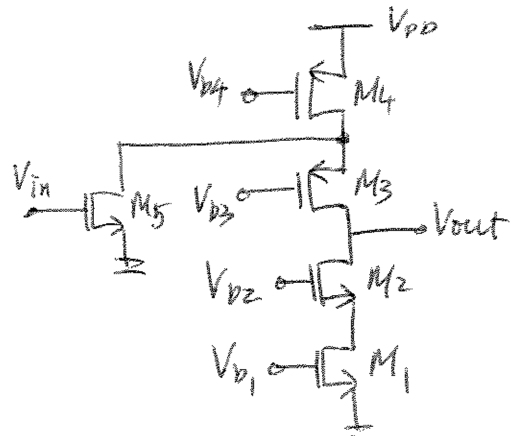
(Realize that r_{o1} & r_{o5} are in parallel.)

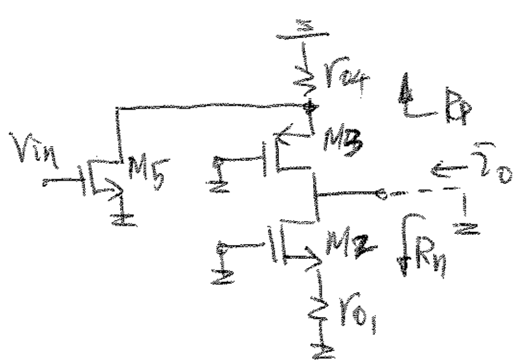
$$G_m = \frac{i_o}{v_{in}} \approx -g_{m5} \qquad (\because g_m r_o \gg 1)$$

$$R_p = g_{m3} r_{o3} r_{o4} \qquad R_n = g_{m2} r_{o2} (r_{o1} \| r_{o5})$$

$$\therefore A_v = -G_m R_{out} = g_{m5} [g_{m3} r_{o3} r_{o4} \| g_{m2} r_{o2} (r_{o1} \| r_{o5})]$$

(d) Equivalent circuit:





$$G_m = \frac{i_o}{v_{in}} \approx g_{m5} \qquad R_p = g_{m3} r_{o3} (r_{o4} \| r_{o5})$$

$$R_n = g_{m2} r_{o2} r_{o1}$$

$$\therefore A_v = -G_m R_{out} = g_{m5} [g_{m3} r_{o3} (r_{o4} \| r_{o5}) \| g_{m2} r_{o2} r_{o1}]$$

35. $$\frac{R_2}{R_1 + R_2} V_{CC} = V_T \ln\left(\frac{I_1}{I_S}\right)$$

$$\Rightarrow I_1 = I_S \cdot \exp\left[\frac{V_{CC}}{V_T} \cdot \frac{R_2}{R_1+R_2}\right]$$

$$\frac{\partial I_1}{\partial V_{CC}} = \frac{I_S}{V_T} \cdot \frac{R_2}{R_1+R_2} \cdot \exp\left[\frac{V_{CC}}{V_T} \cdot \frac{R_2}{R_1+R_2}\right]$$

$$= \frac{I_1}{V_T} \cdot \frac{R_2}{R_1+R_2} = g_m\left(\frac{R_2}{R_1+R_2}\right)$$

Intuitively, we know that an exponential relationship exists between I_C & V_{BE}. Its transconductance is also a function (linear) of I_C. Since V_{BE} comes from a voltage divider (which is also linear), we expect a linear relationship between I_C & V_{CC}.

36. $$I_1 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(\frac{R_2}{R_1+R_2} V_{DD} - V_{TH} \right)^2$$

$$\frac{\partial I_1}{\partial V_{DD}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \cdot 2 \left(\frac{R_2}{R_1+R_2} V_{DD} - V_{TH} \right) \cdot \frac{R_2}{R_1+R_2}$$

$$= \mu_n C_{ox} \frac{W}{L} \left(\frac{R_2}{R_1+R_2} \right) \left(\frac{R_2 \cdot V_{DD}}{R_1+R_2} - V_{TH} \right).$$

$$= g_m \cdot \frac{R_2}{R_1+R_2}$$

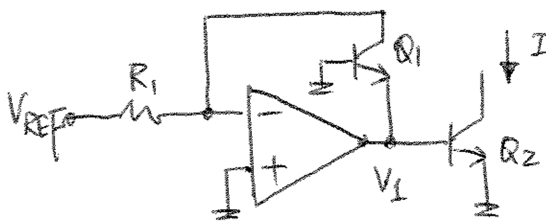
Intuitively, the voltage divider gives a linear relationship between V_{DD} & V_{GS1}. Since g_m of MOS is linearly proportional to $(V_{GS1} - V_{TH})$, we expect the same relationship between V_{DD} & $\frac{\partial I_1}{\partial V_{DD}}$

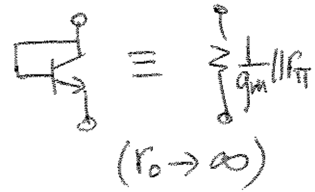
37. $I_1 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(\frac{R_2}{R_1+R_2} V_{DD} - V_{TH} \right)^2$

$$\frac{\partial I_1}{\partial V_{TH}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \cdot 2 \left(\frac{R_2}{R_1+R_2} V_{DD} - V_{TH} \right) \cdot (-1)$$

$$= -\mu_n C_{ox} \frac{W}{L} \left(\frac{R_2}{R_1+R_2} V_{DD} - V_{TH} \right)$$

38.





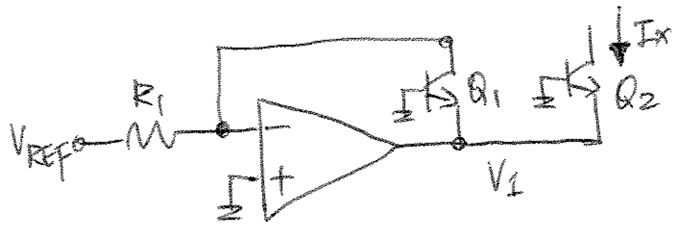
This is a negative feedback circuit.

The inverting input (−) of the op-amp is virtual ground. ($\because$ of feedback) in DC. $\Rightarrow$ Q_1 becomes diode-connected.

$$\Rightarrow \frac{V_{REF}}{R_1} = \frac{0 - V_1}{\left(\frac{1}{g_{m_1}} \| r_{\pi_1}\right)} \Rightarrow V_1 = -\frac{V_{REF}\left(\frac{1}{g_{m_1}} \| r_{\pi_1}\right)}{R_1} < 0$$

This implies $V_{BE_2} < 0 \Rightarrow I_X = 0$!

39.



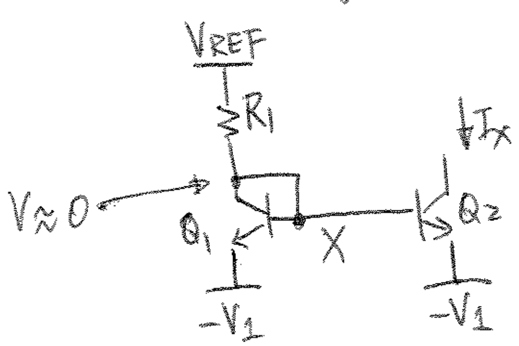
This is a negative feedback circuit.

The inverting input (−) is virtual ground, as a result. Q_1 then becomes diode-connected, and its resistance $= \left(\frac{1}{g_m} \| r_\pi\right)$, assuming $r_o \to \infty$.

$$\Rightarrow \frac{V_{REF}}{R_1} = \frac{-V_1}{\left(\frac{1}{g_{m1}} \| r_{\pi 1}\right)} \Rightarrow V_1 = -\frac{V_{REF}\left(\frac{1}{g_{m1}} \| r_{\pi 1}\right)}{R_1}$$

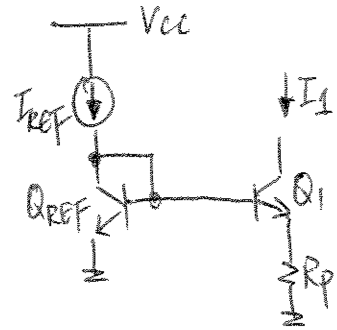
$$\Rightarrow V_{BE1} = V_{BE2} = -V_1$$

This circuit will work if the negative supply voltage of the op-amp allows value of $-V_1$ or lower.



- An equivalent circuit. (without op-amp). The op-amp guarantees a stable voltage at node X. (i.e. inverting input.)

40.



$Q_{REF} = Q_1$

$\beta \to \infty.$

$I_1 = \frac{I_{REF}}{2}$

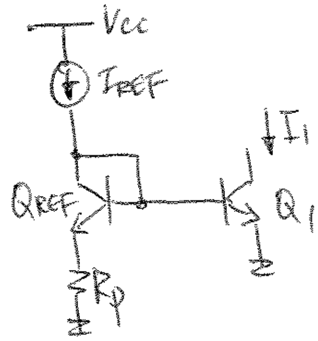
By KVL, $V_{BE,REF} = V_{BE_1} + I_1 R_p$

$$\Rightarrow V_T \ln\left(\frac{I_{REF}}{I_{S,REF}}\right) = V_T \ln\left(\frac{I_{REF}/2}{I_{S,1}}\right) + \frac{I_{REF}}{2} R_p$$

$$V_T \ln(2) = \frac{I_{REF}}{2} R_p$$

$$R_p = 2 \cdot \ln(2) \cdot (V_T / I_{REF})$$

41.



$Q_{REF} = Q_1$

$\beta \to \infty$

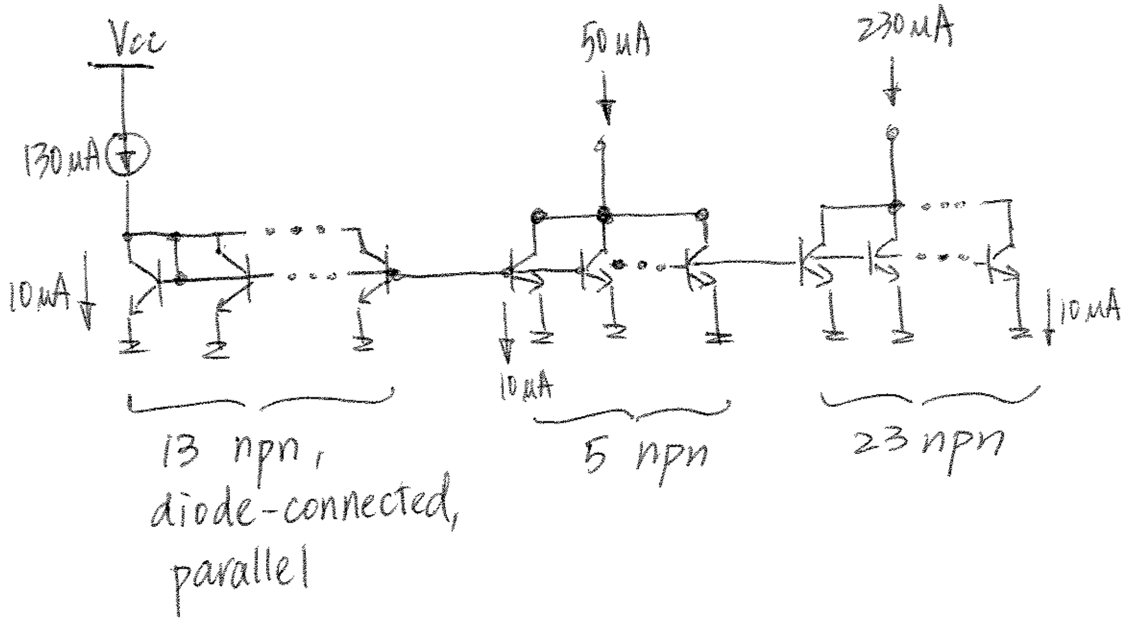
By KVL, $V_{BE,REF} + I_{REF} R_p = V_{BE,1}$

$$\Rightarrow V_T \ln\left(\frac{I_{REF}}{I_{S,REF}}\right) + I_{REF} R_p = V_T \ln\left(\frac{2 I_{REF}}{I_{S,1}}\right)$$

$$I_{REF} R_p = V_T \ln(2)$$

$$R_p = \frac{V_T}{I_{REF}} \ln(2)$$

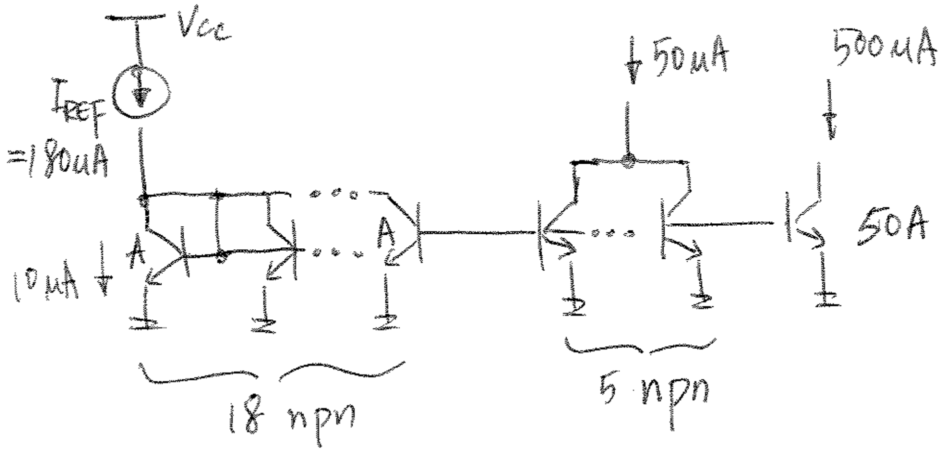
42.



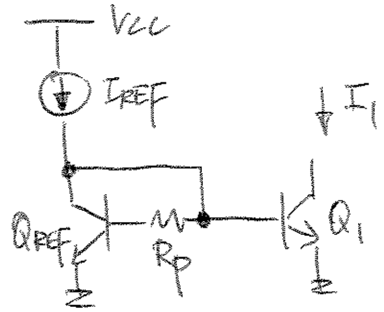
All the bases are the same node.

If the area of BJT is flexible, the 5-npn group can be replaced by one BJT that is 5 times as big in area. Similar concept applies to 23-npn grouping.

43.



44.



$Q_{REF} = Q_1$

I_1 10% larger. ($I_1 = 1.1\, I_{C,REF}$)
Solve for R_P.

By KVL,

$$V_{BE_{REF}} + \frac{I_{C,REF}}{\beta} \cdot R_P = V_{BE_1}$$

$$\Rightarrow V_T \ln\left(\frac{I_1}{I_S}\right) - V_T \ln\left(\frac{I_{C,REF}}{I_S}\right) = \frac{I_{C,REF}}{\beta} \cdot R_P$$

$$V_T \ln\left(\frac{I_1}{I_{C,REF}}\right) = \frac{I_{C,REF}}{\beta} \cdot R_P$$

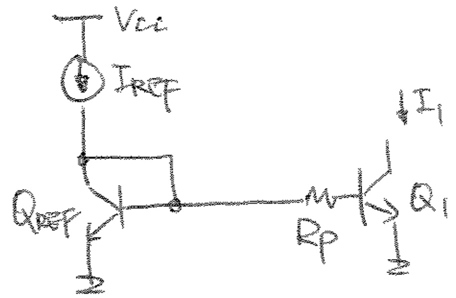
$$\Rightarrow V_T \ln(1.1) = \frac{I_{C,REF}}{\beta} R_P \qquad \Rightarrow I_{C,REF} = \frac{\beta V_T \ln(1.1)}{R_P}$$

By KCL, $I_{REF} = I_{C,REF} + I_{C,REF}/\beta + I_1/\beta$

$$= \frac{\beta}{R_P} V_T \ln(1.1) \cdot \left(1 + 1/\beta\right) + I_1/\beta$$

$$\therefore R_P = \frac{(\beta + 1) V_T \ln(1.1)}{I_{REF} - I_1/\beta}$$

45.



$I_1 = 0.9 I_{C,REF}$

By KVL, $V_{BE_{REF}} = \frac{I_1}{\beta} R_P + V_{BE_1}$

$$\Rightarrow V_T \ln\left(\frac{I_{C,REF}}{I_1}\right) = \frac{I_1}{\beta} R_P$$

$$V_T \ln\left(\frac{1}{0.9}\right) = 0.9\ I_{C,REF}\ \frac{R_P}{\beta}$$

$$\Rightarrow I_{C,REF} = \frac{\beta}{0.9 R_P} V_T \ln\left(\frac{1}{0.9}\right)$$

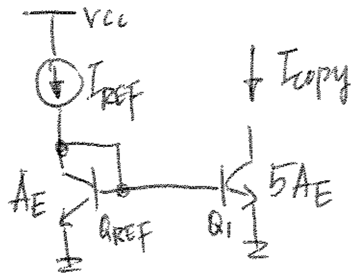
By KCL,

$$I_{REF} = I_{C,REF} + I_{C,REF}/\beta + I_1/\beta$$

$$\therefore I_{REF} - \frac{I_1}{\beta} = \frac{\beta}{0.9 R_P} V_T \ln\left(\frac{1}{0.9}\right)\left(1 + \frac{1}{\beta}\right)$$

$$\Rightarrow R_P = \frac{(\beta+1)\ V_T \ln\left(10/9\right)}{0.9\left(I_{REF} - I_1/\beta\right)}$$

4b (a)



Q_1 has I_S 5 times as that of Q_{REF}

$\Rightarrow I_{C_{REF}} = I_{copy} / 5$

By KCL,

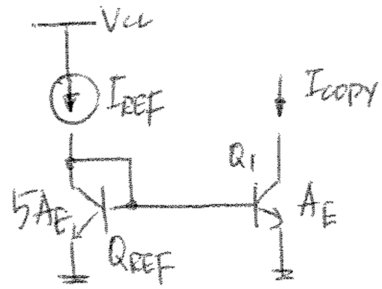
$$I_{REF} = I_{C,REF} + \frac{I_{C_{REF}}}{\beta} + \frac{I_{copy}}{\beta}$$

$$= \frac{I_{copy}}{5}\left(1 + \frac{1}{\beta}\right) + \frac{I_{copy}}{\beta}$$

$$\therefore \ I_{copy} = I_{REF}\left(\frac{5\beta}{\beta+6}\right)$$

$$\therefore \ error = \frac{I_{copy}}{I_{REF}} = \frac{5\beta}{\beta+6}$$

(b)



Q_1 & Q_{REF} have the same V_{BE}, but area of Q_{REF} is 5 times larger

$\Rightarrow I_{C,REF} = 5 \cdot I_{copy}$

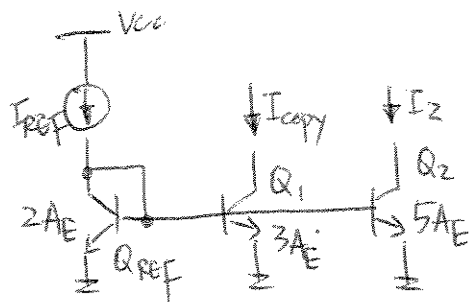
By KCL,

$$I_{REF} = I_{C,REF} + \frac{I_{C,REF}}{\beta} + \frac{I_{COPY}}{\beta}$$

$$= I_{copy} \cdot 5 + \left(1 + \frac{1}{\beta}\right) + I_{COPY}\left(\frac{1}{\beta}\right)$$

$$\therefore I_{copy} = I_{REF}\left(\frac{\beta}{5\beta + 6}\right)$$

(c)



Q_1 & Q_{REF} have identical V_{BE}, but area of Q_1 is 1.5 times larger.

$\Rightarrow 3 I_{C,REF} = 2 I_{copy}$

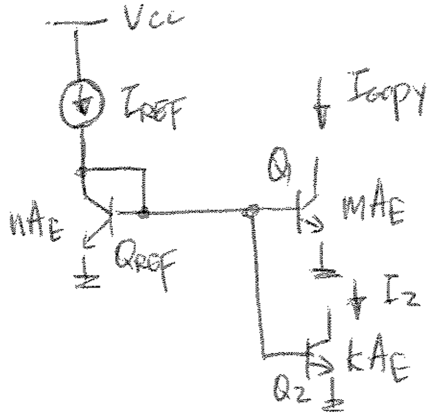
By KCL,

$$I_{REF} = I_{C,REF} + I_{C,REF}/\beta + I_{copy}/\beta + I_2/\beta$$

$$= I_{copy}\left(\frac{2}{3}\right)\left[\left(1 + 1/\beta\right) + 1/\beta + 5/3\left(\frac{1}{\beta}\right)\right]$$

$$\rightarrow I_{copy} = \frac{9\beta}{6\beta + 22} I_{REF}$$

47.



By observing the areas of the BJTs,

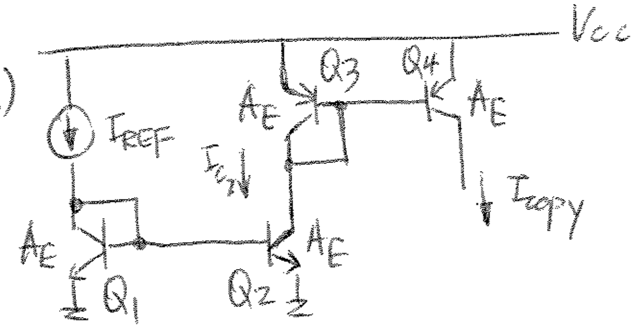
$$I_{C,REF} = \left(\frac{n}{m}\right) I_{copy} = \left(\frac{n}{k}\right) I_2$$

By KCL, $I_{C,REF} = I_{REF} - \dfrac{I_{C,REF}}{\beta} - \dfrac{I_{copy}}{\beta} - \dfrac{I_2}{\beta}$

$$\Rightarrow \frac{n}{m} I_{copy} = I_{REF} - \frac{\left(\frac{n}{m}\right) I_{copy}}{\beta} - \frac{I_{copy}}{\beta} - \frac{\left(\frac{k}{m}\right) I_{copy}}{\beta}$$

$$\therefore I_{copy} = I_{REF}\left[\frac{\beta m}{(\beta+1)n + k + m}\right]$$

48. (a)



$V_{BE_1} = V_{BE_2}$
$\Rightarrow I_{C_1} = I_{C2}$

$V_{BE_3} = V_{BE_4}$
$\Rightarrow I_{B_3} = I_{C_4}$

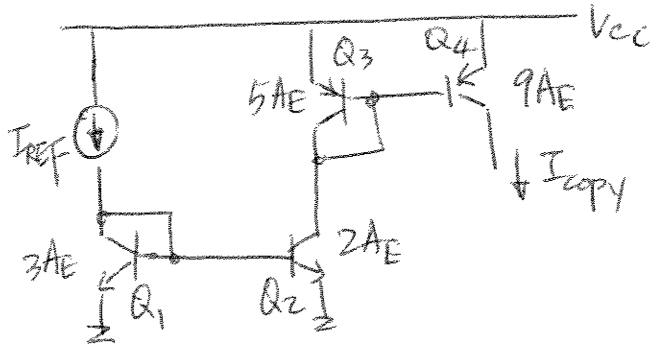
First compute $I_{C_1 2}$:

$$I_{C1} = I_{REF} - \frac{I_{C1}}{\beta} - \frac{I_{C2}}{\beta} \Rightarrow I_{C_2} = \frac{\beta}{\beta+2} \cdot I_{REF}.$$

View I_{C2} as the "I_{REF}" for the Q_3–Q_4 current mirror and apply the equation derived.

$$\Rightarrow I_{copy} = \frac{\beta}{\beta+2}\left[\frac{\beta}{\beta+2} \cdot I_{REF}\right] = I_{REF}\left(\frac{\beta}{\beta+2}\right)^2$$

(b)



$V_{BE_1} = V_{BE_2}$:

$\Rightarrow I_{C_1} = \frac{3}{2} I_{C_2}$

$V_{BE_3} = V_{BE_4}$:

$\Rightarrow I_{copy} = \frac{9}{5} I_{C_3}$

- By KCL,

$$I_{REF} = I_{C_1} + \frac{I_{C_1}}{\beta} + \frac{I_{C_2}}{\beta}$$

$$\Rightarrow I_{C_2} = \frac{2\beta}{3\beta+5} I_{REF} \qquad ①$$

- By KCL,

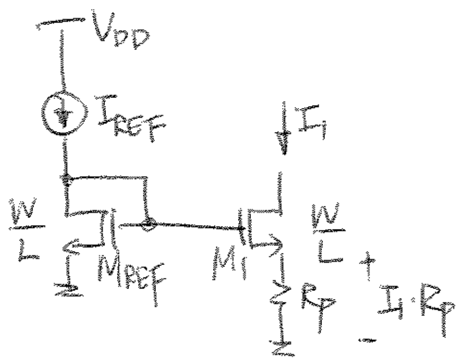
$$I_{C_2} = I_{C_3} + \frac{I_{C_3}}{\beta} + \frac{I_{copy}}{\beta}$$

$$\Rightarrow I_{copy} = \frac{9\beta}{5\beta+14} I_{C_2}$$

Substitute ① into Icopy :

$$\therefore I_{copy} = \frac{9\beta}{5\beta+14} \cdot \frac{2\beta}{3\beta+5} \cdot I_{REF}.$$

49.



Determine R_P such that $I_1 = \frac{I_{REF}}{2}$.

First calculate $V_{GS,REF}$:

$$V_{GS,REF} = \sqrt{\frac{2 I_{REF}}{\mu_n C_{ox} W/L}} + V_{TH} \quad \text{——— (1)}$$

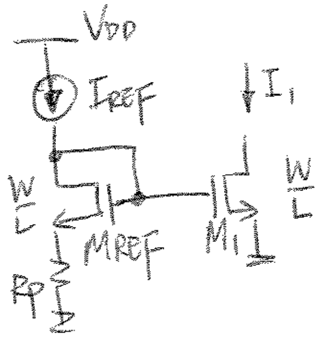
Assuming M_1 in saturation:

$$I_1 = \frac{I_{REF}}{2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[V_{GS_1} - V_{TH} \right]^2$$

$$\Rightarrow \quad \frac{I_{REF}}{2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[V_{GS,REF} - \frac{I_{REF}}{2}(R_P) - V_{TH} \right]^2$$

Rearrange, substitute (1) into equation above and solve for R_P:

$$\therefore R_P = \frac{2(\sqrt{2}-1)}{\sqrt{I_{REF} \cdot \mu_n C_{ox} \frac{W}{L}}}$$

50. 

Determine R_P such that $I_1 = 2I_{REF}$.

First calculate V_{GS_1}:

$$V_{GS_1} = \sqrt{\frac{2I_1}{\mu_n C_{ox}(\frac{W}{L})}} + V_{TH} = 2\sqrt{\frac{I_{REF}}{\mu_n C_{ox}(\frac{W}{L})}} + V_{TH} \quad -①$$

Assuming I_1 is in saturation:

$$I_{REF} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS,REF} - V_{TH})^2$$

$$= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[V_{GS_1} - I_{REF}R_P - V_{TH}\right]^2$$

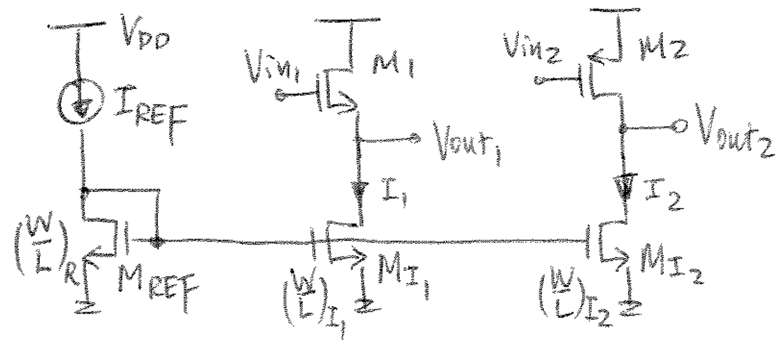
Substitute ① into I_{REF}:

$$I_{REF} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[2\sqrt{\frac{I_{REF}}{\mu_n C_{ox}(\frac{W}{L})}} - I_{REF}R_P\right]^2 \quad -②$$

Solve for R_P: $R_P = \dfrac{(2-\sqrt{2})}{\sqrt{I_{REF}\cdot \mu_n C_{ox}(\frac{W}{L})}}$

From ②, we find that R_P is <u>independent</u> of any change in V_{TH}, ΔV !!

51.



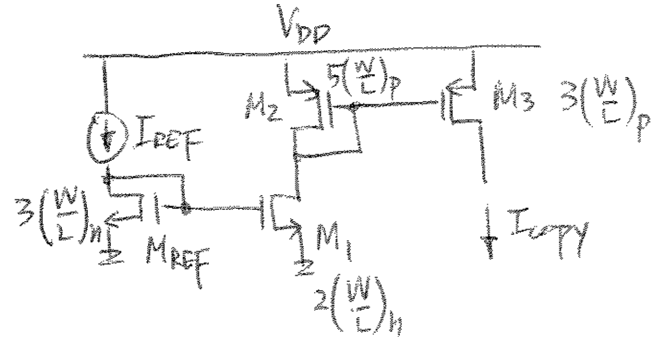
This figure implies that $V_{GS,REF} = V_{GS,I_1} = V_{GS,I_2}$.
Assuming all devices operate in saturation, with $(V_{GS} - V_{TH})$ fixed, $I_D \propto (\frac{W}{L})$

$\Rightarrow$ we have $(\frac{W}{L})_R = 7(\frac{W}{L})$

$$(\frac{W}{L})_{I_1} = 4(\frac{W}{L})$$

$$(\frac{W}{L})_{I_2} = 10(\frac{W}{L})$$

52. (a)

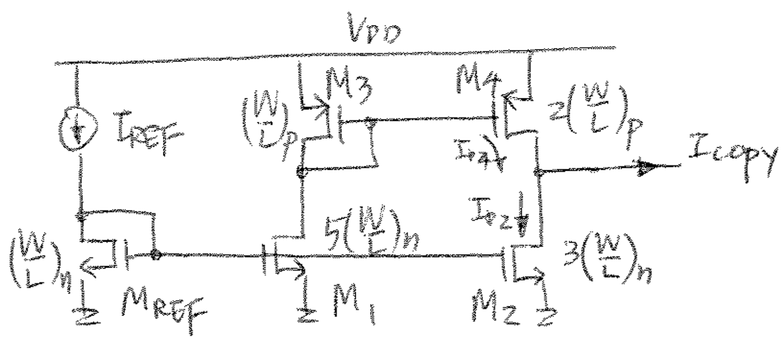


$V_{GS,REF} = V_{GS,1}$: $\Rightarrow I_{D,1} = \frac{2}{3} I_{REF}$

$V_{GS,2} = V_{GS,3}$: $\Rightarrow I_{copy} = \frac{3}{5} I_{D,2} = \frac{3}{5} I_{D,1}$

$= \frac{3}{5} \cdot \left(\frac{2}{3} I_{REF}\right) = \frac{2}{5} I_{REF}$

(b)



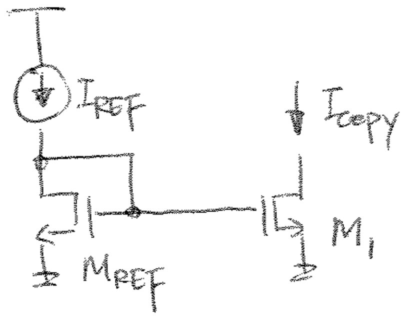
$V_{GS,REF} = V_{GS,1}$: $I_{D,1} = 5 I_{REF}$

$V_{GS,3} = V_{GS,4}$: $I_{D4} = 2 I_{D3} = 2 I_{D,1} = 10 I_{REF}$

$V_{GS,REF} = V_{GS,2}$: $I_{D2} = 3 I_{REF}$

$\therefore I_{copy} = I_{D4} - I_{D2} = 7 I_{REF}$.

53.



$V_{GS,REF} = V_{GS,1} = V_{GS}$

$\lambda \neq 0$

(a) $I_{REF} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2(1+\lambda V_{GS})$

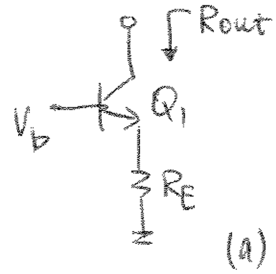
$I_{copy} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2(1+\lambda V_{DS,1})$

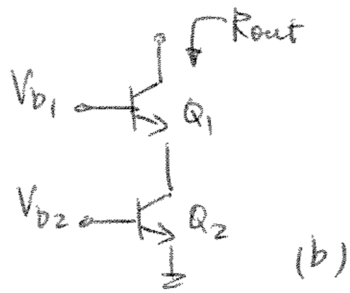
For $I_{REF} = I_{copy} \Rightarrow V_{DS,1} = V_{GS}$

(b) $\frac{I_{REF}}{I_{copy}} = \frac{1+\lambda V_{GS}}{1+\lambda(V_{GS}-V_{TH})}$

$\Rightarrow I_{copy} = I_{REF}\left(1 - \frac{\lambda V_{TH}}{1+\lambda V_{GS}}\right)$

54.





Given $I_{BIAS} = 1mA$, $V_{RE} \approx V_{CE,2} \approx 0.5V$, design the circuit.

R_E can be readily calculated:

$$R_E = \frac{V_{RE}}{I_{BIAS}/\alpha} = \frac{0.5V}{1mA/0.909} = 505\,\Omega$$

$$V_{be_1} = V_T \ln\left(\frac{I_{BIAS}}{I_{S,1}}\right) = (0.026V)\ln\left(\frac{1mA}{6\cdot 10^{-16}A}\right) \approx 0.732V$$

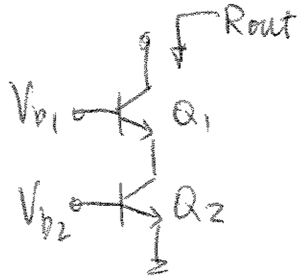
$$\Rightarrow V_b = V_{be_1} + V_{RE} = 0.732V + 0.5V = 1.232V$$

$$R_{out(a)} = \left[1 + g_{m_1}(R_E \parallel r_{\pi_1})\right] r_{o_1} + (R_E \parallel r_{\pi_1})$$

$$R_{out(b)} = \left[1 + g_{m_1}(r_{o2} \parallel r_{\pi_1})\right] r_{o_1} + (r_{o2} \parallel r_{\pi_1})$$

In most cases $r_o > r_\pi > R_E$

$\therefore$ $R_{out(b)}$ is relatively larger than $R_{out(a)}$

55. 

$I_{BIAS} = 1mA$
$\beta = 100$

Given $R_{out} = 50\ k\Omega$, $V_{BC_2} = 100\ mV$, determine V_{b1}.

$$R_{out} = [1 + g_{m1}(r_{o2} \| r_{\pi 1})] r_{o1} + (r_{o2} \| r_{\pi 1})$$
$$\approx g_{m1}(r_{o2} \| r_{\pi 1}) r_{o1}$$
$$= \frac{\beta V_A^2}{(V_A + \beta V_T) I_{BIAS}}$$

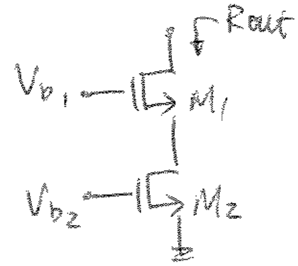
$$\Rightarrow I_{BIAS} = \left[\frac{R_{out}(V_A + \beta V_T)}{\beta V_A^2}\right]^{-1} = \left[\frac{(50k\Omega)(5V + 100 \cdot 0.026V)}{100(5V)^2}\right]^{-1}$$
$$\approx 6.6\ mA.$$

$$V_{b2} = V_{BE2} = V_T \ln\left(\frac{I_{BIAS}}{I_S}\right) = (0.026V) \ln\left(\frac{6.6mA}{6 \cdot 10^{-16} A}\right)$$
$$\approx 0.78\ V$$

$$\Rightarrow V_{C2} = V_{BE2} - 100mV = 0.68V$$

$$\therefore V_{b1} = V_{C2} + V_{BE1} = V_{C2} + V_T \ln\left(\frac{I_{BIAS}}{I_S}\right)$$
$$= 0.68V + (0.026V) \ln\left(\frac{6.6mA}{6 \cdot 10^{-16} A}\right) \approx 1.46\ V.$$

5b. Given $R_{out} = 200k\Omega$
$I_{BIAS} = 0.5mA$



(a) Determine $(W/L)_1 = (W/L)_2$ with $\lambda = 0.1\ V^{-1}$

$$R_{out} = (1 + g_{m1} r_{o2}) r_{o1} + r_{o2}$$
$$= \left[1 + \sqrt{2 I_{BIAS} \mu_n C_{ox} \left(\frac{W}{L}\right)_1} \cdot \frac{1}{\lambda I_{BIAS}}\right] \frac{1}{\lambda I_{BIAS}} + \frac{1}{\lambda I_{BIAS}}$$

$$\therefore \left(\frac{W}{L}\right)_1 \cong \frac{\left[\left(R_{out} - \frac{1}{\lambda I_{BIAS}}\right)(\lambda I_{BIAS})^2\right]^2}{2 I_{BIAS} \mu_n C_{ox}}$$

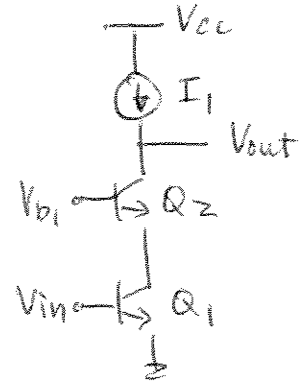
$$= \frac{\left\{\left[(200k\Omega) - \frac{1}{(0.1V^{-1})(0.5mA)}\right](0.1V^{-1})^2(0.5mA)^2\right\}^2}{2(0.5mA)\left(100\ \frac{\mu A}{V^2}\right)}$$

$$\approx 2.0$$

(b) $I_{BIAS} = 0.5\ mA = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b2} - V_{TH,n})^2$

$$\Rightarrow V_{b2} = \sqrt{\frac{2 I_{BIAS}}{\mu_n C_{ox} (W/L)_1}} + V_{TH}$$
$$= \sqrt{\frac{2(0.5mA)}{\left(100\ \frac{\mu A}{V^2}\right)(2.0)}} + (0.4V) \approx 2.62\ V$$

57. Given $|A_v| = 500$
$\beta = 100$



(a) $A_v = -g_{m1} r_{o1} g_{m2} (r_{o1} \| r_{\pi 2})$

$$= -\frac{V_A}{V_T} \times \frac{I_{C2}}{V_T}\left(\frac{V_A}{I_{C1}} \,\|\, \frac{\beta}{g_{m2}}\right)$$

Assume $I_{C1} \approx I_{C2}$. After expanding $(r_{o1} \| r_{\pi 2})$,

$$A_v \approx -\frac{V_A/V_T}{\frac{V_T}{V_A} + \frac{1}{\beta}} \quad \Rightarrow V_A^2 + V_A\left(\frac{V_T A_v}{\beta}\right) + (A_v V_T^2) = 0$$

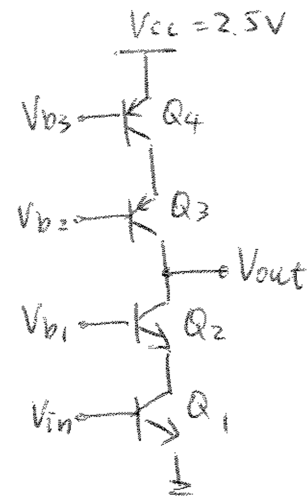
$\Rightarrow V_A \approx 0.65\text{ V}$

(b) $V_{in} = V_T \ln\left(\frac{I_1}{I_S}\right) = (0.026\text{ V}) \ln\left(\frac{0.5\text{ mA}}{6 \cdot 10^{-16}\text{ A}}\right)$

$\approx 0.71\text{ V}$

(c) $V_{b1} = V_{BE2} + 500\text{ mV}$
$= V_T \ln\left(\frac{I_1}{I_S}\right) + 0.5\text{ V}$
$= 0.71\text{ V} + 0.5\text{ V} = 1.21\text{ V}$

58. Given power budget = 2mW
$V_{BC_1} = V_{CB_4} = 200\ mV$,
calculate voltage gain.



$$\alpha_p = \frac{50}{50+1} \approx 0.98$$

$$\alpha_n = \frac{100}{100+1} \approx 0.99$$

$\therefore$ we assume $I_{c,p} \approx I_{e,p}$ & $I_{c,n} \approx I_{e,n}$

This implies that $I_{BIAS} = \frac{Power}{V_{cc}} = \frac{2mW}{2.5V}$

$$\approx 0.8mA$$

$$\Rightarrow V_{BE_1} = V_{in} = V_T \ln\left(\frac{I_{BIAS}}{I_{S,1}}\right) = (0.026V)\cdot\ln\left(\frac{0.8mA}{6\cdot 10^{-16}A}\right) \approx 0.726\,V$$

$$V_{C,1} = V_{BE_1} - V_{BC_1} = 0.726\,V - 0.2V = 0.526\,V$$

$$\therefore V_{b_1} = V_{C_1} + V_{BE_2} = (0.526V) + (0.026V)\ln\left(\frac{0.8mA}{6\cdot 10^{-6}A}\right)$$

$$\approx 1.252\,V$$

$$\Rightarrow V_{EB_4} = V_{cc} - V_{b3} = V_T \ln\left(\frac{I_{BIAS}}{I_{S,4}}\right) = 0.026V \cdot \ln\left(\frac{0.8mA}{6\cdot 10^{-16}A}\right)$$

$$\approx 0.726\,V$$

$$V_{D3} = V_{CC} - 0.726\,V = 1.774\ V$$

$$V_{C4} = V_{D3} + V_{CB4} = 1.774\,V + 0.2\,V = 1.974\ V$$

$$\therefore\ V_{D2} = V_{C4} - V_{EB3} = (1.974\,V) - (0.026)\ln\left(\frac{0.8\,mA}{6\cdot 10^{-16}\,A}\right)$$

$$\approx 1.248\ V$$

$$A_V = -g_{m1}\left\{\left[g_{m2} r_{O2}(r_{O1} \| r_{\pi 2})\right] \| \left[g_{m3} r_{O3}(r_{O4} \| r_{\pi 3})\right]\right\}$$

After simplifying, A_V is independent of I_{BIAS}:

$$A_V \approx \frac{V_{AN}\cdot V_{AP}}{V_T^{\,2}\left(\frac{V_{AP}}{V_{AN}} + \frac{V_{AP}}{\beta_N V_T} + \frac{V_{AN}}{V_{AP}} + \frac{V_{AN}}{\beta_P V_T}\right)}$$

$$= \frac{5\cdot 5}{(0.026\,V)^2\left(\frac{5}{5} + \frac{5}{100\cdot 0.026} + \frac{5}{5} + \frac{5}{50\cdot 0.026}\right)}$$

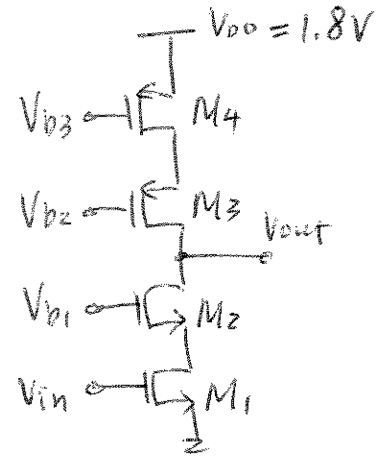
$$\approx 4760$$

59. Given $A_v = 200$

power budget $= 2mW$

all $\left(\frac{W}{L}\right) = \frac{20}{0.18}$

$V_{b1} = V_{b2} = 0.9\ V$



calculate V_{in} & V_{b3}

$\lambda_n = 0.1\ V^{-1}$

$\lambda_p = 0.2\ V^{-1}$

$$A_v \cong -g_{m1}(g_{m2}r_{o1}r_{o2} \parallel g_{m3}r_{o3}r_{o4}) = 200$$

$$\text{power} = V_{DD} \times I_{BIAS} \Rightarrow I_{BIAS} = \frac{\text{power}}{V_{DD}} = \frac{2mW}{1.8V} \approx 1.11\ mA$$

$$g_{m2}r_{o1}r_{o2} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)I_{BIAS}} \cdot \left(\frac{1}{\lambda_n I_{BIAS}}\right)^2$$

$$= \sqrt{2 \cdot 100\ \mu A \cdot \frac{20}{0.18} \cdot 1.11\ mA} \cdot \left[\frac{1}{(0.1V^{-1})(1.11mA)}\right]^2$$

$$\approx 403\ k\Omega$$

$$g_{m3}r_{o3}r_{o4} \approx 71.\ k\Omega$$

We know that $\frac{|A_v|}{(g_{m2}r_{o1}r_{o2} \parallel g_{m3}r_{o3}r_{o4})} = g_{m1} = \frac{2I_D}{V_{GS1} - V_{TH}}$

$$\therefore V_{in} = V_{GS1} = V_{TH} + 2I_D \cdot \frac{g_{m2}r_{o1}r_{o2} \parallel g_{m3}r_{o3}r_{o4}}{A_v}$$

$$= (0.4\,V) + 2(1.11\,mA)\,\frac{(403\,k\Omega \parallel 71.\,k\Omega)}{200}$$

$$\approx 1.07\ V$$

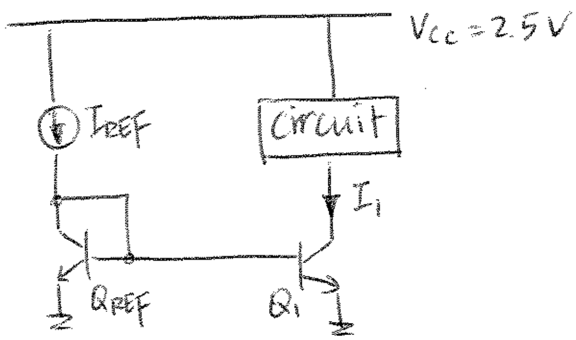
$$g_{m4} = \frac{2I_D}{V_{DD} - V_{D3} - |V_{THp}|} = \sqrt{2\mu_p C_{ox} \frac{W}{L} I_D}$$

$$\therefore\ V_{D3} = V_{DD} - |V_{THp}| - \frac{2I_D}{\sqrt{2\mu_p C_{ox} \frac{W}{L} I_D}}$$

$$= (1.8\,V) - (0.5\,V) - \frac{2(1.22\,mA)}{\sqrt{2 \cdot \left(50\,\frac{\mu A}{V^2}\right)\left(\frac{20}{0.18}\right)(1.22\,mA)}}$$

$$\approx 0.67\ V$$

60.



$I_1 = 0.5mA$

power $= 2mW$

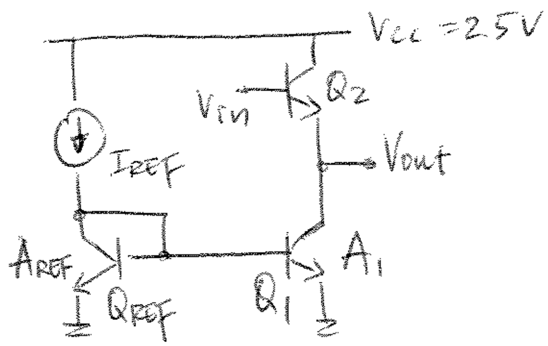
$$\text{Power} = V_{CC}(I_{REF} + I_1)$$

$$\Rightarrow I_{REF} = \frac{\text{Power}}{V_{CC}} - I_1 = \frac{2mW}{2.5V} - 0.5mA = 0.3mA$$

Therefore, if Q_{REF} has area A_E, then Q_1 has area $\frac{5}{3}A_E$ for the currents specified.

i.e. $\frac{A_{REF}}{A_1} = \frac{3}{5}$

61.



power = 3mW
$R_{out} = 50\Omega$

For an emitter follower, $R_{out} = r_{\pi 2} \,||\, \frac{1}{g_{m2}}$

$$\Rightarrow R_{out} = 50\Omega = \frac{1}{\frac{I_{C2}}{V_T}\left(1+\frac{1}{\beta}\right)}$$

$$\therefore I_{C2} = \frac{V_T}{R_{out}} \cdot \frac{1}{1+1/\beta} = \frac{0.026}{50} \frac{1}{1+0.01} \approx 0.51 \text{ mA}$$

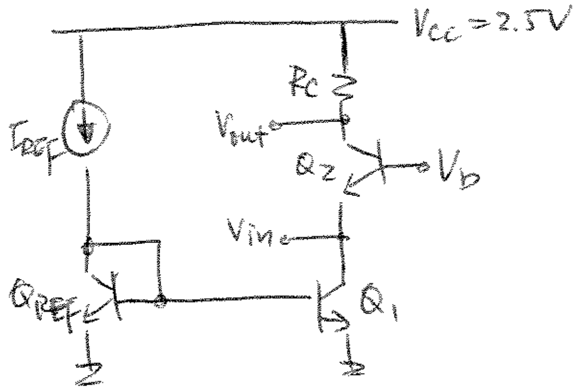
Realize that Vcc is providing current through I_{REF} & I_{C2}, and we are given

$$\text{power} = V_{CC}(I_{REF} + I_{C2}) = 3mW$$

$$\Rightarrow I_{REF} = \frac{\text{power}}{V_{CC}} - I_{C2} = \frac{3mW}{2.5V} - 0.51mA \approx 0.69mA$$

$$\Rightarrow \frac{I_{C2}}{I_{REF}} = \frac{A_1}{A_{REF}} = \frac{0.51}{0.69} \approx \frac{5}{7}$$

b2.



$R_{out} = 50\Omega$
$A_V = 20$
power $= 1.5\,mW$
$\beta \gg 1$, $V_A \to \infty$

$R_{out} = R_C \Rightarrow R_C = 50\Omega$

$A_V = g_m R_C = 20 \Rightarrow g_m = \dfrac{A_V}{R_C} = \dfrac{I_{C2}}{V_T}$

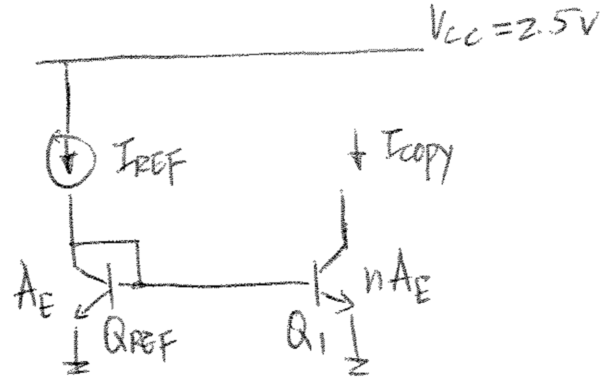
$\Rightarrow I_{C2} = \dfrac{A_V V_T}{R_C} = \dfrac{20\,(0.026V)}{50\Omega} \approx 10.4\,mA$

Realize that Vcc is providing current through I_{REF} & I_{C2}:

power $= V_{CC}(I_{REF} + I_{C2})$

$\Rightarrow I_{REF} = \dfrac{\text{power}}{V_{CC}} - I_{C2} = \dfrac{1.5mW}{2.5V} - 10.4mA$

63.



Given $I_{copy} = 0.5mA$

By KCL, $I_{REF} = I_{C,REF} + \dfrac{I_{C,REF}}{\beta} + \dfrac{I_{copy}}{\beta}$

$$= \frac{I_{copy}}{n} + \frac{I_{copy}/n}{\beta} + \frac{I_{copy}}{\beta}$$

$$\Rightarrow I_{copy} = I_{REF} \cdot \frac{n}{1 + \frac{1}{\beta}(n+1)} = 0.5\ mA$$

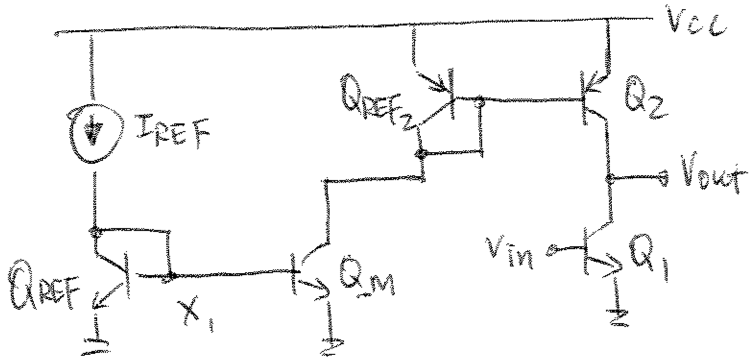
Within 1% implies that:

$$\Rightarrow I_{REF} \geq \frac{0.5mA}{0.99} \approx 0.505\ mA.$$

- For given n and β, $I_{copy} \leq n I_{REF}$. Since the error term causes $I_{copy} < n I_{REF}$ (strictly less than), one needs to increase I_{REF} in order to maintain the desired I_{copy}. This, however, means an increase of power (i.e. $\Delta p = V_{CC} \cdot \Delta I_{REF}$)

$\Rightarrow$ Trade off between accuracy & power dissipation.

64.



$$I_{c2} = I_{REF} \underbrace{\frac{(A_M/A_{REF})}{1+\frac{1}{\beta_n}\left(A_M/A_{REF}+1\right)}}_{X} \cdot \frac{(A_2/A_{REF_2})}{1+\frac{1}{\beta_p}\left(A_2/A_{REF_2}+1\right)}$$

Given $I_{C,M} \geq 0.98\, I_{REF}$ (less than 2% error)

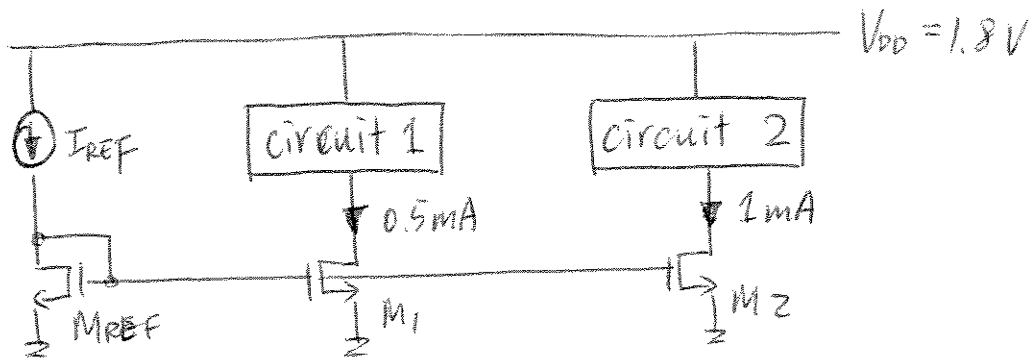
$$I_{c2} = 1\,mA = 0.98\, I_{REF} \cdot \frac{A_2/A_{REF_2}}{1+\frac{1}{50}\left(A_2/A_{REF_2}+1\right)}$$

Suppose $X = 0.98$ & $I_{REF} = 2mA$.

$$\Rightarrow \frac{A_2}{A_{REF_2}} \approx 0.5$$

Solution is not unique because no power constraint is present (i.e. I_{REF} is arbitrary.)

65.



power budget = 3 mW.

$$\text{power} = V_{DD}(I_{REF} + 0.5mA + 1mA)$$

$$\Rightarrow I_{REF} = \frac{\text{power}}{V_{DD}} - 0.5mA - 1mA \approx 0.17mA$$

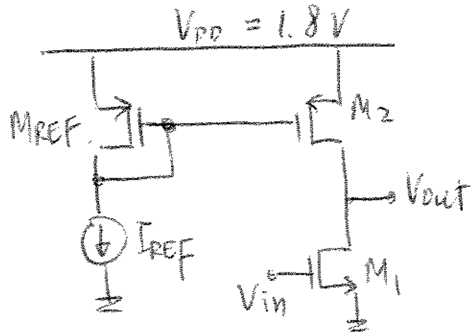
Assuming M_1 & M_2 operate in saturation,

If M_{REF} has $\left(\frac{W}{L}\right)_{REF}$, then

$$\frac{(W/L)_1}{(W/L)_{REF}} = \frac{I_1}{I_{REF}} = \frac{50}{17}$$

$$\frac{(W/L)_2}{(W/L)_{REF}} = \frac{I_2}{I_{REF}} = \frac{100}{17}$$

66.



$A_V = -20$

power = 2mW

$\left(\frac{W}{L}\right)_1 = \frac{20}{0.18}$ $\quad \lambda_n = 0.1\,V^{-1}$

$\lambda_p = 0.2\,V^{-1}$

$$R_{out} = r_{o2} \parallel r_{o1} = \frac{1}{\lambda_n I_{D_1} + \lambda_p I_{D_1}}$$

$$\Rightarrow A_V = -G_m R_{out} = -\frac{g_{m_1}}{\lambda_n I_{D_1} + \lambda_p I_{D_1}} = -\frac{2I_{D_1}/(V_{GS_1} - V_{TH})}{I_{D_1}(\lambda_n + \lambda_p)}$$

$$\Rightarrow -20 = -\frac{2}{(V_{GS_1} - V_{TH})(\lambda_n + \lambda_p)}$$

$$\Rightarrow V_{GS_1} = \frac{1}{10(\lambda_n + \lambda_p)} + V_{TH_n}$$

$$= \frac{1}{10(0.1 + 0.2)V^{-1}} + 0.4V \approx 0.73\,V$$

$$\Rightarrow I_{D_1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS_1} - V_{THn})^2$$

$$= \frac{1}{2}\left(100\,\frac{uA}{V^2}\right)\left(\frac{20}{0.18}\right)(0.33\,V)^2 \approx 0.61\,mA$$

$$\because \text{power} = V_{DD}(I_{REF} + I_{D_1})$$

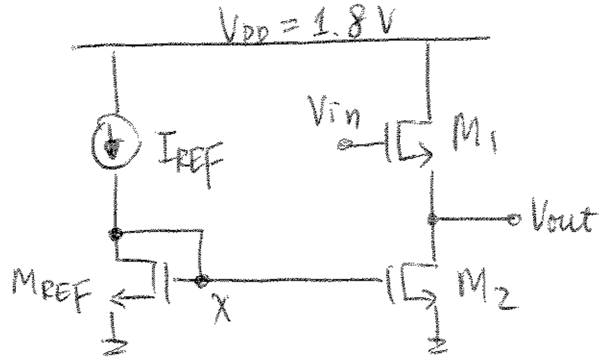
$$\Rightarrow I_{REF} = \frac{\text{power}}{V_{DD}} - I_{D_1} = \frac{2mW}{1.8V} - 0.61mA$$

$$\approx 0.5\,mA$$

$\therefore$ if M_{REF} has $\left(\frac{W}{L}\right)_{REF}$, then

$$\frac{(W/L)_2}{(W/L)_{REF}} = \frac{I_{D2}}{I_{REF}} = \frac{61}{50} \approx 1.2$$

67.



Given:

$A_v = 0.85$

$R_{out} = 100\,\Omega$

$(W/L)_2 = 10/0.18$

$\lambda_n = 0.1\,V^{-1}$, $\lambda_p = 0.2\,V^{-1}$

$$R_{out} = r_{o2} \parallel \left(\frac{1}{g_{m1}} \parallel r_{o1}\right) = \frac{1}{g_{m1} + 1/r_{o2} + 1/r_{o1}} = 100$$

For source follower,

$$A_v = \frac{g_{m1}}{g_{m1} + 1/r_{o2} + 1/r_{o1}} = 0.85$$

$$\Rightarrow g_{m1} = \frac{0.85}{100} = 8.5 \cdot 10^{-3}\,S$$

$$R_{out} = \frac{1}{g_{m1} + 2/r_o} = 100$$

$$\Rightarrow r_o = \frac{200}{1 - 100 g_{m1}} = \frac{200}{1 - 100(8.5 \cdot 10^{-3})}$$

$$\approx 1333\,\Omega$$

$$\Rightarrow I_{D1} = \frac{1}{\lambda_n r_{o1}} = 7.5\,mA.$$

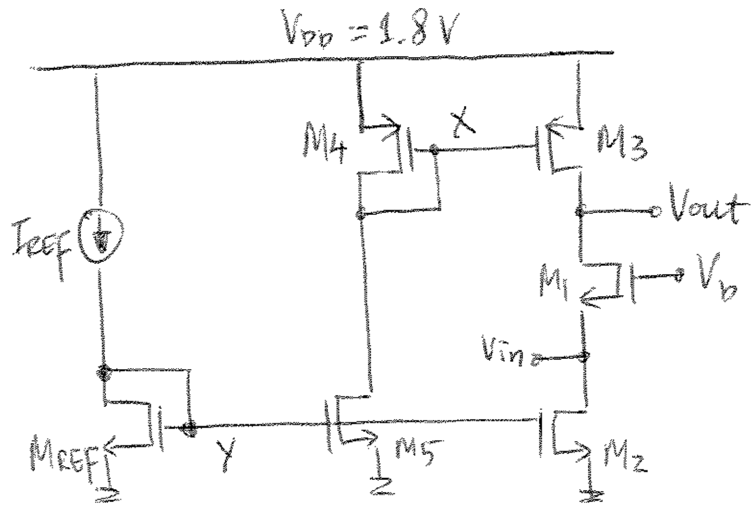
Assume $V_X \approx 1\,V$

$$\left(\frac{W}{L}\right)_2 = \frac{2 I_{D1}}{\mu_n C_{ox} (V_X - V_{TH})^2} \approx 416$$

Set $I_{REF} \approx 0.75$ mA.

$$\Rightarrow \left(\frac{W}{L}\right)_{REF} = \left(\frac{W}{L}\right)_2 \frac{I_{REF}}{I_{D2}} \approx 42.$$

68.



$$\left(\frac{W}{L}\right)_3 = 20/0.18$$
$$\lambda_n = 0.1\ V^{-1}$$
$$\lambda_p = 0.2\ V^{-1}$$

$$A_v = 20$$
$$R_{in} = 50\,\Omega$$

$$R_{in} = 50\,\Omega = r_{o2} \parallel 1/g_{m1} = \frac{1}{\lambda_n I_{D1} + g_{m1}} \quad \text{---- } ①$$

$$R_{out} = r_{o3}$$

$$A_v = g_{m1} r_{o3} = \frac{g_{m1}}{\lambda_p I_{D1}} \quad \text{---- } ②$$

Solve for g_{m1} in ② and substitute it into ①:

$$50 = \frac{1}{\lambda_n I_{D1} + A_v \lambda_p I_{D1}}$$

$$\Rightarrow I_{D1} = \frac{1}{(\lambda_n + A_v \lambda_p)(50\,\Omega)} = \frac{1}{(0.1 + 20(0.2))(50\,\Omega)} \approx 4.88\ mA$$

$$|V_{GS3}| = \sqrt{\frac{2 I_{D1}}{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}} + |V_{THp}| \approx 1.44\ V$$

$$g_{m1} = A_v \lambda_p I_{D1} \Rightarrow \left(\frac{W}{L}\right)_1 = \left[\frac{A_v \lambda_p I_{D1}}{\sqrt{2 \mu_n C_{ox} I_{D1}}}\right]^2$$

$$\approx 390.$$

Since $V_X \approx 0.4\ V$, size up other transistors to allow them to operate in saturation.

Suppose $I_{D4} = 1.2\ mA$ $\Rightarrow \left(\frac{W}{L}\right)_4 = \frac{2 I_{D4}}{\mu_p C_{ox} (|V_{GS3}| - |V_{THp}|)^2}$

$$\approx 10/0.18$$

$$I_{D5} = I_{D4} \Rightarrow \left(\frac{W}{L}\right)_5 = \frac{2 I_{D5}}{\mu_n C_{ox} (V_y - V_{THn})^2} \approx \frac{100}{0.18}$$

(Assume $V_y = 0.6$; this is arbitrary, but must ensure M_5 in saturation.)

$$\text{Set } I_{REF} = I_{D5} \Rightarrow \left(\frac{W}{L}\right)_{REF} \approx \frac{100}{0.18}$$

$$I_{D2} \approx I_{D3} \Rightarrow \left(\frac{W}{L}\right)_2 = \frac{2 I_{D2}}{\mu_n C_{ox} (V_y - V_{THn})^2} \approx \frac{45}{0.18}$$

$$\text{Total power} = V_{DD}(I_{REF} + I_{D4} + I_{D3})$$
$$= 1.8(7.3)\ mW \approx 13\ mW$$