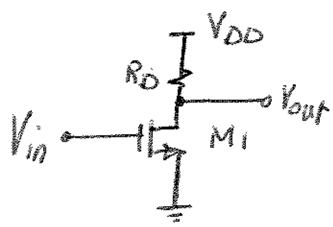
1.



M_1 operates in the triode region

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH})V_{out,min} - V_{out,min}^2\right]$$

$$V_{out} = V_{DD} - R_D I_D$$

$R_D = 10K$

$\left(W/L\right)_1 = 3/0.18$

$V_{out,min} = ?$ when $V_{in} = V_{DD}$

$$V_{out,min} = V_{DD} - R_D I_{D,max}$$

$$= V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH})V_{out,min} - V_{out,min}^2\right] \times R_D$$

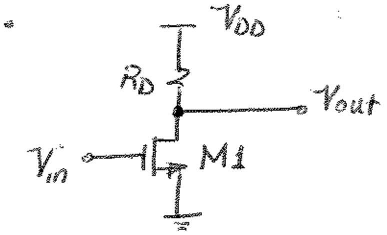
If the second term in the square brackets is neglected, then

$$V_{out,min} \simeq \frac{V_{DD}}{1 + \mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH}) \times R_D}$$

$$= \frac{1.8}{1 + 100 \times 10^{-6} \times 3/0.18 \times (1.8 - 0.4) \times 10^5}$$

$$\boxed{V_{out,min} \simeq 74\ mV}$$

2.



$V_{out,min} \leq 100\,mV$

$R_D = 5K\Omega$

$(W/L)_{1,min} = ?$

Output low level establishes for $V_{in} = V_{DD}$, driving M_1 into the triode region.

$$I_{D\,max} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right]$$

$$V_{out,min} = V_{DD} - R_D \times I_{D,max}$$

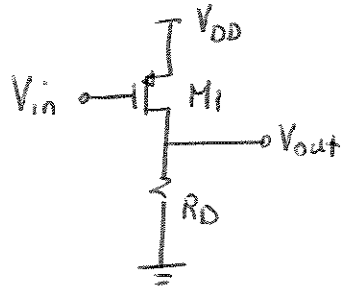
$$= V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right] \times R_D$$

$$\left(\frac{W}{L}\right)_1 = \frac{V_{DD} - V_{out,min}}{\frac{1}{2}\mu_n C_{ox}\left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right] \times R_D}$$

$$\left(\frac{W}{L}\right)_{1,min} = \frac{1.8 - 100\times10^{-3}}{\frac{1}{2}\times100\times10^{-6}\left[2(1.8-0.4)100\times10^{-3} - (100\times10^{-3})^2\right]\times5\times10^3}$$

$$\boxed{\left(\frac{W}{L}\right)_{1,min} = 25}$$

3.



$\left(\frac{W}{L}\right)_1 = 20/0.18$, $R_D = 5K$

$V_{O_L}, V_{O_H} = ?$

(1) $V_{in} = V_{DD} \longrightarrow M_1 \text{ off} \longrightarrow I_D = 0 \longrightarrow \boxed{V_{out} = V_{O_L} = 0}$

(2) $V_{in} = 0 \longrightarrow M_1$ operates in the triode region

$$I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{SG} - |V_{THP}|)V_{SD} - V_{SD}^2\right]$$

$$I_{D,max} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - |V_{THP}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right] \quad (1)$$

$$I_{D,max} = \frac{V_{out}}{R_D} \quad (2)$$

Equating (1) and (2) and neglecting the second order term in the brackets

$$\frac{V_{out}}{R_D} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1 \times 2(V_{DD} - |V_{THP}|)(-V_{out} + V_{DD})$$

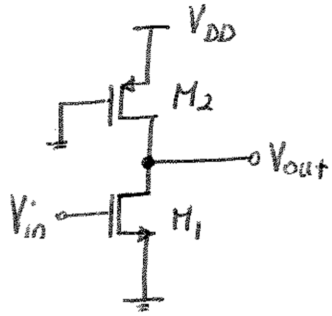
$$V_{out}\left[\frac{1}{R_D} + \mu_p C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - |V_{THP}|)\right] = \mu_p C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - |V_{THP}|) \times V_{DD}$$

$$V_{out} = \frac{R_D}{R_D + \dfrac{1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - |V_{THP}|)}} V_{DD}$$

$$V_{out} = \frac{5000}{5000 + \frac{1}{50\times10^{-6} \times \left(\frac{20}{0.18}\right) \times (1.8 - 0.5)}} \times 1.8$$

$$\boxed{V_{out} = V_{OH} = 1.75\ V}$$

4.



$\left(\frac{W}{L}\right)_1 = 3/0.18 \quad \left(W/L\right)_2 = 2/0.18$

(a) if $V_{in} = V_{DD}$, M_2 Saturated $\rightarrow V_{OL} = ?$

(b) if $V_{in} = V_{out} \longrightarrow V_{in} = ?$

(a) $I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{SG} - |V_{THP}|\right)^2$

$I_{D2} = \frac{1}{2} \times 50 \times 10^{-6} \times \left(\frac{2}{0.18}\right)\left(1.8 - 0.5\right)^2$, Note that $V_{SG} = V_{DD}$

$I_{D2} = 4.7 \times 10^{-4}\ A$

$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2\left(V_{GS} - V_{THN}\right)V_{DS} - V_{DS}^2\right]$

$= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2\left(V_{DD} - V_{THN}\right)V_{OL} - V_{OL}^2\right]$

However $I_{D1} = I_{D2}$

$4.7 \times 10^{-4} = \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{3}{0.18}\right)\left[2\left(1.8 - 0.4\right)V_{OL} - V_{OL}^2\right]$

Neglecting the second-order term yields:

$\boxed{V_{OL} = 0.2\ V}$

As $\left(V_{in} - V_{THN}\right) = \left(V_{DD} - V_{THN}\right) = \left(1.8 - 0.4\right) = 1.4 > V_{DS_1} = V_{OL} = 0.2\ V$

the assumption of M_1 being in Triode region is correct

We define, $V_X = V_{in} - V_{TH,N} \rightarrow V_{in} = V_X + V_{TH,N}$

$$\frac{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1}{\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2} V_X^2 = 2(V_{DD} - |V_{TH,P}|)(V_{DD} - V_{TH,N} - V_X) - (V_{DD} - V_{TH,N} - V_X)^2$$

$$\frac{100}{50} \times \frac{3/0.18}{2/0.18} V_X^2 = 2(1.8-0.5)(1.8-0.4-V_X) - (1.8-0.4-V_X)^2$$

$$3V_X^2 = 2.6(1.4 - V_X) - (1.4 - V_X)^2$$

$$3V_X^2 = 3.64 - 2.6V_X - 1.96 + 2.8V_X - V_X^2$$

$$4V_X^2 - 0.2V_X - 1.68 = 0$$

$$V_X = \frac{0.2 \pm \sqrt{0.2^2 + 4\times 4\times 1.68}}{8} \longrightarrow \boxed{V_X = 0.67\ V}$$

$$V_{in} = V_X + V_{TH,N} = 0.67 + 0.4 \rightarrow \boxed{V_{in} = V_{out} = 1\ V}$$

This value of V_{out} guarantees that M_2 operates in the triode region.

Now, let's investigate the region of operation of M_2

$$V_{SD2} = V_{DD} - V_{out}$$

$$= 1.8 - 0.2$$

$$V_{SD2} = 1.6\,V$$

$$V_{SG2} - |V_{THP}| = V_{DD} - |V_{THP}|$$

$$= 1.8 - 0.5$$

$$V_{SG2} - |V_{THP}| = 1.3$$

As $V_{SD2} > V_{SG2} - |V_{THP}|$, M_2 operates in the saturation region and the initial assumption is valid.

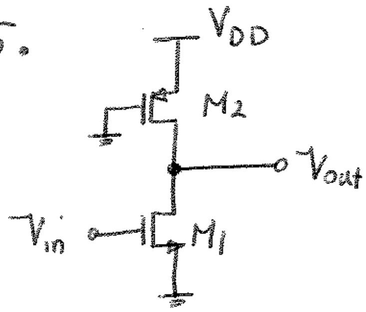
(b) As $V_{in} = V_{out} \longrightarrow M_1$ is saturated.

We assume that M_2 is in the triode region and check the validity of this assumption

$$I_{D1} = I_{D2}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{THN})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \Big[2(V_{DD} - |V_{THP}|) \times (V_{DD} - V_{in}) - (V_{DD} - V_{in})^2\Big]$$

5.



$V_{OL} \leqslant 100\ mV$

$(W/L)_2 = 3/0.18$

$\left(\frac{W}{L}\right)_{1,min} = ?$

$V_{in} = V_{DD} \rightarrow$ M_1 operates in The triode region and M_2 in the saturation.

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{SG} - |V_{TH,p}|\right)^2$$

$$= \frac{1}{2} \times 50 \times 10^{-6} \times \left(\frac{3}{0.18}\right) \times (1.8 - 0.5)^2$$

$$I_{D2} = 7.041 \times 10^{-4} A$$

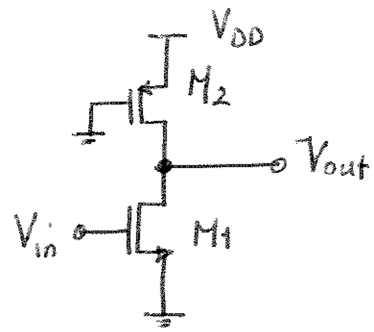
$$I_{D1} = I_{D2} = 7.041 \times 10^{-4} A$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{GS} - V_{TH,N})V_{DS} - V_{DS}^2\right]$$

$$7.041 \times 10^{-4} = \frac{1}{2} \times 100 \times 10^{-6} \left(\frac{W}{L}\right)_1 \left[2(1.8 - 0.4)\,0.1 - (0.1)^2\right]$$

$$\boxed{\left(\frac{W}{L}\right)_{1,min} = 52.16}$$

6.



$V_{OL} \leqslant 80\ mV$

$\left(\frac{W}{L}\right)_1 = 2/0.18$

$\left(\frac{W}{L}\right)_{2,max}$

$V_{in} = V_{DD} \longrightarrow M_1$ operates in the triode region and M_2 in the saturation

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{GS} - V_{TH,N})V_{DS} - V_{DS}^2\right]$$

$$I_{D1} = \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{2}{0.18}\right) \times \left[2(1.8 - 0.4)\,0.08 - 0.08^2\right]$$

$$I_{D1} = 1.2 \times 10^{-4}\,A$$

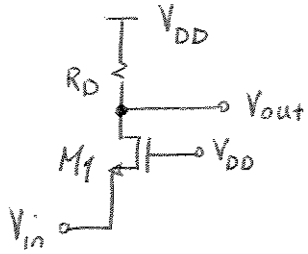
$$I_{D2} = I_{D1} = 1.2 \times 10^{-4}\,A$$

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{SG} - |V_{TH,P}|)^2$$

$$1.2 \times 10^{-4} = \frac{1}{2} \times 50 \times 10^{-6}\left(\frac{W}{L}\right)_2 (1.8 - 0.5)^2$$

$$\boxed{\left(\frac{W}{L}\right)_{2,max} = 2.86}$$

7.



(a) If $V_{in} = 0$, V_{DD}, $V_{out} = ?$

If $V_{in} = 0 \longrightarrow M_1$ operates in the triode region.

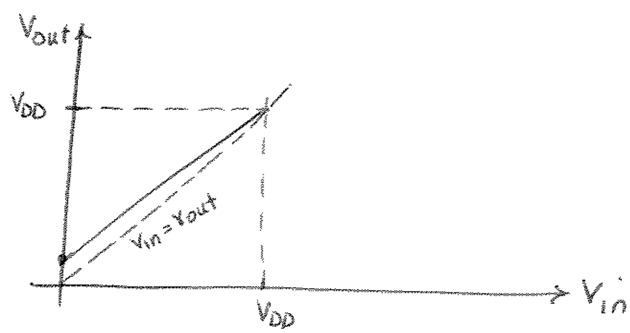
$$R_{on1} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH,N})}$$

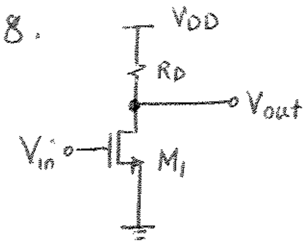
$$V_{out} \cong \frac{R_{on1}}{R_{on1} + R_D} \times V_{DD} \longrightarrow V_{out} \cong \frac{1}{1 + \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH,N}) R_D} \times V_{DD}$$

If $V_{in} = V_{DD} \longrightarrow V_{out} = V_{DD}$

No, this circuit does not invert.

(b) A trip point cannot be found for this circuit because $V_{out} = V_{in}$ line does not intersect the transfer characteristic of this buffer.



8. 

$$\left(\frac{W}{L}\right)_1 = 5/0.18$$

$$R_D = 2K\Omega$$

$$NM_L\ ,\ NM_H = ?$$

Small signal gain of the circuit is equal to $-g_m R_D$

and $g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N})$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N}) R_D = 1 \quad , \quad V_{GS} = V_{IL}$$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{IL} - V_{TH,N}) R_D = 1$$

$$V_{IL} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + V_{TH} = \frac{1}{100 \times 10^{-6} \times \frac{5}{0.18} \times 2000} + 0.4$$

$$\boxed{V_{IL} = 0.58\ V}$$

To determine NM_H, we note that V_{in} drives M_1 into the triode region

$$V_{out} = V_{DD} - R_D I_D$$

$$= V_{DD} - \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2 (V_{in} - V_{TH,N}) V_{out} - V_{out}^2 \right] R_D \quad (1)$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2 (V_{in} - V_{TH,N}) \frac{\partial V_{out}}{\partial V_{in}} - 2 V_{out} \frac{\partial V_{out}}{\partial V_{in}} + 2 V_{out} \right] R_D$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -1 \ @ \ V_{IH}$$

$$-1 = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[-2 (V_{in} - V_{TH,N}) + 2 V_{out} \right] R_D$$

$$1 = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[-V_{in} + V_{TH,N} + 2V_{out}\right] R_D$$

$$\frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 R_D} = -\left(V_{in} - V_{TH,N}\right) + 2V_{out}$$

$$V_{out} = \frac{1}{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 R_D} + \frac{V_{in} - V_{TH,N}}{2} \longrightarrow V_{out} = 0.5V_{in} - 0.11$$

Substituting This in (1) yields:

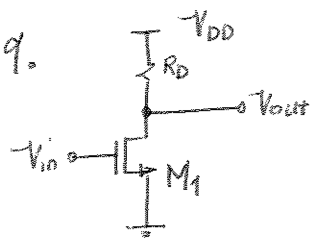
$$0.5V_{in} - 0.11 = 1.8 - \frac{1}{2} \times 100 \times 10^{-6} \times \frac{5}{0.18} \times 2000 \left[2(V_{in} - 0.4)(0.5V_{in} - 0.11) - (0.5V_{in} - 0.11)^2\right]$$

$$0.75V_{in}^2 - 0.33V_{in} - 0.6117 = \emptyset$$

$$V_{in} = V_{IH} = 1.15$$

$$NM_H = V_{DD} - V_{IH} = 1.8 - 1.15 \longrightarrow \boxed{NM_H = 0.65\,V}$$

9.



Small signal gain of the inverter is equal to $-g_m R_D$

and $g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N})$

$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N}) R_D = 1$, $V_{GS} = V_{IL}$

$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{IL} - V_{TH,N}) R_D = 1 \longrightarrow V_{IL} = \dfrac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + V_{TH,N}$

If we double the value of $\left(\frac{W}{L}\right)_1$ or R_D

$$V_{IL} = \frac{1}{100 \times 10^{-6} \times \frac{5}{0.18} \times 2000 \times 2} + 0.4 \longrightarrow \boxed{V_{IL} = 0.49}$$

To determin NM_H, we note that V_{in} drives M_1 into the triode region

$V_{out} = V_{DD} - R_D I_D$

$$= V_{DD} - \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH}) V_{out} - V_{out}^2\right] R_D. \quad (1)$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2 V_{out} + 2(V_{in} - V_{TH}) \frac{\partial V_{out}}{\partial V_{in}} - 2 V_{out} \frac{\partial V_{out}}{\partial V_{in}}\right]$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -1 \ @ \ V_{IH}$$

$$V_{out} = \frac{1}{2 \mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + \frac{V_{in} - V_{TH,N}}{2}$$

Doubling $\left(\frac{W}{L}\right)_1$ or R_D leads to $\boxed{V_{out} = 0.5V_{in} - 0.155}$

Substituting in (1) yields:

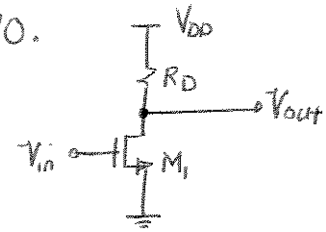
$$0.5V_{in} - 0.155 = 1.8 - \frac{1}{2} \times 100 \times 10^{-6} \times \frac{5}{0.18} \times 2000 \times 2\left[2(V_{in} - 0.4)(0.5V_{in} - 0.155) - (0.5V_{in} - 0.155)^2\right]$$

$$0.75V_{in}^2 - 0.465V_{in} - 0.251925 = 0$$

$V_{in} = 0.967\,V \longrightarrow NM_H = 1.8 - 0.967$

$$\boxed{NM_H = 0.833\,V}$$

10.



$$\left(\frac{W}{L}\right)_1 = \frac{5}{0.18}$$

$R_D = 2K$

NM_L and $NM_H = ?$ if $\frac{\partial V_{out}}{\partial V_{in}} = -0.5$ instead of -1

Small signal gain of the inverter is equal to "$-g_m R_D$"

and $g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N})$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{IL} - V_{TH,N}) R_D = 0.5$$

$$V_{IL} = \frac{1}{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + V_{TH,N} = \frac{1}{2 \times 100 \times 10^{-6} \times \frac{5}{0.18} \times 2000} + 0.4$$

$\boxed{V_{IL} = 0.49}$ which is less than 0.58 obtained in problem 8.

To determine NM_H, note that M_1 operates in the triode region

$$V_{out} = V_{DD} - R_D I_D$$

$$= V_{DD} - \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH}) V_{out} - V_{out}^2\right] R_D \quad (1)$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2V_{out} + 2(V_{in} - V_{TH}) \frac{\partial V_{out}}{\partial V_{in}} - 2V_{out} \frac{\partial V_{out}}{\partial V_{in}}\right] R_D$$

$\frac{\partial V_{out}}{\partial V_{in}} = -0.5$ @ V_{IH}

$$-0.5 = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[-(V_{in} - V_{TH,N}) + 3V_{out}\right] R_D$$

$$V_{out} = \frac{1}{3\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + \frac{V_{in} - V_{THN}}{3} \longrightarrow V_{out} = -73.33 \times 10^{-3} + 0.33 V_{in}^{o}$$

or $V_{out} = -\frac{0.22}{3} + \frac{V_{in}}{3}$

Substituting in (1) yields:

$$-\frac{0.22}{3} + \frac{V_{in}}{3} = 1.8 - \frac{1}{2} \times 100 \times 10^{-6} \times \frac{5}{0.18} \times 2000 \left[2\left(V_{in} - 0.4\right)\left(-\frac{0.22}{3} + \frac{V_{in}}{3}\right) - \left(-\frac{0.22}{3} + \frac{V_{in}}{3}\right)^2 \right]$$

$$5V_{in}^2 - 2.2V_{in} - 5.59 = 0$$

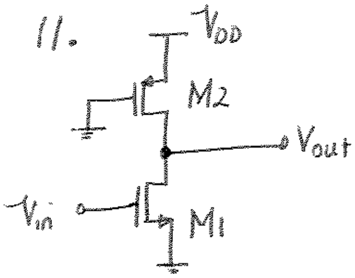
$$V_{in} = V_{IH} = 1.3$$

$$NM_H = 1.8 - 1.3$$

$$\boxed{NM_H = 0.5\,V}$$

less than 0.65 V obtained in problem 8 because V_{IH} is now further pushed up toward V_{DD}.

11.



$$\left(\frac{W}{L}\right)_1 = \frac{4}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{9}{0.18}$$

To calculate V_{IL}, we assume that M_1 and M_2 operate in saturation and triode region respectively.

$$I_{D1} = I_{D2} \tag{1}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH,N})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH,P}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right]$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{THN}) = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH,P}|)\left(-\frac{\partial V_{out}}{\partial V_{in}}\right) - 2(V_{DD} - V_{out})\left(-\frac{\partial V_{out}}{\partial V_{in}}\right)\right]$$

By substituting $\frac{\partial V_{out}}{\partial V_{in}}$ with "-1" in the above relationship:

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH,N}) = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH,P}|) - 2(V_{DD} - V_{out})\right]$$

$$V_{out} = \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2}(V_{in} - V_{THN}) + |V_{THP}| = \frac{100\times10^{-6}\times 4/0.18}{50\times10^{-6}\times 9/0.18}(V_{in} - 0.4) + 0.5$$

$V_{out} = 0.144 + 0.88\, V_{in}^{o}$ or $\boxed{V_{out} = \frac{8}{9}V_{in}^{o} + \frac{1.3}{9}}$

Substituting V_{out} in (1) by the derivation versus V_{in} gives:

$$136\, V_{in}^{2} - 108.8\, V_{in}^{o} - 115.13 = 0$$

$$\boxed{V_{in}^{o} = V_{IL} = NM_L = 1.4\,V}$$

To calculate V_{IH}, we assume that M_1 and M_2 operate in the triode and saturation region respectively.

$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{THN})V_{out}-V_{out}^2\right]=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD}-|V_{THP}|\right)^2 \quad (2)$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out}+2(V_{in}-V_{THN})\frac{\partial V_{out}}{\partial V_{in}}-2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right]=\emptyset$$

$$\frac{\partial V_{out}}{\partial V_{in}}=-1$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out}-2(V_{in}-V_{THN})+2V_{out}\right]=\emptyset$$

$V_{out}=\dfrac{V_{in}-V_{THN}}{2}$ supstituted in (2) yields:

$$V_{in}=\sqrt{\frac{3}{2}}\left(V_{DD}-|V_{THP}|\right)+V_{THN}$$

$V_{in}=2 \longrightarrow V_{out}=0.8$ This value of V_{out} puts M_2 into the triode region so our initial assumption is not correct

Now we assume that both M_1 and M_2 operate in the triode region.

$I_{D1}=I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{THN})V_{out}-V_{out}^2\right]=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2\left(V_{DD}-|V_{THP}|\right)\left(V_{DD}-V_{out}\right)-\left(V_{DD}-V_{out}\right)^2\right] \quad (3)$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out}+2(V_{in}-V_{THN})\frac{\partial V_{out}}{\partial V_{in}}-2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right]=\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \times$$

$$\left[2(V_{DD}-|V_{THP}|)\left(-\frac{\partial V_{out}}{\partial V_{in}}\right)-2(V_{DD}-V_{out})\left(-\frac{\partial V_{out}}{\partial V_{in}}\right)\right]$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out}-2(V_{in}-V_{THN})+2V_{out}\right]=\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \times$$

$$\left[2(V_{DD}-|V_{THP}|)-2(V_{DD}-V_{out})\right]$$

$$V_{out}=\frac{\frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2}(V_{in}-V_{THN})-|V_{THP}|}{2\frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2}-1}$$

$$V_{out}=\frac{8}{7}V_{in}-1.1$$

After substituting in (3) it leads to :

$$2.1769\,V_{in}^2-4.19\,V_{in}^{\circ}+0.576=\phi$$

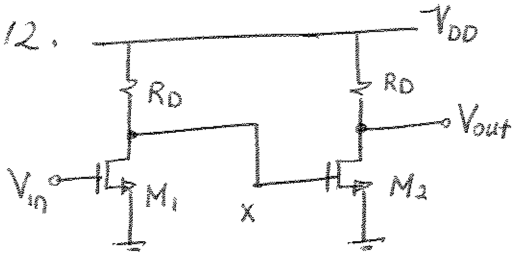
$$\boxed{V_{in}=1.77\,V}$$

$V_{out}=0.93\,V \longrightarrow$ The assumption is correct

$V_{IH}=1.77\,V \longrightarrow NM_H=1.8-1.77$

$$\boxed{NM_H=0.03\,V}$$

12.



The small signal gain of the circuit is equal to $-g_m R_D$ and since

$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{THN})$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{IL} - V_{THN}) R_D = 1$$

$$V_{IL} = \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 R_D} + V_{THN} = \frac{2}{S} + 0.4 \quad ; \left(\frac{W}{L}\right)_{1,2} = S$$

Now we calculate the output of M_1 for $V_{in} = V_{DD}$:

$$V_{DD} - R_D I_D = V_{out}$$

$$V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{THN}) V_{out} - V_{out}^2\right] R_D = V_{out} \quad ; \left(\frac{W}{L}\right)_{1,2} = S$$

$$1.8 - \frac{1}{2} \times 100 \times 10^{-6} \times S \left[2(1.8 - 0.4)\left(\frac{2}{S} + 0.4\right) - \left(\frac{2}{S} + 0.4\right)^2\right] \times 5000 = \left(\frac{2}{S} + 0.4\right)$$

$$1.8 - 0.25 \times \left[2.8\,(2 + 0.4S) - S\left(\frac{2}{S} + 0.4\right)^2\right] = \frac{2}{S} + 0.4$$

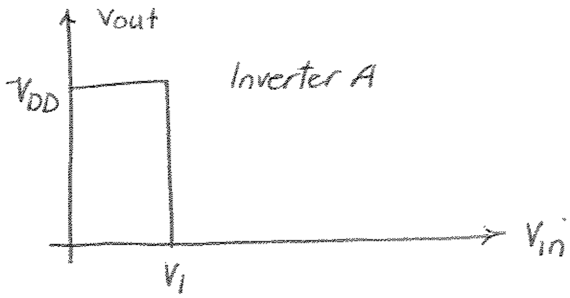
$$1.8S - 0.25 \times \left[2.8\,(2S + 0.4S^2) - S^2\left(\frac{2}{S} + 0.4\right)^2\right] = 2 + 0.4S$$

$$1.8S - 0.25 \times \left[5.6S + 1.12S^2 - 4 - 1.6S - 0.16S^2\right] = 2 + 0.4S$$

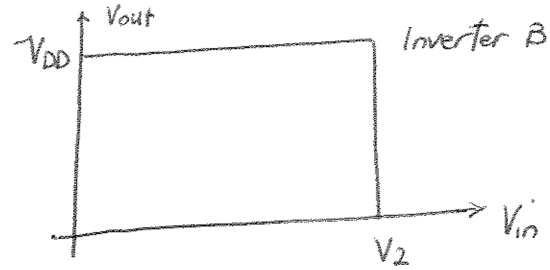
$$0.24S^2 - 0.4S + 1 = 0$$

$$\Delta < 0 \,!$$

13.



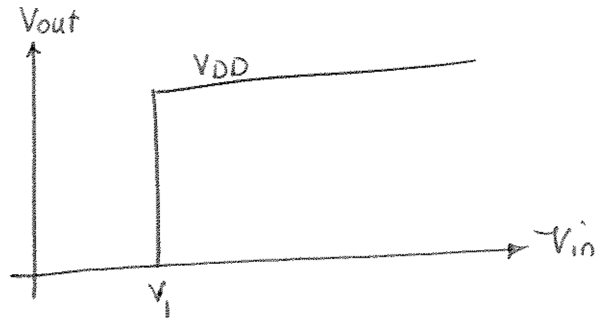
Vout
VDD
Inverter A
V1
Vin



Vout
VDD
Inverter B
V2
Vin

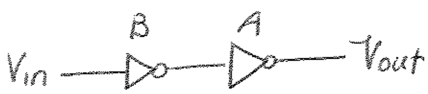
(a)

Vin
A
B
Vout

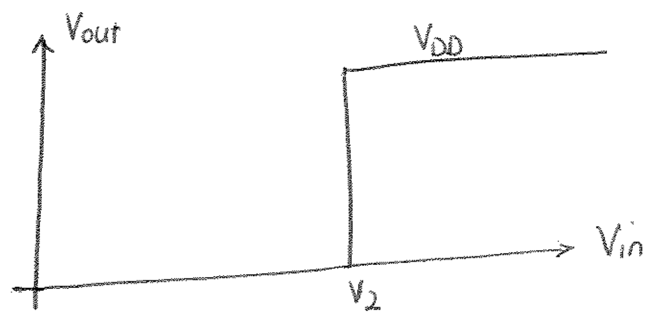


Vout
VDD
V1
Vin

(b)

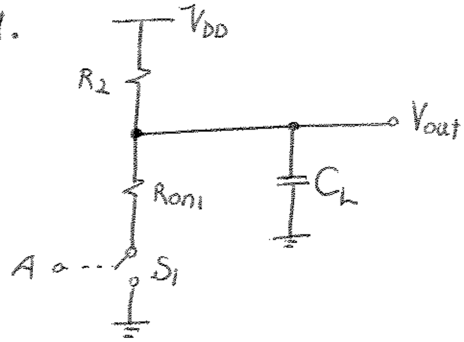


Vin
B
A
Vout



Vout
VDD
V2
Vin

14.



$R_{on1} << R_2 \longrightarrow V_{out,min} \approx 0$

(a) $V_{out}(t) = V_{out}(0^-) + [V_{DD} - V_{out}(0^-)]\left(1 - \exp\frac{-t}{R_2 C_L}\right) \quad t > 0$

Note that $V_{out}(0^-) = 0$, $V_{out}(\infty) = V_{DD}$

$V_{out}(t) = V_{DD} \times \left(1 - \exp\frac{-t}{R_2 C_L}\right) \quad t > 0$

$0.95\, V_{DD} = V_{DD} \times \left(1 - \exp\frac{-T_{95\%}}{R_2 C_L}\right)$

$$\boxed{T_{95\%} = 3 R_2 C_L}$$

(b) $V_{out}(t) = V_{out}(0^-) + [V_{out}(\infty) - V_{out}(0^-)] \times \left(1 - \exp\frac{-t}{R_2 C_L}\right)$

$V_{out}(t) = V_{DD} + [0 - V_{DD}] \times \left(1 - \exp\frac{-t}{R_2 C_L}\right)$

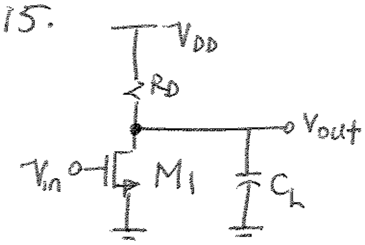
$V_{out}(t) = V_{DD} \exp\frac{-t}{R_2 C_L}$

$0.05\, V_{DD} = V_{DD} \exp\frac{-T_{0.05}}{R_2 C_L}$

$$\boxed{T_{5\%} = 3 R_2 C_L}$$

If $R_{on1} << R_2$, inverter exhibits equal rise and fall time (or low-to-high and high-to-low delay) at the output.

15.



$C_L = 50\,fF$

$T_R = 100\,pS$

$T_R = 3\,\tau_{out}$

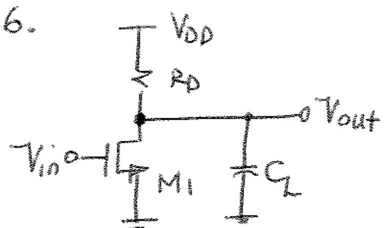
$R_{D,max} = ?$

$$T_R = 3 R_D C_L = 100\,pS$$

$$R_D \leqslant \frac{100\,pS}{3 \times 50\,fF}$$

$$\boxed{R_D \leqslant 666.67\,\Omega}$$

16.



$C_L = 100\,fF$

$V_{out,min} = 50\,mV$

$T_R = 200\,pS$

$R_D\,,\ \left(\frac{W}{L}\right)_1 = ?$

$T_R = 3\tau_{out}$

$T_R = 3R_D C_L$

$200\times10^{-12} = 3\times R_D \times 100\times10^{-15}$

$$\boxed{R_D = 666.667\ \Omega}$$

$V_{in} = V_{DD}$ places M_1 in the triode region

$$V_{out,min} = V_{DD} - R_D I_{D,max}$$

$$= V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 R_D\left[2(V_{DD}-V_{THN})V_{out,min} - V_{out,min}^2\right]$$

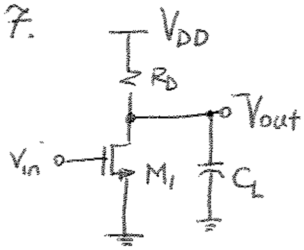
Neglecting the 2^{nd}-order term in the square brackets yields:

$$V_{out,min} = \frac{V_{DD}}{1+\mu_n C_{ox}\left(\frac{W}{L}\right)_1 R_D (V_{DD}-V_{TH})}$$

$$50\times10^{-3} = \frac{1.8}{1+100\times10^{-6}\times\left(\frac{W}{L}\right)_1\times 666.7\times(1.8-0.4)}$$

$$\boxed{\left(\frac{W}{L}\right)_1 = 375}$$

17.



$C_L = 100\,fF$ $\quad V_{out,min} \approx 0$

$I_{D_{max}} \leq 1mA$

$T_{R,min} = ?$

$$I_{D,max} = \frac{V_{DD} - V_{out,min}}{R_D}$$

$$10^{-3} = \frac{1.8 - 0}{R_D}$$

$$\boxed{R_D = 1.8\,K\Omega}$$

$$V_{out}(t) = V_{out}(0^-) + \left[V_{out}(\infty) - V_{out}(0^-)\right]\left(1 - \exp\frac{-t}{R_D C_L}\right) \quad t > 0$$

$$V_{out}(t) = V_{out,min} + \left[V_{DD} - V_{out,min}\right] \times \left(1 - \exp\frac{-t}{R_D C_L}\right) \quad t > 0$$

$$V_{out}(t) = V_{DD}\left(1 - \exp\frac{-t}{R_D C_L}\right) \quad t > 0$$

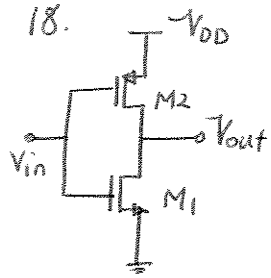
$$0.1 V_{DD} = V_{DD}\left(1 - \exp\frac{-T_{10\%}}{R_D C_L}\right) \longrightarrow T_{10\%} = 0.105\, R_D C_L$$

$$0.9 V_{DD} = V_{DD}\left(1 - \exp\frac{-T_{90\%}}{R_D C_L}\right) \longrightarrow T_{90\%} = 2.3\, R_D C_L$$

$$T_R = T_{90\%} - T_{10\%} = 2.197\, R_D C_L = 2.197 \times 1.8 \times 10^{3} \times 100 \times 10^{-15}$$

$$\boxed{T_R = 395.5\, pS}$$

18.



$$\left(\frac{W}{L}\right)_1 = \frac{2}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{3}{0.18}$$

$I_{D1} = I_{D2}$

At the trip point $V_{in} = V_{out}$; therefore, both M_1 and M_2 operate in the Saturation region.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(V_{in}^{o} - V_{THN}\right)^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in}^{o} - |V_{THP}|\right)^2$$

$$V_{in}^{o} = \frac{V_{DD} - |V_{THP}| + \sqrt{\frac{\mu_n C_{ox}(W/L)_1}{\mu_p C_{ox}(W/L)_2}} \times V_{THN}}{1 + \sqrt{\frac{\mu_n C_{ox}(W/L)_1}{\mu_p C_{ox}(W/L)_2}}}$$

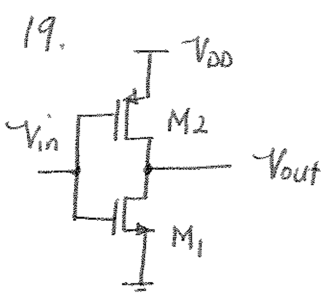
$$V_{in}^{o} = \frac{1.8 - 0.5 + \left(\frac{100 \times 2}{50 \times 3}\right)^{1/2} \times 0.4}{1 + \left(\frac{100 \times 2}{50 \times 3}\right)^{1/2}}$$

$$\boxed{V_{in} = V_{out} = 0.82\,V}$$

$$I_{D1} = I_{D2} = \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{2}{0.18}\right)\left(0.82 - 0.4\right)^2$$

$$\boxed{I_{D1} = I_{D2} = 97\,\mu A}$$

19.



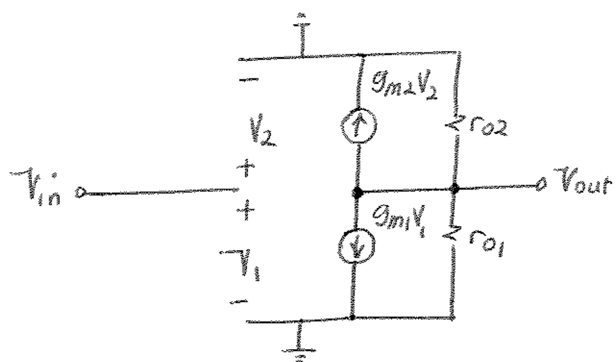
$$\left(\frac{W}{L}\right)_1 = \frac{2}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{3}{0.18}$$

$$\lambda_N = 0.1\,V^{-1}$$

$$\lambda_P = 0.2\,V^{-1}$$

Replacing M_1 and M_2 with their small-signal model in the saturation region yields:



$$V_{out} = (-g_{m1}V_1 - g_{m2}V_2)(r_{o1} \| r_{o2})$$

$$V_1 = V_2 = V_{in}$$

$$V_{out} = -(g_{m1} + g_{m2})(r_{o1} \| r_{o2})V_{in}$$

$$\frac{V_{out}}{V_{in}} = -(g_{m1} + g_{m2})(r_{o1} \| r_{o2})$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{THN})^2$$

$$g_{m_1} = \frac{\partial I_{D1}}{\partial V_{in}} = \mu_n C_{ox}\left(\frac{W}{L_1}\right)_1 (V_{in} - V_{THN}) = \frac{2I_{D1}}{V_{in} - V_{THN}}$$

$$g_{m1} = \frac{2 \times 9.7 \times 10^{-5}}{(0.817 - 0.4)} \rightarrow \boxed{g_{m1} = 4.641 \times 10^{-4}\ \mho}$$

$$g_{m2} = \frac{2 I_{D2}}{(V_{SG} - |V_{THP}|)} = \frac{2 \times 9.7 \times 10^{-5}}{(1.8 - 0.817 - 0.5)} \rightarrow \boxed{g_{m2} = 4.02 \times 10^{-4}\ \mho}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$$

$$g_o = \frac{\partial I_D}{\partial V_{DS}} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH})^2 \lambda \simeq \lambda I_D$$

$$r_o \cong \frac{1}{\lambda I_D}$$

$$r_{oN} \simeq \frac{1}{0.1 \times 9.7 \times 10^{-5}} = 103.17\,K\Omega$$

$$r_{oP} \simeq \frac{1}{0.2 \times 9.7 \times 10^{-5}} = 51.58\,K\Omega$$

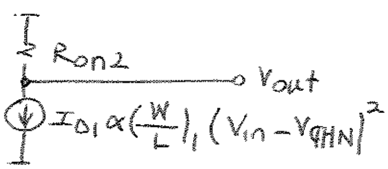
$$\text{Gain} = \frac{V_{out}}{V_{in}} = -(4.641 \times 10^{-4} + 4.02 \times 10^{-4})(51.58K \,\|\, 103.17K)$$

$$\boxed{\text{Gain} = -29.8}$$

20.

(a) Length of M_1 is increased

Let's assume that $V_{in} < V_{TH1}$, as a result M_1 is off and M_2 is on operating in the triode region. As V_{in} increases beyond V_{TH1}, M_1 starts pulling current (Conducting) in the saturation region while M_2 is still in the triode region. operating as a resistor; therefore, CMOS inverter can be modelled as follows:

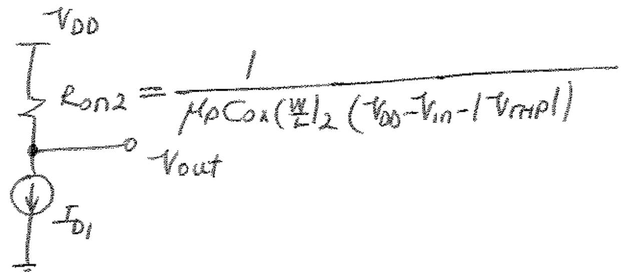


By increasing L_1, I_{D1} is weakened due to the inverse proportionality; as a result, an excess V_{in} is required to drop V_{out} to the point where $V_{out} = V_{in} + |V_{TH2}|$ and M_2 is placed at the edge of saturation. Therefore characteristic is shifted to the right and it will be steeper at the gain region where both M_1 and M_2 are in saturation region.

(b) Length of M_2 is increased

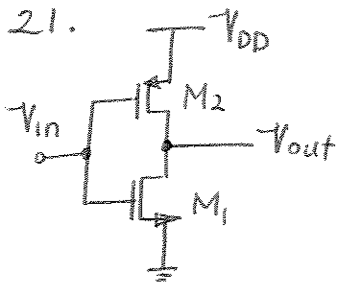
Again if we assume that $V_{in} < V_{TH1}$, M_1 is off and M_2 is operating in the triode region with no current. By increasing

V_{in} above V_{TH1}, M_1 conducts in the saturation region while M_2 is operating in the triode region. Using the same model as used in part (a) yields:



By increasing L_2, R_{on2} becomes larger; as a result, lower value of I_{D1} causes comparable voltage drop at the output. This will drive M_2 into the saturation with lower current (I_{D1}) and; hence, lower value of V_{in}. Therefore, characteristic is shifted to the left and small signal gain will be higher.

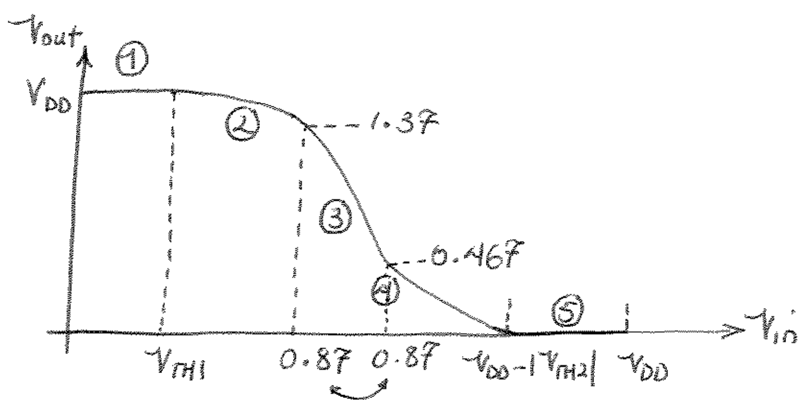
21.



$$\left(\frac{W}{L}\right)_1 = \frac{3}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{7}{0.18}$$

VTC looks like the following figure



① M_1 off, M_2 in triode region

$$I_{D1} = 0$$

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2\left(V_{DD}-V_{in}-|V_{TH2}|\right)V_{SD} - V_{SD}^2\right] = 0$$

$$V_{SD} = 0 \rightarrow V_{out} = V_{DD} \quad (1)$$

② M_1 in saturation, M_2 in triode region

$$I_{D1} = I_{D2}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2\left(V_{DD}-V_{in}-|V_{THP}|\right)(V_{DD}-V_{out}) - (V_{DD}-V_{out})^2\right]$$

$$\frac{1}{2}\times 100\times 10^{-6}\times\frac{3}{0.18}\times(V_{in}-0.4)^2=\frac{1}{2}\times 50\times 10^{-6}\times\frac{7}{0.18}\times\left[2(1.8-V_{in}-0.5)\times(1.8-V_{out})-(1.8-V_{out})^2\right]$$

$$6(V_{in}-0.4)^2=7\left[2(1.3-V_{in})(1.8-V_{out})-(1.8-V_{out})^2\right] \quad (2)$$

If V_{out} falls significantly, M_2 enters saturation. That is $V_{out}=V_{in}+|V_{TH2}|$.

then M_2 is about to exit the triode region.

Replacing V_{out} by $V_{in}+|V_{TH2}|$ in (2) leads to:

$$6(V_{in}-0.4)^2=7\left[2(1.3-V_{in})(1.8-V_{in}-0.5)-(1.8-V_{in}-0.5)^2\right]$$

$$6(V_{in}-0.4)^2=7(1.3-V_{in})^2 \rightarrow \sqrt{\frac{6}{7}}(V_{in}-0.4)=(1.3-V_{in})$$

$V_{in}=0.867\,V$, $V_{out}=1.37$

③ $$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1(V_{in}-V_{TH1})^2(1+\lambda_1 V_{out})=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2(V_{DD}-V_{in}-|V_{TH2}|)^2\times\left[1+\lambda_2(V_{DD}-V_{out})\right]$$

$$V_{out}=\frac{\mu_p C_{ox}\left(\frac{W}{L}\right)_2(V_{DD}-V_{in}-|V_{TH2}|)^2-\mu_n C_{ox}\left(\frac{W}{L}\right)_1(V_{in}-V_{TH1})^2}{\lambda_2\mu_p C_{ox}\left(\frac{W}{L}\right)_2(V_{DD}-V_{in}-|V_{TH2}|)^2+\lambda_1\mu_n C_{ox}\left(\frac{W}{L}\right)_1(V_{in}-V_{TH1})^2}$$

$$V_{out}=\frac{\cancel{3}\,7(1.3-V_{in})^2-6(V_{in}-0.4)^2}{7\lambda_2(1.3-V_{in})^2+6\lambda_1(V_{in}-0.4)^2} \quad (3)$$

in region ③ M_1 and M_2 are both in saturation.

④ M_1 in triode region, M_2 in saturation

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{TH1})V_{out}-V_{out}^2\right]=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \times (V_{DD}-V_{in}-|V_{TH2}|)^2 \quad (4)$$

$$6\left[2(V_{in}-0.4)V_{out}-V_{out}^2\right]=7(1.3-V_{in})^2$$

If V_{out} falls sufficiently, M_1 enters the triode region. That is, if $V_{in}=V_{out}+V_{TH1}$. Then M_1 is about to enter the triode region.

By substituting V_{in} with $V_{out}+0.4$ in (4), we have:

$$6\left[2V_{out}^2-V_{out}^2\right]=7(0.9-V_{out})^2$$

$V_{out}=0.467$, $V_{in}=0.867$

As channel length modulation has been neglected in this calculation tha value of input voltage that makes CMOS inverter transition from region ② to ③ is the same as that which makes inverter transition from region ③ to ④.

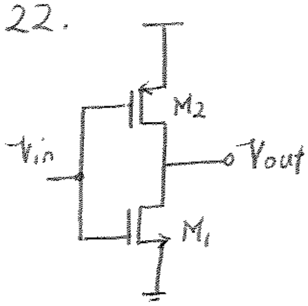
The slope in region ③ is infinit ; however, we assume a finite slope in that region to emphasize the behavior of inverter as to producing a high gain.

⑤ M_1 in triode region, M_2 off

$I_{D2}=0$; $I_{D1}=0$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{TH1})V_{out}-V_{out}^2\right]=0 \longrightarrow V_{out}=0$$

22.



$V_{in} = V_{out} = 0.5\,V$

M_1 and M_2 are both in saturation region

$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD}-V_{in}-|V_{TH2}|)^2$$

$$\frac{1}{2}\times 100\times 10^{-6}\times\left(\frac{W}{L}\right)_1 (0.5-0.4)^2 = \frac{1}{2}\times 50\times 10^{-6}\times\left(\frac{W}{L}\right)_2 (1.8-0.5-0.5)^2$$

$$\left(\frac{W}{L}\right)_1 \Big/ \left(\frac{W}{L}\right)_2 = 32$$

23. The value of the trip point has to be larger than the threshold voltage of NMOS transistor, 0.4 V. Therefore, 0.3 V Cannot be the trip point of such an inverter.

24.

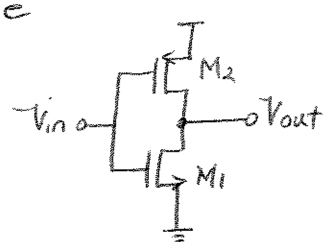
(a) If the inverter exhibits a very high voltage gain around the trip point, the range of input voltage values which guarantees that M_1 and M_2 are in saturation region is very narrow. Therefore this range can be fairly approximated with only one value of input voltage.

(b) $(W/L)_1 = 3/0.18$ and $(W/L)_2 = 7/0.18$

To calculate the minimum input voltage at which both transistors operate in saturation we assume

M_1 saturation

M_2 triode



$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - V_{in} - |V_{TH2}|)(V_{out} - V_{DD}) - (V_{out} - V_{DD})^2\right]$$

$V_{out} = V_{in} + |V_{TH2}|$ places M_2 at the edge of saturation

$$2 \times 3 \times (V_{in} - 0.4)^2 = 7 \times \left[2(1.8 - V_{in} - 0.5)(1.8 - V_{in} - 0.5) - (1.8 - V_{in} - 0.5)^2\right]$$

$$\boxed{V_{in,min} = 0.867}$$

To calculate $V_{in,max}$, we assume that M_1 and M_2 are in triode and saturation region respectively

$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH1})V_{out} - V_{out}^2\right] = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2$$

when M_1 is just going to leave the saturation and enters the triode region

$$V_{in} = V_{out} + 0.4^{(V_{TH1})}$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \frac{3}{0.18} \times \left[2(V_{out} + V_{TH1} - V_{TH1}) V_{out} - V_{out}^2\right] = \frac{1}{2} \times 50 \times 10^{-6} \times \frac{7}{0.18} \times (V_{DD} - V_{in} - |V_{TH2}|)^2$$

$$\frac{6}{7} V_{out}^2 = (V_{DD} - V_{out} - V_{TH1} - |V_{TH2}|)^2$$

$$\frac{6}{7} V_{out}^2 = (0.9 - V_{out})^2$$

$$V_{out} = 0.467\,V, \boxed{V_{in_{max}} = 0.867}$$

To find the trip point, M_1 and M_2 are assumed to be in saturation.

$$I_{D1} = I_{D2}$$

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2$$

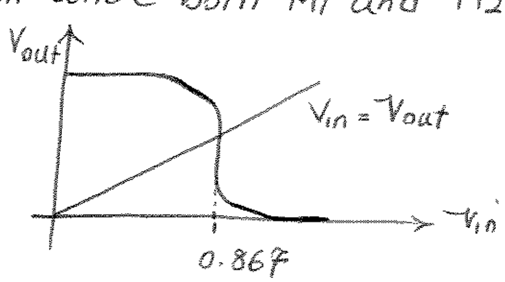
$$2 \times 3 \times (V_{in} - 0.4)^2 = 7 \times (1.8 - V_{in} - 0.5)^2$$

$$\boxed{V_{in}^{o} = 0.867 \text{ @ trip point}}$$

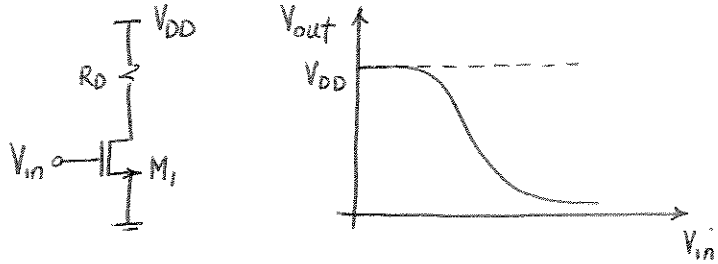
$$V_{in,trip} - V_{in,min} = \emptyset$$

$$V_{in,max} - V_{in,trip} = \emptyset$$

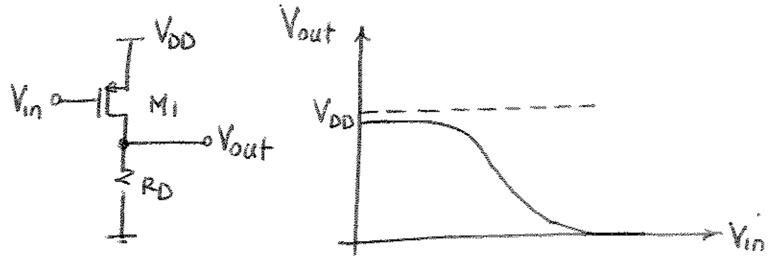
This result is not surprising because VTC of inverter has infinite slope at the region where both M_1 and M_2 are in saturation region



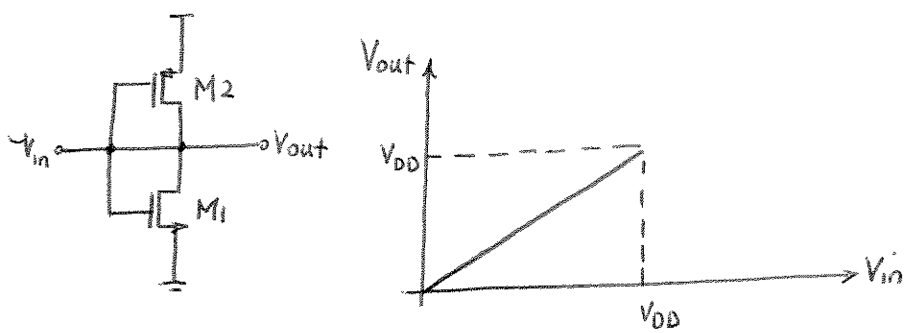
25.



VDD
RD
Vin
M1
Vout
VDD
Vin

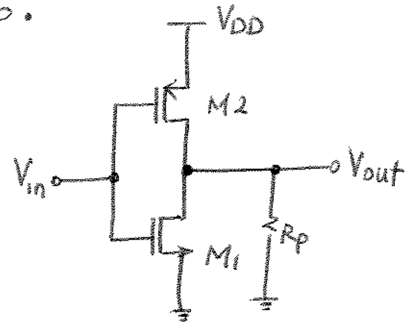


VDD
Vin
M1
Vout
RD
Vout
VDD
Vin



M2
Vin
Vout
M1
Vout
VDD
Vin
VDD

26.



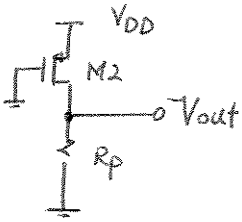
$R_p = 2K$

$\left(\frac{W}{L}\right)_1 = 3/0.18$

$\left(\frac{W}{L}\right)_2 = 5/0.18$

$V_{OL}, V_{OH}, V_{in,trip} = ?$

To calculate V_{OH}, V_{in} is assumed to be $0V$



$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD}-|V_{TH2}|)(V_{DD}-V_{out}) - (V_{DD}-V_{out})^2\right]$$

$$I_{D2} = \frac{V_{out}}{R_p}$$

$$\frac{V_{out}}{R_p} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD}-|V_{TH2}|)(V_{DD}-V_{out}) - (V_{DD}-V_{out})^2\right]$$

$$V_{out}^2 - 0.28V_{out} - 1.44 = 0$$

$$\boxed{V_{out} = V_{OH} = 1.348\,V}$$

$\boxed{V_{OL} = 0}$ because M_2 is off for $V_{in} = V_{DD}$

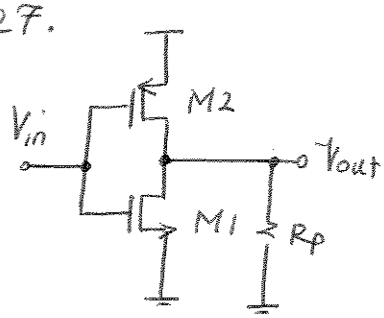
(a) trip point $V_{in} = V_{out}$

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_p}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD}-V_{in}-|V_{TH2}|\right)^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left(V_{in}-V_{TH1}\right)^2 + \frac{V_{out}}{2000}$$

$$0.05V_{out}^2 + 0.59V_{out} - 0.3745 = 0 \longrightarrow \boxed{V_{in} = V_{out} = 0.6\,V}$$

27.



$R_P = 2K$

$\left(W/L\right)_1 = 3/0.18$

$\left(W/L\right)_2 = 5/0.18$

$V_{in} = V_{out} = 0.6V$ @ trip point

With R_P

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_P}$$

$$\frac{1}{2}\mu_P C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} - |V_{TH2}|\right)^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(V_{in} - V_{TH1}\right)^2 + \frac{V_{out}}{2000}$$

$$0.05 V_{out}^2 + 0.59 V_{out} - 0.3745 = 0$$

$$V_{in} = V_{out} = 0.6V$$

$$I_{D1} = \frac{1}{2} \times 100 \times 10^{-6} \times \frac{3}{0.18} \times (0.6 - 0.4)^2$$

$$\boxed{I_{D1} = 3.33 \times 10^{-5} A}$$

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_P} = 3.33 \times 10^{-5} + \frac{0.6}{2000}$$

$$\boxed{I_{D2} = 3.35 \times 10^{-4} A}$$

$$g_{m1} = \frac{2 I_{D1}}{V_{eff1}} = \frac{2 \times 3.33 \times 10^{-5}}{(0.6 - 0.4)} \rightarrow \boxed{g_{m1} = 3.3 \times 10^{-3}\ \mho}$$

$$g_{m2} = \frac{2 I_{D2}}{V_{eff2}} = \frac{2 \times 3.35 \times 10^{-4}}{(1.8 - 0.6 - 0.5)} \rightarrow \boxed{g_{m2} = 9.4 \times 10^{-3}}$$

$$A_V = -(g_{m1} + g_{m2}) \times R_P$$

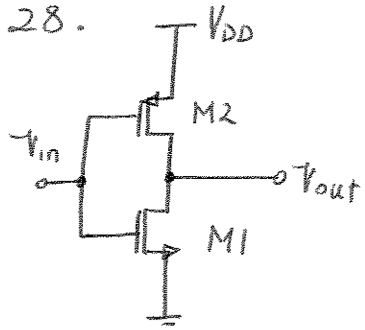
$$A_V = -(3.3 \times 10^{-3} + 9.4 \times 10^{-3}) \times 2000$$

$$\boxed{A_V = -25.4}$$

Without R_P

$$A_V \Rightarrow -\infty$$

28.



$$\left(\frac{W}{L}\right)_1 = 5/0.18$$

$$\left(\frac{W}{L}\right)_2 = 11/0.18$$

NM_L and $NM_H = ?$

To calculate NM_L, M_1 and M_2 are assumed to operate in the saturation and triode region respectively.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD}-V_{in}-|V_{TH2}|)(V_{DD}-V_{out}) - (V_{DD}-V_{out})^2\right] \quad (1)$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1}) = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[-2(V_{DD}-V_{out}) - 2(V_{DD}-V_{in}-|V_{TH2}|)\times\frac{\partial V_{out}}{\partial V_{in}} + 2(V_{DD}-V_{out})\frac{\partial V_{out}}{\partial V_{in}}\right]$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -1, \quad V_{in} = V_{IL}$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{IL}-V_{TH1}) = \mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2V_{OH}-V_{IL}-|V_{TH2}|-V_{DD}\right] \quad (2)$$

Obtaining V_{OH} from (2) and substituting in (1) yields:

$$V_{IL} = \frac{2\sqrt{a}\,(V_{DD}-V_{TH1}-|V_{TH2}|)}{(a-1)\sqrt{a+3}} - \frac{V_{DD}-aV_{TH1}-|V_{TH2}|}{a-1}$$

$$a = \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2} = \frac{100}{50}\times\frac{5}{11} = \frac{10}{11}$$

$$V_{IL} = \frac{2\sqrt{10/11}\,(1.8-0.4-0.5)}{(10/11-1)\sqrt{10/11+3}} - \frac{1.8-(10/11)\times 0.4-0.5}{10/11-1}$$

$$\boxed{V_{IL} = 0.7516\ V}$$

To determine NM_H, M_1 and M_2 are assumed to operate in the triode and saturation region respectively.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{TH1})V_{out}-V_{out}^2\right]=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD}-|V_{TH2}|-V_{in}\right)^2 \quad (3)$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out}+2(V_{in}-V_{TH1})\frac{\partial V_{out}}{\partial V_{in}}-2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right]=-\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \times \left(V_{DD}-V_{in}-|V_{TH2}|\right)$$

$\frac{\partial V_{out}}{\partial V_{in}}=-1$ yields

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[V_{out}-V_{in}+V_{TH1}+V_{out}\right]=-\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD}-V_{in}-|V_{TH2}|\right)$$

$$100\times 5\times\left[2V_{out}-(V_{in}-0.4)\right]=-50\times 11\times(1.8-0.5-V_{in})$$

$$V_{out}=1.05V_{in}-0.915 \quad (4)$$

Substituting (4) in (3) yields an equation versus V_{in} as follows:

$$10\times\left[2(V_{in}-0.4)(1.05V_{in}-0.915)-(1.05V_{in}-0.915)^2\right]=11\times(1.3-V_{in})^2$$

$$1.025V_{in}^2-21.115V_{in}+19.64225=0$$

$$V_{in}=V_{IH}=0.9765\ V$$

$$NM_H=V_{DD}-V_{IH}$$

$$\boxed{NM_H=0.823\ V}$$

29. $NM_L = 0.6\,V$

$$(W/L)_1 / (W/L)_2 = ?$$

$$V_{IL} = \frac{2\sqrt{a}\,(V_{DD} - V_{TH1} - |V_{TH2}|)}{(a-1)\sqrt{a+3}} - \frac{V_{DD} - aV_{TH1} - |V_{TH2}|}{a-1}$$

$$a = \frac{\mu_n C_{ox} \left(\frac{W}{L}\right)_1}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2}$$

$$0.6 = \frac{2\sqrt{a}\,(1.8 - 0.4 - 0.5)}{(a-1)\sqrt{a+3}} - \frac{1.8 - a\,0.4 - 0.5}{a-1}$$

$$a = 3\sqrt{\frac{a}{a+3}} - \frac{1.3 - 0.4a}{0.6} + 1$$

$$a = \frac{a+3}{9} \times \left[a - 1 + \frac{1.3 - 0.4a}{0.6}\right]^2$$

$$\boxed{a = 1}$$

$$(W/L)_1 / (W/L)_2 = \frac{\mu_p C_{ox}}{\mu_n C_{ox}} = \frac{1}{2}$$

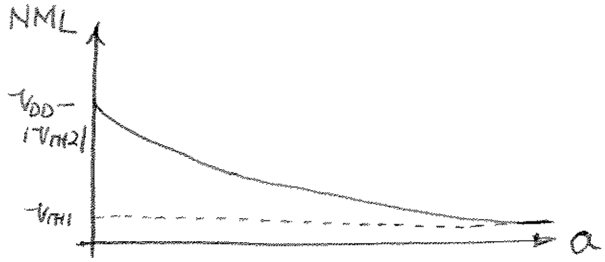
$$\boxed{(W/L)_1 / (W/L)_2 = \frac{1}{2}}$$

30. $$V_{IL} = \frac{2\sqrt{a}\,(V_{DD} - V_{TH1} - |V_{TH2}|)}{(a-1)\sqrt{a+3}} - \frac{V_{DD} - aV_{TH1} - |V_{TH2}|}{a-1}$$

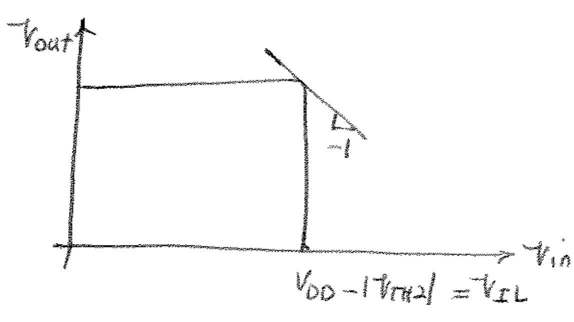
$$a = \frac{\mu_n \left(\frac{W}{L}\right)_1}{\mu_p \left(\frac{W}{L}\right)_2}$$

$a \to 0 \quad V_{IL} = V_{DD} - |V_{TH2}|$

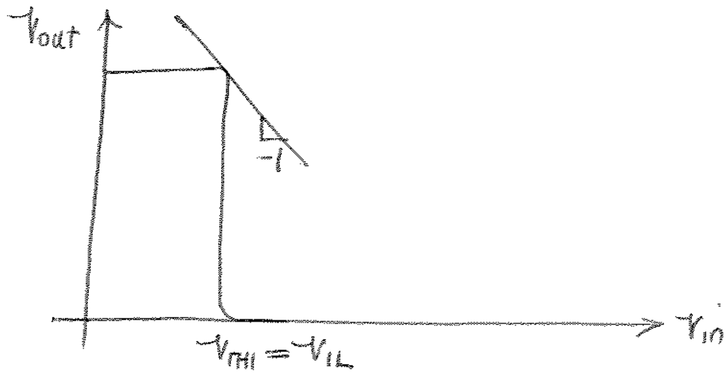
$a \to \infty \quad V_{IL} = V_{TH1}$



If $a = \frac{\mu_n}{\mu_p} \times \frac{(W/L)_1}{(W/L)_2} \to 0$, it implies that PMOS transistor is extremely stronger than NMOS. Therefor, as V_{in} increases from 0V, the output of inverter stays at V_{DD} until input reaches $V_{DD} - |V_{TH2}|$. At that point, PMOS is cut off and V_{out} sharply drops to 0V.



When $a \to \infty$, NMOS is prevailing and once input voltage hits the threshold voltage of NMOS, output voltage falls sharply to 0V.



V_{out}
-1
V_{in}
$V_{TH} = V_{IL}$

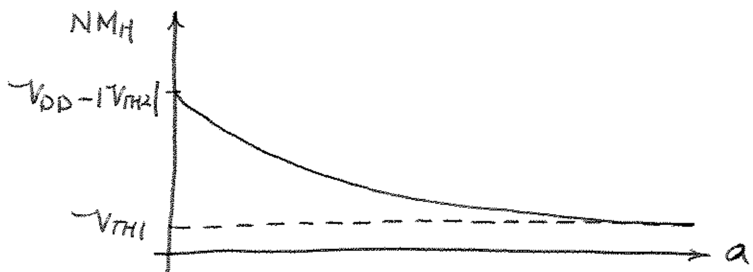
31.

$$NM_H = V_{DD} - \frac{2a\,(V_{DD} - V_{TH1} - |V_{TH2}|)}{(a-1)\sqrt{1+3a}} + \frac{V_{DD} - aV_{TH1} - |V_{TH2}|}{a-1}$$

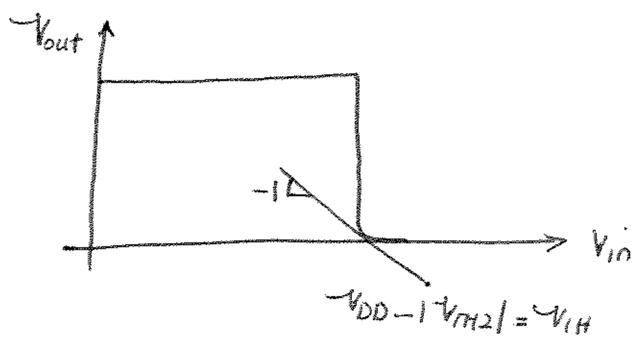
$$a = \frac{\mu_n \left(\frac{W}{L}\right)_1}{\mu_p \left(\frac{W}{L}\right)_2}$$

$a \to 0 \quad NM_H = |V_{TH2}|\ , \quad V_{IH} = V_{DD} - |V_{TH2}|$

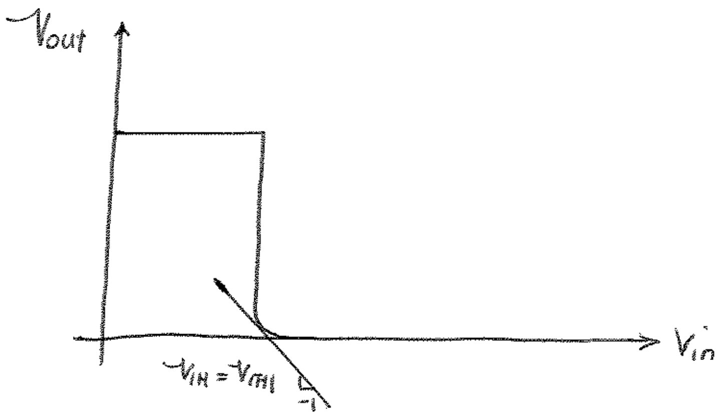
$a \to \infty \quad NM_H = V_{DD} - V_{TH1}\ , \quad V_{IH} = V_{TH1}$



If $a = \frac{\mu_n}{\mu_p} \times \frac{(W/L)_1}{(W/L)_2} \to 0$, it implies that PMOS transistor is much stronger than NMOS. Therefor, as V_{in} increases from $0\,V$, the output of inverter remains at V_{DD} until input reaches $V_{DD} - |V_{TH2}|$. At that point, PMOS is cutoff and V_{out} sharply drops to $0\,V$.

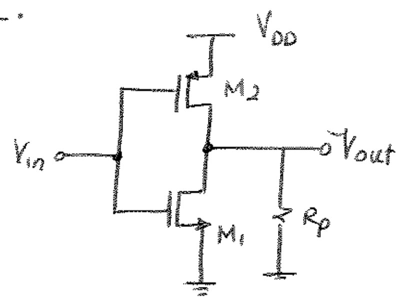


When "a" approaches infinity, NMOS is prevailing and once input voltage hits the threshold voltage of NMOS, output voltage falls sharply to 0V.



Note that the separation between V_{IH} and V_{IL} depends on the slope of VTC in the transition region. If "a" approaches either "0" or infinity, VTC exhibits infinite gain in its transition region. Therefore V_{IL} and V_{IH} coincide.

32.



$R_p = 2K$

$\left(W/L\right)_1 = 3/0.18$

$\left(W/L\right)_2 = 5/0.18$

$NM_L,\ NM_H = ?$

To calculate NM_L, M_1 and M_2 are assumed to be in the saturation and triode region respectively.

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_p} \quad (V_{in} = V_{IL})$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2\left(V_{DD}-V_{in}-|V_{TH2}|\right)\left(V_{DD}-V_{out}\right)-\left(V_{DD}-V_{out}\right)^2\right] =$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left(V_{in}-V_{TH1}\right)^2 + \frac{V_{out}}{R_p} \quad (1)$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -1, \quad V_{in} = V_{IL}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[-2\left(V_{DD}-V_{out}\right)-2\left(V_{DD}-V_{in}-|V_{TH2}|\right)\frac{\partial V_{out}}{\partial V_{in}}+2\left(V_{DD}-V_{out}\right)\frac{\partial V_{out}}{\partial V_{in}}\right] =$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left(V_{in}-V_{TH1}\right)+\frac{1}{R_p}\frac{\partial V_{out}}{\partial V_{in}}$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left(V_{IL}-V_{TH1}\right)-\frac{1}{R_p} = \mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2V_{OH}-V_{IL}-|V_{TH2}|-V_{DD}\right] \quad (2)$$

$$\boxed{V_{OH} = 1.01\,V_{IL} + 0.73}$$

Replacing V_{out} in (1) with its equivalent versus V_{IL} obtained from (2) yields:

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD}-V_{IL}-|V_{TH2}|)(V_{DD}-1.1V_{IL}-0.73)-(V_{DD}-1.1V_{IL}-0.73)^2\right]=$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1(V_{IL}-V_{TH1})^2+\frac{1.1V_{IL}+0.73}{R_p}$$

$$\frac{1}{2}\times 50\times 10^{-6}\times\frac{5}{0.18}\left[2(1.8-V_{IL}-0.5)(1.8-1.1V_{IL}-0.73)-(1.8-1.1V_{IL}-0.73)^2\right]=$$

$$\frac{1}{2}\times 100\times 10^{-6}\times\frac{3}{0.18}(V_{IL}-0.4)^2+\frac{1.1V_{IL}+0.73}{2000}$$

$$-52.5\times 10^{-3}V_{IL}-0.6195V_{IL}+0.229875=0$$

$$\boxed{V_{IL}=NM_L=0.36\,V}\quad < V_{TH1} \quad \text{Not Acceptable !}$$

This is less than threshold voltage of M_1; therefore, this answer is not acceptable. It means that M_1 is off and should be left out in this calculation.

$$I_{D1}=0,\quad I_{D2}=\frac{V_{out}}{R_p}$$

$$\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2V_{OH}-V_{IL}-|V_{TH2}|-V_{DD}\right]=-\frac{1}{R_p}$$

$$50\times 10^{-6}\times\frac{5}{0.18}\times\left[2V_{OH}-V_{IL}-0.5-1.8\right]=-\frac{1}{2000}$$

$$\boxed{V_{OH}=V_{out}=0.5V_{IL}+0.97}\quad(3)$$

$$\frac{1}{2}\times 50\times 10^{-6}\times\frac{5}{0.18}\times\left[2(1.8-V_{IL}-0.5)(1.8-0.5V_{IL}-0.97)-(1.8-0.5V_{IL}-0.97)^2\right]=$$

$$\frac{0.5V_{IL}+0.97}{2000}$$

$$0.1875V_{IL}^2-0.6225V_{IL}+0.192675=0$$

$$\boxed{V_{IL}=NM_L=0.345\,V}$$

To determine NM_H, M_1 and M_2 are assumed to operate in the triode and saturation region respectively.

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_p} \quad (V_{in} = V_{IH})$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} - |V_{TH2}|\right)^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2\left(V_{in} - V_{TH1}\right)V_{out} - V_{out}^2\right] + \frac{V_{out}}{R_p} \quad (4)$$

$$-\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} - |V_{TH2}|\right) = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2V_{out} + 2\left(V_{in} - V_{TH1}\right)\frac{\partial V_{out}}{\partial V_{in}} - 2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right] + \frac{\partial V_{out}}{\partial V_{in}}\frac{1}{R_p}$$

$$-\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} - |V_{TH2}|\right) = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[V_{out} - V_{in} + V_{TH1} + V_{out}\right] - \frac{1}{R_p}$$

$$-50\times10^{-6}\times\frac{5}{0.18}\times\left(1.8 - V_{in} - 0.5\right) = 100\times10^{-6}\times\frac{3}{0.18}\times\left(2V_{out} - V_{in} + 0.4\right) - \frac{1}{2000}$$

$$\boxed{V_{out} = \frac{0.1V_{in} + 0.59}{1.2}} \quad (5)$$

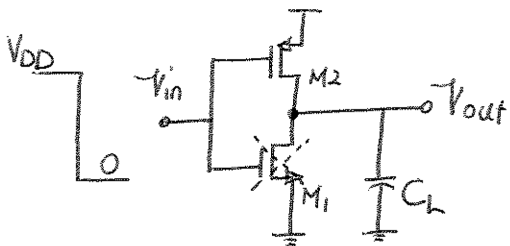
Combining equs (4) and (5) yields:

$$\frac{1}{2}\times 50\times10^{-6}\times\frac{5}{0.18}\left(1.8 - V_{in} - 0.5\right)^2 = \frac{1}{2}\times100\times10^{-6}\times\frac{3}{0.18}\left[2\left(V_{in} - 0.4\right)\frac{0.1V_{in}+0.59}{1.2} - \frac{\left(0.1V_{in}+0.59\right)^2}{1.2^2}\right] + \frac{0.1V_{in}+0.59}{1.2\times2000}$$

$$-0.291V_{in}^2 + 1.3182V_{in} - 0.75531 = 0$$

$$V_{in} = V_{IH} = 0.673\,V \longrightarrow \boxed{NM_H = V_{DD} - V_{IH} = 1.127\,V}$$

33.



$V_{out}(t=0)=0$

$(W/L)_2 = 6/0.18$

$C_L = 50fF$

$0 < V_{out} < |V_{TH2}|$; M_2 in the saturation

$$C_L \frac{dV_{out}}{dt} = I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD}-V_{in}-|V_{TH2}|)^2 = \frac{1}{2}\times 50\times10^{-6}\times\frac{6}{0.18}(1.8-0.5)^2$$
$$= 1.4\times10^{-3} A$$

$$V_{out}(t) = \frac{I_{D2}}{C_L}\times t$$

$$|V_{TH2}| = \frac{I_{D2}}{C_L}\cdot T_1 \longrightarrow T_1 = \frac{C_L\times|V_{TH2}|}{I_{D2}} = 50\times10^{-15}\times(1.4\times10^{-3})^{-1}\times0.5$$

$$\boxed{T_1 = 17.75\ pS}$$

$|V_{TH2}| < V_{out} < V_{DD}/2$; M_2 in Triode

$$C_L\frac{dV_{out}}{dt} = I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD}-|V_{TH2}|)(-V_{out}+V_{DD}) - (V_{DD}-V_{out})^2\right]$$

$$\frac{dV_{out}}{2(V_{DD}-|V_{TH2}|)(V_{DD}-V_{out})-(V_{DD}-V_{out})^2} = \frac{1}{2}\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 dt$$

$$\frac{1}{(V_{DD}-V_{out})\left[2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})\right]} = \frac{1}{2(V_{DD}-|V_{TH2}|)}\left[\frac{1}{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})} + \frac{1}{V_{DD}-V_{out}}\right]$$

$$\frac{1}{\cancel{2}(V_{DD}-|V_{TH2}|)}\left[\frac{dV_{out}}{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}+\frac{dV_{out}}{V_{DD}-V_{out}}\right]=\frac{1}{\cancel{2}}\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 dt$$

$$Ln\,\frac{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}{V_{DD}-V_{out}}=\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)t+C$$

$$\frac{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}{V_{DD}-V_{out}}=K.\exp\left[\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)t\right]$$

Time origin is assumed to be at $t=T_1=17.75\,pS$

$V_{out}(t=0)=|V_{TH2}| \longrightarrow K=1$

$$\frac{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}{V_{DD}-V_{out}}=e^{\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)t}$$

ⓐ $V_{out}=V_{DD}/2$ $\quad T_2=\dfrac{Ln(3-4|V_{TH2}|/V_{DD})}{\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)}$

$$=\frac{Ln(3-4\times0.5/1.8)}{50\times10^{-6}\times\frac{1}{50\times10^{-15}}\times\frac{6}{0.18}\times(1.8-0.5)}$$

$$\boxed{T_2=1.467\times10^{-11}}$$

$T_{0\to V_{DD}/2}=T_1+T_2=17.75+14.67$

$$\boxed{T_{0\to V_{DD}/2}=32.43\,pS}$$

34. $|V_{TH2}| < V_{out} < 0.95V_{DD}$ M_2 in Triode

$$\frac{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}{V_{DD}-V_{out}} = e^{\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 (V_{DD}-|V_{TH2}|)t}$$

(a) $V_{out} = 0.95V_{DD}$, $T_2 = \dfrac{\ln(39-40|V_{TH2}|/V_{DD})}{\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 (V_{DD}-|V_{TH2}|)}$

$$= \frac{\ln(39-40\times 0.5/1.8)}{50\times10^{-6} \times \frac{1}{50\times10^{-15}} \times \frac{6}{0.18} \times (1.8-0.5)}$$

$$\boxed{T_2 = 7.68\times10^{-11}}$$

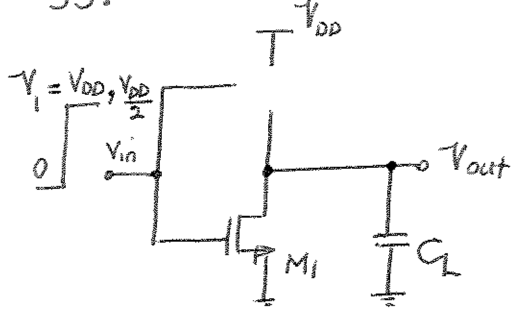
$T_1 = 17.75$ ps from previous problem

$T_{0\to0.95V_{DD}} = T_1 + T_2 = 17.75 + 76.8$

$$\boxed{T_{0\to0.95V_{DD}} = 94.55 \text{ ps}}$$

$$\boxed{(T_{0\to0.95V_{DD}})/(T_{0\to V_{DD}/2}) \approx 3}$$

35.



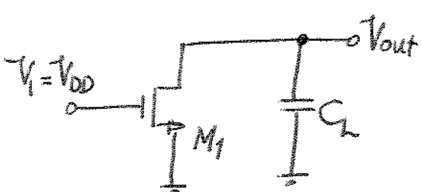
$V_{out}(t=0) = V_{DD}$ $\qquad C_L = 30fF$

$\left(\frac{W}{L}\right)_1 = 1/0.18$ $\qquad T_{V_{DD} \to V_{DD}/2} = ?$

$\left(\frac{W}{L}\right)_2 =$ Not necessary $\qquad V_1 = V_{DD}$ & $V_{DD}/2$

(a) $V_1 = V_{DD}$

$V_{DD} - V_{TH1} \leq V_{out} \leq V_{DD}$ $\quad M_1$ saturation



$\frac{V_{DD}}{2} \leq V_{out} \leq V_{DD} - V_{TH1}$ $\quad M_1$ Triode

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})^2 = -\frac{1}{2} \times 100 \times 10^{-6} \times \frac{1}{0.18}(1.8 - 0.4)^2 =$$
$$5.44 \times 10^{-4} A$$

$$dV_{out} = -\frac{I_{D1}}{C_L} \cdot dt$$

$$V_{out}(t) - V_{DD} = -\frac{I_{D1}}{C_L} t \longrightarrow V_{out}(t) = V_{DD} - \frac{I_{D1}}{C_L} t$$

$$T_{V_{DD} \to V_{DD} - V_{TH1}} = \frac{V_{TH1} \times C_L}{I_{D1}} = \frac{0.4 \times 30 \times 10^{-15}}{5.44 \times 10^{-4}} = 2.2 \times 10^{-11} S$$

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH1})V_{out} - V_{out}^2\right]$$

$$\frac{dV_{out}}{2(V_{DD} - V_{TH1})V_{out} - V_{out}^2} = -\frac{1}{2}\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 dt$$

$$\frac{1}{\left[2(V_{DD} - V_{TH1}) - V_{out}\right]V_{out}} = \frac{1}{2(V_{DD} - V_{TH1})}\left[\frac{1}{2(V_{DD} - V_{TH1}) - V_{out}} + \frac{1}{V_{out}}\right]$$

$$\frac{1}{\cancel{2}(V_{DD}-V_{TH1})}\left[\frac{dV_{out}}{2(V_{DD}-V_{TH1})-V_{out}}+\frac{dV_{out}}{V_{out}}\right]=-\frac{1}{\cancel{2}}\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 dt$$

$$-\ln\left[2(V_{DD}-V_{TH1})-V_{out}\right]+\ln V_{out}=-\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})t+C$$

$$\frac{V_{out}}{2(V_{DD}-V_{TH1})-V_{out}}=K\cdot\exp\left[-\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})t\right]$$

$V_{out}(t=0)=V_{DD}-V_{TH1}$ Note that time origin is assumed to be 2.2×10^{-11}

$K=1 \longrightarrow$

$$\frac{V_{out}}{2(V_{DD}-V_{TH1})-V_{out}}=e^{-\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})t}$$

$$V_{out}=V_{DD}/2 \longrightarrow \frac{V_{DD}/2}{2(V_{DD}-V_{TH1})-V_{DD}/2}=e^{-\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})T_{(V_{DD}-V_{TH1})\to V_{DD}/2}}$$

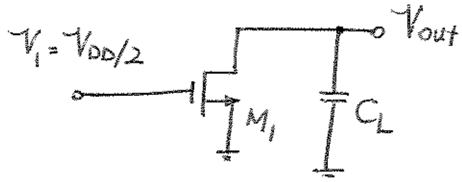
$$T_{(V_{DD}-V_{TH1})\to V_{DD}/2}=\frac{\ln\left(3-\frac{4V_{TH1}}{V_{DD}}\right)}{\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})}$$

$$=\frac{\ln(3-4\times0.4/1.8)}{100\times10^{-6}\times\frac{1}{30\times10^{-15}}\times\frac{1}{0.18}\times(1.8-0.4)}=2.88\times10^{-11}\,S$$

$$T_{V_{DD}\to V_{DD}/2}=T_{V_{DD}\to V_{DD}-V_{TH1}}+T_{(V_{DD}-V_{TH1})\to V_{DD}/2}$$

$$\boxed{T_{V_{DD}\to V_{DD}/2}=5\times10^{-11}=50.86\,pS}$$

(b) $V_1 = V_{DD}/2$



$V_{DD}/2 - V_{TH1} < V_{out} < V_{DD}$ M_1 in Saturation

$V_{DD}/2 < V_{out} < V_{DD}$ M_1 in Saturation

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left(V_{DD}/2 - V_{TH1}\right)^2 = -\frac{1}{2} \times 100 \times 10^{-6} \times \frac{1}{0.18}(0.9-0.4)^2$$

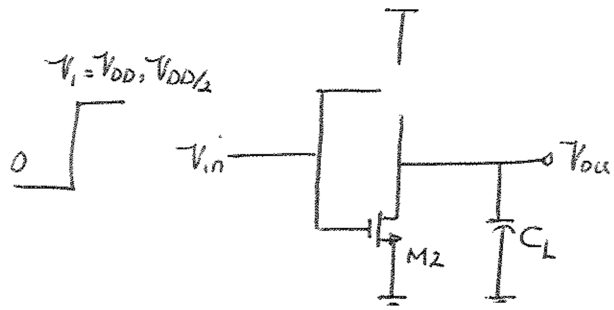
$$= 6.944 \times 10^{-5}$$

$$V_{out}(t) = V_{DD} - \frac{I_{D1}}{C_L} \times t$$

$$V_{DD}/2 = V_{DD} - \frac{I_{D1}}{C_L} \times T_{(V_{DD} \to V_{DD}/2)} \rightarrow T_{(V_{DD} \to V_{DD}/2)} = \frac{(V_{DD}/2) \times C_L}{I_{D1}}$$

$$\boxed{T_{(V_{DD} \to V_{DD}/2)} = 3.888 \times 10^{-10}}$$

36.



$V_{out}(0) = V_{DD}$

$(W/L)_1 = 1/0.18$

$C_L = 30fF$

$T_{V_{DD} \to 0.05 V_{DD}} = ?$

(a) $V_1 = V_{DD}$ $\quad V_{DD} - V_{TH1} < V_{out} < V_{DD}$ M_1 in saturation

$0.05 V_{DD} < V_{out} < V_{DD} - V_{TH1}$ M_1 in Triode

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})^2 = -\frac{1}{2} \times 100 \times 10^{-6} \times \frac{1}{0.18}(1.8 - 0.4)^2$$
$$= 5.44 \times 10^{-4} A$$

$$V_{out}(t) = V_{DD} - \frac{I_{D1}}{C_L} \times t$$

$$T_{V_{DD} \to V_{DD} - V_{TH1}} = \frac{V_{TH1} \times C_L}{I_{D1}} = \frac{0.4 \times 30 \times 10^{-15}}{5.44 \times 10^{-4}} = 2.2 \times 10^{-11} S$$

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH1})V_{out} - V_{out}^2\right]$$

$$\frac{1}{2(V_{DD} - V_{TH1})}\left[\frac{dV_{out}}{2(V_{DD} - V_{TH1}) - V_{out}} + \frac{dV_{out}}{V_{out}}\right] = -\frac{1}{2}\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 dt$$

$V_{out}(t = 0) = V_{DD} - V_{TH1}$ Note that time origin is assumed to be $2.2 \times 10^{-11} S$

$$\frac{V_{out}}{2(V_{DD} - V_{TH1}) - V_{out}} = e^{-\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1}) t}$$

$V_{out} = 0.05 V_{DD}$

$$\frac{0.05 V_{DD}}{2(V_{DD} - V_{TH1}) - 0.05 V_{DD}} = e^{-\mu_n \frac{C_{ox}}{C_L} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1}) T_{(V_{DD} - V_{TH1}) \rightarrow 0.05 V_{DD}}}$$

$$T_{(V_{DD} - V_{TH1}) \longrightarrow 0.05 V_{DD}} = \frac{\ln(39 - 40 V_{TH1}/V_{DD})}{\mu_n \frac{C_{ox}}{C_L} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})}$$

$$= \frac{\ln(39 - 40 \times 0.4/1.8)}{100 \times 10^{-6} \times \frac{1}{30 \times 10^{-15}} \times \frac{1}{0.18} (1.8 - 0.4)}$$

$$T(V_{DD} - V_{TH1}) \longrightarrow 0.05 V_{DD} = 131.33\ pS$$

$$T_{(V_{DD} \rightarrow 0.05 V_{DD})} = T_{(V_{DD} \rightarrow V_{DD} - V_{TH1})} - T_{(V_{DD} - V_{TH1} \rightarrow 0.05 V_{DD})}$$

$$= 2.2 \times 10^{-11} + 1.3133 \times 10^{-10}$$

$$\boxed{T_{(V_{DD} \rightarrow 0.05 V_{DD})} = 153.33\ pS}$$

(b) $V_1 = V_{DD}/2$ $\quad V_{DD}/2 - V_{TH1} < V_{out} < V_{DD}$ M_1 in Saturation

$0.05 V_{DD} < V_{out} < V_{DD}/2 - V_{TH1}$ M_1 in Triode

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left(V_{DD}/2 - V_{TH1}\right)^2 = -\frac{1}{2} \times 100 \times 10^{-6} \times \frac{1}{0.18} (0.9 - 0.4)^2$$

$$= 6.944 \times 10^{-5} A$$

$$V_{out}(t) = V_{DD} - \frac{I_{D1}}{C_L} \times t$$

$$V_{DD}/_2 - V_{TH1} = V_{DD} - \frac{I_{D1}}{C_L} T_{(V_{DD} \rightarrow V_{DD}/_2 - V_{TH1})}$$

$$T_{(V_{DD} \rightarrow V_{DD}/_2 - V_{TH1})} = \frac{(V_{DD}/2 + V_{TH1}) \times C_L}{I_{D1}}$$

$$T_{(V_{DD} \longrightarrow V_{DD}/_2 - V_{TH1})} = 5.616 \times 10^{-10}$$

<u>for $0.05V_{DD} < V_{out} < V_{DD}/_2 - V_{TH1}$</u>

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2\left(V_{DD}/_2 - V_{TH1}\right) V_{out} - V_{out}^2\right]$$

$$\frac{V_{out}}{2(V_{DD}/_2 - V_{TH1}) - V_{out}} = e^{-\mu_n \frac{C_{ox}}{C_L} \left(\frac{W}{L}\right)_1 \left(V_{DD}/_2 - V_{TH1}\right) t}$$

$$V_{out} = 0.05 V_{DD} \rightarrow \frac{0.05 V_{DD}}{2(V_{DD}/_2 - V_{TH1}) - 0.05 V_{DD}} = e^{-\mu_n \frac{C_{ox}}{C_L} \left(\frac{W}{L}\right)_1 \left(V_{DD}/_2 - V_{TH1}\right) \times T}$$

$$T_{(V_{DD}/_2 - V_{TH1} \rightarrow 0.05 V_{DD})} = \frac{\ln(19 - 40 V_{TH1}/V_{DD})}{\mu_n \frac{C_{ox}}{C_L} \left(\frac{W}{L}\right)_1 \left(V_{DD}/_2 - V_{TH1}\right)}$$

$$= \frac{\ln(19 - 40 \times 0.4/1.8)}{100 \times 10^{-6} \times \frac{1}{30 \times 10^{-15}} \times \frac{1}{0.18} (0.9 - 0.4)}$$

$$= 2.5 \times 10^{-10}$$

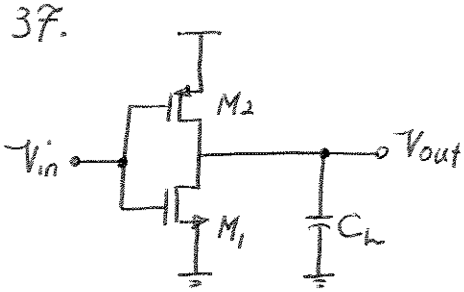
$$T_{(V_{DD} \longrightarrow 0.05 V_{DD})} = T_{(V_{DD} \rightarrow V_{DD}/_2 - V_{TH1})} + T_{(V_{DD}/_2 - V_{TH1} \longrightarrow 0.05 V_{DD})}$$

$$\boxed{T_{(V_{DD} \longrightarrow 0.05 V_{DD})} = 5.616 \times 10^{-10} + 2.5 \times 10^{-10} = 811.5 \, ps}$$

By decreasing V_{in} from V_{DD} to $V_{DD}/2$, the time it takes the output to reach $0.05V_{DD}$ will be 5.3 time larger!

$$\frac{T(V_{DD} \rightarrow 0.05V_{DD})(V_{in}=V_{DD})}{T(V_{DD} \longrightarrow 0.05V_{DD})(V_{in}=V_{DD}/2)} = \frac{811.5p}{153.33p} \approx 5.3$$

37.



$\left(\frac{W}{L}\right)_1 = \frac{1}{0.18}$

$\left(\frac{W}{L}\right)_2 = \frac{3}{0.18}$

$C_L = 80fF$

$T_{PHL}, T_{PLH} = ?$

To calculate T_{PLH}

$0 < V_{out} < |V_{TH2}|$ M_2 in Saturation

$|V_{TH2}| < V_{out} < V_{DD}/2$ M_2 in Triode

$$|I_{D2}| = \frac{1}{2}\mu_P C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2$$

$$V_{out}(t) = \frac{|I_{D2}|}{C_L} t$$

$$= \frac{1}{2}\mu_P \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2 t.$$

$$V_{out}(T_{PLH1}) = |V_{TH2}|$$

$$T_{PLH1} = \frac{|V_{TH2}| \times C_L}{\frac{1}{2}\mu_P C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2}.$$

for M_2 operating in Triode region

$$C_L \frac{dV_{out}}{dt} = \frac{1}{2}\mu_P C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right]$$

$$\frac{dV_{out}}{2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2} = \frac{1}{2}\mu_P \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 dt.$$

Defining $V_{DD} - V_{out} = u$ and noting that $\int \frac{du}{au - u^2} = \frac{1}{a} \ln \frac{u}{a-u}$,

$$\frac{-1}{2(V_{DD} - |V_{TH2}|)} \ln \frac{V_{DD} - V_{out}}{V_{DD} - 2|V_{TH2}| + V_{out}} \Bigg|_{V_{out} = |V_{TH2}|}^{V_{out} = V_{DD}/2} = \frac{1}{2} \mu_P \frac{C_{ox}}{C_L} \left(\frac{W}{L}\right)_2 T_{PLH2}$$

$$T_{PLH2} = \frac{C_L}{\mu_P C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \ln \left(3 - 4 \frac{|V_{TH2}|}{V_{DD}}\right)$$

$$T_{PLH} = T_{PLH1} + T_{PLH2} = \frac{C_L}{\mu_P C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \left[\frac{2|V_{TH2}|}{V_{DD} - |V_{TH2}|} + \ln \left(3 - 4 \frac{|V_{TH2}|}{V_{DD}}\right)\right]$$

$$T_{PLH} = \frac{80 \times 10^{-15}}{50 \times 10^{-6} \times \frac{3}{0.18} (1.8 - 0.5)} \left[\frac{2 \times 0.5}{1.8 - 0.5} + \ln \left(3 - 4 \frac{0.5}{1.8}\right)\right]$$

$$\boxed{T_{PLH} = 1.0377 \times 10^{-10}}$$

To calculate T_{PHL} $\quad V_{DD} - V_{TH1} < V_{out} < V_{DD}$ $\quad M_1$ in Saturation

$V_{DD}/2 < V_{out} < V_{DD} - V_{TH1}$ $\quad M_1$ in Triode

$$T_{PHL1} = \frac{-\Delta V_{out} \times C_L}{I_{D1}} = \frac{V_{TH1} \times C_L}{\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})^2}$$

after this point in time.

$$C_L \frac{dV_{out}}{dt} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH1}) V_{out} - V_{out}^2\right]$$

$$V_{out}(t=0) = V_{DD} - V_{TH1}$$

$$\frac{1}{2(V_{DD}-V_{TH1})} Ln \frac{V_{out}}{2(V_{DD}-V_{TH1})-V_{out}} \Bigg|_{V_{out}=V_{DD}-V_{TH1}}^{V_{out}=V_{DD}/2} = -\frac{1}{2}\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 T_{HL2}$$

$$T_{PHL2} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH1})} \times Ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)$$

$$T_{PHL} = T_{PHL1} + T_{PHL2} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH1})} \times \left[\frac{2V_{TH1}}{V_{DD}-V_{TH1}} + Ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$$T_{PHL} = \frac{80\times10^{-15}}{100\times10^{-6}\times\frac{1}{0.18}(1.8-0.4)} \times \left[\frac{2\times0.4}{1.8-0.4} + Ln\left(3-4\frac{0.4}{1.8}\right)\right]$$

$$\boxed{T_{PHL} = 1.3563\times10^{-10}}$$

38. $V_{DD} = 1.8 + 1.8 \times 0.1 = 1.98$

$$T_{PLH} = \frac{C_L}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \left[\frac{2|V_{TH2}|}{V_{DD} - |V_{TH2}|} + \ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right) \right]$$

$$= \frac{80 \times 10^{-15}}{50 \times 10^{-6} \times \frac{3}{0.18} \times (1.98 - 0.5)} \times \left[\frac{2 \times 0.5}{1.98 - 0.5} + \ln\left(3 - 4 \times \frac{0.5}{1.98}\right) \right]$$

$$\boxed{T_{PLH} = 8.846 \times 10^{-11}}$$

$$\text{Decrease in } T_{PLH} = \left| \frac{8.846 \times 10^{-11} - 1.0377 \times 10^{-10}}{1.0377 \times 10^{-10}} \right| \times 100$$

$$= 14.75\%$$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} \times \left[\frac{2 V_{TH1}}{V_{DD} - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right) \right]$$

$$= \frac{80 \times 10^{-15}}{100 \times 10^{-6} \times \frac{1}{0.18} \times (1.98 - 0.4)} \left[\frac{2 \times 0.4}{1.98 - 0.4} + \ln\left(3 - 4\frac{0.4}{1.98}\right) \right]$$

$$\boxed{T_{PHL} = 1.1767 \times 10^{-10}}$$

$$\text{Decrease in } T_{PHL} = \left| \frac{1.1767 \times 10^{-10} - 1.3563 \times 10^{-10}}{1.3563 \times 10^{-10}} \right| \times 100$$

$$= 13.24\%$$

39. $V_{DD} = 0.9\,V$

$$C_L \frac{dV_{out}}{dt} = I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - |V_{TH2}|\right)^2$$

$$T_{PLH} = \frac{\Delta V_{out} \times C_L}{I_{D2}} = \frac{\left(V_{DD}/2\right) \times C_L}{\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - |V_{TH2}|\right)^2}$$

$$= \frac{0.45 \times 80 \times 10^{-15}}{\frac{1}{2} \times 50 \times 10^{-6} \times \frac{3}{0.18} \times (0.9 - 0.5)^2}$$

$$\boxed{T_{PLH} = 5.4 \times 10^{-10} = 540\,pS}$$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(V_{DD} - V_{TH1}\right)} \left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$$= \frac{80 \times 10^{-15}}{100 \times 10^{-6} \times \frac{1}{0.18} \times (0.9 - 0.4)} \times \left[\frac{2 \times 0.4}{0.9 - 0.4} + \ln\left(3 - 4\frac{0.4}{0.9}\right)\right]$$

$$\boxed{T_{PHL} = 5.186 \times 10^{-10} = 518.6\,pS}$$

$$\text{Increase in } T_{PLH} = \left|\frac{5.4 \times 10^{-10} - 1.0377 \times 10^{-10}}{1.0377 \times 10^{-10}}\right| \times 100$$

$$= 420.38\,\%$$

$$\text{Increase in } T_{PHL} = \left|\frac{5.186 \times 10^{-10} - 1.3563 \times 10^{-10}}{1.3563 \times 10^{-10}}\right| \times 100$$

$$= 282.36\,\%$$

40. $T_{PLH} = T_{PHL} = 80\,pS$

$C_L = 50\,fF$

$(W/L)_1 , (W/L)_2 = ?$

$$T_{PLH} = \frac{C_L}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)}\left[\frac{2|V_{TH2}|}{V_{DD}-|V_{TH2}|} + \ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right)\right]$$

$$80\times10^{-12} = \frac{50\times10^{-15}}{50\times10^{-6}\times(1.8-0.5)\times\left(\frac{W}{L}\right)_2}\times\left[\frac{2\times0.5}{1.8-0.5} + \ln\left(3-4\times\frac{0.5}{1.8}\right)\right]$$

$$\boxed{\left(\frac{W}{L}\right)_2 = \frac{2.4}{0.18}}$$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})}\times\left[\frac{2V_{TH1}}{V_{DD}-V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$$80\times10^{-12} = \frac{50\times10^{-15}}{100\times10^{-6}\times(1.8-0.4)\times\left(\frac{W}{L}\right)_1}\times\left[\frac{2\times0.4}{1.8-0.4} + \ln\left(3-4\times\frac{0.4}{1.8}\right)\right]$$

$$\boxed{\left(\frac{W}{L}\right)_1 = \frac{1}{0.18}}$$

41.

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH1})}\left[\frac{2V_{TH1}}{V_{DD}-V_{TH1}} + Ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$V_{TH1} = 0.4$

$$\frac{2V_{TH1}}{V_{DD}-V_{TH1}} = Ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right) \longrightarrow V_{DD} = V_{TH1}\left[1+\frac{2}{Ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)}\right]$$

$V_{TH1} = 0.4 \longrightarrow \boxed{V_{DD} = 1.57}$

$$\frac{2V_{TH1}}{V_{DD}-V_{TH1}} = 0.1 \times Ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right) \longrightarrow V_{DD} = V_{TH1}\left[1+\frac{20}{Ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)}\right]$$

$V_{TH1} = 0.4 \longrightarrow \boxed{V_{DD} = 8.16}$

42. $\left(\frac{W}{L}\right)_1 = 1/0.18$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})}\left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$T_{PHL} = 100\,pS$

$C_L = 80\,fF$

$V_{DD} = ?$

$$100\times10^{-12} = \frac{80\times10^{-15}}{100\times10^{-6}\times\frac{1}{0.18}\times(V_{DD} - 0.4)} \times \left[\frac{2\times0.4}{V_{DD} - 0.4} + \ln\left(3 - 4\frac{0.4}{V_{DD}}\right)\right]$$

$$V_{DD} = 0.4 + 1.44\left[\frac{0.8}{V_{DD} - 0.4} + \ln\left(3 - \frac{1.6}{V_{DD}}\right)\right]$$

$$\boxed{V_{DD} = 2.22}$$

43. $T_{PHL} = 120\ ps$ $\quad (W/L)_1 = ?$

$C_L = 90\ fF$ $\quad V_{TH1} = ?$

$V_{DD} = 1.8$

$T_{PHL} = 160\ ps$ $\quad T_{PHL} = \dfrac{C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} \left[\dfrac{2V_{TH1}}{V_{DD} - V_{TH1}} + Ln\left(3 - 4\dfrac{V_{TH1}}{V_{DD}}\right)\right]$

$V_{DD} = 1.5\ V$

$C_L = 90\ fF$

$$120\times 10^{-12} = \frac{90\times 10^{-15}}{100\times 10^{-6}\left(\frac{W}{L}\right)_1 (1.8 - V_{TH1})} \times \left[\frac{2V_{TH1}}{1.8 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.8}\right)\right] \quad (1)$$

$$160\times 10^{-12} = \frac{90\times 10^{-15}}{100\times 10^{-6}\left(\frac{W}{L}\right)_1 (1.5 - V_{TH1})} \times \left[\frac{2V_{TH1}}{1.5 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.5}\right)\right] \quad (2)$$

Dividing Equations (1) and (2) yields:

$$0.75 = \frac{1.5 - V_{TH1}}{1.8 - V_{TH1}} \times \frac{\dfrac{2V_{TH1}}{1.8 - V_{TH1}} + Ln\left(3 - 4\dfrac{V_{TH1}}{1.8}\right)}{\dfrac{2V_{TH1}}{1.5 - V_{TH1}} + Ln\left(3 - 4\dfrac{V_{TH1}}{1.5}\right)}$$

$$V_{TH1} = 1.8 - \left(\frac{1.5 - V_{TH1}}{0.75}\right) \times \frac{\dfrac{2V_{TH1}}{1.8 - V_{TH1}} + Ln\left(3 - 4\dfrac{V_{TH1}}{1.8}\right)}{\dfrac{2V_{TH1}}{1.5 - V_{TH1}} + Ln\left(3 - 4\dfrac{V_{TH1}}{1.5}\right)}$$

This equation does not lead to a real Value for V_{TH1} so we use another derivation

$$V_{TH1} = 0.45 \times \left\{ 3 - e^{\left[0.75\,\frac{1.8 - V_{TH1}}{1.5 - V_{TH1}} \times \left[\frac{2V_{TH1}}{1.5 - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{1.5}\right)\right] - \frac{2V_{TH1}}{1.8 - V_{TH1}}\right]} \right\}$$

$$\boxed{V_{TH1} = 0.39}$$

$$\boxed{\left(\frac{W}{L}\right)_1 = \frac{1.26}{0.18}}$$

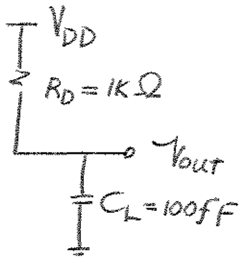
44.

$$\tau_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH1})}\left[\frac{2V_{TH1}}{V_{DD}-V_{TH1}} + \ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$\ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)$ is meaningless if $V_{DD} < 4V_{TH1}/3$.

Let's consider the case where $V_{DD} = \frac{4}{3}V_{TH1}$; then, τ_{PHL} is the time it takes for the output to drop from $V_{DD} = \frac{4}{3}V_{TH1}$ to $\frac{V_{DD}}{2} = \frac{2}{3}V_{TH1}$. However, $\left(V_{in}^{0} = V_{DD} = \frac{4}{3}V_{TH1}\right) - \left(V_{out} = \frac{2}{3}V_{TH1}\right) = \frac{2}{3}V_{TH1} < V_{TH1}$. In other words, M_1 never enters the triode region in the region where τ_{PHL} is calculated. The logrithmic term is derived from equation in which M_1 was assumed to be in Triode region. Therefore the logarithmic term is meaningless for $V_{DD} < \frac{4}{3}V_{TH1}$.

45.



$$V_{R_D} = (V_{DD} - V_{out})$$

$$I_{R_D} = C_L \frac{dV_{out}}{dt}$$

$$P_{R_D}(t) = V_{R_D} \cdot I_{R_D} = C_L (V_{DD} - V_{out}) \frac{dV_{out}}{dt}$$

$$E_{R_D} = \int_{t=0}^{\infty} P_{R_D}(t)\, dt = \int_{V_{out}=0}^{V_{DD}} (V_{DD} - V_{out})\, dV_{out} = \frac{1}{2} C_L V_{DD}^2$$

$$= \frac{1}{2} \times 100 \times 10^{-15} \times (1.8)^2$$

$$\boxed{E_{R_D} = 0.162\ pJ}$$

46. 10^6 Gates

$f = 2\,GHz$

20% of gates switch in every clock cycle

$C_L = 20fF$ for each gate

$P_{av} = ?$

$$P_{av,\,gate} = f_{in} C_L V_{DD}^2$$

$$P_{av,\,total} = 0.2 \times 10^6 \times f_{in} C_L V_{DD}^2$$

$$= 0.2 \times 10^6 \times 2 \times 10^9 \times 20 \times 10^{-15} \times (1.8)^2$$

$$\boxed{P_{av,\,total} = 25.92\ W}$$

47. $f = 2\,GHz$

5×10^6 Transistors with $W = 1\,\mu m$, $L = 0.18\,\mu m$, $C_{ox} = 10\,fF/\mu m^2$

$C_{gate} = WLC_{ox}$

$C_{Load} = 5\times10^6\, C_{gate}$

$= 5\times10^6 \times WLC_{ox}$

$= 5\times10^6 \times 1\,\mu m \times 0.18\,\mu m \times 10\,fF/\mu m^2$

$\boxed{C_{Load} = 9\,pF}$

$P_{av} = f_{in} C_L V_{DD}^2$

$= 2\times10^9 \times 9\times10^{-9} \times (1.8)^2$

$\boxed{P_{av} = 58.32\,W}$

48.

$$V_{DD} = V_{DD} + 0.1\,V_{DD} = 1.98$$

$$\left(\frac{W}{L}\right)_1 = 2/0.18$$

$$\left(\frac{W}{L}\right)_2 = 4/0.18$$

$$I_{Peak}\Big|_{V_{DD}=1.8} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left(\frac{V_{DD}}{2} - V_{TH1}\right)^2\left(1+\lambda_1\frac{V_{DD}}{2}\right)$$

$$= \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{2}{0.18}\right)\left(0.9 - 0.4\right)^2$$

$$I_{Peak}\Big|_{V_{DD}=1.8} = 1.388 \times 10^{-4}$$

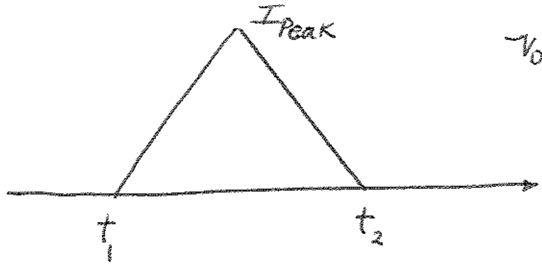
$$I_{Peak}\Big|_{V_{DD}=1.98} = \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{2}{0.18}\right)\left(0.99 - 0.4\right)^2$$

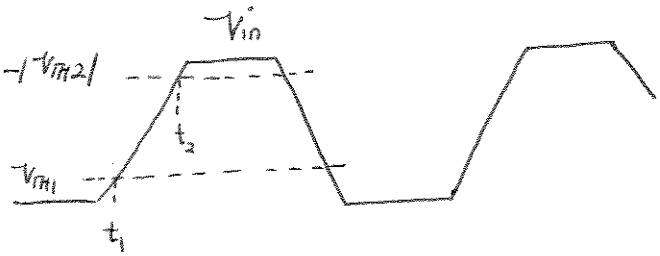
$$I_{Peak}\Big|_{V_{DD}=1.98} = 1.9338 \times 10^{-4}$$

$$\text{Change in Crowbar Current} = \frac{1.9338\times10^{-4} - 1.388\times10^{-4}}{1.388\times10^{-4}}$$

$$\boxed{\text{Change in Crowbar Current} = 39.24\%}$$

49.





Total Energy drawn from V_{DD} during the interval $[t_1, t_2]$ is:

$$E = V_{DD} \times I_{Peak} \times \frac{t_2 - t_1}{2}$$

In a periode the total energy is:

$$E_{tot} = 2 \times V_{DD} \times I_{Peak} \times \frac{t_2 - t_1}{2}$$

$$P_{av} = V_{DD} I_{Peak} (t_2 - t_1) f_1$$

$$\text{Slope of input voltage} = \frac{0.9V_{DD} - 0.1V_{DD}}{t_r} = \frac{0.8V_{DD}}{t_r}$$

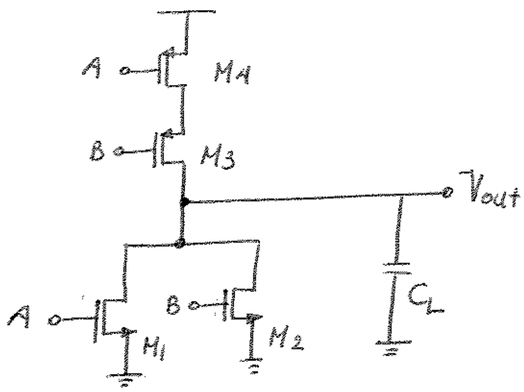
$$(t_2 - t_1) = \frac{(V_{DD} - V_{TH1} - |V_{TH2}|)}{0.8V_{DD}} \times t_r$$

$$P_{av} = V_{DD} \times \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left(\frac{V_{DD}}{2} - V_{TH1}\right)^2 \times \frac{(V_{DD} - V_{TH1} - |V_{TH2}|)}{0.8V_{DD}} t_r \times f_1$$

$$P_{av} = \frac{1}{1.6} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left(\frac{V_{DD}}{2} - V_{TH1}\right)^2 (V_{DD} - V_{TH1} - |V_{TH2}|) f_1 \cdot t_r$$

$$\boxed{P_{av} = 1.4 \times 10^{-5} \left(\frac{W}{L}\right)_1 \times t_r \times f_1}$$

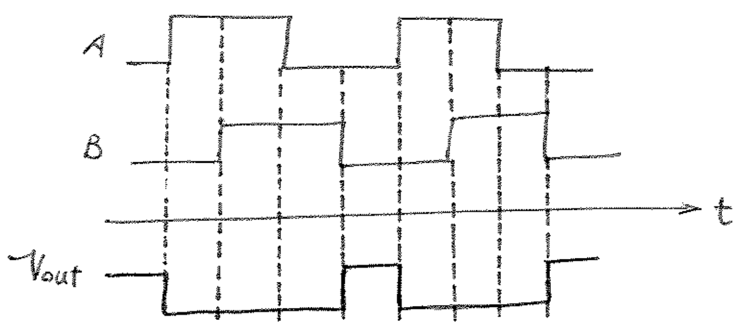
50.



$C_L = 20\,fF$

$f_1 = 500\,MHz$

$P_{av} = ?$

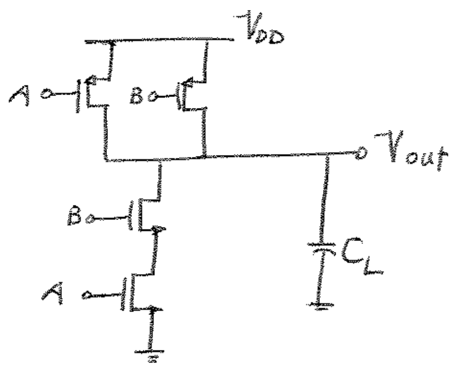


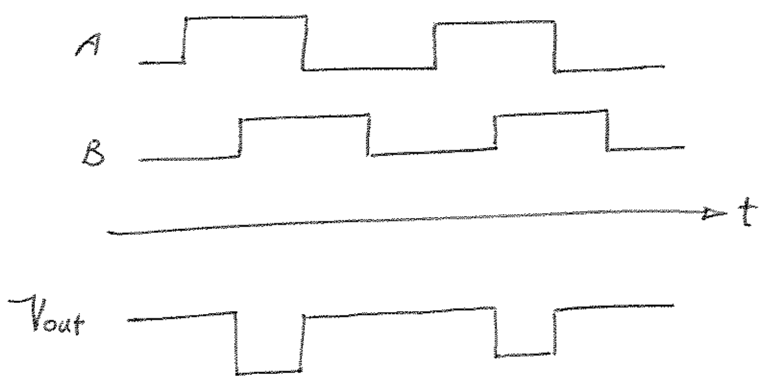
$$P_{av} = f_{in} C_L V_{DD}^2$$

$$= 500\times10^{6} \times 20\times10^{-15} \times (1.8)^2$$

$$\boxed{P_{av} = 3.24\times10^{-5}\,W}$$

51.



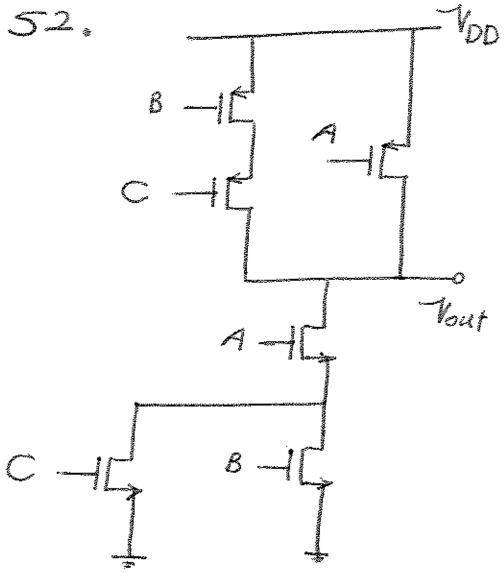


$$P_{av} = f_{in} C_L V_{DD}^2$$

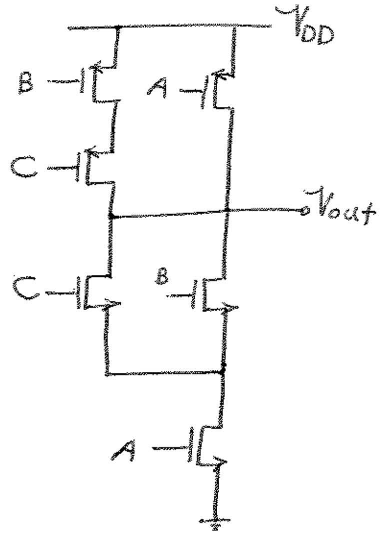
$$= 500\times10^{6}\times20\times10^{-15}\times(1.8)^2$$

$$\boxed{P_{av} = 3.24\times10^{-5}\,W}$$

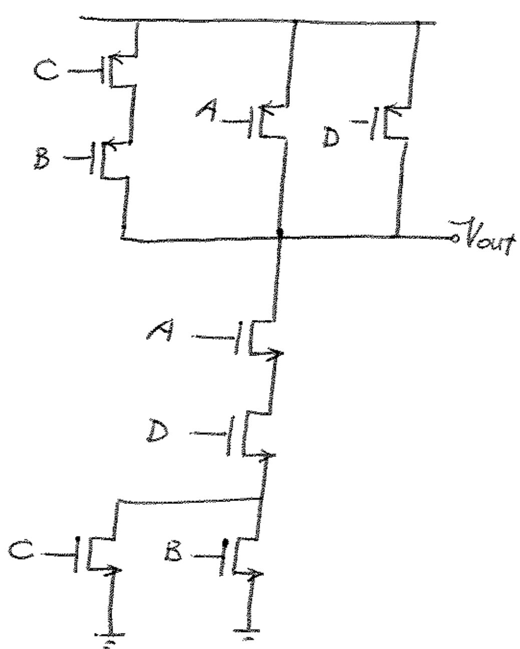
52.



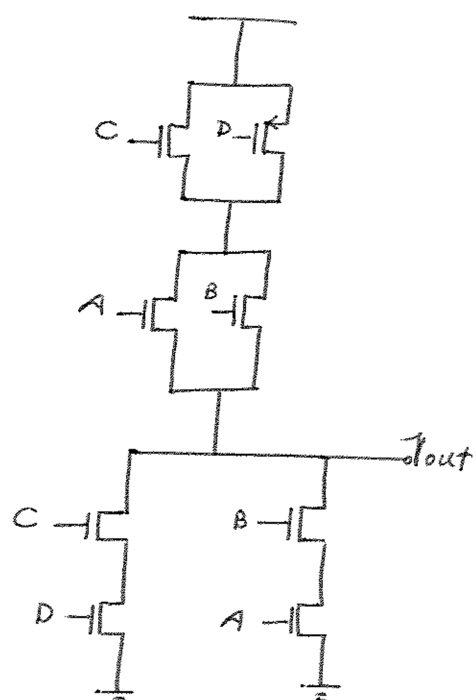
$V_{out} = \overline{(B+C)A}$



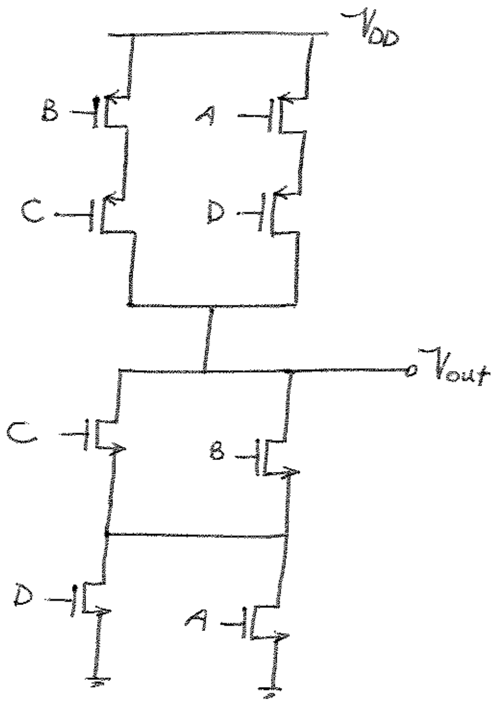
$V_{out} = \overline{(B+C) \cdot A}$



$V_{out} = \overline{(B+C)\, D \cdot A}$

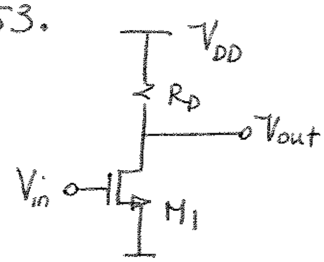


$V_{out} = \overline{A \cdot B + C.D}$



$$V_{out} = \overline{(A+D)\cdot(B+C)}$$

53.



$P_{static} = 0.5\,mW$

$V_{OL} = 100\,mV$

$$\frac{(V_{DD}-V_{OL})^2}{R_D} + V_{OL} \times \frac{V_{DD}-V_{OL}}{R_D} = 0.5 \times 10^{-3}$$

$$\frac{(1.8-0.1)^2}{R_D} + 0.1 \times \frac{1.8-0.1}{R_D} = 0.5 \times 10^{-3}$$

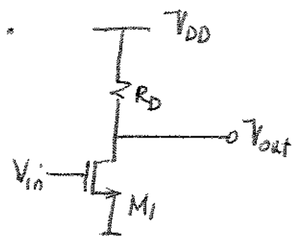
$$\frac{1}{R_D} \times 3.06 = 0.5 \times 10^{-3}$$

$$\boxed{R_D = 6120\,\Omega}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD}-V_{TH1})V_{OL} - V_{OL}^2\right] = \frac{V_{DD}-V_{OL}}{R_D}$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{W}{L}\right)_1 \times \left[2(1.8-0.4)0.1 - 0.1^2\right] = \frac{1.8-0.1}{6120}$$

$$\boxed{\left(\frac{W}{L}\right)_1 = \frac{3.7}{0.18}}$$

54. 

$P_{Static} = 0.25\ mW$

$NM_L = 600\ mV$

Small signal gain $= -g_m R_D$

$g_m = \mu_n C_{ox} (W/L)(V_{GS} - V_{TH})$

$\mu_n C_{ox} \frac{W}{L} (V_{IL} - V_{TH1}) R_D = 1$

$$V_{IL} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right) R_D} + V_{TH1}$$

$$NM_L = V_{IL} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right) R_D} + V_{TH1}$$

$$\frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right) R_D} = (NM_L - V_{TH1}) \rightarrow \left(\frac{W}{L}\right) R_D = \frac{1}{\mu_n C_{ox} (NM_L - V_{TH1})}$$

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) \left[2(V_{DD} - V_{TH1}) V_{OL} - V_{OL}^2 \right] = \frac{V_{DD} - V_{OL}}{R_D}$$

$$\frac{1}{2} \cancel{\mu_n C_{ox}} \frac{1}{\cancel{\mu_n C_{ox}} (NM_L - V_{TH1})} \times \left[2(V_{DD} - V_{TH1}) V_{OL} - V_{OL}^2 \right] = (V_{DD} - V_{OL})$$

$$2(V_{DD} - V_{TH1}) V_{OL} - V_{OL}^2 = 2(NM_L - V_{TH1})(V_{DD} - V_{OL})$$

$$V_{OL}^2 - 2(V_{DD} - V_{TH1}) V_{OL} - 2(NM_L - V_{TH1}) V_{OL} + 2(NM_L - V_{TH1}) V_{DD} = \emptyset$$

$$V_{OL}^2 - 2(V_{DD} + NM_L - 2V_{TH1}) V_{OL} + 2(NM_L - V_{TH1}) V_{DD} = 0$$

$$V_{OL}^2 - 3.2 V_{OL} + 0.72 = 0$$

$$\boxed{V_{OL} = 0.2435}$$

$$\frac{(V_{DD} - V_{OL})^2}{R_D} + V_{OL} \times \frac{V_{DD} - V_{OL}}{R_D} = 0.25 \times 10^{-3}$$

$$\frac{(1.8 - 0.24)^2 + 0.24 \times (1.8 - 0.24)}{R_D} = 0.25 \times 10^{-3}$$

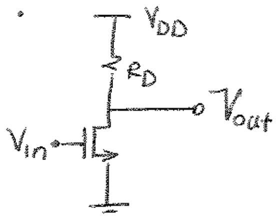
$$\boxed{R_D = 11206.55\,\Omega}$$

$$\left(\frac{W}{L}\right) = \frac{1}{\mu_n C_{ox} (NM_L - V_{TH1}) R_D}$$

$$\left(\frac{W}{L}\right) = \frac{1}{100 \times 10^{-6} (0.6 - 0.4)\, 11206.55}$$

$$\boxed{\left(\frac{W}{L}\right) = \frac{0.8}{0.18}}$$

55.



$V_{OL} = 100mV$

$P_{av} = 0.25mW$

$$\frac{(V_{DD} - V_{OL})^2}{R_D} + \frac{V_{OL}(V_{DD} - V_{OL})}{R_D} = P_{av}$$

$$\frac{(1.8 - 0.1)^2 + 0.1 \times (1.8 - 0.1)}{0.25 \times 10^{-3}} = R_D$$

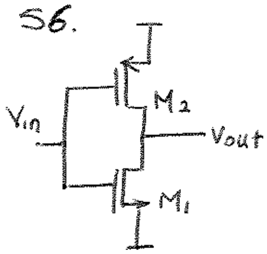
$$\boxed{R_D = 12240}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[2(V_{DD} - V_{TH1})V_{OL} - V_{OL}^2\right] = \frac{V_{DD} - V_{OL}}{R_D}$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{W}{L}\right) \times \left[2(1.8 - 0.4) \times 0.1 - 0.1^2\right] = \frac{1.8 - 0.1}{12240}$$

$$\boxed{\left(\frac{W}{L}\right) = \frac{1.85}{0.18}}$$

56.



$V_{in} = V_{out} = 0.8V$, $I_{D1} = I_{D2} = 0.5mA$

$\lambda_n = 0.1 V^{-1}$

$\lambda_p = 0.2 V^{-1}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 (1 + \lambda_N V_{out}) = I_{D1}$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{W}{L}\right)_1 (0.8 - 0.4)^2 (1 + 0.1 \times 0.8) = 0.5 \times 10^{-3}$$

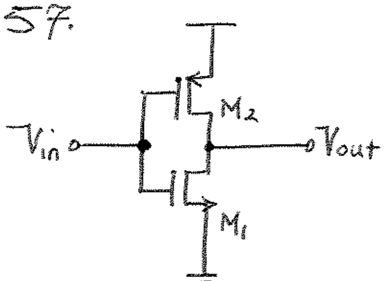
$$\boxed{\left(\frac{W}{L}\right)_1 = \frac{10.4}{0.18}}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2 \left[1 + \lambda_p (V_{DD} - V_{out})\right] = I_{D2}$$

$$\frac{1}{2} \times 50 \times 10^{-6} \times \left(\frac{W}{L}\right)_2 (1.8 - 0.8 - 0.5)^2 \left[1 + 0.2 \times (1.8 - 0.8)\right] = 0.5 \times 10^{-3}$$

$$\boxed{\left(\frac{W}{L}\right)_2 = \frac{12}{0.18}}$$

57.



$NM_L = NM_H = 0.7\,V$

NM_L : M_1 in Saturation and M_2 in triode

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD}-V_{in}-|V_{TH2}|)(V_{DD}-V_{out})-(V_{DD}-V_{out})^2\right] \quad (1)$$

Differentiating both sides with respect to V_{in}

$$2\mu_n\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1}) = \mu_p\left(\frac{W}{L}\right)_2 \left[-2(V_{DD}-V_{out})-2(V_{DD}-V_{in}-|V_{TH2}|)\frac{\partial V_{out}}{\partial V_{in}} + 2(V_{DD}-V_{out})\frac{\partial V_{out}}{\partial V_{in}}\right]$$

ⓐ $V_{in}=V_{IL}$, $\dfrac{\partial V_{out}}{\partial V_{in}}=-1$

$$\mu_n\left(\frac{W}{L}\right)_1 (V_{IL}-V_{TH1}) = \mu_p\left(\frac{W}{L}\right)_2 \left(2V_{OH}-V_{IL}-|V_{TH2}|-V_{DD}\right) \quad (2)$$

obtaining V_{OH} from (2), substituting in (1), we arrive at

$$V_{IL} = \frac{2\sqrt{a}\,(V_{DD}-V_{TH1}-|V_{TH2}|)}{(a-1)\sqrt{a+3}} - \frac{V_{DD}-aV_{TH1}-|V_{TH2}|}{a-1}$$

$$a = \frac{\mu_n\left(\frac{W}{L}\right)_1}{\mu_p\left(\frac{W}{L}\right)_2}$$

NM_H, M_1 in triode and M_2 in Saturation

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{TH1})V_{out}-V_{out}^2\right]=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD}-V_{in}-|V_{TH2}|\right)^2$$

Differentiating both sides with respect to V_{in}:

$$\mu_n\left(\frac{W}{L}\right)_1\left[2V_{out}+2(V_{in}-V_{TH1})\frac{\partial V_{out}}{\partial V_{in}}-2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right]=2\mu_p\left(\frac{W}{L}\right)_2\times$$

$$\left(V_{in}-V_{DD}-|V_{TH2}|\right)$$

Assuming $\frac{\partial V_{out}}{\partial V_{in}}=-1$, $V_{in}=V_{IH}$, and $V_{out}=V_{OL}$ obtaining

$$V_{IH}=\frac{2a\left(V_{DD}-V_{TH1}-|V_{TH2}|\right)}{(a-1)\sqrt{1+3a}}-\frac{V_{DD}-aV_{TH1}-|V_{TH2}|}{a-1}$$

$$V_{IL}=NM_L=0.7$$

$$V_{IH}=V_{DD}-NM_H=1.8-0.7=1.1$$

$$0.7=\frac{2\sqrt{a}\,(1.8-0.4-0.5)}{(a-1)\sqrt{a+3}}-\frac{1.8-0.4a-0.5}{a-1}$$

$$0.7(a-1)=\frac{1.8\sqrt{a}}{\sqrt{a+3}}-\frac{1.3-0.4a}{1}$$

$$0.7a-0.7+1.3-0.4a=\sqrt{\frac{a}{a+3}}\times 1.8$$

$$\frac{0.6+0.3a}{1.8}=\sqrt{\frac{a}{a+3}}\longrightarrow a^3+7a^2-20a+12=0$$

$$a=\begin{cases}-9.3\\ 1.3\\ 1\end{cases}\longrightarrow \boxed{a=1.3}$$

$$1.1 = \frac{2a(1.8-0.4-0.5)}{(a-1)\sqrt{1+3a}} - \frac{1.8-0.4a-0.5}{a-1}$$

$$1.1(a-1) = \frac{1.8a}{\sqrt{1+3a}} - 1.3 + 0.4a$$

$$1.1a - 1.1 + 1.3 - 0.4a = \frac{1.8a}{\sqrt{1+3a}}$$

$$0.2 + 0.7a = \frac{1.8a}{\sqrt{1+3a}}$$

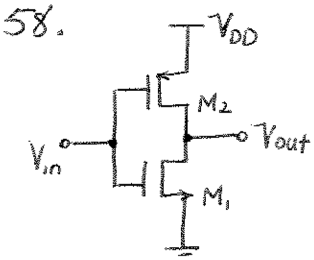
$$147a^3 - 191a^2 + 40a + 4 = 0 \longrightarrow \begin{cases} a_1 = 1 \\ a_2 = 0.37 \\ a_3 = -0.073 \end{cases} \longrightarrow \boxed{a = 0.37}$$

No it is not possible to design a CMOS inverter with $NM_L = NM_H = 0.7$.

The reason is that each value of $a = \dfrac{\mu_n C_{ox} (W/L)_1}{\mu_p C_{ox} (W/L)_2}$ specifies a unique set of noise margins (NM_L, NM_H).

Remember, the relative strength of NMOS and PMOS determines the noise margins interdependently.

58.



$T_{PLH} = T_{PHL} = 100\,ps$

$C_L = 50\,fF$

T_{PLH}

$$|I_{D2}| = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - |V_{TH2}|\right)^2$$

$$V_{out}(t) = \frac{|I_{D2}|}{C_L}t$$

$$= \frac{1}{2}\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 \left(V_{DD} - |V_{TH2}|\right)^2 t.$$

$$T_{PLH1} = \frac{2|V_{TH2}|C_L}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - |V_{TH2}|\right)^2}.$$

$$|I_{D2}| = C_L \frac{dV_{out}}{dt}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2\left(V_{DD} - |V_{TH2}|\right)\left(V_{DD} - V_{out}\right) - \left(V_{DD} - V_{out}\right)^2\right] = C_L \frac{dV_{out}}{dt}$$

$$\frac{dV_{out}}{2\left(V_{DD} - |V_{TH2}|\right)\left(V_{DD} - V_{out}\right) - \left(V_{DD} - V_{out}\right)^2} = \frac{1}{2}\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 dt$$

$$\frac{-1}{2\left(V_{DD} - |V_{TH2}|\right)} \ln \frac{V_{DD} - V_{out}}{V_{DD} - 2|V_{TH2}| + V_{out}}\Bigg|_{V_{out} = |V_{TH2}|}^{V_{out} = V_{DD}/2} = \frac{1}{2}\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 T_{PLH2}$$

$$T_{PLH2} = \frac{C_L}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - |V_{TH2}|\right)} \ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right)$$

$$T_{PLH} = T_{PLH1} + T_{PLH2}$$

$$= \frac{C_L}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \left[\frac{2|V_{TH2}|}{V_{DD} - |V_{TH2}|} + Ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right) \right]$$

$$100 \times 10^{-12} = \frac{50 \times 10^{-15}}{50 \times 10^{-6} \left(\frac{W}{L}\right)_2 (1.8 - 0.5)} \left[\frac{2 \times 0.5}{1.8 - 0.5} + Ln\left(3 - 4\frac{0.5}{1.8}\right) \right]$$

$$\boxed{\left(\frac{W}{L}\right)_2 = \frac{1.9}{0.18}}$$

T_{PHL}

$$T_{PHL1} = \frac{2 V_{TH1} C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})^2}$$

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH1}) V_{out} - V_{out}^2 \right] = -C_L \frac{dV_{out}}{dt}$$

$$\frac{-1}{2(V_{DD} - V_{TH1})} Ln \frac{V_{out}}{2(V_{DD} - V_{TH1}) - V_{out}} \Bigg|_{V_{out} = V_{DD} - V_{TH1}}^{V_{out} = V_{DD}/2} = \frac{1}{2} \mu_n C_{ox}/C_L \left(\frac{W}{L}\right)_1 T_{PHL2}$$

$$T_{PHL2} = \frac{C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)$$

$$T_{PHL} = T_{PHL1} + T_{PHL2} = \frac{C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} \left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right) \right]$$

$$100 \times 10^{-12} = \frac{50 \times 10^{-15}}{100 \times 10^{-6} \times \left(\frac{W}{L}\right)_1 \times (1.8 - 0.4)} \times \left[\frac{2 \times 0.4}{1.8 - 0.4} + Ln\left(3 - 4 \times \frac{0.4}{1.8}\right) \right], \quad \boxed{\left(\frac{W}{L}\right)_1 = \frac{0.85}{0.18}}$$