4. Suppose we have reason to believe that our data is sampled from a two-dimensional uniform density

$$p(\mathbf{x}|\boldsymbol{\theta}) \sim U(\mathbf{x}_l, \mathbf{x}_u) = \begin{cases} \frac{1}{|x_{u1}-x_{l1}||x_{u2}-x_{l2}|} & \text{for } x_{l1} \le x_1 \le x_{u1} \text{ and } x_{l2} \le x_2 \le x_{u2} \\ 0 & \text{otherwise,} \end{cases}$$

where x_{l1} is the x_1 component of the "lower" bounding point $\mathbf{x}_l$, and analogously for the x_2 component and for the upper point. Suppose we have reliable prior information that the density is zero outside the box defined by $\mathbf{x}_l = \binom{-6}{-6}$ and $\mathbf{x}_u = \binom{+6}{+6}$. Write a program that calculates $p(\mathbf{x}|\mathcal{D})$ via recursive Bayesian estimation and apply it to the $x_1 - x_2$ components of ω_1, in sequence, from the table above. For each expanding data set $\mathcal{D}^n$ $(2 \le n \le 10)$ plot your posterior density.