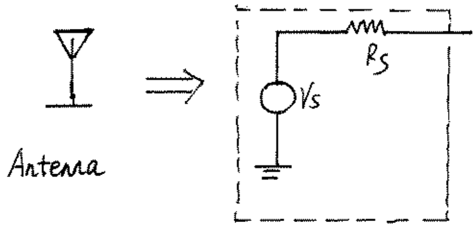
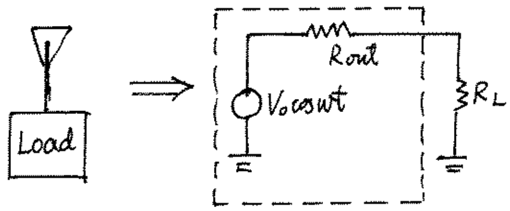
1)



Thevenin Equivalent:
$V_S = V_0 \cos \omega t$
$R_S = R_{out}$



Average power delivered to load $= (I_{RMS})^2 R_L$,

$$I_{RMS} = \frac{V_{RMS}}{R_{out}+R_L}, \quad V_{RMS} = \frac{V_0}{\sqrt{2}} \Rightarrow I_{RMS} = \frac{V_0}{\sqrt{2}\,(R_{out}+R_L)}$$

$$\text{Average power} = (I_{RMS})^2 R_L = \frac{V_0^2 R_L}{2(R_{out}+R_L)^2} \qquad \text{(Eq. 1)}$$

Plot of Average Power

When R_L is small, Eq. 1 is small.

When R_L is large, Eq. 1 is also small.

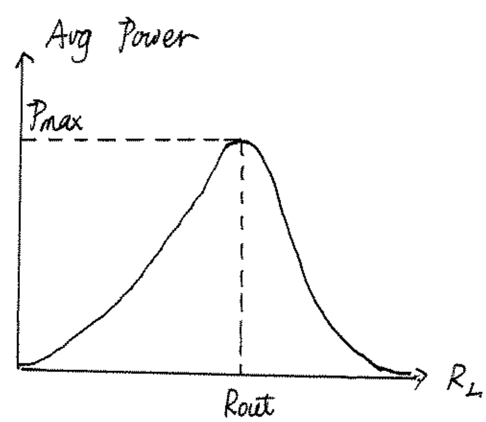
So for some R_L between zero and infinity, the average power will reach its peak. Let's take the derivative of Eq. 1 with respect to R_L to find the optimum R_L.

$$\frac{\partial}{\partial R_L}\left[\frac{V_0^2 R_L}{2(R_{out}+R_L)^2}\right] = \frac{V_0^2}{2(R_{out}+R_L)^2} - \frac{V_0^2 R_L}{(R_{out}+R_L)^3}$$

Setting it to zero and solve for R_L

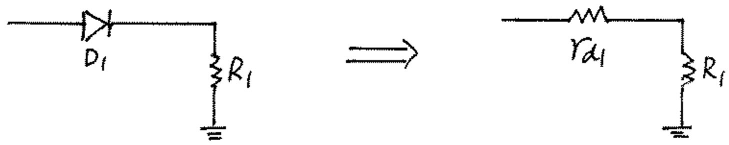
$$\frac{V_0^2}{2(R_{out}+R_L)^2} = \frac{V_0^2 R_L}{(R_{out}+R_L)^3} \Rightarrow \frac{(R_{out}+R_L)}{2} = R_L$$

$$\Rightarrow R_{out}+R_L = 2R_L \Rightarrow R_L = R_{out}$$



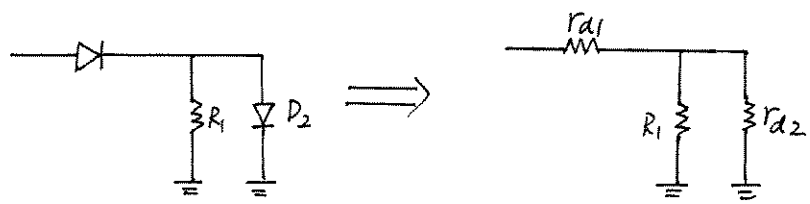
2) In small signal operation, a diode can be replaced by a linear resistor if charges are small.

a).



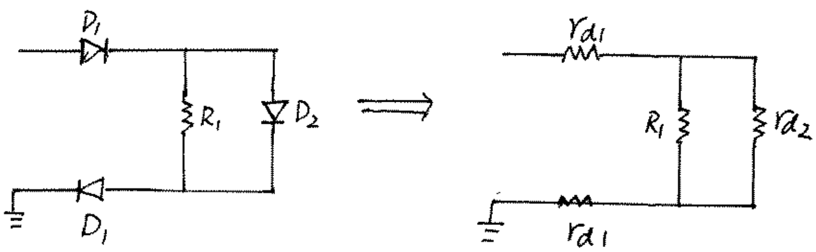
$R_{in} = r_{d1} + R_1$

b).



$R_{in} = r_{d1} + R_1 // r_{d2}$ (// means in parallel)

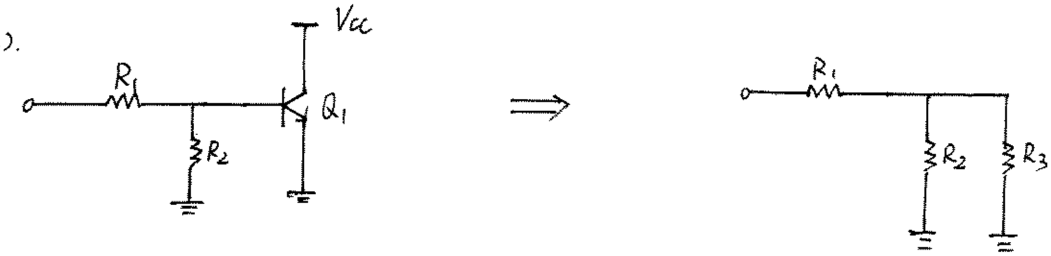
c).



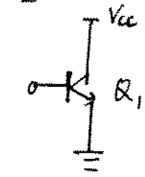
$R_{in} = 2r_{d1} + R_1 // r_{d2}$

3). When $V_A = \infty$, $V_o = \infty$.

a).



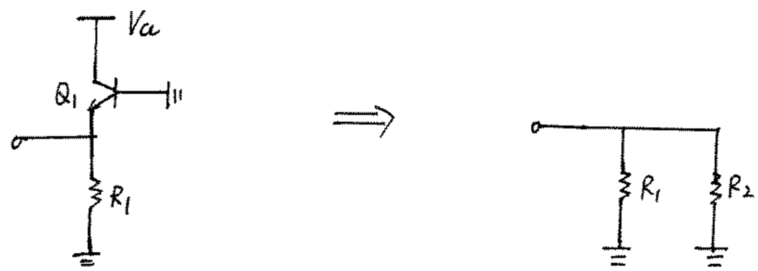
Replacing Q_1 by its equivalent resistance seen at base



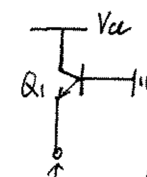
$R_3 = r_{\pi 1}$

So $R_{in} = R_1 + R_2 // R_3 = R_1 + R_2 // r_{\pi 1}$

b).



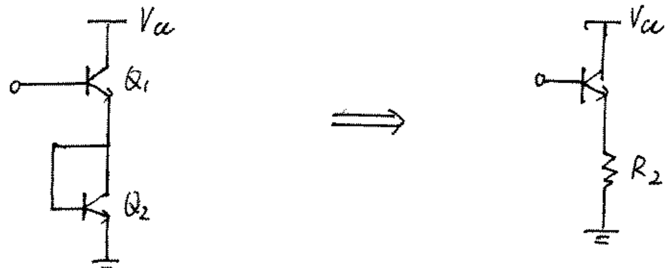
Replacing Q_1 by its equivalent resistance see at emitter



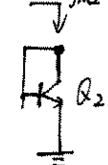
$R_2 = \frac{1}{g_{m1}} // r_{\pi 2}$

So $R_{in} = R_1 // R_2 = R_{in} // (\frac{1}{g_{m1}} // r_{\pi 1})$

c).



$R_2 = (\frac{1}{g_{m2}} // r_{\pi 2})$

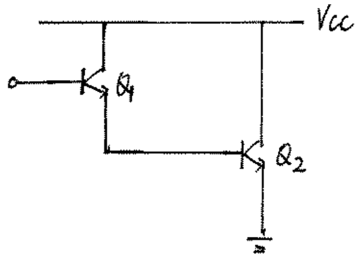


Replacing Q_2 by its equivalent diode-connected resistante

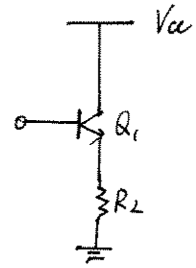
So $R_{in} = r_{\pi 1} + (1+\beta) R_2 = r_{\pi 1} + (1+\beta)(\frac{1}{g_{m2}} // r_{\pi 2})$

3).

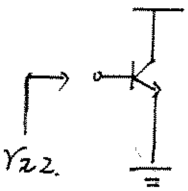
d).



$\Longrightarrow$



Replacing Q_2 by its equivalent resistance seen at Base.



$R_2 = r_{\pi 2}$.

So $R_{in} = r_{\pi 1} + (1+\beta) R_2 = r_{\pi 1} + (1+\beta) r_{\pi 2}$.

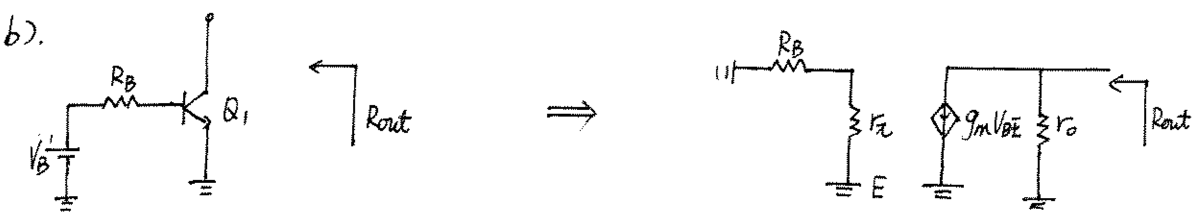
(Please refer to the textbook for all the equivalent resistances)

4). Since the problem doesn't say $V_A = \infty$, r_o must be considered in derivation.

a). Short V_B since it's a DC source, and replace Q_1 with an ideal transistor with it's output resistance.

So $R_{out} = R_1 // r_o // \infty = R_1 // r_o$

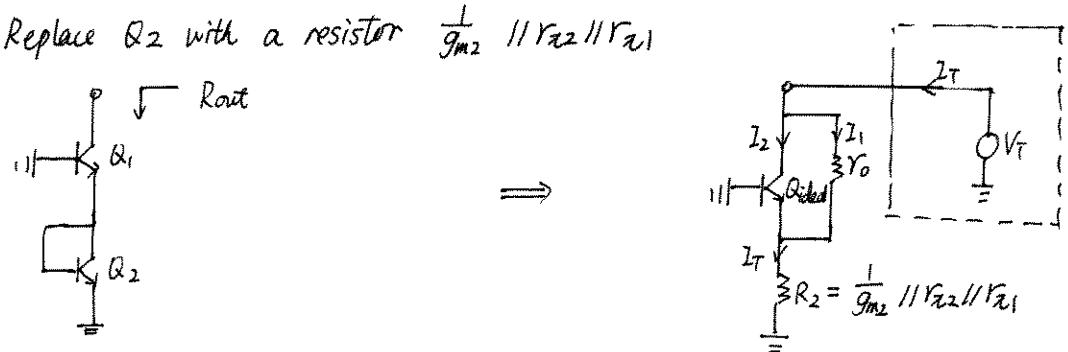
b).



By drawing the small-signal model, it's easy to tell $V_{BE} = 0$ and $R_{out} = r_o$

c). Replace Q_1 with an ideal transistor and an output impedance r_{o1}.

Replace Q_2 with a resistor $\frac{1}{g_{m2}} // r_{\pi 2} // r_{\pi 1}$



Here, $r_{\pi 1}$ is included in R_2 because it is also connected from emitter to ground and it accounts for the base current of Q_1.

4) $I_1 = \dfrac{V_T - I_T R_2}{r_o}$, $I_2 = g_{m1}(0 - I_T R_2)$

$$I_T = I_1 + I_2 = \frac{V_T - I_T R_2}{r_o} - g_{m1} I_T R_2$$

$$\Rightarrow I_T + \frac{I_T R_2}{r_o} + g_m I_T R_2 = \frac{V_T}{r_o}$$

$$\Rightarrow \frac{V_T}{I_T} = r_o\left(1 + \frac{R_2}{r_o} + g_{m1} R_2\right)$$

$$\Rightarrow R_{out} = \frac{V_T}{I_T} = r_o(1 + g_{m1} R_2) + R_2$$

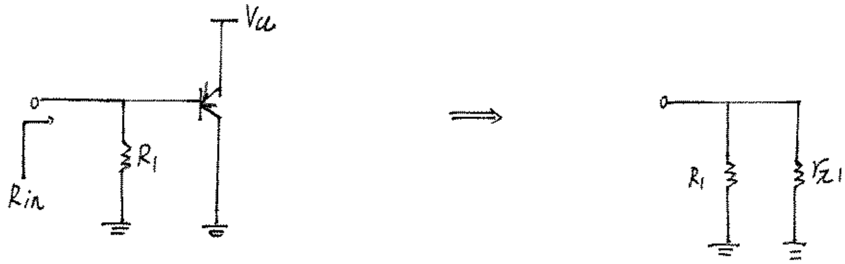
$$= r_o\left[1 + g_{m1}\left(\frac{1}{g_{m2}} // r_{\pi 2} // r_{\pi 1}\right)\right] + \frac{1}{g_{m2}} // r_{\pi 2} // r_{\pi 1}$$

Usually $\frac{1}{g_m} << r_\pi$, and if $Q_1 = Q_2$

$$R_{out} \approx \frac{1}{g_m} + 2 r_o$$

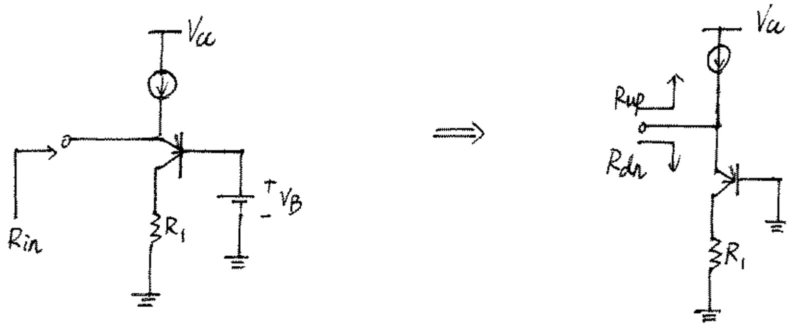
5). $V_A = \infty$, $r_o = \infty$

a).



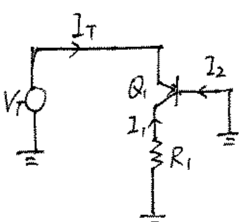
$R_{in} = R_1 // r_{\pi 1}$

b).



$R_{in} = R_{up} // R_{dn}$. $R_{up} = \infty$, since a DC current source is open.

Finding R_{dn}:



$$I_T = -(I_1 + I_2)$$

$$I_1 = g_m(0 - V_T) = -g_m V_T$$

$$I_2 = \frac{I_1}{\beta}.$$

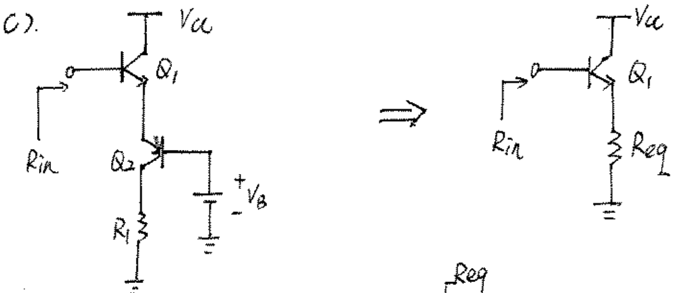
So $$I_T = -\left(-g_m V_T - \frac{g_m V_T}{\beta}\right) = \left(g_m + \frac{g_m}{\beta}\right) V_T$$

$$\frac{V_T}{I_T} = \frac{1}{\left(g_m + \frac{g_m}{\beta}\right)} = \frac{1}{g_m} // r_\pi$$

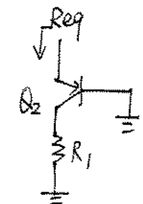
$$R_{dn} = \frac{V_T}{I_T} = \frac{1}{g_m} // r_\pi$$

So $$R_{in} = R_{up} // R_{dn} = \infty // \frac{1}{g_m} // r_\pi = \frac{1}{g_m} // r_\pi$$

5). c).



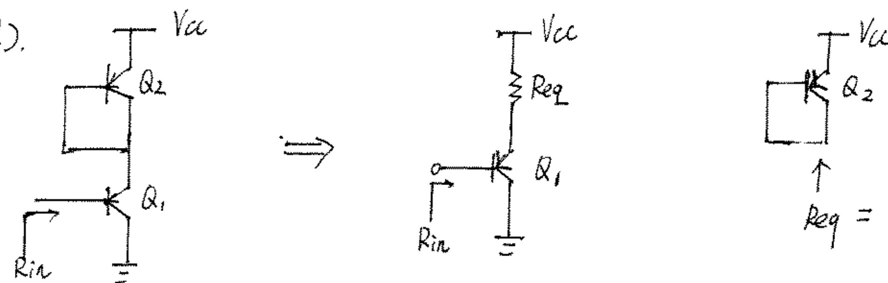
From b), we know that



$R_{eq} = \frac{1}{g_{m2}} // r_{\pi 2}$.

So $R_{in} = r_{\pi 1} + (1+\beta) R_{eq} = r_{\pi 1} + (1+\beta)\left(\frac{1}{g_{m2}} // r_{\pi 2}\right)$.

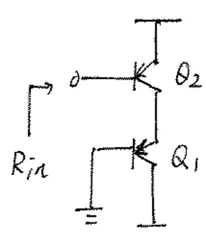
d).



$R_{eq} = \frac{1}{g_{m2}} // r_{\pi 2}$

$R_{in} = r_{\pi 1} + (1+\beta) R_{eq} = r_{\pi 1} + (1+\beta)\left(\frac{1}{g_{m2}} // r_{\pi 2}\right)$

e)

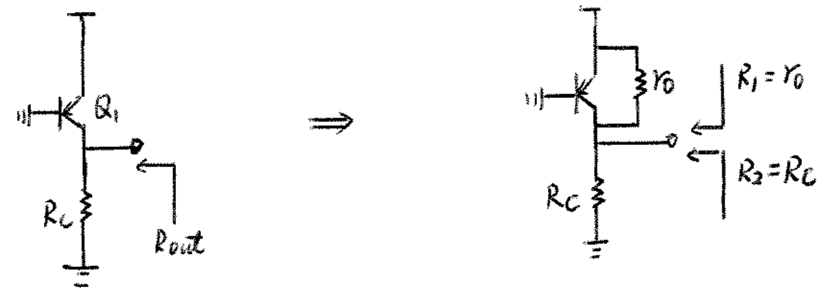


$R_{in} = r_{\pi 2}$. Q_1 plays no role here since it's connected to the collector of Q_2.

It can not be seen from the Base of Q_2.

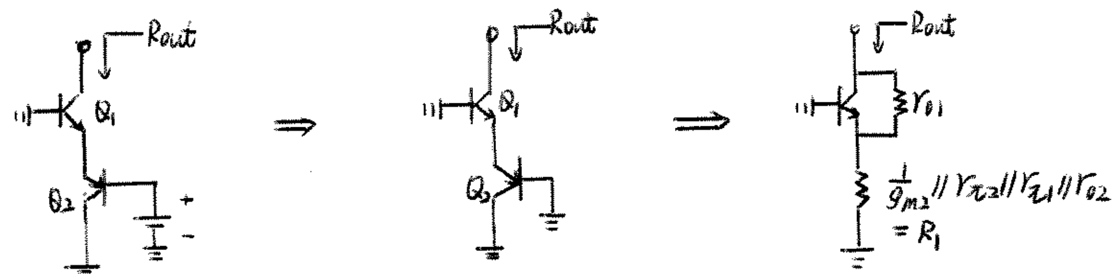
b) Since the problem doesn't state $V_A = \infty$, r_o is not ∞.

a)



$$R_{out} = R_1 // R_2 = R_C // r_o$$

b)

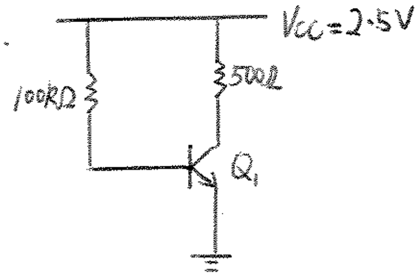


As shown is problem 4).c).

$$R_{out} = R_1 + r_{o1} + g_{m2} r_{o1} R_1 = r_{o1} + (1 + g_{m1} r_{o1}) R_1$$

$$= r_{o1} + (1 + g_{m1} r_{o1}) \left(\frac{1}{g_{m2}} // r_{\pi 2} // r_{\pi 1} // r_{o2} \right)$$

7) a).



$$I_B = \frac{V_{cc} - V_{BE}}{100\ k\Omega}, \quad I_c = \beta I_B$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right)$$

Guess $V_{BE} = 0.7V$,

$$I = \beta\left(\frac{V_{cc} - V_{BE}}{100k\Omega}\right) = 1.8mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.747V$$, not $0.7V$, reiterate

$V_{BE} = 0.747V$, $I_c = 1.753mA$

Verify V_{BE}, $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.746V$, converged

$$V_{CE} = 2.5 - (1.753)(0.5k) = 1.62V$$

$V_{CE} > V_{BE}$, Q_1 in forward active region.

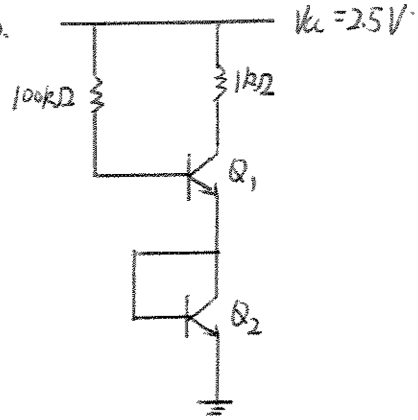
$I_c = 1.754mA$ $\quad V_{CE} = 1.62V$

$I_B = 17.54\mu A$ $\quad V_{BE} = 0.746V$

↑

operating point

7). b).



$$I_{B1} = \frac{2.5 - (V_{BE1} + V_{BE2})}{100K\Omega}$$

$$I_{C1} = \beta I_{B1}$$

$$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right), \quad V_{BE2} = V_T \ln\left(\frac{I_{C2}}{I_S}\right)$$

Assume $V_{BE1} = V_{BE2} = 0.8V$

$$I_{C1} = \beta\left(\frac{2.5 - 1.6}{100K}\right) = 0.9mA$$

$$V_{BE1} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.728V$$, not 0.8V, reiterate

$I_{C2} = 0.9mA$, since β's are the same

$V_{BE2} = 0.728V$

$$I_{C1} = \beta\left(\frac{2.5 - (2)(0.728)}{100K\Omega}\right) = 1.042mA = I_{C2}$$

$$V_{BE1} = V_{BE2} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.733V$$, iterate once more

$$I_{C1} = I_{C2} = \beta\left(\frac{2.5 - (2)(0.733)}{100K\Omega}\right) = 1.034mA$$

7) b)

$$I_{C_1} = I_2 = 1.034\ mA$$

$$V_{BE_1} = V_{BE_2} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.733, \quad \text{Converges.}$$

$$V_{CE_1} = 2.5 - 0.733 - (1.034)(1K\Omega) = 0.733V$$

$V_{CE} = V_{BE}$, Q_1 at the edge of active region.

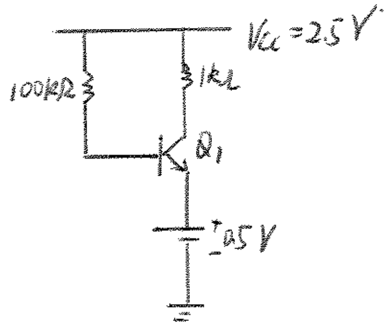
$$V_{BE_2} = V_{CE_2} = 0.733V$$

Operating Point:

$I_{C_1} = 1.034mA$	$I_{C2} = 1.034mA$
$I_{B_1} = 0.01\ mA$	$I_{B2} = 0.01\ mA$
$V_{BE_1} = 0.733\ V$	$V_{BE_2} = 0.733\ V$
$V_{CE_1} = 0.733V$	$V_{CE_2} = 0.733V$

Although, for Q_2 $V_{BE} = V_{CE}$, it is at the edge of active region, the situation is not as severe as Q_1's. Since Q_2's configuration will always render $V_{BE} = V_{CE}$, whereas for Q_1, V_{CE} may drop below V_{BE}.

7) c).



$$I_B = \frac{V_{cc} - (V_{BE} + 0.5)}{100K}$$

$$I_c = \beta I_B$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right)$$

Guess $V_{BE} = 0.8V$,

$$I_c = \beta\left(\frac{2.5-1.3}{100K}\right) = 1.2\,mA$$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.736V$, not 0.8, reiterate

$V_{BE} = 0.736V$, $I_c = \beta\left(\frac{2.5-(0.736+0.5)}{100K\Omega}\right) = 1.26\,mA$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.738V$, converges.

$$V_{CE} = (2.5-0.5) - (1.26)(1K\Omega) = 0.74$$

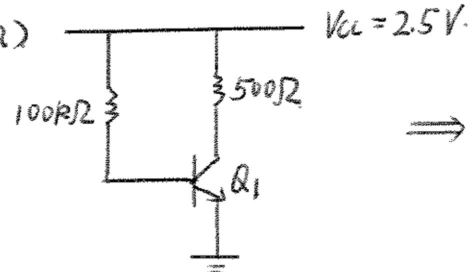
$V_{CE} > V_{BE}$, Q_1 in forward active region

Operating point

$I_c = 1.26mA$ $\quad V_{BE} = 0.738V$

$I_B = 0.0126mA$ $\quad V_{CE} = 0.74V$

8). a)



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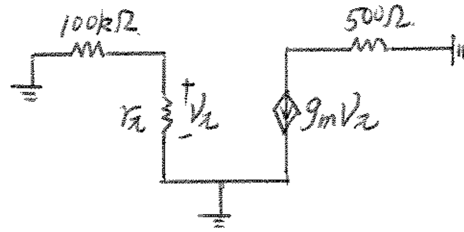
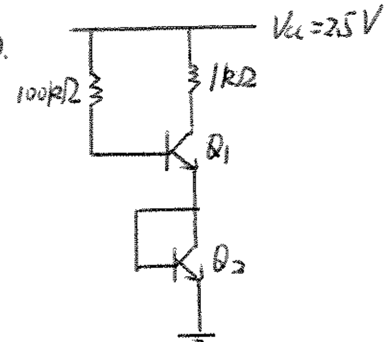


$$g_m = \frac{I_c}{V_T} = \frac{1.754mA}{26mV} = 0.0675\ S$$

$$r_{\pi 1} = \frac{\beta}{g_m} = \frac{100}{0.0675}\ \Omega = 1482.3\ \Omega$$

b).



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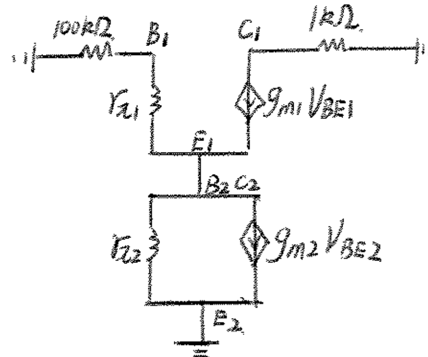


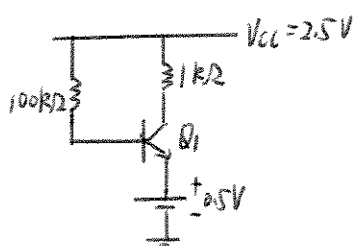
$$g_{m1} = \frac{I_c}{V_T} = \frac{1.034mA}{26mV} = 0.04\ S$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{0.04}\ \Omega = 2500\ \Omega$$

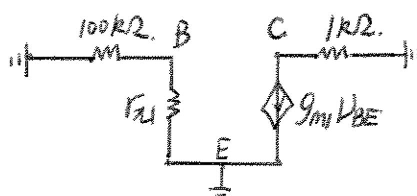
$$g_{m2} = \frac{I_c}{V_T} = \frac{1.034mA}{26mV} = 0.04\ S$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} = \frac{100}{0.04}\ \Omega = 2500\ \Omega$$

c).



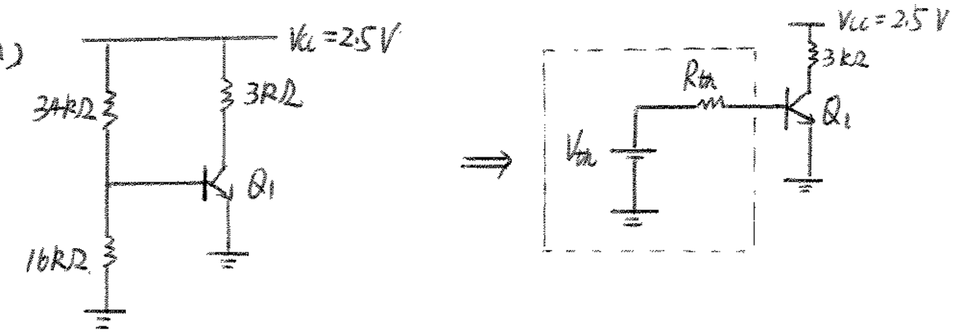
$\Longrightarrow$



$$g_{m1} = \frac{I_c}{V_T} = \frac{1.26mA}{26mV} = 0.048\ S$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{0.048}\ \Omega = 2083\ \Omega$$

9) a)



Thevenin Equivalent $\quad R_{th} = \frac{34 \times 16}{34+16} k\Omega = 10.88\,k\Omega$

$$V_{th} = \frac{2.5V \times 16}{34+16} = 0.8\,V.$$

$$I_c = \beta\left(\frac{0.8 - V_{BE}}{10.88K}\right), \quad V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right)$$

Assume $V_{BE} = 0.7$,

$$I_c = \beta\left(\frac{0.8-0.7}{10.88K\Omega}\right) = 0.92\,mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.734V$$

Iterate, $V_{BE} = 0.734V$

$$I_c = \beta\left(\frac{0.8-0.734}{10.88K\Omega}\right) = 0.61\,mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.724\,V$$

Iterate, $V_{BE} = 0.724\,V$

$$I_c = \beta\left(\frac{0.8-0.724}{10.88\,K\Omega}\right) = 0.699\,mA$$

9)

a) $I_c = 0.699mA$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.727V$$

Iterate, $V_{BE} = 0.727V$

$$I_c = \beta\left(\frac{0.8-0.727}{10.88K\Omega}\right) = 0.67mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.726V, \text{ Converged !!}$$

$V_{CE} = 2.5 - (0.67)(3K\Omega) = 0.49$

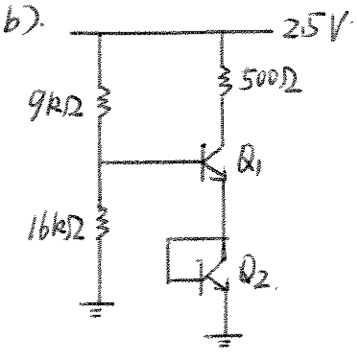
$V_{BE} - V_{CE} = 0.236V$, Soft-saturation, still ok.

Operating point:

$I_c = 0.67mA$ $V_{BE} = 0.726V$

$I_B = 6.7\mu A$ $V_{CE} = 0.49V$

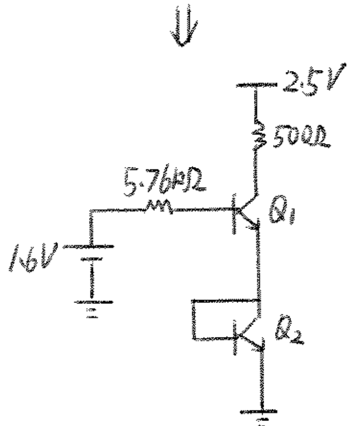
9) b).



$\Longrightarrow$

$$R_{th} = \frac{9 \times 16}{9+16}\,k\Omega = 5.76\,k\Omega$$

$$V_{th} = 2.5V \times \frac{16}{9+16} = 1.6V$$



$$I_{C_1} = \beta\left(\frac{1.6 - (V_{BE_1} + V_{BE_2})}{5.76\,k\Omega}\right)$$

$$V_{BE} = V_{BE_1} = V_{BE_2} = V_T \ln\left(\frac{I_c}{I_s}\right)$$

$$I_{C_1} = I_{C_2} = I_c$$

Guess $V_{BE1} = V_{BE2} = 0.7V$

$$I_c = \beta\left(\frac{1.6-1.4}{5.76\,k\Omega}\right) = 3.47\,mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.769V$$

Iterate, $V_{BE} = 0.769V$

$$I_c = \beta\left(\frac{1.6-(2)(0.769)}{5.76\,k\Omega}\right) = 1.08\,mA$$

9)

b)

$$I_c = 1.08mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.738V$$

Iterate, $V_{BE} = 0.738V$

$$I_c = \beta\left(\frac{1.6 - 2(0.738)}{5.76K\Omega}\right) = 2.15mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.756V$$

Iterate, $V_{BE} = 0.756V$

$$I_c = \beta\left(\frac{1.6 - 2(0.756)}{5.76K\Omega}\right) = 1.53mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.747V$$

Iterate ... (For 3 more times)

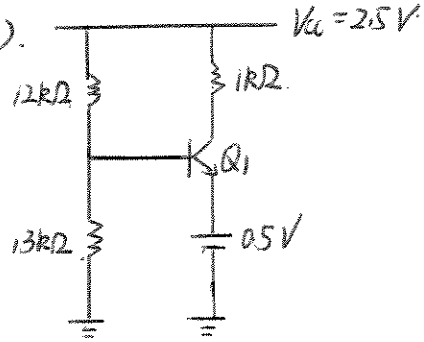
$V_{BE} = 0.75V$, $I_c = 1.74mA$ converged

$V_{CE} = 2.5 - 0.75 - (1.74)(0.5) = 0.88V$

Operating Point

$I_{c_1} = 1.74mA$		$I_{c_2} = 1.74mA$	
$I_{B_1} = 17.4\mu A$		$I_{B_2} = 17.4\mu A$	
$V_{BE} = 0.75V$	(Forward active)	$V_{BE_2} = 0.75V$	(Edge of forward active)
$V_{CE} = 0.88V$		$V_{CE_2} = 0.75V$	

9). c).

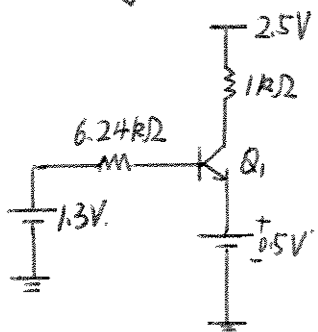


$\Rightarrow$

$$V_{th} = 2.5V \times \frac{13}{12+13} = 1.3V$$

$$R_{th} = \frac{12 \times 13}{12+13} k\Omega = 6.24k\Omega$$

$\Downarrow$



$$I_c = \beta \left(\frac{1.3 - (V_{BE} + 0.5)}{6.24 K\Omega} \right)$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right)$$

Guess $V_{BE} = 0.743V$

$$I_c = \beta \left(\frac{1.3 - (0.743 + 0.5)}{6.24 K\Omega} \right) = 0.913 mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.734V$$

Iterate, $V_{BE} = 0.734V$

$$I_c = \beta \left(\frac{1.3 - (0.734 + 0.5)}{6.24 K\Omega} \right) = 1.06 mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.738V$$

9)

c)

Iterate, $V_{BE} = 0.738V$

$$I_c = \beta\left(\frac{1.3-(0.738+0.5)}{6.24K\Omega}\right) = 0.99mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.736V$$

$$V_{CE} = 2.5-0.5-(0.99)(1K\Omega) = 1.01V$$

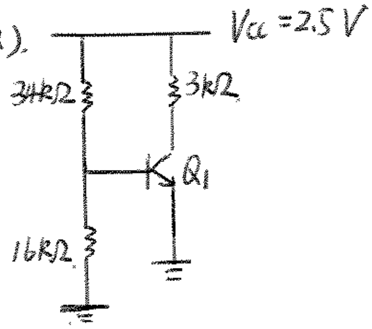
$V_{CE} > V_{BE} \Rightarrow$ Forward Active Region

Operating Point

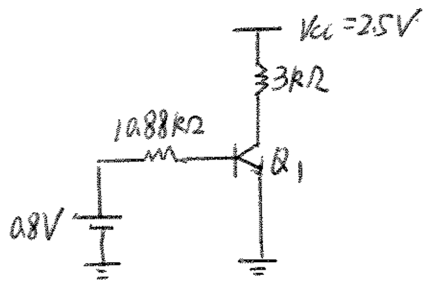
$I_c = 0.99mA$ $\qquad$ $V_{BE} = 0.736V$

$I_B = 9.9\mu A$ $\qquad$ $V_{CE} = 1.01V$

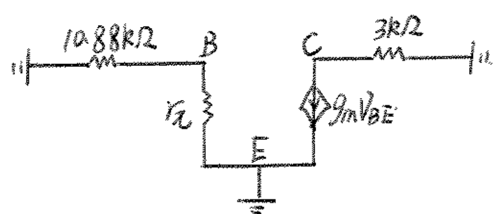
10). a).



$\Longrightarrow$



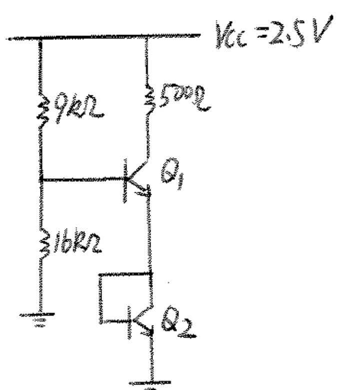
$\Longrightarrow$



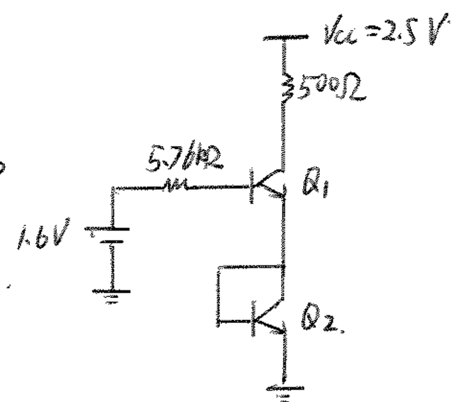
$$g_m = \frac{I_c}{V_T} = \frac{0.67mA}{26mV} = 0.026\,S$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.026}\,\Omega = 3846\,\Omega$$

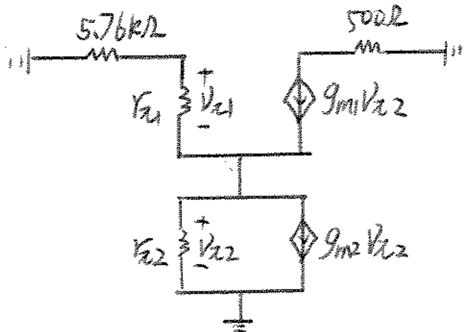
b).



$\Longrightarrow$



$\Longrightarrow$



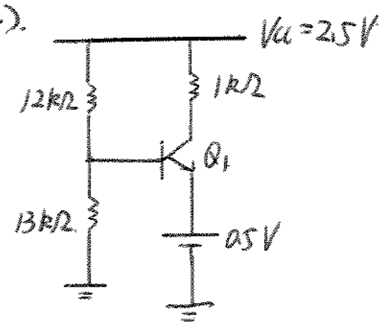
$$g_{m1} = \frac{1.74mA}{26mV} = 0.067S$$

$$r_{\pi1} = \frac{\beta_1}{g_{m1}} = 1494.3\,\Omega$$

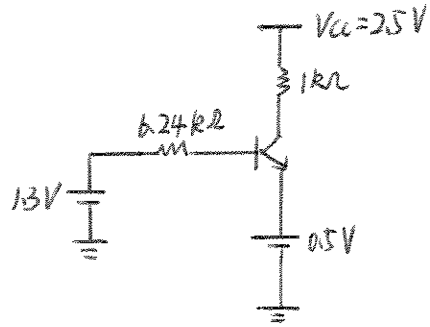
$$g_{m2} = 0.067S$$

$$r_{\pi2} = 1494.3\,\Omega.$$

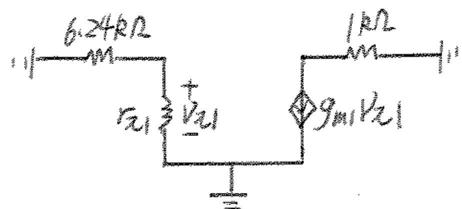
10) c).



$\Longrightarrow$



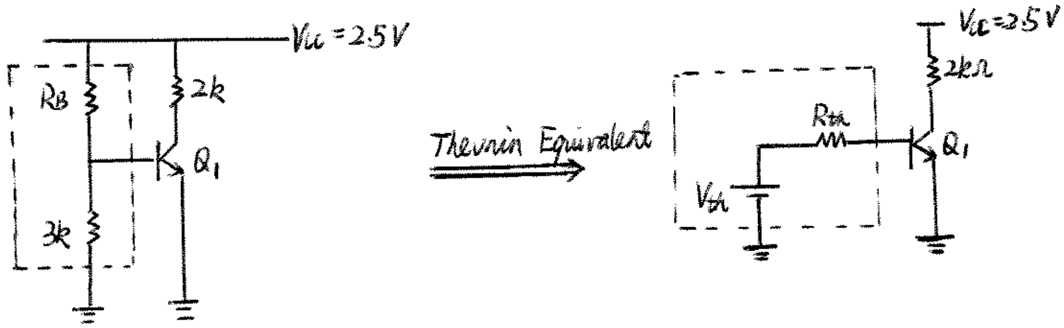
$\Longrightarrow$



$$g_{m1} = \frac{I_C}{V_T} = \frac{0.99mA}{26mV} = 0.038\,S$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{0.038}\Omega = 2632\Omega.$$

11). a). Find the minimum R_B that guarantees forward active region.



$$R_{th} = \frac{R_B \times 3}{R_B + 3}, \qquad V_{th} = \frac{2.5 \times 3}{R_B + 3}$$

To maintain Q_1 in forward-active region, $V_{CE} \geq V_{BE}$ (*)

$$V_{CE} = V_{cc} - I_C \cdot 2k, \quad I_C = \beta I_B, \quad I_B = \frac{V_{th} - V_{BE}}{R_{th}}$$

So $V_{CE} = V_{cc} - \beta \left(\frac{V_{th} - V_{BE}}{R_{th}} \right) \cdot 2k$

From (*)

$$V_{cc} - \beta \left(\frac{V_{th} - V_{BE}}{R_{th}} \right) \cdot 2k \geq V_{BE} \qquad (1)$$

And $V_{BE} = V_T \ln(I_C / I_S) = V_T \ln[\beta (V_{th} - V_{BE}) / R_{th} / I_S]$ (2)

Find the minimum R_B by iteration. Guess $V_{BE} = 0.8$ as initial condition.

Use $V_{BE} = 0.8$, and subsitute R_{th} and V_{th} into (1), it can be calculated

$$R_B \geq 6.178\,k$$

Check the validity of V_{BE}. With $R_B \geq 6.178k$, from (2)

$$V_{BE} = 0.727V$$

So the initial guess of V_{BE} is not accurate.

Reiterate with $V_{BE} = 0.727$, it can be calculated from (1)

$$R_B \geq 7.058\,k$$

II) With $R_B \geqslant 7.06k$, from ②

$$V_{BE} = 0.728$$

It's very close to 0.727. So the results have converged. (Satisfy both ① & ②)

The final answer is

$$R_B \geqslant 7.06k$$

b). β changes from 100 to 200, so $\partial\beta$ is 100

$$V_{CB} = 2.5 - I_C(2k) - V_{BE} = 2.5 - \left(\frac{V_{th} - V_{BE}}{R_{th}}\right)\beta \cdot (2k) - V_{BE}$$

$$\frac{\partial V_{CB}}{\partial \beta} = -\left(\frac{V_{th} - V_{BE}}{R_{th}}\right) \cdot 2k$$

$$\partial V_{CB} = -\left(\frac{V_{th} - V_{BE}}{R_{th}}\right) \cdot 2k \cdot (\partial\beta) = -1.6627$$

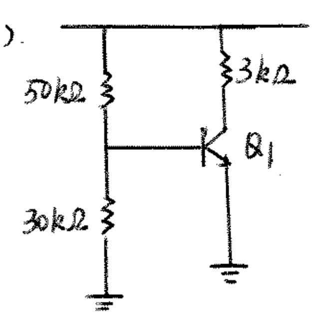
(Forward bias sustained during β's rising : 1.663V)

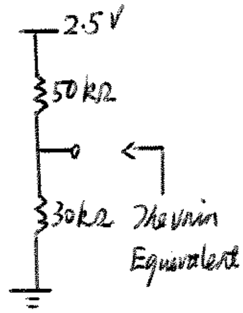
Original $V_{CB} = 0.01428$

Total net forward bias after β has rose to 200 :

$$1 - 1.6627 + 0.01428 = 1.648 \text{ (V)}$$

12). a).





$V_{th} = 2.5 \times \frac{30}{50+30}$
$= 0.9375$

$R_{th} = \frac{30 \times 50}{30+50} k$
$= 18.75 k$

Since $I_C = 0.5mA$, $I_B = \frac{I_C}{\beta} = 0.005mA$.

$$I_B = \frac{V_{th} - V_{BE}}{R_{th}} \Rightarrow V_{BE} = V_{th} - I_B \cdot R_{th} = 0.84375$$

$$I_C = I_S \, e^{(\frac{V_{BE}}{V_T})} \Rightarrow I_S = \frac{I_C}{e^{(V_{BE}/V_T)}} = 4.03 \times 10^{-15} (mA)$$

b). At the edge of saturation means $V_{BE} - V_{CE} = 0$.

(soft saturation not allowed)

$V_{CE} = 2.5 - I_C \cdot (3k)$, in which $I_C = \beta I_B = \beta (\frac{V_{th} - V_{BE}}{R_{th}})$

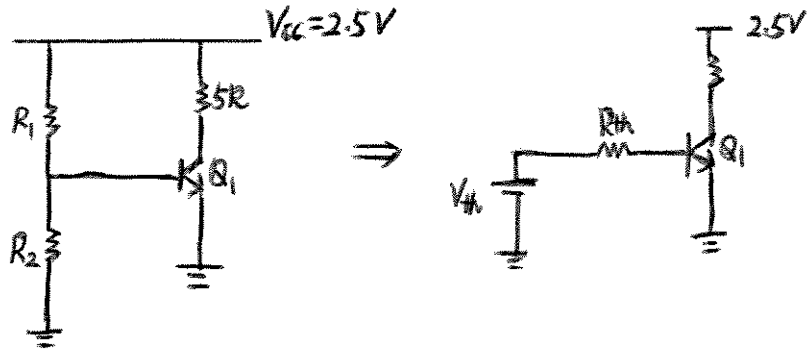
so $V_{BE} = 2.5 - \beta(\frac{V_{th} - V_{BE}}{R_{th}}) \cdot (3k)$

Solve this equation:

$$V_{BE} = 0.83.$$

$$I_S = \frac{I_C}{e^{(V_{BE}/V_T)}} = \frac{\beta \left(\frac{V_{th} - V_{BE}}{R_{th}}\right)}{e^{(V_{BE}/V_T)}} = 7.84 \times 10^{-15} (mA)$$

13)



$V_{th} = 2.5 \times \frac{R_2}{R_1+R_2}$

$R_{th} = \frac{R_1R_2}{R_1+R_2}$

$R_{in} = R_{th} \parallel r_\pi = R_{th} \parallel \frac{\beta}{g_m} = R_{th} \parallel V_T\beta/I_C > 10k\Omega$

$g_m \geq \frac{1}{260\Omega} = 0.0038S$

Let's choose g_m to be $0.0038S$

$g_m = \frac{I_C}{V_T} \Rightarrow I_C = g_m V_T = 0.104(mA)$

$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.761(V)$

Let $R_{in} = 10k\Omega$

$R_{in} = R_{th} \parallel V_T\beta/I_C \Rightarrow R_{th} = 16.13k\Omega$ ①

$I_B = \frac{I_C}{\beta} = \frac{V_{th} - V_{BE}}{R_{th}} \Rightarrow V_{th} = V_{BE} + \frac{I_C \cdot (R_{th})}{\beta} = 0.78V$ ②

② $\Rightarrow 2.5 \times \frac{R_2}{R_1+R_2} = 0.78V$

① $\Rightarrow \frac{R_1R_2}{R_1+R_2} = 16.13k\Omega$

It can be solved that $R_1 = 51.7k\Omega$, $R_2 = 23.44k\Omega$

This is only one possible solution set. The thought process is more important.

14). If g_m at least $\frac{1}{26} = 0.03848\ (\Omega^{-1})$

Let $g_m = 0.03848 = \frac{I_C}{V_T} \Rightarrow I_C = 0.99996\ (mA)$

$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.82\ V$

$V_{CE} = V_{CC} - I_C \cdot 5k = -2.5$

No solution exists because the transistor is in saturation mode

where g_m is essentially zero.

Whereas for problem 13),

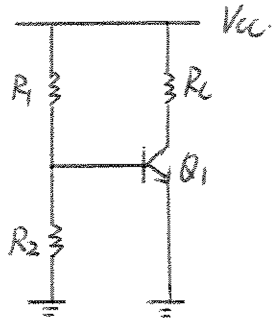
$V_{CE} = V_{CC} - I_C \cdot 5k = 2.5 - 0.104 \times 5 = 1.98\ V$

$V_{BE} = 0.761$

$V_{CE} > V_{BE}$

So Q_1 is still in forward-active region.

15).



$$\text{Gain} = A_0$$

$$R_{out} = R_0 = R_C$$

$$R_{in} = \frac{R_1 R_2}{R_1 + R_2} \,//\, r_\pi \quad , \quad r_\pi = \frac{\beta}{g_m}$$

$$R_{in} = \frac{R_1 R_2}{R_1 + R_2} \,//\, \frac{\beta}{g_m}$$

$$\text{Gain} = A_0 = g_m R_0 \Rightarrow g_m = \frac{A_0}{R_0} = \frac{I_c}{V_T} \Rightarrow I_c = \frac{A_0}{R_0} V_T$$

(I_c is set)

Bias point analysis:

$$\frac{\dfrac{V_{cc} R_2}{R_1 + R_2} - V_{BE}}{\dfrac{R_1 R_2}{R_1 + R_2}} = \frac{A_0}{R_0} V_T$$

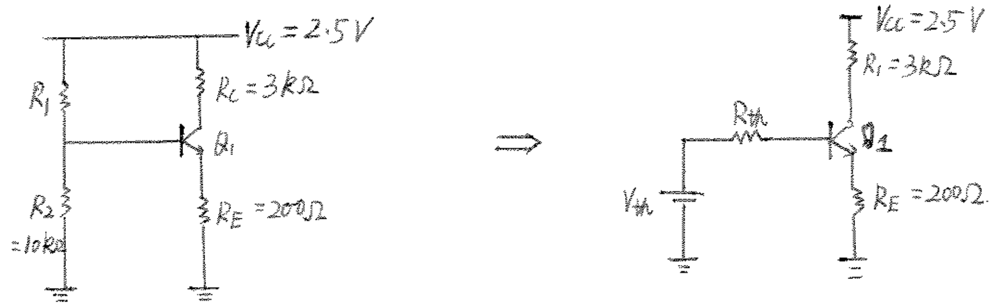
$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = V_T \ln\left(\frac{A_0 V_T}{R_0 I_s}\right)$$

15)
$$\frac{\dfrac{V_{cc}R_2}{R_1+R_2} - V_T \ln\left(\dfrac{A_0 V_T}{R_0 I_S}\right)}{\dfrac{R_1R_2}{R_1+R_2}} = \frac{A_0}{R_0}V_T$$

$$\frac{R_1R_2}{R_1+R_2} = \frac{\dfrac{V_{cc}R_2}{R_1+R_2} - V_T \ln\left(\dfrac{A_0 V_T}{R_0 I_S}\right)}{\dfrac{A_0}{R_0}V_T}$$

Max R_{in}:
$$\frac{\dfrac{V_{cc}R_2}{R_1+R_2} - V_T \ln\left(\dfrac{A_0 V_T}{R_0 I_S}\right)}{\dfrac{A_0}{R_0}V_T} \,//\, \beta\frac{R_0}{A_0}$$

16).



a) $V_{th} = V_{cc} \cdot \frac{R_2}{R_1 + R_2} = 2.5 \times \frac{10k}{10k + R_1}$,

$R_{th} = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 \cdot (10k)}{R_1 + 10k}$

$V_{th} = I_B R_{th} + V_{BE} + I_E R_E$ (*)

Since $I_C = 0.25mA$, $I_B = 0.0025mA$, $I_E = \frac{0.25mA}{99} = 0.2525mA$.

$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.696V$

(*) becomes

$$2.5 \times \frac{10k}{10k + R_1} = 0.0025m \times \frac{R_1 \cdot (10k)}{R_1 + 10k} + 0.696 + 0.2525 \times 0.2$$

So

$R_1 = 22.73k$.

b) If R_E deviates by 5%, changes in R_E is 10Ω.

$$I_B = \frac{V_{th} - (V_{BE} + I_E R_E)}{R_{th}} \Rightarrow \frac{I_C}{\beta} = \frac{V_{th} - (V_{BE} + \frac{I_C}{\alpha} R_E)}{R_{th}}$$

$$\Rightarrow I_C = \frac{\beta\alpha\,(V_{th} - V_{BE})}{\alpha R_{th} + \beta R_E}$$

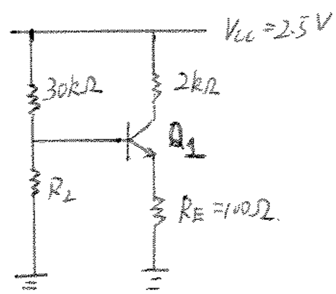
$$\Rightarrow \partial I_C = -\frac{\beta^2 \alpha\,(V_{th} - V_{BE})}{(\alpha R_{th} + \beta R_E)^2}\,\partial R_E$$

16) $\partial R_E = 10$, $V_{Th} = 0.764$, $V_{BE} = 0.7465$, $R_{Th} = 6.94k$, $\alpha = 0.99$, $\beta = 100$

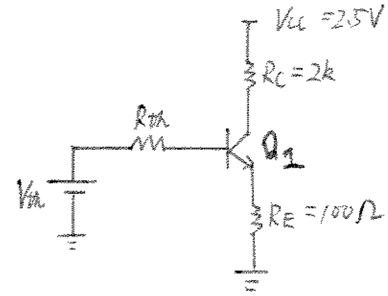
So $\partial I_C = -0.0024\ (mA)$

The error is $\frac{0.0024}{0.25} \times 100\% = 0.96\%$ in I_C.

17).



$\Longrightarrow$



$$V_{th} = \frac{R_2 \times 2.5}{30k + R_2}, \quad R_{th} = \frac{30k \times R_2}{30k + R_2}$$

$V_{CE} \geq V_{BE}$ (To be guaranteed in active mode, soft saturation is not allowed.)

$$V_{CE} = V_{CC} - (I_C \cdot 2k + I_E \cdot 100)$$

$$I_C = \frac{\beta\alpha(V_{th} - V_{BE})}{\alpha R_{th} + \beta R_E} \quad \left(\because I_C = \frac{V_{th} - (V_{BE} + \frac{I_C}{\alpha} R_E)}{R_{th}} \right)$$

$$\text{So } V_{CE} = V_{CC} - \left[\frac{\beta\alpha(V_{th} - V_{BE})}{\alpha R_{th} + \beta R_E} \cdot 2k + \frac{\beta(V_{th} - V_{BE})}{\alpha R_{th} + \beta R_E} \times 100 \right]$$

$V_{CE} \geq V_{BE}$ means

$$2.5 - \left[\frac{99\left(\frac{R_2 \times 2.5}{30k + R_2} - V_{BE}\right)}{0.99 \times \frac{30k \times R_2}{30k + R_2} + 100 \times 100} \times 2k + \frac{100\left(\frac{R_2 \times 2.5}{30k + R_2} - V_{BE}\right)}{0.99 \times \frac{30k \times R_2}{30k + R_2} + 100 \times 100} \times 100 \right] \geq V_{BE} \quad ①$$

And

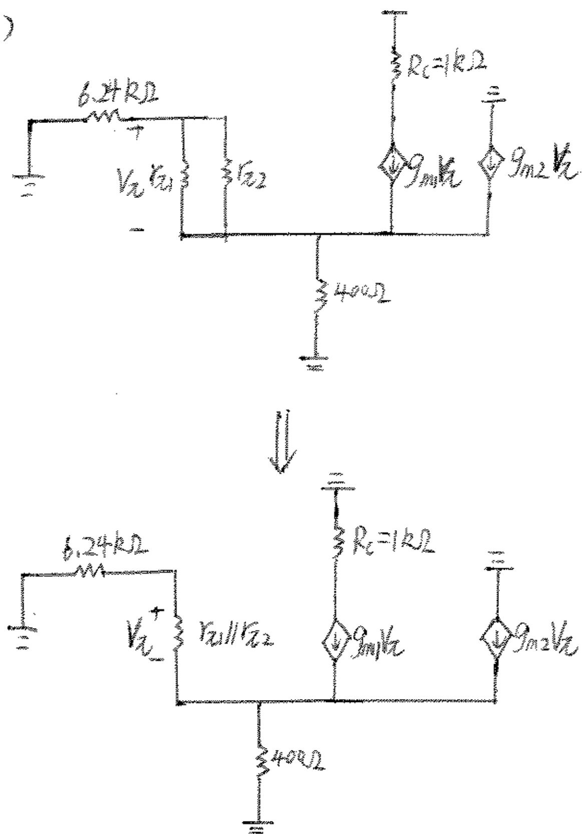
$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = V_T \ln\left[\frac{\beta\alpha(V_{th} - V_{BE})}{I_S(\alpha R_{th} + \beta R_E)}\right] \quad ②$$

There are two unknowns (R_2 and V_{BE}) and two equations (① and ②)

Since ② is a nonlinear equation, the problem can be solved by iteration.

Maximum $R_2 = 20.343k$.

18b)



Small - Signal

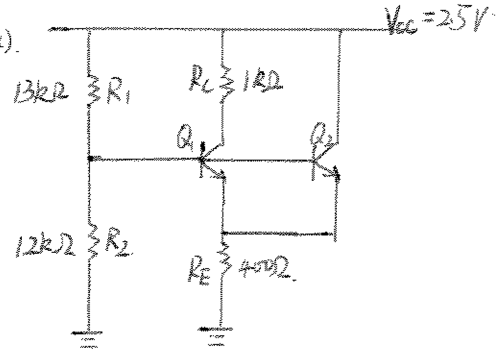
$$g_{m1} = \frac{I_C}{V_T} = 0.02855 \ (S)$$

$$g_{m2} = 0.0142 \ (S)$$

$$r_{\pi 1} = 3571.4 \ (\Omega)$$

$$r_{\pi 2} = 7042.3 \ (\Omega)$$

18) a).



$I_{S1} = 2 I_{S2} = 5\times10^{-16}\,A$

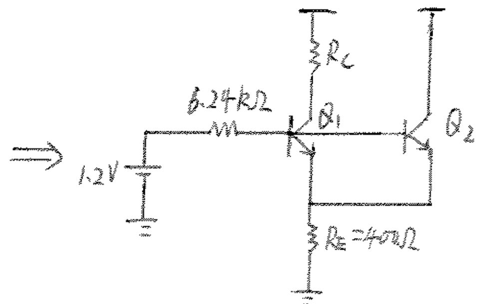
$\beta_1 = \beta_2 = 100$

$I_{S2} = 2.5\times10^{-16}\,A$

$V_{th} = V_{cc} \times \frac{R_2}{R_1+R_2} = 1.2\,V$

$R_{th} = R_1 \parallel R_2 = 6.24k\Omega$

$\Rightarrow$



$$I_{B2} = \frac{1.2 - (V_{BE} + 3I_{E2}\cdot R_E)}{6.24k}, \quad \text{and} \quad I_{B2} = \frac{I_{C2}}{\beta}$$

so $$\frac{I_{C2}}{\beta} = \frac{1.2 - (V_{BE} + 3I_{C2}/\alpha \cdot 0.4k)}{6.24k}$$

$\beta = 100, \ \alpha = 0.99$

$$\Rightarrow I_{C2} = \frac{(1.2 - V_{BE})\cdot(\beta\alpha)}{(\alpha\cdot 6.24k + 3\beta\cdot 0.4k)} = \frac{(1.2 - V_{BE})(99)}{126.1776}\ (mA)$$

Guess $V_{BE} = 0.8$, $I_{C2} = 0.314\,mA$

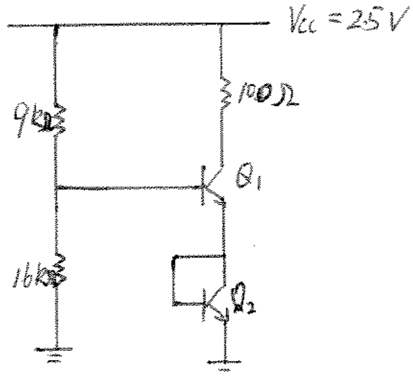
$V_{BE} = V_T \ln\left(\frac{I_C}{I_{S2}}\right) = 0.724$, not 0.8, so reiterate.

$$I_{C2} = \frac{(1.2 - 0.724)(99)}{126.1776} = 0.3735$$

$$V_{BE} = 26 \ln\left(\frac{0.3735}{2.5\times10^{-13}}\right) = 0.728,$$ close. iterate again

$\Rightarrow V_{BE} \approx 0.729\,(V)$, $I_{C2} = 0.371\,(mA)$, $I_{C1} = 0.74\,(mA)$

19).a)



$I_{S1} = I_{S2} = 4\times10^{-16}A$

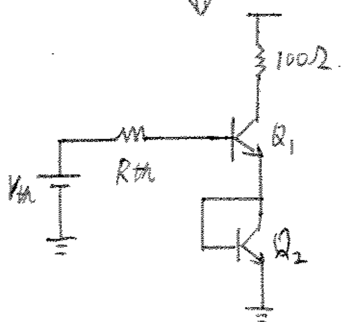
$\beta_1 = \beta_2 = 100$

$V_A = \infty$

$$V_{th} = \frac{(2.5)(16k)}{9k+16k} = 1.61(V)$$

$$R_{th} = 9k // 16k = 5.76k(\Omega)$$

$\Downarrow$



$$I_{B1} = \frac{V_{th} - 2(V_{BE})}{R_{th}} \quad , \quad I_{C1} = \beta I_{B1} = \beta \frac{V_{th} - 2(V_{BE})}{R_{th}} \qquad ①$$

$$V_{BE} = V_T \ln\left(\frac{I_{C1}}{I_{S1}}\right). \qquad ②$$

Guess $V_{BE} = 0.7$,

$① \Rightarrow I_{C1} = 100 \times \frac{1.6 - 2\times0.7}{5.76} = 3.47 (mA)$

$② \Rightarrow V_{BE} = V_T \ln\left(\frac{3.47}{4\times10^{-13}}\right) = 0.7746$, not 0.7, reiterate

$① \Rightarrow I_{C1} = 0.8819$

$② \Rightarrow V_{BE} = 0.739$, not 0.7746, reiterate ...

After several iterations, V_{BE} converges to $\underline{\underline{0.755}}$

19) a).

$V_{BE} = 0.755\ (V)$

$$I_{B1} = \frac{V_{th} - 2V_{BE}}{R_{th}} = 0.0156\ (mA)$$

$I_{C2} = \beta I_{B1} = 1.56 (mA)$

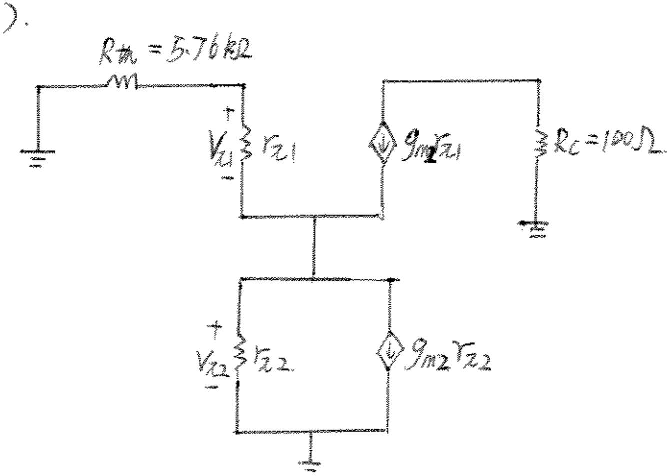
$V_{CE} = V_{CC} - [I_C \cdot (0.1) + V_{BE}] = 1.589\ (V)$

$I_{C2} = 1.56 (mA)$

$I_{B2} = (1/\beta) I_{C2} = 0.0156 (mA)$

$V_{CE2} = V_{BE} = 0.755 (\quad V)$

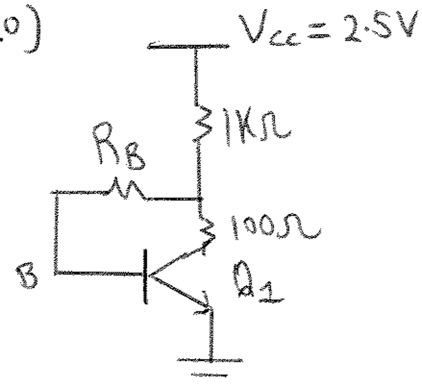
b).



$$g_{m1} = g_{m2} = \frac{I_C}{V_T} = 0.065\ (S)$$

$$r_{\pi 1} = r_{\pi 2} = \frac{\beta}{g_m} = 1666.7 (\Omega)$$

20)



$I_c = 1mA$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.750V$

$V_B = 2.5 - (I_E(1K\Omega) + I_B R_B) = 0.750V$

$I_E = 1.01mA$

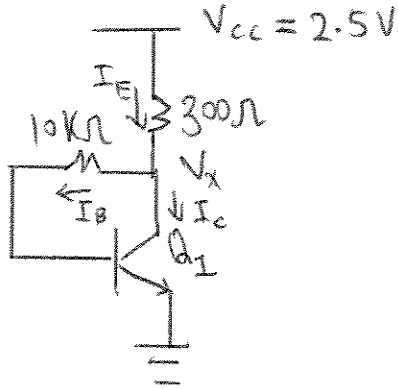
$I_B = 0.01mA$

$V_B = 2.5 - 1.01 - 0.01 R_B = 0.750$

$0.74 = 0.01 R_B$

$R_B = 74K\Omega$

21)



$V_X = 1.1V$

$\beta = 100$

$I_s = ?$

$I_E = I_B + I_c$

$$I_E = \frac{2.5 - 1.1}{300\Omega} = 4.67\,mA$$

$$I_B = \frac{I_c}{\beta}$$

$$I_E = \frac{I_c}{\beta} + I_c = 4.67\,mA$$

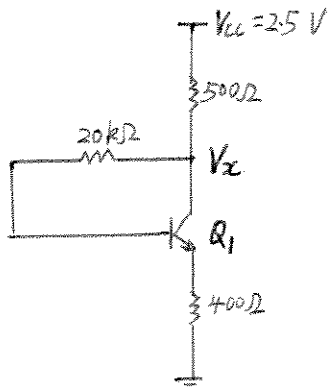
$I_c = 4.624\,mA$

$$I_s = \frac{I_c}{e^{\left(\frac{V_{BE}}{V_T}\right)}} \quad , \quad V_{BE} = 1.1 - \frac{4.624}{100}(10K) = 0.6376V$$

$I_s = 1.035 \times 10^{-10}\,mA$

$I_s = 1.035 \times 10^{-13}\,A$

22.



$I_S = 6 \times 10^{-16}$ A

$\beta = 100$

$V_A = \infty$

$$\frac{V_{CC} - V_x}{0.5k} = I_C + I_B = I_C\left(1 + \frac{1}{\beta}\right) \Rightarrow V_x = 2.5 - 0.5k \cdot \frac{I_C}{\alpha} \qquad ①$$

$$\frac{V_x - (V_{BE} + I_E \cdot 0.4k)}{20k} = \frac{I_C}{\beta} \Rightarrow V_x = (20k)\left(\frac{I_C}{\beta}\right) + V_{BE} + \frac{I_C}{\alpha} \cdot (0.4k) \qquad ②$$

Equating V_x in ① and ②

$$2.5 - (0.5k)\left(\frac{I_C}{\alpha}\right) = (20k)\left(\frac{I_C}{\beta}\right) + V_{BE} + \frac{I_C}{\alpha}(0.4k)$$

$$I_C = \frac{2.5 - V_{BE}}{\frac{0.9k}{\alpha} + \frac{20k}{\beta}} = \frac{2.5 - V_{BE}}{1.11k} \qquad ③$$

First iteration $V_{BE} = 0.8$.

③ $\Rightarrow$ $I_C = 1.53$ (mA)

$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.743$, not 0.8, reiterate

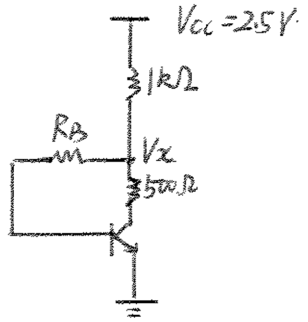
③ $\Rightarrow$ $I_C = \frac{2.5 - 0.743}{1.11} = 1.583$ (mA)

$V_{BE} = V_T \ln\left(\frac{1.583}{I_S}\right) = 0.744$, converged.

So. $V_{BE} = 0.74$ (V) $\quad I_C = 1.58$ (mA), $I_B = I_C/\beta = 0.0158$ (mA)

$V_C = 2.5 - \frac{1.583}{0.99} \times 0.5 = 1.7$ (V), $V_E = V_C - (I_B \cdot 20k + V_{BE}) = 0.644$ (V), $V_{CE} = V_C - V_E$
$= 1.056$ (V)

23).



$$I_c = \beta\left(\frac{2.5 - I_E(1K) - V_{BE}}{R_B}\right)$$

$$\frac{I_c R_B}{\beta} = 2.5 - I_E(1K) - V_{BE}$$
$$\left(I_E = \frac{I_c}{\alpha}\right)$$

$$I_c = \frac{2.5 - V_{BE}}{\frac{R_B}{\beta} + \frac{1K}{\alpha}} \quad (1)$$

$V_{BC} \leq 0.2V$

$$(V_x - I_B R_B) - (V_x - I_c 0.5) \leq 0.2V$$

$$I_c\left(0.5 - \frac{R_B}{\beta}\right) \leq 0.2V$$

$$\left(\frac{2.5 - V_{BE}}{\frac{R_B}{\beta} + \frac{1K}{\alpha}}\right)\left(0.5 - \frac{R_B}{\beta}\right) \leq 0.2V \quad (2)$$

Guess $V_{BE} = 0.75V \Rightarrow R_B \geq 34.513K\Omega$ (From (2))

Let $R_B = 34.513K\Omega$

$I_c = 1.291mA$, (From (1))

$V_{BE} = V_T \ln\left(\frac{I_c}{I_S}\right) = 0.7564V$, not 0.75, reiterate

$V_{BE} = 0.7564V \Rightarrow R_B \geq 34.461K\Omega$

23)

Let $R_B = 34.461\ k\Omega$

$I_c = 1.287\ mA$ (from (1))

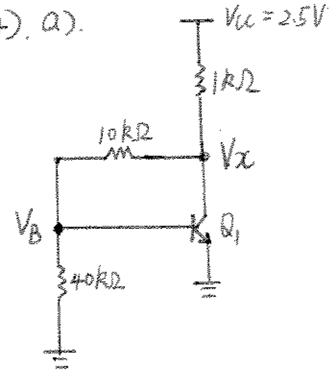
$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.7563\ V$, converged !!

So $I_c = 1.287mA$, $R_B = 34.46\ k\Omega$

Check V_{BC} : $V_{BC} = (1.287)(0.5) - \left(\frac{1.287}{100}\right)(34.46)$

$V_{BC} = 0.1999998$, less than $0.2V$

24). a).



$I_S = 8 \times 10^{-16} A$

$\beta = 100$

$V_A = \infty$

$V_x = 2.5 - \left(\frac{I_c}{\alpha} + \frac{V_B}{40k}\right) \cdot 1k$

$V_x = \left(\frac{V_B}{40k} + I_B\right) 10k + V_B = \left(\frac{V_B}{40k} + \frac{I_c}{\beta}\right) 10k + V_B$

Equating V_x $\Rightarrow$ $2.5 - \left(V_B + \frac{V_B \cdot 1k}{40k} + \frac{V_B \cdot 10k}{40k}\right) = \frac{I_c}{\alpha} \cdot 1k + \frac{I_c}{\beta} \cdot 10k$

$\Rightarrow$ $I_c = \dfrac{2.5 - 1.275 V_B}{\frac{1k}{\alpha} + \frac{10k}{\beta}}$

Guess $V_B = 0.8$

$$I_C = \frac{1.48}{\frac{1k}{0.99} + \frac{10k}{100}} = 1.33\, m(A)$$

Then

$$V_B = V_T \ln\left(\frac{I_c}{I_S}\right) = 0.732, \text{ not } 0.8$$

Reiterate

$$I_C = \frac{1.5667}{1.11} = 1.4113\,(mA)$$

$$V_B = V_T \ln\left(\frac{I_c}{I_S}\right) = 0.733$$

So V_B converges to $0.73V$

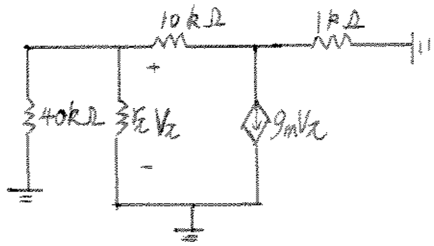
$I_C = 1.41\, mA$

$I_B = 14.1\, uA$

$V_{CE} = 2.5V - \left(\frac{1.41}{0.99} + \frac{0.73}{40}\right) \times 1V = 1.06V$

$V_{BE} = 0.73V$

24 b) Small Signal

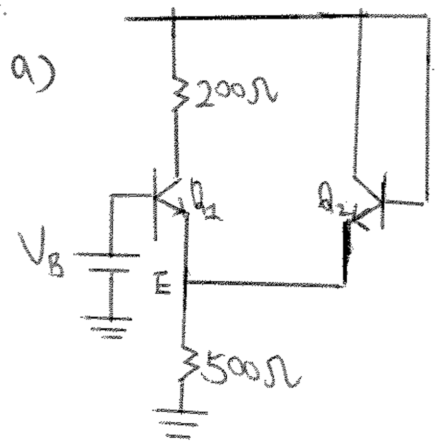


$$g_m = \frac{I_C}{V_T} = 0.054\ S$$

$$r_\pi = \frac{\beta}{g_m} = 1844\Omega$$

25).

a)



$I_{C_1} = 1mA$, $I_{E_1} = 1.01mA$

$I_{S_1} = I_{S_2} = 3\times10^{-16}A$

$V_A = \infty$

$\beta = 100$

$V_E = (I_{E_1} + I_{E_2})0.5K$, $V_{BE_1} = V_T \ln\left(\frac{I_{C_1}}{I_S}\right) = 0.75V$

$V_E = 2.5 - V_{BE2}$

$V_B - (1.01 + I_{E_2})0.5 = 0.75V.$

Guess $V_{BE_2} = 0.7V$

$V_E = 1.8 \Rightarrow I_{E_1} + I_{E_2} = 3.6mA \Rightarrow I_{E_2} = 2.59mA$

$I_{C_2} = 2.5641mA \Rightarrow V_{BE_2} = V_T \ln\left(\frac{I_{C2}}{I_S}\right) = 0.774V$

Reiterate

$V_E = 1.726 \Rightarrow I_{E_2} = 2.442mA,\ I_{C_2} = 2.4176mA$

$V_{BE_2} = V_T \ln\left(\frac{I_{C_2}}{I_S}\right) = 0.773$, Converged !!

$V_{BE} = 0.773V,\ I_{C_2} = 2.42mA,\ I_{E_2} = 2.44mA$

$V_B = 0.75 + (1.01 + 2.44)0.5 = 2.475V$

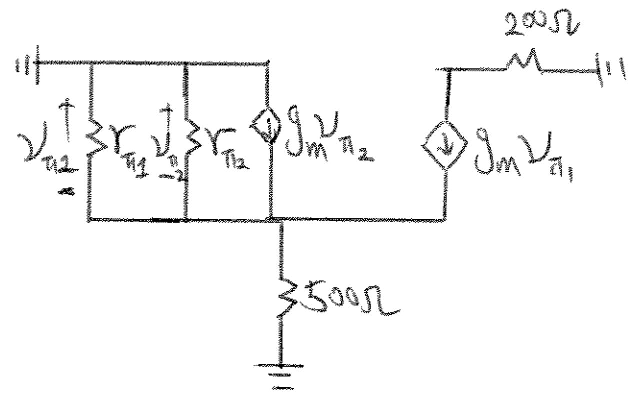
$V_C = 2.5 - (1)(0.2) = 2.3$

Q_1 in soft-saturation region.

25

b)

Small Signal Model



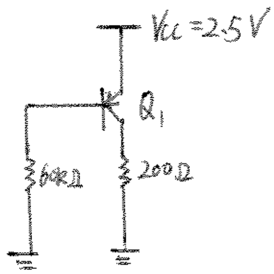
$$g_{m_1} = \frac{1mA}{26mV} = 0.0385 \left(\frac{1}{\Omega}\right) S$$

$$r_{\pi_1} = \frac{100}{0.0385} = 2.6 K\Omega$$

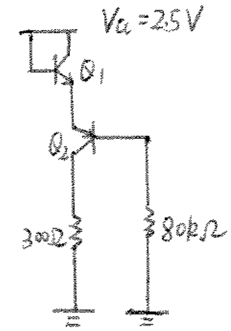
$$g_{m_2} = \frac{2.42mA}{26\ mV} = 0.0931 \left(\frac{1}{\Omega}\right) S$$

$$r_{\pi_2} = \frac{100}{0.0931} = 1.07 K\Omega$$

26). $\beta_{npn} = 2\beta_{pnp} = 100$ $\quad I_S = 9\times10^{-16}$ A $\quad V_A = \infty$



(a)



(b)

a) $I_C = \dfrac{2.5 - |V_{BE}|}{60k}\beta_{pnp}$, $\quad V_{BE} = V_T \ln\left(\dfrac{I_C}{I_S}\right)$

Guess $|V_{BE}| = 0.8 \Rightarrow I_C = 1.42m(A)$

$|V_{BE}| = 26\times10^{-3}\ln\left(\dfrac{1.42}{9\times10^{-13}}\right) = 0.730(V)$, not 0.8

Reiterate, $\quad I_C = \dfrac{2.5-0.73}{60k}\times 50 = 1.475m(A)$

$|V_{BE}| = 26\times10^{-3}\ln\left(\dfrac{1.42}{9\times10^{-13}}\right) = 0.731(V)$

Reiterate, $\quad I_C = \dfrac{2.5-0.731}{60k}\times 50 = 1.474m(A)$

$|V_{BE}| = 26\times10^{-3}\ln\left(\dfrac{1.474}{9\times10^{-13}}\right) = 0.731(V)$, converged.

Q_1 : $\quad |V_{BE}| = 0.731V$, $\quad I_C = 1.47mA$, $\quad I_B = 29.4\mu A$.

$|V_{CE}| = 2.206V$.

In forward active region.

2b)

b). $$I_{C2} = \frac{2.5 - (V_{BE1} + |V_{BE2}|)}{80k}$$ ①

$$\left.\begin{array}{l} I_{C2} \cdot \frac{\beta_{pnp}+1}{\beta_{pnp}} = \frac{I_{C1}(\beta_{npn}+1)}{\beta_{npn}} \\ \beta_{npn} = 2\beta_{pnp} = 100 \end{array}\right\} \Rightarrow I_{C1} = \frac{2(\beta_{pnp}+1)}{2\beta_{pnp}+1} I_{C2} = 1.0099\, I_{C2}$$ ②

$$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right)$$ ③

$$V_{BE2} = V_T \ln\left(\frac{I_{C2}}{I_S}\right)$$ ④

Four unknowns : I_{C1}, I_{C2}, V_{BE1}, V_{BE2}, Four equations : ①.②.③.④

Solve by iteration since ③ and ④ are exponential equations.

Guess $V_{BE2} = V_{BE1} = 0.8$

① $\Rightarrow$ $I_{C2} = 50 \times \left(\frac{2.5-1.6}{80k}\right) A = 0.5625\,mA$

② $\Rightarrow$ $I_{C1} = 0.568\,mA$

③ $\Rightarrow$ $V_{BE1} = V_T \ln\left(\frac{0.568}{9\times10^{-13}}\right) V = 0.706\,V$

④ $\Rightarrow$ $V_{BE2} = V_T \ln\left(\frac{0.5625}{9\times10^{-13}}\right) V = 0.706\,V$

Reiterate,

$I_{C2} = 0.68\,mA$, $I_{C1} = 0.6867\,mA$, $V_{BE1} = 0.711\,V$, $V_{BE2} = 0.711\,V$.

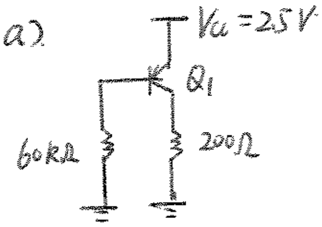
Reiterate,

$I_{C2} = 0.674\,mA$, $I_{C1} = 0.680\,mA$, $V_{BE1} = 0.711\,V$, $V_{BE2} = 0.711\,V$

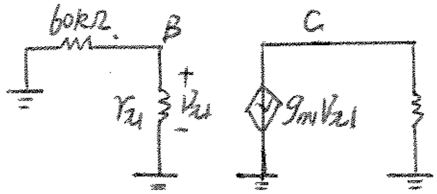
So.

$I_{C1} = 0.680\,mA$	$I_{C2} = 0.674\,mA$
$I_{B1} = 6.8\,uA$	$I_{B2} = 13.48\,uA$
$V_{BE1} = 0.711\,V$	$\|V_{BE2}\| = 0.711\,V$
$V_{CE1} = 0.711\,V$	$\|V_{CE2}\| = 2.5\,V - 0.711\,V - (0.674)(0.3)\,V = 1.5868\,V$

27. a).



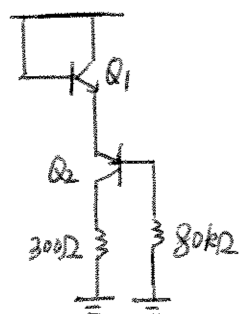
$\Rightarrow$



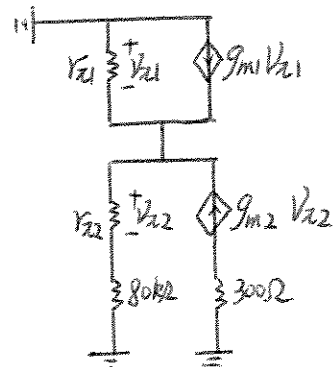
$$g_{m1} = \frac{I_c}{V_T} = \frac{1.47mA}{26mV} = 0.0565\,S$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = 884\Omega$$

b).



$\Rightarrow$



$$g_{m1} = \frac{I_{c1}}{V_T} = 0.02615\,S$$

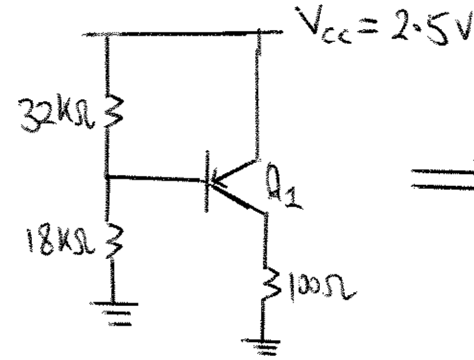
$$r_{\pi 1} = 3823.5\Omega$$

$$g_{m2} = \frac{I_{c2}}{V_T} = 0.0259\,S$$

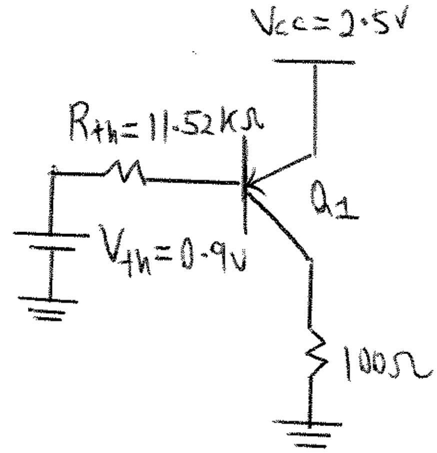
$$r_{\pi 2} = 1928.8\Omega$$

28)

a)



$\Longrightarrow$



$$I_c = \beta_{pnp}\left(\frac{2.5 - |V_{BE}| - V_{th}}{R_{th}}\right)$$

Guess $|V_{BE}| = 0.7\text{ V}, \quad I_c = 3.91\text{mA}$

$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.757\text{ V}$

Reiterate, $|V_{BE}| = 0.757\text{V}, \quad I_c = 3.66\text{mA}$

$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.755\text{V}$

Reiterate, $|V_{BE}| = 0.755\text{ V}, \quad I_c = 3.67\text{mA}$

$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.755\text{V}$, Converged!!

$V_c = (3.67\text{mA})(0.1\text{K}\Omega) = 0.367\text{V}, \quad V_B \approx 2.5 - 0.755 = 1.745\text{V}$

Q_1 in forward active.

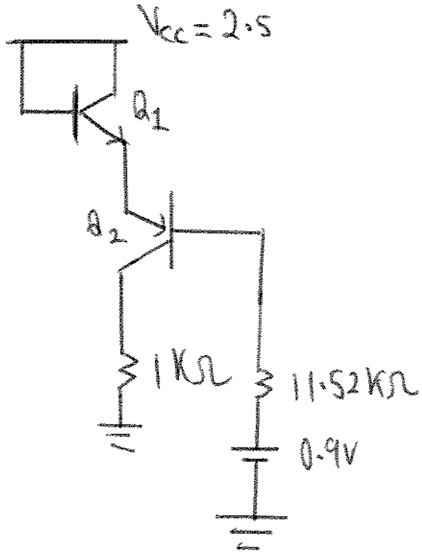
Bias point:

$I_c = 3.67\text{mA} \qquad |V_{BE}| = 0.755$

$I_B = 73.4\,\mu\text{A} \qquad |V_{CE}| = 2.5 - 0.367 = 2.133\text{V}$

28)

b)



$$I_{C2} = \left(\frac{2.5 - (V_{BE1} + V_{BE2}) - 0.9}{11.52K}\right)50$$

$$I_{C1} = I_{C2}(1.0099)$$

(From β relation)

$$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right)$$

$$|V_{BE2}| = V_T \ln\left(\frac{I_{C2}}{I_S}\right)$$

Guess, $V_{BE1} = V_{BE2} = 0.7V$

$I_{C2} = 0.868\,mA, \quad I_{C1} = 0.877\,mA$

$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right) = 0.718V, \quad |V_{BE2}| = V_T \ln\left(\frac{I_{C2}}{I_S}\right) = 0.717$

Reiterate, $V_{BE1} = 0.718V, \quad |V_{BE2}| = 0.717V$

$I_{C2} = 0.716\,mA, \quad I_{C1} = 0.723\,mA$

$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right) = 0.713V, \quad |V_{BE2}| = V_T \ln\left(\frac{I_{C2}}{I_S}\right) = 0.712V$

Reiterate, $V_{BE1} = 0.713V, \quad |V_{BE2}| = 0.712V$

$I_{C2} = 0.760\,mA, \quad I_{C1} = 0.767\,mA$

$V_{BE1} = 0.714V, \quad |V_{BE2}| = 0.714V$

28)

b)

Reiterate, $V_{BE_1} = 0.714V$, $|V_{BE_2}| = 0.714V$

$$I_{C_2} = 0.747mA, \quad I_{C_1} = 0.754mA$$

$$V_{BE_1} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.714V,$$

$$|V_{BE_2}| = 0.714\ V$$

$$V_{B_2} = \frac{(0.747mA)}{50}(11.52K\Omega) + 0.9 = 1.07\ V$$

$$V_{C_2} = (0.747mA)(1K\Omega) = 0.747\ V$$

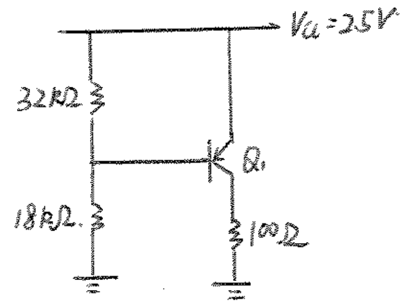
Q_2 is in forward-active region. Q_1 is always in forward-active region.

Bias point:

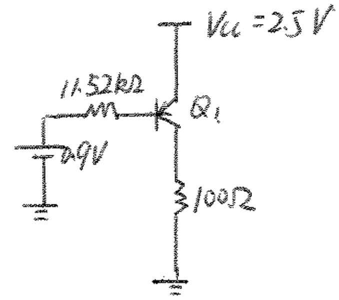
$V_{BE_1} = 0.714V$	$\lvert V_{BE_2}\rvert = 0.714V$
$I_{C_1} = 0.754\ mA$	$I_{C2} = 0.747mA$
$I_{B_1} = 7.54\mu A$	$I_{B2} = 14.94\ \mu A$
$V_{CE_1} = 0.714V$	$\lvert V_{CE_2}\rvert = 2.5 - 0.714 - 0.747 = 1.039V$

29).

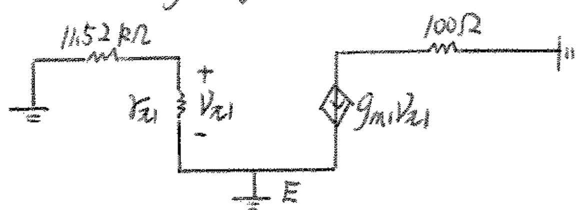
a).



⇒



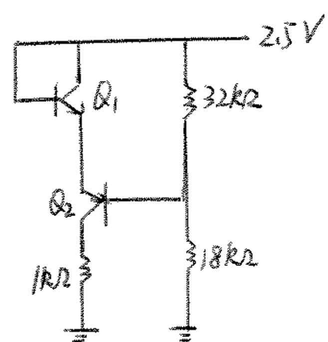
Small Signal:



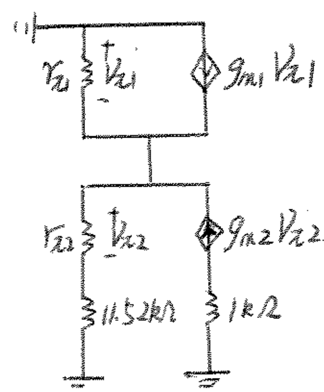
$$g_{m1} = \frac{3.67mA}{26mV} = 0.141\ S$$

$$r_\pi = \frac{50}{0.141}\ \Omega = 354.2\ \Omega$$

b).



⇒



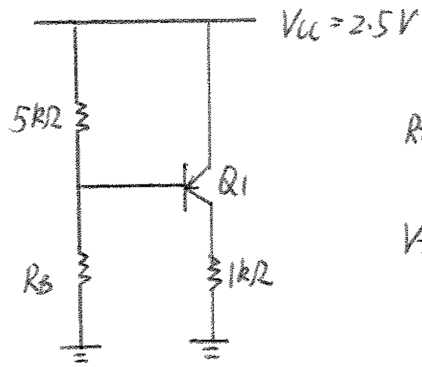
$g_{m1} = 0.029\ S$

$r_{\pi 1} = 3448.3\ \Omega$

$g_{m2} = 0.0287\ S$

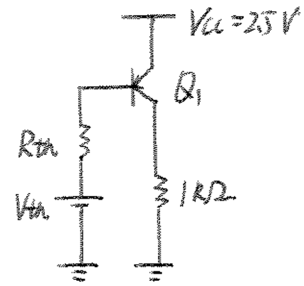
$r_{\pi 2} = 1740.3\ \Omega$

30)



$$R_{th} = \frac{(R_B)(5k\Omega)}{R_B + 5k\Omega}$$

$$V_{th} = \frac{R_B}{R_B + 5k\Omega} \cdot 2.5V$$



$\beta = 50$, $I_s = 8 \times 10^{-16} A$, $V_A = \infty$

Edge of saturation: $|V_{BE}| = |V_{CE}|$

$$I_c = \frac{50(2.5 - |V_{BE}| - V_{th})}{R_{th}}, \quad |V_{CE}| = 2.5 - I_c \, 1K\Omega = |V_{BE}|$$

$$2.5 - \frac{50(2.5 - |V_{BE}| - V_{th})}{R_{th}}(1K\Omega) = |V_{BE}|$$

Subsitute in R_{th} and V_{th} and rearrange:

$$12.5 R_B + 50|V_{BE}|R_B - |V_{BE}|(5K)R_B = 625 - |V_{BE}|250 \quad (1)$$

Guess $|V_{BE}| = 0.7V$, $(1) \Rightarrow 44 R_B = 450 \Rightarrow R_B = 10.23K\Omega$

$V_{th} = 1.68V$ $\quad I_c = 1.7857\ mA$

$R_{th} = 3.36K\Omega$ $\quad |V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.739V$, not 0.7V, reiterate

$|V_{BE}| = 0.739V$, $(1) \Rightarrow 45.755 R_B = 440.25$

$R_B = 9.62K\Omega$

$V_{th} = 1.645V$ $\quad I_c = 1.763mA$

$R_{th} = 3.29K\Omega$ $\quad |V_{BE}| = 0.739V$, Converged

3°)

So ±5% of 9.62KΩ.

+5% Case:

$9.62K\Omega + 5\% = 10.101K\Omega$

$V_{th} = 1.67\ V$, $R_{th} = 3.345\ K\Omega$

$I_c = \dfrac{(2.5 - 0.74 - 1.67)}{3.345} 50 = 1.3455\ mA$ (Assume $|V_{BE}| = 0.74V$)

Check for $|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.732v$, iterate once

$I_c = 1.4651\ mA$, $|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.734V$, converged

$|V_{BE}| \approx 0.734$, $|V_{CE}| = 2.5 - 1.4651(1k\Omega) = 1.0349V$

$V_{BC} = 0.3009\ V$ (Reverse bias)

-5% Case:

$9.62k\Omega - 5\% = 9.139\ K\Omega$

$V_{th} = 1.616V$, $R_{th} = 3.23184K\Omega$

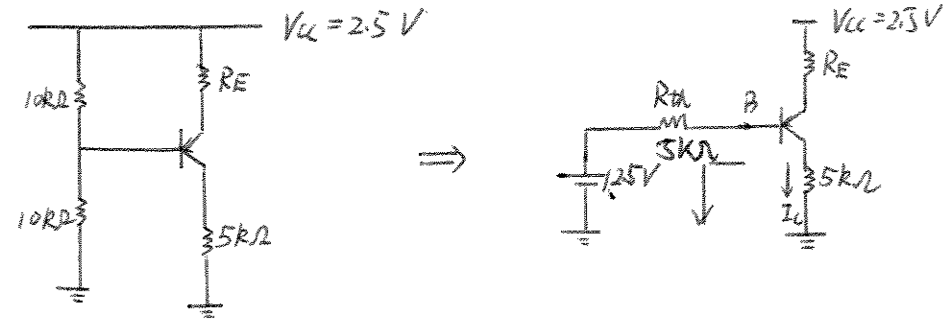
$I_c = \dfrac{(2.5 - 0.74 - 1.616)}{(3.23184)} 50 = 2.228mA$ ($|V_{BE}| = 0.7$), $|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.745V$

reiterate: $|V_{BE}| = 0.745\ V$, $I_c = 2.150\ mA$,

Verify V_{BE}, $|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.744\ V$, Converged

$|V_{CE}| = 2.5 - 2.150(1K\Omega) = 0.35$, $|V_{BE}| = 0.744V$, $V_{BC} = -0.394V$ (Forward Bias)

31)



$$V_{BC} = 1.25 + I_B R_{th} - I_c 5K = 0.3$$

$$1.25 + \frac{I_c}{\beta} 5 - I_c 5K = 0.3$$

$$\beta = 50 \Rightarrow I_c = 0.1939\,mA$$

$$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.682\,V$$

$$I_B = \frac{\left(2.5 - \frac{I_c}{\alpha} R_E - |V_{BE}|\right) - 1.25}{5K}, \quad \alpha = 0.9804$$

$I_B = 0.003878\,mA$

$|V_{BE}| = 0.682\,V$

$I_c = 0.1939\,mA$

$$R_E = 2.89\,k\Omega$$

If R_E is halved $\Rightarrow$ $R_E = 1.44\,k\Omega$

$$I_c = \beta\left(\frac{2.5 - |V_{BE}| - 1.25 - \alpha I_c R_E}{5k}\right)$$

$$I_c = \frac{62.5 - 50|V_{BE}|}{78.44}, \quad \text{Guess } |V_{BE}| = 0.682\,V$$

31)

$I_c = 0.3621\ mA$

Verify $|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.698V$, not $0.682V$

reiterate

$$I_c = \frac{62.5 - 50(0.698)}{78.44} = 0.352\ mA$$

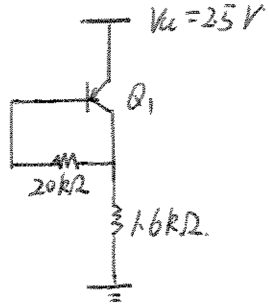
Verify $|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.697V$, converged

so $I_c = 0.352\ mA$, which is 1.82 times of $0.1939\ mA$

$$V_{BC} = 1.25 + \frac{(0.352)(5k\Omega)}{50} - (0.352)(5k\Omega) = -0.4748V$$

which drive Q_1 into saturation.

32)



$\beta = 80$

$V_A = \infty$

$$V_B = (I_B)(20k\Omega) + I_E \cdot (1.6k\Omega)$$

$$I_C = 1mA$$

$$I_B = \frac{1}{80} mA, \quad I_E = \frac{1}{0.98765} mA = 1.0125 mA$$

$$V_B = \left(\frac{1}{80}\right)(20)V + (1.0125)(1.6)V$$

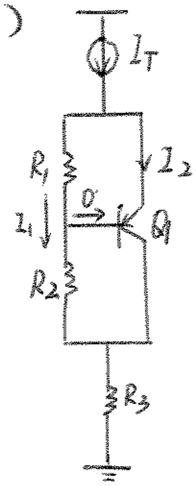
$$= 1.87V$$

$$|V_{BE}| = 2.5V - 1.87V = 0.63V$$

$$I_C = I_S e^{\left(\frac{V_{BE}}{V_T}\right)} \Rightarrow I_S = \frac{I_C}{e^{\left(\frac{V_{BE}}{V_T}\right)}} = \frac{1mA}{e^{\left(\frac{0.63}{0.026}\right)}}$$

$$I_S = 3 \times 10^{-11} mA$$

33)



If Base current is neglected, $I_C = I_E$

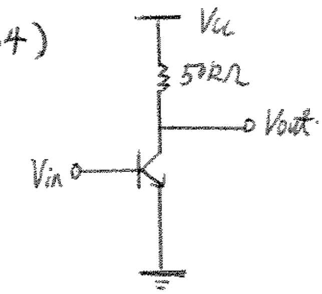
$$I_1 = \frac{V_E - V_C}{R_1 + R_2}$$

$$|V_{BE}| = I_1 R_1 = \frac{V_E - V_C}{R_1 + R_2} R_1 = \frac{|V_{CE}|}{R_1 + R_2} R_1$$

So $$\frac{|V_{CE}|}{|V_{BE}|} = \frac{R_1 + R_2}{R_1}$$

Let $A = \frac{R_1 + R_2}{R_1}$, $|V_{CE}| = A\,|V_{BE}|$, thus $|V_{BE}|$ is multiplied.

34)



$A_v = g_m R_C = 20$

$\frac{I_C R_C}{V_T} = 20 \Rightarrow I_C = \frac{20 V_T}{R_C}$

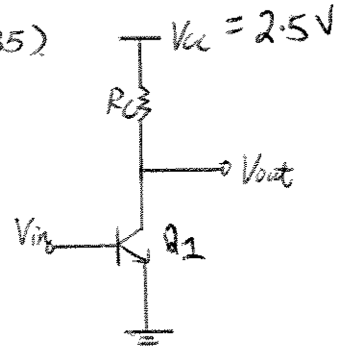
$I_C = 0.0104 mA$

$V_{CC} - (50k\Omega)\cdot(0.0104 mA) = V_{BE}$

$\Rightarrow V_{CC} - 50 \times 0.0104 V = 0.8 V$

$\Rightarrow V_{CC} = 1.32 V$

35)



$V_A = 10V$, $r_o = \dfrac{V_A}{I_C}$, $g_m = \dfrac{I_c}{V_T}$

$$\left|\frac{V_{out}}{V_{in}}\right| = g_m (R_C // r_o) = g_m\left(\frac{R_C r_o}{R_C + r_o}\right) = \frac{R_C V_A}{V_T\left(R_C + \frac{V_A}{I_C}\right)}$$

As the equation above shows, a large gain means a large I_c. However, a large I_c will drive Q_1 into saturation. So a tradeoff must be made. The maximum limit for I_c is when it drives Q_1 into the edge of saturation, namely, $V_{BE} = V_{CE}$.

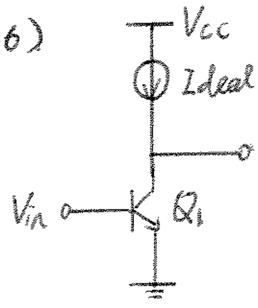
$$V_{CE} = V_{cc} - I_c (1K)$$

$V_{BE} = 0.8V$, $V_{cc} = 2.5V$

$$0.8 = 2.5 - I_c 1K$$

$$I_c = 1.7 mA$$

36)



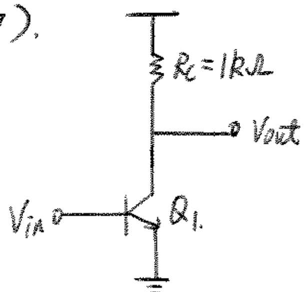
$A_v = 50$

$R_{out} = r_o = 10k\Omega$

$$A_v = g_m R_{out} = \frac{I_c}{V_T} R_{out} = 50$$

$$I_c = 50\left(\frac{V_T}{R_{out}}\right) = 0.13mA$$

37).



$$I_c = I_s \exp\left(\frac{V_{BE}}{2V_T}\right)$$

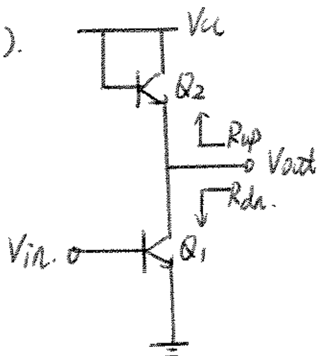
$$g_m = \frac{\partial I_c}{\partial V_{BE}} = \frac{I_c}{2V_T}$$

$$R_{out} = R_c$$

$$\left|\frac{V_{out}}{V_{in}}\right| = g_m R_{out} = \frac{I_c R_c}{2V_T} = \frac{(1mA)(1K\Omega)}{(2)(0.026V)} = 19.23$$

38). (Find A_v, R_{in}, R_{out})

a).



$V_A = \infty$

$R_{out} = R_{up} \parallel R_{dn}$

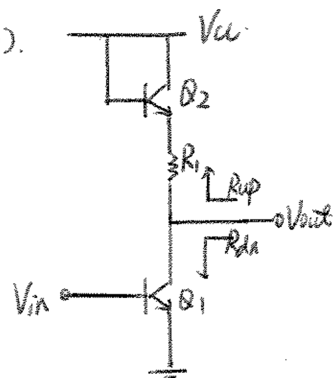
$R_{up} = \frac{1}{g_{m2}} \parallel r_{\pi 2}$, $R_{dn} = \infty$

$$R_{out} = \frac{1}{g_{m2}} \parallel r_{\pi 2}$$

$R_{in} = r_{\pi 1}$

$$A_v = \left|\frac{V_{out}}{V_{in}}\right| = g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$$

b).



$V_A = \infty$

$R_{up} = R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}$

$R_{dn} = \infty$

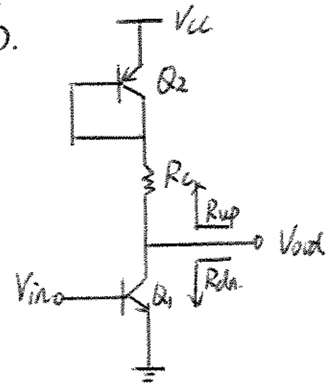
$$R_{out} = R_{up} \parallel R_{dn} = R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}$$

$R_{in} = r_{\pi 1}$

$$A_v = \left|\frac{V_{out}}{V_{in}}\right| = g_{m1}\left(R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$$

38

c).



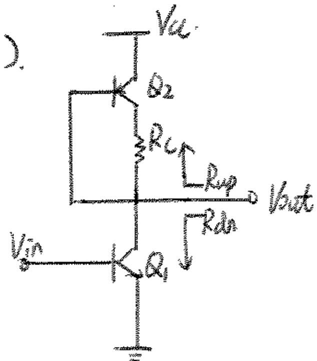
$V_A = \infty$

$R_{up} = R_c + \left(\frac{1}{g_{m_2}} // r_{\pi_2}\right)$, $R_{dn} = \infty$

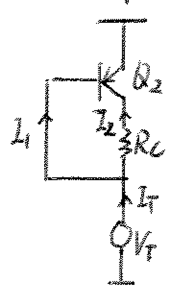
$R_{out} = R_c + \left(\frac{1}{g_{m_2}} // r_{\pi 2}\right)$, $R_{in} = r_{\pi 1}$

$A_v = g_{m1}\left(R_c + \left(\frac{1}{g_{m2}} // r_{\pi 2}\right)\right)$

d).



Find R_{up}:



$I_T = I_1 + I_2 = \frac{I_2}{\beta} + I_2$

$I_2 = g_m V_T$, $I_T = \frac{g_m V_T}{\beta} + g_m V_T$

$\frac{V_T}{I_T} = R_{up} = \frac{1}{\left(\frac{g_m}{\beta} + g_m\right)} = r_{\pi_2} // \frac{1}{g_{m_2}}$

$R_{dn} = \infty$

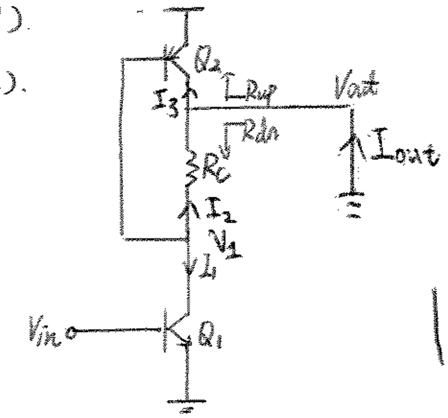
$R_{out} = R_{up} // R_{dn} = r_{\pi 2} // \frac{1}{g_{m2}}$

$R_{in} = r_{\pi 1}$

$A_v = g_{m1}\left(r_{\pi 2} // \frac{1}{g_{m2}}\right)$

38).

e).



$R_{up} = \infty$

$R_{dn} = R_c + r_{\pi_2}$

$R_{out} = R_c + r_{\pi_2}$

$|A_v| = G_m R_{out}$

where $G_m = \left| \frac{I_{out}}{V_{in}} \right|$

$I_{out} + I_2 = I_3$, $I_2 = \frac{V_1}{R_c}$

$I_3 = V_1 g_{m2}$

$V_1 = -I_1 (R_c // r_{\pi_2}) = -g_{m_1} V_{in} (R_c // r_{\pi_2})$

$$I_{out} = I_3 - I_2 = V_1 \left(g_{m2} - \frac{1}{R_c} \right)$$

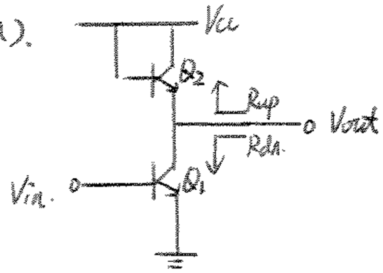
$$I_{out} = -g_{m_1} V_{in} (R_c // r_{\pi_2}) \left(g_{m2} - \frac{1}{R_c} \right)$$

$$G_m = \left| \frac{I_{out}}{V_{in}} \right| = g_{m_1} (R_c // r_{\pi_2}) \left(g_{m2} - \frac{1}{R_c} \right)$$

$$|A_v| = g_{m1} (R_c // r_{\pi_2}) \left(g_{m_2} - \frac{1}{R_c} \right) (R_c + r_{\pi_2})$$

39). $V_A < \infty$, find A_v, R_{in}, R_{out}

a).



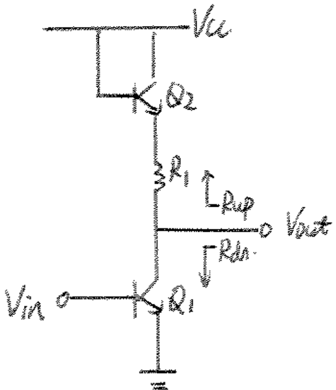
$$R_{up} = \frac{1}{g_{m2}} \,//\, r_{\pi 2} \,//\, r_{o2}, \quad R_{dn} = r_{o1}$$

$$R_{out} = \frac{1}{g_{m2}} \,//\, r_{\pi 2} \,//\, r_{o1} \,//\, r_{o2}$$

$$R_{in} = r_{\pi 1}$$

$$|A_v| = g_{m1}\left(\frac{1}{g_{m2}} \,//\, r_{\pi 1} \,//\, r_{o1} \,//\, r_{o2}\right)$$

b).



$$R_{up} = R_1 + \frac{1}{g_{m2}} \,//\, r_{\pi 2} \,//\, r_{o2}$$

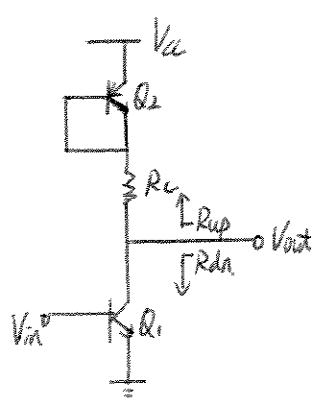
$$R_{dn} = r_{o1}$$

$$R_{out} = r_{o1} \,//\, \left(R_1 + \frac{1}{g_{m2}} \,//\, r_{\pi 2} \,//\, r_{o2}\right)$$

$$R_{in} = r_{\pi 1}$$

$$|A_v| = g_{m1}\left[r_{o1} \,//\, \left(R_1 + \frac{1}{g_{m2}} \,//\, r_{\pi 2} \,//\, r_{o2}\right)\right]$$

c).



$$R_{up} = R_C + \frac{1}{g_{m2}} \,//\, r_{\pi 2} \,//\, r_{o2}$$

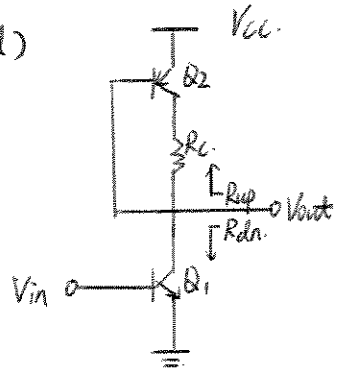
$$R_{dn} = r_{o1}$$

$$R_{in} = r_{\pi 1}$$

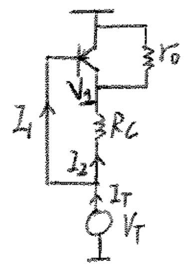
$$R_{out} = r_{o1} \,//\, \left(R_C + \frac{1}{g_{m2}} \,//\, r_{\pi 2} \,//\, r_{o2}\right)$$

$$|A_v| = g_{m1}\left[r_{o1} \,//\, \left(R_C + \frac{1}{g_{m2}} \,//\, r_{\pi 2} \,//\, r_{o2}\right)\right]$$

39 d)



Find R_{up}:



$$I_T = I_1 + I_2$$

$$I_2 = \frac{V_T - V_1}{R_C}, \quad I_1 = \frac{g_m V_T}{\beta}$$

$$V_1 = (I_2 - g_m V_T) r_o$$

$$I_2 = \frac{V_T - (I_2 - g_m V_T) r_o}{R_C}$$

$$I_2 = \frac{(1 + g_m r_o)}{R_c + r_o} V_T$$

$$I_T = \frac{g_m V_T}{\beta} + \frac{(1 + g_m r_o)}{R_c + r_o} V_T$$

$$\frac{V_T}{I_T} = R_{up} = \frac{1}{\frac{g_m}{\beta} + \frac{(1 + g_m r_o)}{R_c + r_o}}$$

$$R_{up} = r_\pi \,//\, \frac{R_c + r_o}{(1 + g_m r_o)}$$

$$R_{up} = r_{\pi_2} \,//\, \frac{(R_c + r_{o2})}{1 + g_{m2} r_{o2}}$$

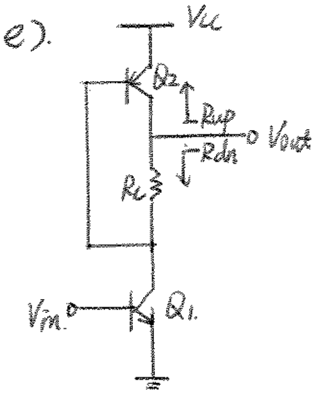
$$R_{dn} = r_{o1}$$

$$R_{out} = r_{\pi_2} \,//\, \frac{(R_c + r_{o2})}{1 + g_{m2} r_{o2}} \,//\, r_{o1}$$

$$R_{in} = r_{\pi 1}$$

$$|A_v| = \left| \frac{V_{out}}{V_{in}} \right| = g_{m1} \left(r_{\pi_2} \,//\, \frac{(R_c + r_{o2})}{1 + g_{m2} r_{o2}} \,//\, r_{o1} \right)$$

39 e).



$R_{up} = r_{o2}$

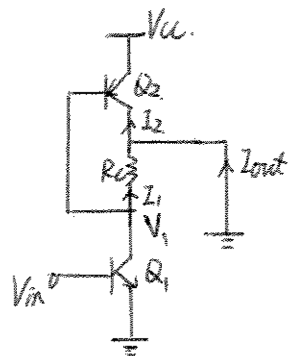
$R_{dn} = R_C + (r_{o1} \parallel r_{\pi 2})$

$R_{in} = r_{\pi 1}$

$R_{out} = r_{o2} \parallel [R_C + (r_{o1} \parallel r_{\pi 2})]$

$|A_v| = G_m R_{out}$

Finding G_m:



$G_m = \left| \frac{I_{out}}{V_{in}} \right|$

$I_{out} + I_1 = I_2$

$I_{out} = I_2 - I_1$

$I_1 = \frac{V_1}{R_C} \quad I_2 = V_1 g_{m2}$

$V_1 = -(g_{m1} V_{in})(r_{o1} \parallel R_C \parallel r_{\pi 2})$

$I_{out} = V_1 (g_{m2} - \frac{1}{R_C}) = -g_{m1} V_{in} (r_{o1} \parallel R_C \parallel r_{\pi 2})(g_{m2} - \frac{1}{R_C})$

$G_m = \left| \frac{I_{out}}{V_{in}} \right| = g_{m1} (r_{o1} \parallel R_C \parallel r_{\pi 2})(g_{m2} - \frac{1}{R_C})$

$|A_v| = g_{m1} (r_{o1} \parallel R_C \parallel r_{\pi 2})(g_{m2} - \frac{1}{R_C}) \{ r_{o2} \parallel [R_C + (r_{o1} \parallel r_{\pi 2})] \}$

40)

Gain of a degenerated CE stage ($V_A = \infty$)

$$A_V = \frac{-R_C}{\frac{1}{g_m} + R_E} = \frac{-R_C g_m}{1 + R_E g_m}$$

$$\frac{\partial A_v}{\partial I_C} = R_C \left(\frac{g_m R_E}{(1+R_E g_m)^2} \frac{\partial g_m}{\partial I_C} - \frac{\partial g_m / \partial I_C}{1 + g_m R_E} \right)$$

$$\frac{\partial g_m}{\partial I_C} = \frac{1}{V_T} = \frac{1}{26mV} = 38.46 \left(\frac{1}{V}\right)$$

a) $g_m R_E = 3$

$$\frac{\partial A_v}{\partial I_C} = R_C(-2.404) \quad , \quad \partial I_C = 0.1 I_C$$

$$\partial A_v = -R_C I_C (0.24)$$

$$\text{Relative change in gain} = \frac{\partial A_v}{A_v} = \frac{-0.24(R_C I_C)}{-\frac{R_C I_C}{V_T(1+R_E g_m)}} = 2.5\%$$

40)

b) $g_m R_E = 7$

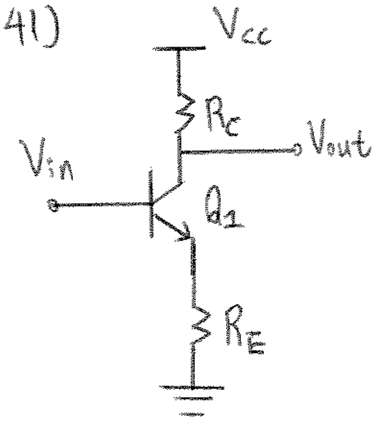
$$\frac{\partial A_v}{\partial I_c} = -R_c\, 0.6$$

$$\partial A_v = -R_c I_c (0.06)$$

Relative change in gain

$$\frac{\partial A_v}{A_v} = \frac{-0.06\,(R_c I_c)}{\dfrac{-R_c I_c}{V_T(1+R_E g_m)}} = 1.25\%$$

41)



$V_A = \infty$

$R_C I_C = 20 V_T$

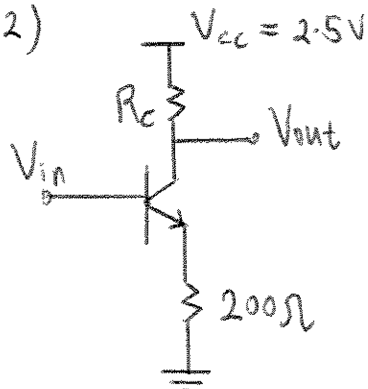
$R_E I_C = 5 V_T$

$$|A_v| = \frac{R_C}{R_E + \frac{1}{g_m}} = \frac{R_C}{R_E + \frac{V_T}{I_C}} = \frac{R_C I_C}{R_E I_C + V_T}$$

Assume β is large, so $I_C = I_E$.

$R_C I_C = 20 V_T$, $R_E I_C = 5 V_T$

$$|A_v| = \frac{20 V_T}{5 V_T + V_T} = \frac{20 V_T}{6 V_T} = 3.33$$

42) 

$$|A_v| = \frac{R_C I_C}{R_E I_C + V_T} = 10$$

Edge of Saturation

$V_{CE} = V_{BE} = 2.5 - I_C(R_C + R_E)$

$V_{BE} = 0.8V \Rightarrow I_C R_C = 1.7 - I_C 0.2$ (operating point)

$|A_v| = 10 \Rightarrow R_C I_C = 10(R_E I_C + V_T)$ (Gain Equation)

Equating the two equations above $\Rightarrow$

$1.7 - 0.2 I_C = 2 I_C + 0.26 \Rightarrow I_C = 0.655mA$

Check for $V_{BE} \Rightarrow V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.725$, not 0.8, Reiterate

$I_C R_C = 1.775 - I_C 0.2$ (operating point)

$I_C R_C = 2I_C + 0.26$ (Gain equation)

Equating the two equations $\Rightarrow I_C = 0.689mA$

Check for $V_{BE} \Rightarrow V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.727V$, iterate 1 more time

$I_C R_C = 1.773 - I_C 0.2$ (operating point)

$I_C R_C = 2I_C + 0.26$ (Gain equation)

42)

Equating the two equations $\Rightarrow I_c = 0.688\ mA$

Check for $V_{BE} \Rightarrow V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.727V$, converged

$I_c = 0.688mA$

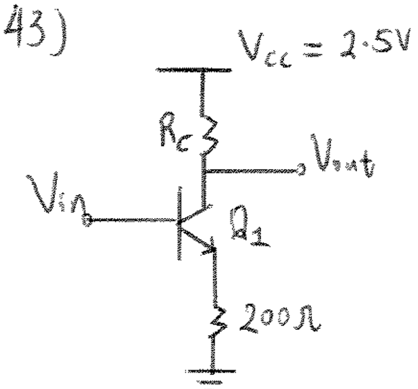
$$R_c = \frac{2I_c + 0.26}{I_c} = \frac{(2)(0.688) + 0.26}{0.688}$$

$R_c = 2.38K\Omega$

$R_{in} = r_\pi + (1+\beta) R_E$

$$R_{in} = \frac{\beta}{g_m} + (101)(0.2) = 24.0K\Omega$$

43)



$|A_v| = 100$

(Voltage gain)

$|A_v| = 100 \Rightarrow R_c I_c = 100(R_E I_c + V_T)$

$R_c I_c = 20 I_c + 2.6 \quad (1)$

$R_c I_c = 1.7 - I_c 0.2 \quad (2)$ (Assume $V_{BE} = 0.8$)

Equating (1) and (2) yield

$1.7 - I_c 0.2 = 20 I_c + 2.6 \Rightarrow I_c = -0.04455\,mA$

A negative I_c in forward active region is impossible, therefore, a solution does not exist. The reason is because $R_c I_c$ is too large to produce a gain of 100 that drive Q_1 into saturation region.

Maximum gain achievable:

$$\frac{R_c I_c}{R_E I_c + V_T} = |A_v| \quad \text{(Gain Equation)}$$

$2.5 = R_c I_c + V_{CE} + R_E I_c$ (Operating Point Equation)

Let A = Maximum gain

43)

$$AI_c 0.2 + A0.026 = 1.7 - I_c 0.2$$

$$\Rightarrow I_c = \frac{1.7 - A0.026}{A0.2 + 0.2}$$

Since I_c cannot be zero, set

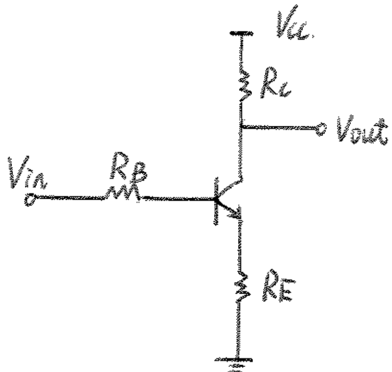
$$\frac{1.7 - A0.026}{A0.2 + 0.2} > 0$$

$$1.7 - A0.026 > 0$$

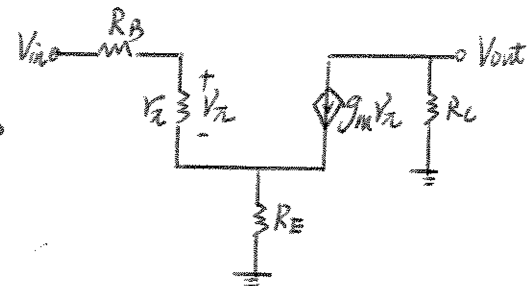
$$1.7 > A0.026$$

$$A < \frac{1.7}{0.026} = 65.4$$ (Maximum gain achievable)

44) $V_A = \infty$



Small Signal



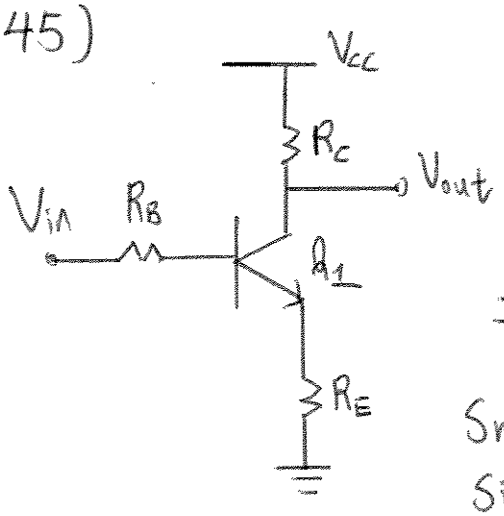
$$V_{out} = -g_m V_\pi R_C$$

$$V_\pi = \frac{V_{in} r_\pi}{R_B + r_\pi + (\beta+1) R_E}$$

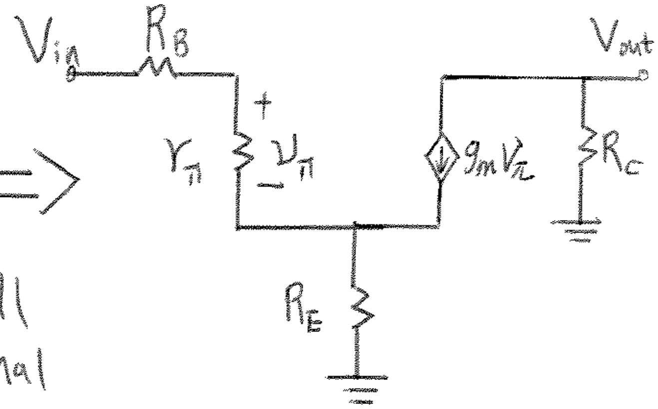
$$V_{out} = \frac{-g_m r_\pi R_C V_{in}}{R_B + r_\pi + (\beta+1) R_E} = \frac{-\beta R_C V_{in}}{R_B + r_\pi + (\beta+1) R_E} = \frac{-R_C V_{in}}{\frac{R_B}{\beta} + \frac{1}{g_m} + \frac{\beta+1}{\beta} R_E}$$

$$\frac{V_{out}}{V_{in}} \approx \frac{-R_C}{\frac{R_B}{\beta+1} + \frac{1}{g_m} + R_E}$$

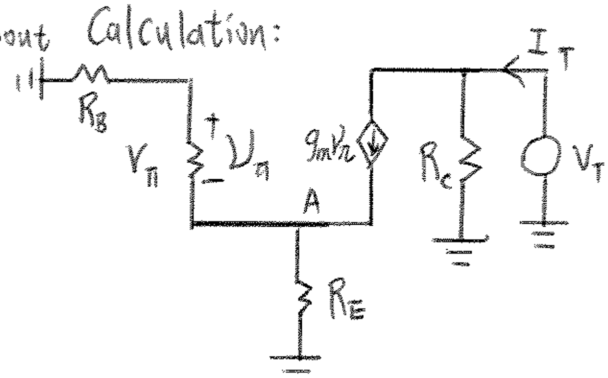
45)



$\Longrightarrow$ Small Signal



Rout Calculation:



$$V_A = g_m v_\pi \,(R_E \,//\, R_B + r_\pi) \qquad (1)$$

$$v_\pi = -\frac{V_A r_\pi}{r_\pi + R_B} \Rightarrow V_A = -\frac{v_\pi (r_\pi + R_B)}{r_\pi} \qquad (2)$$

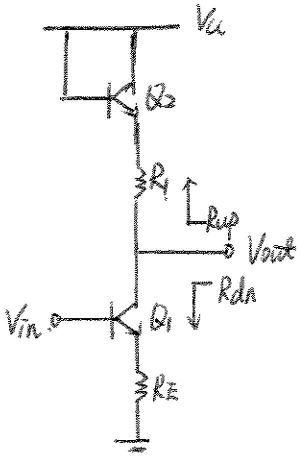
The only possible solution for 1) and 2) is $v_\pi = V_A = 0$, since 1) is positive and 2) is negative.

$$v_\pi = 0 \Rightarrow g_m v_\pi \Rightarrow 0 \Rightarrow \frac{V_T}{I_T} = R_c$$

Therefore, $R_{out} = R_c$

46) $V_A = \infty$

a)



$$R_{up} = R_1 + \frac{1}{g_{m2}} \,//\, r_{\pi 2}$$

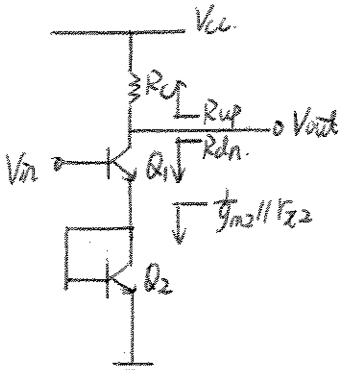
$$R_{dn} = \infty$$

$$R_{out} = R_1 + \frac{1}{g_{m2}} \,//\, r_{\pi 2}$$

$$R_{in} = r_\pi + (1+\beta) R_E$$

$$|A_v| = \frac{R_1 + \frac{1}{g_{m2}} \,//\, r_{\pi 2}}{R_E + \frac{1}{g_{m1}}}$$

b).



$$R_{up} = R_C$$

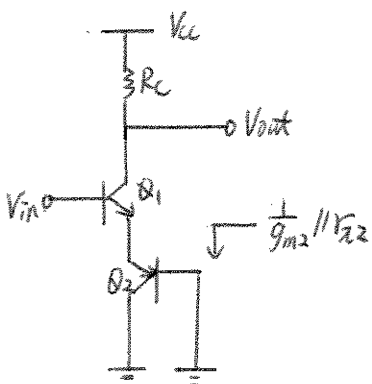
$$R_{dn} = \infty$$

$$R_{out} = R_C$$

$$R_{in} = r_{\pi 1} + (\beta+1)\left(\frac{1}{g_{m2}} \,//\, r_{\pi 2}\right)$$

$$|A_v| = \frac{R_C}{\frac{1}{g_{m2}} \,//\, r_{\pi 2} + \frac{1}{g_{m1}}}$$

c)



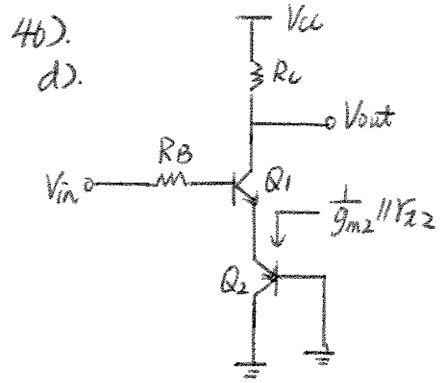
$$R_{out} = R_C$$

$$R_{in} = r_{\pi 1} + (\beta+1)\left(\frac{1}{g_{m2}} \,//\, r_{\pi 2}\right)$$

$$|A_v| = \frac{R_C}{\frac{1}{g_{m2}} \,//\, r_{\pi 2} + \frac{1}{g_{m1}}}$$

4b).

d).

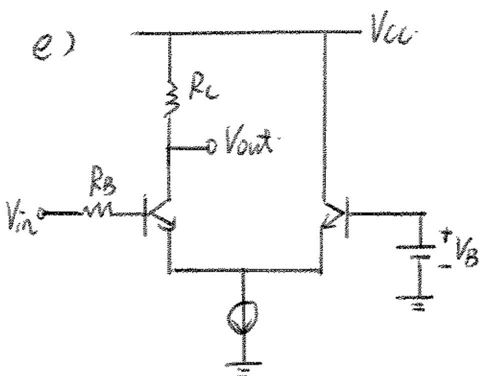


$$R_{out} = R_C$$

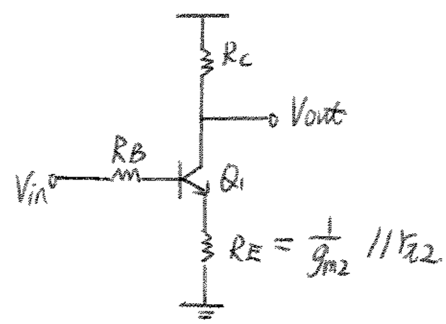
$$R_{in} = R_B + r_{\pi 1} + (\beta+1)\left(\frac{1}{g_{m2}} \| r_{\pi 2}\right)$$

$$|A_v| = \frac{R_C}{\frac{1}{g_{m2}} \| r_{\pi 2} + \frac{1}{g_{m1}} + \frac{R_B}{\beta+1}}$$

e)



$\Longrightarrow$



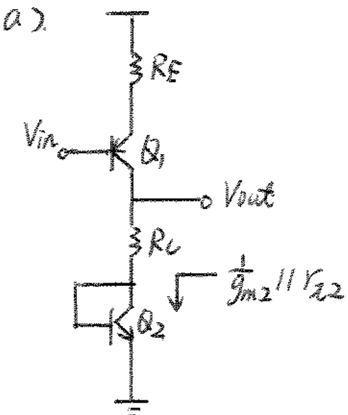
$$R_{out} = R_C$$

$$R_{in} = R_B + r_{\pi 1} + (\beta+1)\left(\frac{1}{g_{m2}} \| r_{\pi 2}\right)$$

$$|A_v| = \frac{R_C}{\frac{1}{g_{m2}} \| r_{\pi 2} + \frac{1}{g_{m1}} + \frac{R_B}{\beta+1}}$$

47). $V_A = \infty$.

a).

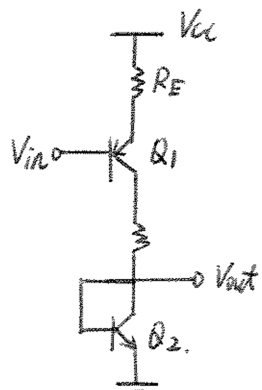


$$R_{out} = R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}$$

$$R_{in} = r_{\pi 1} + (1+\beta) R_E$$

$$|A_v| = \frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{R_E + \frac{1}{g_{m1}}}$$

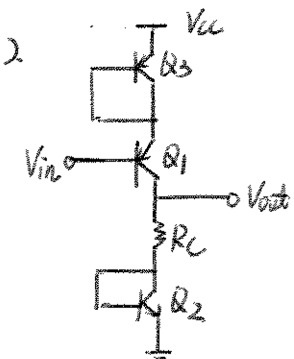
b). $R_{out} = \frac{1}{g_{m2}} \parallel r_{\pi 2}$.



$$R_{in} = r_{\pi 1} + (1+\beta) R_E$$

$$|A_v| = \frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{R_E + \frac{1}{g_{m1}}}$$

c).

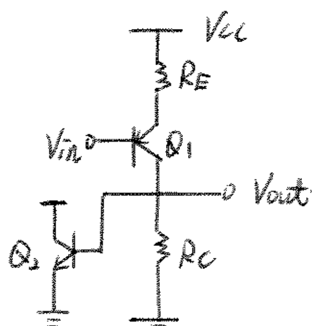


$$R_{out} = R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}$$

$$R_{in} = r_{\pi 1} + (1+\beta)\left(\frac{1}{g_{m3}} \parallel r_{\pi 3}\right)$$

$$|A_v| = \frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + \frac{1}{g_{m3}} \parallel r_{\pi 3}}$$

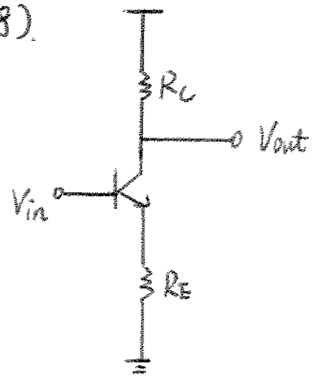
d) $R_{out} = R_C \parallel r_{\pi 2}$



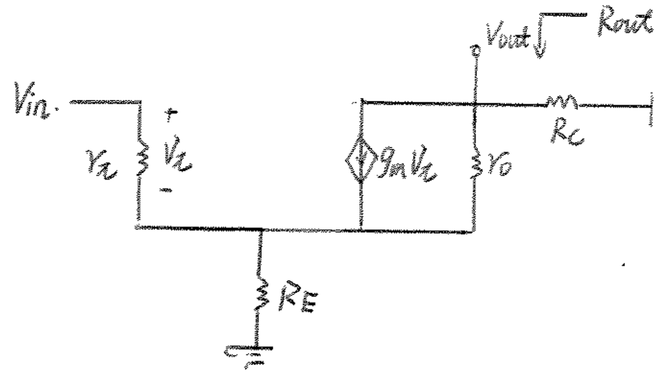
$$R_{in} = r_{\pi 1} + (\beta+1) R_E$$

$$|A_v| = \frac{R_C \parallel r_{\pi 2}}{R_E + \frac{1}{g_{m1}}}$$

48)

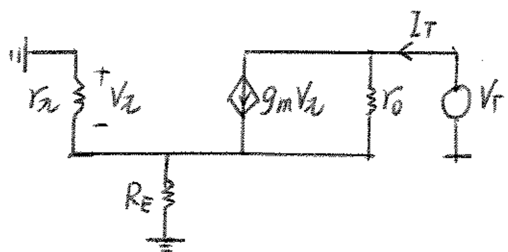


$\Longrightarrow$



$R_{out} = R_C // R_{eq}$

Solve for R_{eq}.



$$I_T = g_m v_\pi + \frac{(V_T + v_\pi)}{r_o}$$

$$v_\pi = -I_T (r_\pi // R_E)$$

$$I_T = -g_m I_T (r_\pi // R_E) + \frac{(V_T - I_T (r_\pi // R_E))}{r_o}$$

$$\frac{V_T}{I_T} = r_o \left(1 + \frac{(r_\pi // R_E)}{r_o} \right) + g_m (r_\pi // R_E) \Big)$$

$$\frac{V_T}{I_T} = r_o + (1 + g_m r_o)(r_\pi // R_E)$$

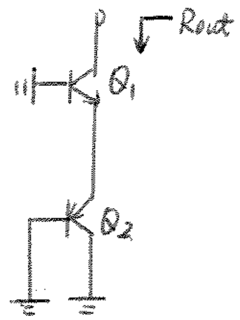
$$R_{eq} = r_o + (1 + g_m r_o)(r_\pi // R_E)$$

$$R_{out} = R_C // r_o + (1 + g_m r_o)(r_\pi // R_E)$$

$$R_{out} \approx R_C // r_o (1 + g_m (r_\pi // R_E)) \quad \text{since } g_m r_o >> 1$$

49). $\beta \gg 1$ and $V_A < \infty$ to have meaningful result.

a).

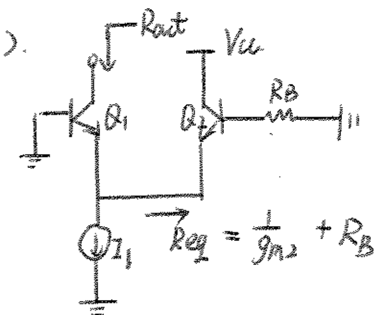


$\frac{1}{g_{m2}} \parallel r_{\pi 2} \approx \frac{1}{g_{m2}}$, since $\beta \gg 1$

$$R_{out} = r_{o1} + (1 + g_{m1} r_{o1}) \left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$$

$$= r_{o1} (1 + g_{m1}/g_{m2})$$

b).

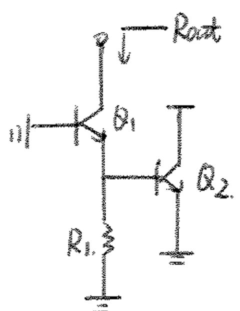


$$R_{out} = r_{o1} + (1 + g_{m1} r_{o1}) \left[\left(\frac{1}{g_{m2}} + \frac{R_B}{\beta+1}\right) \parallel r_{\pi 1}\right]$$

$$\approx r_{o1} + (1 + g_{m1} r_{o1}) \left(\frac{1}{g_{m2}} + \frac{R_B}{\beta}\right)$$

$$\approx r_{o1} \left[1 + g_{m1}\left(\frac{1}{g_{m2}} + \frac{R_B}{\beta}\right)\right].$$

c).



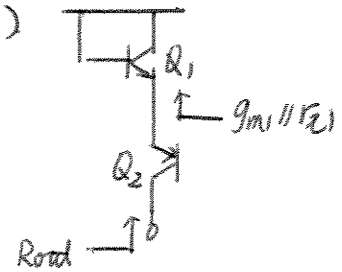
$$R_{out} = r_o + (1 + g_{m1} r_o)(R_1 \parallel r_{\pi 2} \parallel r_{\pi 1})$$

$R_1 \parallel r_{\pi 1} \parallel r_{\pi 2} \approx R_1$, since $\beta \gg 1$.

$$R_{out} \approx r_o (1 + g_{m1} R_1)$$

50) $\beta >> 1$, $r_A \to \infty$, for meaningful results

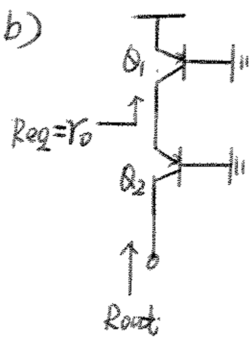
a)



$$R_{out} = r_{o2} + (1+g_{m2} r_{o2})\left(\frac{1}{g_{m1}} // r_{\pi 2}\right)$$

$$\approx r_{o2}(1+g_{m2}/g_{m1})$$

b)



$$R_{out} = r_{o2} + (1+g_{m2} r_{o2})(r_{o1} // r_{\pi 2})$$

$$\approx r_{o2}\left[1+g_{m2}(r_{o1} // r_{\pi 2})\right]$$

The output impedance in b) is larger than a) because Q_2's connected for a high impedance load, whereas in a) it's connected to a low impedance load.

51). $r_\pi = \beta V_T / I_c$.

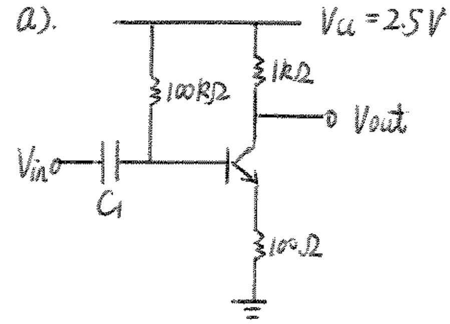
$$R_{in} = r_\pi \,//\, R_B = \frac{\frac{\beta V_T}{I_c} R_B}{\frac{\beta V_T}{I_c} + R_B} = \frac{V_T R_B}{V_T + \frac{I_c}{\beta} R_B} = \frac{V_T R_B}{V_T + I_B R_B}$$

Since $I_B R_B >> V_T \Rightarrow R_{in} \approx \frac{V_T R_B}{I_B R_B} = \frac{V_T}{I_B} = \frac{V_T}{\frac{I_c}{\beta}} = \frac{\beta V_T}{I_c} \approx r_\pi$

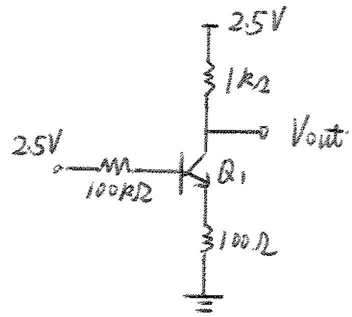
So $R_{in} = r_\pi \,//\, R_B \approx r_\pi$.

52). $I_S = 8\times10^{-6}A,\quad \beta = 100,\quad V_A = \infty$

a).



DC Analysis $\longrightarrow$



$$I_c = \frac{\beta\left(2.5 - \left(V_{BE} + \frac{I_c}{\alpha}0.1\right)\right)}{100K} \Rightarrow I_c = \frac{100\,(2.5 - V_{BE})}{100K + 10.1K}$$

Guess $V_{BE} = 0.75V$, $I_c = 1.59mA$

Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.736V$, not $0.75V$, reiterate

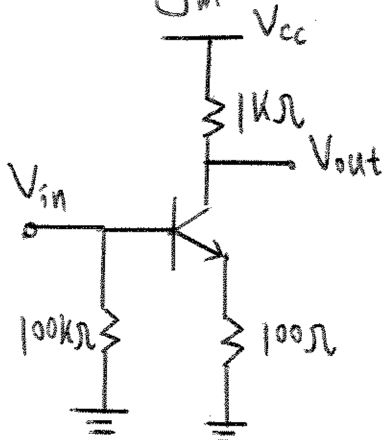
$V_{BE} = 0.736V$, $I_c = 1.60\,mA$

Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.736V$, converged !!

$I_c = 1.60mA$

$$g_m = \frac{I_c}{V_T} = \frac{1.60mA}{26mV} = 0.0615\left(\frac{1}{\Omega}\right)s$$

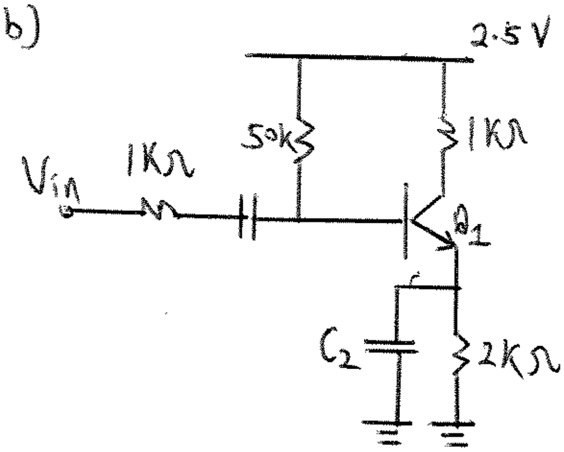
$$r_\pi = \frac{\beta}{g_m} = 1.63\,K\Omega$$



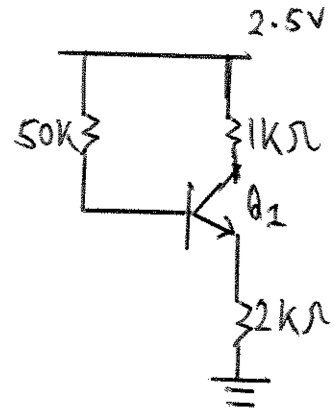
$$A_v = \left|\frac{V_{out}}{V_{in}}\right| = \frac{1K}{0.1 + \frac{1}{g_m}} = 8.6$$

52)

b)



DC Analysis $\Longrightarrow$



$$I_c = \beta\left(\frac{2.5 - (V_{BE} + I_E 2K)}{50K}\right) \Longrightarrow I_c = \frac{100(2.5 - V_{BE})}{50K + 202K}$$

Guess $V_{BE} = 0.7V$, $I_c = 0.714\ mA$

Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.7155V$, reiterate

$V_{BE} = 0.7155V$, $I_c = 0.708\ mA$

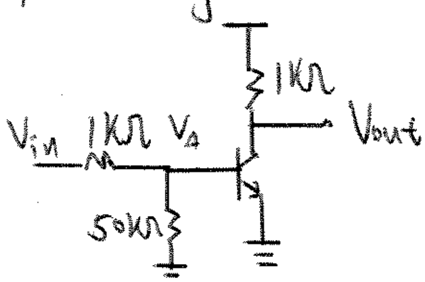
Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.715V$, converged!!

$I_c = 0.708mA$

$g_m = \frac{I_c}{V_T} = 0.02723\left(\frac{1}{\Omega}\right)$ S

$V_{BE} = 0.715V$

AC Analysis:



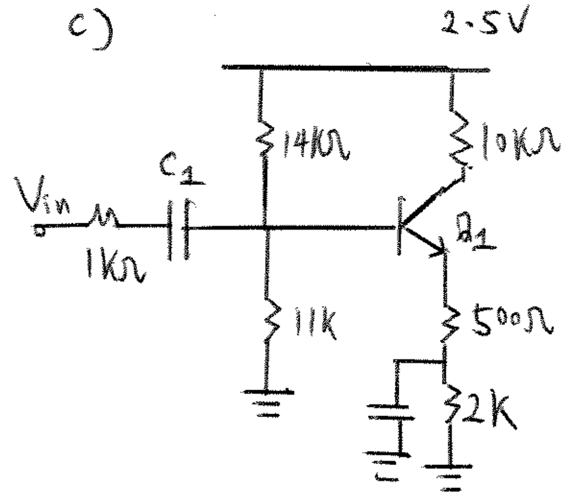
$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{V_{out}}{V_A}\right|\left|\frac{V_A}{V_{in}}\right| = 21.1$$

$$\left|\frac{V_{out}}{V_A}\right| = g_m\, 1K\Omega = 27.23$$

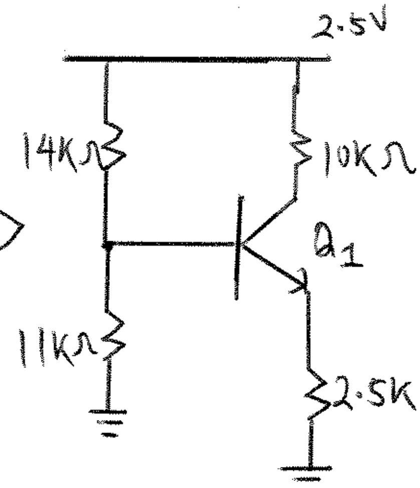
$$\left|\frac{V_A}{V_{in}}\right| = \frac{50K // V_\pi}{50K // V_\pi + 1K} = 0.77$$

52)

c)



$\Longrightarrow$



$$I_c = \beta\left(\frac{1.1 - (V_{BE} + \frac{I_c}{\alpha}2.5)}{14K\Omega // 11K\Omega}\right) \Longrightarrow I_c = \frac{100\,(1.1 - V_{BE})}{6.16 + 252.53}$$

Guess $V_{BE} = 0.7V$, $I_c = 0.1546\ mA$

Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.676\ V$, not $0.7V$, reiterate

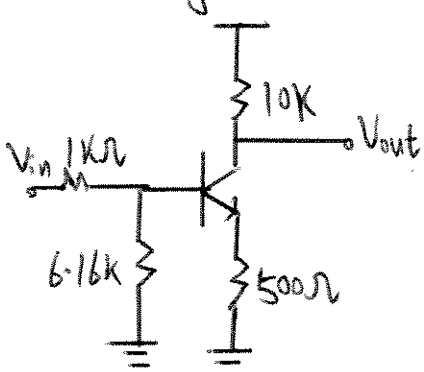
$V_{BE} = 0.676\ V$, $I_c = 0.164\ mA$

Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.677V$, converged !!

$I_c = 0.164\ mA$, $V_{BE} = 0.677V$, $g_m = 0.00631\ (\frac{1}{\Omega})S$,

$r_\pi = 15.85k$

AC Analysis:

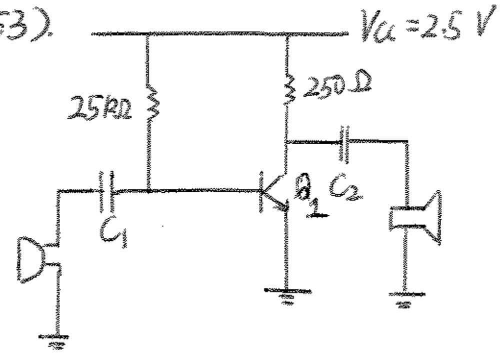


$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{V_{out}}{V_A}\right|\left|\frac{V_A}{V_{in}}\right| = 12.9$$

$$\left|\frac{V_{out}}{V_A}\right| = \frac{10K}{\frac{1}{g_m} + 0.5} = 15.2$$

$$\left|\frac{V_A}{V_{in}}\right| = \frac{6.16K//(15.85K + 101(0.5))}{6.16K//(15.85K + 101(0.5)) + 1K} = 0.85$$

53).



$R_B = 25\,K\Omega$

$R_C = 250\Omega$

$I_S = 5 \times 10^{-17} A$

$V_A = \infty$

DC Analysis: Assume collector bias voltage is still 1.5 V. So 1V across $R_C \Rightarrow I_C = 4mA$.

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.832$$

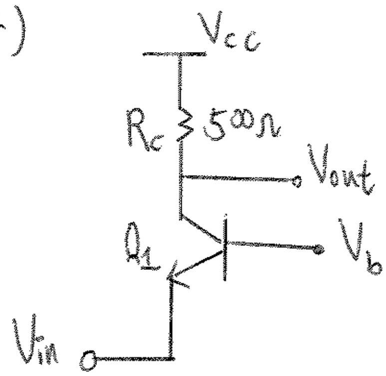
$$I_B = \frac{2.5 - V_{BE}}{25K} = 0.06673\ mA$$

$$\beta = \frac{I_C}{I_B} = \frac{0.832\ mA}{0.06673\ mA} = 60$$

$$A_v = \left|\frac{V_{out}}{V_{in}}\right| = g_m(250\Omega // 8\Omega) = 1.2, \text{(Greater than unity)}$$

$$g_m = \frac{4mA}{26mV} = 0.1538\left(\frac{1}{\Omega}\right) S$$

54)



$I_c = 2mA$

$V_A = \infty$

$$g_m = \frac{I_c}{V_T} = 0.0769 \left(\frac{1}{\Omega}\right) S \,, \quad \frac{1}{g_m} = 13\Omega$$

a)

$$|A_v| = g_m R_c = \frac{0.5}{0.013} = 38.5$$

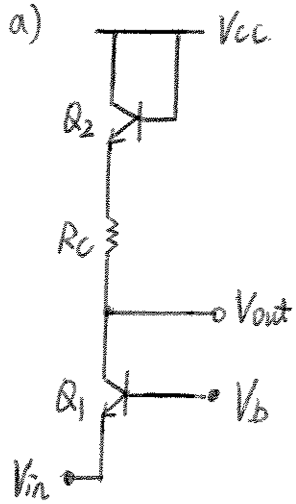
$$R_{in} = \frac{1}{g_m} // r_\pi \approx \frac{1}{g_m} = 13\Omega$$ (Since β is usually large)

$$R_{out} = 500\Omega$$

b) Since $|A_v| = g_m R_c$, and g_m is fixed by I_c. The only way to maximize $|A_v|$ is to maximize R_c. However a large R_c will push Q_1 into saturation, losing its gain altogether. Therefore, V_B has to be as small as possible to provide enough room for V_c to drop $\Rightarrow$ large $R_c \Rightarrow$ large gain.

55) $V_A = \infty$

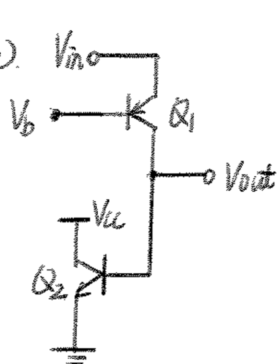
a)



$$|A_v| = \frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}}}$$

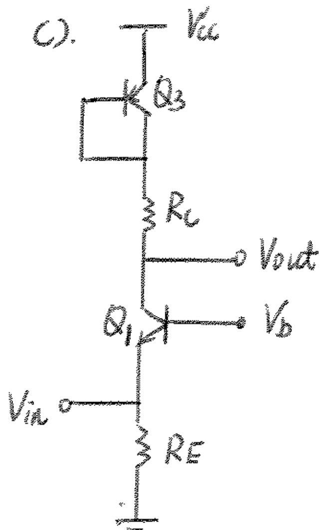
$$= g_{m1}\left(R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$$

b).



$$|A_v| = \frac{r_{\pi 2}}{\frac{1}{g_{m1}}} = g_{m1} r_{\pi 2}$$

c).

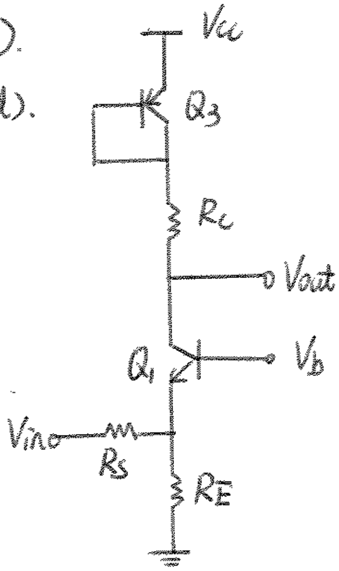


$$|A_v| = \frac{R_C + \frac{1}{g_{m3}} \parallel r_{\pi 3}}{\frac{1}{g_{m1}}}$$

$$= g_{m1}\left(R_C + \frac{1}{g_{m3}} \parallel r_{\pi 3}\right)$$

55).

d).

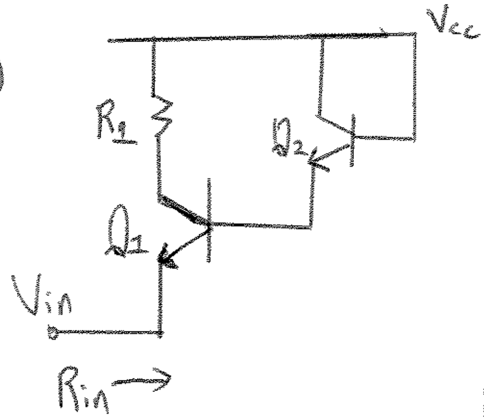


$$|A_v| = \left|\frac{V_{out}}{V_A}\right| \left|\frac{V_A}{V_{in}}\right|$$

$$= \left[g_{m1} \left(R_C + \frac{1}{g_{m3}} // r_{\pi 3} \right) \right] \left(\frac{R_E // \frac{1}{g_{m1}}}{R_E // \frac{1}{g_{m1}} + R_S} \right)$$

56) $V_A = \infty$

a)

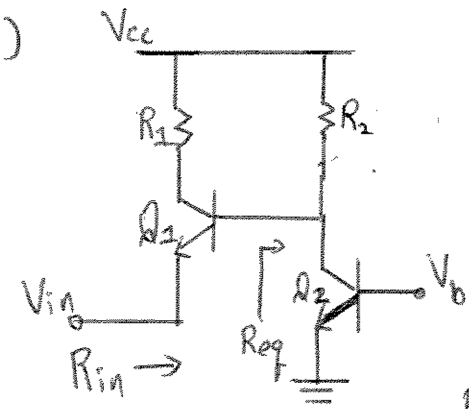


$$R_{in} = \frac{1}{g_{m1}} \parallel r_{\pi 1} + \frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{\beta_1 + 1}$$

Since β is usually very large

$$R_{in} \approx \frac{1}{g_{m1}} + \frac{1}{g_{m2}(\beta_1 + 1)}$$

b)



$$R_{eq} = R_2 \parallel \infty = R_2$$

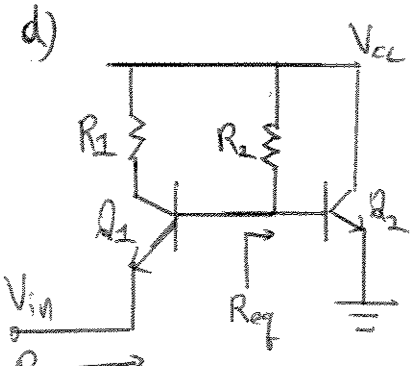
$$R_{in} = \frac{1}{g_{m1}} \parallel r_{\pi 1} + \frac{R_2}{\beta_1 + 1}$$

Since β is usually very large

$$R_{in} = \frac{1}{g_{m1}} + \frac{R_2}{\beta_1 + 1}$$

* Note, part c) and d) have swapped places.

d)



$$R_{eq} = R_2 \parallel r_{\pi 2}$$

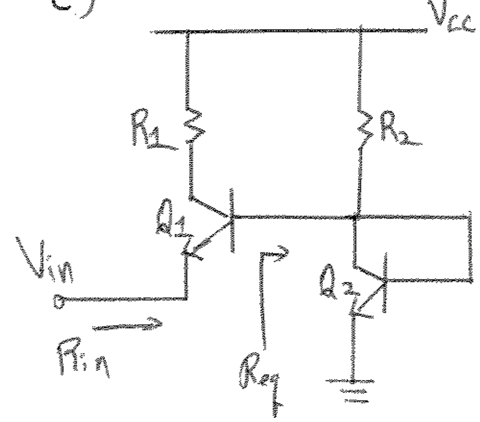
$$R_{in} = \frac{1}{g_{m1}} \parallel r_{\pi 1} + \frac{R_2 \parallel r_{\pi 2}}{\beta_1 + 1}$$

Since β is usually very large

$$R_{in} \approx \frac{1}{g_{m1}} + \frac{R_2 \parallel r_{\pi 2}}{\beta_1 + 1}$$

56) * Note, part c) and d) have swapped places

c)



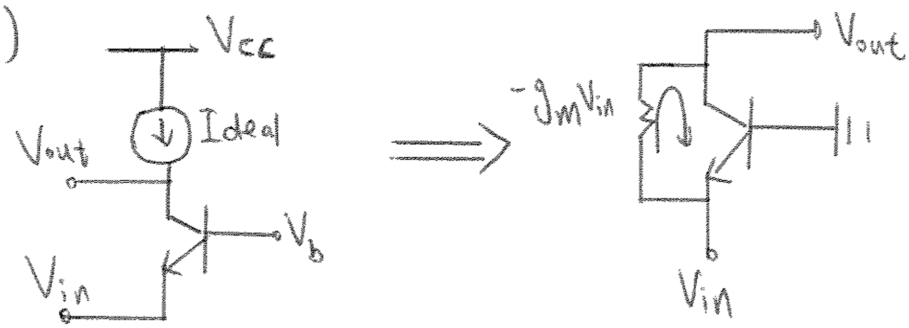
$$R_{eq} = R_2 \,//\, \frac{1}{g_{m_2}} \,//\, r_{\pi_2}$$

$$R_{in} = \frac{1}{g_{m_1}} \,//\, r_{\pi_1} + \frac{R_2 \,//\, \frac{1}{g_{m_2}} \,//\, r_{\pi_2}}{\beta_1 + 1}$$

Since β is usually very large

$$R_{in} \approx \frac{1}{g_{m_1}} + \frac{R_2 \,//\, \frac{1}{g_{m_2}}}{\beta_1 + 1}$$

57)



Since an ideal current source is an open circuit, the signal current produced by the transistor has no where to go but r_o.

So $V_{out} = -(g_m(0 - V_{in}))r_o + V_{in}$

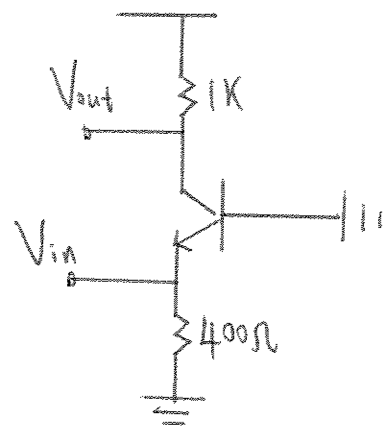
$V_{out} = g_m r_o V_{in} + V_{in}$

$V_{out} = V_{in}(g_m r_o + 1)$

$$\frac{V_{out}}{V_{in}} = 1 + g_m r_o$$

58)

b) AC Analysis



$$A_v = \left|\frac{V_{out}}{V_{in}}\right| = g_m 1K$$

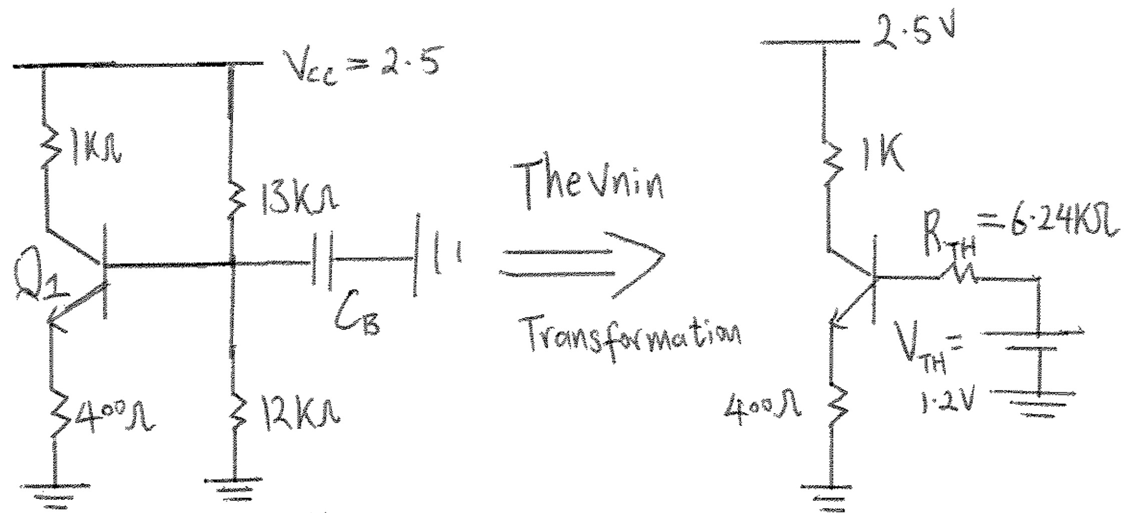
$$g_m = 0.0391 \left(\frac{1}{\Omega}\right) S$$

$$A_v = 39.1$$

$$R_{in} = 400\Omega // \frac{1}{g_m} = 400\Omega // 25.583\Omega = 24.0\Omega$$

$$R_{out} = 1K$$

58)



$\beta = 100, \quad I_s = 8 \times 10^{-16} A, \quad V_A = \infty, \quad C_B = $ Very large

a) DC Analysis:

$$I_c = \beta\left(\frac{1.2 - (V_{BE} + I_E 0.4)}{6.24}\right) \Rightarrow \frac{\beta(1.2 - V_{BE})}{6.24 + \frac{0.4\beta}{\alpha}}$$

Guess $V_{BE} = 0.7 \Rightarrow I_c = 1.072$ mA.

Verify V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.726$ V, not 0.7V, reiterate.

$V_{BE} = 0.726$ V; $I_c = 1.0163$ mA

Verify V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.725$V, Converged!!

$V_{BE} = 0.725$, $V_{CE} = 2.5 - \left[(1.0163)(1K) + 0.4\left(\frac{1.0163}{0.99}\right)\right]$

$V_{CE} = 1.07$

$I_c = 1.0163$mA, $I_B = 10.163\ \mu A$

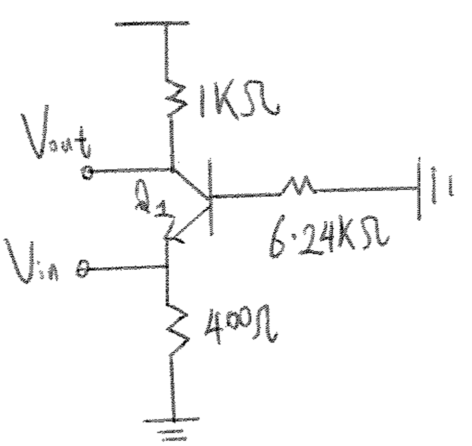
59)

$C_B = 0$

a) Since C_B was not considered during DC analysis, it has no effect on operating point analysis. So it is still the same as 58).

$$V_{BE} = 0.725\text{ V}$$
$$I_c = 1.0163\text{ mA}$$
$$I_B = 10.163\ \mu\text{A}$$
$$V_{CE} = 1.07\text{ V}$$

b) Since capacitor is frequency dependent, the circuit's AC analysis will be different.



$$|A_v| = \frac{1k}{\frac{1}{g_m} + \frac{6.24K\Omega}{\beta+1}} = 11.4$$

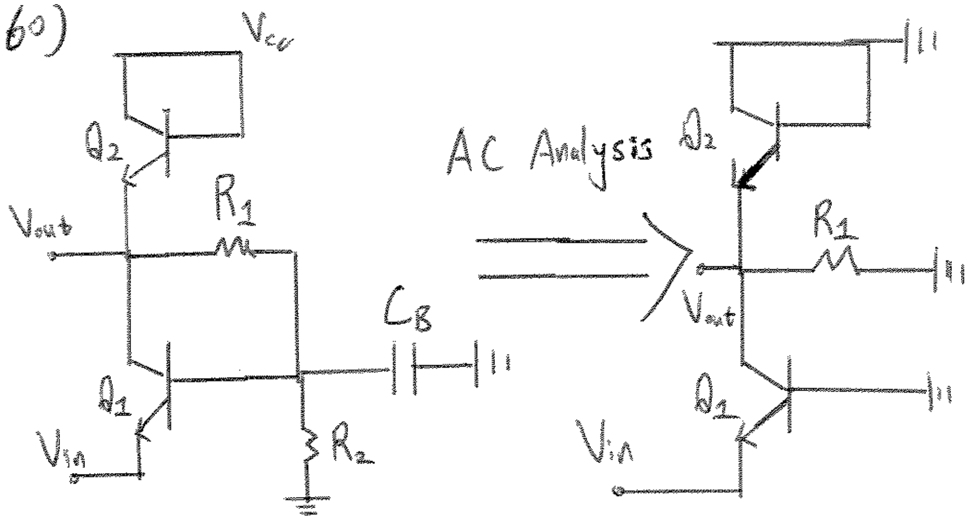
$$R_{in} = 400\Omega \,//\, \left(\frac{1}{g_m} + \frac{6.24K\Omega}{\beta+1}\right)$$

$$R_{in} = 71.7\Omega$$

Note: 6.24 KΩ is R_{THEV} of 13KΩ and 12KΩ combination.

$$R_{out} = 1K\Omega$$

60)

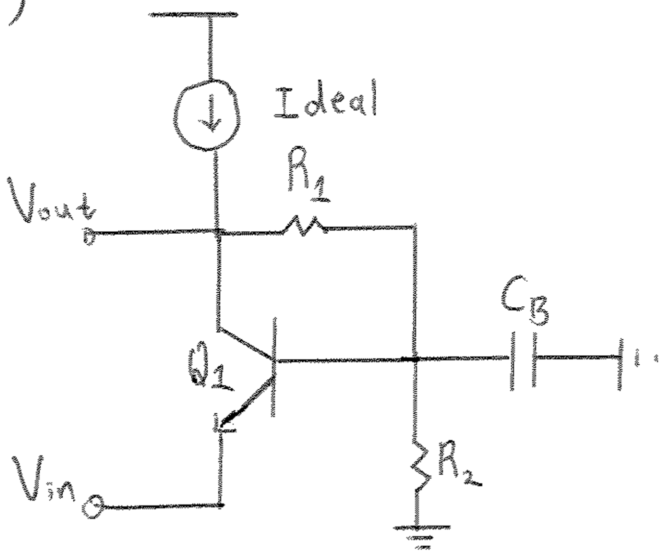


$$R_{out} = \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel R_1 \approx \frac{1}{g_{m2}} \parallel R_1$$

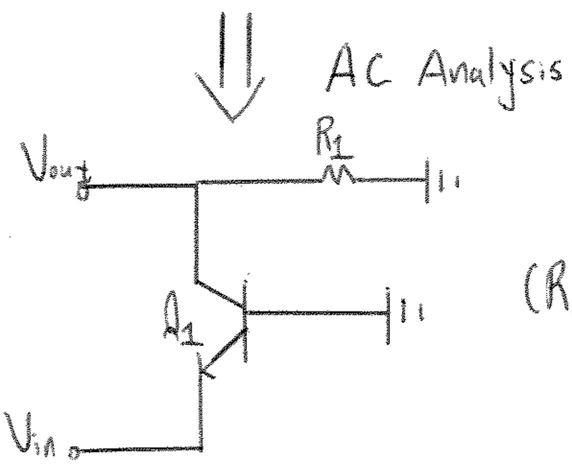
$$A_v = \left|\frac{V_{out}}{V_{in}}\right| = g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel R_1\right) \approx g_{m1}\left(\frac{1}{g_{m2}} \parallel R_1\right)$$

$$R_{in} = \frac{1}{g_m} \parallel r_{\pi 1} \approx \frac{1}{g_{m1}}$$

61)



$V_A = \infty$
C_B large



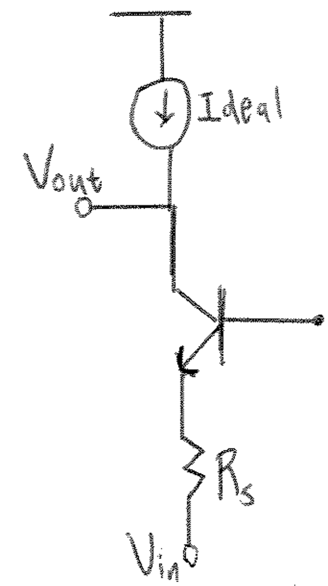
(R_2 shorted out)

$R_{out} = R_1$

$R_{in} = \frac{1}{g_{m_1}} // r_{\pi_1} \approx \frac{1}{g_{m_1}}$

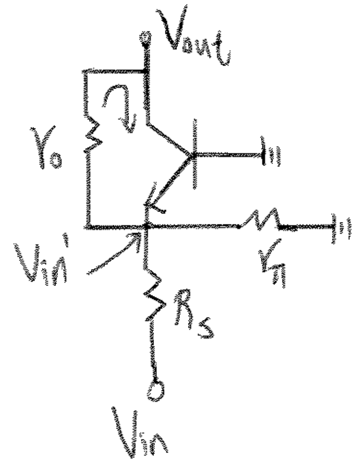
$|A_v| = g_{m_1} R_1$

62)



$V_A < \infty$

AC Analysis $\Longrightarrow$



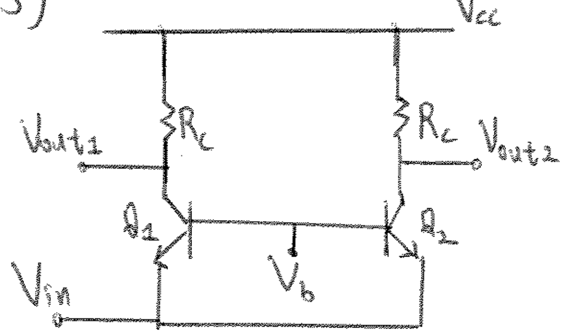
$$A_v = \frac{V_{out}}{V_{in}} = \left(\frac{V_{in}'}{V_{in}}\right)\left(\frac{V_{out}}{V_{in}'}\right), \quad \left(\frac{V_{in}'}{V_{in}}\right) = \frac{r_\pi}{r_\pi + R_s}$$

Since V_{out} is float, so looking at emitter and r_o, we will see an infinite impedance.

$$\frac{V_{out}}{V_{in}'} \Rightarrow \; -g_m(-V_{in}')r_o + V_{in}' = V_{out} \Rightarrow \frac{V_{out}}{V_{in}'} = (g_m r_o + 1)$$

$$A_v = (g_m r_o + 1)\left(\frac{r_\pi}{r_\pi + R_s}\right).$$

63)



$V_A = \infty$

$I_{S1} = 2I_{S2}$

$$\left|\frac{V_{out1}}{V_{in}}\right| = g_{m1} R_c \quad , \quad \left|\frac{V_{out2}}{V_{in}}\right| = g_{m2} R_c$$

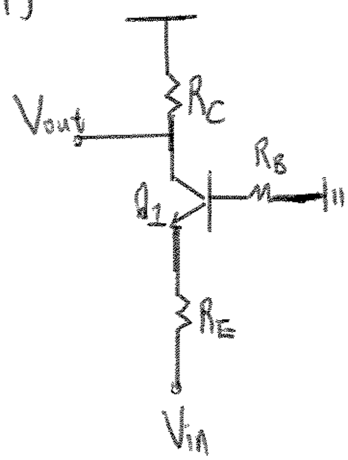
$$g_{m1} = \frac{V_T}{I_{c1}} = \frac{V_T}{2I_{S2}e^{(V_{BE}/V_T)}} \quad , \quad \text{Since } I_{S1} = 2I_{S2}$$

$$g_{m2} = \frac{V_T}{I_{c2}} = \frac{V_T}{I_{S2}e^{(V_{BE}/V_T)}}$$

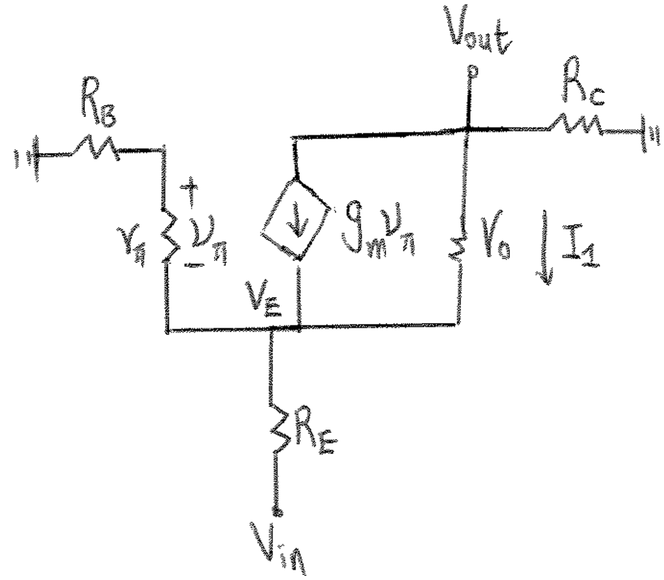
$(V_{BE1} = V_{BE2} = V_{BE})$

$$\Rightarrow \quad g_{m1} = \frac{g_{m2}}{2} \Rightarrow \left|\frac{V_{out1}}{V_{in}}\right| = \frac{1}{2}\left|\frac{V_{out2}}{V_{in}}\right|$$

64)



$\Longrightarrow$



$$V_{out} = -(I_1 + g_m v_\pi)R_c \quad , \quad I_1 = \frac{V_{out} - V_E}{r_o}$$

$$V_{out} = -\left(\frac{V_{out} - V_E}{r_o} + g_m v_\pi\right) R_c \quad , \quad V_E = -\frac{g_m v_\pi}{\beta}(r_\pi + R_B)$$

$$V_{out} = -\left(\frac{V_{out} + \frac{g_m v_\pi (r_\pi + R_B)}{\beta}}{r_o} + g_m v_\pi\right) R_c$$

Rearranging

$$v_\pi = \frac{-\left(1 + \frac{R_c}{r_o}\right)}{\frac{g_m (r_\pi + R_B) R_c}{\beta r_o} + g_m R_c} V_{out} = A V_{out}$$

Summing the voltage at node E.

$$V_E - \left(\left(1 + \frac{1}{\beta}\right) g_m v_\pi + \frac{(V_{out} - V_E)}{r_o}\right) R_E = V_{in} \qquad (1)$$

64)

Writing V_E in terms of V_π, and V_π in terms of V_{out}

1) becomes

$$-\frac{g_m A V_{out}}{\beta}(r_\pi + R_B)\left(1+\frac{R_E}{r_o}\right) - \left(1+\frac{1}{\beta}\right) g_m A V_{out} R_E - \frac{V_{out}}{r_o} R_E = V_{in}$$

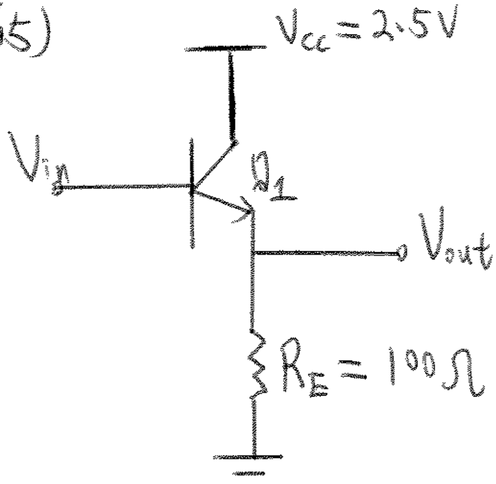
Solving V_{out}/V_{in} =>

$$\frac{V_{out}}{V_{in}} = \frac{1}{-\frac{g_m A}{\beta}(r_\pi + R_B)\left(1+\frac{R_E}{r_o}\right) - \left(1+\frac{1}{\beta}\right) g_m A R_E - \frac{R_E}{r_o}}$$

subsituting A into equation

$$\frac{V_{out}}{V_{in}} = \frac{\frac{g_m (r_\pi + R_B) R_C}{\beta r_o} + g_m R_C}{g_m\left(1+\frac{R_C}{r_o}\right)(r_\pi + R_B)\left(1+\frac{R_E}{r_o}\right) + \left(1+\frac{1}{\beta}\right) g_m \left(1+\frac{R_C}{r_o}\right) R_E - \frac{R_E}{r_o}\left(\frac{g_m (r_\pi + R_B) R_C}{\beta r_o} + g_m R_C\right)}$$

65)



$R_E = 100\Omega$
$V_A = \infty$
$|A_v| = 0.8$

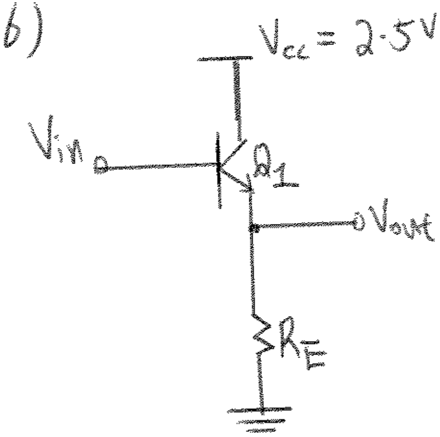
$$|A_v| = \frac{R_E}{R_E + \frac{1}{g_m}} = \frac{R_E I_c}{R_E I_c + V_T} = 0.8$$

$\Rightarrow R_E I_c = 0.8(R_E I_c + V_T)$, $R_E = 100\Omega$

$\Rightarrow 0.1 I_c = 0.08 I_c + 0.0208 \Rightarrow 0.02 I_c = 0.0208$

$\Rightarrow I_c = 1.04\,mA$

6b)



$|A_v| > 0.9$
$R_{in} > 10K\Omega$

$$|A_v| = \frac{R_E I_c}{R_E I_c + V_T} > 0.9 \Rightarrow R_E I_c > 0.9[R_E I_c + V_T]$$

$$\Rightarrow R_E I_c > 9V_T = 234\,mV, \text{ Let } R_E I_c = 240\,mV$$

$$R_{in} = r_\pi + (1+\beta)R_E > 10K \Rightarrow 100V_T + (101)R_E I_c > 10K\Omega\, I_c$$

Subsituting $R_E I_c = 240\,mV \Rightarrow I_c < 2.684\,mA$

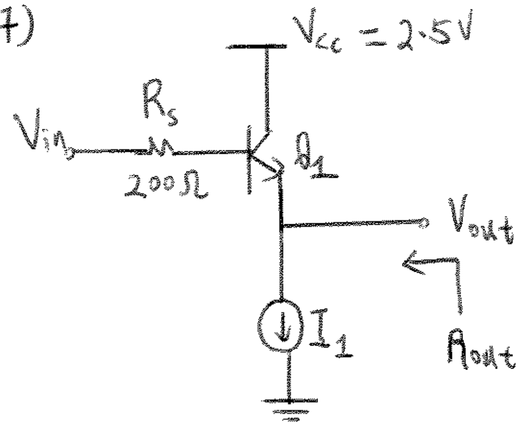
Choose I_c to be $2.5\,mA \Rightarrow R_E = 96\Omega$

To Verify:

$$R_{in} = 100\frac{(0.026)}{2.5} + (101)0.096 = 10.74K\Omega$$

$$|A_v| = \frac{(0.096)(2.5)}{(0.096)(2.5) + 0.026} = 0.902$$

67)



$\beta = 100$

$V_A = \infty$

$$R_{out} = \frac{1}{g_m} + \frac{R_s}{(\beta+1)} \leq 5\,\Omega \quad (\text{Assuming } r_\pi >> \frac{1}{g_m})$$

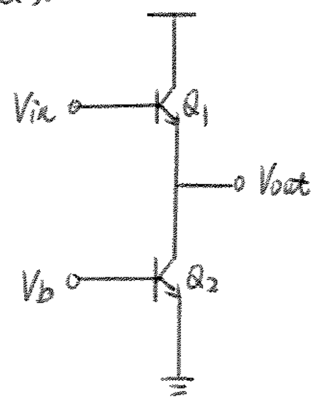
$$R_{out} = 0.026 + \frac{200\Omega I_c}{101} \leq 5\Omega I_c$$

$$\Rightarrow I_c \geq 0.0086A$$

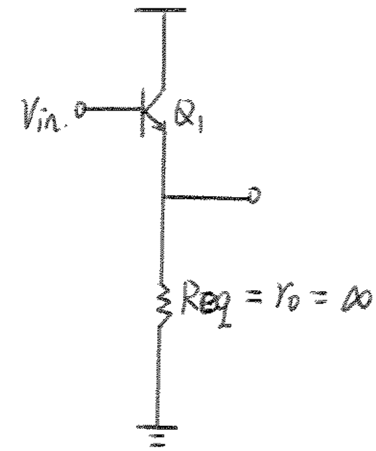
Pick $I_c = 0.009\,A$

$$R_{out} = \frac{0.026V}{0.009A} + \frac{200}{101} = 4.87\Omega$$

68). a). $V_A = \infty$



$\Longrightarrow$



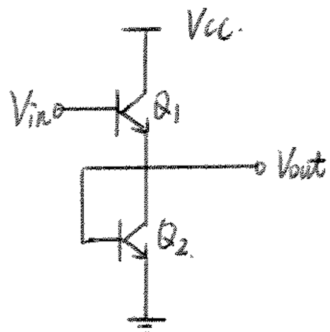
$$|A_v| = \frac{r_o}{r_o + \frac{1}{g_m}}, \quad \text{since } r_o = \infty$$

$$|A_v| = 1.$$

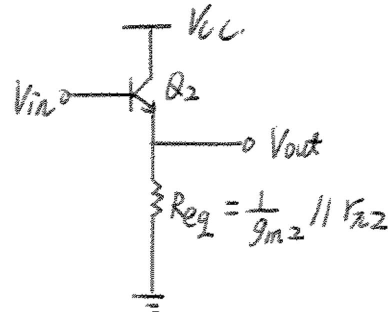
$$R_{in} = \infty \quad (\text{since } r_o = \infty)$$

$$R_{out} = \infty \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1} = \frac{1}{g_{m1}} \,//\, r_{\pi 1}$$

b).



$\Longrightarrow$



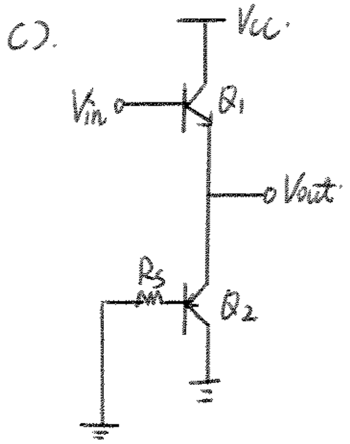
$$|A_v| = \frac{\frac{1}{g_{m2}} // r_{\pi 2}}{\frac{1}{g_{m2}} // r_{\pi 2} + \frac{1}{g_{m1}}}$$

$$R_{in} = r_{\pi 1} + (1+\beta) \frac{1}{g_{m2}} // r_{\pi 2}$$

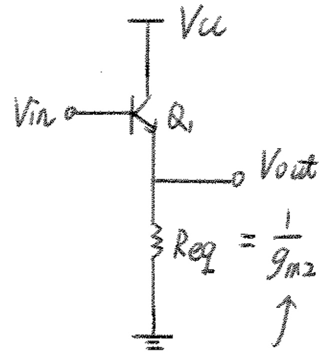
$$R_{out} = \frac{1}{g_{m2}} \,//\, r_{\pi 2} \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1}$$

(If $I_{S1} = I_{S2}$, $g_{m1} = g_{m2} = g_m$, $r_{\pi 1} = r_{\pi 2} = r_\pi$, $R_{out} = \frac{1}{2g_m} // \frac{r_\pi}{2}$)

c).



$\Longrightarrow$



$$R_{eq} = \frac{1}{g_{m2}} + \frac{R_S}{\beta+1}$$

$$\uparrow$$

(Assuming $r_\pi >> \frac{1}{g_m}$)

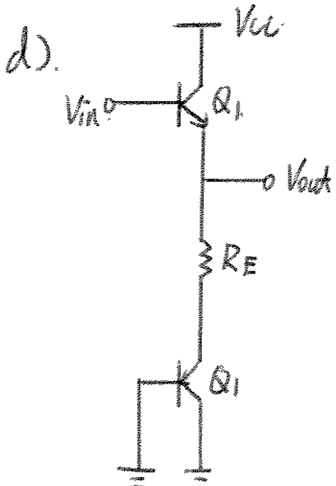
$$|A_v| = \frac{\frac{1}{g_{m2}} + \frac{R_S}{\beta+1}}{\frac{1}{g_{m2}} + \frac{R_S}{\beta+1} + \frac{1}{g_{m1}}}$$

$$R_{in} = r_\pi + (1+\beta)\left(\frac{1}{g_{m2}} + \frac{R_S}{\beta+1}\right)$$

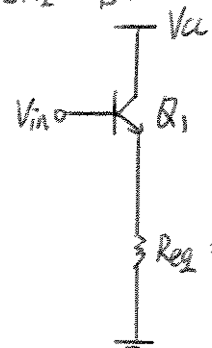
$$R_{out} = \left(\frac{1}{g_{m2}} + \frac{R_S}{\beta+1}\right) // \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)$$

$$R_{out} \approx \left(\frac{1}{g_{m2}} + \frac{R_S}{\beta+1}\right) // \frac{1}{g_{m1}}$$

d).



$\Longrightarrow$



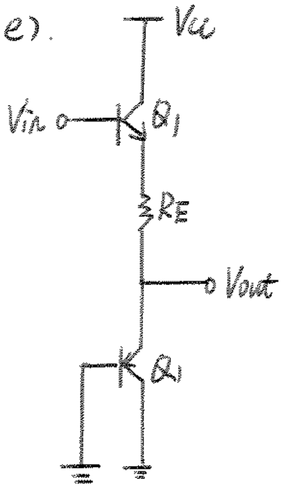
$$R_{eq} = R_E + \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)$$

$$|A_v| = \frac{R_E + \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)}{R_E + \left(\frac{1}{g_{m1}} // r_{\pi 1}\right) + \frac{1}{g_{m1}}}$$

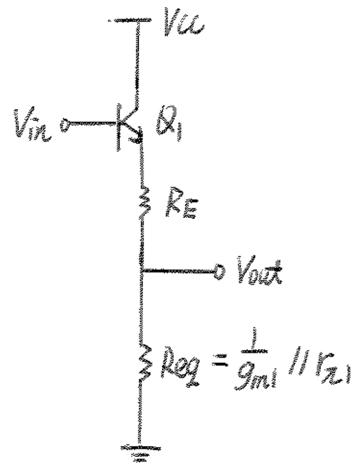
$$R_{in} = r_\pi + (1+\beta)\left[R_E + \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)\right]$$

$$R_{out} = \left[R_E + \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)\right] // \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)$$

68). e).



$\Longrightarrow$

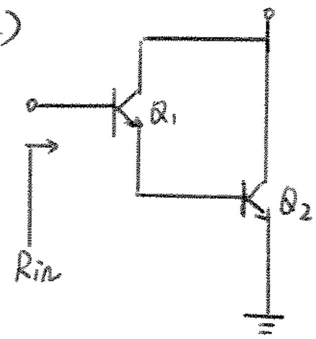


$$|A_v| = \frac{\frac{1}{g_{m1}} \parallel r_{\pi 1}}{\frac{1}{g_{m1}} \parallel r_{\pi 1} + R_E + \frac{1}{g_{m1}}}$$

$$R_{in} = r_\pi + (1+\beta)\left[R_E + \frac{1}{g_{m1}} \parallel r_{\pi 1}\right]$$

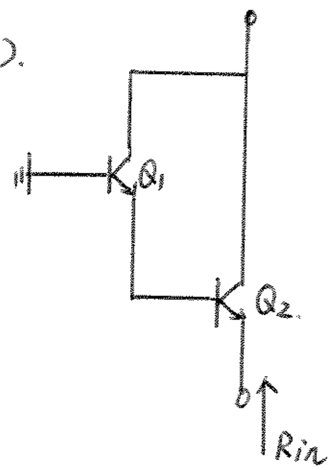
$$R_{out} = \left(\frac{1}{g_{m1}} \parallel r_{\pi 1}\right) \parallel \left(R_E + \frac{1}{g_{m1}} \parallel r_{\pi 1}\right).$$

69 a)



$$R_{in} = r_{\pi_1} + (1+\beta)\, r_{\pi_2}$$

b).



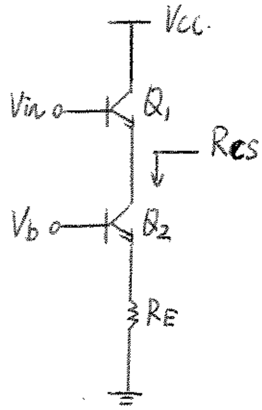
$$R_{in} = \frac{1}{g_{m_2}} + \frac{1/g_{m_1}}{(\beta+1)} \quad \left(\text{Assume } r_\pi \gg \frac{1}{g_m}\right)$$

c) Current Gain $= \dfrac{(I_{C1} + I_{C2})}{I_{B1}} = \beta + \dfrac{I_{C2}}{I_{B1}} = \beta + \dfrac{\beta I_{B2}}{I_{B1}}$

Since $I_{B_2} = I_{C1} = \beta I_{B1}$

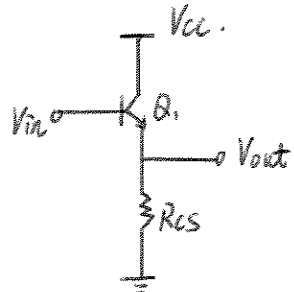
Current Gain $= \beta + \beta^2 = \beta(\beta+1)$, (Assuming $\beta_1 = \beta_2 = \beta$)

70).



a) $R_{cs} = r_{o2} + (1 + g_{m2} r_{o2})(R_E // r_{\pi 2})$

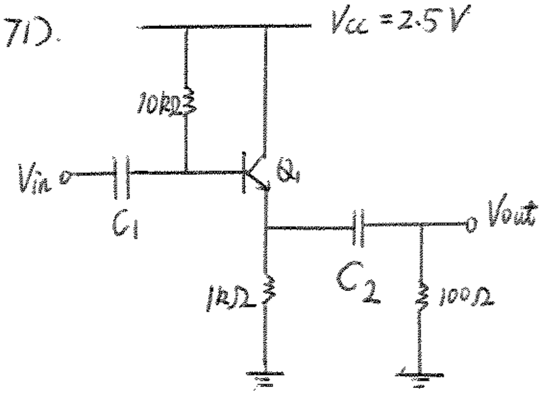
b).



$$A_v = \frac{R_{cs} // r_{o1}}{R_{cs} // r_{o1} + \frac{1}{g_{m1}}}$$

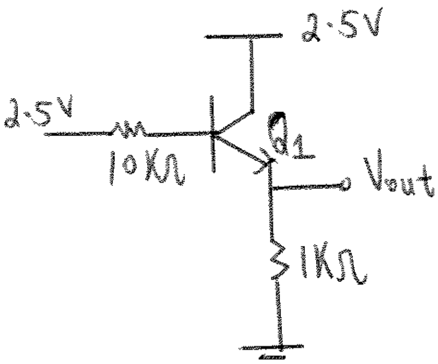
$$A_v = \frac{(r_{o2} + (1 + g_{m2} r_{o2})(R_E // r_{\pi 2})) // r_{o1}}{(r_{o2} + (1 + g_{m2} r_{o2})(R_E // r_{\pi 2})) // r_{o1} + \frac{1}{g_{m1}}}$$

71).



$I_s = 7 \times 10^{-16} A$
$\beta = 100$
$V_A = 5V$

DC Analysis: (Ignore r_o's effect)



$$I_c = \beta \left(\frac{2.5 - \left(V_{BE} + \frac{I_c}{\alpha} 1K\Omega\right)}{10K\Omega} \right)$$

rearrange

$$I_c = \frac{2.5 - V_{BE}}{\frac{10K\Omega}{\beta} + \frac{1K\Omega}{\alpha}}$$

Guess: $V_{BE} = 0.7V$, $I_c = 1.621mA$

check for V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.740V$, not 0.7, reiterate

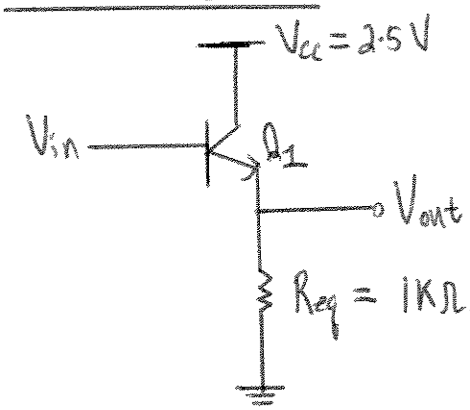
$V_{BE} = 0.740V$, $I_c = 1.59mA$

Check for V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.740V$, converged.

So $I_c = 1.59mA$, $g_m = 0.0612 \left(\frac{1}{\Omega}\right) S$, $\frac{1}{g_m} = 16.34\Omega$,
$r_o = 3.14K\Omega$

71)

AC Analysis: (Include r_o)

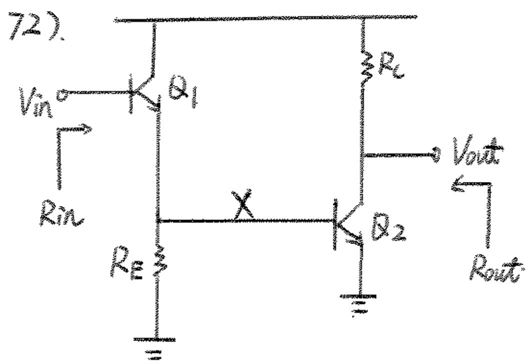


(Assume C_1 and C_2 are large)

$R_{eq} = 1K\Omega // 100\Omega // 3.14K\Omega \ (r_o)$

$$A_v = \frac{(1K\Omega // 100\Omega // 3.14K\Omega)}{16.34\Omega + (1K\Omega // 100\Omega / 3.14K\Omega)} = 0.84$$

72).



$V_A < \infty$

a) $R_{in} = r_{\pi 1} + (1+\beta)(R_E \,//\, r_{\pi 2} \,//\, r_{o1})$

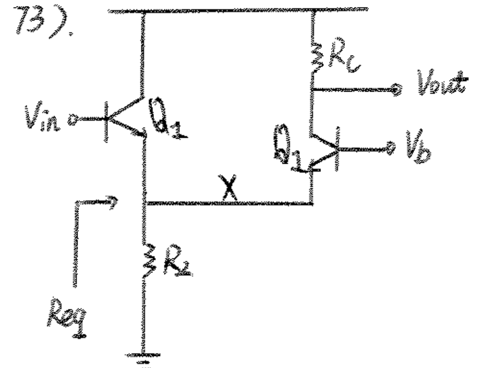
$R_{out} = R_C \,//\, r_{o2}$

b) $\left|\dfrac{V_{out}}{V_{in}}\right| = \left|\dfrac{V_x}{V_{in}}\right| \left|\dfrac{V_{out}}{V_x}\right|$

$\left|\dfrac{V_x}{V_{in}}\right| = \dfrac{(R_E \,//\, r_{\pi 2} \,//\, r_{o1})}{\frac{1}{g_{m1}} + R_E \,//\, r_{\pi 2} \,//\, r_{o1}}$, $\left|\dfrac{V_{out}}{V_x}\right| = g_{m2} R_C$

$\left|\dfrac{V_{out}}{V_{in}}\right| = (g_{m2} R_C)\left[\dfrac{R_E \,//\, r_{\pi 2} \,//\, r_{o1}}{\frac{1}{g_{m1}} + R_E \,//\, r_{\pi 2} \,//\, r_{o1}}\right]$

73).



$V_A = \infty$

a) $R_{eq} = R_E \,//\, r_{\pi 1} \,//\, \frac{1}{g_{m1}}$

$R_{in} = r_{\pi 1} + (1+\beta)\left[R_E \,//\, r_{\pi 1} \,//\, \frac{1}{g_{m1}}\right]$

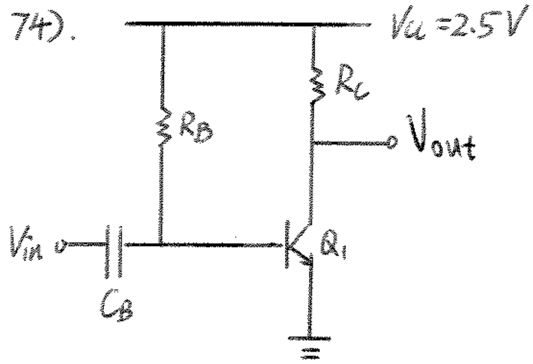
$R_{out} = R_C$

b) $\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{V_X}{V_{in}}\right|\left|\frac{V_{out}}{V_X}\right|$

$\left|\frac{V_X}{V_{in}}\right| = \dfrac{R_E \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1}}{\frac{1}{g_{m1}} + R_E \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1}}$, $\left|\frac{V_{out}}{V_X}\right| = g_{m1} R_C$

$$\left|\frac{V_{out}}{V_{in}}\right| = (g_{m1} R_C)\left(\frac{R_E \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1}}{R_E \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1} + \frac{1}{g_{m1}}}\right)$$

74).



$A_V = 10$

$R_{in} > 5K\Omega$

$R_{out} = 1K$, $R_C = 1K\Omega$

$$A_V = \frac{R_C}{\frac{1}{g_m}} = 10 = \frac{I_C R_C}{V_T} \Rightarrow I_C = 0.26mA$$

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.697V.$$

$$I_C = 100\left(\frac{2.5 - 0.697}{R_B}\right) \Rightarrow R_B = 693K\Omega, \quad r_\pi = \beta\frac{V_T}{I_C} = 10K\Omega$$

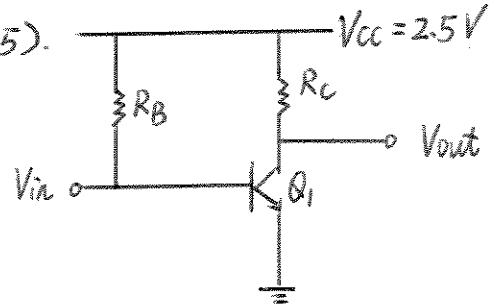
$$R_{in} = 693K // 10K = 9.86K\Omega$$

$$\frac{1}{2\pi(200)C_B} = \frac{1}{10}\frac{1}{g_m} = 10\Omega \Rightarrow C_B = 80\mu f$$

(To avoid gain degradation).

$R_C = 1K\Omega$ | | $A_V = 10$
$R_B = 693K$ | $\Rightarrow$ | $R_{out} = 1K\Omega$
$C_B = 80\mu f$ | | $R_{in} = 9.86K\Omega$

75).



$A_V = \text{Maximum}$
$R_{out} \leq 500\Omega$
$V_{BC} \leq 400\text{ mV}$

$A_v = g_m R_c = \frac{I_c R_c}{V_T}$, gain is maximized by maximize $I_c R_c$

$R_{out} = R_c \leq 500\Omega$, Choose $R_c = 450\Omega$, $R_{out} = 450\Omega$

$V_{BC} = V_{BE} - (2.5 - I_c R_c) \leq 400\text{ mV}$

Guess $V_{BE} = 0.7$, and let $V_{BC} = 400\text{ mV}$ to maximize $I_c R_c$

$$0.7 - (2.5 - I_c\, 0.450) = 0.4$$

$$I_c = 4.89\text{mA}, \quad V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.773$$

Not 0.7, reiterate.

$$0.773 - 2.5 + I_c\, 0.450 = 0.4$$

$$I_c = 4.73\text{mA}, \quad V_{BE} = 0.772 \quad \text{converged!!}$$

$$A_v = \left(\frac{I_c}{V_T}\right)(R_c) = \left(\frac{4.73}{26}\right)(450) = 81.9$$

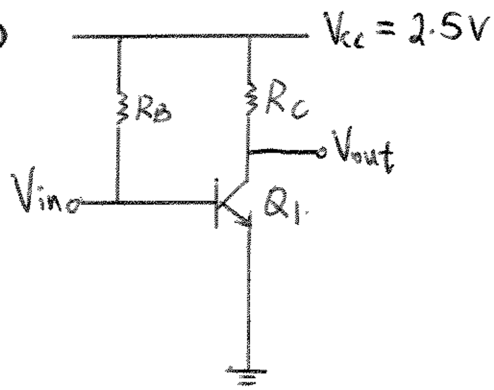
$$R_B = 100\left(\frac{2.5 - 0.772}{4.73}\right) = 36.5K$$

$R_B = 36.5K$
$R_c = 450\Omega$

$\Longrightarrow$

$A_v = 81.9$
$V_{BC} = 0.4V$
$R_{out} = 450\Omega$

76)



R_{in} : Maximum

$A_v \geq 20$

$R_{out} = R_c = 1K$

$$A_v = g_m R_c = \frac{I_c R_c}{V_T} \geq 20 \Rightarrow I_c \geq 0.52\,mA$$

$$R_{in} = R_B // r_\pi = \frac{\beta R_B V_T}{R_B I_c + V_T \beta} \quad 1), \quad I_c = \beta \left(\frac{2.5 - V_{BE}}{R_B} \right) \quad 2)$$

As we can see from 1), higher I_c means lower R_{in}.

So set I_c as low as possible, $I_c = 0.52mA$.

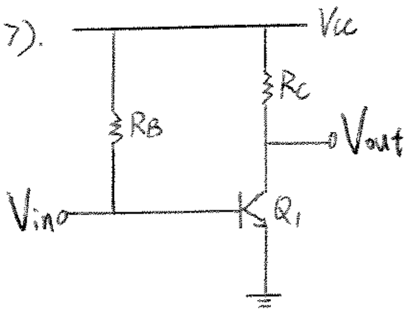
$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.715V$$

From 2), $R_B = \frac{100(2.5 - 0.715)}{0.52} = 343.3K\Omega$, $r_\pi = 5K\Omega$

$R_{in} = 4.93K\Omega$

$R_c = 1K\Omega$
$R_B = 343.3K\Omega$ $\Rightarrow$ $A_v = 20$
$R_{in} = 4.93K\Omega$
$R_{out} = 1K\Omega$

77).



Minimum Supply

$A_v = 15$

$R_{out} = 2k\Omega$, $R_c = 2k\Omega$

$V_{BC} \leq 0.4V$

$$A_v = g_m R_c = \frac{I_c}{V_T} R_c = 15 \Rightarrow I_c = 0.195mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.689$$

$$V_{BC} = V_{BE} - (V_{cc} - I_c R_c) \leq 0.4V \text{ , } I_c R_c = 0.39V$$

$$V_{cc} \geq 0.689 + 0.39 - 0.4 = 0.679V$$

Since the problem is concerned with minimum power supply,
let $V_{cc} = 0.69V$, Since $V_{BE} = 0.679V$ $(V_{cc} > V_{BE})$

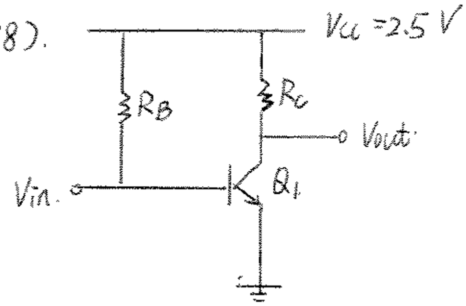
$$I_c = \beta\left(\frac{V_{cc} - 0.689}{R_B}\right) \Rightarrow R_B = 100\left(\frac{0.69 - 0.689}{0.195}\right) = 512.8\Omega$$

$R_c = 2k\Omega$

$R_B = 512.8\Omega$ $\Rightarrow$ $A_v = 15$

$V_{cc} = 0.69V$ $\quad R_{out} = 2k\Omega$

78).

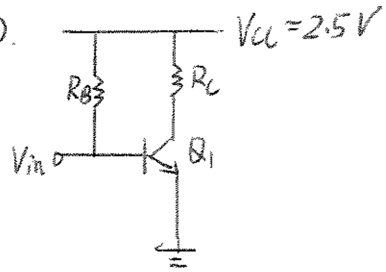


$$A_o = g_m R_C$$

$$A_o = \frac{I_C R_C}{V_T}, \quad \text{Power Dissipation} = I_C V_{CC}$$

$$R_{out} = R_C = \frac{A_o V_T}{I_C}$$

For large R_{out}, I_c has to be small, which decreases power. So small power dissipation and small output impedance cannot be satisfied simultaneously.

79). 

Power Budget = 1mW
$A_v = 20$

$$A_v = g_m R_c = \frac{I_c R_c}{V_T} = 20 \;, \quad V_{cc} I_c = 1mW$$

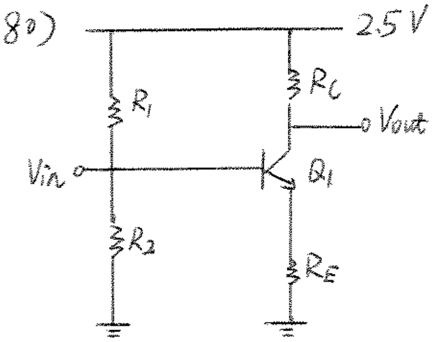
$$I_c = 0.4mA \;, \; R_c = 1.3K$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.708V \;, \quad I_c = \beta\left(\frac{V_{cc} - V_{BE}}{R_B}\right) = 100\left(\frac{2.5 - 0.708}{R_B}\right)$$

$$\Rightarrow R_B = 448K \;, \quad R_{in} = 448 \parallel (100)\left(\frac{26}{0.4}\right) = 6.4K$$

$R_B = 448K$
$R_c = 1.3K$ $\Rightarrow$
$A_v = 20$
Power Budget = 1mW
$R_{out} = 1.3K\Omega$
$R_{in} = 6.4K$

80)



$A_V = 5$
$R_{out} = R_c = 500\Omega$
$R_E I_c \approx 300\,mV$

$$A_V = \frac{R_c I_c}{R_E I_c + V_T} = \frac{R_c I_c}{300+26} \Rightarrow \begin{array}{l} R_c I_c = 1.63V \Rightarrow I_c = 3.26\,mA \\ R_E I_c \approx 300\,mV \Rightarrow R_E = 92\,\Omega \end{array}$$

$$R_1 = \frac{2.5 - (V_{BE} + 0.3)}{10\, I_B}, \quad V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.7624$$
$$10 I_B = 0.326\,mA$$

$$R_1 = \frac{2.5 - (0.7624 + 0.3)}{0.326} = 4.41K$$

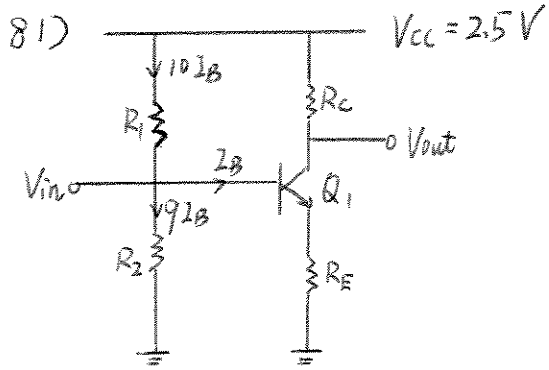
$$R_2 = \frac{(0.7624 + 0.3)}{(9)(0.0326)} = 3.62\,K$$

$V_{CE} = 2.5 - 1.63 - 0.3 = 0.57, \quad V_{BE} = 0.7624.$

Q_1 is in soft saturation region, so active region characteristics still apply.

$$\begin{array}{l} R_c = 500\Omega \\ R_1 = 4.41K\Omega \\ R_2 = 3.62K\Omega \\ R_E = 92\Omega \end{array} \Rightarrow \begin{array}{l} A_V = 5 \\ R_{out} = 500\Omega \end{array}$$

8 1)



A_v = Maximum

$R_{out} = R_c \leq 1K\Omega$

$V_{BC} = 0.4V$

$R_E I_c \approx 200mV$

$$V_{BC} = (V_{BE} + 0.2) - (2.5 - I_c R_c) = 0.4 \quad \text{i)}$$

$$A_v = \frac{R_c I_c}{R_E I_c + V_T} = \frac{R_c I_c}{0.226}$$

Rearrange, i) becomes $I_c R_c = 0.4 + 2.5 - (V_{BE} + 0.2)$

Guess $V_{BE} = 0.7 \Rightarrow I_c R_c = 2V$

Let $R_c = 1K \Rightarrow I_c = 2mA$

Check for V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.750$, not 0.7, reiterate

$V_{BE} = 0.75 \Rightarrow I_c R_c = 1.95V$

$R_c = 1K \Rightarrow I_c = 1.95mA$

Check for V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.750$, Converged!!

$I_c = 1.95mA$, $R_E I_c = 200mA \Rightarrow R_E = 103\Omega$

$I_B = 0.0195mA$

$$R_1 = \frac{2.5 - (0.750 + 0.2)}{(10)(0.0195)} = 7.95K$$

$$R_2 = \frac{(0.750 + 0.2)}{(9)(0.0195)} = 5.41K$$

81)

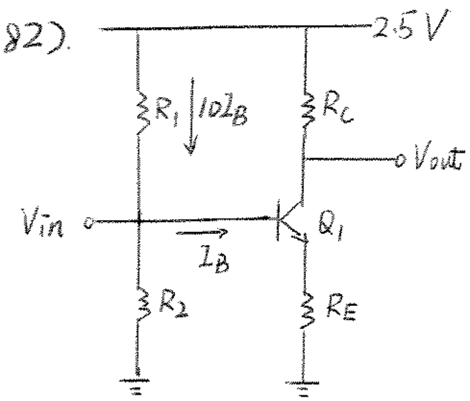
$$A_V = \frac{R_c I_c}{R_E I_c + V_T} = \frac{1.95}{0.226} = 8.63$$

This is the maximum gain we would get when R_{out} is $1K\Omega$ and V_{BC} is at 0.4 V. Since anything larger will violate either requirement.

$R_c = 1K\Omega$
$R_E = 103\Omega$
$R_1 = 7.95K\Omega$
$R_2 = 5.41K\Omega$

$\Longrightarrow$ $A_V = 8.63$
$R_{out} = 1K\Omega$

82).



Power budget = 5 mW

$A_V = 5$

$R_E I_C \approx 200\,mV$

$$V_{cc}\left(I_c + \frac{I_c}{10}\right) = 5\,mW \Rightarrow I_C = 1.82\,mA\ ,\ I_B = 0.0182\,mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.747$$

$$A_V = \frac{R_c I_c}{R_E I_c + V_T} = \frac{R_c I_c}{0.226} = 5 \Rightarrow R_c I_c = 1.13V \Rightarrow R_c = 621\,\Omega$$

$$R_1 = \frac{2.5 - (0.747 + 0.2)}{(10)(0.0182)} = 8.53K\Omega$$

$$R_2 = \frac{(0.747 + 0.2)}{(9)(0.0182)} = 5.78K\Omega$$

$$R_E I_c \approx 200\,mV \Rightarrow R_E = 110\,\Omega$$

$R_c = 621\,\Omega$

$R_E = 110\,\Omega$

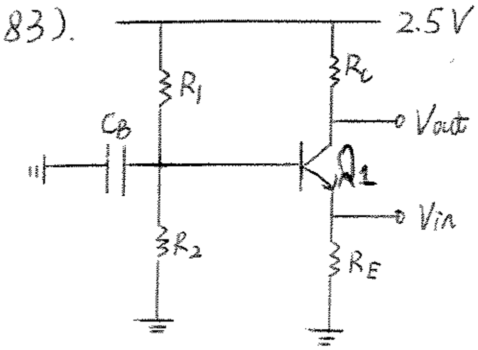
$R_1 = 8.53\,K\Omega$

$R_2 = 5.78\,K\Omega$

$\Longrightarrow$ $A_V = 5$

Power Budget = 5mW

83).



$A_V = 20$
$R_{in} = 50\,\Omega$
$R_E I_C \approx 10 V_T = 260\text{ mV}$

$R_{in} = \dfrac{1}{g_m} = 50\Omega$, since R_E doesn't affect input impedance.

$\dfrac{V_T}{I_C} = 50\Omega \Rightarrow I_C = \dfrac{V_T}{50\Omega} = 0.52\text{ mA}$, $I_B = 0.0052\text{ mA}$

$A_V = \dfrac{R_C}{1/g_m} = \dfrac{I_C R_C}{V_T} = 20 \Rightarrow R_C = 1K\Omega$

$R_1 = \dfrac{2.5 - (0.715 + 0.260)}{(10)(0.0052)} = 29.3K$

$R_2 = \dfrac{(0.715 + 0.260)}{(9)(0.0052)} = 20.83K$

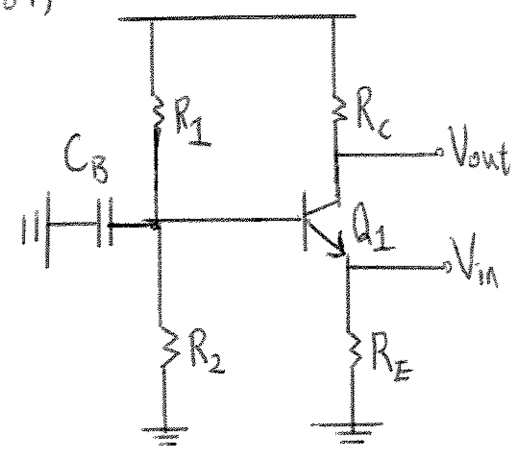
$R_E I_C \approx 260\text{ mV} \Rightarrow R_E \approx 500\Omega$

$\dfrac{1}{C_B(2\pi)(200)} = \dfrac{1}{10}\dfrac{1}{g_m} = 5\Omega \Rightarrow C_B = 159.1\,\mu f$

$R_C = 1K\Omega$, $R_E = 500\Omega$, $R_1 = 29.3K\Omega$, $R_2 = 20.83K$, $C_B = 159.1\,\mu f$

$\Rightarrow A_V = 20$, $R_{in} = 50\Omega$

84)



$A_v = 8$

$R_{out} = 500\Omega$

$R_{out} = R_c = 500\Omega$

$$A_v = \frac{I_c}{V_T} R_c = 8 \implies I_c = 0.416\ mA,\ I_B = 0.00416\ mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.709$$

$$R_E \approx \frac{260\ mV}{I_c} = 625\Omega, \quad R_1 = \frac{2.5 - (0.709 + 0.260)}{(10)(0.00416)} = 36.8K\Omega$$

$$R_2 = \frac{(0.709 + 0.260)}{(9)(0.00416)} = 25.9K\Omega$$

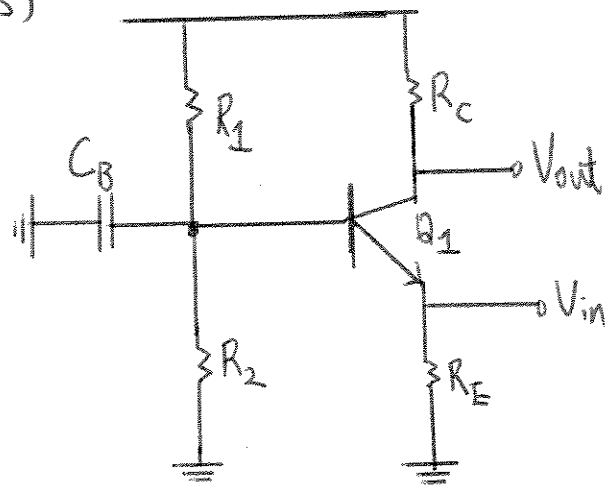
$$\frac{1}{g_m} = \frac{V_T}{I_c} = 62.5\Omega, \quad \frac{1}{C_B\, 200(2\pi)} = \frac{62.5}{10} \implies C_B = 127.3\ \mu f$$

$C_B = 127.3\mu f$
$R_1 = 36.8\ K$
$R_2 = 25.9\ K$
$R_c = 500\Omega$
$R_E = 625\Omega$

$\implies$ $A_v = 8$

$R_{out} = 500\Omega$

85)



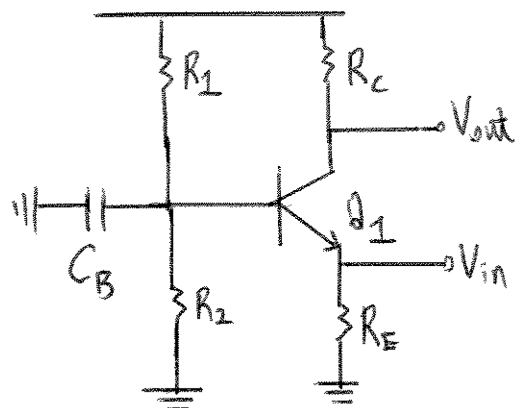
$A_v = 20$
$R_c = 200\Omega$
$(R_c = R_{out})$

$$A_v = \frac{I_c R_c}{V_T} = 20 \Rightarrow \quad I_c = 2.6\,mA$$
$$I_B = 0.026\,mA, \quad 10 I_B = 0.26\,mA$$

$$\text{Power} = V_{cc}(I_c + 10 I_B) = 2.5(0.26\,mA + 2.6\,mA) = 7.15\,mW$$

This is the minimum power dissipation, since anything lower will lower the voltage gain.

86)



Power = 5mW

$A_v = 10$

$V_{cc} I_c + V_{cc} \frac{I_c}{10} = 5mW$, $V_{cc} I_c 1.1 = 5mW$, $I_c = 1.82\,mA$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.747$

$A_v = g_m R_c = \frac{I_c R_c}{V_T} = 10 \Rightarrow R_c = 0.143K\Omega$

$I_c R_E \approx 260\,mV$, $R_E \approx 142.9\Omega$

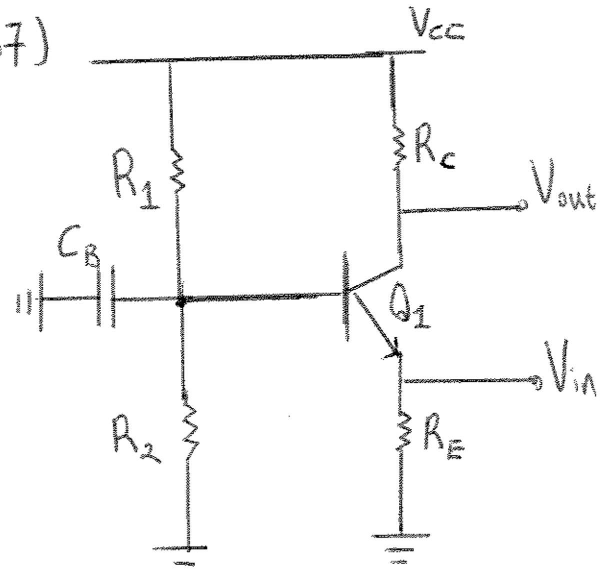
$R_1 = \frac{2.5 - (0.747 + 0.260)}{(10)(0.0182)} = 8.2K$, $R_2 = \frac{(0.747 + 0.260)}{(9)(0.0182)} = 6.15K$

$\frac{1}{g_m} = \frac{V_T}{I_c} = 14.3\Omega$, $\frac{1}{C_B 2\pi(200)} = \frac{14.3}{10} \Rightarrow C_B = 556.5\,\mu f$

$R_c = 143\Omega$, $R_E = 143\Omega$, $R_1 = 8.2K$, $R_2 = 6.15K$, $C_B = 556.5\mu f$.

$\Rightarrow$ $A_v = 10$, Power = 5mW

87)



$R_{in} = 50\Omega$

$A_v = 20$

Assume R_E doesn't affect R_{in} significantly,

$R_{in} \approx \dfrac{1}{g_m} = 50\Omega$

$$A_v = \frac{R_c}{1/g_m} = 20 \Rightarrow R_c = 1K\Omega, \quad \frac{1}{g_m} = \frac{V_T}{I_c} \Rightarrow I_c = \frac{26mV}{50\Omega} = 0.52\,mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.715V, \quad R_E I_c = 260\,mV \Rightarrow R_E = 500\Omega$$

$$V_{cc} = I_c R_c + V_{CE} + I_c R_E = 0.52 + V_{CE} + 0.260$$

V_{BC} is forward biased to $0.4V$, $V_{CE} = V_{BE} - 0.4 = 0.315\,V$

$V_{cc} = 0.52 + 0.315 + 0.260 = 1.1V$. (Minimum Supply Voltage)

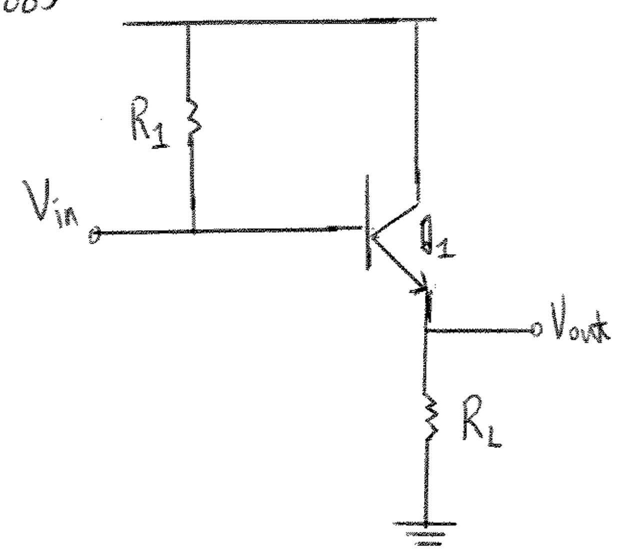
$$R_1 = \frac{1.1 - (0.715 + 0.260)}{0.052} = 2.4K, \quad R_2 = \frac{(0.715 + 0.260)}{(9)(0.0052)} = 20.83K$$

$$\frac{1}{C_B 2\pi 200} = \frac{1}{10}\frac{1}{g_m} = 5 \Rightarrow C_B = 159.2\,\mu f$$

$V_{cc} = 1.1V$, $R_1 = 2.4K$, $R_2 = 20.83K$, $R_c = 1K$, $R_E = 500\Omega$, $C_B = 159.2\mu f$

$\Rightarrow R_{in} = 50\Omega$, $A_v = 20$

88)



$A_v = 0.85$
$R_{in} > 10k\Omega$
$R_L = 200\Omega$

$$A_v = \frac{R_L}{R_L + \frac{1}{g_m}} = 0.85 \Rightarrow \frac{200}{200 + \frac{1}{g_m}} = 0.85$$

$$\Rightarrow 200 = 0.85\left(200 + \frac{1}{g_m}\right) \Rightarrow \frac{1}{g_m} = 35.294\Omega$$

$$\Rightarrow I_c = \frac{26\,mV}{35.294\Omega} = 0.737\,mA, \quad V_{BE} = V_T \ln\left(\frac{0.737}{6\times10^{-13}}\right) = 0.724V$$

$$R_{in} = R_1 \,//\, (r_\pi + (1+\beta)(200\Omega))$$

$$R_{in} = R_1 \,//\, 23.73K$$

$$R_{in} = \frac{R_1\, 23.73K}{R_1 + 23.73K} > 10K \Rightarrow R_1 > 17.28K$$ (Input Impedance requirement)

To support an I_c of 0.737, R_1 must be determined.

88)

$$R_1 = \frac{2.5 - (0.724 + (0.737)(0.2)/0.99)}{0.737/100}$$

$$R_1 = 220.77K\Omega$$

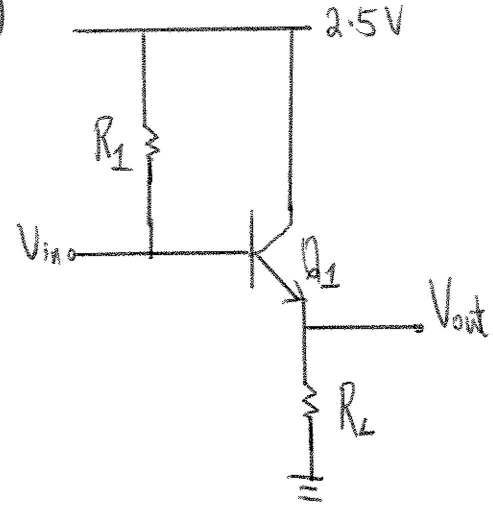
$R_1 = 220.77K\Omega \Rightarrow R_{in} = 220.77K\Omega // 23.73K\Omega$

$R_{in} = 21.43K\Omega > 10K\Omega$

$R_1 = 220.77K\Omega$ $\Rightarrow$ $A_v = 0.85$

$R_L = 200\Omega$ $R_{in} = 21.43K\Omega$

89)



Power = 5 mW

$A_v = 0.9$

$$A_v = \frac{R_L}{R_L + \frac{1}{g_m}} = 0.9 \Rightarrow R_L = 0.9\left(R_L + \frac{1}{g_m}\right)$$

$$R_L = 9\frac{1}{g_m}$$

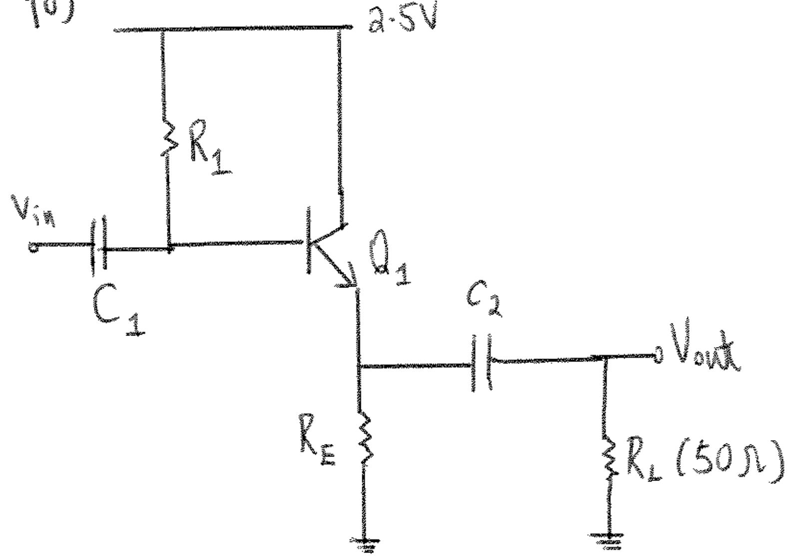
$$\text{Power} = 2.5\left(I_c + \frac{I_c}{\beta}\right) \Rightarrow I_c = 1.98\,mA$$

$$\frac{1}{g_m} = \frac{V_T}{I_c} = \frac{26mV}{1.98mA} = 13.13\,\Omega$$

$$R_L = (9)(13.13) = 118.17\,\Omega.$$

This is the minimum load resistance, since anything lower will lower the voltage gain.

40)



$A_v = 0.8$

Since R_E doesn't affect voltage gain significantly.

$$A_v \approx \frac{R_L}{R_L + \frac{1}{g_m}} = 0.8$$

$$R_L = 0.8\left(R_L + \frac{1}{g_m}\right)$$

$$0.2R_L = 0.8\frac{1}{g_m}$$

$$R_L = 4\frac{1}{g_m} \Rightarrow \frac{R_L}{4} = \frac{1}{g_m} = 12.5 = \frac{V_T}{I_c}$$

$$I_c = \frac{26mV}{12.5\Omega} = 2.08mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.751\,V$$

Let $R_E I_c = 20V_T$ (0.52V), $R_E = \frac{20}{g_m} = 250\Omega$

$$R_1 = \frac{2.5 - (0.751 + 0.52)}{0.0208mA} = 59.1K$$

9c)

$$\frac{1}{(2\pi)(100\times10^6)C_1} = \frac{1}{10}\frac{1}{g_m} \Rightarrow C_1 = 1.27nf$$

$$\frac{1}{(2\pi)(100\times10^6)C_2} = \frac{1}{10}50 \Rightarrow C_2 = 0.32nf$$

(So C_2 will not load Q_1).

$C_1 = 1.27\ nf$
$C_2 = 0.32\ nf$
$R_1 = 59.1\ K\Omega \quad \Rightarrow \quad A_v = 0.8$
$R_E = 250\ \Omega$
$R_L = 50\Omega$