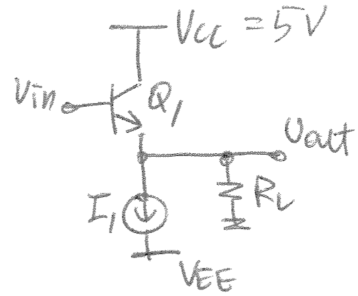
1. $$A_v = \frac{g_{m1} R_L}{1 + g_{m1} R_L}$$



$P_{LOAD} = 0.5\,W$

$R_L = 8\Omega$

(Assume V_{out} biased at $V_{BE(ON),1} \approx 800\,mV$

(a) $$0.8 = \frac{g_{m1}(8\Omega)}{1 + g_{m1}(8\Omega)}$$

$$\Rightarrow g_{m1} = 0.5 = \frac{I_C}{V_T} = \frac{I_1}{V_T}$$

$$\therefore I_1 = 13\,mA$$

(b) When $V_{in} = V_p = V_{CC}$, $V_{out} \approx V_{CC} - V_{BE(ON),1}$

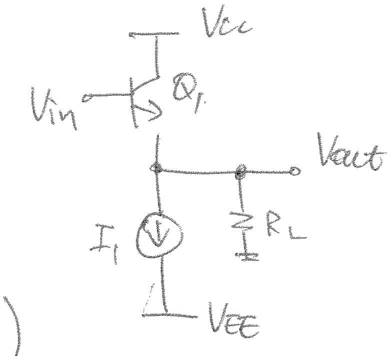
$$I_{C1} = I_1 + \frac{V_{out}}{R_L} \Rightarrow I_{C1} = I_1 + \frac{5-0.8}{8} \approx 0.54\,A$$

$$\Rightarrow g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.54A}{0.026V} = 20.8\,S$$

$$\Rightarrow A_v\Big|_{V_{in}=V_p} = \frac{g_{m1} R_L}{1 + g_{m1} R_L} = \frac{(20.8S)(8\Omega)}{1 + (20.8S)(8\Omega)} \approx 0.99$$

2.

(a) $I_1 = V_p / R_L \qquad V_p \gg V_T$



$$A_V = \frac{I_C R_L}{I_C R_L + V_T}$$

$$= \frac{\frac{I_C}{I_1} V_p}{\frac{I_C}{I_1} V_p + V_T} = \frac{V_p}{V_p + V_T} \quad (\approx 1)$$

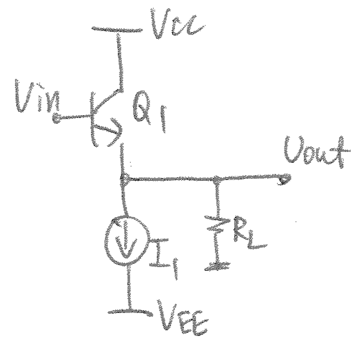
(b) When $V_{out} = V_p$, $I_{C_1} = I_1 + \frac{V_{out}}{R_L} = \frac{V_p}{R_L} + \frac{V_p}{R_L}$

$$= \frac{2V_p}{R_L}$$

$$\therefore A_V = \frac{\left(2\frac{V_p}{R_L}\right) R_L}{\left(\frac{2V_p}{R_L}\right) R_L + V_T} = \frac{2V_p}{2V_p + V_T} \left(\approx \frac{2V_p}{2V_p} = 1.\right)$$

$$\Delta A_V = \frac{\frac{2V_p}{2V_p + V_T} - \frac{V_p}{V_p + V_T}}{\frac{V_p}{V_p + V_T}} = \frac{V_T}{2V_p + V_T} \quad \left(\approx \frac{V_T}{V_p}\right)$$

3. $A_v = 0.7$ $R_L = 4\Omega$



Q_1 shuts off when:

$I_1 = \frac{V_P}{R_L}$

- Suppose $V_{out} = V_p \sin\omega t$. $\left(\omega = \frac{2\pi}{T}\right)$

$$P_{R_L, AVG} = \frac{1}{T}\int_0^T \frac{(V_{out})^2}{R_L}\,dt = \frac{1}{T}\int_0^T \frac{V_p^2 \sin^2\omega t}{R_L}\,dt$$

$$= \frac{1}{2} V_P^2 / R_L$$

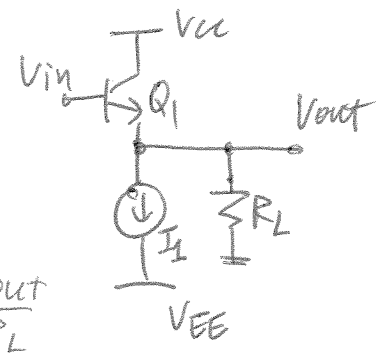
$\therefore$ Largest power (average) $= \frac{1}{2}\frac{(I_1 R_L)^2}{R_L}$

$$A_v = 0.7 = \frac{g_{m_1} R_L}{1 + g_{m_1} R_L} \Rightarrow g_{m_1} = \frac{A_v}{(1 - A_v)R_L} = \frac{0.7}{(1-0.7)(4)} = 0.58\,S$$

$$\Rightarrow I_{C_1}\ (= I_1) = g_{m_1} V_T = 0.015\,A$$

$$\therefore P_{AV, MAX} = \frac{1}{2} I_1^2 R_L = \frac{1}{2}(0.015\,A)^2(4\Omega) = 0.45\,W$$

4. $A_v = \dfrac{g_{m1} R_L}{1 + g_{m1} R_L}$ $\quad \left(g_m = \dfrac{I_C}{V_T}\right)$



- Q_1 shuts off when $I_1 = -\dfrac{V_{out}}{R_L}$

$\Rightarrow V_p = I_1 \times R_L$

$$g_{m1} = \frac{A_v}{(1-A_v)R_L} = \frac{I_{C1}}{V_T} \Rightarrow I_{C1} = \frac{V_T A_v}{R_L(1-A_v)} \quad (= I_1)$$

- Power delivered to R_L:

$$P_{R_L} = \frac{1}{T}\int_0^T \frac{V_{out}^2}{R_L}\,dt = \frac{1}{T}\int_0^T \frac{V_p^2 \sin^2 \omega t}{R_L}\,dt$$

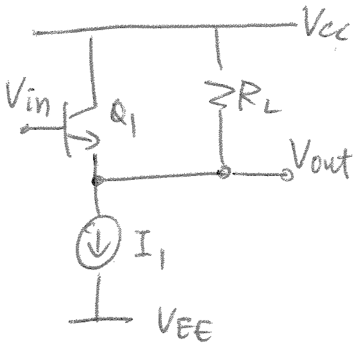
$$= \frac{1}{2}\frac{V_p^2}{R_L}$$

$\therefore$ Maximum power $= \dfrac{1}{2}\dfrac{(I_1 R_L)^2}{R_L}$

$$= \frac{1}{2}\left[\frac{V_T A_v}{(1-A_v)}\right]^2 \cdot \frac{1}{R_L}$$

5.

Assume $V_{cc} = 5V$



$I_{S1} = 5 \cdot 10^{-17} A$

$R_L = 8\Omega$

$I_1 = 20mA$.

(a) By KCL,

$$I_1 = I_{S1} \cdot \exp\left(\frac{V_{in}-V_{out}}{V_T}\right) + \frac{V_{cc}-V_{out}}{R_L}$$

$$\Rightarrow V_{in} = V_{out} + V_T \ln\left(\frac{I_1}{I_{S1}} - \frac{V_{cc}-V_{out}}{I_{S1}R_L}\right)$$

$= 0$ (X) — no solution

$\therefore V_{out} \approx 5 - I_1 R_L = 4.84V$

(i.e. Q_1 is off.)

(b) $(0.01)I_1 = I_1 - \frac{V_{cc}-V_{out}}{R_L}$

$\Rightarrow V_{out} = 4.84 \ V$

$I_{C1} = (0.01)I_1 = I_{S1} \exp\left(\frac{V_{in}-V_{out}}{V_T}\right)$

$\Rightarrow V_{in} = V_{out} + V_T \ln\left(0.01 \frac{I_1}{I_S}\right)$

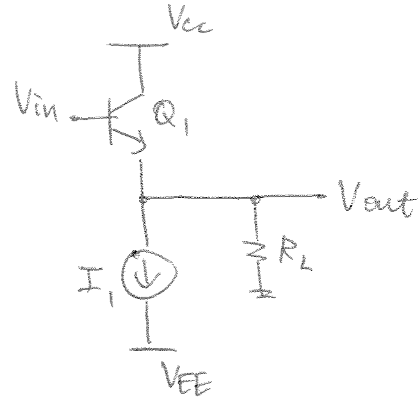
$= 4.84 + (0.026) \ln\left(0.01 \frac{20mA}{5 \cdot 10^{-17}A}\right)$

$\approx 5.59 \ V$

(exceeds V_{cc})

b.

(a) Calculate V_{BE} for



$I_{S_1} = 6 \cdot 10^{-17} A$

$R_L = 8\Omega$

$I_1 = 25\,mA$

$V_p = 1\,V$

$\underline{V_{in} = 1\,V:}$

$$I_{C_1} = I_1 + \frac{V_{out}}{R_L}$$

$$\Rightarrow I_{S_1} \cdot \exp\left(\frac{V_{in} - V_{out}}{V_T}\right) = I_1 + \frac{V_{out}}{R_L}$$

Solving for Vout gives:

$V_{out} \approx 0.113\,V$

$$\therefore V_{BE}\Big|_{V_{in}=1V} = V_{in} - V_{out} = 1 - 0.113 = 0.887\,V$$

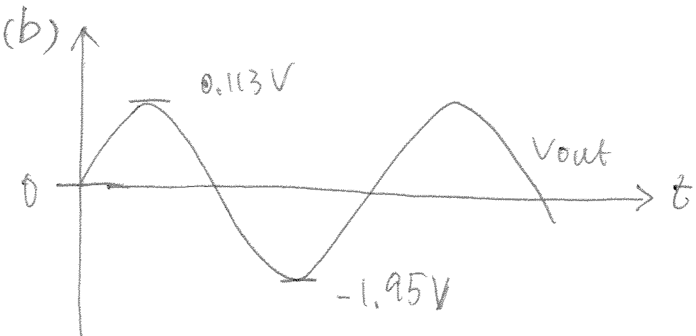
$\underline{V_{in} = -1\,V:}$

$$I_{C_1} = I_1 + \frac{-V_{out}}{R_L} \Rightarrow I_{S_1} \exp\left(\frac{V_{in} - V_{out}}{V_T}\right) = I_1 - \frac{V_{out}}{R_L}$$

Solving for Vout gives: $V_{out} \approx -1.95\,V$

$$\therefore V_{BE}\Big|_{V_{in}=-1V} = V_{in} - V_{out} = -1 - (-1.95V) = 0.95\,V$$

(b)



7. Determine V_p such that

$$V_{BE}\Big|_{V_{in}=+V_p} - V_{BE}\Big|_{V_{in}=-V_p} = 10\text{ mV}$$

$$\Rightarrow \left(V_p^{+} - V_{out,+}\right) - \left(V_p^{-} - V_{out,-}\right) = 10\text{ mV}$$

$$I_s \exp\left(\frac{V_p^{+} - V_{out,+}}{V_T}\right) = I_1 + \frac{V_{out,+}}{R_L} \quad \text{— ①}$$

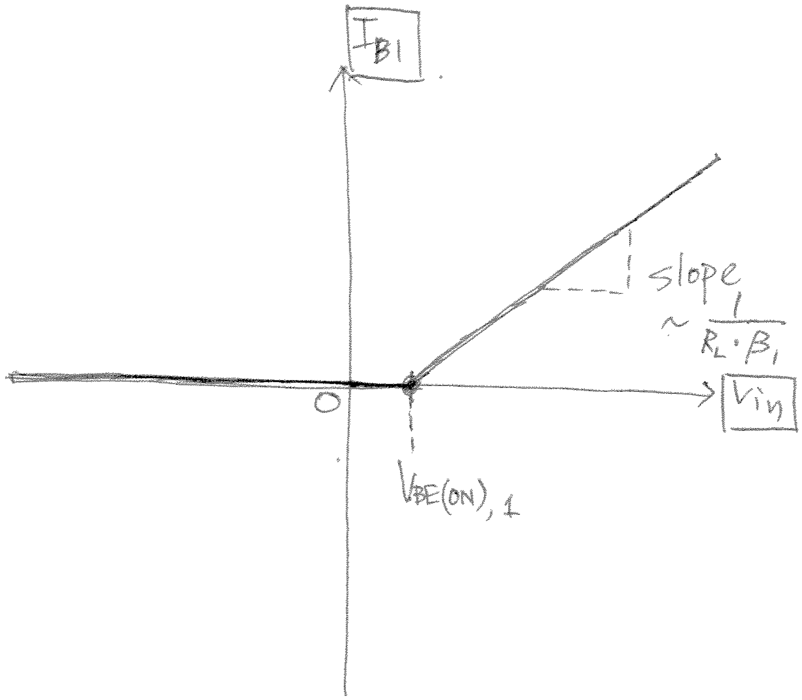
$$I_s \exp\left(\frac{V_p^{-} - V_{out,-}}{V_T}\right) = I_1 - \frac{V_{out,-}}{R_L} \quad \text{— ②}$$

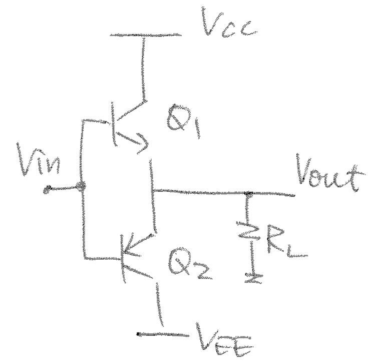
Iterate ① & ②. This gives:

$V_p \approx 0.7$ V

$$\Rightarrow \text{Nonlinearity} = \frac{10\text{ mV}}{0.7 \times 2} = 0.007.$$

8.





β_1 : current gain of Q_1

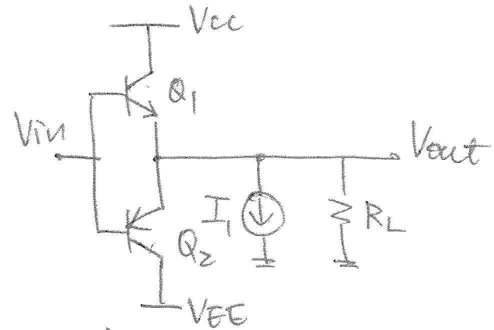
- Q_1 is on whenever $V_{in} \geq V_{BE(ON),1}$. In this region,

$$V_{out} = V_{in} - V_{BE(ON)1} \qquad I_{C_1} = \frac{V_{out}}{R_L}$$

$$\therefore I_{B_1} = \frac{I_{C1}}{\beta} = \frac{V_{out}}{\beta R_L} = \frac{V_{in} - V_{BE(ON)1}}{\beta R_L}.$$

9.

(a) To guarantee Q_1 on,



- $V_{out} \approx V_{in} - V_{BE(ON)1}$
  $= -800\,mV$

$\Rightarrow I_{C_1} = I_1 + \dfrac{V_{out}}{R_L}$ (Q_2 is off)

$I_{S_2} = 6 \cdot 10^{-17} A$

$R_L = 8\,\Omega$

- $I_{C_1} \geq 0 \Rightarrow I_1 + \dfrac{V_{out}}{R_L} \geq 0$

$\Rightarrow I_1 + \dfrac{-800\,mV}{R_L} \geq 0$

$\therefore I_1 R_L \geq 800\,mV$ ——————— ①

(b) When Q_2 turns on,

$$-\frac{V_{out}}{R_L} - I_1 = I_{C_2}$$

$$\Rightarrow -\frac{V_{out}}{R_L} - \left(\frac{800\,mV}{R_L}\right) = I_{S_2} \exp\left(\frac{V_{BE2}}{V_T}\right)$$

$$\Rightarrow V_{out} = -R_L I_{S2} \cdot \exp\left(\frac{V_{BE2}}{V_T}\right) - 0.8$$

$$= -(8\,\Omega)(6 \times 10^{-17} A) \exp\left(\frac{0.8}{0.026}\right) - 0.8$$

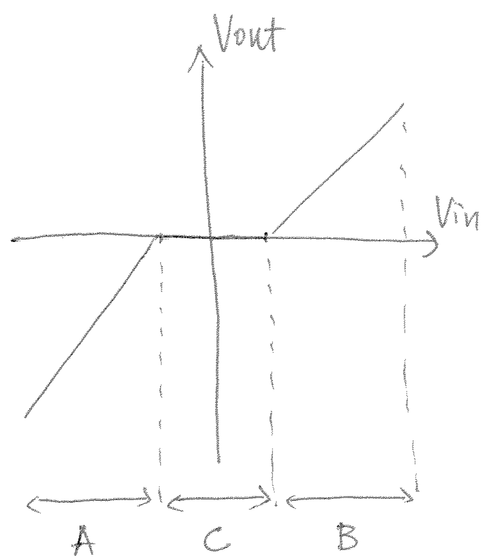
$$\approx -0.81\ V$$

$$\therefore V_{in} = V_{out} - |V_{BE(ON)2}| = -0.81 - 0.8 = -1.61\ V$$

10. Consider two scenarios:

- In gain regions ($|V_{in}| \geq |V_{BE(ON)}|$), V_{out} tracks V_{in}.
- In dead zone, both transistors shut off.

In both cases, V_{out} has an important role. Current source I_1 affects the input/output characteristic by modulating V_{out}:



I/O characteristic of Push-pull stage.

Consider region A:

$$I_{C2} + I_1 = \frac{-V_{out}}{R_L}$$

$\therefore I_1 \uparrow\downarrow \Rightarrow |V_{BE2}| = |V_{out} - V_{in}|$ stays relatively constant.

(Q_2 absorbs/sinks all the currents from I_1 in order to have the same $|V_{BE2}|$)

Consider region B:

$$I_{C1} = I_1 + \frac{V_{out}}{R_L}$$

$\therefore I_1 \uparrow\downarrow \Rightarrow |V_{BE1}| = |V_{in} - V_{out}|$ stays relatively constant.

(Q_1 provides/sources current to I_1 in order to have $|V_{BE1}|$ constant.)

Consider region C: (Dead zone).

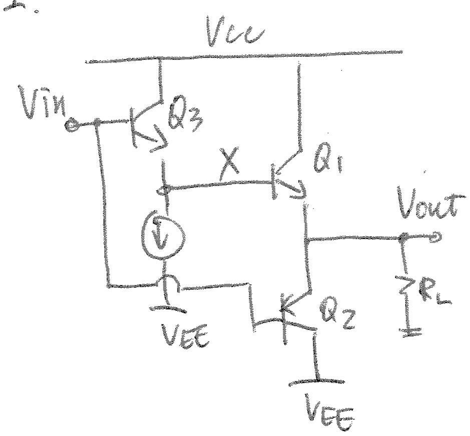
$I_1 = -\frac{V_{out}}{R_L}$ (Both transistors off)

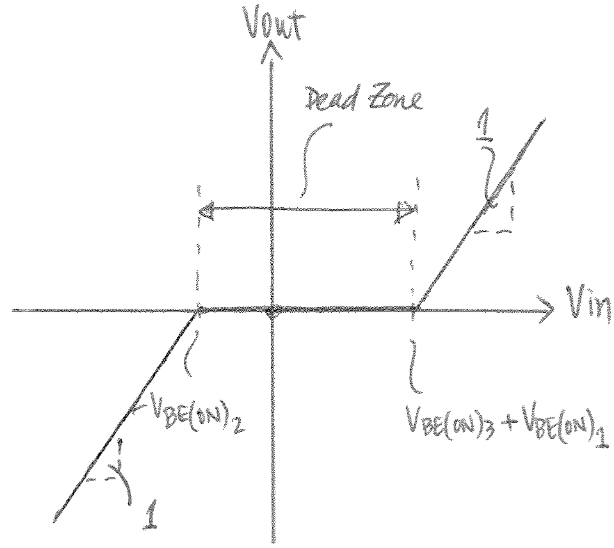
$\therefore I_1 \uparrow \Rightarrow V_{out} \downarrow$

$I_1 \downarrow \Rightarrow V_{out} \uparrow$

i.e. In the dead zone, Vout is predominantly controlled by I_1. One can use this to control Vout and effectively shift the region of dead zone.
($\because V_{out}|_{V_{in}=0} \neq 0$ anymore)

11.





Dead Zone
$= |V_{BE(ON)_2}| + V_{BE(ON)_3} + V_{BE(ON)_1}$

Analysis

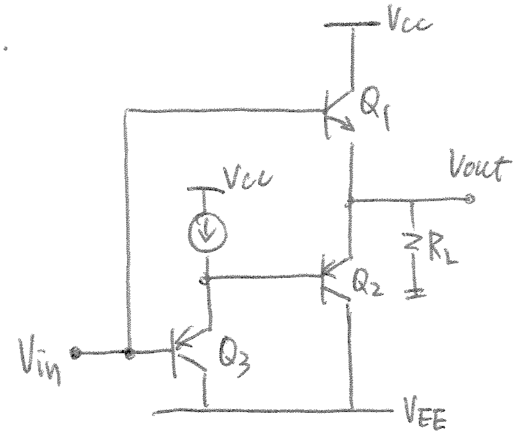
- $(0 < V_{in} < V_{BE(ON)_3} + V_{BE(ON)_1})$:
  Q_1 is OFF $(V_{in} < V_{BE(ON)_1})$
  Q_2 is OFF (V_{BE_2} reverse-biased)
  $\Rightarrow V_{out} = 0$

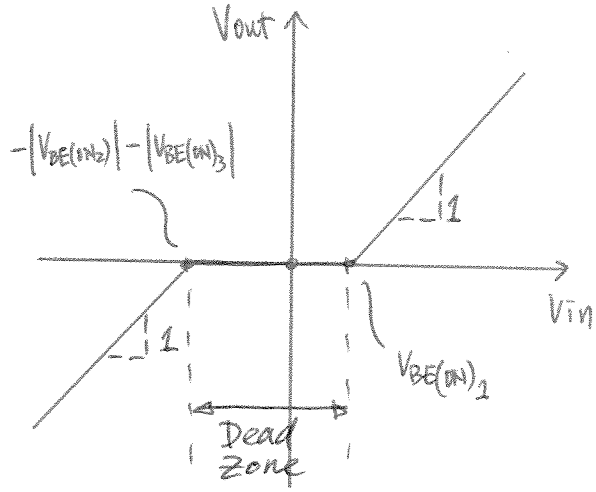
- $(-|V_{BE(ON)_2}| < V_{in} < 0)$:
  Q_1, Q_2 OFF.
  $V_{out} = 0$

- $(V_{BE(ON)_3} + V_{BE(ON)_1} < V_{in} < V_{cc})$
  Q_1 ON
  Q_2 OFF
  $V_{out} = V_{in} - V_{BE(ON)_3} - V_{BE(ON)_1}$

- $(-|V_{EE}| < V_{in} < -|V_{BE(ON)_2}|)$
  Q_2 ON
  Q_1 OFF
  $V_{out} = V_{in} + |V_{BE(ON)_2}|$

12.





$-V_{EE} < V_{in} < -(|V_{BE(ON)_2}| + |V_{BE(ON)_3}|)$:

$\Rightarrow$ Q_2, Q_3 ON, Q_1 OFF $\Big\}$ $V_{out} = V_{in} + |V_{BE(ON)_3}| + |V_{BE(ON)_2}|$

$-(|V_{BE(ON)_2}| + |V_{BE(ON)_3}|) < V_{in} < V_{BE(ON)_1}$:

$\Rightarrow$ Q_1, Q_2 OFF $\Rightarrow$ $V_{out} \cong 0$

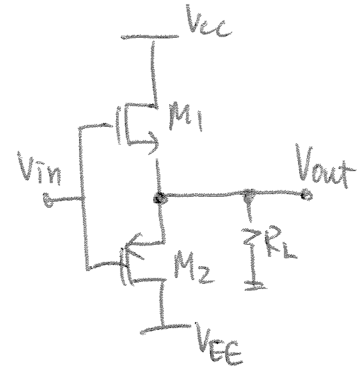
$V_{BE(ON)_1} < V_{in} < V_{cc}$:

$\Rightarrow$ Q_1 ON, Q_2, Q_3 OFF $\Big\}$ $V_{out} = V_{in} - V_{BE(ON)_1}$

$$\text{Dead Zone} = V_{BE(ON)_1} + |V_{BE(ON)_2}| + |V_{BE(ON)_3}|$$

13.

(a)



$-|V_{EE}| < V_{in} < -|V_{t,p}|$:

$\Rightarrow$ M_1 OFF, M_2 ON (saturation) $\Big\}$ $V_{out} = V_{in} + V_{SG,2}$

Ignore body effect.

$V_{CC} > V_{in} > V_{t,n}$:

$\Rightarrow$ M_1 ON, M_2 OFF $\Big\}$ $V_{out} = V_{in} - V_{GS,1}$

$\Rightarrow$ M_1 & M_2 can never on at the same time.

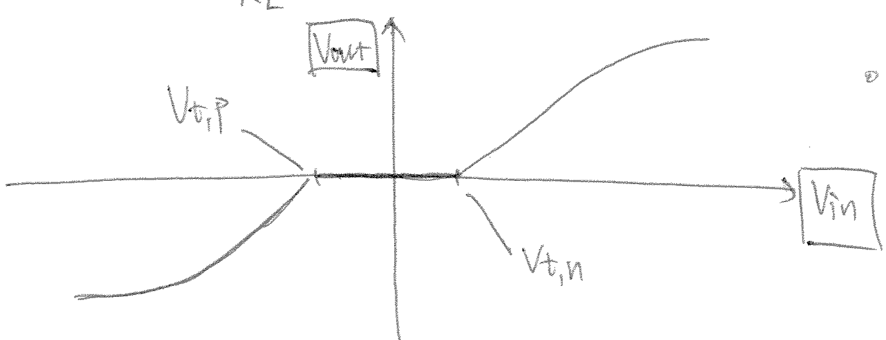
$-|V_{t,p}| < V_{in} < V_{t,n}$:

M_1, M_2 OFF $\Rightarrow V_{out} = 0$

For MOS, $I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}(|V_{GS}|-|V_t|)^2$ — saturation region.

$\Rightarrow$ M_1 ON: $\frac{V_{out}}{R_L} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{in}-V_{out}-V_{t,n})^2$, $V_{out} > 0$

M_2 ON: $-\frac{V_{out}}{R_L} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{out}-V_{in}-V_{t,p})^2$, $V_{out} < 0$

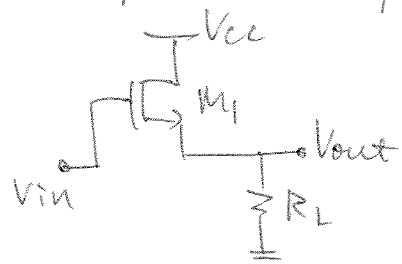


- Solve for Vout in both cases.

(b) Outside dead zone

$\Rightarrow$ either M_1 or M_2 is on.

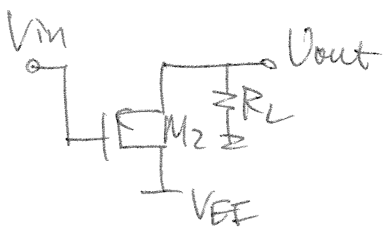
• For positive inputs:



Source follower:

$$\therefore \frac{v_{out}}{v_{in}} = \frac{g_{m1}}{1+g_{m1}R_L}$$

• For negative inputs:

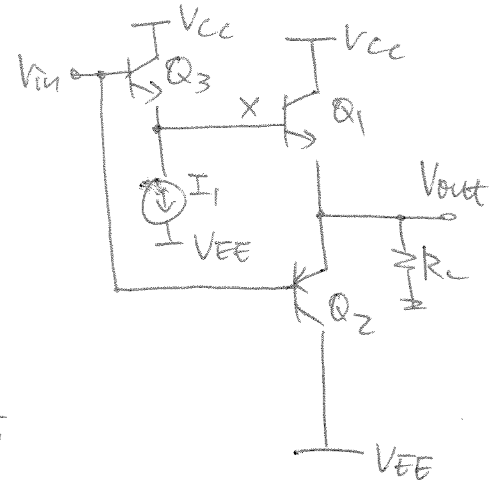


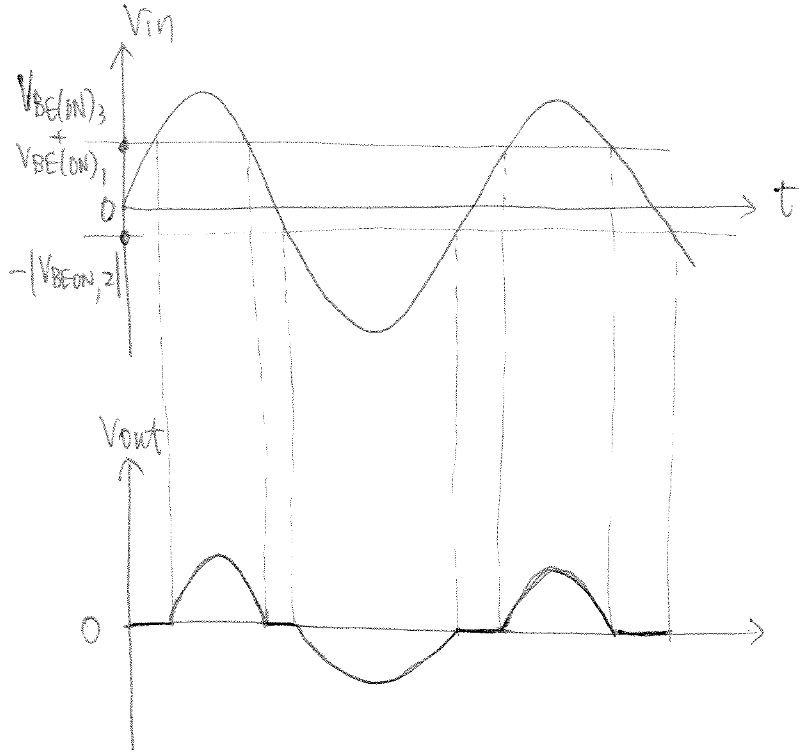
Source follower:

$$\therefore \frac{v_{out}}{v_{in}} = \frac{g_{m2}}{1+g_{m2}R_L}$$

14. Dead zone :

$V_{out} \in \left[-|V_{BE(ON)_2}|,\ V_{BE(ON)_3} + V_{BE(ON)_1}\right]$

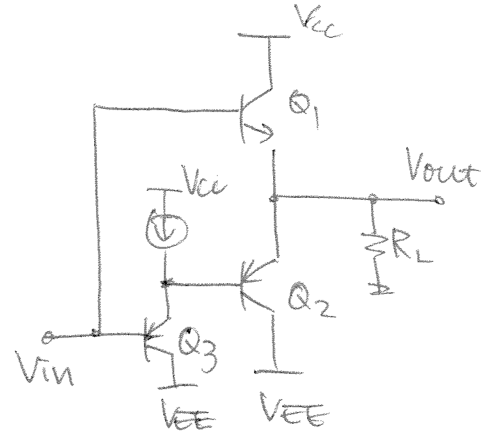


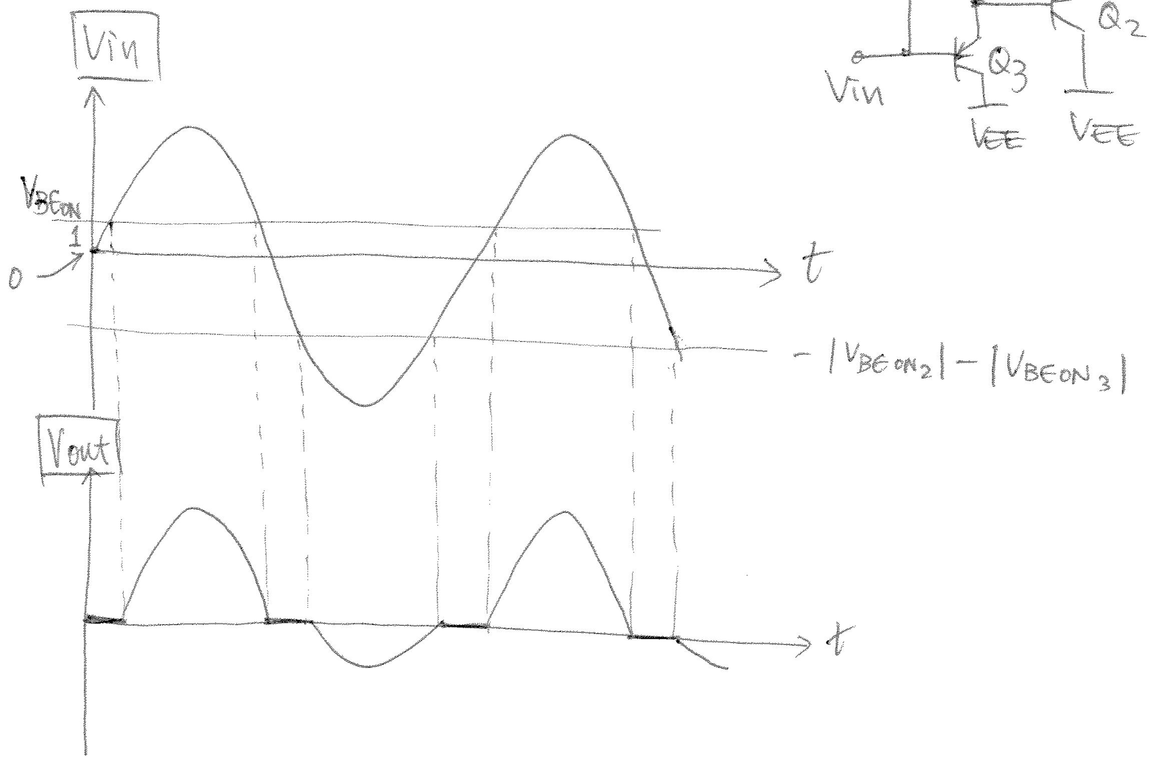


15.

Dead zone:

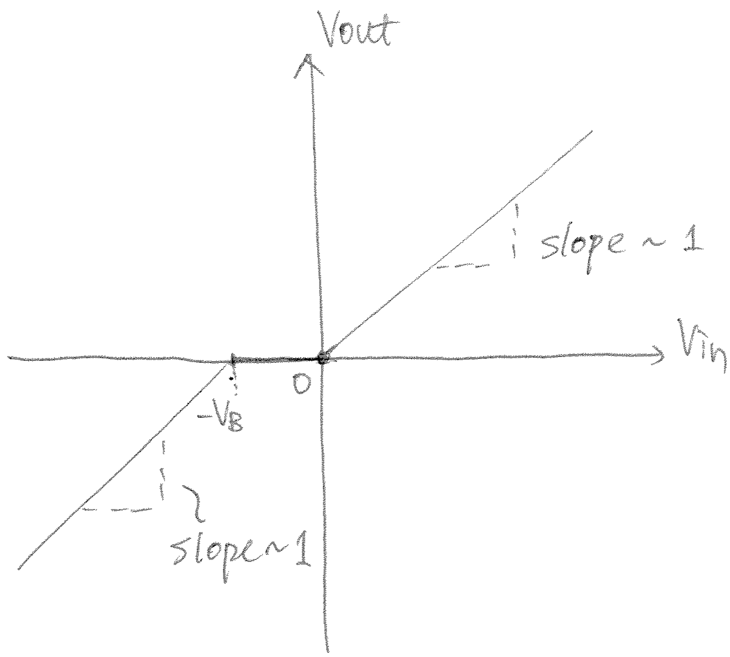
$V_{out} \in \left[-\left(|V_{BE,ON,2}| + |V_{BE,ON,3}| \right), V_{BE,ON,1} \right]$

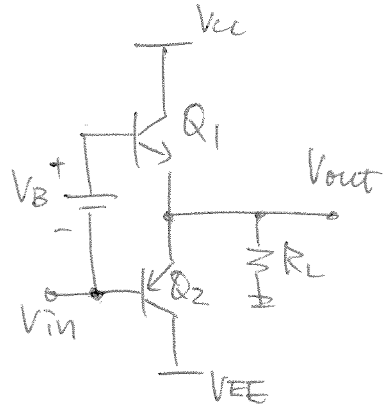




16.

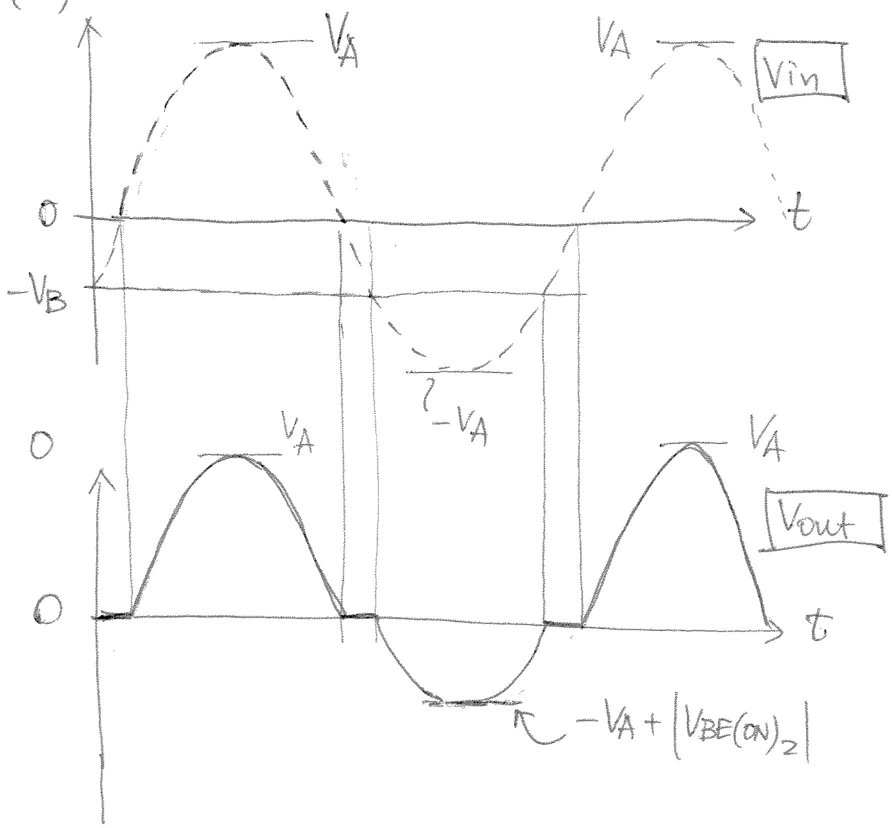
(a)


Vout
Vin
0
$-V_B$
slope ~ 1
slope ~ 1

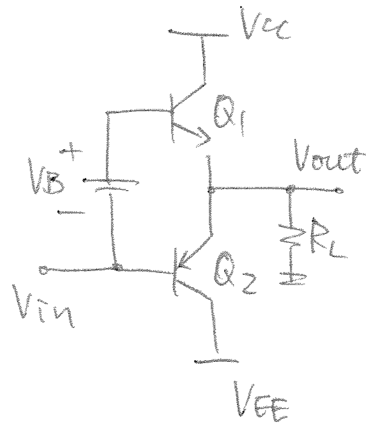

Vcc
Q_1
V_B
Vout
R_L
Q_2
Vin
VEE

$(V_B = V_{BE})$

(b)


V_A
V_A
Vin
0
t
$-V_B$
$-V_A$
V_A
V_A
Vout
0
t
$-V_A + |V_{BE(ON)_2}|$

17.



- $V_{out} = 0$:

$\Rightarrow I_{C_1} = I_{C_2} = I_{BIAS}$

$$\Rightarrow I_{S_1} \exp\left(\frac{V_{in}+V_B-V_{out}}{V_T}\right) = I_{S_2} \exp\left(\frac{|V_{out}-V_{in}|}{V_T}\right)$$

$$\ln\left(\frac{I_{S1}}{I_{S2}}\right) + \frac{V_{in}+V_B-V_{out}}{V_T} = \frac{V_{out}-V_{in}}{V_T}$$

$I_{S_1} = 5\cdot 10^{-17} A$

$I_{S2} = 8\cdot 10^{-17} A$

$I_{BIAS} = 5 mA$ ($V_{out} = 0$)

- For $V_{out} = 0$, $V_T = 0.026\ V$:

$$\Rightarrow \ln\left(\frac{5}{8}\right) + \frac{V_{in}+V_B}{0.026} = +\frac{V_{in}}{0.026}$$

- Given $I_{C2} = 5 mA$

$$\Rightarrow I_{S2} \exp\left(\frac{-V_{in}}{0.026}\right) = 5mA \Rightarrow V_{in} = -0.83\ V$$

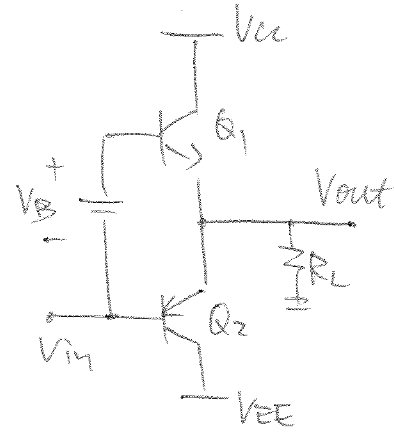
$$I_{C_1} = I_{S_1} \exp\left(\frac{V_{in}+V_B-V_{out}}{V_T}\right) = (5\cdot 10^{-17}A) \exp\frac{-0.83+V_B}{V_T}$$

$$\Rightarrow V_B = 0.83 + 0.026 \ln\left(\frac{5mA}{5\cdot 10^{-17}A}\right)$$

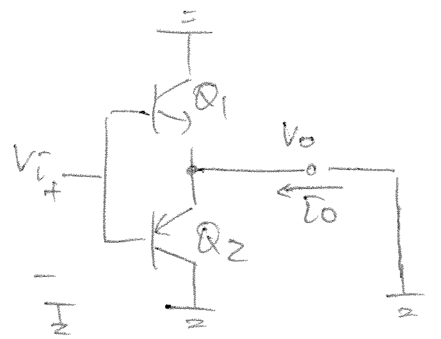
$$\approx 1.67\ V.$$

18.

(a) Equivalent circuit (small-signal) around $V_{out} = 0$:



$V_{P_{in}} = 2\,V$
$R_L = 8\,\Omega$



$$i_o = -g_{m1} v_i + (-v_i) g_{m2}$$
$$= -(g_{m1} + g_{m2}) v_i$$

$$\therefore G_m = \frac{i_o}{v_i} = -(g_{m1} + g_{m2})$$

$$\therefore A_v = \frac{v_o}{v_i} = \frac{i_o \times R_L}{v_i} = -(g_{m1} + g_{m2}) R_L$$

(b) $A_v = -(g_{m1} + g_{m2}) R_L = -\left(\frac{I_{C1}}{V_T} + \frac{I_{C2}}{V_T}\right) R_L$

$$= -\left(\frac{5mA}{0.026V} + \frac{5mA}{0.026V}\right)(8\,\Omega) = -3.08$$

$$\Rightarrow |V_o|_p = |v_i A_v|_p = |(2V)(-3.08)| = 6.16\,V$$

(Assume Vcc is large enough)

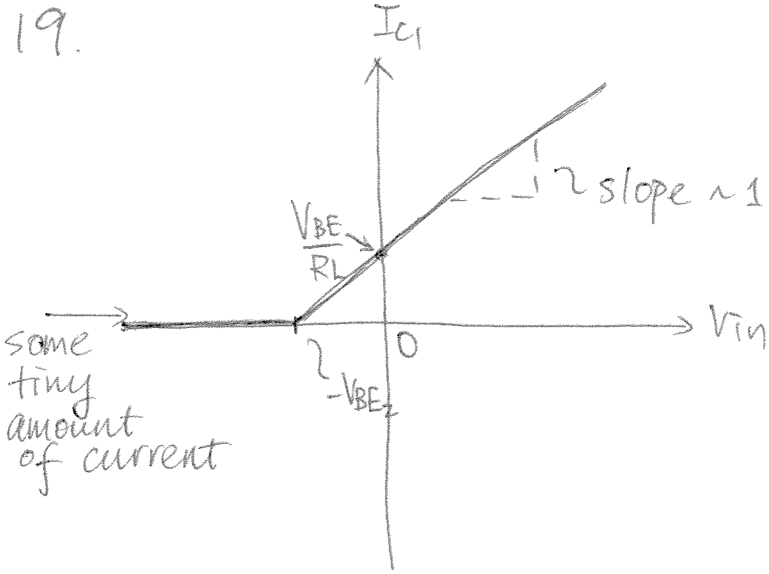
(c) $I_{C1} = I_{C2} + \frac{V_{out}}{R_L}$

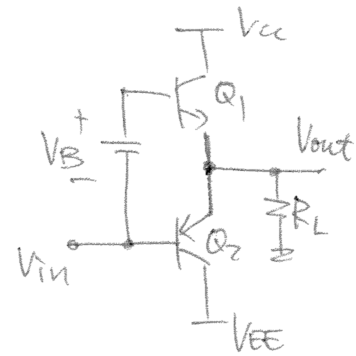
$$I_{C1,peak} = I_{C2} + \frac{V_P}{R_L}$$

$$= 5mA + \frac{6.16\,V}{8\Omega}$$

$$= 775\,mA$$

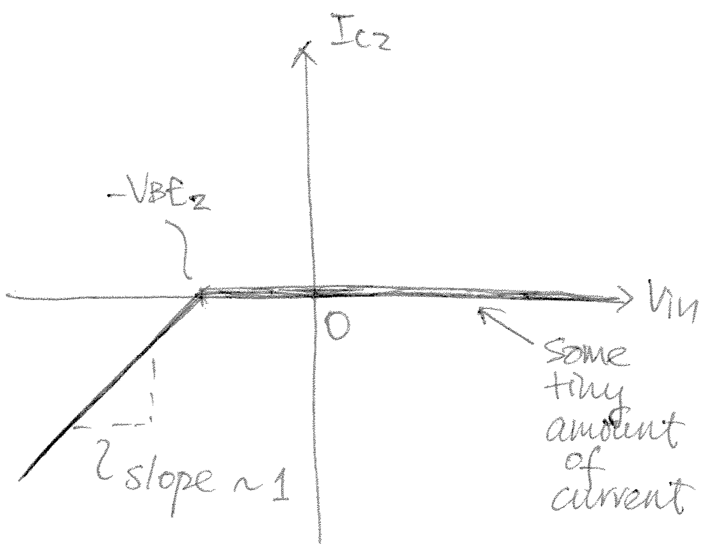
19.



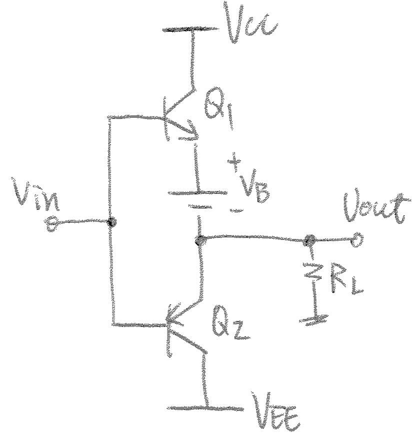


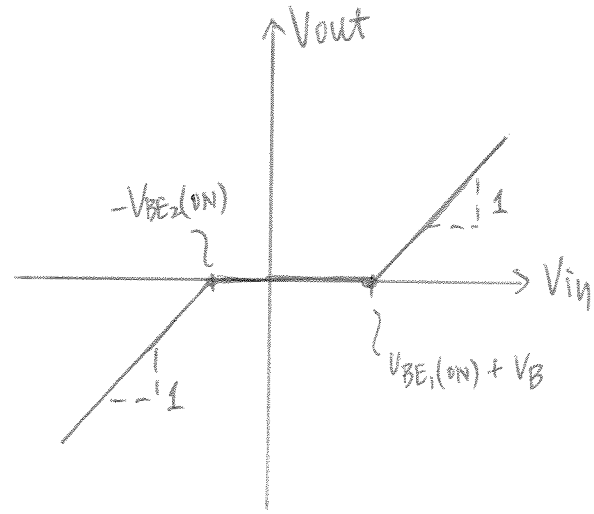
$$V_{out} = V_{in} + |V_{BE2}|$$

$$\Rightarrow I_{C1} = I_{C2} + \frac{V_{out}}{R_L}$$



20.





- To analyze such circuit, assume $V_{out} = 0$:

$\Rightarrow$ $-V_{BE_2(ON)} < V_{in} < V_{BE_1(ON)} + V_B$.

$(V_{BE_1(ON)} + V_B) < V_{in}$: $V_{out} = V_{in} - V_{BE_1(ON)} - V_B$

$V_{in} < -V_{BE_2(ON)}$: $V_{out} = V_{in} + |V_{BE_2(ON)}|$

21. $V_{BE_1} + |V_{BE_2}| = V_{D_1} + V_{D_2}$

$$\Rightarrow V_T \left[\ln \frac{I_{C1}}{I_{S_{Q1}}} + \ln \frac{I_{C2}}{I_{S_{Q2}}} \right] = V_T \left[\ln \frac{I_{D_1}}{I_{S_{D_1}}} + \ln \frac{I_{D_2}}{I_{S_{D_2}}} \right]$$

$$\Rightarrow \frac{I_{C_1} I_{C_2}}{I_{S_{Q1}} I_{S_{Q2}}} = \frac{I_{D_1} I_{D_2}}{I_{S_{D_1}} I_{S_{D_2}}}$$

$\therefore$ If $I_{S_{Q1}} I_{S_{Q2}} = I_{S_{D_1}} I_{S_{D_2}}$,

then $I_{C_1} I_{C_2} = I_{D_1} I_{D_2}$

22. $V_{BE_1} + |V_{BE_2}| = V_{D_1} + V_{D_2}$

$$\Rightarrow V_T \ln\left(\frac{I_{C_1} I_{C_2}}{I_{S,Q_1} I_{S,Q_2}}\right) = V_T \ln\left(\frac{I_{D_1} I_{D_2}}{I_{S,D_1} I_{S,D_2}}\right)$$

$$\Rightarrow \frac{I_{C_1} I_{C_2}}{I_{S,Q_1} I_{S,Q_2}} = \frac{I_{D_1} I_{D_2}}{I_{S,D_1} I_{S,D_2}} \quad \text{——— } (1)$$

$I_1 = I_{D_1} = I_{D_2} = 1mA$; $I_{S,Q} = 16 I_{S,D}$

$V_{out} = 0 \Rightarrow I_{C_1} = I_{C_2}$ ——— (2)

Substitute all into (1):

$$\frac{I_{C_1} I_{C_1}}{(16 I_{S,D})^2} = \frac{(1mA)^2}{(I_{S,D})^2} \Rightarrow I_{C_1} = I_{C_2} = 16 \text{ mA}$$

23. $V_{BE_1} + |V_{BE2}| = V_{D_1} + V_{D2}$

$$\Rightarrow \frac{I_{C_1} I_{C2}}{I_{S,Q_1} I_{S,Q_2}} = \frac{I_{D_1} I_{D2}}{I_{S,D_1} I_{S,D_2}} \qquad \text{———} \quad (1)$$

$$I_{C_1} = I_{C_2} = 5mA \qquad \text{———} \quad (2)$$

$$I_{S,Q} = 8 I_{S,D} \qquad \text{———} \quad (3)$$

Substitute all into (1):

$$\frac{(5mA)^2}{(8 I_{S,D})^2} = \frac{I_{D_1} I_{D_2}}{(I_{S,D})^2} \Rightarrow I_1 = I_D = 0.625\ mA$$

24. $V_{BE_1} + |V_{BE_2}| = V_{D_1} + V_{D_2}$

$$\Rightarrow \frac{I_{C_1} I_{C_2}}{I_{S,Q_1} I_{S,Q_2}} = \frac{I_{D_1} I_{D_2}}{I_{S,D_1} I_{S,D_2}} \quad \text{———} \quad (1)$$

$I_1 = I_D = 2mA$

$I_{S,Q_1} = 8 I_{S,D_1}$; $I_{S,Q_2} = 16 I_{S,D_2}$

Substitute all into (1):

$$\frac{I_{C_1} I_{C_2}}{(8 I_{S,D_1})(16 I_{S,D_2})} = \frac{(2mA)^2}{I_{S,D_1} I_{S,D_2}}$$

$\Rightarrow I_{C_1} = I_{C_2} \cong 22.6\ mA$

25. $V_{BE_1} + |V_{BE_2}| = V_{D_1} + V_{D_2}$

$$\Rightarrow \frac{kT_Q}{q}\left[\ln\left(\frac{I_{C_1} I_{C_2}}{I_{S,Q_1} I_{S,Q_2}}\right)\right] = \frac{kT_D}{q}\left[\ln\left(\frac{I_{D_1} I_{D_2}}{I_{S,D_1} I_{S,D_2}}\right)\right]$$

Suppose $T_D = (T_Q + \Delta T)$:

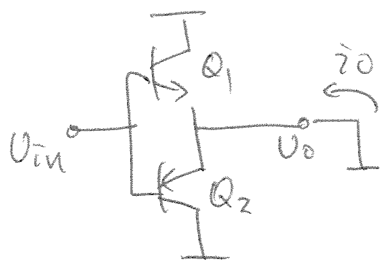
$$\Rightarrow T_Q\left[\ln\frac{I_{C_1} I_{C_2}}{I_{SQ_1} I_{SQ_2}} - \ln\frac{I_{D_1} I_{D_2}}{I_{SD_1} I_{SD_2}}\right] = \Delta T \cdot \ln\frac{I_{D_1} I_{D_2}}{I_{SD_1} I_{SD_2}}$$

$$\Rightarrow I_{C_1} \cdot I_{C_2} = I_{SQ_1} \cdot I_{SQ_2} \cdot \left(\frac{I_{D_1} I_{D_2}}{I_{SD_1} I_{SD_2}}\right)^{1+\frac{\Delta T}{T_Q}}$$

Typically, $\frac{I_{D_1} I_{D_2}}{I_{SD_1} I_{SD_2}} > 1$

$\Rightarrow$ A ΔT introduces a factor $\left(\frac{I_{D_1} I_{D_2}}{I_{SD_1} I_{SD_2}}\right)^{\frac{\Delta T}{T_Q}} < 0$, implying that the $I_{C_1} I_{C_2}$ product drops corresponding to a change (positive) in temperature.

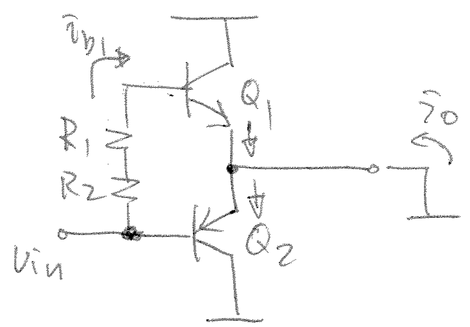
26. Small Signal:



$$G_m = \frac{i_o}{v_{in}} = -(g_{m1} + g_{m2})$$

$$\Rightarrow \frac{v_o}{v_{in}} = \frac{i_o R_L}{v_{in}} = +(g_{m1} + g_{m2}) R_L$$

27. <u>Small-Signal:</u>



$$i_o = -g_{m1} v_{be1} + g_{m2}|v_{be2}| \qquad \left(i_o = i_{c2} - i_{c1}\right)$$

$$|v_{be2}| = v_{in}$$

$$v_{be1} = v_{in} - i_{b1}(R_1+R_2) = v_{in} - \frac{i_{c1}}{\beta_1}(R_1+R_2)$$

$$= v_{in} - \frac{i_{c2} - i_o}{\beta_1}(R_1+R_2)$$

$$= v_{in} + \frac{g_{m2} v_{in} + i_o}{\beta_1}(R_1+R_2)$$

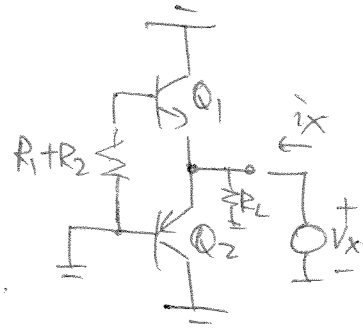
$$\therefore\ g_{m1}\left[v_{in} + \frac{g_{m2} v_{in} + i_o}{\beta_1}(R_1+R_2)\right] + i_o = -g_{m2} v_{in}$$

Solving for $\frac{i_o}{v_{in}}$ gives:

$$G_m = \frac{i_o}{v_{in}} = -\frac{\left[g_{m1} + \frac{g_{m1} g_{m2}}{\beta_1}(R_1+R_2) + g_{m2}\right]}{1 + \frac{g_{m1}(R_1+R_2)}{\beta_1}}$$

Rout:

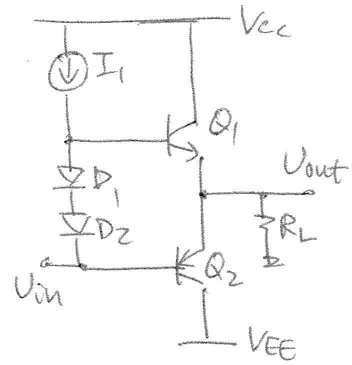
$$\frac{v_x}{i_x} = R_{out} = \left(r_{\pi_2} \,\|\, \frac{1}{g_{m_2}}\right) \,\|\, \left[(r_{\pi_1} + R_1 + R_2) \,\|\, \frac{1}{g_{m_1}}\right] \,\|\, R_L$$



$$\therefore A_v = G_m R_{out}$$

$$= -\left[\frac{g_{m_1} + \dfrac{g_{m_1} g_{m_2}(R_1+R_2)}{\beta_1} + g_{m_2}}{1 + \dfrac{g_{m_1}(R_1+R_2)}{\beta_1}}\right] \cdot \left\{\left[r_{\pi_2} \,\|\, \frac{1}{g_{m_2}}\right] \,\|\, \left[(r_{\pi_1} + R_1 + R_2) \,\|\, \frac{1}{g_{m_1}}\right] \,\|\, R_L\right\}$$

28. Small signal gain around $V_{out} = 0$:



$R_L = 8\Omega$

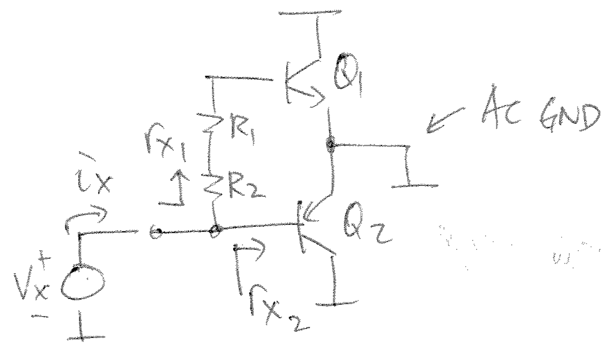
$$A_v = +(g_{m1} + g_{m2}) R_L$$

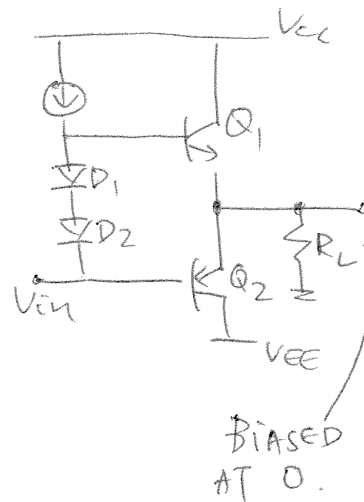
$$0.8 = (I_{C1} + I_{C2}) \frac{R_L}{V_T}$$

If $I_{C1} = I_{C2} = I_{BIAS}$, then

$$I_C = \frac{0.8}{2} \times \frac{V_T}{R_L} = 0.4 \frac{V_T}{R_L} = 0.01 \cdot R_L = 0.08 \text{ A}$$

29. Small-signal equivalent:





$$R_{in} = \frac{v_x}{i_x} = r_{x_1} \parallel r_{x_2}$$

$$= (R_1 + R_2 + r_{\pi_1}) \parallel r_{\pi_2}$$

- R_1 & R_2 can be neglected when $r_{\pi_1} \gg (R_1 + R_2)$

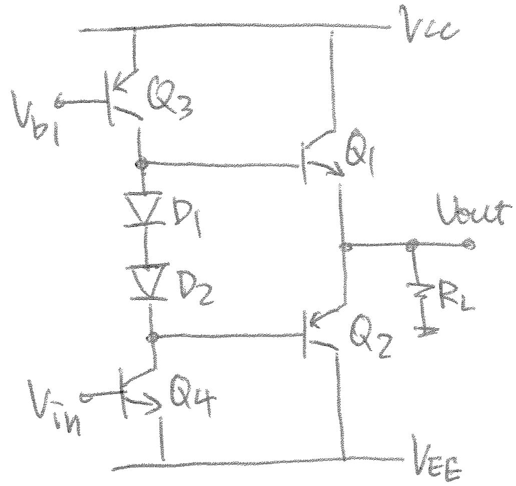
30. $I_{C_1} = I_{C_2} = 10\ mA$

$I_{C_3} = I_{C_4} = 1\ mA$

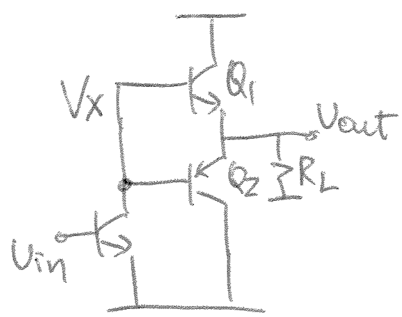
$\beta_1 = 40 \qquad \beta_2 = 20$

$R_L = 8\Omega$

$R_{D_1} = R_{D_2} = 0$



Small-signal



$$A_V = \frac{v_{out}}{v_x} \cdot \frac{v_x}{v_{in}}$$

$$= -g_{m4}\left[(g_{m1}+g_{m2})(r_{\pi_1} \| r_{\pi_2})R_L + (r_{\pi_1} \| r_{\pi_2})\right] \times \frac{R_L}{R_L + \frac{1}{g_{m1}+g_{m2}}}$$

$$= -g_{m4}(r_{\pi_1} \| r_{\pi_2})(g_{m1}+g_{m2})R_L$$

$$\therefore A_V = -\frac{I_{C4}}{V_T}\left(\frac{\beta_1 V_T}{I_{C1}} \| \frac{\beta_2 V_T}{I_{C2}}\right)\left(\frac{I_{C1}}{V_T} + \frac{I_{C2}}{V_T}\right)R_L$$

$$= -\frac{1mA}{0.026}\left[35\right] \cdot \left(2 \times \frac{10mA}{V_T}\right)(8)$$

$$\approx -8.3$$

31.

$$\frac{v_{out}}{v_{in}} = -g_{m4}(r_{\pi 1} \parallel r_{\pi 2})(g_{m1} + g_{m2})R_L \qquad \left(r_\pi = \frac{\beta}{g_m}\right)$$

When $g_{m1} \approx g_{m2}$: $(\Rightarrow r_\pi$

$$\frac{v_{out}}{v_{in}} \approx -g_{m4}R_L(2g_{m1})\left(\frac{\beta_1}{g_{m1}} \parallel \frac{\beta_2}{g_{m1}}\right)$$

$$= -g_{m4}R_L(2g_{m1})\left[\frac{1}{g_{m1}} \cdot \frac{\beta_1\beta_2}{\beta_1+\beta_2}\right]$$

$$= -\frac{2\beta_1\beta_2}{\beta_1+\beta_2} g_{m4}R_L$$

32. From lecture, small-signal gain of the output stage is:

$$\left|\frac{v_{out}}{v_{in}}\right| = +g_{m4}(r_{\pi 1} \parallel r_{\pi 2})(g_{m1}+g_{m2})R_L$$

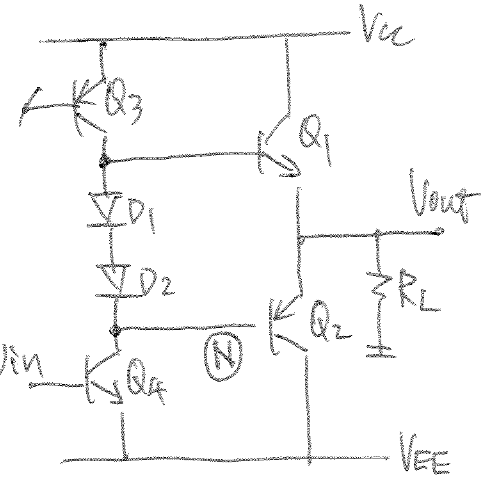
$$\approx +g_{m4}R_L \times \frac{2\beta_1\beta_2}{\beta_1+\beta_2}$$

$$\Rightarrow 4 = +\frac{I_{C4}}{V_T}(8\Omega) \times \frac{2(40)(20)}{40+20}$$

$$\Rightarrow I_{C4} \approx I_{C3}$$

$$= \frac{4V_T}{(8\Omega)} \cdot \frac{40+20}{2(40)(20)}$$

$$= 0.49\ mA$$



$A_V = \frac{v_{out}}{v_{in}} = 4$

$\beta_1 = 40$

$\beta_2 = 20$

$R_L = 8\Omega$

33. From lecture,

$$\frac{v_x}{i_x} = \frac{1}{g_{m1}+g_{m2}} + \frac{r_{o3} \| r_{o4}}{(g_{m1}+g_{m2})(r_{\pi_1} \| r_{\pi_2})}$$

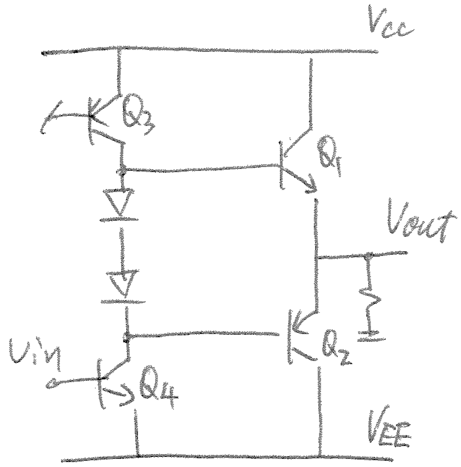
If $g_{m1} \approx g_{m2} = g_m$:

$$\frac{v_x}{i_x} \approx \frac{1}{2g_m} + \frac{r_{o3} \| r_{o4}}{2g_m\left(\frac{\beta_1}{g_m} \| \frac{\beta_2}{g_m}\right)}$$

$$= \frac{1}{2g_m} + \frac{r_{o3} \| r_{o4}}{2g_m\left(\frac{1}{g_m} \frac{\beta_1\beta_2}{\beta_1+\beta_2}\right)}$$

$$= \frac{1}{2g_m} + \frac{r_{o3} \| r_{o4}}{2\beta_1\beta_2}(\beta_1+\beta_2)$$

34.



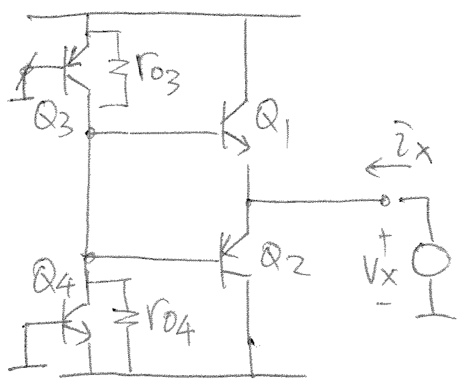
$I_3 = I_4 = 1mA$

$I_1 = I_2 = 8mA$

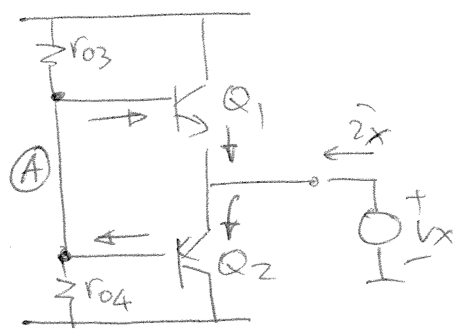
$V_{A3} = 10V$

$V_{A4} = 15V$

(a) Small-signal equivalent:



$\Rightarrow$



$$v_{eb} = v_x \frac{(r_{\pi_1} \| r_{\pi_2})}{(r_{\pi_1} \| r_{\pi_2}) + (r_{o3} \| r_{o4})} \qquad v_{be} = v_A - v_x$$

$$i_x + i_{c_1} = i_{c_2} \Rightarrow i_x = i_{c_2} - i_{c_1} = g_{m2} v_{eb} - g_{m1} v_{be}$$

$$\therefore i_x = [g_{m2} + g_{m1}]\, v_x \frac{(r_{\pi_1} \| r_{\pi_2})}{(r_{\pi_1} \| r_{\pi_2}) + (r_{o3} \| r_{o4})}$$

$$\Rightarrow \frac{v_x}{i_x} = R_{out} = \frac{(r_{\pi_1} \| r_{\pi_2}) + (r_{o3} \| r_{o4})}{[g_{m1} + g_{m2}]\,(r_{\pi_1} \| r_{\pi_2})}$$

$$r_{\pi_1} = \frac{\beta_1 V_T}{I_{C1}} = 130\,\Omega \qquad r_{\pi_2} = \frac{\beta_2 V_T}{I_{C2}} = 65\,\Omega$$

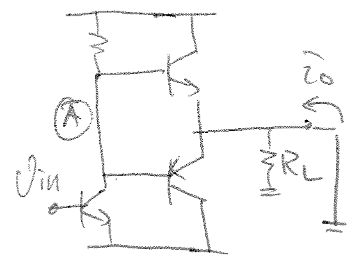
$$r_{O3} = \frac{V_{A3}}{I_{C3}} = 10\,k\Omega \qquad r_{O4} = \frac{V_{A4}}{I_{C4}} = 15\,k\Omega$$

$$g_{m1} = 0.31\,S \qquad g_{m2} = 0.31\,S$$

$$\Rightarrow R_{out} = \frac{43.3 + 6000}{(0.62)(43.3)} \approx 6001\,\Omega$$

(b) Effective $R_{out} = R_{out,a} \parallel 8\Omega \approx 8\,\Omega$.

$$G_m = \frac{i_o}{v_A} \cdot \frac{v_A}{v_{in}}$$
$$= -g_{m4}(r_{\pi_1} \parallel r_{\pi_2} \parallel r_{o3}) \cdot (g_{m1} + g_{m2})$$



$$g_{m4} = \frac{I_{C4}}{V_T} = 0.038\,S$$

$$\therefore A_v = G_m R_{out}$$
$$= -g_{m4}(r_{\pi_1} \parallel r_{\pi_2} \parallel r_{o3})(g_{m1} + g_{m2}) R_{out}$$
$$= -0.038[130 \parallel 65 \parallel 10k][0.62](8)$$
$$\approx -8.1$$

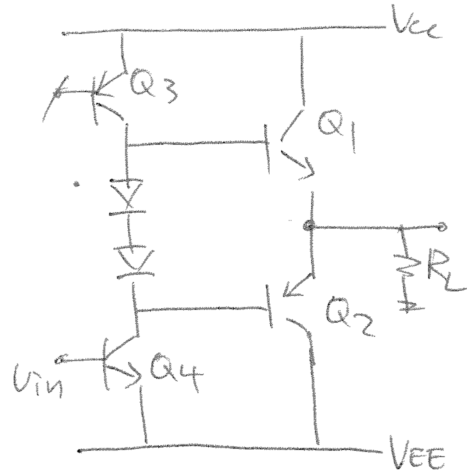
35. Max current delivered by $Q_1 = I_{C3} \cdot \beta_1 = 1mA \cdot 40$

$= 40\ mA$. (Q_4 off)

Max current delivered by $Q_2 = I_{C4} \cdot \beta_2$

$= 1mA \cdot 20$

$= 20\ mA$. (Q_3 off)

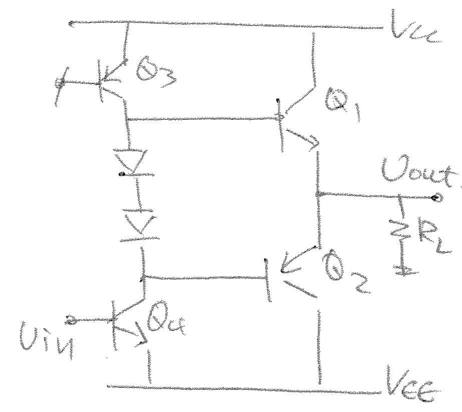


$I_{C_3} = I_{C4} = 1mA$

$\beta_1 = 40 \quad \beta_2 = 20$

36. $P = 0.5\,W \qquad R_L = 8\,\Omega$

$\beta_1 = 40 \qquad \beta_2 = 20.$



$$P_{AVG} = \frac{1}{2}\frac{V_p^2}{R_L} = 0.5$$

$$\Rightarrow V_p^2 = 2(0.5)R_L$$

$$\Rightarrow V_p = \sqrt{R_L} = 2\sqrt{2}$$

At positive V_p, $I_{C_1} = \frac{V_p}{R_L} = \frac{2\sqrt{2}}{8} = 0.35\,A$.

At negative V_p, $I_{C_2} = \frac{V_p}{R_L} \Rightarrow I_{C_2} = 0.35\,A$.

- At $+V_p$, all of I_{C_3} supports the base current of Q_1

$$\Rightarrow I_{C_3} = I_{B_1} = \frac{I_{C_1}}{\beta_1} = \frac{0.35A}{40} = 8.75\,mA$$

- At $-V_p$, all of I_{C_4} supports the base current of Q_2

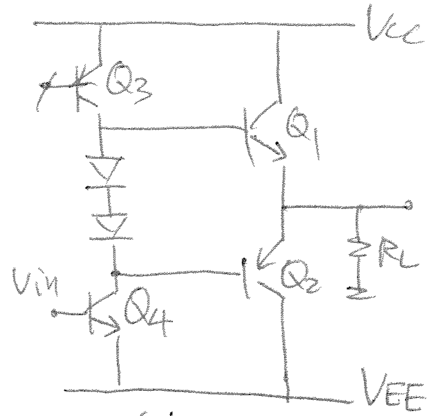
$$\Rightarrow I_{C_4} = I_{B_2} = \frac{I_{C_2}}{\beta_2} = \frac{0.35A}{20} = 17.5\,mA$$

37. $P_{AVG} = 0.5\,W \qquad R_L = 8\Omega$

$V_{CC} = 5V$

$\Rightarrow 0.5W = \frac{1}{2}\frac{V_p^2}{R_L}$

$\Rightarrow V_p = 2\sqrt{2}\ V$



(Assume negligible currents at Vout=0)

$$P_{Q_1} = \frac{1}{T}\int_0^{T/2} I_{C_1} V_{CE_1}\,dt$$

$$= \frac{1}{T}\int_0^{T/2}\left(\frac{V_p \sin\omega t}{R_L}\right)\cdot\left(V_{CC} - V_p\sin\omega t\right)dt$$

$$= \frac{1}{T}\int_0^{T/2}\left[\frac{V_{CC}V_p}{R_L}\sin\omega t - \frac{V_p^2}{2R_L}\right]dt$$

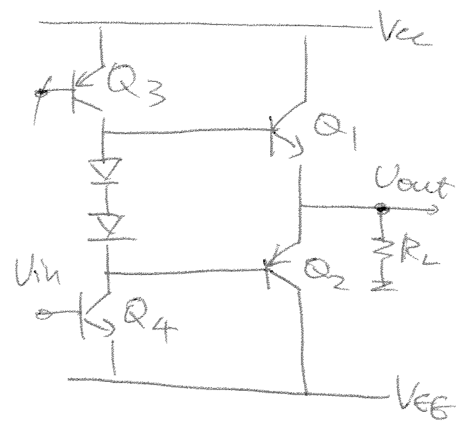
$$= \frac{V_p}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_p}{4}\right) = \frac{2\sqrt{2}}{8}\left(\frac{5}{\pi} - \frac{2\sqrt{2}}{4}\right)$$

$$\approx 0.31\ W$$

38. $P_{Q,MAX} = 0.75W$

$R_L = 8\,\Omega$

$V_{CC} = 5V$



- Out of all 4 transistors, Q_1 & Q_2 must sustain the most currents.

$$\underset{(INST)}{P_{Q_1,MAX}} = V_{CE} \times I_{C_1,MAX} = (V_{CC} - V_{out})\, I_{C_1,MAX}$$

$$\Rightarrow \underset{(AVG)}{P_{Q_1,MAX}} = \frac{1}{T}\int_0^{T/2} \frac{V_p \sin\omega t}{R_L} \cdot (V_{CC} - V_p \sin\omega t)\, dt$$

$$= \frac{1}{T}\int_0^{T/2} \left(\frac{V_{CC} V_p}{R_L} \sin\omega t - \frac{V_p^2}{2R_L} \right) dt$$

$$= \frac{V_p}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_p}{4} \right)$$

$$\Rightarrow \frac{dP_Q}{dV_p} = \frac{V_{CC}}{\pi R_L} - \frac{V_p}{2R_L}$$

$$= 0 \quad \text{when} \quad V_p = \frac{2V_{CC}}{\pi} = 3.18\,V$$

$$P_Q \Big|_{V_p = \frac{2V_{CC}}{\pi}} = 0.32\,W$$

$$\therefore\ P_{R_L,MAX} = \frac{1}{2}\frac{V_p^2}{R_L} = 0.63\,W$$

39. $P_{Q_1,MAX} = \left(\frac{V_{CC}}{\pi} - \frac{2V_{CC}}{4\pi}\right) \cdot \frac{2V_{CC}}{\pi R_L} \leq 0.75\,W$

$$\Rightarrow V_{CC}\big|_{MAX} = 7.7\ V$$

$$\Rightarrow V_{P,MAX} = \frac{2V_{CC\,MAX}}{\pi} = 4.9\ V$$

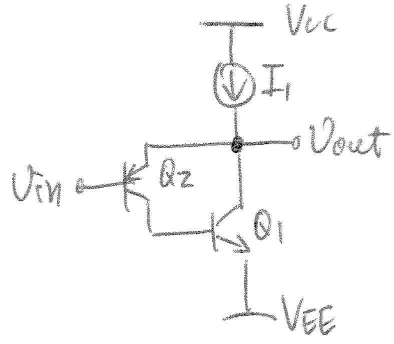
$$\Rightarrow P_{R_L,MAX} = \frac{1}{2}\frac{V_{P\,max}^2}{R_L} = 1.5\ W$$

40. $I_1 = I_{C1} + I_{E2}$

$$= I_{C1} + \frac{\beta_1+1}{\beta_1} I_{C2}$$

$$= I_{C1} + \frac{\beta_1+1}{\beta_1} I_{B1}$$

$$= \beta_1 I_{B1} + \frac{\beta_1+1}{\beta_1} I_{B1}$$



$I_1 = 5mA$

$\beta_1 = 40$

$\beta_2 = 50.$

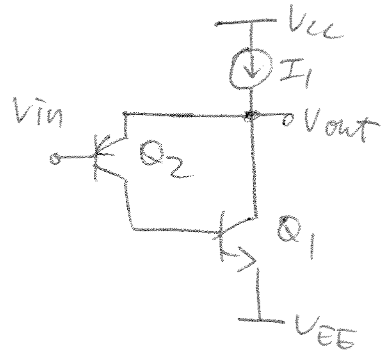
$$\Rightarrow I_{B1} = \frac{I_1}{\beta_1 + \frac{\beta_1+1}{\beta_1}} = \frac{0.005}{40 + \frac{41}{40}}$$

$$\approx 0.12\,mA$$

$$\Rightarrow I_{B2} = \frac{I_{C2}}{\beta_2} = \frac{I_{B1}}{\beta_2} = 0.0024\,mA$$

41. $V_{in} = 0.5\,V$

$I_{S2} = 6 \cdot 10^{-17} A.$



$I_{B_1} = I_{C2} = 0.12\ mA$

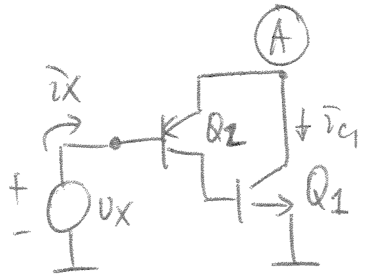
$\Rightarrow I_{C2} = I_{S2} \cdot \exp\left(\frac{V_{out} - V_{in}}{V_T}\right)$

$\therefore V_{out} = V_T \ln\left(\frac{I_{C2}}{I_{S2}}\right) + V_{in}$

$= 0.026 \ln\left(\frac{0.12\ mA}{6 \cdot 10^{-17} A}\right) + 0.5$

$\approx 1.24\ V$

42.



$$i_{c2} = i_x \beta_2$$

$$i_{c1} = -g_{m2} v_{eb2} = i_{e2} = i_{c2} + i_{b2}$$
$$= i_x(\beta_2 + 1)$$

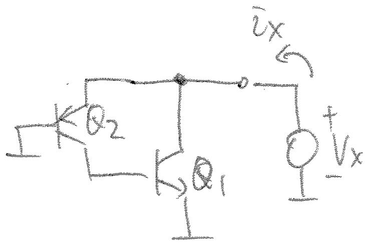
$$v_{eb2} = v_A - v_x$$

where $v_A = v_x - i_x r_{\pi 2}$

$$\therefore \; i_{b1} = -g_{m2}(i_x \cdot r_{\pi 2})$$

$$i_{c1} = i_{b1} + i_{b1} \cdot \beta_1 = -g_m i_x \cdot r_{\pi 2}(1+\beta_1)$$

$$\Rightarrow \frac{v_x}{i_x} \to \infty \quad (R_{in})$$



$$i_x = i_{e2} + i_{c1}$$
$$= i_{e2} + i_{b1}\beta_1$$
$$= i_{e2} + i_{c2}\beta_1$$
$$= i_{c2} + i_{b2} + i_{c2}\beta_1$$
$$= i_{c2}\left(1+\beta_1+\frac{1}{\beta_1}\right)$$
$$= v_x g_{m2}\left(1+\beta_1+\frac{1}{\beta_1}\right)$$

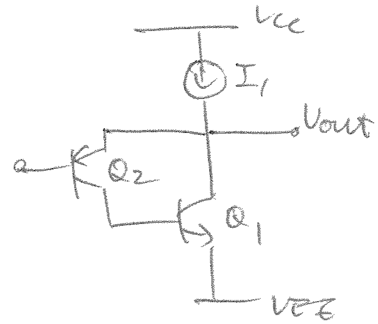
$$g_{m2} = \frac{I_{B2}\beta_2}{V_T}$$
$$= 4.6\ S$$

$$\Rightarrow R_{out} = \frac{v_x}{i_x} = \frac{1}{g_{m2}\left(1+\beta_1+\frac{1}{\beta_1}\right)}$$

$$= 0.005\ \Omega.$$

43. $R_{out} = 1\,\Omega$.

$\beta_1 = 40 \qquad \beta_2 = 50$.



$$R_{out} = 1 = \frac{1}{g_{m2}\left(1+\beta_1+\frac{1}{\beta_1}\right)}$$

$$\Rightarrow g_{m2} = 0.024\,S = \frac{I_{B2}\beta_2}{V_T}$$

$$\Rightarrow I_{B_2} = 0.012\,mA.$$

$$I_1 = I_{C_1} + I_{E2} = I_{B_1}\beta_1 + (I_{C2}+I_{B2})$$

$$= I_{C2}\beta_1 + I_{B2}(\beta_2+1)$$

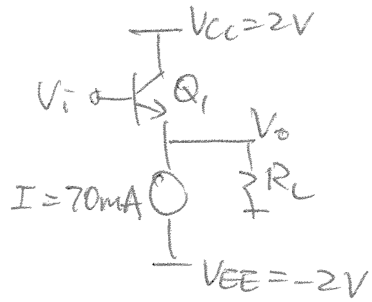
$$= I_{B2}\beta_2\beta_1 + I_{B_2}(\beta_2+1)$$

$$= 0.012\left[50\times 40 + 50 + 1\right]$$

$$= 25.6\ mA$$

44. $V_p = 0.5V$

$R_L = 8\Omega$.



$$P_{R_L} = \frac{V_p^2}{2R_L} = \frac{0.25}{16} = 0.0156\ W$$

$$P_I = -I \times V_{EE} = 0.14\ W$$

$$P_{Q_1} = I_1\left(V_{cc} - \frac{V_p}{2}\right) = 0.1225\ W$$

$$\therefore \eta = \frac{P_{R_L}}{P_{R_L} + P_I + P_{Q_1}} = \frac{0.0156}{0.2781} = 5.6\%$$

45. $P_{R_L} = \frac{V_P^2}{2R_L} = \frac{(V_{CC}-V_{BE})^2}{2R_L}$

$P_{Q_1} = I_1\left(V_{CC} - \frac{V_{CC}-V_{BE}}{2}\right)$

$P_I = +I_1|V_{EE}|$

- Assume $|V_{CC}| = |V_{EE}|$,

$I_1 = V_P/R_L$
$= \frac{V_{CC}-V_{BE}}{R_L}$

$$\therefore \eta = \frac{P_{R_L}}{P_{R_L}+P_{Q_1}+P_I} = \frac{\frac{(V_{CC}-V_{BE})^2}{2R_L}}{\frac{(V_{CC}-V_{BE})^2}{2R_L} + I_1\left[V_{CC} - \frac{V_{CC}-V_{BE}}{2} + |V_{EE}|\right]}$$

$$= \frac{\frac{1}{2R_L}}{\frac{1}{2R_L} + \frac{3V_{CC}+V_{BE}}{2R_L(V_{CC}-V_{BE})}}$$

$$= \frac{1}{1 + \frac{3V_{CC}-V_{BE}}{V_{CC}-V_{BE}}} \approx \frac{V_{CC}-V_{BE}}{3V_{CC}-V_{BE}}$$

46. $$\eta = \frac{\frac{V_p^2}{2R_L}}{\frac{V_p^2}{2R_L} + \frac{2V_p}{R_L}\left(\frac{V_{cc}}{\pi} - \frac{V_p}{4}\right)}$$

$$= \frac{\pi}{4}\frac{V_p}{V_{cc}}$$

$$\Rightarrow \eta\Big|_{V_p = V_{cc} - V_{BE}} = \frac{\pi}{4} - \frac{\pi}{4}\cdot\frac{V_{BE}}{V_{cc}}$$

47. $$\eta = \frac{\dfrac{(V_p/2)^2}{2R_L}}{\dfrac{(V_p/2)^2}{2R_L} + \dfrac{2(V_p/2)}{R_L}\left(\dfrac{V_{cc}}{\pi} - \dfrac{V_p/2}{4}\right)}$$

$$= \frac{V_p^2/8R_L}{\dfrac{V_p^2}{8R_L} + \dfrac{V_p}{R_L}\left(\dfrac{V_{cc}}{\pi} - \dfrac{V_p}{8}\right)} = \frac{1/8R_L}{1/8R_L + \dfrac{1}{R_L}\left(\dfrac{V_{cc}}{V_p\pi} - \dfrac{1}{8}\right)}$$

$$= \frac{1}{1 + \left(\dfrac{8V_{cc}}{V_p\pi} - 1\right)} = \frac{\pi}{8}\frac{V_p}{V_{cc}} \approx 39\%.$$

48. $V_{cc} = 3V$ $\quad P_{R_L} = 0.2W$ $\quad R_L = 8\Omega$.

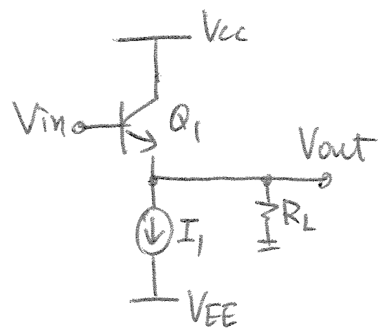
$$P_{R_L} = \frac{1}{2}\frac{V_p^2}{R_L} \Rightarrow V_p = \sqrt{2P_{R_L} \times R_L} = 1.8\ V$$

$$\therefore \eta = \frac{P_{R_L}}{P_{R_L} + \frac{2V_p}{R_L}\left(\frac{V_{cc}}{\pi} - \frac{V_p}{4}\right)} = \frac{0.2}{0.2 + \frac{3.6}{8}\left(\frac{3}{\pi} - \frac{1.8}{4}\right)}$$

$$\approx 18\%.$$

49. Power = 1 W

$R_L = 8\Omega$



$P_{LOAD} = \frac{1}{2}\frac{V_P^2}{R_L} = 1W$

$\Rightarrow V_P = 4\,V \Rightarrow I_1 = \frac{V_P}{R_L} = 0.5\ mA$

(Note: the problem does not specify small-signal voltage gain, so choose $V_P = I_1 R_L$)

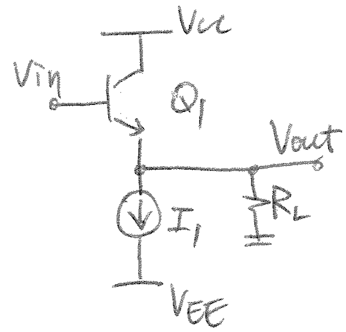
P_{Q_1} (power rating) $= I_1 (V_{CC})$

$= (0.5mA)(5V)$

$= 2.5\ mW$

50. $A_v = 0.8$

$R_L = 4\Omega$



$$A_v = \frac{R_L}{R_L + \frac{1}{g_{m1}}} = \frac{4}{4 + \frac{0.026}{I_{C1}}} = 0.8$$

$\Rightarrow I_{C1} = 26 \text{ mA}$

$\therefore I_1 = I_{C1} = 26\text{mA}$ (Vout biased at 0 V.)

Max Output Swing $= I_1 R_L$

$\approx (26\text{mA})(8\Omega)$

$= 0.208\text{V}$

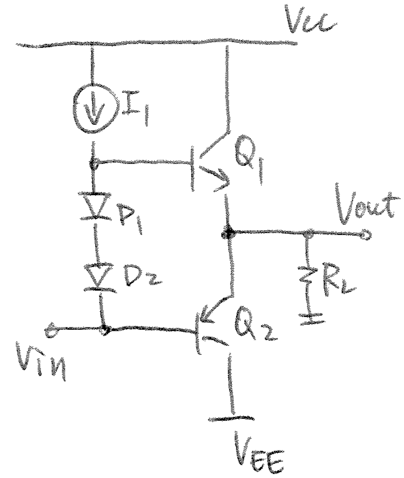
P_{Q_1} (power rating) $= I_1 V_{CC}$ ($V_p = 0$)

$= (26\text{ mA})(5\text{V}) = 130 \text{ mW}$

51. $A_v = 0.6$

$R_L = 8\ \Omega$

$r_{D_1} = r_{D_2} = 0$



(Vout biased at 0V.)

$$A_v = \frac{R_L}{R_L + \frac{1}{g_{m_1}}} = \frac{(8\Omega)}{(8\Omega) + \frac{0.026\,V}{I_{Q_1}}}$$

$= 0.6$

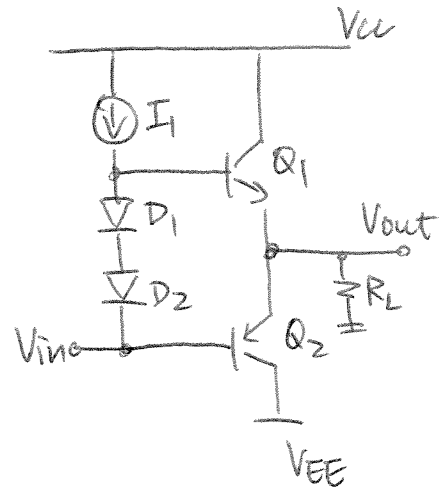
$\Rightarrow I_{Q_1} = I_{Q_2} = 4.8\ mA$

52. Power $= 1\ W$ (to load)

$R_L = 8\,\Omega$

$|V_{BE}| \approx 0.8\ V$

$\beta_1 = 40$



$$P_L = \frac{1}{2}\frac{V_p^2}{R_L} = 1\ W$$

$$\Rightarrow V_p = 4\ V$$

$\therefore$ Min allowable supply voltage $= V_p + |V_{BE}| = 4.8\ V$

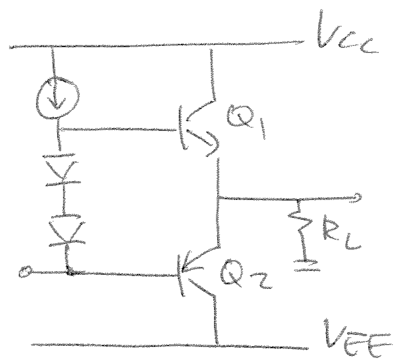
- At $+V_p$, all of I_1 goes to base of Q_1

$$\Rightarrow I_1 = I_{B1} = \frac{I_{C1}}{\beta_1} = \frac{V_p}{R_L}\cdot\frac{1}{\beta_1} \quad (Q_2 \text{ off})$$

$$= \frac{4}{8}\cdot\frac{1}{40} = \frac{1}{80} = 12.5\ mA.$$

53. $P_{Q,MAX} = 2W$

$R_L = 8\Omega$.



For this circuit,

$$P_{AVG,MAX} = \frac{V_{CC}^2}{\pi^2 R_L} \quad \left(V_p = \frac{2V_{CC}}{\pi}\right)$$

$$= 2W$$

$$\Rightarrow V_{CC}\Big|_{MAX} = 12.6\ V \quad \Rightarrow V_p\Big|_{max} = \frac{25.2}{\pi} = 8.02\ V$$

$$\therefore P_{R_L\,MAX} = \frac{V_{Pmax}^2}{2R_L} = \frac{(8.02)^2}{2 \cdot 8} = 4.02\ W$$

54. For this circuit,

$P_{Q,MAX} = 2W$

$R_L = 4\Omega$

$$P_{AVG,MAX} = \frac{V_{cc}^2}{\pi^2 R_L} \qquad \left(V_p = \frac{2V_{cc}}{\pi}\right)$$

$$\Rightarrow V_{cc\,max} = \sqrt{\frac{\pi^2 R_L P_{Q\,MAX}}{1}} = 8.9\ V$$

$$\Rightarrow V_{P_{max}} = \frac{2V_{cc\,max}}{\pi} = 5.6\ V$$

$$\therefore P_{R_L,MAX} = \frac{V_{P,MAX}^2}{2R_L} = \frac{32}{2(4)} = 4W$$

55. $A_V = 4$ $\quad R_L = 8\Omega$ $\quad I_{C1} \approx I_{C2}$

$\beta_1 = 40$ $\quad \beta_2 = 20$

Suppose we want 1st-stage (CE amplifier) to have gain $= 5$ $\Rightarrow$ 2nd stage gain $= 0.8$.

$$\Rightarrow 0.8 = \frac{R_L}{R_L + \frac{1}{g_{m1}+g_{m2}}}$$

$$0.8 = \frac{8}{8 + \frac{1}{2g_{m1}}} \Rightarrow g_{m1} = .5 \Rightarrow I_{C1} = I_{C2} = 6.5mA$$

$$r_{\pi 1} \| r_{\pi 2} = \frac{\beta_1 V_T}{I_{C1}} \| \frac{\beta_2 V_T}{I_{C2}} = \frac{40(0.026)}{6.5mA} \| \frac{20(0.026)}{6.5mA} \approx 133\Omega$$

- $A_V = 4 = g_{m4}(r_{\pi 1} \| r_{\pi 2})(g_{m1} + g_{m2}) R_L$

$$= \frac{I_{C4}}{V_T}(133)(0.5)8$$

$$\Rightarrow I_{C4} = I_{C3} = \frac{4V_T}{8(133)(0.5)} = 0.195 \text{ mA}$$

Max I_{Q1} when all of I_{C3}/I_{C4} supports base current of Q_1

$\Rightarrow I_{Q1,MAX} = I_{C4} = 0.195 \text{mA}$

56. $A_v = 4$ $\quad R_L = 4\,\Omega$ $\quad I_{C_1} \approx I_{C_2}$

$\beta_1 = 40$ $\quad \beta_2 = 20$

1st stage gain $= 5$ (CE amplifier)

2nd " " $= 0.8$

- $$0.8 = \frac{R_L}{R_L + \frac{1}{g_{m_1} + g_{m_2}}} = \frac{4}{4 + \frac{1}{2g_{m_1}}}$$

$\Rightarrow g_{m_1} = 0.5\,S \Rightarrow I_{C_1} = I_{C_2} = 13\,mA$

$$r_{\pi_1} \| r_{\pi_2} = \frac{\beta_1 V_T}{I_{C_1}} \| \frac{\beta_2 V_T}{I_{C_2}} = 80 \| 40 = 26.7\,\Omega$$

- $$A_v = 4 = g_{m_4}(r_{\pi_1} \| r_{\pi_2})(g_{m_1} + g_{m_2}) R_L$$
$$= \frac{I_{C4}}{V_T}(26.7)(1)(4)$$

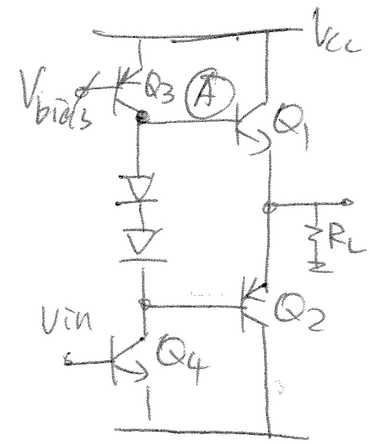
$$\Rightarrow I_{C4} = I_{C3} = \frac{4 V_T}{(26.7)(1)(4)} = 0.974\ mA.$$

- Max I_{Q_1} ($I_{Q_1 MAX}$) when $I_{C4} = I_{Q MAX} = 0.974\,mA$.
- For a reduction of $2\times$ the R_L, we have to provide $\sim 5\times$ current to base of Q_1. $\left(\frac{0.974}{0.195} \approx 5\right)$

57. $P_{R_L} = 2W$ $\quad \beta_1 = 40$

$R_L = 8\,\Omega$ $\quad \beta_2 = 20$

$|V_{BE}| = 0.8V$



(a) $P_{R_L} = \frac{1}{2}\frac{V_p^2}{R_L} \Rightarrow V_p \approx 5.6\ V$

- At $+V_p$, $V_A = V_p + |V_{BE}|$.
- For Q_3 in active region, $V_A \le V_{bias}$

$\Rightarrow V_{CC} \ge V_{bias} + |V_{BE}| = V_p + 2|V_{BE}|$

$\ge 5.6 + 1.6 = 7.2\ V.$

(b) $I_p = \frac{V_p}{R_L} = 0.7\ A.\ (\approx I_{E_1}),\ (\approx I_{E_2})$

$\Rightarrow I_{B_1} = \frac{I_{E_1}}{1+\beta_1} = 17\ mA.$

$\therefore$ We bias Q_3 & Q_4 with $I_C = 17\ mA$.

(c) $P_{AV} = \frac{V_P}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_P}{4}\right)$

$= \frac{5.6}{8}\left(\frac{5}{\pi} - \frac{5.6}{4}\right) = 3.66\ W$

(d) $P_{IQ_3} = 2V_{CC} \times I_{Q_3} = 10 \times 17mA = 170\,mW$

$P_{AV,Q_1} = \frac{V_P}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_P}{4}\right) = 3.66\ W$

$P_{R_L} = 2W$

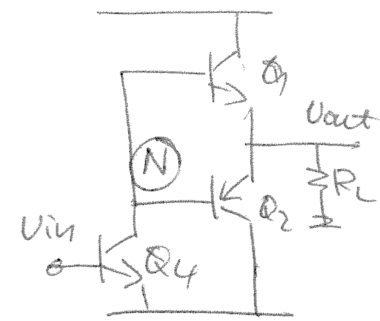
$\Rightarrow \eta = \frac{P_{R_L}}{P_{IQ_3} + 2 \cdot P_{AV,Q_1} + P_{R_L}}$

$= \frac{2}{170m + 3.66 \times 2 + 2} = 0.21 = 21\%$.

58.

(a) $A_v = 5 \quad R_L = 4\Omega \quad \beta_1 = 40 \quad \beta_2 = 20.$

Assume $I_{C1} \approx I_{C2}$.



$$\frac{v_{out}}{v_N} = \frac{R_L}{(g_{m1}+g_{m2})^{-1} + R_L} = 0.8$$

$$\Rightarrow 2g_{m1}^{-1} = 1 \Rightarrow I_{C1} = 2V_T$$

$$= 0.052\ A.$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = +g_{m4}(r_{\pi1} \| r_{\pi2})(g_{m1}+g_{m2})R_L = 5$$

Assume $g_{m1} \approx g_{m2}$:

$$\Rightarrow I_{C4} = V_T \frac{5}{(r_{\pi1} \| r_{\pi2})(g_{m1}+g_{m2})R_L}$$

$$= V_T \times \frac{5}{(r_{\pi1} \| r_{\pi2})(g_{m1} \times 2)R_L}$$

$$= 0.026 \frac{5}{(6.7\Omega)(2\times 2)(4)}$$

$$\approx 1.2\ mA.$$

$\Rightarrow$ Max I by $Q_1 = \beta_1 \times I_{C4} = 48\ mA$

$\Rightarrow P_{R_L} = \frac{1}{2} I^2 R_L = 24 \times 4\ mW = 96\ mW$, BELOW requirement!

(b) $P = 5\,W = \frac{1}{2}\frac{V_p^2}{R_L} \Rightarrow V_p = 6.3\,V$

$\Rightarrow I_p = \frac{V_p}{R_L} = 1.6\,A$

$\Rightarrow I_{B2,MAX} = \frac{I_p}{\beta_2} = \frac{1.6}{20} = 79\,mA$

$\Rightarrow I_{C4}$ must equal 79 mA to allow max output swing V_p

$\Rightarrow g_{m4} = \frac{I_{C4}}{V_T} = 3.04\,S$

Suppose 2nd stage gain $= 0.8$ $(I_{C1} \simeq I_{C2})$

$\Rightarrow \frac{v_{out}}{v_N} = \frac{R_L}{R_L + \frac{1}{g_{m1}+g_{m2}}} = 0.8$ $\Rightarrow g_{m1} = 0.5\,S$

$\Rightarrow I_{C1} = I_{C2} = 13\,mA.$

$r_{\pi 1} \| r_{\pi 2} = \frac{\beta_1 V_T}{I_{C1}} \| \frac{\beta_2 V_T}{I_{C2}} = 26.7\,\Omega.$

$\therefore \frac{v_{out}}{v_{in}} = -(3.04)(26.7\,\Omega)(0.5+0.5)4$

$= -324$!! (Huge! Impractical)

- Even when the 2nd stage gets close to 1, we still need <u>huge</u> gain from first stage.