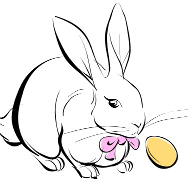
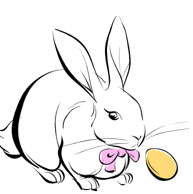
期末试题B卷



一、判断题

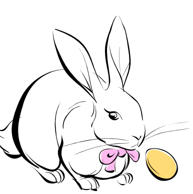
1. 单调函数的原函数仍然是单调函数。

1. 错误，$y=x^3$。



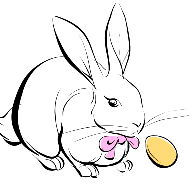
2. 设 $I_1=\int_{0.3}^{0.7}\sqrt{x}dx$， $I_2=\int_{0.3}^{0.7}x^3dx$，则 $I_1<I_2$。

2. 错误，可以分别做出函数 $f_1(x)=\sqrt{x}$ 和 $f_2(x)=x^3$ 的图像，比较二者在区间 $[0.3,0.7]$ 上与 x 轴围成的面积，可得 $I_1>I_2$。



3. 若定积分 $\int_a^b f(x)dx$ 存在，则 $f(x)$ 在 $[a,b]$ 上是有界的。

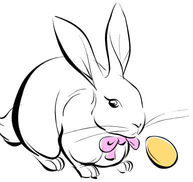
3. 正确。



4. 对二元函数 $f(x,y)$，如果在点 (x_0,y_0) 它是可微的，则它在该点连续，但偏导数 $\dfrac{\partial f}{\partial x},\dfrac{\partial f}{\partial y}$ 不一定存在。

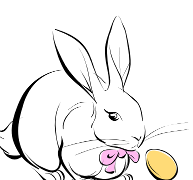
4. 错误，偏导数一定存在。

考点：可微的定义



5. 设区域 $D_1=\left\{(x,y)\middle|x^2+y^2\le 4\right\}$，$D_2=\left\{(x,y)\middle||x|+|y|\le 2\sqrt{2}\right\}$，则 $\iint_{D_1}dxdy\ge\iint_{D_2}dxdy$。

5. 错误。可以分别做出区域 D_1 和 D_2 的图像，比较二者的面积大小，可得 $\iint_{D_1}dxdy<\iint_{D_2}dxdy$。



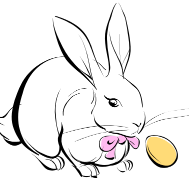
二、选择题

1. $\lim\limits_{x\to 0}\dfrac{\int_0^{x^2}\sin t^2dt}{x^6}=$ (　　)

(A) 0　(B) 1/2　(C) 1/3　(D) ∞

C

$$\lim_{x\to 0}\frac{\int_0^{x^2}\sin t^2dt}{x^6}=\lim_{x\to 0}\frac{\sin x^4\bullet 2x}{6x^5}=\frac{1}{3}$$



2. 设 $f(x)$ 有原函数 $x\ln x$，则 $\int xf(x)dx=$（　）

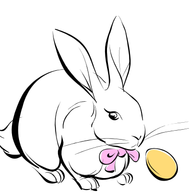
(A) $x^2\left(\frac{1}{2}+\frac{1}{4}\ln x\right)+C$　(B) $x^2\left(\frac{1}{4}+\frac{1}{2}\ln x\right)+C$

(C) $x^2\left(\frac{1}{4}-\frac{1}{2}\ln x\right)+C$　(D) $x^2\left(\frac{1}{2}-\frac{1}{4}\ln x\right)+C$

B

$$f(x)=(x\ln x)'=\ln x+1$$

$$\int xf(x)dx=\int x(\ln x+1)dx=x^2\left(\frac{1}{4}+\frac{1}{2}\ln x\right)+C$$



3. 隐函数 $\ln\sqrt{x^2+y^2}=\arctan\frac{y}{x}$ 的导数 $\frac{dy}{dx}=$（ ）

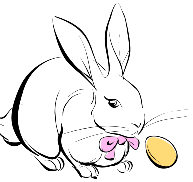
(A) $\frac{x+y}{x-y}$ (B) $\frac{x-y}{x+y}$ (C) $\frac{x-y}{x^2+y^2}$ (D) $\frac{x^2-y^2}{x+y}$

A

原式左右对x求导

$$\frac{x+y\bullet y'}{x^2+y^2}=\frac{x\bullet y'-y}{x^2+y^2}$$

故 $y'=\frac{x+y}{x-y}$



4. 设数列 x_n, y_n 满足 $\lim\limits_{n\to\infty} x_n y_n = 0$，则下列断言正确的是（　）

(A)若 x_n 发散，则 y_n 必发散

(B)若 x_n 无界，则 y_n 必有界

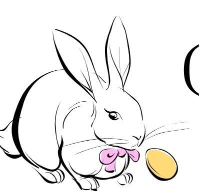
(C)若 x_n 有界，则 y_n 必为无穷小

(D)若 $1/x_n$ 为无穷小，则 y_n 必为无穷小

D

$$(A) x_n = n^2, y_n = \frac{1}{n^3}$$

$$(B) x_n = e^{\frac{1}{n-1}} - 1, y_n = e^{\frac{1}{n-2}}$$



$$(C) x_n = \frac{1}{n}, y_n = 1$$

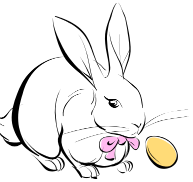
5. 设函数 $f(x)$ 有连续导数，则以下公式不正确的是（　）

(A) $\frac{d}{dx}\int f(x)dx = f(x)$　　(B) $\int \frac{d}{dx} f(x)dx = f(x)$

(B) $\int_a^b f'(x)dx = f(b) - f(a)$　　(D) $\frac{d}{dx}\int_a^x f(t)dt = f(x)$

B

$$\int \frac{d}{dx} f(x)dx = f(x) + C$$



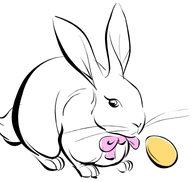
三、填空题

1．对隐函数$\left(x^2-y^2\right)+\sin xy=y^x$，则$\dfrac{dy}{dx}=$ ________。

原式左右对x求导

$$2x-2yy'+\cos(xy)(y+xy')=y^x(\ln y+\frac{x}{y}y')$$

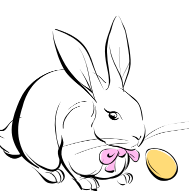
$$y'=\frac{y^{x+1}\ln y-2xy-y^2\cos xy}{xy\cos(xy)-2y^2-xy^x}$$



2. 二元函数 $f(x,y)=x^3-y^3+3x^2+3y^2-9x$ 的驻点为__________。

$$\begin{cases} f_x' = 0 \\ f_y' = 0 \end{cases} \Rightarrow \begin{cases} 3x^2+6x-9=0 \\ -3y^2+6y=0 \end{cases}$$

解得驻点（1, 0），(1, 2), (−3, 0), (−3, 2)

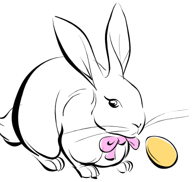


3．设函数 $f(x,y)=\ln xy$，则 $\dfrac{\partial^2 f}{\partial x^2}+\dfrac{\partial^2 f}{\partial y^2}=$ ________。

$$\frac{\partial f}{\partial x}=\frac{1}{x},\frac{\partial^2 f}{\partial x^2}=-\frac{1}{x^2}$$

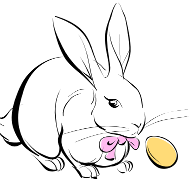
$$\frac{\partial f}{\partial y}=\frac{1}{y},\frac{\partial^2 f}{\partial y^2}=-\frac{1}{y^2}$$

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=-\frac{1}{x^2}-\frac{1}{y^2}$$



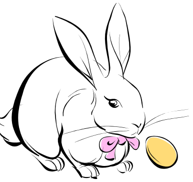
4. 多元函数$u=\ln\left(x^2+y^2+z^2\right)$的全微分$du=$__________。

$$du=\frac{2}{x^2+y^2+z^2}\left(xdx+ydy+zdz\right)$$



5. 不定积分$\int 2xe^{x^2}dx =$__________。

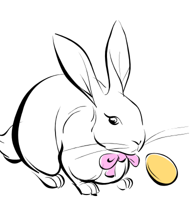
$$e^{x^2}+C$$



四、计算题

$$\int \frac{x^3}{\sqrt{1+x^2}}dx = \frac{1}{2}\int \frac{x^2}{\sqrt{1+x^2}}dx^2 = \frac{1}{2}\int\left(\sqrt{1+u} - \frac{1}{\sqrt{1+u}}\right)du$$

$$= \frac{\left(1+x^2\right)^{\frac{3}{2}}}{3} - \sqrt{1+x^2} + C$$



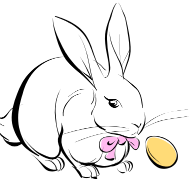
$$\int \frac{xdx}{x^8-1} = \int \frac{xdx}{(x^4-1)(x^4+1)} = \int \frac{1}{2}\left(\frac{1}{x^4-1} - \frac{1}{x^4+1}\right)xdx$$

$$= \frac{1}{4}\int \left(\frac{1}{x^4-1} - \frac{1}{x^4+1}\right)dx^2$$

$$= \frac{1}{4}\int \left[\frac{1}{2}\left(\frac{1}{x^2-1} - \frac{1}{x^2+1}\right) - \frac{1}{x^4+1}\right]dx^2$$

$$= \frac{1}{8}\left[\int \frac{1}{x^2-1}d(x^2-1) - \int \frac{1}{x^2+1}d(x^2+1)\right] - \frac{1}{4}\int \frac{1}{(x^2)^2+1}dx^2$$

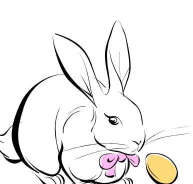
$$= \frac{1}{8}\ln\left|\frac{x^2-1}{x^2+1}\right| - \frac{1}{4}\arctan x^2 + C.$$



$$\int \frac{\ln x}{x^2}dx = \int \ln x d(-\frac{1}{x}) = -\frac{1}{x}\ln x + \int \frac{1}{x^2}dx$$

$$= -\frac{1}{x}(\ln x + 1) + c$$

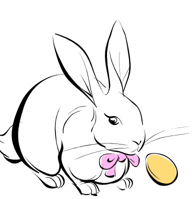
考点：分部积分



$$\int e^x \sin x dx = \int \sin x d\left(e^x\right)$$
$$= e^x \sin x - \int e^x \cos x dx$$
$$= e^x \sin x - \int \cos x d\left(e^x\right)$$
$$= e^x \sin x - e^x \cos x - \int e^x \sin x dx$$

移项，整理可得 $\int e^x \sin x dx = \frac{1}{2} e^x \left(\sin x - \cos x\right) + C$

考点：分部积分



$$\int_0^{\pi}\sqrt{1-\sin x}dx$$

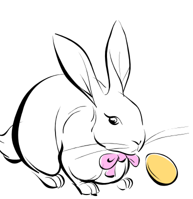
$$=\int_0^{\pi}\sqrt{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}dx$$

$$=\int_0^{\pi}\left|\sin\frac{x}{2}-\cos\frac{x}{2}\right|dx=2\int_0^{\frac{\pi}{2}}\left|\sin x-\cos x\right|dx$$

$$=2[-\int_0^{\frac{\pi}{4}}(\sin x-\cos x)dx+\int_{\frac{\pi}{4}}^{\pi}(\sin x-\cos x)dx]$$

$$=2[-(-\cos x-\sin x)\Big|_0^{\frac{\pi}{4}}+(-\cos x-\sin x)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}]$$

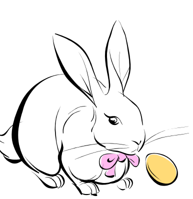
$$=4(\sqrt{2}-1)$$



$$\int_0^{\frac{\pi}{2}} x^2 \sin x dx = -x^2 \cos x \Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} 2x \cos x dx = 2x \sin x \Big|_0^{\frac{\pi}{2}} - 2\int_0^{\frac{\pi}{2}} \sin x dx$$

$$= \pi + 2\cos \Big|_0^{\frac{\pi}{2}} = \pi - 2$$

考点：分部积分



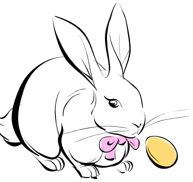
设函数 $z=f(xy,g(x))$,函数 f 具有二阶连续偏导数，函数 g(x)可导且在 x=1 处取得极值 g(1)=1,求 $\left.\frac{\partial^2 z}{\partial x \partial y}\right|_{\substack{x=1,\\ y=1}}$。

$$\frac{\partial z}{\partial x}=f_1'y+f_2'g'(x)$$

$$\frac{\partial^2 z}{\partial x \partial y}=f_1'+xyf_{11}''+xf_{21}''g'(x)$$

$$g'(1)=0,$$

$$\left.\frac{\partial^2 z}{\partial x \partial y}\right|_{\substack{x=1\\ y=1}}=f_1'(1,1)+f_{11}''(1,1)$$

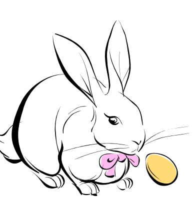


求由方程$2xz-2xyz+\ln(xyz)=0$所确定的隐函数$z=z(x,y)$在点（1，1）处的全微分。

解：x=1,y=1 时，z=1

$$\left.\frac{\partial z}{\partial x}\right|_{(1,1,1)}=\left.\frac{2z-2yz+1/x}{2xy-2x-1/z}\right|_{(1,1,1)}=-1,\quad \left.\frac{\partial z}{\partial y}\right|_{(1,1,1)}=\left.\frac{-2xz+1/y}{2xy-2x-1/z}\right|_{(1,1,1)}=1$$

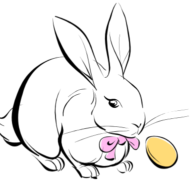
$$dz\big|_{(1,1)}=\left.\frac{\partial z}{\partial x}\right|_{(1,1,1)}dx+\left.\frac{\partial z}{\partial y}\right|_{(1,1,1)}dy=-dx+dy$$



计算 $\iint\limits_D |xy|dxdy$，其中 D 是圆域：$x^2+y^2\le a^2$。

根据对称性

$$\iint\limits_D |xy|\,dxdy = 4\int_0^a x\int_0^{\sqrt{a^2-x^2}} ydydx = 4\int_0^a x\frac{a^2-x^2}{2}dx = \frac{a^4}{2}$$



计算 $\iint_D y\sqrt{1+x^2+y^2}\,d\sigma$，其中 D 是由直线 $y=x$、$x=-1$ 和 $y=1$ 所围成的闭区域。

$$\iint_D y\sqrt{1+x^2-y^2}\,d\sigma=\int_{-1}^{1}\left[\int_x^1 y\sqrt{1+x^2-y^2}\,dy\right]dx$$

$$=-\frac{1}{2}\int_{-1}^{1}\left[\int_x^1\sqrt{1+x^2-y^2}\,d(1+x^2-y^2)\right]dx$$

$$=-\frac{1}{3}\int_{-1}^{1}\left[\left(1+x^2-y^2\right)^{\frac{3}{2}}\right]\Bigg|_x^1 dx$$

$$=-\frac{1}{3}\int_{-1}^{1}\left(\left|x\right|^3-1\right)dx$$

$$=-\frac{2}{3}\int_0^1\left(x^3-1\right)dx$$

$$=\frac{1}{2}$$