Question 1:

(a) $P(x=x)=(1-p)^{x-1}p$, where $p=\frac{1}{3}$ because of $N=3$

$E(x)=\sum_{x=1}^{n} xP(x=x)=\sum_{x=1}^{n} x(1-p)^{x-1}p=\sum_{x=1}^{n} x\left(1-\frac{1}{3}\right)^{x-1}\cdot\frac{1}{3}=\frac{1}{3}\sum_{x=1}^{n} x\left(\frac{2}{3}\right)^{x-1}$

Let $S=\sum_{x=1}^{n} x\left(\frac{2}{3}\right)^{x-1}=1\times\left(\frac{2}{3}\right)^{0}+2\times\left(\frac{2}{3}\right)^{1}+\cdots+(n-1)\times\left(\frac{2}{3}\right)^{n-2}+n\times\left(\frac{2}{3}\right)^{n-1}$,

hence $\frac{2}{3}S=\sum_{x=1}^{n} x\left(\frac{2}{3}\right)^{x}=1\times\left(\frac{2}{3}\right)^{1}+2\times\left(\frac{2}{3}\right)^{2}+\cdots+(n-1)\times\left(\frac{2}{3}\right)^{n-1}+n\times\left(\frac{2}{3}\right)^{n}$,

So, we have $S-\frac{2}{3}S=\left(\frac{2}{3}\right)^{0}+\left(\frac{2}{3}\right)^{1}+\cdots+\left(\frac{2}{3}\right)^{n-2}+\left(\frac{2}{3}\right)^{n-1}-n\times\left(\frac{2}{3}\right)^{n}$

$=\frac{1\left(1-\left(\frac{2}{3}\right)^{n}\right)}{1-\frac{2}{3}}-n\times\left(\frac{2}{3}\right)^{n}$

$=3-3\times\left(\frac{2}{3}\right)^{n}-n\times\left(\frac{2}{3}\right)^{n}$

$=3-(3+n)\times\left(\frac{2}{3}\right)^{n}$

$\because \lim_{n\to\infty}(3+n)\times\left(\frac{2}{3}\right)^{n}=\lim_{n\to\infty}\frac{(3+n)}{\left(\frac{3}{2}\right)^{n}}=\lim_{n\to\infty}\frac{1}{\left(\frac{3}{2}\right)^{n-1}\ln\frac{3}{2}}=0$

$\therefore E(x)=\frac{1}{3}S=3$

(b) $E(x)=\sum_{x=1}^{n} xP(x=x)=\sum_{x=1}^{n} x(1-p)^{x-1}p$

$\therefore (1-p)E(x)=\sum_{x=1}^{n} x(1-p)^{x}p$

$\therefore pE(x)=p(\sum_{x=1}^{n} x(1-p)^{x-1}-\sum_{x=1}^{n} x(1-p)^{x})$

$=p(1+2(1-p)+\cdots+(n-1)(1-p)^{n-2}+n(1-p)^{n-1}-((1-p)+2(1-p)^{2}+\cdots+(n-1)(1-p)^{n-1}+n(1-p)^{n})$

$=p(1+(1-p)+\cdots+(1-p)^{n-1}-n(1-p)^{n})$

$=p(\frac{1(1-(1-p)^{n})}{p}-n(1-p)^{n})$

$=p(\frac{1}{p}-(\frac{1}{p}+n)(1-p)^{n})$

$\because \lim_{n\to\infty}\left(\frac{1}{p}+n\right)\times(1-p)^{n}=\lim_{n\to\infty}\frac{\left(\frac{1}{p}+n\right)}{\left(\frac{1}{1-p}\right)^{n}}=\lim_{n\to\infty}\frac{1}{\left(\frac{1}{1-p}\right)^{n-1}\ln\frac{1}{1-p}}=0$

$\therefore pE(x)=p*\frac{1}{p}=1,\ E(x)=\frac{1}{p}$

$\therefore E(x)=N$ when $p=\frac{1}{N}$