COMP 2432 Operating Systems
Tutorial 10 Solution

1. Virtual Memory Page Replacement.

(*a*) FIFO: 10 page faults

Ref string	0	1	2	3	0	2	3	4	0	1	2	3	4	2	3
Frame 1	0*	0	0	3*	3	3	3	3	3	1*	1	1	4*	4	4
Frame 2	-	1*	1	1	0*	0	0	0	0	0	2*	2	2	2	2
Frame 3	-	-	2*	2	2	2	2	4*	4	4	4	3*	3	3	3

(*b*) Optimal: 7 page faults

Ref string	0	1	2	3	0	2	3	4	0	1	2	3	4	2	3
Frame 1	0*	0	0	0	0	0	0	0	0	1*	1	3*	3	3	3
Frame 2	-	1*	1	3*	3	3	3	4*	4	4	4	4	4	4	4
Frame 3	-	-	2*	2	2	2	2	2	2	2	2	2	2	2	2

(*c*) LRU: 11 page faults

Ref string	0	1	2	3	0	2	3	4	0	1	2	3	4	2	3
Frame 1	0*	0	0	3*	3	3	3	3	3	1*	1	1	4*	4	4
Frame 2	-	1*	1	1	0*	0	0	4*	4	4	2*	2	2	2	2
Frame 3	-	-	2*	2	2	2	2	2	0*	0	0	3*	3	3	3

It can be oserved that FIFO is performing better than LRU in this example, but they sometimes produce the same number of page faults and sometimes LRU is performing better than FIFO.

2. Virtual Memory Page Replacement.

Using 3 frames.

(*a*) FIFO: 18 page faults

Ref str	0	1	2	3	4	0	2	3	4	5	0	1	3	5	4	3	2	1	2	3
Frame 1	0*	0	0	3*	3	3	2*	2	2	5*	5	5	3*	3	3	3	2*	2	2	2
Frame 2	-	1*	1	1	4*	4	4	3*	3	3	0*	0	0	5*	5	5	5	1*	1	1
Frame 3	-	-	2*	2	2	0*	0	0	4*	4	4	1*	1	1	4*	4	4	4	4	3*

(*b*) Optimal: 10 page faults

Ref str	0	1	2	3	4	0	2	3	4	5	0	1	3	5	4	3	2	1	2	3
Frame 1	0*	0	0	0	0	0	0	0	0	0	0	1*	1	1	1	1	1	1	1	1
Frame 2	-	1*	1	3*	4*	4	4	4	4	5*	5	5	5	5	4*	4	2*	2	2	2
Frame 3	-	-	2*	2	2	2	2	3*	3	3	3	3	3	3	3	3	3	3	3	3

(*c*) LRU-counter: 17 page faults

Ref str	0	1	2	3	4	0	2	3	4	5	0	1	3	5	4	3	2	1	2	3
Frame 1	0*	0	0	3*	3	3	2*	2	2	5*	5	5	3*	3	3	3	3	3	3	3
Frame 2	-	1*	1	1	4*	4	4	3*	3	3	0*	0	0	5*	5	5	2*	2	2	2
Frame 3	-	-	2*	2	2	0*	0	0	4*	4	4	1*	1	1	4*	4	4	1*	1	1
Counter	1	1	1	4	4	4	7	7	7	10	10	10	13	13	13	16	16	16	16	20
	-	2	2	2	5	5	5	8	8	8	11	11	11	14	14	14	17	17	19	19
	-	-	3	3	3	6	6	6	9	9	9	12	12	12	15	15	15	18	18	18

(*d*) LRU-stack: 17 page faults

Ref str	0	1	2	3	4	0	2	3	4	5	0	1	3	5	4	3	2	1	2	3
Frame 1	0*	0	0	3*	3	3	2*	2	2	5*	5	5	3*	3	3	3	3	3	3	3
Frame 2	-	1*	1	1	4*	4	4	3*	3	3	0*	0	0	5*	5	5	2*	2	2	2
Frame 3	-	-	2*	2	2	0*	0	0	4*	4	4	1*	1	1	4*	4	4	1*	1	1
Stack	-	-	2	3	4	0	2	3	4	5	0	1	3	5	4	3	2	1	2	3
	-	1	1	2	3	4	0	2	3	4	5	0	1	3	5	4	3	2	1	2
	0	0	0	1	2	3	4	0	2	3	4	5	0	1	3	5	4	3	3	1

Using 4 frames.

(*a*) FIFO: 11 page faults

Ref str	0	1	2	3	4	0	2	3	4	5	0	1	3	5	4	3	2	1	2	3
Frame 1	0*	0	0	0	4*	4	4	4	4	4	4	4	3*	3	3	3	3	3	3	3
Frame 2	-	1*	1	1	1	0*	0	0	0	0	0	0	0	0	4*	4	4	4	4	4
Frame 3	-	-	2*	2	2	2	2	2	2	5*	5	5	5	5	5	5	2*	2	2	2
Frame 4	-	-	-	3*	3	3	3	3	3	3	3	1*	1	1	1	1	1	1	1	1

(*b*) Optimal: 8 page faults

Ref str	0	1	2	3	4	0	2	3	4	5	0	1	3	5	4	3	2	1	2	3
Frame 1	0*	0	0	0	0	0	0	0	0	0	0	1*	1	1	1	1	1	1	1	1
Frame 2	-	1*	1	1	4*	4	4	4	4	4	4	4	4	4	4	4	2*	2	2	2
Frame 3	-	-	2*	2	2	2	2	2	2	5*	5	5	5	5	5	5	5	5	5	5
Frame 4	-	-	-	3*	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3

Optimal: 8 page faults (a second possible answer on the choice of victim frame for admitting page 2)

Ref str	0	1	2	3	4	0	2	3	4	5	0	1	3	5	4	3	2	1	2	3
Frame 1	0*	0	0	0	0	0	0	0	0	0	0	1*	1	1	1	1	1	1	1	1
Frame 2	-	1*	1	1	4*	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
Frame 3	-	-	2*	2	2	2	2	2	2	5*	5	5	5	5	5	5	2*	2	2	2
Frame 4	-	-	-	3*	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3

(*c*) LRU-counter: 13 page faults

Ref str	0	1	2	3	4	0	2	3	4	5	0	1	3	5	4	3	2	1	2	3
Frame 1	0*	0	0	0	4*	4	4	4	4	4	4	4	3*	3	3	3	3	3	3	3
Frame 2	-	1*	1	1	1	0*	0	0	0	5*	5	5	5	5	5	5	5	1*	1	1
Frame 3	-	-	2*	2	2	2	2	2	2	2	0*	0	0	0	4*	4	4	4	4	4
Frame 4	-	-	-	3*	3	3	3	3	3	3	3	1*	1	1	1	1	2*	2	2	2
Counter	1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	16	16	16	16	20
	-	2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14	18	18	18
	-	-	3	3	3	3	7	7	7	7	11	11	11	11	15	15	15	15	15	15
	-	-	-	4	4	4	4	8	8	8	8	12	12	12	12	12	17	17	19	19

(*d*) LRU-stack: 13 page faults

Ref str	0	1	2	3	4	0	2	3	4	5	0	1	3	5	4	3	2	1	2	3
Frame 1	0*	0	0	0	4*	4	4	4	4	4	4	4	3*	3	3	3	3	3	3	3
Frame 2	-	1*	1	1	1	0*	0	0	0	5*	5	5	5	5	5	5	5	1*	1	1
Frame 3	-	-	2*	2	2	2	2	2	2	2	0*	0	0	0	4*	4	4	4	4	4
Frame 4	-	-	-	3*	3	3	3	3	3	3	3	1*	1	1	1	1	2*	2	2	2
Stack	0	1	2	3	4	0	2	3	4	5	0	1	3	5	4	3	2	1	2	3
	-	0	1	2	3	4	0	2	3	4	5	0	1	3	5	4	3	2	1	2
	-	-	0	1	2	3	4	0	2	3	4	5	0	1	3	5	4	3	3	1
	-	-	-	0	1	2	3	4	0	2	3	4	5	0	1	1	5	4	4	4

It is clear that the frame contents will be the same for (*c*) and (*d*) when 3 frames and 4 frames are provided, since both are based on LRU replacement. The only difference is the implementation approach, i.e. using counter versus using stack in LRU. It can be observed that as the number of frames incrases, the number of page faults would improve in this example. When comparing among the algorithms, it can be observed that with 3 frames, LRU is performing better than FIFO, but with 4 frames, FIFO is performing better than LRU. Optimal is always the best, but LRU and FIFO may beat each other under different occasions.

3. Virtual Memory Page Replacement.

Using 3 frames.

(*a*) FIFO: 16 page faults

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	3*	3	3	2*	2	2	2	2	1*	1	1	1	1	1	1	5*	5	5	4*	4	4
-	1*	1	1	4*	4	4	4	4	3*	3	3	3	3	2*	2	2	2	2	0*	0	0	0	0
-	-	2*	2	2	1*	1	1	1	1	5*	5	5	5	5	3*	3	3	3	3	2*	2	2	2

(*b*) Optimal: 11 page faults (first acceptable answer, and you can answer *any one* of them).

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	3*	4*	4	4	4	4	3*	3	3	3	3	3	3	3	3	5*	0*	0	4*	4	4
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
-	-	2*	2	2	2	2	2	2	2	5*	5	5	5	2*	2	2	2	2	2	2	2	2	2

Optimal: 11 page faults (second acceptable answer).

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	3*	4*	4	4	4	4	3*	3	3	3	3	3	3	3	3	5*	0*	0	0	0	0
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4*	4	4
-	-	2*	2	2	2	2	2	2	2	5*	5	5	5	2*	2	2	2	2	2	2	2	2	2

Optimal: 11 page faults (third acceptable answer).

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	3*	4*	4	4	4	4	3*	3	3	3	3	3	3	3	3	5*	5	5	4*	4	4
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0*	0	0	0	0
-	-	2*	2	2	2	2	2	2	2	5*	5	5	5	2*	2	2	2	2	2	2	2	2	2

Optimal: 11 page faults (fourth acceptable answer).

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	3*	4*	4	4	4	4	3*	3	3	3	3	3	3	3	3	5*	5	5	5	5	5
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0*	0	4*	4	4
-	-	2*	2	2	2	2	2	2	2	5*	5	5	5	2*	2	2	2	2	2	2	2	2	2

Optimal: 11 page faults (fifth acceptable answer).

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	3*	4*	4	4	4	4	3*	3	3	3	3	3	3	3	3	3	0*	0	4*	4	4
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5*	5	5	5	5	5
-	-	2*	2	2	2	2	2	2	2	5*	5	5	5	2*	2	2	2	2	2	2	2	2	2

Optimal: 11 page faults (sixth acceptable answer).

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	3*	4*	4	4	4	4	3*	3	3	3	3	3	3	3	3	3	0*	0	0	0	0
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5*	5	5	4*	4	4
-	-	2*	2	2	2	2	2	2	2	5*	5	5	5	2*	2	2	2	2	2	2	2	2	2

Optimal: 11 page faults (seventh acceptable answer).

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	3*	4*	4	4	4	4	3*	3	3	3	3	3	3	3	3	3	3	3	4*	4	4
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5*	0*	0	0	0	0
-	-	2*	2	2	2	2	2	2	2	5*	5	5	5	2*	2	2	2	2	2	2	2	2	2

Optimal: 11 page faults (eighth acceptable answer).

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	3*	4*	4	4	4	4	3*	3	3	3	3	3	3	3	3	3	3	3	3	3	3
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5*	0*	0	4*	4	4
-	-	2*	2	2	2	2	2	2	2	5*	5	5	5	2*	2	2	2	2	2	2	2	2	2

(*c*) LRU: 15 page faults

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	3*	3	3	2*	2	2	3*	3	3	3	3	3	3	3	3	3	3	2*	2	2	2
-	1*	1	1	4*	4	4	4	4	4	4	1*	1	1	1	1	1	1	1	0*	0	0	0	0
-	-	2*	2	2	1*	1	1	1	1	5*	5	5	5	2*	2	2	2	5*	5	5	4*	4	4

Using 4 frames.

(*a*) FIFO: 11 page faults

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	0	4*	4	4	4	4	4	4	4	4	4	4	3*	3	3	3	3	3	3	3	3
-	1*	1	1	1	1	1	1	1	1	5*	5	5	5	5	5	5	5	5	0*	0	0	0	0
-	-	2*	2	2	2	2	2	2	2	2	1*	1	1	1	1	1	1	1	1	1	4*	4	4
-	-	-	3*	3	3	3	3	3	3	3	3	3	3	2*	2	2	2	2	2	2	2	2	2

(*b*) Optimal: 8 page faults (note that there could be 9 acceptable answers starting the request of page 0).

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	0	4*	4	4	4	4	4	5*	5	5	5	5	5	5	5	5	0*	0	4*	4	4
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
-	-	2*	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
-	-	-	3*	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3

Optimal: 8 page faults (acceptable answer 2).

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	0	4*	4	4	4	4	4	5*	5	5	5	5	5	5	5	5	0*	0	0	0	0
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4*	4	4
-	-	2*	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
-	-	-	3*	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3

Optimal: 8 page faults (acceptable answer 3).

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	0	4*	4	4	4	4	4	5*	5	5	5	5	5	5	5	5	0*	0	0	0	0
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
-	-	2*	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
-	-	-	3*	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4*	4	4

(*c*) LRU: 10 page faults

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	0	4*	4	4	4	4	4	4	4	4	4	2*	2	2	2	2	0*	0	0	0	0
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2*	2	2	2
-	-	2*	2	2	2	2	2	2	2	5*	5	5	5	5	5	5	5	5	5	5	5	5	5
-	-	-	3*	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4*	4	4

Using 5 frames.

(*a*) FIFO: 7 page faults

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	0	0	0	0	0	0	0	5*	5	5	5	5	5	5	5	5	5	5	5	5	5
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0*	0	0	0	0
-	-	2*	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
-	-	-	3*	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
-	-	-	-	4*	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4

(*b*) Optimal: 7 page faults (note that there could be 4 acceptable answers at the request of page 4)

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4*	4	4
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
-	-	2*	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
-	-	-	3*	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
-	-	-	-	4*	4	4	4	4	4	5*	5	5	5	5	5	5	5	5	5	5	5	5	5

(*c*) LRU: 8 page faults

0	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	0	0	0	0	0	0	0	5*	5	5	5	5	5	5	5	5	5	5	5	5	5
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4*	4	4
-	-	2*	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
-	-	-	3*	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
-	-	-	-	4*	4	4	4	4	4	4	4	4	4	4	4	4	4	4	0*	0	0	0	0

Observation: Providing *more frames* normally results in *fewer* page faults. The degree of improvement in the number of page faults is *not* proportional to increased number of frames, but depends on actual reference string.

4. Virtual Memory Page Replacement.

Using a slightly modified reference string.

(*a*) FIFO: **13** page faults (16 page faults in question 3)

0	1	2	1	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	0	4*	4	4	4	4	4	4	1*	1	1	1	1	1	1	5*	5	5	4*	4	4
-	1*	1	1	1	1	1	1	1	3*	3	3	3	3	2*	2	2	2	2	0*	0	0	0	0
-	-	2*	2	2	2	2	2	2	2	5*	5	5	5	5	3*	3	3	3	3	2*	2	2	2

(*b*) Optimal: **10** page faults (11 page faults in question 3)

0	1	2	1	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	0	4*	4	4	4	4	3*	3	3	3	3	3	3	3	3	5*	0*	0	4*	4	4
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
-	-	2*	2	2	2	2	2	2	2	5*	5	5	5	2*	2	2	2	2	2	2	2	2	2

(*c*) LRU: **12** page faults (15 page faults in question 3)

0	1	2	1	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	0	4*	4	4	4	4	4	4	1*	1	1	1	1	1	1	1	0*	0	0	0	0
-	1*	1	1	1	1	1	1	1	1	5*	5	5	5	2*	2	2	2	5*	5	5	4*	4	4
-	-	2*	2	2	2	2	2	2	3*	3	3	3	3	3	3	3	3	3	3	2*	2	2	2

Some interesting observations: The reference string is just one item different, but the number of page faults to both FIFO and LRU ***drastically decrases*** by 3! The number of page faults for Optimal is also improved. This is quite a lucky case. This situation is not common, but could occur sometimes.

Using yet another slightly modified reference string.

(*a*) FIFO: **18** page faults (16 page faults in question 3)

0	1	2	3	4	0	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	3*	3	3	2*	2	2	3*	3	3	3	3	2*	2	2	2	2	0*	0	0	0	0
-	1*	1	1	4*	4	4	1*	1	1	5*	5	5	5	5	3*	3	3	3	3	2*	2	2	2
-	-	2*	2	2	0*	0	0	4*	4	4	1*	1	1	1	1	1	1	5*	5	5	4*	4	4

(*b*) Optimal: **12** page faults (11 page faults in question 3)

0	1	2	3	4	0	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	0	0	0	0	1*	1	1	1	1	1	1	1	1	1	1	5*	0*	0	4*	4	4
-	1*	1	3*	4*	4	4	4	4	3*	3	3	3	3	3	3	3	3	3	3	3	3	3	3
-	-	2*	2	2	2	2	2	2	2	5*	5	5	5	2*	2	2	2	2	2	2	2	2	2

(*c*) LRU: **17** page faults (15 page faults in question 3)

0	1	2	3	4	0	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	3*	3	3	2*	2	2	3*	3	3	3	3	3	3	3	3	3	3	2*	2	2	2
-	1*	1	1	4*	4	4	1*	1	1	5*	5	5	5	2*	2	2	2	5*	5	5	4*	4	4
-	-	2*	2	2	0*	0	0	4*	4	4	1*	1	1	1	1	1	1	1	0*	0	0	0	0

Some interesting observations: The reference string is just one item different, but the number of page faults to both FIFO and LRU ***drastically increases*** by 2! The number of page faults for Optimal is also worsened. This is quite an unlucky case. This situation is also not common, but could occur sometimes.

5. Virtual Memory Page Replacement.

Using a slightly longer reference string.

(*a*) FIFO: **14** page faults (16 page faults in question 3)

0	3	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	2*	2	2	2	2	2	2	2	5*	5	5	5	5	3*	3	3	3	3	2*	2	2	2
-	3*	3	3	3	4*	4	4	4	4	4	4	1*	1	1	1	1	1	1	5*	5	5	4*	4	4
-	-	1*	1	1	1	1	1	1	1	3*	3	3	3	3	2*	2	2	2	2	0*	0	0	0	0

(*b*) Optimal: 11 page faults (11 page faults in question 3)

0	3	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	2*	2	2	2	2	2	2	2	5*	5	5	5	2*	2	2	2	2	2	2	2	2	2
-	3*	3	3	3	4*	4	4	4	4	3*	3	3	3	3	3	3	3	3	5*	0*	0	4*	4	4
-	-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

(*c*) LRU: 15 page faults (15 page faults in question 3)

0	3	1	2	3	4	1	2	1	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	2*	2	2	1*	1	1	1	1	5*	5	5	5	2*	2	2	2	5*	5	5	4*	4	4
-	3*	3	3	3	3	3	2*	2	2	3*	3	3	3	3	3	3	3	3	3	3	2*	2	2	2
-	-	1*	1	1	4*	4	4	4	4	4	4	1*	1	1	1	1	1	1	1	0*	0	0	0	0

Some interesting observation: The reference string is one item longer than the string in question 3. It is normally expected that the number of page faults would either ***stay the same***, or ***increase***. However, it is interesting to note that there are ***two fewer page faults*** for FIFO despite a ***longer*** reference string. This is due to the occurrence of a "lucky" history (in the form of the reference string) that makes predicting the future "better" with respect to the real page access pattern. Such a situation is not common, but could occur sometimes. Note that there may also be some other situations in which there are fewer page faults for LRU yet without an increase in the number of page faults for FIFO. However, such a situation of fewer page faults for a longer reference string would not occur in Optimal.

Using a slightly shorter reference string.

(*a*) FIFO: 11 page faults (11 page faults in question 3)

0	1	2	3	4	1	2	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	0	4*	4	4	4	4	4	4	4	4	4	3*	3	3	3	3	3	3	3	3
-	1*	1	1	1	1	1	1	1	5*	5	5	5	5	5	5	5	5	0*	0	0	0	0
-	-	2*	2	2	2	2	2	2	2	1*	1	1	1	1	1	1	1	1	1	4*	4	4
-	-	-	3*	3	3	3	3	3	3	3	3	3	2*	2	2	2	2	2	2	2	2	2

(*b*) Optimal: 8 page faults (8 page faults in question 3)

0	1	2	3	4	1	2	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	0	4*	4	4	4	4	5*	5	5	5	5	5	5	5	5	0*	0	4*	4	4
-	1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
-	-	2*	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
-	-	-	3*	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3

(*c*) LRU: **11** page faults (10 page faults in question 3)

0	1	2	3	4	1	2	4	3	5	1	3	1	2	3	1	3	5	0	2	4	2	4
0*	0	0	0	4*	4	4	4	4	4	4	4	4	2*	2	2	2	2	0*	0	0	0	0
-	1*	1	1	1	1	1	1	1	5*	5	5	5	5	5	5	5	5	5	5	5	5	5
-	-	2*	2	2	2	2	2	2	2	1*	1	1	1	1	1	1	1	1	2*	2	2	2
-	-	-	3*	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4*	4	4

Some interesting observation: The reference string is now one item shorter than the string in the previous question. It is normally expected that the number of page faults would either ***stay the same***, or ***decrease***. However, it is interesting to note that there is ***one more page fault*** with 4 frames for LRU despite a ***shorter*** reference string. This is due to the occurrence of an "unlucky" history (in the form of the reference string) that makes predicting the future "worse" with respect to the real page access pattern. Such a situation is not common, but could occur sometimes. However, such a situation of more page faults for a shorter reference string would not occur in Optimal.