Digital Signal Processing
Lab3
Convolution

Objective:

- To be able to perform convolution of two given signal using basic formula
- To be able to perform convolution of two given signals using Matlab function.

Functions Used:

Conv, *Sinc* etc.

Background:

<u>Convolution Sum:</u>

The output of any Linear Time Invariant (**LTI**) system is some sort of operation between input and system response; the operation is nothing but convolution, denoted by symbol '*', and defined as

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau).h(t-\tau)\,d\tau$$ ----For continuous time

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k].h[n-k]$$ ----For discrete time

For a causal LTI system, convolution sum is given by

$$y(n) = \sum_{k=0}^{n} x(k)h(n-k)$$

The process of computing the convolution between x(k) and h(k) involves the following four steps:

1. Folding : Fold h(k) about k=0 to obtain h(-k)
2. Shifting: Shift h(-k) by n_0 to the right (left if n_0 is positive (negative), to obtain h(n_0-k).
3. Multiplication: Multiply x(k) by h(n_0 –k) to obtain the product sequence V_{n0}(k)=x(k) h(n_0-k).
4. Summation: Sum all the values of the product sequence V_{n0}(k) to obtain the value of the output at times n=n_0.

Don't worry! You can use Matlab's built in function to calculate those.

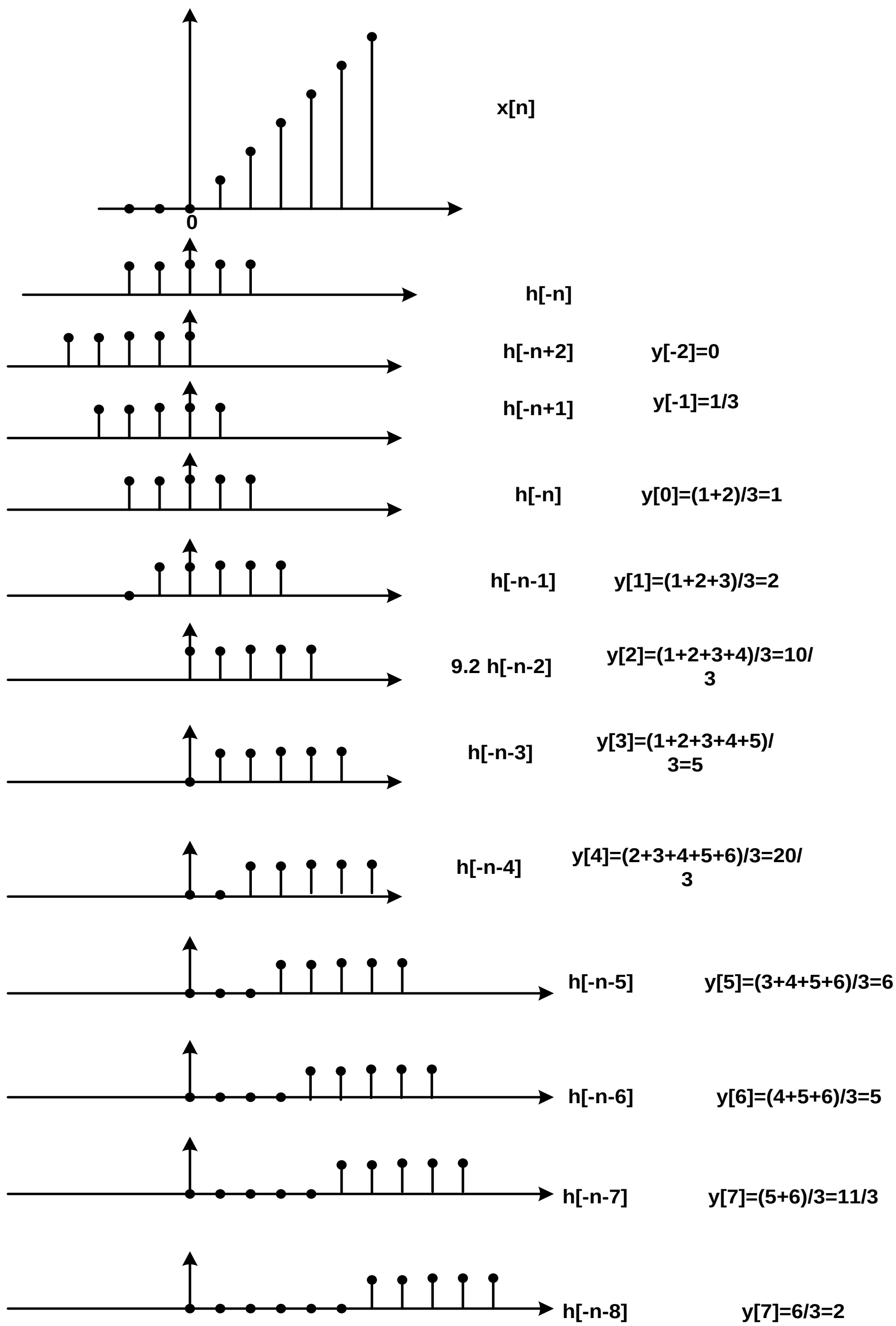
Illustration of convolution :

Convolution of signals x[n] and h[n] are obtained by two methods:

$$x[n]=\begin{cases}\frac{1}{3}n & for\, 0\le n\le 6\\ 0 & else\end{cases}$$ **and**

$$h[n]=\begin{cases}1 & for -2\le n\le 2\\ 0 & else\end{cases}$$

Graphical Method



	1	1	1	1	1
0	0	0	$\boxed{0}$	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6

y[n]={0,1,3,6,10,15,20,18,15,11,6}/3={0,1/3,1,2,10/3,5,20/3,6,5,11/3,2}

Tabular Method

References:

1. For the folding operation the function is to be formed as follows:

```
function [y,n]=sigfold(x,n)
y=fliplr(x); n=-fliplr(n);
```

2. For shifting operation

```
function[y,n]=sigshift_m(x,m,n0);
n=m+n0; y=x;
```

3. For Multiplication

```
function[y,n]=sigmulti(x1,n1,x2,n2);
n=min(min(n1),min(n2)):
max(max(n1),max(n2));

y1=zeros(1,length(n));
y2=y1;
y1(find((n>=min(n1))&(n<=max(n1))==1))=x1;
y2(find((n>=min(n2))&(n<=max(n2))==1))=x2;
y=y1.*y2;
```

4. Using Conv() function

```
x=[1,0,-1,1,2,1]
n1=[-2,-1,0,1,2,3]
nx=length(x)
h=[1,1,1,1,1]
n2=[0,1,2,3,4]
nh=length(h)
y=conv(x,h)
n=-2:1:-7
% nmin = min(min(n1),min(n2))
% n= nmin:1:nx+nh-2+ nmin
stem(n,y)
```

Problems

1. Find the convolution result of the following signal using basic convolution formula :

   $X_1(n1) = [1,1,1,1,1]$
   n1= [-2,-1,0,1,2]

   X2(n2) = [1,0,0,0,0,0,0,0,0,0]
   n2 = [-4,-3,-2,-1,0,1,2,3,4,5]
   X2 is a periodic signal.
   Y2= X1*X2

2. Find the convolution of using **conv** function

   a. $x[n] = \begin{cases} \frac{1}{3}n & for\ 0 \le n \le 6 \\ 0 & else \end{cases}$ **and**

   $h[n] = \begin{cases} 1 & for\ -2 \le n \le 2 \\ 0 & else \end{cases}$

   b. *x(t)= u(t)*
   $h(t)= e^{-at} u(t)$, where $a>0$

3. Consider two discrete time sequences **x[n]** and **h[n]** given by

   x[n] = 1 for $0 \le n \le 4$, elsewhere 0
   h[n] = 2^n for $0 \le n \le 6$, elsewhere 0

   a. Find the response of the LTI system with impulse response **h[n]** to input **x[n]**.
   b. Plot the signals and comment on the result.

4. If the impulse response of a LTI system is given by **sinc** function as

   $h[n] = 2\tau/T_p \ \mathrm{sinc}(k\ 2\tau/T_p)$

   and input signal is a rectangular wave given by

   x(t) = 1 for $1 \le t \le 100$

   0 elsewhere,

   Find output of the system for different values of τ. Comment on the result.