Digital Signal Processing
Lab6

Design of FIR Digital Filters using Window method

Background:

In previous experiment we discussed the most commonly used techniques for design of IIR filters based on transformations of continuous-time IIR systems into discrete time IIR systems. The major difficulty lies in the implementation of the non-iterative direct design method for IIR filters. However FIR filters are almost entirely restricted to discrete-time implementations. Thus the design techniques for FIR filters are based on directly approximating the desired frequency response of the discrete time system. Furthermore, most techniques for approximating the magnitude response of the FIR system assume a linear phase constraint; thereby avoiding the problem of spectrum factorization that complicates the direct design of IIR filters.

The simplest method of FIR design is called the window method. This method generally begins with an ideal desired frequency response $H_d(w)$. The impulse response $h_d(n)$ of the filter exhibiting this desired frequency response can be obtained from inverse Fourier transform of $H_d(w)$. However this impulse response exists for $n = -\infty$ to $+\infty$ and hence truncation is needed to make the finite duration impulse response. The truncation is similar to the multiplication of the $h_d(n)$ with the window function $w(n)$. The multiplication in discrete time domain is equivalent to the convolution of the two in frequency domain, which actually gives the frequency response of the truncated FIR filter.

But depending on the tapering of the window to zero at each end, the nature of the window differs. For example, the rectangular window exhibits the most abrupt changes while approaching to zero at each end. For the specific value of length of the window, the rectangular window exhibits main lobe with the greatest width and the lowest side lobe attenuation, than the other windows like Bartlett, Hanning, Hamming, Blackman etc. There are also adjustable windows like Kaiser windows whose windowing function w(n) can be adjusted by changing the value of parameter β according to the stop band attenuation A as given below:

$$\beta = \begin{cases} 0.1102(A-8.7), & \text{if } A>50, \\ 0.5842(A-21)^{0.4}+0.07886(A-21), & \text{if } 21 \le A \le 50, \\ 0.0, & \text{if } A<21, \end{cases}$$

where $A = -20\log_{10} \delta$.

The length of the Kaiser window is given by;

$$M = ((\delta-7.95)/(14.36*\Delta w)) +1, \text{ where } \Delta w = (w_s - w_p)/2$$

MATLAB Signal Processing Toolbox provides built-in functions to determine and plot the different types of windowing functions and their frequency responses. For this the functions like Bartlett( ), Hamming( ), Hanning( ), Blackman( ) , Kaiser( ) etc. are available. There is a function fir1( ) that can be used to obtain the frequency response of the designed FIR filter. Use MATLAB ‘Help’ for further information of the functions.

Problems:

1. Design an FIR linear phase digital filter approximating the ideal frequency response

$$H(w) = \begin{cases} 1, & \text{for } |w| \le \pi/6 \\ 0, & \text{for } \pi/6 < |w| \le \pi \end{cases}$$

   a. Plot the window function for Hamming and its frequency response for length of M=31.

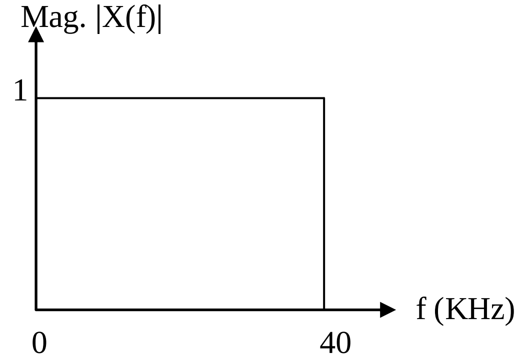
   b. Using the Hamming window plot the frequency response of the truncated FIR filter.

   c. Repeat parts (a) and (b) for the Hanning, Blackman and Bartlett windows.

   d. Repeat parts (a), (b) and (d) for filter length of M=61.

2. Discuss the effects of different types of the windowing functions on the frequency response of the FIR filter. Carry out the comparative study on the basis of the peak side-lobe level, the approximate transition width of the main lobe etc. Also discuss on the effects of increasing value of M.

3. An analog signal x(t) consists of the sum of two components $x_1(t)$ and $x_2(t)$. The spectral characteristics of x(t) is shown in fig 2.1. The signal x(t) is band limited to 40kHz and is sampled at the rate of 100kHz to yield the sequence x(n) .



It is desired to suppress the signal $x_2(t)$ by passing the sequence x(n) through a digital low pass filter. The allowable distortion on $|X_1(f)|$ is ±2% (δ_1=0.02) over the range $0 \le |F| \le 15$ kHz. Above 20 kHz, the filter must have an attenuation of at least 40 dB (δ_2=0.01).

   a. For obtaining the filter with above specifications, use the Kaiser window and determine the length of the required window. Plot the frequency response of the filter and its impulse response also.

   b. If the same filter is to be designed using Hamming window what would be the length of the required window. Plot the frequency and impulse response of the filter.