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1 Contest Setup

1.1 vimrc

```

1 set number      " Show line numbers
2 set mouse=a     " Enable inaction via mouse
3 set showmatch   " Highlight matching brace
4 set cursorline  " Show underline
5 set cursorcolumn " highlight vertical column
6
7 filetype on "enable file detection
8 syntax on    "syntax highlight
9
10 set autoindent    " Auto-indent new lines
11 set shiftwidth=4  " Number of auto-indent spaces
12 set smartindent  " Enable smart-indent
13 set smarttab     " Enable smart-tabs
14 set tabstop=4    " Number of spaces per Tab
15
16 " -----Optional-----
17
18 set undolevels=10000 " Number of undo levels
19 set scrolloff=5     " Auto scroll
20
21 set hlsearch      " Highlight all search results
22 set smartcase     " Enable smart-case search
23 set ignorecase    " Always case-insensitive
24 set incsearch     " Searches for strings incrementally
25
26 highlight Comment ctermfg=cyan
27 set showmode
28
29 set encoding=utf-8
30 set fileencoding=utf-8
31 set scriptencoding=utf-8

```

1.2 bashrc

```

1 alias g++="g++ -Wall -Wextra -std=c++11 -O2"

```

1.3 Grep Error and Warnings

```

1 g++ main.cpp 2>&1 | grep -E 'warning|error'

```

1.4 C++ template

```

1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 typedef long long int ll;
6 typedef pair<int, int> ii;
7

```

```

8 int main()
9 {
10     return 0;
11 }

```

1.5 Java template

```

1 import java.io.*;
2 import java.util.*;
3
4 public class Main
5 {
6     public static void main(String[] args)
7     {
8         MyScanner sc = new MyScanner();
9         out = new PrintWriter(new BufferedOutputStream(System.out));
10        // Start writing your solution here.
11
12        // Stop writing your solution here.
13        out.close();
14    }
15
16    public static PrintWriter out;
17
18    public static class MyScanner
19    {
20        BufferedReader br;
21        StringTokenizer st;
22
23        public MyScanner()
24        {
25            br = new BufferedReader(new InputStreamReader(System.in));
26        }
27
28        boolean hasNext()
29        {
30            while (st == null || !st.hasMoreElements()) {
31                try {
32                    st = new StringTokenizer(br.readLine());
33                } catch (Exception e) {
34                    return false;
35                }
36            }
37            return true;
38        }
39
40        String next()
41        {
42            if (hasNext())
43                return st.nextToken();
44            return null;
45        }
46
47        int nextInt()
48        {

```

```

49     return Integer.parseInt(next());
50 }
51
52 long nextLong()
53 {
54     return Long.parseLong(next());
55 }
56
57 double nextDouble()
58 {
59     return Double.parseDouble(next());
60 }
61
62 String nextLine()
63 {
64     String str = "";
65     try {
66         str = br.readLine();
67     } catch (IOException e) {
68         e.printStackTrace();
69     }
70     return str;
71 }
72 }
73 }

```

1.5.1 Java Issues

1. Random Shuffle before sorting: `Random rnd = new Random(); rnd.nextInt();`
2. Use StringBuilder for large output

2 System Testing

1. Setup bashrc and vimrc
2. Look for compilation parameter and code it into bashrc
3. Test if c++ and java templates work properly on local and judge machine
4. Test "divide by 0" → RE/TLE?
5. Test stack size

3 Reminder

1. 隊友的建議，要認真聽！通常隊友的建議都會突破你盲點
2. Read the problem statements carefully. Input and output specifications and constraints are crucial!
3. Estimate the **time complexity** and **memory complexity** carefully.
4. Time penalty is 20 minutes per WA, **don't rush!**
5. Sample test cases must all be tested and passed before every submission!
6. Test the corner cases, such as 0, 1, -1. Test all edge cases of the input specification.
7. Bus error: the code has `scanf, fgets` but have nothing to read! Check if you have early termination but didn't handle it properly.
8. Binary search? 數學算式移項合併後查詢?
9. Two Pointer <-> Binary Search
10. Directed graph connectivity -> DFS. Undirected graph -> Union Find
11. Check connectivity of the graph if the problem statement doesn't say anything
12. `longlong = int * int` won't work!
13. Shifting for `longlongint` should be something like `1LL << 35`
14. For continuous input problems, be sure to read in all input BEFORE terminating and start processing next the input.
15. Don't use anonymous struct

16. 因式分解

4 Topic list

1. enumeration
2. greedy
3. sorting, topological sort
4. binary search

5 Useful code

5.1 Leap year

```
1 || year % 400 == 0 || (year % 4 == 0 && year % 100 != 0)
```

5.2 Fast Exponentiation $O(\log(\exp))$

```

1 ll fast_pow(ll base, ll exp, ll mod)
2 {
3     if (exp == 0)
4         return 1LL;
5     ll res = 1;
6     while (exp > 0) {
7         if (exp & 1) {
8             res = ((res % mod) * (base % mod)) % mod;
9         }
10        exp >>= 1;
11        base = (base * base) % mod;
12    }
13    return res;
14 }

```

5.3 GCD $O(\log(a + b))$

注意負數的 case! C++ 是看被除數決定正負號的。

```

1 ll gcd(ll a, ll b)
2 {
3     return b == 0 ? a : gcd(b, a % b);
4 }

```

5.4 Extended Euclidean Algorithm

Bezout identity $ax + by = \gcd(a, b)$, where $\gcd(a, b)$ is the smallest positive integer that can be written as $ax + by$, and every integer of the form $ax + by$ is a multiple of $\gcd(a, b)$.

```

1 ll ext_gcd(ll a, ll b, ll &x, ll &y)
2 {
3     if (a == 0) {
4         x = 0;
5         y = 1;
6         return b;
7     }
8 }

```

```

9  ll x1, y1;
10 ll gcd = ext_gcd(b % a, a, x1, y1);
11
12 x = y1 - (b / a) * x1;
13 y = x1;
14
15 return gcd;
16 }

```

5.5 Mod Inverse

Case 1 $\gcd(a, m) = 1$: $ax + my = \gcd(a, m) = 1$ (use `ext_gcd`)

Case 2 m is prime: $a^{m-2} \equiv a^{-1} \pmod m$ (use Fermat's little theorem)

5.6 Prime Generator

```

1 bool is_prime[N];
2 vector<ll> primes;
3 void init()
4 {
5     fill(is_prime, is_prime + N, true);
6     for (int i = 2; i < N; i++) {
7         if (is_prime[i] == true) {
8             primes.push_back(i);
9             for (int j = i * i; j < N; j += i)
10                 is_prime[j] = false;
11         }
12     }
13 }

```

5.7 Binomial Coefficient

Add recursive version

```

1 int binomialCoeff(int n, int k)
2 {
3     int res = 1;
4
5     if ( k > n - k ) // Since C(n, k) = C(n, n-k)
6         k = n - k;
7
8     for (int i = 0; i < k; ++i) // n...n-k / 1...k
9     {
10         res *= (n - i);
11         res /= (i + 1);
12     }
13
14     return res;
15 }

```

5.8 scanf/printf reference

5.9 STL quick reference

5.9.1 Map

```

1 map<T1, T2> m; // iterable
2 void clear();
3 void erase(T1 key);
4 it find(T1 key); // <key, val>
5 void insert(pair<T1, T2> P);
6 T2 &[](T1 key); // if key not in map, new key will be inserted with
   default val
7 it lower_bound(T1 key); // = m.end() if not found, *it = <key, val>
8 it upper_bound(T1 key); // = m.end() if not found, *it = <key, val>

```

5.9.2 Set

```

1 set<T> s; // iterable
2 void clear();
3 size_t count(T val); // number of val in set
4 void erase(T val);
5 it find(T val); // = s.end() if not found
6 void insert(T val);
7 it lower_bound(T val); // = s.end() if not found, *it = <key, val>
8 it upper_bound(T val); // = s.end() if not found, *it = <key, val>

```

5.9.3 Algorithm

```

1 // return if i is smaller than j
2 comp = [&](const T &i, const T &j) -> bool;
3 vector<T> v;
4 bool any_of(v.begin(), v.end(), [&](const T &i) -> bool);
5 bool all_of(v.begin(), v.end(), [&](const T &i) -> bool);
6 void copy(inp.begin(), inp.end(), out.begin());
7 int count(v.begin(), v.end(), int val); // number of val in v
8 it unique(v.begin(), v.end()); // it - v.begin() = size
9 // after calling, v[nth] will be n-th smallest elem in v
10 void nth_element(v.begin(), nth_it, bin_comp);
11 void merge(in1.begin(), in1.end(), in2.begin(), in2.end(), out.begin(),
   comp);
12 // include union, intersection, difference, symmetric_difference(xor)
13 void set_union(in1.begin(), in1.end(), in2.begin(), in2.end(), out.
   begin(), comp);
14 bool next_permutation(v.begin(), v.end());
15 // v1, v2 need sorted already, whether v1 includes v2
16 bool inclues(v1.begin(), v1.end(), v2.begin(), v2.end());
17 it find(v.begin(), v.end(), T val); // = v.end() if not found
18 it search(v1.begin(), v1.end(), v2.begin(), v2.end());
19 it lower_bound(v.begin(), v.end(), T val);
20 it upper_bound(v.begin(), v.end(), T val);
21 bool binary_search(v.begin(), v.end(), T val); // exist in v ?
22 void sort(v.begin(), v.end(), comp);
23 void stable_sort(v.begin(), v.end(), comp);

```

5.9.4 String

5.9.5 Priority Queue

```

1 bool cmp(ii a, ii b)
2 {
3     if(a.first == b.first)
4         return a.second > b.second;
5     return b.first > a.first;
6 }
7
8 priority_queue< ii, vector<ii>, function<bool(ii, ii)> > pq(cmp);

```

6 Search

6.1 Ternary Search

6.2 折半完全列舉

6.3 Two-pointer 爬行法

7 Basic data structure

7.1 1D BIT

```

1 // BIT is 1-based
2 const int MAX_N = 20000; //這個記得改!
3 ll bit[MAX_N + 1];
4
5 int sum(int i) {
6     int s = 0;
7     while (i > 0) {
8         s += bit[i];
9         i -= (i & -i);
10    }
11    return s;
12 }
13
14 void add(int i, int x) {
15     while (i <= MAX_N) {
16         bit[i] += x;
17         i += (i & -i);
18    }
19 }

```

7.2 2D BIT

```

1 // BIT is 1-based
2 const int MAX_N = 20000, MAX_M = 20000; //這個記得改!
3 ll bit[MAX_N + 1][MAX_M + 1];
4
5 ll sum(int a, int b) {
6     ll s = 0;

```

```

7     for (int i = a; i > 0; i -= (i & -i))
8         for (int j = b; j > 0; j -= (j & -j))
9             s += bit[i][j];
10    return s;
11 }
12
13 void add(int a, int b, ll x) {
14     // MAX_N, MAX_M 須適時調整!
15     for (int i = a; i <= MAX_N; i += (i & -i))
16         for (int j = b; j <= MAX_M; j += (j & -j))
17             bit[i][j] += x;
18 }

```

7.3 Union Find

```

1 #define N 20000 // 記得改
2 struct UFDS {
3     int par[N];
4
5     void init() {
6         memset(par, -1, sizeof(par));
7     }
8
9     int root(int x) {
10        return par[x] < 0 ? x : par[x] = root(par[x]);
11    }
12
13    void merge(int x, int y) {
14        x = root(x);
15        y = root(y);
16
17        if (x != y) {
18            if (par[x] > par[y])
19                swap(x, y);
20            par[x] += par[y];
21            par[y] = x;
22        }
23    }
24 }

```

7.4 Segment Tree

7.5 Sparse Table

```

1 struct {
2     int sp[MAX_LOG_N][MAX_N]; // MAX_LOG_N = ceil(lg(MAX_N))
3
4     void build(int inp[], int n) {
5         for (int j = 0; j < n; j++) {
6             sp[0][j] = inp[j];
7         }
8
9         for (int i = 1; (1 << i) <= n; i++)
10            for (int j = 0; j + (1 << i) <= n; j++)

```

```

11         sp[i][j] =
12             min(sp[i - 1][j], sp[i - 1][j + (1 << (i - 1))]);
13     }
14
15     int query(int l, int r) { // [l, r)
16         int k = floor(log2(r - l));
17
18         return min(sp[k][l], sp[k][r - (1 << k)]);
19     }
20 } sptb;

```

8 Dynamic Programming

9 Tree

9.1 LCA

9.2 Tree Centroid

9.3 Treap

10 Graph

10.1 Articulation point / edge

10.2 CC

10.2.1 BCC vertex

10.2.2 BCC edge

10.2.3 SCC

10.3 Shortest Path

10.3.1 Dijkstra

10.3.2 Dijkstra (next-to-shortest path)

10.3.3 SPFA

10.3.4 Bellman-Ford

10.3.5 Floyd-Warshall

10.4 MST

10.4.1 Kruskal

1. Store the graph by *(weight, (from, to))*
2. Sort the graph by *weight*
3. Start from the smallest weight, and keep adding edges that won't form a cycle with the current MST set
4. Early termination condition: $n - 1$ edges has been added, NOT size of the union-find set

10.4.2 Prim

10.5 Flow

10.5.1 Max Flow (Dinic)

10.5.2 Min-Cut

10.5.3 Min Cost Max Flow

10.5.4 Maximum Bipartite Graph

11 String

11.1 Rolling Hash

1. Use two rolling hashes if needed.
2. The prime for pre-calculation can be 137 and 257, for modulo can be $1e9 + 7$ and $0xdefaced$

```

1  #define N 1000100
2  #define B 137
3  #define M 1000000007
4
5  typedef long long ll;
6
7  char inp[N];
8  int len;
9  ll p[N], h[N];
10
11 void init()
12 { // build polynomial table and hash value
13     p[0] = 1; // b to the ith power
14     for (int i = 1; i <= len; i++) {
15         h[i] = (h[i - 1] * B % M + inp[i - 1]) % M; // hash value
16         p[i] = p[i - 1] * B % M;
17     }
18 }
19
20 ll get_hash(int l, int r) // [l, r) of the inp string array
21 {
22     return ((h[r + 1] - (h[l] * p[r - l + 1])) % M + M) % M;
23 }

```

11.2 KMP

```

1 void fail()
2 {
3     int len = strlen(pat);
4
5     f[0] = 0;
6     int j = 0;
7     for (int i = 1; i < len; i++) {
8         while (j != 0 && pat[i] != pat[j])
9             j = f[j - 1];
10
11         if (pat[i] == pat[j])
12             j++;

```

```

13     f[i] = j;
14 }
15 }
16 }
17
18 int match()
19 {
20     int res = 0;
21     int j = 0, plen = strlen(pat), tlen = strlen(text);
22
23     for (int i = 0; i < tlen; i++) {
24         while (j != 0 && text[i] != pat[j])
25             j = f[j - 1];
26
27         if (text[i] == pat[j]) {
28             if (j == plen - 1) { // find match
29                 res++;
30                 j = f[j];
31             } else {
32                 j++;
33             }
34         }
35     }
36
37     return res;
38 }

```

11.3 Z Algorithm

11.4 Trie

```

1 #define N 600010
2 struct node {
3     int child[26];
4     bool ending;
5 } trie[N];
6
7 /*
8 root is 0
9 memset(trie, 0, sizeof(trie));
10 freeNode = 1;
11 */
12 int freeNode;
13 void insert(string &str, int pos, int node)
14 {
15     if (pos == (int)str.length()) {
16         trie[node].ending = true;
17     } else { // find which way to go
18         int c = str[pos] - 'a';
19         if (trie[node].child[c] == 0) // give a new node
20             trie[node].child[c] = freeNode++;
21         insert(str, pos + 1, trie[node].child[c]);
22     }
23 }

```

11.5 Suffix Array

12 Geometry

1. Keep things in integers as much as possible!
2. Try not to divide
3. If you have decimals, if they are fixed precision, you can usually just multiply all the input and use integers instead

12.1 EPS

$a > b \rightarrow a - b > 0 \rightarrow a - b > EPS$ (stands for positive)
 $a \geq b \rightarrow a - b \geq 0 \rightarrow a - b > -EPS$ (stands for positive or zero)

12.2 Template

```

1 // if the points are given in doubles form, change the code accordingly
2
3 typedef long long ll;
4
5 typedef pair<ll, ll> pt; // points are stored using long long
6 typedef pair<pt, pt> seg; // segments are a pair of points
7
8 #define x first
9 #define y second
10
11 #define EPS 1e-9
12
13 pt operator+(pt a, pt b)
14 {
15     return pt(a.x + b.x, a.y + b.y);
16 }
17
18 pt operator-(pt a, pt b)
19 {
20     return pt(a.x - b.x, a.y - b.y);
21 }
22
23 pt operator*(pt a, int d)
24 {
25     return pt(a.x * d, a.y * d);
26 }
27
28 ll cross(pt a, pt b)
29 {
30     return a.x * b.y - a.y * b.x;
31 }
32
33 int ccw(pt a, pt b, pt c)
34 {
35     ll res = cross(b - a, c - a);
36     if (res > 0) // left turn
37         return 1;
38     else if (res == 0) // straight
39         return 0;
40     else // right turn

```

```

41     return -1;
42 }
43
44 double dist(pt a, pt b)
45 {
46     double dx = a.x - b.x;
47     double dy = a.y - b.y;
48     return sqrt(dx * dx + dy * dy);
49 }
50
51 bool zero(double x)
52 {
53     return fabs(x) <= EPS;
54 }
55
56 bool overlap(seg a, seg b)
57 {
58     return ccw(a.x, a.y, b.x) == 0 && ccw(a.x, a.y, b.y) == 0;
59 }
60
61 bool intersect(seg a, seg b)
62 {
63     if (overlap(a, b) == true) { // non-proper intersection
64         double d = 0;
65         d = max(d, dist(a.x, a.y));
66         d = max(d, dist(a.x, b.x));
67         d = max(d, dist(a.x, b.y));
68         d = max(d, dist(a.y, b.x));
69         d = max(d, dist(a.y, b.y));
70         d = max(d, dist(b.x, b.y));
71
72         // d > dist(a.x, a.y) + dist(b.x, b.y)
73         if (d - (dist(a.x, a.y) + dist(b.x, b.y)) > EPS)
74             return false;
75         return true;
76     }
77     // Equal sign for -----| case
78     // non equal sign => proper intersection
79     if (ccw(a.x, a.y, b.x) * ccw(a.x, a.y, b.y) <= 0 &&
80         ccw(b.x, b.y, a.x) * ccw(b.x, b.y, a.y) <= 0)
81         return true;
82     return false;
83 }
84
85 double area(vector<pt> pts)
86 {
87     double res = 0;
88     int n = pts.size();
89     for (int i = 0; i < n; i++)
90         res += (pts[i].y + pts[(i + 1) % n].y) * (pts[(i + 1) % n].x -
91             pts[i].x);
92     return res / 2.0;
93 }
94
95 vector<pt> halfHull(vector<pt> &points)

```

```

96 {
97     vector<pt> res;
98
99     for (int i = 0; i < (int)points.size(); i++) {
100         while ((int)res.size() >= 2 &&
101             ccw(res[res.size() - 2], res[res.size() - 1], points[i])
102             < 0)
103             res.pop_back(); // res.size() - 2 can't be assign before
104                             // size() >= 2
105                             // check, bitch
106         res.push_back(points[i]);
107     }
108     return res;
109 }
110
111 vector<pt> convexHull(vector<pt> &points)
112 {
113     vector<pt> upper, lower;
114
115     // make upper hull
116     sort(points.begin(), points.end());
117
118     upper = halfHull(points);
119     // make lower hull
120     reverse(points.begin(), points.end());
121     lower = halfHull(points);
122
123     // merge hulls
124     if ((int)upper.size() > 0) // yes sir~
125         upper.pop_back();
126     if ((int)lower.size() > 0)
127         lower.pop_back();
128
129     vector<pt> res(upper.begin(), upper.end());
130     res.insert(res.end(), lower.begin(), lower.end());
131
132     return res;
133 }
134
135 bool completelyInside(vector<pt> &outer, vector<pt> &inner)
136 {
137     int even = 0, odd = 0;
138     for (int i = 0; i < (int)inner.size(); i++) {
139         // y = slope * x + offset
140         int cntIntersection = 0;
141         ll slope = rand() % INT_MAX + 1;
142         ll offset = inner[i].y - slope * inner[i].x;
143
144         ll farx = 111111 * (slope >= 0 ? 1 : -1);
145         ll fary = farx * slope + offset;
146         seg a = seg(pt(inner[i].x, inner[i].y), pt(farx, fary));
147         for (int j = 0; j < (int)outer.size(); j++) {
148             seg b = seg(outer[j], outer[(j + 1) % (int)outer.size()]);
149

```



```

150         if ((b.x.x * slope + offset == b.x.y) ||
151             (b.y.x * slope + offset == b.y.y)) { // on-line
152             i--;
153             break;
154         }
155
156         if (intersect(a, b) == true)
157             cntIntersection++;
158     }
159
160     if (cntIntersection % 2 == 0) // outside
161         even++;
162     else
163         odd++;
164 }
165
166 return odd == (int)inner.size();
167 }
168
169 // srand(time(NULL))
170 // rand()

```

13 Math

13.1 Euclid's formula (Pythagorean Triples)

$$\begin{aligned}
 a &= p^2 - q^2 \\
 b &= 2pq \text{ (always even)} \\
 c &= p^2 + q^2
 \end{aligned}$$

13.2 Difference between two consecutive numbers' square is odd

$$(k+1)^2 - k^2 = 2k + 1$$

13.3 Summation

$$\begin{aligned}
 \sum_{k=1}^n 1 &= n \\
 \sum_{k=1}^n k &= \frac{n(n+1)}{2} \\
 \sum_{k=1}^n k^2 &= \frac{n(n+1)(2n+1)}{6} \\
 \sum_{k=1}^n k^3 &= \frac{n^2(n+1)^2}{4}
 \end{aligned}$$

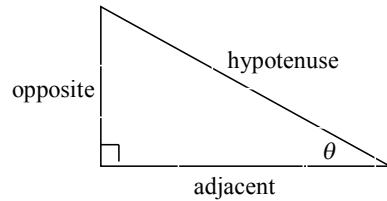
Trig Cheat Sheet

Definition of the Trig Functions

Right triangle definition

For this definition we assume that

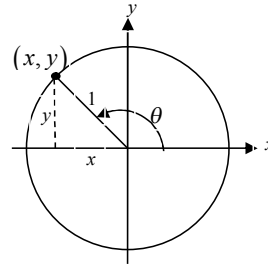
$$0 < \theta < \frac{\pi}{2} \text{ or } 0^\circ < \theta < 90^\circ.$$



$$\begin{aligned} \sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} & \csc \theta &= \frac{\text{hypotenuse}}{\text{opposite}} \\ \cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} & \sec \theta &= \frac{\text{hypotenuse}}{\text{adjacent}} \\ \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} & \cot \theta &= \frac{\text{adjacent}}{\text{opposite}} \end{aligned}$$

Unit circle definition

For this definition θ is any angle.



$$\begin{aligned} \sin \theta &= \frac{y}{1} = y & \csc \theta &= \frac{1}{y} \\ \cos \theta &= \frac{x}{1} = x & \sec \theta &= \frac{1}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y} \end{aligned}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$$\begin{aligned} \sin \theta, \quad \theta &\text{ can be any angle} \\ \cos \theta, \quad \theta &\text{ can be any angle} \\ \tan \theta, \quad \theta &\neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \csc \theta, \quad \theta &\neq n\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \sec \theta, \quad \theta &\neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \cot \theta, \quad \theta &\neq n\pi, \quad n = 0, \pm 1, \pm 2, \dots \end{aligned}$$

Range

The range is all possible values to get out of the function.

$$\begin{aligned} -1 &\leq \sin \theta \leq 1 & \csc \theta &\geq 1 \text{ and } \csc \theta \leq -1 \\ -1 &\leq \cos \theta \leq 1 & \sec \theta &\geq 1 \text{ and } \sec \theta \leq -1 \\ -\infty &< \tan \theta < \infty & -\infty &< \cot \theta < \infty \end{aligned}$$

Period

The period of a function is the number, T , such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\begin{aligned} \sin(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \cos(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \tan(\omega\theta) &\rightarrow T = \frac{\pi}{\omega} \\ \csc(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \sec(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \cot(\omega\theta) &\rightarrow T = \frac{\pi}{\omega} \end{aligned}$$

Formulas and Identities

Tangent and Cotangent Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Reciprocal Identities

$$\begin{aligned} \csc \theta &= \frac{1}{\sin \theta} & \sin \theta &= \frac{1}{\csc \theta} \\ \sec \theta &= \frac{1}{\cos \theta} & \cos \theta &= \frac{1}{\sec \theta} \\ \cot \theta &= \frac{1}{\tan \theta} & \tan \theta &= \frac{1}{\cot \theta} \end{aligned}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin \theta \quad \csc(-\theta) = -\csc \theta$$

$$\cos(-\theta) = \cos \theta \quad \sec(-\theta) = \sec \theta$$

$$\tan(-\theta) = -\tan \theta \quad \cot(-\theta) = -\cot \theta$$

Periodic Formulas

If n is an integer.

$$\sin(\theta + 2\pi n) = \sin \theta \quad \csc(\theta + 2\pi n) = \csc \theta$$

$$\cos(\theta + 2\pi n) = \cos \theta \quad \sec(\theta + 2\pi n) = \sec \theta$$

$$\tan(\theta + \pi n) = \tan \theta \quad \cot(\theta + \pi n) = \cot \theta$$

Double Angle Formulas

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas (alternate form)

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}} \quad \sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}} \quad \cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} \quad \tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

Product to Sum Formulas

$$\sin \alpha \sin \beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

Sum to Product Formulas

$$\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$$

$$\sin \alpha - \sin \beta = 2 \cos \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right)$$

$$\cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$$

$$\cos \alpha - \cos \beta = -2 \sin \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right)$$

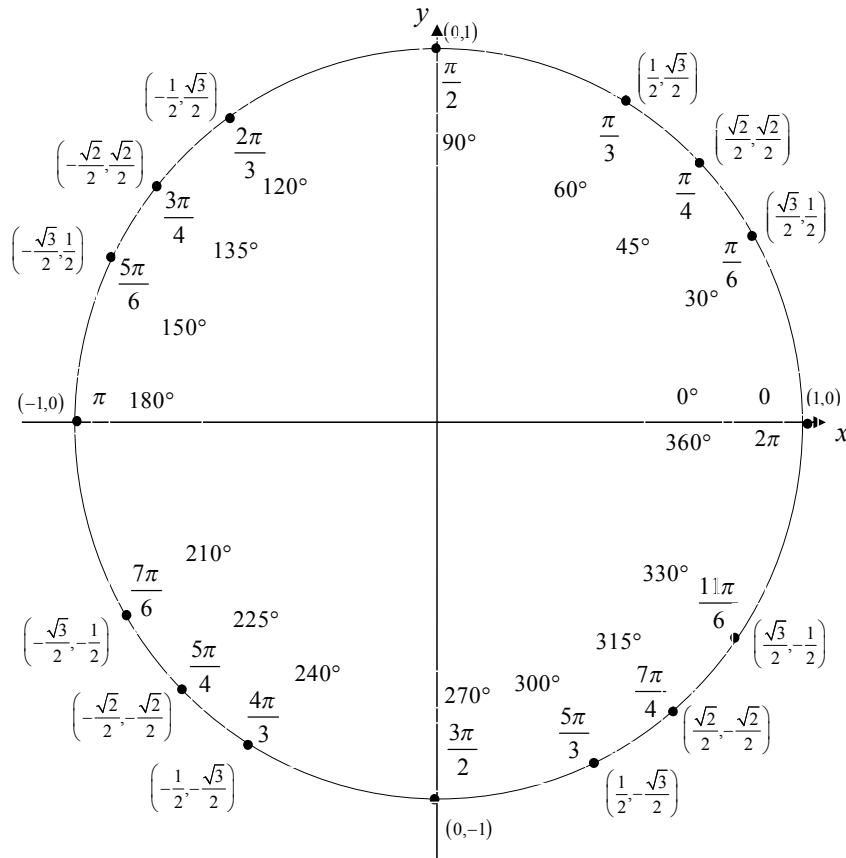
Cofunction Formulas

$$\sin \left(\frac{\pi}{2} - \theta \right) = \cos \theta \quad \cos \left(\frac{\pi}{2} - \theta \right) = \sin \theta$$

$$\csc \left(\frac{\pi}{2} - \theta \right) = \sec \theta \quad \sec \left(\frac{\pi}{2} - \theta \right) = \csc \theta$$

$$\tan \left(\frac{\pi}{2} - \theta \right) = \cot \theta \quad \cot \left(\frac{\pi}{2} - \theta \right) = \tan \theta$$

Unit Circle



For any ordered pair on the unit circle (x, y) : $\cos \theta = x$ and $\sin \theta = y$

Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \quad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Inverse Trig Functions

Definition

$y = \sin^{-1} x$ is equivalent to $x = \sin y$

$y = \cos^{-1} x$ is equivalent to $x = \cos y$

$y = \tan^{-1} x$ is equivalent to $x = \tan y$

Inverse Properties

$$\cos(\cos^{-1}(x)) = x \quad \cos^{-1}(\cos(\theta)) = \theta$$

$$\sin(\sin^{-1}(x)) = x \quad \sin^{-1}(\sin(\theta)) = \theta$$

$$\tan(\tan^{-1}(x)) = x \quad \tan^{-1}(\tan(\theta)) = \theta$$

Domain and Range

Function	Domain	Range
$y = \sin^{-1} x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

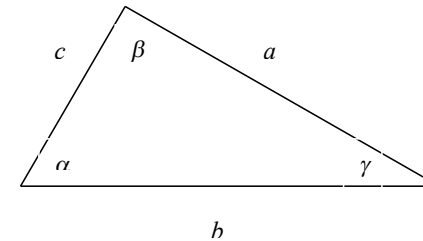
Alternate Notation

$$\sin^{-1} x = \arcsin x$$

$$\cos^{-1} x = \arccos x$$

$$\tan^{-1} x = \arctan x$$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos \frac{1}{2}(\alpha - \beta)}{\sin \frac{1}{2}\gamma}$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(\alpha - \beta)}{\tan \frac{1}{2}(\alpha + \beta)}$$

$$\frac{b-c}{b+c} = \frac{\tan \frac{1}{2}(\beta - \gamma)}{\tan \frac{1}{2}(\beta + \gamma)}$$

$$\frac{a-c}{a+c} = \frac{\tan \frac{1}{2}(\alpha - \gamma)}{\tan \frac{1}{2}(\alpha + \gamma)}$$