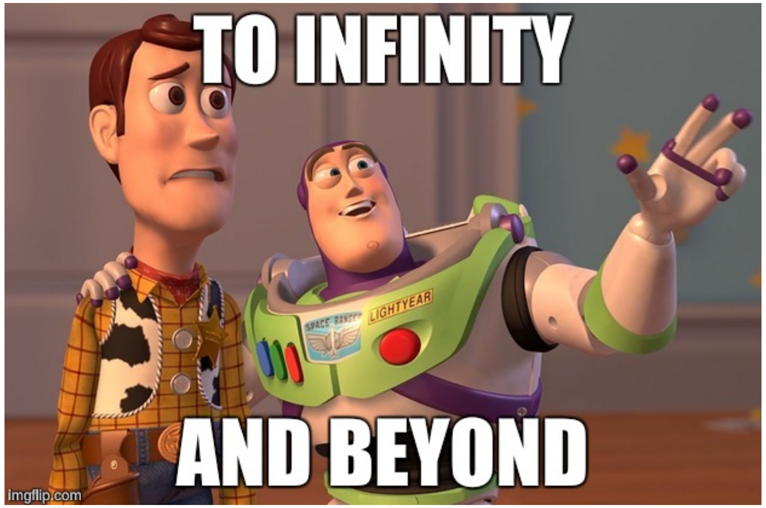
Contents



1 Contest Setup

1.1 vimrc

```
set number       " Show line numbers
set mouse=a      " Enable inaction via mouse
set showmatch        " Highlight matching brace
set cursorline       " Show underline
set cursorcolumn     " highlight vertical column
set ruler    " Show row and column ruler information

filetype on "enable file detection
syntax on   "syntax highlight

set expandtab   " For python code indentation to work correctly
set autoindent       " Auto-indent new lines
```

```
set shiftwidth=4    " Number of auto-indent spaces
set smartindent     " Enable smart-indent
set smarttab        " Enable smart-tabs
set tabstop=4   " Number of spaces per Tab

" ---------Optional----------

set undolevels=10000    " Number of undo levels
set scrolloff=5     " Auto scroll

set hlsearch    " Highlight all search results
set smartcase   " Enable smart-case search
set ignorecase  " Always case-insensitive
set incsearch   " Searches for strings incrementally

highlight Comment ctermfg=cyan
set showmode

set encoding=utf-8
set fileencoding=utf-8
scriptencoding=utf-8
```

1.2 Java template

```java
import java.io.*;
import java.util.*;

public class Main
{
    public static void main(String[] args)
    {
        MyScanner sc = new MyScanner();
        out = new PrintWriter(new BufferedOutputStream(System.out));
        // Start writing your solution here.

        // Stop writing your solution here.
        out.close();
    }

    public static PrintWriter out;

    public static class MyScanner
    {
        BufferedReader br;
        StringTokenizer st;

        public MyScanner()
        {
            br = new BufferedReader(new InputStreamReader(System.in));
        }

        boolean hasNext()
        {
            while (st == null || !st.hasMoreElements()) {
                try {
                    st = new StringTokenizer(br.readLine());
                } catch (Exception e) {
                    return false;
                }
            }
            return true;
        }

        String next()
        {
            if (hasNext())
                return st.nextToken();
            return null;
        }

        int nextInt()
        {
            return Integer.parseInt(next());
        }

        long nextLong()
        {
            return Long.parseLong(next());
        }

        double nextDouble()
        {
            return Double.parseDouble(next());
        }

        String nextLine()
        {
            String str = "";
            try {
                str = br.readLine();
            } catch (IOException e) {
                e.printStackTrace();
            }
            return str;
        }
    }
}
```

1.2.1 Java Issues

1. Random Shuffle before sorting:
   `Random rnd = new Random(); rnd.nextInt();`
2. Use `StringBuilder` for large output
3. For class sorting, use code `implements Comparable<Class name>`. Or, use code `new Comparator<Interval>() {}` at `Collections.sort()` second argument

2 System Testing

1. Test g++ (`-Wall -Wextra -Wshadow -std=c++11`) and Java 8 compiler
2. Test if c++ and Java templates work properly on local and judge machine (`bits/stdc++.h`, `auto`)
3. Test "divide by 0" $\rightarrow$ RE/TLE?
4. Make a complete graph and run Floyd warshall, to test time complexity of the judge machine

3 Reminder

1. 請不要排擠隊友！要記得心平氣和的小聲討論喔！通常隊友的建議都會突破你盲點。
2. 每一題都要小心讀，尤其是 IO 的格式和限制都要看清楚。
3. 小心估計**時間複雜度**和 **空間複雜度**
4. Coding 要兩人一組，要相信你隊友的實力！
5. 1WA 罰 20 分鐘！**放輕鬆**，**不要急**，多產幾組測資後再丟。
6. 範測一定要過！產個幾組極端測資，例如 input 下限、特殊 cases 0, 1, -1、空集合等等
7. 比賽是連續測資，一定要全部讀完再開始 solve 喔！
8. Bus error: 有`scanf`, `fgets` 但是卻沒東西可以讀取了！可能有 early termination 但是時機不對。
9. 圖論一定要記得檢查連通性。最簡單的做法就是 loop 過所有的點
10. `long long = int * int` 會完蛋
11. `long long int` 的位元運算要記得用 `1LL << 35`
12. 記得清理 Global variable （煒杰要記得清圖喔!）
13. 建圖時要注意有無重邊!
14. c++ priority queue 是 max heap，Java 是 Min heap
15. 注意要不要建立反向圖

4 Topic list

1. 列舉、窮舉 enumeration
2. 貪心 greedy
3. 排序 sorting, topological sort
4. 二分搜 binary search (數學算式移項合併後查詢)
5. 爬行法（右跑左追）Two Pointer
6. 離散化
7. Dynamic programming, 矩陣快速冪
8. 鴿籠原理 Pigeonhole
9. 最近共同祖先 LCA (倍增法, LCA 轉 RMQ)
10. 折半完全列舉 (能用 vector 就用 vector)
11. 離線查詢 Offline (DFS, LCA)
12. 圖的連通性 Directed graph connectivity -> DFS. Undirected graph -> Union Find
13. 因式分解
14. 從答案推回來
15. 寫出數學式，有時就馬上出現答案了！
16. 奇偶性質
17. 串接、反轉、兩倍長度

5 Useful code

5.1 Leap year $O(1)$

```
(year % 400 == 0 || (year % 4 == 0 && year % 100 != 0))
```

5.2 Fast Exponentiation $O(log(exp))$

Fermat's little theorem: 若 p 是質數，則 $a^{p-1} \equiv 1 \pmod p$

```
ll fast_pow(ll a, ll b, ll P) {
    // b %= (P - 1)
    ll ans = 1;
    ll base = a % P;
    while (b) {
        if (b & 1)
            ans = ans * base % P;
        base = base * base % P;
        b >>= 1;
    }
    return ans;
}
```

5.3 Mod Inverse $O(logn)$

Case 1: $gcd(a, m) = 1$: ax + my = gcd(a, m) = 1 (use ext_gcd)

Case 2: $p\ is\ prime$: $a^{p-2} \equiv a^{-1} mod\ p$

5.4 GCD $O(log(min(a + b)))$

注意負數的 case! C++ 是看被除數決定正負號的。

```
ll gcd(ll a, ll b)
{
    return b == 0 ? a : gcd(b, a % b);
}
```

5.5 Extended Euclidean Algorithm GCD $O(log(min(a + b)))$

Bezout identity $ax + by = gcd(a, b)$, where $|\ x\ | \le \frac{b}{d}$ and $|\ y\ | \le \frac{a}{d}$.

```
ll extgcd(ll a, ll b, ll& x, ll&y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    else {
        ll d = extgcd(b, a % b, y, x);
        y -= (a / b) * x;
        return d;
    }
}
```

5.6 Prime Generator $O(nloglogn)$

```
const ll MAX_NUM = 1e6; // 要是合數
bool is_prime[MAX_NUM];
vector<ll> primes;

void init_primes() {
    fill(is_prime, is_prime + MAX_NUM, true);
    is_prime[0] = is_prime[1] = false;
    for (ll i = 2; i < MAX_NUM; i++) {
        if (is_prime[i]) {
            primes.push_back(i);
            for (ll j = i * i; j < MAX_NUM; j += i)
                is_prime[j] = false;
        }
    }
}
```

5.7 C++ Reference

```
algorithm
    ::find: [it s, it t, val] -> it
    ::count: [it s, it t, val] -> int
    ::unique: [it s, it t] -> it (it = new end)
    ::merge: [it s1, it t1, it s2, it t2, it o] -> void (o allocated)

string::
    ::replace(idx, len, string) -> void
    ::find (str, pos = 0) -> idx
    ::substr (pos = 0, len = npos) -> string
string <-> int
    ::stringstream; // remember to clear
    ::sscanf(s.c_str(), "%d", &i);
    ::sprintf(result, "%d", i); string s = result;

math/cstdlib
    ::atan2(y=0, x=-1) -> pi

io printf/scanf
    ::int:                 "%d"       /   "%d"
    ::double:              "%lf","f"  /   "%lf"
    ::string:              "%s"       /   "%s"
    ::long long:           "%lld"     /   "%lld"
    ::long double:         "%Lf"      /   "%Lf"
    ::unsigned int:        "%u"       /   "%u"
    ::unsigned long long:  "%ull"     /   "%ull"
    ::oct:                 "0%o"
    ::hex:                 "0x%x"
    ::scientific:          "%e"
    ::width:               "%05d"
    ::precision:           "%.5f"
    ::adjust left:         "%-5d"

io cin/cout
    ::oct:                 cout << oct << showbase;
    ::hex:                 cout << hex << showbase;
    ::scientific:          cout << scientific;
    ::width:               cout << setw(5);
    ::precision:           cout << fixed << setprecision(5);
    ::adjust left:         cout << setw(5) << left;
```

6 Search

6.1 Ternary Search $O(nlogn)$

```
double l = ..., r = ....; // input
for(int i  = 0; i < 100; i++) {
    double m1 = l + (r - l) / 3, m2 = r - (r - l) / 3;
    if (f (m1) < f (m2)) // f - convex function
        l = m1;
    else
        r = m2;
}
f(r) - maximum of function
```

7 Basic data structure

7.1 1D BIT

```
// BIT is 1-based
const int MAX_N = 20000; //這個記得改!
ll bit[MAX_N + 1];

ll sum(int i) {
    int s = 0;
    while (i > 0) {
        s += bit[i];
        i -= (i & -i);
    }
    return s;
}

void add(int i, ll x) {
    while (i <= MAX_N) {
        bit[i] += x;
        i += (i & -i);
    }
}
```

7.2 2D BIT

```
// BIT is 1-based
const int MAX_N = 20000, MAX_M = 20000; //這個記得改!
ll bit[MAX_N + 1][MAX_M + 1];

ll sum(int a, int b) {
    ll s = 0;
    for (int i = a; i > 0; i -= (i & -i))
        for (int j = b; j > 0; j -= (j & -j))
            s += bit[i][j];
        return s;
```

```
}

void add(int a, int b, ll x) {
    // MAX_N, MAX_M 須適時調整!
    for (int i = a; i <= MAX_N; i += (i & -i))
        for (int j = b; j <= MAX_M; j += (j & -j))
            bit[i][j] += x;
}
```

7.3 Union Find

```
const int MAX_N = 20000; // 記得改
struct UFDS {
    int par[MAX_N];

    void init(int n) {
        memset(par, -1, sizeof(int) * n);
    }

    int root(int x) {
        return par[x] < 0 ? x : par[x] = root(par[x]);
    }

    void merge(int x, int y) {
        x = root(x);
        y = root(y);

        if (x != y) {
            if (par[x] > par[y])
            swap(x, y);
            par[x] += par[y];
            par[y] = x;
        }
    }
};
```

7.4 Segment Tree

```
typedef long long ll;
const int MAX_N = 100000;
const int MAX_NN = (1 << 20); // bigger than MAX_N

struct SegTree {
    int NN;               // size of tree
    ll dflt;              // default val
    ll seg[2 * MAX_NN];   // 0-based index, 2 * MAX_NN - 1 in fact
    ll lazy[2 * MAX_NN];  // 0-based index, 2 * MAX_NN - 1 in fact
    // lazy[u] != 0 <->
    // substree of u (u not inclued) is not up-to-date (it's dirty)

    void init(int n, ll val)
    {
        dflt = val;
        NN = 1;
        while (NN < n)
            NN <<= 1;
        fill(seg, seg + 2 * NN, dflt);
        fill(lazy, lazy + 2 * NN, dflt);
    }

    void gather(int u, int l, int r)
    {
        seg[u] = seg[u * 2 + 1] + seg[u * 2 + 2];
    }

    void push(int u, int l, int r)
    {
        if (lazy[u] != 0) {
            int m = (l + r) / 2;

            seg[u * 2 + 1] += (m - l) * lazy[u];
            seg[u * 2 + 2] += (r - m) * lazy[u];

            lazy[u * 2 + 1] += lazy[u];
            lazy[u * 2 + 2] += lazy[u];
            lazy[u] = 0;
        }
    }

    void build(int u, int l, int r)
    {
        if (r - l == 1)
            return;
        int m = (l + r) / 2;
        build(u * 2 + 1, l, m);
        build(u * 2 + 2, m, r);
        gather(u, l, r);
    }

    ll query(int a, int b, int u, int l, int r)
    {
        if (l >= b || r <= a)
            return dflt;
        if (l >= a && r <= b)
            return seg[u];
        int m = (l + r) / 2;
        push(u, l, r);
        ll res1 = query(a, b, u * 2 + 1, l, m);
        ll res2 = query(a, b, u * 2 + 2, m, r);
        gather(u, l, r); // data is dirty since previous push
        return res1 + res2;
    }

    void update(int a, int b, int x, int u, int l, int r)
    {
        if (l >= b || r <= a)
            return;
        if (l >= a && r <= b) {
            seg[u] += (r - l) * x; // update u and
            lazy[u] += x;          // set subtree u is not up-to-date
            return;
```

```
        }
        int m = (l + r) / 2;
        push(u, l, r);
        update(a, b, x, u * 2 + 1, l, m);
        update(a, b, x, u * 2 + 2, m, r);
        gather(u, l, r); // remember this
    }
};
```

7.5 Sparse Table

```
struct Sptb {
    int sp[MAX_LOG_N][MAX_N]; // MAX_LOG_N = ceil(lg(MAX_N))

    void build(int inp[], int n)
    {
        for (int j = 0; j < n; j++)
            sp[0][j] = inp[j];

        for (int i = 1; (1 << i) <= n; i++)
            for (int j = 0; j + (1 << i) <= n; j++)
                sp[i][j] = min(sp[i - 1][j], sp[i - 1][j + (1 << (i - 1))]);
    }

    int query(int l, int r) // [l, r)
    {
        int k = floor(log2(r - l));
        return min(sp[k][l], sp[k][r - (1 << k)]);
    }
};
```

8 Tree

8.1 LCA

```
const int MAX_N = 10000;
const int MAX_LOG_N = 14; // (1 << MAX_LOG_N) > MAX_N

int N;
int root;
int dep[MAX_N];
int par[MAX_LOG_N][MAX_N];

vector<int> child[MAX_N];

void dfs(int u, int p, int d) {
    dep[u] = d;
    for (int i = 0; i < int(child[u].size()); i++) {
        int v = child[u][i];
        if (v != p) {
            dfs(v, u, d + 1);
        }
    }
}

void build() {
    // par[0][u] and dep[u]
    dfs(root, -1, 0);

    // par[i][u]
    for (int i = 0; i + 1 < MAX_LOG_N; i++) {
        for (int u = 0; u < N; u++) {
            if (par[i][u] == -1)
                par[i + 1][u] = -1;
            else
                par[i + 1][u] = par[i][par[i][u]];
        }
    }
}

int lca(int u, int v) {
    if (dep[u] > dep[v]) swap(u, v); // 讓 v 較深
    int diff = dep[v] - dep[u]; // 將 v 上移到與 u 同層
    for (int i = 0; i < MAX_LOG_N; i++) {
        if (diff & (1 << i)) {
            v = par[i][v];
        }
    }

    if (u == v) return u;

    for (int i = MAX_LOG_N - 1; i >= 0; i--) { // 必需倒序
        if (par[i][u] != par[i][v]) {
            u = par[i][u];
            v = par[i][v];
        }
    }
    return par[0][u];
}
```

8.2 Tree Center

```
int diameter = 0, radius[N], deg[N]; // deg = in + out degree
int findRadius()
{
    queue<int> q; // add all leaves in this group
    for (auto i : group)
        if (deg[i] == 1)
            q.push(i);

    int mx = 0;
    while (q.empty() == false) {
        int u = q.front();
        q.pop();

        for (int v : g[u]) {
            deg[v]--;
            if (deg[v] == 1) {
                q.push(v);
```

```
                radius[v] = radius[u] + 1;
                mx = max(mx, radius[v]);
            }
        }
    }

    int cnt = 0; // crucial for knowing if there are 2 centers or not
    for (auto j : group)
        if (radius[j] == mx)
            cnt++;

    // add 1 if there are 2 centers (radius, diameter)
    diameter = max(diameter, mx * 2 + (cnt == 2));
    return mx + (cnt == 2);
}
```

8.3 Treap

```
// Remember srand(time(NULL))
struct Treap { // val: bst, pri: heap
    int pri, size, val;
    Treap *lch, *rch;
    Treap() {}
    Treap(int v) {
        pri = rand();
        size = 1;
        val = v;
        lch = rch = NULL;
    }
};

inline int size(Treap* t) {
    return (t ? t->size : 0);
}
// inline void push(Treap* t) {
//     push lazy flag
// }
inline void pull(Treap* t) {
    t->size = 1 + size(t->lch) + size(t->rch);
}

int NN = 0;
Treap pool[30000];

Treap* merge(Treap* a, Treap* b) { // a < b
    if (!a || !b) return (a ? a : b);
    if (a->pri > b->pri) {
        // push(a);
        a->rch = merge(a->rch, b);
        pull(a);
        return a;
    }
    else {
        // push(b);
        b->lch = merge(a, b->lch);
        pull(b);
        return b;
    }
}

void split(Treap* t, Treap*& a, Treap*& b, int k) {
    if (!t) { a = b = NULL; return; }
    // push(t);
    if (size(t->lch) < k) {
        a = t;
        split(t->rch, a->rch, b, k - size(t->lch) - 1);
        pull(a);
    }
    else {
        b = t;
        split(t->lch, a, b->lch, k);
        pull(b);
    }
}

// get the rank of val
// result is 1-based
int get_rank(Treap* t, int val) {
    if (!t) return 0;
    if (val < t->val)
        return get_rank(t->lch, val);
    else
        return get_rank(t->rch, val) + size(t->lch) + 1;
}

// get kth smallest item
// k is 1-based
Treap* get_kth(Treap*& t, int k) {
    Treap *a, *b, *c, *d;
    split(t, a, b, k - 1);
    split(b, c, d, 1);
    t = merge(a, merge(c, d));
    return c;
}

void insert(Treap*& t, int val) {
    int k = get_rank(t, val);
    Treap *a, *b;
    split(t, a, b, k);
    pool[NN] = Treap(val);
    Treap* n = &pool[NN++];
    t = merge(merge(a, n), b);
}

// Implicit key treap init
void insert() {
    for (int i = 0; i < N; i++) {
        int val; scanf("%d", &val);
        root = merge(root, new_treap(val)); // implicit key(index)
    }
}
```

9 Graph

9.1 Articulation point / Bridge

```cpp
const int MAX_N = 1111;
vector<int> g[MAX_N];

// for bridge
typedef pair<int, int> ii;
vector<ii> ans;

// for articulation point
int root;                // set it before dfs() call
bool isCutVertex[MAX_N]; // init to false

int tt = 0, dfn[MAX_N], low[MAX_N]; // init array to -1
void dfs(int u, int p)
{
    dfn[u] = low[u] = tt++;

    // for articulation point, root needs to have >= 2 childrens
    int child = 0;
    for (auto v : g[u]) {
        if (v == p)
            continue;
        child++;

        if (dfn[v] == -1) {
            dfs(v, u);
            low[u] = min(low[u], low[v]);

            if (low[v] > dfn[u]) // bridge
                ans.push_back(ii(min(u, v), max(u, v)));

            if (u != root && low[v] >= dfn[u]) { // articulation point
                isCutVertex[u] = true;
            } else if (u == root && child >= 2) { // articulation point
                isCutVertex[u] = true;
            }
        } else {
            // u -> v, u has direct access to v -> back edge
            low[u] = min(low[u], dfn[v]);
        }
    }
}
```

9.2 2-SAT

$$
\begin{aligned}
&p \lor (q \land r) \\
&= ((p \land q) \lor (p \land r)) \\
&p \oplus q \\
&= \neg((p \land q) \lor (\neg p \land \neg q)) \\
&= (\neg p \lor \neg q) \land (p \lor q)
\end{aligned}
$$

```cpp
// 建圖
// (x1 or x2) and ... and (xi or xj)
// (xi or xj) 建邊
// ~xi -> xj
// ~xj -> xi

tarjan(); // scc 建立的順序是倒序的拓璞排序
for (int i = 0; i < 2 * N; i += 2) {
    if (belong[i] == belong[i ^ 1]) {
        // 無解
    }
}
for (int i = 0; i < 2 * N; i += 2) { // 迭代所有變數
    if (belong[i] < belong[i ^ 1]) { // i 的拓璞排序比 ~i 的拓璞排序大
        // i = T
    }
    else {
        // i = F
    }
}
```

9.3 BCC

一張無向圖上，不會產生關節點 (articulation point) 的連通分量，稱作「雙連通分量」(Biconnected Component)。
一張無向圖上，不會產生橋 (bridge) 的連通分量，稱作「橋連通分量」(Bridge-connected Component)。

9.3.1 Biconnected Component

以 Edge 做分界的話，stack 要裝入 (u - v), 並 pop 終止條件為!= (u - v)
以 Articulation point 做為分界 (code below)，注意有無坑人的重邊

用 SCC 的 code 的話，只要多判一個 u 是否為 p, 如果是的話就直接 return (加在第 21 行之後)

9.3.2 Bridge-connected Component

```cpp
const int MAX_N = 5555;
vector<int> g[MAX_N];
int tt, dfn[MAX_N], low[MAX_N];
int bcc;
int belong[MAX_N]; // 縮點用
stack<int> s;
void dfs(int u, int p)
{
    dfn[u] = low[u] = tt++;
    s.push(u);
    for (int i = 0; i < (int)g[u].size(); i++) {
        int v = g[u][i];
        if (v == p)
            continue;
        if (dfn[v] == -1) {
            dfs(v, u);
```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```
            low[u] = min(low[u], low[v]);
        } else {
            low[u] = min(low[u], dfn[v]);
        }
    }
    if (low[u] == dfn[u]) {
        bcc++;
        while (1) {
            int v = s.top();
            s.pop();
            belong[v] = bcc;
            if (v == u)
                break;
        }
    }
}
```

9.4 SCC

First of all we run DFS on the graph and sort the vertices in decreasing of their finishing time (we can use a stack).
Then, we start from the vertex with the greatest finishing time, and for each vertex v that is not yet in any SCC, do : for each u that v is reachable by u and u is not yet in any SCC, put it in the SCC of vertex v. The code is quite simple.

```
const int MAX_V = ...;
const int INF = 0x3f3f3f3f;
int V;
vector<int> g[MAX_V];

int dfn_idx = 0;
int scc_cnt = 0;
int dfn[MAX_V];
int low[MAX_V];
int belong[MAX_V];
bool in_st[MAX_V];
vector<int> st;

void scc(int v)
{
    dfn[v] = low[v] = dfn_idx++;
    st.push_back(v);
    in_st[v] = true;

    for (int i = 0; i < int(g[v].size()); i++) {
        const int u = g[v][i];
        if (dfn[u] == -1) {
            scc(u);
            low[v] = min(low[v], low[u]);
        } else if (in_st[u]) {
            low[v] = min(low[v], dfn[u]);
        }
    }

    if (dfn[v] == low[v]) {
        int k;
        do {
            k = st.back();
            st.pop_back();
            in_st[k] = false;
            belong[k] = scc_cnt;
        } while (k != v);
        scc_cnt++;
    }
}

void tarjan() // scc 建立的順序即為反向的拓璞排序
{
    st.clear();
    fill(dfn, dfn + V, -1);
    fill(low, low + V, INF);
    dfn_idx = 0;
    scc_cnt = 0;
    for (int v = 0; v < V; v++) {
        if (dfn[v] == -1) {
            scc(v);
        }
    }
}
```

9.5 Shortest Path

Time complexity notations: V = vertex, E = edge
Minimax: $dp[u][v] = min(dp[u][v], max(dp[u][k], dp[k][v]))$

9.5.1 Dijkatra (next-to-shortest path) $O(VlogE)$

密集圖別用 priority queue!

```
struct Edge {
    int to, cost;
};

typedef pair<int, int> P; // <d, v>
const int INF = 0x3f3f3f3f;

int N, R;
vector<Edge> g[5000];

int d[5000];
int sd[5000];

int solve()
{
    fill(d, d + N, INF);
    fill(sd, sd + N, INF);
    priority_queue<P, vector<P>, greater<P>> pq;

    d[0] = 0;
    pq.push(P(0, 0));
```


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```cpp

    while (!pq.empty()) {
        P p = pq.top();
        pq.pop();
        int v = p.second;

        if (sd[v] < p.first) // 比次短距離還大, 沒用, 跳過
            continue;

        for (size_t i = 0; i < g[v].size(); i++) {
            Edge &e = g[v][i];
            int nd = p.first + e.cost;
            if (nd < d[e.to]) { // 更新最短距離
                swap(d[e.to], nd);
                pq.push(P(d[e.to], e.to));
            }
            if (d[e.to] < nd && nd < sd[e.to]) { // 更新次短距離
                sd[e.to] = nd;
                pq.push(P(sd[e.to], e.to));
            }
        }
    }

    return sd[N - 1];
}
```

9.5.2 SPFA

```cpp
typedef pair<int, int> ii;
vector<ii> g[N];

bool SPFA()
{
    vector<ll> d(n, INT_MAX);
    d[0] = 0; // origin

    queue<int> q;
    vector<bool> inqueue(n, false);
    vector<int> cnt(n, 0);
    q.push(0);
    inqueue[0] = true;
    cnt[0]++;

    while (q.empty() == false) {
        int u = q.front();
        q.pop();
        inqueue[u] = false;

        for (auto i : g[u]) {
            int v = i.first, w = i.second;
            if (d[u] + w < d[v]) {
                d[v] = d[u] + w;
                if (inqueue[v] == false) {
                    q.push(v);
                    inqueue[v] = true;
                    cnt[v]++;

                    if (cnt[v] == n) { // loop!
                        return true;
                    }
                }
            }
        }
    }

    return false;
}
```

9.5.3 Bellman-Ford $O(VE)$

```cpp
struct Edge {
    int from, to, cost;
};

const int MAX_V = ...;
const int MAX_E = ...;
const int INF = 0x3f3f3f3f;
int V, E;
Edge edges[MAX_E];
int d[MAX_V];

bool bellman_ford()
{
    fill(d, d + V, INF);

    d[0] = 0;
    for (int i = 0; i < V; i++) {
        for (int j = 0; j < E; j++) {
            Edge &e = edges[j];
            if (d[e.to] > d[e.from] + e.cost) {
                d[e.to] = d[e.from] + e.cost;

                if (i == V - 1) // negative cycle
                    return true;
            }
        }
    }

    return false;
}
```

9.5.4 Floyd-Warshall $O(V^3)$

The graph is stored using adjacency matrix. The initial state is $diagnal = 0$ and $others = INF$. (If INF is int, use long long for the matrix)
If diagonal numbers are negative $\leftarrow$ cycle .

```cpp
for(int k = 0; k < N; k++)
    for(int i = 0; i < N; i++)
        for(int j = 0; j < N; j++)
            dp[i][j] = min(dp[i][j], dp[i][k] + dp[k][j]);
```

9.6 MST

9.6.1 Kruskal

1. Store the graph by $(weight, (from, to))$
2. Sort the graph by $weight$
3. Start from the smallest weight, and keep adding edges that won't form a cycle with the current MST set
4. Early termination condition: $n-1$ edges has been added, NOT size of the union-find set

9.6.2 Second MST

```
const int INF = 0x3f3f3f3f;
const int MAX_V = 100;
const int MAX_LOG_V = 7;
int V, E; // 記得初使化

struct Edge {
    int u, v, w;
};
vector<Edge> edges;

// btn[i][u] = u 前往它 2^i parent 的路上經過的最大權重
// par[i][u] = u 的 2^i parent 是誰
int dep[MAX_V]; // should be init to -1
int btn[MAX_LOG_V][MAX_V];
int par[MAX_LOG_V][MAX_V];

// mst
struct AdjE {
    int to, w;
};
vector<AdjE> g[MAX_V];

void dfs(int u, int p, int d) {
    dep[u] = d;
    par[0][u] = p;
    for (auto e : g[u]) {
        if (e.to != p) {
            btn[0][e.to] = e.w;
            dfs(e.to, u, d + 1);
        }
    }
}

void build() {
    for (int u = 0; u < V; u++) {
        if (dep[u] == -1) {
            dfs(u, -1, 0);
        }
    }

    for (int i = 0; i + 1 < MAX_LOG_V; i++) {
        for (int u = 0; u < V; u++) {
            if (par[i][u] == -1 || par[i][par[i][u]] == -1) {
                par[i + 1][u] = -1;
                btn[i + 1][u] = 0;
            }
            else {
                par[i + 1][u] = par[i][par[i][u]];
                btn[i + 1][u] = max(btn[i][u], btn[i][par[i][u]]);
            }
        }
    }
}

int lca(int u, int v) { // 回傳 u, v 之間的最大權重
    int mx = -INF; // u, v 之間的最大權重

    if (dep[u] > dep[v]) swap(u, v);
    int diff = dep[v] - dep[u];
    for (int i = MAX_LOG_V - 1; i >= 0; i--) {
        if (diff & (1 << i)) {
            mx = max(mx, btn[i][v]);
            v = par[i][v];
        }
    }

    if (u == v) return mx;

    for (int i = MAX_LOG_V - 1; i >= 0; i--) {
        if (par[i][u] != par[i][v]) {
            mx = max(mx, btn[i][u]);
            mx = max(mx, btn[i][v]);
            u = par[i][u];
            v = par[i][v];
        }
    }
    // lca = par[0][u] = par[0][v];
    mx = max(mx, max(btn[0][u], btn[0][v]));

    return mx;
}

// second mst
build();
int ans = INF;
for (auto e: non_mst_edges) {
    int mx_w = lca(e.u, e.v);
    ans = min(ans, (total_w + e.w - mx_w));
}
```

9.6.3 Prim

```
int ans = 0; bool used[n];
memset(used, false, sizeof(used));
priority_queue<ii, vector<ii>, greater<ii>> pq;
pq.push(ii(0, 0)); // push (0, origin)
while (!pq.empty())
{
```

```
    ii cur = pq.top(); pq.pop();

    int u = cur.second;
    if (used[u]) continue;
    ans += cur.first;
    used[u] = true;
    for (int i = 0; i < (int)g[u].size(); i++) {
        int v = g[u][i].first, w = g[u][i].second;
        if (used[v] == false) pq.push(ii(w, v));
    }
}
```

10 Flow

10.1 Max Flow (Dinic)

```
struct Edge {
    int to, cap, rev;
    Edge(int a, int b, int c) {
        to = a;
        cap = b;
        rev = c;
    }
};

const int INF = 0x3f3f3f3f;
const int MAX_V = 20000 + 10;
// vector<Edge> g[MAX_V];
vector< vector<Edge> > g(MAX_V);
int level[MAX_V];
int iter[MAX_V];

inline void add_edge(int u, int v, int cap) {
    g[u].push_back((Edge){v, cap, (int)g[v].size()});
    g[v].push_back((Edge){u, 0, (int)g[u].size() - 1});
}

void bfs(int s) {
    memset(level, -1, sizeof(level)); // 用 fill
    queue<int> q;

    level[s] = 0;
    q.push(s);

    while (!q.empty()) {
        int v = q.front(); q.pop();
        for (int i = 0; i < int(g[v].size()); i++) {
            const Edge& e = g[v][i];
            if (e.cap > 0 && level[e.to] < 0) {
                level[e.to] = level[v] + 1;
                q.push(e.to);
            }
        }
    }
}

int dfs(int v, int t, int f) {
    if (v == t) return f;
    for (int& i = iter[v]; i < int(g[v].size()); i++) { // & 很重要
        Edge& e = g[v][i];
        if (e.cap > 0 && level[v] < level[e.to]) {
            int d = dfs(e.to, t, min(f, e.cap));
            if (d > 0) {
                e.cap -= d;
                g[e.to][e.rev].cap += d;
                return d;
            }
        }
    }
    return 0;
}

int max_flow(int s, int t) { // dinic
    int flow = 0;
    for (;;) {
        bfs(s);
        if (level[t] < 0) return flow;
        memset(iter, 0, sizeof(iter));
        int f;
        while ((f = dfs(s, t, INF)) > 0) {
            flow += f;
        }
    }
}
```

10.2 Min Cost Flow

```
#define st first
#define nd second

typedef pair<double, int> pii; // 改成用 int
const double INF = 1e10;

struct Edge {
    int to, cap;
    double cost;
    int rev;
};

const int MAX_V = 2 * 100 + 10;
int V;
vector<Edge> g[MAX_V];
double h[MAX_V];
double d[MAX_V];
int prevv[MAX_V];
int preve[MAX_V];
// int match[MAX_V];

void add_edge(int u, int v, int cap, double cost) {
    g[u].push_back((Edge){v, cap, cost, (int)g[v].size()});
```

```cpp
    g[v].push_back((Edge){u, 0, -cost, (int)g[u].size() - 1});
}

double min_cost_flow(int s, int t, int f) {
    double res = 0;
    fill(h, h + V, 0);
    fill(match, match + V, -1);
    while (f > 0) {
        // dijkstra 找最小成本增廣路徑
        // without h will reduce to SPFA = O(V*E)
        fill(d, d + V, INF);
        priority_queue< pii, vector<pii>, greater<pii> > pq;

        d[s] = 0;
        pq.push(pii(d[s], s));

        while (!pq.empty()) {
            pii p = pq.top(); pq.pop();
            int v = p.nd;
            if (d[v] < p.st) continue;
            for (size_t i = 0; i < g[v].size(); i++) {
                const Edge& e = g[v][i];
                if (e.cap > 0 && d[e.to] > d[v] + e.cost + h[v] - h[e.to]) {
                    d[e.to] = d[v] + e.cost + h[v] - h[e.to];
                    prevv[e.to] = v;
                    preve[e.to] = i;
                    pq.push(pii(d[e.to], e.to));
                }
            }
        }

        // 找不到增廣路徑
        if (d[t] == INF) return -1; // double 時不能這樣判

        // 維護 h[v]
        for (int v = 0; v < V; v++)
            h[v] += d[v];

        // 找瓶頸
        int bn = f;
        for (int v = t; v != s; v = prevv[v])
            bn = min(bn, g[prevv[v]][preve[v]].cap);

        // // find match
        // for (int v = prevv[t]; v != s; v = prevv[prevv[v]]) {
        //     int u = prevv[v];
        //     match[v] = u;
        //     match[u] = v;
        // }

        // 更新剩餘圖
        f -= bn;
        res += bn * h[t]; // SPFA: res += bn * d[t]
        for (int v = t; v != s; v = prevv[v]) {
            Edge& e = g[prevv[v]][preve[v]];
            e.cap -= bn;
            g[v][e.rev].cap += bn;
        }
    }
    return res;
}
```

10.3 Bipartite Matching, Unweighted

最大匹配數：最大匹配的匹配邊的數目
最小點覆蓋數：選取最少的點，使任意一條邊至少有一個端點被選擇
最大獨立數：選取最多的點，使任意所選兩點均不相連
最小路徑覆蓋數：對於一個 DAG（有向無環圖），選取最少條路徑，使得每個頂點屬於且僅屬於一條路徑。路徑長可以為 0（即單個點）
定理 1：最大匹配數 = 最小點覆蓋數（這是 Konig 定理）
定理 2：最大匹配數 = 最大獨立數
定理 3：最小路徑覆蓋數 = 頂點數 - 最大匹配數

```cpp
const int MAX_V = ...;
int V;
vector<int> g[MAX_V];
int match[MAX_V];
bool used[MAX_V];

void add_edge(int u, int v) {
    g[u].push_back(v);
    g[v].push_back(u);
}

// 回傳有無找到從 v 出發的增廣路徑
//（首尾都為未匹配點的交錯路徑）
// [待確認] 每次遞迴都找一個未匹配點 v 及匹配點 u
bool dfs(int v) {
    used[v] = true;
    for (size_t i = 0; i < g[v].size(); i++) {
        int u = g[v][i], w = match[u];
        // 尚未配對或可從 w 找到增廣路徑（即路徑繼續增長）
        if (w < 0 || (!used[w] && dfs(w))) {
            // 交錯配對
            match[v] = u;
            match[u] = v;
            return true;
        }
    }
    return false;
}

int bipartite_matching() { // 匈牙利演算法
    int res = 0;
    memset(match, -1, sizeof(match));
    for (int v = 0; v < V; v++) {
        if (match[v] == -1) {
            memset(used, false, sizeof(used));
            if (dfs(v)) {
```

```
                res++;
            }
        }
    }
    return res;
}
```

11 String

11.1 Rolling Hash

1. Use two rolling hashes if needed.
2. The prime for pre-calculation can be 137 and 257, for modulo can be $1e9+7$ and $0xdefaced$

```
#define N 1000100
#define B 137
#define M 1000000007

typedef long long ll;

char inp[N];
int len;
ll p[N], h[N];

void init()
{ // build polynomial table and hash value
    p[0] = 1; // b to the ith power
    for (int i = 1; i <= len; i++) {
        h[i] = (h[i - 1] * B % M + inp[i - 1]) % M; // hash value
        p[i] = p[i - 1] * B % M;
    }
}

ll get_hash(int l, int r) // [l, r] of the inp string array
{
    return ((h[r + 1] - (h[l] * p[r - l + 1])) % M + M) % M;
}
```

11.2 KMP

```
void fail()
{
    int len = strlen(pat);

    f[0] = 0;
    int j = 0;
    for (int i = 1; i < len; i++) {
        while (j != 0 && pat[i] != pat[j])
            j = f[j - 1];

        if (pat[i] == pat[j])
            j++;

        f[i] = j;
    }
}

int match()
{
    int res = 0;
    int j = 0, plen = strlen(pat), tlen = strlen(text);

    for (int i = 0; i < tlen; i++) {
        while (j != 0 && text[i] != pat[j])
            j = f[j - 1];

        if (text[i] == pat[j]) {
            if (j == plen - 1) { // find match
                res++;
                j = f[j];
            } else {
                j++;
            }
        }
    }

    return res;
}
```

11.3 Z Algorithm

```
int len = strlen(inp), z[len];
z[0] = 0; // initial

int l = 0, r = 0; // z box bound [l, r]
for (int i = 1; i < len; i++)
{
    if (i > r) { // i not in z box
        l = r = i; // z box contains itself only
        while (r < len && inp[r - l] == inp[r])
            r++;
        z[i] = r - l;
        r--;
    } else { // i in z box
        if (z[i - l] + i < r) // over shoot R bound
            z[i] = z[i - l];
        else {
            l = i;
            while (r < len && inp[r - l] == inp[r])
                r++;
            z[i] = r - l;
            r--;
        }
    }
}
```

11.4 Trie

注意 count 的擺放位置，視題意可以擺在迴圈外

```cpp
struct Node {
    int cnt;
    Node* nxt[2];
    Node() {
        cnt = 0;
        fill(nxt, nxt + 2, nullptr);
    }
};

const int MAX_Q = 200000;
int Q;

int NN = 0;
Node data[MAX_Q * 30];
Node* root = &data[NN++];

void insert(Node* u, int x) {
    for (int i = 30; i >= 0; i--) {
        int t = ((x >> i) & 1);
        if (u->nxt[t] == nullptr) {
            u->nxt[t] = &data[NN++];
        }

        u = u->nxt[t];
        u->cnt++;
    }
}

void remove(Node* u, int x) {
    for (int i = 30; i >= 0; i--) {
        int t = ((x >> i) & 1);
        u = u->nxt[t];
        u->cnt--;
    }
}

int query(Node* u, int x) {
    int res = 0;
    for (int i = 30; i >= 0; i--) {
        int t = ((x >> i) & 1);
        // if it is possible to go the another branch
        // then the result of this bit is 1
        if (u->nxt[t ^ 1] != nullptr && u->nxt[t ^ 1]->cnt > 0) {
            u = u->nxt[t ^ 1];
            res |= (1 << i);
        }
        else {
            u = u->nxt[t];
        }
    }
    return res;
}
```


11.5 Suffix Array

```cpp
#include <bits/stdc++.h>
#define rank rk
using namespace std;
const int MXN = 1e5 + 5;
int n, k;
int rank[MXN], tmp[MXN];
bool cmp_sa(int i, int j)
{
    if (rank[i] != rank[j])
        return rank[i] < rank[j];
    int _i = i + k <= n ? rank[i + k] : -1;
    int _j = j + k <= n ? rank[j + k] : -1;
    return _i < _j;
}

void build_sa(string s, int *sa)   // O(nlg2n)
{
    n = s.length();
    for (int i = 0; i <= n; i++) {
        sa[i] = i;                      // 先填入 sa
        rank[i] = i < n ? s[i] : -1; // ascii 當排名用
    }
    for (k = 1; k <= n; k <<= 1) {
        sort(sa, sa + n + 1, cmp_sa); // 依照排名 sort sa
        tmp[sa[0]] = 0;               // 初始化第 0 名
        for (int i = 1; i <= n; i++)  // 依照 sa 重新排名
            tmp[sa[i]] = tmp[sa[i - 1]] + (cmp_sa(sa[i - 1], sa[i]) ? 1 :
             ↪  0);
        for (int i = 0; i <= n; i++) // 儲存排名結果
            rank[i] = tmp[i];
    }
}

void build_lcp(string s, int *sa, int *lcp)
{
    int n = s.length(), h = 0;
    /* 自行製造 rank 數列
    for(int i=0;i<=n;i++) rank[sa[i]] = i;
    */
    lcp[0] = 0;
    for (int i = 0; i < n; i++) {
        int j = sa[rank[i] - 1]; // 存下排名在 i 之前
        if (h > 0)
            h--;
        for (; j + h < n && i + h < n; h++)
            if (s[j + h] != s[i + h])
                break;
        lcp[rank[i] - 1] = h;
    }
}
int main()
{
    string str = "abracadabra";
    int suffix[10000], lcp[10000];
```

```
    build_sa(str, suffix);
    build_lcp(str, suffix, lcp);
}
```

12 Matrix

12.1 Gauss Jordan Elimination

```
typedef long long ll;
typedef vector<ll> vec;
typedef vector<vec> mat;

vec gauss_jordan(mat A) {
    int n = A.size(), m = A[0].size(); // 增廣矩陣
    for (int i = 0; i < n; i++) {
        // float: find j s.t. A[j][i] is max
        // mod: find min j s.t. A[j][i] is not 0
        int pivot = i;
        for (int j = i; j < n; j++) {
            // if (fabs(A[j][i]) > fabs(A[pivot])) {
            //     pivot = j;
            // }
            if (A[pivot][i] != 0) {
                pivot = j;
                break;
            }
        }

        swap(A[i], A[pivot]);
        if (A[i][i] == 0) { // if (fabs(A[i][i]) < eps)
            // 無解或無限多組解
            // 可改成 continue, 全部做完後再判
            return vec();
        }

        ll divi = inv(A[i][i]);
        for (int j = i; j < m; j++) {
            // A[i][j] /= A[i][i];
            A[i][j] = (A[i][j] * divi) % MOD;
        }

        for (int j = 0; j < n; j++) {
            if (j != i) {
                for (int k = i + 1; k < m; k++) {
                    // A[j][k] -= A[j][i] * A[i][k];
                    ll p = (A[j][i] * A[i][k]) % MOD;
                    A[j][k] = (A[j][k] - p + MOD) % MOD;
                }
            }
        }
    }

    vec x(n);
    for (int i = 0; i < n; i++)
        x[i] = A[i][m - 1];
    return x;
}
```

12.2 Determinant

整數版本

```
typedef long long ll;
typedef vector<ll> vec;
typedef vector<vec> mat;

ll determinant(mat m) { // square matrix
    const int n = m.size();
    ll det = 1;
    for (int i = 0; i < n; i++) {
        for (int j = i + 1; j < n; j++) {
            int a = i, b = j;
            while (m[b][i]) {
                ll q = m[a][i] / m[b][i];
                for (int k = 0; k < n; k++)
                    m[a][k] = m[a][k] - m[b][k] * q;
                swap(a, b);
            }

            if (a != i) {
                swap(m[i], m[j]);
                det = -det;
            }
        }

        if (m[i][i] == 0)
            return 0;
        else
            det *= m[i][i];
    }
    return det;
}
```

13 Geometry

1. Keep things in integers as much as possible!
2. Try not to divide
3. If you have decimals, if they are fixed precision, you can usually just multiply all the input and use integers instead

13.1 EPS

$= 0$: $fabs \leq eps$
< 0: $< -eps$
> 0: $> +eps$

```
// if the points are given in doubles form, change the code accordingly

typedef long long ll;
```

```cpp

typedef pair<ll, ll> pt;  // points are stored using long long
typedef pair<pt, pt> seg; // segments are a pair of points

#define x first
#define y second

#define EPS 1e-9

pt operator+(pt a, pt b)
{
    return pt(a.x + b.x, a.y + b.y);
}

pt operator-(pt a, pt b)
{
    return pt(a.x - b.x, a.y - b.y);
}

pt operator*(pt a, int d)
{
    return pt(a.x * d, a.y * d);
}

ll cross(pt a, pt b)
{
    return a.x * b.y - a.y * b.x;
}

int ccw(pt a, pt b, pt c)
{
    ll res = cross(b - a, c - a);
    if (res > 0) // left turn
        return 1;
    else if (res == 0) // straight
        return 0;
    else // right turn
        return -1;
}

double dist(pt a, pt b)
{
    double dx = a.x - b.x;
    double dy = a.y - b.y;
    return sqrt(dx * dx + dy * dy);
}

bool zero(double x)
{
    return fabs(x) <= EPS;
}

bool overlap(seg a, seg b)
{
    return ccw(a.x, a.y, b.x) == 0 && ccw(a.x, a.y, b.y) == 0;
}

bool intersect(seg a, seg b)
{
    if (overlap(a, b) == true) { // non-proper intersection
        double d = 0;
        d = max(d, dist(a.x, a.y));
        d = max(d, dist(a.x, b.x));
        d = max(d, dist(a.x, b.y));
        d = max(d, dist(a.y, b.x));
        d = max(d, dist(a.y, b.y));
        d = max(d, dist(b.x, b.y));

        // d > dist(a.x, a.y) + dist(b.x, b.y)
        if (d - (dist(a.x, a.y) + dist(b.x, b.y)) > EPS)
            return false;
        return true;
    }
    //                        |
    // Equal sign for -----| case
    // non qeual sign => proper intersection
    if (ccw(a.x, a.y, b.x) * ccw(a.x, a.y, b.y) <= 0 &&
        ccw(b.x, b.y, a.x) * ccw(b.x, b.y, a.y) <= 0)
        return true;
    return false;
}

double area(vector<pt> pts)
{
    double res = 0;
    int n = pts.size();
    for (int i = 0; i < n; i++)
        res += (pts[i].y + pts[(i + 1) % n].y) * (pts[(i + 1) % n].x - pts[i].x);
    return res / 2.0;
}

vector<pt> halfHull(vector<pt> &points)
{
    vector<pt> res;

    for (int i = 0; i < (int)points.size(); i++) {
        while ((int)res.size() >= 2 &&
                ccw(res[res.size() - 2], res[res.size() - 1], points[i]) < 0)
            res.pop_back(); // res.size() - 2 can't be assign before size() >= 2
        // check, bitch

        res.push_back(points[i]);
    }

    return res;
}

vector<pt> convexHull(vector<pt> &points)
{
```

```cpp
    vector<pt> upper, lower;

    // make upper hull
    sort(points.begin(), points.end());

    upper = halfHull(points);
    // make lower hull
    reverse(points.begin(), points.end());
    lower = halfHull(points);

    // merge hulls
    if ((int)upper.size() > 0) // yes sir~
        upper.pop_back();
    if ((int)lower.size() > 0)
        lower.pop_back();

    vector<pt> res(upper.begin(), upper.end());
    res.insert(res.end(), lower.begin(), lower.end());

    return res;
}
```

13.2 Rectangle area

```cpp
#define sz(x) (int(x.size()))

const int MAX_NN = (1 << 17);

struct Rect {
    double x1, y1, x2, y2;
};

struct Event {
    double y; int x1, x2, type;
    bool operator < (const Event& e) const {
        if (y == e.y)
            return type < e.type;
        return y < e.y;
    }
};

vector<double> xs;

struct SegTree {
    int NN;
    int cnt[MAX_NN];
    double len[MAX_NN];

    void init(int n) {
        NN = 1;
        while (NN < n)
            NN <<= 1;
        fill(cnt, cnt + 2 * NN, 0);
        fill(len, len + 2 * NN, double(0.0));
    }

    void maintain(int u, int l, int r) {
        if (cnt[u] > 0) len[u] = xs[r] - xs[l];
        else {
            if (u >= NN - 1)
                len[u] = 0;
            else
                len[u] = len[u * 2 + 1] + len[u * 2 + 2];
        }
    }

    void update(int a, int b, int x, int u, int l, int r) { // [a, b), [l, r)
        if (r <= a || l >= b) return;
        if (a <= l && r <= b) {
            cnt[u] += x;
            maintain(u, l, r);
            return;
        }
        int m = (l + r) / 2;
        update(a, b, x, u * 2 + 1, l, m);
        update(a, b, x, u * 2 + 2, m, r);
        maintain(u, l, r);
    }
};

double get_union_area(const vector<Rect>& rect) {
    // 離散化 x
    xs.clear();
    for (int i = 0; i < sz(rect); i++) {
        xs.push_back(rect[i].x1);
        xs.push_back(rect[i].x2);
    }
    sort(xs.begin(), xs.end());
    xs.resize(unique(xs.begin(), xs.end()) - xs.begin());

    // sweep line events
    vector<Event> es;
    for (int i = 0; i < sz(rect); i++) {
        int x1 = lower_bound(xs.begin(), xs.end(), rect[i].x1) - xs.begin();
        int x2 = lower_bound(xs.begin(), xs.end(), rect[i].x2) - xs.begin();
        es.push_back((Event) {rect[i].y1, x1, x2, +1}); // bottom
        es.push_back((Event) {rect[i].y2, x1, x2, -1}); // top
    }
    sort(es.begin(), es.end());

    // find total area
    SegTree seg;
    seg.init(sz(xs));
    seg.update(es[0].x1, es[0].x2, es[0].type, 0, 0, seg.NN);

    double res = 0;
    for (int i = 1; i < sz(es); i++) {
        res += seg.len[0] * (es[i].y - es[i - 1].y);
```


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```
        seg.update(es[i].x1, es[i].x2, es[i].type, 0, 0, seg.NN);
    }

    return res;
}
```

14 Math

14.1 Euclid's formula (Pythagorean Triples)

$a = p^2 - q^2$
$b = 2pq$ **(always even)**
$c = p^2 + q^2$

14.2 Difference between two consecutive numbers' square is odd

$(k+1)^2 - k^2 = 2k + 1$

14.3 Summation

$\sum_{k=1}^{n} 1 = n$
$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$
$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$

14.4 FFT

```
typedef unsigned int ui;
typedef long double ldb;
const ldb pi = atan2(0, -1);

struct Complex {
    ldb real, imag;
    Complex(): real(0.0), imag(0.0) {;}
    Complex(ldb a, ldb b) : real(a), imag(b) {;}
    Complex conj() const {
        return Complex(real, -imag);
    }
    Complex operator + (const Complex& c) const {
        return Complex(real + c.real, imag + c.imag);
    }
    Complex operator - (const Complex& c) const {
        return Complex(real - c.real, imag - c.imag);
    }
    Complex operator * (const Complex& c) const {
        return Complex(real*c.real - imag*c.imag, real*c.imag + imag*c.real);
    }
    Complex operator / (ldb x) const {
        return Complex(real / x, imag / x);
    }
    Complex operator / (const Complex& c) const {
        return *this * c.conj() / (c.real * c.real + c.imag * c.imag);
    }
};

inline ui rev_bit(ui x, int len){
    x = ((x & 0x55555555u) << 1)  | ((x & 0xAAAAAAAAu) >> 1);
    x = ((x & 0x33333333u) << 2)  | ((x & 0xCCCCCCCCu) >> 2);
    x = ((x & 0x0F0F0F0Fu) << 4)  | ((x & 0xF0F0F0F0u) >> 4);
    x = ((x & 0x00FF00FFu) << 8)  | ((x & 0xFF00FF00u) >> 8);
    x = ((x & 0x0000FFFFu) << 16) | ((x & 0xFFFF0000u) >> 16);
    return x >> (32 - len);
}

// flag = -1 if ifft else +1
void fft(vector<Complex>& a, int flag = +1) {
    int n = a.size(); // n should be power of 2

    int len = __builtin_ctz(n);
    for (int i = 0; i < n; i++) {
        int rev = rev_bit(i, len);

        if (i < rev)
            swap(a[i], a[rev]);
    }

    for (int m = 2; m <= n; m <<= 1) { // width of each item
        auto wm = Complex(cos(2 * pi / m), flag * sin(2 * pi / m));
        for (int k = 0; k < n; k += m) { // start idx of each item
            auto w = Complex(1, 0);
            for (int j = 0; j < m / 2; j++) { // iterate half
                Complex t = w * a[k + j + m / 2];
                Complex u = a[k + j];
                a[k + j] = u + t;
                a[k + j + m / 2] = u - t;
                w = w * wm;
            }
        }
    }

    if (flag == -1) { // if it's ifft
        for (int i = 0; i < n; i++)
            a[i].real /= n;
    }
}

vector<int> mul(const vector<int>& a, const vector<int>& b) {
    int n = int(a.size()) + int(b.size()) - 1;
    int nn = 1;
    while (nn < n)
        nn <<= 1;

    vector<Complex> fa(nn, Complex(0, 0));
    vector<Complex> fb(nn, Complex(0, 0));
```

```cpp
    for (int i = 0; i < int(a.size()); i++)
        fa[i] = Complex(a[i], 0);
    for (int i = 0; i < int(b.size()); i++)
        fb[i] = Complex(b[i], 0);

    fft(fa, +1);
    fft(fb, +1);
    for (int i = 0; i < nn; i++) {
        fa[i] = fa[i] * fb[i];
    }
    fft(fa, -1);

    vector<int> c;
    for(int i = 0; i < nn; i++) {
        int val = int(fa[i].real + 0.5);
        if (val) {
            while (int(c.size()) <= i)
                c.push_back(0);
            c[i] = 1;
        }
    }

    return c;
}
```

14.5 Combination

14.5.1 Pascal triangle

```cpp
#define N 210
ll C[N][N];

void Combination() {
    for(ll i=0; i<N; i++) {
        C[i][0] = 1;
        C[i][i] = 1;
    }

    for(ll i=2; i<N; i++) {
        for(ll j=1; j<=i; j++) {
            C[i][j] = (C[i-1][j] + C[i-1][j-1])%M; // if needed, mod it
        }
    }
}
```

14.5.2 Lucus

$$\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod{p} \tag{1}$$

where

$$n = n_k p^k + n_{k-1} p^{k-1} + \cdots + n_1 p + n_0,$$

$$m = m_k p^k + m_{k-1} p^{k-1} + \cdots + m_1 p + m_0$$

p is prime

```cpp
typedef long long ll;

ll fast_pow(ll a, ll b, ll p) {
    ll ans = 1;
    ll base = a % p;
    b = b % (p - 1); // Fermat's little theorem
    while (b) {
        if (b & 1) {
            ans = (ans * base) % p;
        }
        base = (base * base) % p;
        b >>= 1;
    }
    return ans;
}

ll inv(ll a, ll p) {
    return fast_pow(a, p - 2, p);
}

ll C(ll n, ll m, ll p) {
    if (n < m) return 0;
    m = min(m, n - m);
    ll nom = 1, den = 1;
    for (ll i = 1; i <= m; i++) {
        nom = (nom * (n - i + 1)) % p;
        den = (den * i) % p;
    }
    return (nom * inv(den, p)) % p;
}

// To make C(n, m) % p computed in O(log(p, n) * p) instead of O(m)
// https://en.wikipedia.org/wiki/Lucas's_theorem
ll lucas(ll n, ll m, ll p) {
    if (m == 0) return 1;
    return C(n % p, m % p, p) * lucas(n / p, m / p, p) % p;
}
```

14.5.3 線性

```cpp
ll binomialCoeff(ll n, ll k)
{
    ll res = 1;

    if ( k > n - k ) // Since C(n, k) = C(n, n-k)
    k = n - k;

    for (int i = 0; i < k; ++i) // n...n-k / 1...k
    {
        res *= (n - i);
        res /= (i + 1);
    }

    return res;
}
```

14.6 Chinese remainder theorem

$$\begin{cases} x \equiv r_1 & \pmod{m_1} \\ x \equiv r_2 & \pmod{m_2} \\ \dots \\ x \equiv r_n & \pmod{m_n} \end{cases} \tag{2}$$

```cpp
typedef long long ll;

struct Item {
    ll m, r;
};

Item extcrt(const vector<Item> &v)
{
    ll m1 = v[0].m, r1 = v[0].r, x, y;

    for (int i = 1; i < int(v.size()); i++) {
        ll m2 = v[i].m, r2 = v[i].r;
        ll g = extgcd(m1, m2, x, y); // now x = (m/g)^(-1)

        if ((r2 - r1) % g != 0)
            return {-1, -1};

        ll k = (r2 - r1) / g * x % (m2 / g);
        k = (k + m2 / g) % (m2 / g); // for the case k is negative

        ll m = m1 * m2 / g;
        ll r = (m1 * k + r1) % m;

        m1 = m;
        r1 = (r + m) % m; // for the case r is negative
    }

    return (Item) {
        m1, r1
    };
}
```

14.7 2-Circle relations

d = 圓心距, R, r 為半徑 ($R \geq r$)
內切: $d = R - r$
外切: $d = R + r$
內離: $d < R - r$
外離: $d > R + r$
相交: $d < R + r$ 且 $d > R - r$

14.8 Fun Facts

1. 如果 $\frac{b}{a}$ 是最簡分數，則 $1 - \frac{b}{a}$ 也是

14.9 偏序集與 Dilworth

14.9.1 原理

Dilworth 與其對偶定理適用於嚴格偏序，也適用於非嚴格偏序。(Hasse Diagram)

1. Dilworth：最小正鏈覆蓋 = 最長反鏈長度
2. 對偶定理：最小反鏈覆蓋 = 最長正鏈長度

14.9.2 題目

R x C 的格子中 N 個點 (r[i], c[i])。
定義一次旅行為從任一個點出發，只能 **往右走或往下走**。
請問：
1. 經過最多點的那次旅行是經過了多少個點?
2. 最少須要旅行幾次才能走完所有點?

即非嚴格偏序集 $P = (S, \leq)$

$$\begin{cases} S & = \{(r_i, c_i) | 0 \leq i < N\} \\ \leq & = (r_i \leq r_j \wedge c_i \leq c_j) \end{cases} \tag{3}$$

題目所求分別為
1. 最長正鏈長度，即為最小反鏈覆蓋。
2. 最小正鏈覆蓋，即為最長反鏈長度。

14.9.3 只能往右走或往下走

最長正鏈長度

1. 將點排序，r 由小到大，相同時 c **由小到大**
2. 在 c 上求最長 **單調遞增**子序列，長度即為所求

最長反鏈長度

1. 將點排序，r 由小到大，相同時 c **由小到大**
2. 在 c 上求最長 **嚴格遞減**子序列，長度即為所求

只能往斜右下走

如果題目改成 **只能往斜右下走，不能向右或向下**，則演算法改成

最長正鏈長度

1. 將點排序，r 由小到大，相同時 c **由大到小**
2. 上求最長 **嚴格遞增**子序列，長度即為所求

最長反鏈長度

1. 將點排序，r 由小到大，相同時 c **由大到小**
2. 在 c 上求最長 **單調遞減**子序列，長度即為所求

14.10 排列組合

14.10.1 排列 P

$P_k^n = \frac{n!}{(n-k)!}$

1. 從 n 個不同球中取 k 個，**不同取出順序視為不同**，方法數為 P_k^n
2. $P_2^4 = 12$

14.10.2 圓排列 Q

$Q_k^n = \frac{n!}{k(n-k)!} = \frac{P_k^n}{k}$

1. 從 n 個不同球中取 k 個，**排成一個圓**，圓只考慮相對位置，方法數為 Q_k^n
2. $Q_2^4 = 6$

14.10.3 組合 C

$$\begin{cases} C_n^n &= C_0^n = 1 \\ C_k^n &= C_{n-k}^n = \frac{k!}{(n-k)!k!} = \frac{P_k^n}{k!} \\ C_k^n &= C_{k-1}^{n-1} + C_k^{n-1} \end{cases} \tag{4}$$

1. 從 n 個不同球中取 k 個，**取出順序不影響**，方法數為 C_k^n
2. 考慮第 k 顆球取或不取：取的話轉移成從 $n-1$ 個不同球中取出 $k-1$ 個；不取的話，轉移成從 $n-1$ 個球中取 k 個，即 $C_k^n = C_{k-1}^{n-1} + C_k^{n-1}$
3. $C_2^4 = 6$

14.10.4 重複組合 H

$H_k^n = C_k^{n+k-1} = C_{n-1}^{n+k-1}$

1. 將 n 顆**相同的球**丟進 k 個**可空、不同**箱子，方法數為 H_k^n
2. $x_1 + x_2 + \cdots + x_n = k$，$(x_1, x_2, \ldots, x_n)$ 非負整數解的個數為 H_k^n
3. $x_1 + x_2 + \cdots + x_n = k$，$(x_1, x_2, \ldots, x_n)$ 正整數解的個數為 H_{k-n}^n
4. $H_2^4 = 10$

14.10.5 Stirling Number（Type I）

$$\begin{cases} S_0^n &= 0 \\ S_1^n &= 1 \\ S_k^n &= S_{k-1}^{n-1} + (n-1)S_k^{n-1} \end{cases} \tag{5}$$

1. 將 n 顆**不同的球**丟進 k 個 **非空、相同**箱子，每個箱子為一 **圓排列**，方法數為 S_k^n
2. 考慮第 n 顆球：可以自己一個箱子，即其他 $n-1$ 個球丟進 $k-1$ 個箱子；也可以 $n-1$ 個球丟進 k 個箱子，這顆球插到任一顆球旁邊。所以得 $S_k^n = S_{k-1}^{n-1} + (n-1)S_k^{n-1}$
3. $S_2^4 = 11$

14.10.6 Stirling Number（Type II）

$$\begin{cases} S_n^n &= S_1^n = 1 \\ S_k^n &= S_{k-1}^{n-1} + kS_k^{n-1} \end{cases} \tag{6}$$

1. 將 n 顆 **不同的球**丟進 k 個 **非空、相同**箱子，每一個箱子為一 **集合**，方法數為 S_k^n
2. 考慮第 n 顆球：可以自己一個箱子，即其他 $n-1$ 個球丟進 $k-1$ 個箱子；也可以 $n-1$ 個球丟進 k 個箱子，這顆球丟進其中一個箱子。所以得 $S_k^n = S_{k-1}^{n-1} + kS_k^{n-1}$
3. $S_2^4 = 7$

15 Dynamic Programming - Problems collection

```
# 零一背包 (poj 1276)
fill(dp, dp + W + 1, 0);
for (int i = 0; i < N; i++)
    for (int j = W; j >= items[i].w; j--)
        dp[j] = max(dp[j], dp[j - w[i]] + v[i]);
return dp[W];

# 多重背包二進位拆解 (poj 1276)
for_each(ll v, w, num) {
    for (ll k = 1; k <= num; k *= 2) {
        items.push_back((Item) {k * v, k * w});
        num -= k;
    }
    if (num > 0)
        items.push_back((Item) {num * v, num * w});
}

# 完全背包
dp[i][j] = 前 i + 1 個物品，在重量 j 下所能組出的最大價值
第 i 個物品，不放或至少放一個
dp[i][j] = max(dp[i - 1][j], dp[i][j - w[i]] + v[i])
fill(dp, dp + W + 1, 0);
for (int i = 0; i < N; i++)
    for (int j = w[i]; j <= W; j++)
        dp[j] = max(dp[j], dp[j - w[i]] + v[i]);
return dp[W];

# Coin Change (2015 桂冠賽 E)
dp[i][j] = 前 i + 1 個物品，組出 j 元的方法數
第 i 個物品，不用或用至少一個
dp[i][j] = dp[i - 1][j] + dp[i][j - coin[i]]

# Cutting Sticks (2015 桂冠賽 F)
補上二個切點在最左與最右
dp[i][j] = 使 (i, j) 區間中的所有切點都被切的最小成本
dp[i][j] = min(dp[i][c] + dp[c][j] + (p[j] - p[i]) for i < c < j)
dp[i][i + 1] = 0
ans = dp[0][N + 1]

# Throwing a Party (itsa dp 06)
給定一棵有根樹，代表公司職位層級圖，每個人有其權重，現從中選一個點集合出來，
且一個人不能與其上司一都在集合中，並最大化集合的權重和，輸出該總和。
dp[u][0/1] = u 在或不在集合中，以 u 為根的子樹最大權重和
dp[u][0] = max(max(dp[c][0], dp[c][1]) for children c of u) + val[u]
dp[u][1] = max(dp[c][0] for children c of u)
bottom up dp
```


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```
# LIS (O(N^2))
dp[i] = 以 i 為結尾的 LIS 的長度
dp[i] = max(dp[j] for 0 <= j < i) + 1
ans = max(dp)

# LIS (O(nlgn)), poj 1631
dp[i] = 長度為 i + 1 的 LIS 的最後一項的最小值，不存在時為 INF
fill(dp, dp + N, INF);
for (int i = 0; i < N; i++)
    *lower_bound(dp, dp + N, A[i]) = A[i];
ans = lower_bound(dp, dp + N, INF) - dp;

# Maximum Subarray

# Not equal on a Segment (cf edu7 C)
給定長度為 n 的陣列 a[] 與 m 個詢問。
針對每個詢問 l, r, x 請輸出 a[l, r] 中不等於 x 的任一位置。
不存在時輸出 -1
dp[i] = max j such that j < i and a[j] != a[i]
dp[0] = -1
dp[i] = dp[i - 1] if a[i] == a[i - 1] else i - 1
針對每筆詢問 l, r, x
1. a[r] != x               -> 輸出 r
2. a[r] = x && dp[r] >= l   -> 輸出 dp[r]
3. a[r] = x && dp[r] < l    -> 輸出 -1

# bitmask dp, poj 2686
給定一個無向帶權圖，代表 M 個城市之間的路，與 N 張車票，
每張車票有一個數值 t[i]，若欲使用車票 t[i] 從城市 u 經由路徑 d[u][v] 走到城市 v，
所花的時間為 d[u][v] / t[i]。請問，從城市 A 走到城市 B 最快要多久?
dp[S][v] = 從城市 A 到城市 v 的最少時間，其中 S 為用過的車票的集合
考慮前一個城市 u 是誰，使用哪個車票 t[i] 而來，可以得到轉移方程式:
dp[S][v] = min([
    dp[S - {v}][u] + d[u][v] / t[i]
    for all city u has edge to v, for all ticket in S
])

# Tug of War
N 個人參加拔河比賽，每個人有其重量 w[i]，欲使二隊的人數最多只差一，雙方的重量和越接近越好
請問二隊的重量和分別是多少?
dp[i][j][k] = 只考慮前 i + 1 個人，可不可以使左堆的重量為 j，且左堆的人數為 k
dp[i][j][k] = dp[i - 1][j - w[i][k - 1] or dp[i - 1][j][k]
dp[i][j] = (dp[i - 1][j - w[i]] << 1) | (dp[i - 1][j])

# Modulo Sum (cf 319 B)
給定長度為 N 的序列 A 與一正整數 M，請問該序列中有無一個子序列，子序列的總合是 M 的倍數
若 N > M，則根據鴿籠原理，必有至少兩個前綴和的值 mod M 為相同值，解必定存在
dp[i][j] = 前 i + 1 個數可否組出 mod m = j 的數
dp[i][j] = true if
    dp[i - 1][(j - (a[i] mod m)) mod m] or
    dp[i - 1][j] or
    j = a[i] % m

# POJ 2229
給定正整數 N，請問將 N 拆成一堆 2^x 之和的方法數
dp[i] = 拆解 N 的方法數
dp[i] = dp[i / 2] if i is odd
      = dp[i - 1] + dp[i / 2] if i is even

# POJ 3616
給定 N 個區間 [s, t)，每個區間有權重 w[i]，從中選出一些不相交的區間，使權重和最大
dp[i] = 考慮前 i + 1 個區間，且必選第 i 個區間的最大權重和
dp[i] = max(dp[j] | 0 <= j < i) + w[i]
ans = max(dp)

# POJ 2184
N 隻牛每隻牛有權重 <s, f>，從中選出一些牛的集合，
使得 sum(s) + sum(f) 最大，且 sum(s) > 0, sum(f) > 0。
枚舉 sum(s) ，將 sum(s) 視為重量對 f 做零一背包。

# POJ 3666
給定長度為 N 的序列，請問最少要加多少值，使得序列單調遞增
dp[i][j] = 使序列前 i+1 項變為單調，且將 A[i] 變為「第 j 小的數」的最小成本
dp[i][j] = min(dp[i - 1][k] | 0 <= k <= j) + abs(S[j] - A[i])
min(dp[i - 1][k] | 0 <= k <= j) 動態維護
for (int j = 0; j < N; j++)
    dp[0][j] = abs(S[j] - A[0]);
for (int i = 1; i < N; i++) {
    int pre_min_cost = dp[i][0];
    for (int j = 0; j < N; j++) {
        pre_min_cost = min(pre_min_cost, dp[i-1][j]);
        dp[i][j] = pre_min_cost + abs(S[j] - A[i]);
    }
}
ans = min(dp[N - 1])

# POJ 3734
N 個 blocks 上色，R, G, Y, B，上完色後紅色的數量與綠色的數量都要是偶數。請問方法數。
dp[i][0/1/2/3] = 前 i 個 blocks 上完色，紅色數量為奇數/偶數，綠色數量為數/偶數
用遞推，考慮第 i + 1 個 block 的顏色，找出個狀態的轉移，整理可發現
dp[i + 1][0] = dp[i][2] + dp[i][1] + 2 * dp[i][0]
dp[i + 1][1] = dp[i][3] + dp[i][0] + 2 * dp[i][1]
dp[i + 1][2] = dp[i][0] + dp[i][3] + 2 * dp[i][2]
dp[i + 1][3] = dp[i][1] + dp[i][2] + 2 * dp[i][3]
矩陣快速冪加速求 dp[N - 1][0][0]

# POJ 3171
數線上，給定 N 個區間 [s[i], t[i]]，每個區間有其代價，求覆蓋區間 [M, E] 的最小代價。
dp[i][j] = 最多使用前 i + 1 個區間，使 [M, j] 被覆蓋的最小代價
考慮第 i 個區間用或不用，可得:
dp[i][j] =
    1. min(dp[i - 1][k] for k in [s[i] - 1, t[i]]) + cost[i] if j = t[i]
    2. dp[i - 1][j] if j ≠ t[i]
壓空間，使用線段樹加速。
dp[t[i]] = min(dp[t[i]],
    min(dp[i - 1][k] for k in [s[i] - 1, t[i]]) + cost[i]
)
fill(dp, dp + E + 1, INF);
seg.init(E + 1, INF);
int idx = 0;
while (idx < N && A[idx].s == 0) {
    dp[A[idx].t] = min(dp[A[idx].t], A[idx].cost);
    seg.update(A[idx].t, A[idx].cost);
    idx++;
}
for (int i = idx; i < N; i++) {
    ll v = min(dp[A[i].t], seg.query(A[i].s - 1, A[i].t + 1) + A[i].cost);
    dp[A[i].t] = v;
    seg.update(A[i].t, v);
}
```

```cpp
// dp[S][v] = 訪問過的點集合為 S，且從目前所在點 v，回到頂點 0 的路徑的最小權重和。
// （頂點 0 尚未訪問）
//
// 從所有尚未訪問過的集合中找轉移的最小值
// dp[V][0] = 0
// dp[S][v] = min([
//     dp[S 連集 {u}][u] + d(v, u) for u not in S
// ])

const int MAX_N = ...;
const int INF = 0x3f3f3f3f;
int N;
int dp[1 << MAX_N][MAX_N];

int tsp() {
    for (int S = 0; S < (1 << N); S++)
        fill(dp[S], dp[S] + N, INF);

    dp[(1 << N) - 1][0] = 0;
    for (int S = (1 << N) - 2; S >= 0; S--)
        for (int v = 0; v < N; v++)
            for (int u = 0; u < N; u++)
                if (!((S >> u) & 1))
                    dp[S][v] = min(dp[S][v], dp[S | (1 << u)][u] + d[v][u]);

    return dp[0][0];
}
```