CSE 592 - Convex Optimization
HW 3

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1 Log-Barrier function

1.1 Implementation

Done in hw3-functions.py

1.2 Experiments

1. **main_quadratic()**

   - We can observe that the number of iterations taken by Gradient descent (193 iterations) are much larger than that required by Newton's (6 iterations) algorithm. This is just as expected, the reason being that the functions is Quadratic, and Newton's algorithm works very well for Quadratic functions (Quadratic convergence). For every iteration, gradient descent points in the negative direction of the gradient which may not be the exact direction of the optimum. So, it takes many more iterations that Newton's method. Moreover, if we zoom in to the optimum, we can see that Newton and GD do not converge to the exact same value, this is because Newton's method is a second order approximation algorithm and GD is a first order approximation algorithm.
   - Plot of number of iterations taken by GD and Newton's

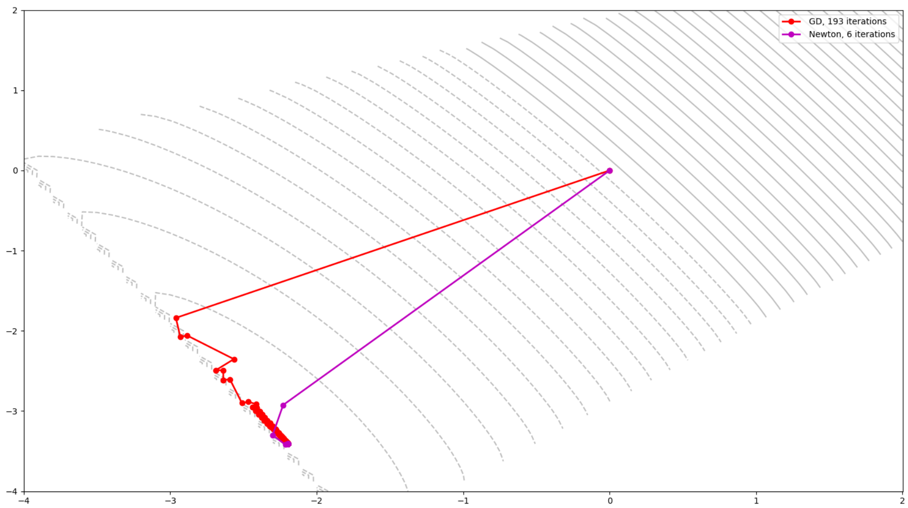

Figure 1: Number of iterations for Newton and GD on main_quadratic()

2. **main_quadratic2()**

- The log barrier is an approximation of the Indicator function and is dependent on the value of parameter t. The effect of t is such that, as $t \to \infty$ the approximation becomes closer to the Indicator function. Here, the Indicator function is:

$$I_{-}(u) = \begin{cases} 0, & u \leq 0 \\ \infty, & u > 0 \end{cases}$$

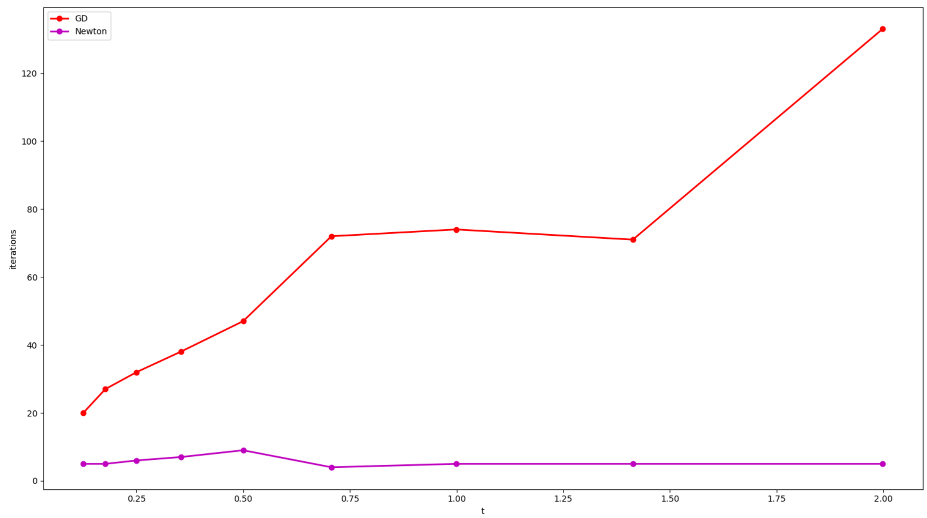

Figure 2: Number of iterations for Newton and GD wrt t

- The plot shows the effect of t on number of iterations taken by GD and Newton

  We can see that as the value of t increases, the number of iterations for Gradient Descent increases. This is because, as we increase the value of t, the optimum becomes closer to the boundary. Consequently, the number of iterations for GD also increases.

  For Newton's method, as the value of t increases, the number of iterations do not increase with t as at every iteration, the newton's step gives the direction of the actual optimum, so the whether the point is closer to the boundary or not does not affect it's number of iterations.

2 Log Barrier method

2.1 Implementation

Done in code

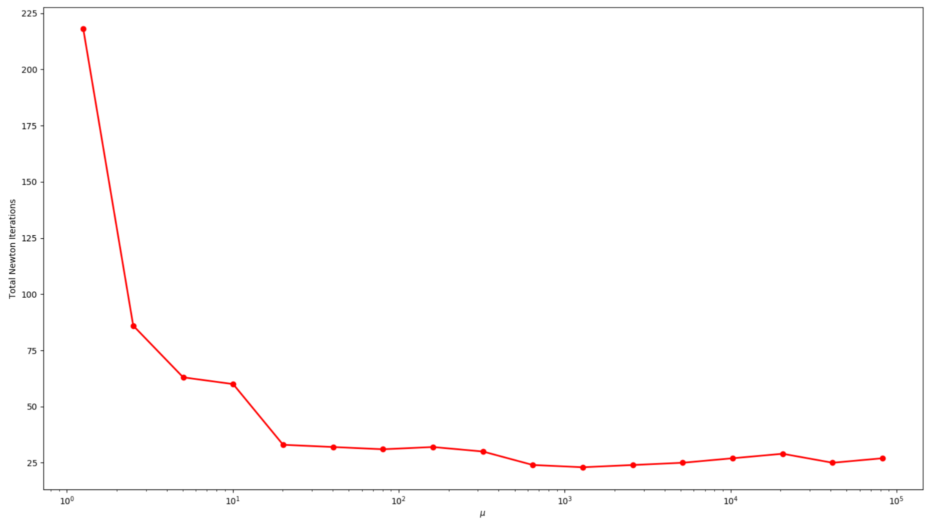

Figure 3: Number of iterations for Newton vs μ

2.2 Experiments

1. **Describe the dependence which you expect to see here?**

   For small values of μ, the number of inner Newton iterations should be small.
   As the value of μ increases, the number of inner Newton iterations should also increase.
   So, there should be a direct dependence of number of inner Newton iterates on μ
   However, the total number of Newton Iterations (inner and outer(centering step)) changes inversely with μ

2. **Does the plot meet your expectations?**
   The plot is of total Newton iterations (outer + inner) vs μ. The plot meets the theoretical explanation that the total inner (Newton) iterations and Outer(centering) iterations decrease as value of μ increases. However, after a certain value (μ greater than 10), the number of iterations do not change much. As μ increases over this range, the decrease in the number of iterations is overshadowed by an increase in the number of Newton steps per outer iteration.

3. **What is the solution which is found for this LP?**
   Solution found: 30610.00006064