Q Learning

COMP4211

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Q Function

At state s, take action a, then follow policy π

- assume deterministic rewards and actions
- discounted cumulative reward:

$$r(s, a) + \gamma V^{\pi}(\delta(s, a)) \equiv Q^{\pi}(s, a)$$

- action value function for policy π

For the optimal policy, $Q^*(s, a) = r(s, a) + \gamma V^*(\delta(s, a))$

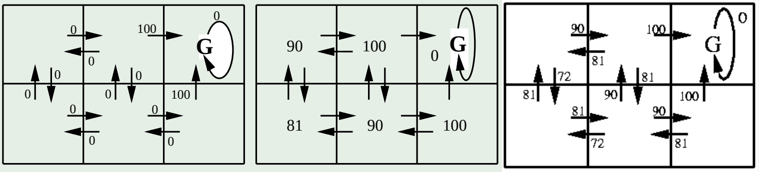
- maximum discounted cumulative reward

Optimal policy:

$$\pi^*(s) = \arg\max_a [r(s, a) + \gamma V^*(\delta(s, a))] = \arg\max_a Q^*(s, a)$$

Example

problem; state value function; action value function ($\gamma = 0.9$)

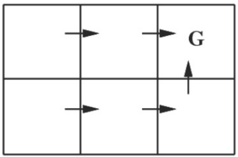


e.g., $(s, a) =$ (bottom left state; move up)

- Q: $0 + \gamma \times 90 = 81$

e.g., $(s, a) =$ (bottom right state; move up)

- Q: $100 + \gamma \times 0 = 100$



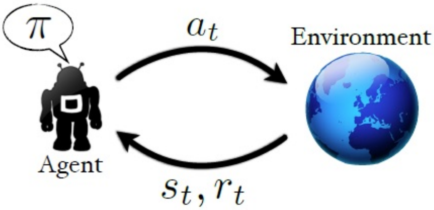
One optimal policy

Unknown Environment

For the optimal policy

$$
\begin{aligned}
Q^*(s,a) &= r(s,a) + \gamma V^*(\delta(s,a)) \\
&= \boxed{r(s,a) + \gamma \max_{a'} Q^*(\delta(s,a), a')}
\end{aligned}
$$

What if the agent does not know δ and r?



learn an approximation of Q^*

- if the agent knows Q^*, it can choose optimal action even without knowing r and δ!

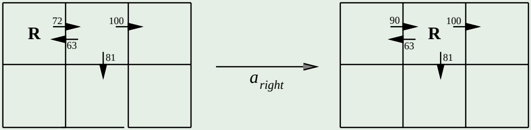
How to Approximate Q^*?

$$Q^*(s, a) = r(s, a) + \gamma \max_{a'} Q^*(\delta(s, a), a')$$

Learn directly from experience:

- observe the current state s
- choose and execute an action a
- observe the resulting reward $r = r(s, a)$ and the new state $s' = \delta(s, a)$
  - does not need to know $\delta(s, a)$ and $r(s, a)$ explicitly
- update $\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a')$

Example ($\hat{Q}$)



Initial state: s_1 Next state: s_2

$$\begin{aligned} \hat{Q}(s_1, a_{right}) &\leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a') \\ &\leftarrow 0 + 0.9 \max\{63, 81, 100\} = 90 \end{aligned}$$

Q Learning

```
begin
    for each s, a; initialize table entry Q̂(s, a) ← 0
    observe current state s;
    repeat
        select an action a and execute it;
        receive immediate reward r;
        observe the new state s′;
        update the table entry for Q̂(s, a) as:

                    Q̂(s, a) ← r + γ max_a′ Q̂(s′, a′)

        s ← s′;
    until;
end
```

begin

- for each s, a; initialize table entry $\hat{Q}(s, a) \leftarrow 0$
- observe current state s;
- **repeat**
  - select an action a and execute it;
  - receive immediate reward r;
  - observe the new state s';
  - update the table entry for $\hat{Q}(s, a)$ as:

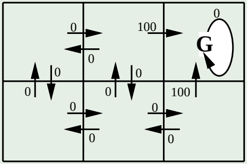
$$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a')$$

  - $s \leftarrow s'$;
- **until**;

end

$\hat{Q}$ converges to Q in the limit

Example (numbers are the rewards)



$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

With all the $\hat{Q}$ values initialized to zero

- the agent will make no changes to any $\hat{Q}$ entry until it happens to reach the goal state and receive a nonzero reward
- refine the $\hat{Q}$ value for the single transition leading into the goal state

On the next episode

- if the agent passes through this state adjacent to the goal state, its nonzero $\hat{Q}$ value will allow refining the value for some transition two steps from the goal, and so on

Given a sufficient number of training episodes, the information will propagate through the entire state-action space

Non-Deterministic Rewards and Actions

Reward function $r(s,a)$ and action transition function $\delta(s,a)$ may have probabilistic outcomes

$$
\begin{aligned}
V^{\pi}(s) &\equiv E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots] \\
Q^{\pi}(s,a) &\equiv E[r(s,a) + \gamma V^{\pi}(\delta(s,a))] \\
Q^{*}(s,a) &= E[r(s,a) + \gamma V^{*}(\delta(s,a))] \\
&= E[r(s,a)] + \gamma \sum_{s'} P(s,s',a) V^{*}(s') \\
&= E[r(s,a)] + \gamma \sum_{s'} P(s,s',a) \max_{a'} Q^{*}(s',a')
\end{aligned}
$$

Learning Q^*

$$Q^*(s,a) = E[r(s,a)] + \gamma \sum_{s'} P(s,s',a) \max_{a'} Q^*(s',a')$$

Deterministic case:

$$Q(s_t,a_t) \leftarrow r_t + \gamma \max_a Q(s_{t+1},a)$$

Non-deterministic case:

- a nondeterministic reward function will produce different rewards each time the transition (s_t,a_t) is repeated

$$Q(s_t,a_t) \leftarrow (1-\alpha) Q(s_t,a_t) + \alpha[r_t + \gamma \max_a Q(s_{t+1},a)]$$

  - cf. perceptron learning
  - the new $Q(s_t,a_t)$ is an average between $Q(s_t,a_t)$ and $r_t + \gamma \max_a Q(s_{t+1},a)$

$$Q(s_t,a_t) \leftarrow Q(s_t,a_t) + \alpha[r_t + \gamma \max_a Q(s_{t+1},a) - Q(s_t,a_t)]$$

- can still prove convergence to Q^*

Learning Q^*...

How to set α?

- as training progresses, reduce α

Example

$\alpha_n = \frac{1}{1+visits_n(s,a)}$

- $visits_n(s, a)$: total number of times this state-action pair has been visited up to and including the nth iteration

demo on Q learning

Action Selection

begin

for each s, a; initialize table entry $\hat{Q}(s, a) \leftarrow 0$

observe current state s;

repeat

select an action a and execute it;

receive immediate reward r;

observe the new state s';

update the table entry for $\hat{Q}(s, a)$ as:;

$$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a')$$

$s \leftarrow s'$;

until *convergence*;

end

How to select actions when the agent is in state s?

Greedy Action Selection

Obvious strategy:

- always select the action that looks best

$$\pi(s) = \arg\max_a \hat{Q}(s, a)$$

- greedy policy

Problem

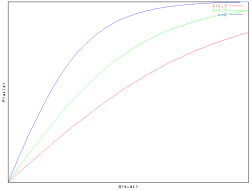
- over-commit to actions that are found during early training to have high $\hat{Q}$ values
- fail to explore other actions that may have even higher values

Be less greedy!

Exploit vs Explore

Select action a_i with probability

- $\pi(s, a_i) = k^{\hat{Q}(s,a_i)} / \sum_j k^{\hat{Q}(s,a_j)}, \quad k > 0$



Large k:

- assign higher probabilities to actions with above average $\hat{Q}$
- exploit

Small k:

- assign higher probabilities to other actions
- explore actions that do not currently have high $\hat{Q}$ values

ϵ-Greedy Policy

Alternatively

- be greedy most of the time (exploit)
- occasionally (with probability ϵ), take a random action (explore)
- surprising, this is the state of the art