

# Introduction to Aerial Robotics

## Lecture 4

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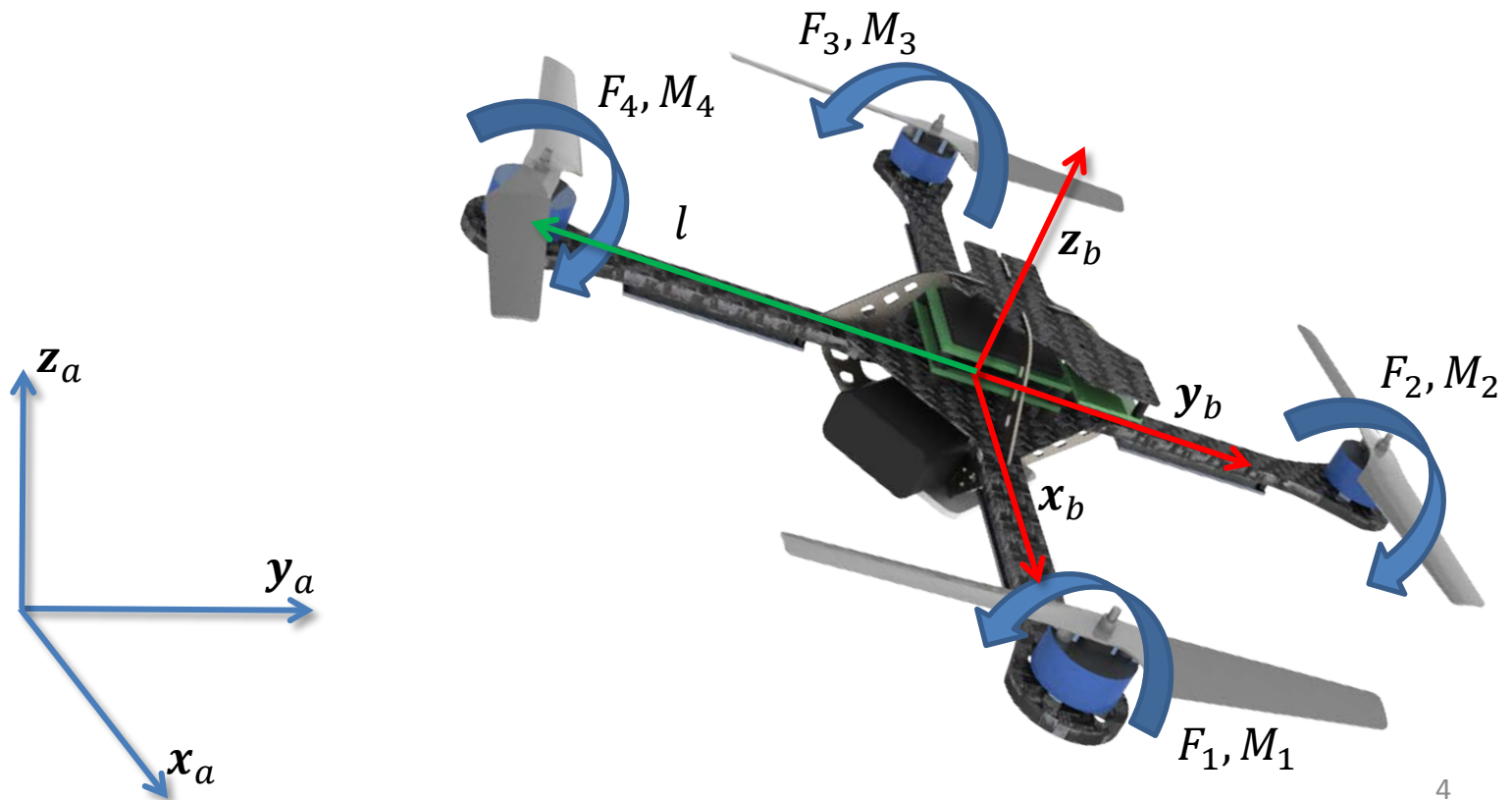
29 September 2015

# Outline

- Review: Quadrotor Control
- Trajectory Generation

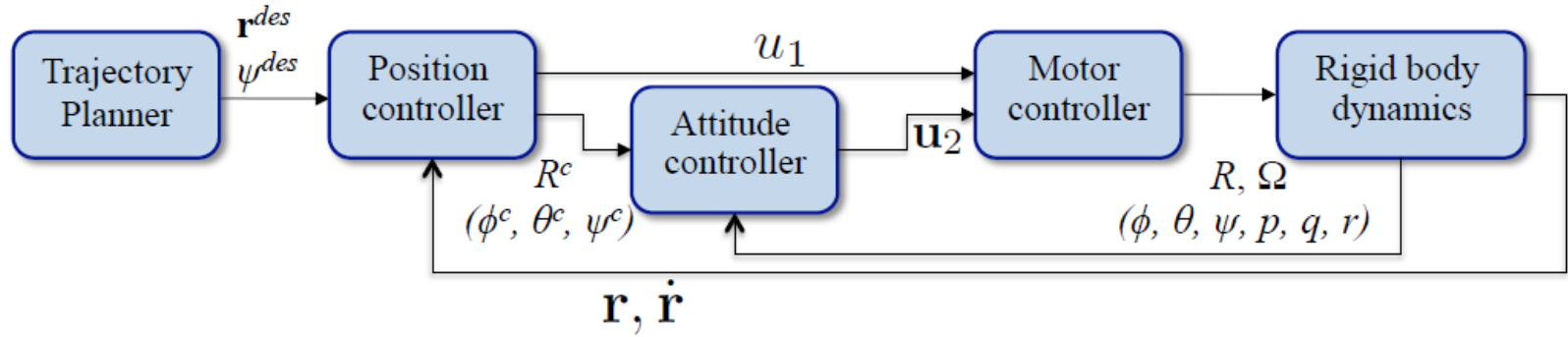
# Review: Quadrotor Control

# Quadrotor Dynamics



# Quadrotor Dynamics

- Motor model:  $\dot{\omega}_i = k_m(\omega_i^{des} - \omega_i)$
- Thrust from individual motor:  $F_i = k_F \omega_i^2$
- Moment from individual motor:  $M_i = k_M \omega_i^2$
- Newton Equation:  $m\ddot{\mathbf{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + \mathbf{R} \begin{bmatrix} 0 \\ 0 \\ F_1 + F_2 + F_3 + F_4 \end{bmatrix}$
- Euler Equation:  $\mathbf{I} \cdot \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} + \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \times \mathbf{I} \cdot \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} l(F_2 - F_4) \\ l(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix}$



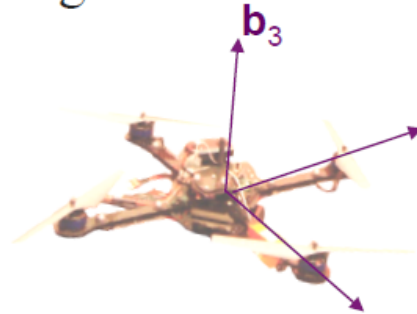
$$m\ddot{\mathbf{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ F_1 + F_2 + F_3 + F_4 \end{bmatrix}$$

$u_1$

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$u_2$

# Control for Hovering



$$m\ddot{\mathbf{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ F_1 + F_2 + F_3 + F_4 \end{bmatrix}$$

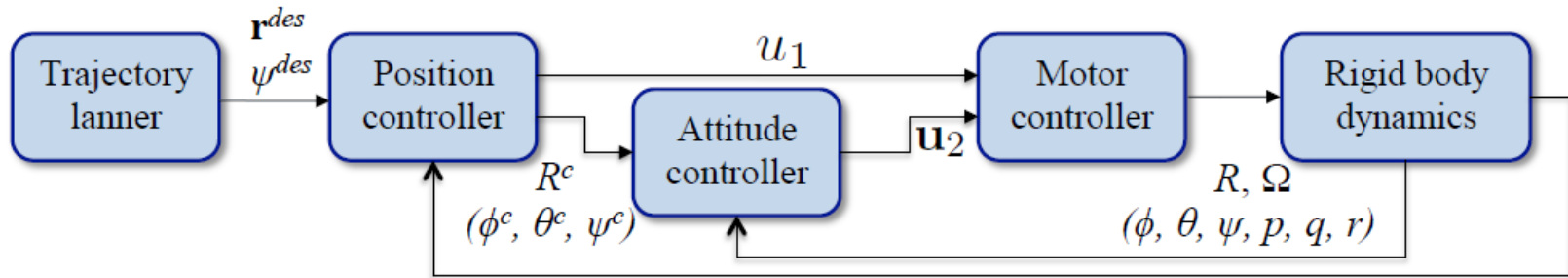
$u_1$

Linearization

$$(u_1 \sim mg, \theta \sim 0, \phi \sim 0, \psi \sim \psi_0)$$

$$\ddot{r}_1 = \ddot{x} = g(\theta \cos \psi + \phi \sin \psi)$$

$$\ddot{r}_2 = \ddot{y} = g(\theta \sin \psi - \phi \cos \psi)$$



$$(\ddot{r}_{i,des} - \ddot{r}_{i,c}) + k_{d,i}(\dot{r}_{i,des} - \dot{r}_i) + k_{p,i}(r_{i,des} - \dot{r}_i) = 0$$

The equation is derived from the following inputs:

- commanded**:  $\ddot{r}_{i,des}$
- actual (feedback)**:  $\dot{r}_i$
- specified**:  $\ddot{r}_{i,c}$

$$u_1 = m(g + \ddot{r}_{3,c})$$

$$\phi_c = \frac{1}{g}(\ddot{r}_{1,c} \sin \psi_{des} - \ddot{r}_{2,c} \cos \psi_{des})$$

$$\theta_c = \frac{1}{g}(\ddot{r}_{1,c} \cos \psi_{des} + \ddot{r}_{2,c} \sin \psi_{des})$$

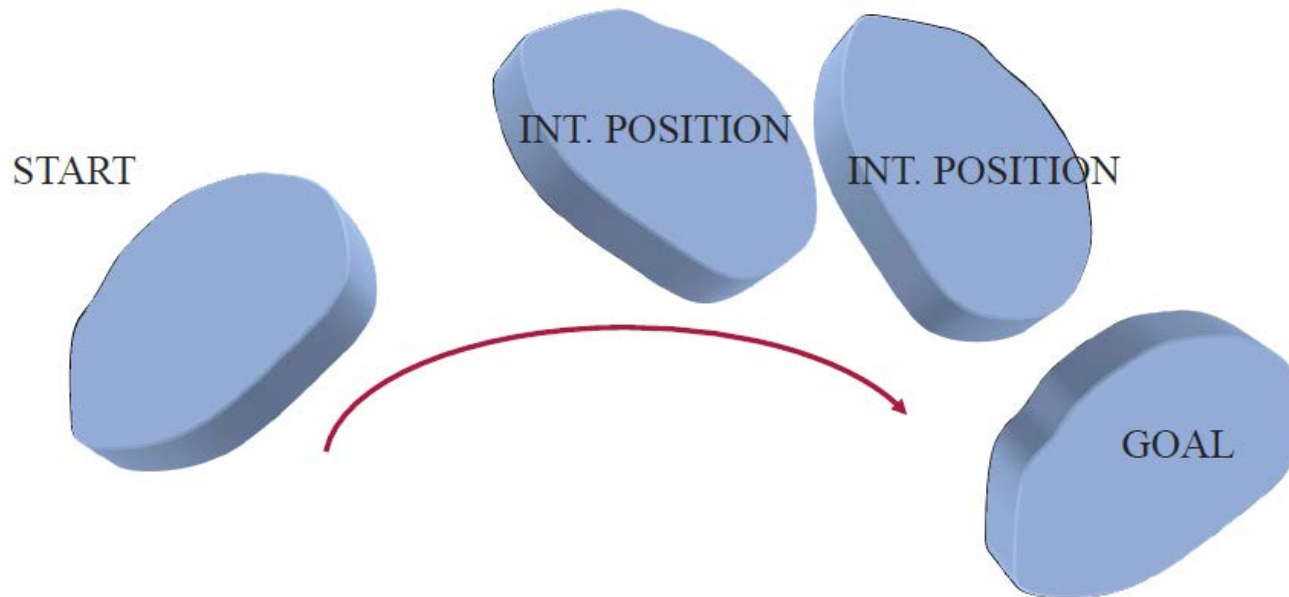
$$\mathbf{u}_2 = \begin{bmatrix} k_{p,\phi}(\phi_c - \phi) + k_{d,\phi}(p_c - p) \\ k_{p,\theta}(\theta_c - \theta) + k_{d,\theta}(q_c - q) \\ k_{p,\psi}(\psi_c - \psi) + k_{d,\psi}(r_c - r) \end{bmatrix}$$



# Trajectory Generation

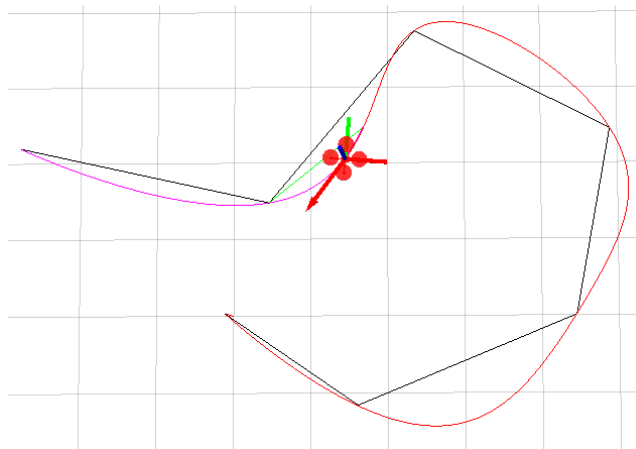
# Smooth 3D Trajectories

- Smooth trajectory is beneficial for autonomous flight
  - Smooth trajectories respect the continuous nature of aerial robots
  - The robot should not stop at turns



# Smooth 3D Trajectories

- General setup
  - Start, goal positions (orientations)
  - Waypoint positions (orientations)
    - Waypoints can be found by path planning ( $A^*$ ,  $RRT^*$ , etc)
  - Smoothness criterion
    - Generally translates into minimizing rate of change of “input”



# Differential Flatness

- The states and the inputs of a quadrotor can be written as algebraic functions of four carefully selected flat outputs and their derivatives
  - Enables automated generation of trajectories
  - Any smooth trajectory in the space of flat outputs (with reasonably bounded derivatives) can be followed by the under-actuated quadrotor
  - A possible choice:
    - $\sigma = [x, y, z, \psi]^T$
  - Trajectory in the space of flat outputs:
    - $\sigma(t) = [T_0, T_M] \rightarrow \mathbb{R}^3 \times SO(2)$

# Differential Flatness

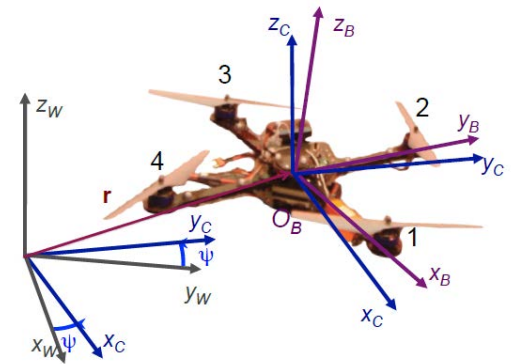
- Quadrotor states
  - Position, orientation, linear velocity, angular velocity

$$\mathbf{x} = [x, y, z, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, p, q, r]^T$$

- Equation of motions:

$$m\ddot{\mathbf{r}} = -mg\mathbf{z}_W + u_1\mathbf{z}_B.$$

$$\dot{\omega}_{B\mathcal{W}} = \mathcal{I}^{-1} \left[ -\omega_{B\mathcal{W}} \times \mathcal{I}\omega_{B\mathcal{W}} + \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix} \right]$$



- Position, velocity, and acceleration are simply derivatives of the flat outputs

# Differential Flatness

- Orientation

- Quadrotor state:

$$\mathbf{x} = [x, y, z, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, p, q, r]^T$$

- From the equation of motion:

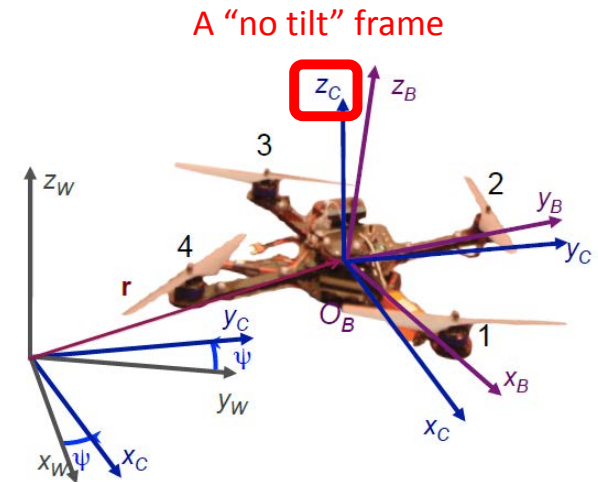
$$\mathbf{z}_B = \frac{\mathbf{t}}{\|\mathbf{t}\|}, \quad \mathbf{t} = [\ddot{\sigma}_1, \ddot{\sigma}_2, \ddot{\sigma}_3 + g]^T$$

- Define the yaw vector (Z-X-Y Euler):

$$\mathbf{x}_C = [\cos \sigma_4, \sin \sigma_4, 0]^T$$

- Orientation can be expressed in terms of flat outputs

$$\mathbf{y}_B = \frac{\mathbf{z}_B \times \mathbf{x}_C}{\|\mathbf{z}_B \times \mathbf{x}_C\|}, \quad \mathbf{x}_B = \mathbf{y}_B \times \mathbf{z}_B \quad {}^W R_B = [\mathbf{x}_B \quad \mathbf{y}_B \quad \mathbf{z}_B]$$



# Differential Flatness

- Angular velocity

- Quadrotor state:

$$\mathbf{x} = [x, y, z, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, p, q, r]^T$$

- Take the derivative of the equation of motion

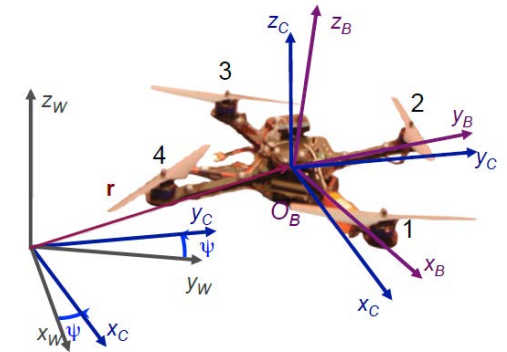
$$m\ddot{\mathbf{r}} = -mg\mathbf{z}_W + u_1\mathbf{z}_B. \quad \longrightarrow \quad m\dot{\mathbf{a}} = \dot{u}_1\mathbf{z}_B + \omega_{BW} \times u_1\mathbf{z}_B$$

- Quadrotors only have vertical thrust:

$$\dot{u}_1 = \mathbf{z}_B \cdot m\dot{\mathbf{a}}$$

- We have:

$$\mathbf{h}_\omega = \omega_{BW} \times \mathbf{z}_B = \frac{m}{u_1}(\dot{\mathbf{a}} - (\mathbf{z}_B \cdot \dot{\mathbf{a}})\mathbf{z}_B).$$



Body angular velocity  
viewed in the world frame

# Differential Flatness

- Angular velocity

- Quadrotor state:

$$\mathbf{x} = [x, y, z, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, p, q, r]^T$$

- We have:

$$\mathbf{h}_\omega = \omega_{B\mathcal{W}} \times \mathbf{z}_B = \frac{m}{u_1} (\dot{\mathbf{a}} - (\mathbf{z}_B \cdot \dot{\mathbf{a}}) \mathbf{z}_B).$$

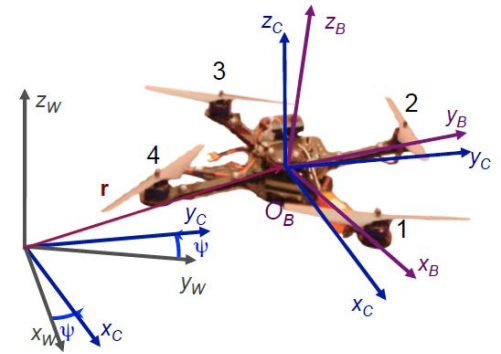
- This is the projection of  $\frac{m}{u_1} \dot{\mathbf{a}}$  onto the  $x_B - y_B$  plane

- We know that:

$$\omega_{B\mathcal{W}} = p\mathbf{x}_B + q\mathbf{y}_B + r\mathbf{z}_B.$$

- Angular velocities along  $x_B$  and  $y_B$  directions can be found as:

$$p = -\mathbf{h}_\omega \cdot \mathbf{y}_B, \quad q = \mathbf{h}_\omega \cdot \mathbf{x}_B$$





# Differential Flatness

- Angular velocity

- Quadrotor state:

$$\mathbf{x} = [x, y, z, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, p, q, r]^T$$

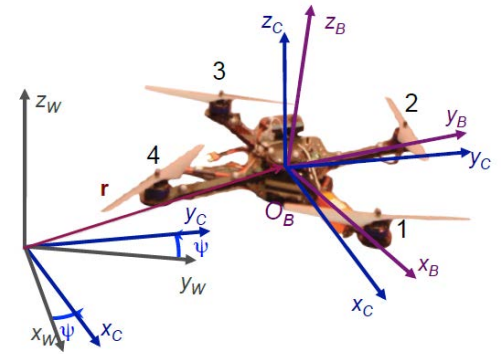
- We have:

$$\mathbf{h}_\omega = \omega_{BW} \times \mathbf{z}_B = \frac{m}{u_1} (\dot{\mathbf{a}} - (\mathbf{z}_B \cdot \dot{\mathbf{a}}) \mathbf{z}_B).$$

- This is the projection of  $\frac{m}{u_1} \dot{\mathbf{a}}$  onto the  $x_B - y_B$  plane

- Since  $\omega_{BW} = \omega_{BC} + \omega_{CW}$ , where  $\omega_{BC}$  has no  $z_B$  component:

$$r = \omega_{CW} \cdot \mathbf{z}_B = \dot{\psi} \mathbf{z}_W \cdot \mathbf{z}_B.$$



# Differential Flatness

- Summary

- Quadrotor state:

$$\mathbf{x} = [x, y, z, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, p, q, r]^T$$

- Flat outputs:

- $\sigma = [x, y, z, \psi]^T$

- Position, velocity, acceleration

- Derivatives of flat outputs

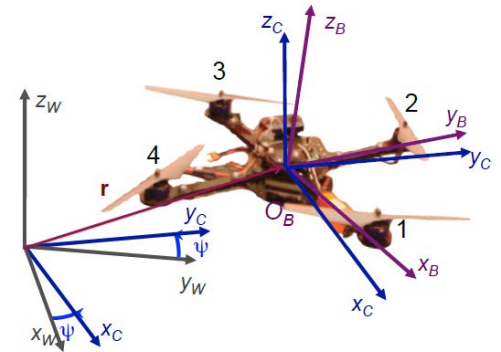
- Orientation

$$\mathbf{x}_C = [\cos \sigma_4, \sin \sigma_4, 0]^T \longrightarrow {}^W R_B = [\mathbf{x}_B \ \mathbf{y}_B \ \mathbf{z}_B]$$

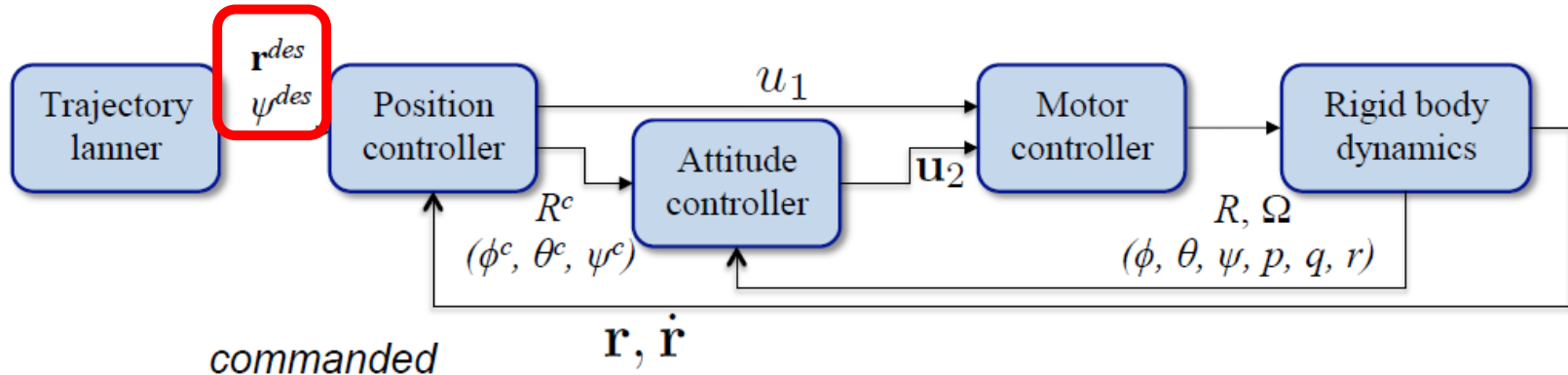
- Angular velocity

$$p = -\mathbf{h}_\omega \cdot \mathbf{y}_B, \quad q = \mathbf{h}_\omega \cdot \mathbf{x}_B$$

$$r = \omega_{CW} \cdot \mathbf{z}_B = \dot{\psi} \mathbf{z}_W \cdot \mathbf{z}_B.$$



**How about Inputs ( $u_1, u_2$ )??**



$$(\ddot{r}_{i,des} - \ddot{r}_{i,c}) + k_{d,i}(\dot{r}_{i,des} - \dot{r}_i) + k_{p,i}(r_{i,des} - \overset{\text{actual (feedback)}}{\downarrow} \dot{r}_i) = 0$$

$\uparrow$  *specified*

$$u_1 = m(g + \ddot{r}_{3,c})$$

$$\phi_c = \frac{1}{g}(\ddot{r}_{1,c} \sin \psi_{des} - \ddot{r}_{2,c} \cos \psi_{des})$$

$$\theta_c = \frac{1}{g}(\ddot{r}_{1,c} \cos \psi_{des} + \ddot{r}_{2,c} \sin \psi_{des})$$

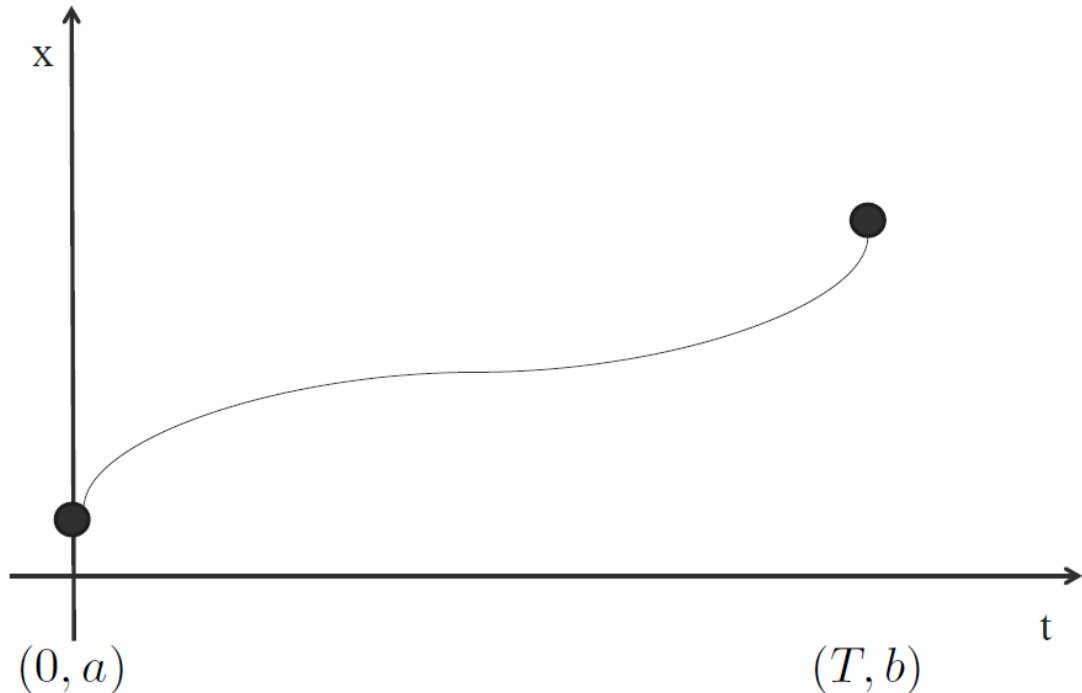
$$\mathbf{u}_2 = \begin{bmatrix} k_{p,\phi}(\phi_c - \phi) + k_{d,\phi}(p_c - p) \\ k_{p,\theta}(\theta_c - \theta) + k_{d,\theta}(q_c - q) \\ k_{p,\psi}(\psi_c - \psi) + k_{d,\psi}(r_c - r) \end{bmatrix}$$

# Polynomial Trajectories

- Flat outputs:
  - $\sigma = [x, y, z, \psi]^T$
- Trajectory in the space of flat outputs:
  - $\sigma(t) = [T_0, T_M] \rightarrow \mathbb{R}^3 \times SO(2)$
- Polynomial functions can be used to specify trajectories in the space of flat outputs
  - Easy determination of smoothness criterion with polynomial orders
  - Easy and closed form calculation of derivatives
  - Decoupled trajectory generation in three dimensions

# Smooth 1D Trajectory

- Design a trajectory  $x(t)$  such that:
  - $x(0) = a$
  - $x(T) = b$



# Smooth 1D Trajectory

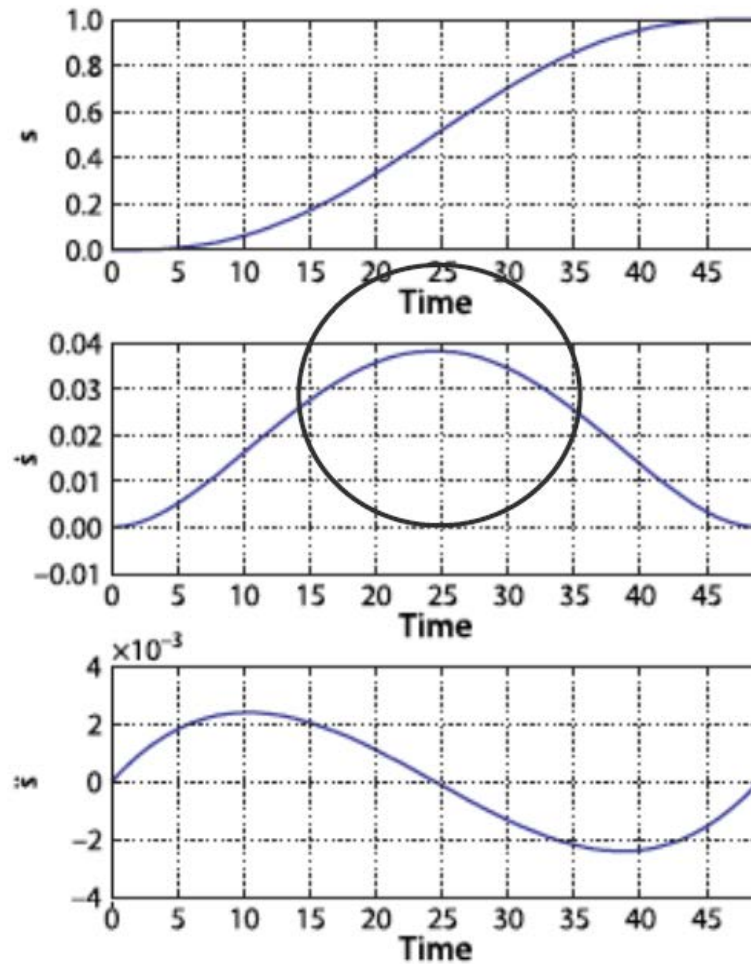
- 5<sup>th</sup> order polynomial trajectory:
  - $x(t) = c_5t^5 + c_4t^4 + c_3t^3 + c_2t^2 + c_1t + c_0$
- Boundary conditions

	Position	Velocity	Acceleration
t = 0	a	0	0
t = T	b	0	0

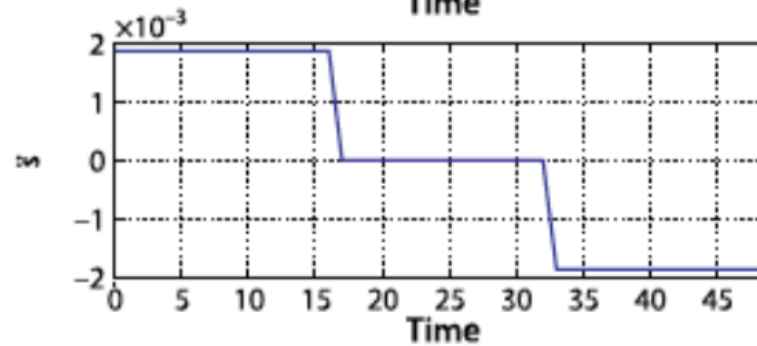
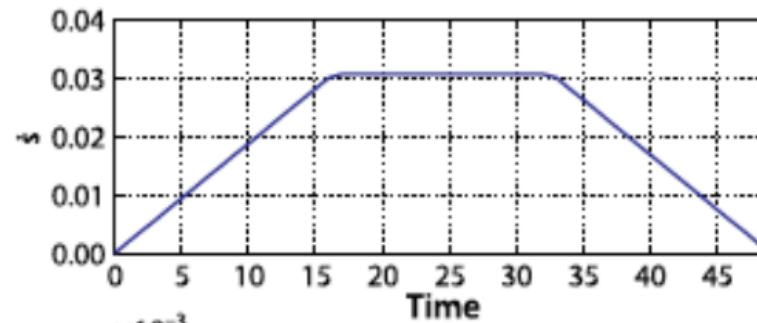
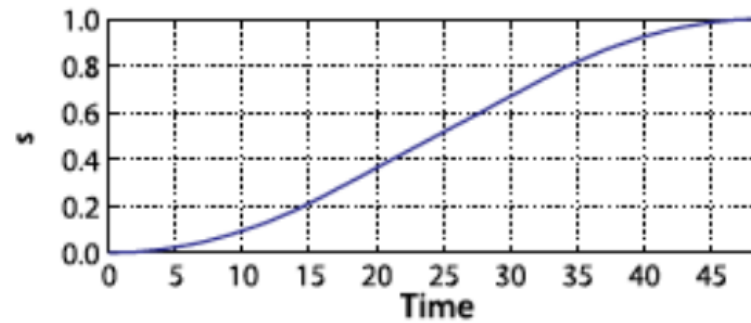
- Solve:

$$\begin{bmatrix} a \\ b \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ T^5 & T^4 & T^3 & T^2 & T & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 5T^4 & 4T^3 & 3T^2 & 2T & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 20T^3 & 12T^2 & 6T & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_5 \\ c_4 \\ c_3 \\ c_2 \\ c_1 \\ c_0 \end{bmatrix}$$

# Smooth 1D Trajectory



# Bang-Bang Trajectory





# Smooth Multi-Segment Trajectory

- Smooth the corners of straight line segments
- Preferred constant velocity motion at  $v$
- Preferred zero acceleration
- Requires special handling of short segments



# Smooth 1D Trajectory

- Generate each 5<sup>th</sup> order polynomial indepently:
  - $x(t) = c_5t^5 + c_4t^4 + c_3t^3 + c_2t^2 + c_1t + c_0$
- Boundary conditions

	Position	Velocity	Acceleration
t = 0	a	$v_0$	0
t = T	b	$v_T$	0

- Solve:

$$\begin{bmatrix} a \\ b \\ v_0 \\ v_T \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ T^5 & T^4 & T^3 & T^2 & T & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 5T^4 & 4T^3 & 3T^2 & 2T & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 20T^3 & 12T^2 & 6T & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_5 \\ c_4 \\ c_3 \\ c_2 \\ c_1 \\ c_0 \end{bmatrix}$$

# Optimization-based Trajectory Generation

- Explicitly minimize certain derivatives in the space of flat outputs
- Quadrotor dynamics

Derivative	Translation	Rotation	Thrust
0	Position		
1	Velocity		
2	Acceleration	Rotation	
3	Jerk	Angular Velocity	
4	Snap	Angular Acceleration	Differential Thrust
5	Crackle	Angular Jerk	Change in Thrust

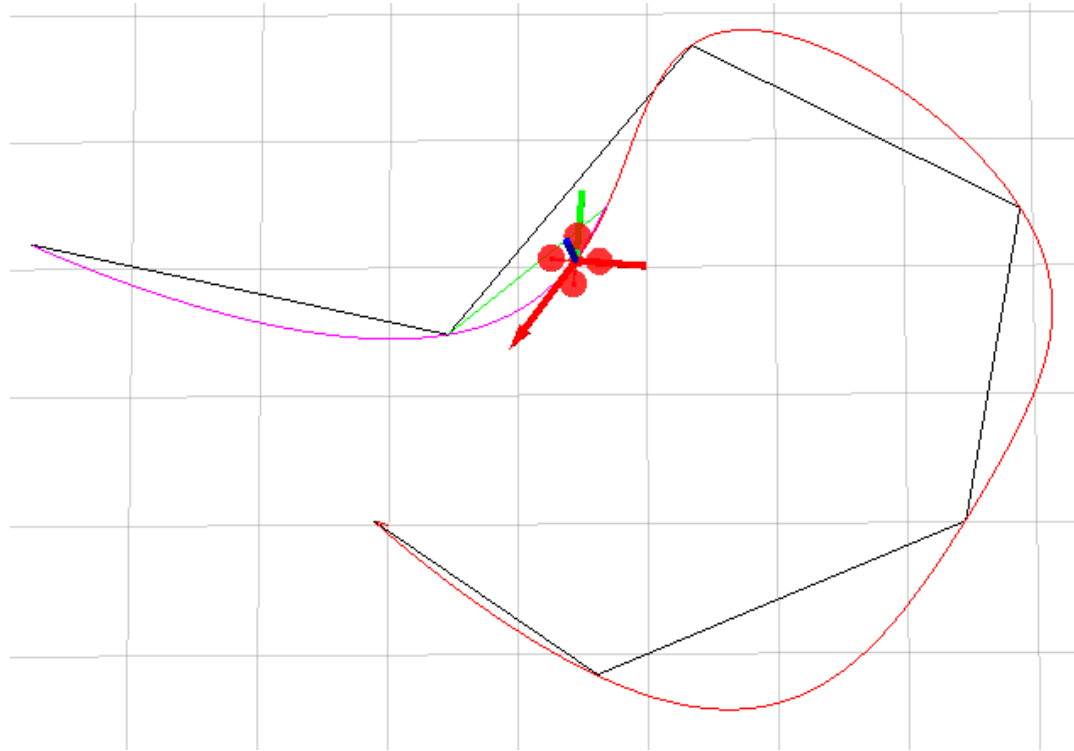
# Optimization-based Trajectory Generation

- Explicitly minimize certain derivatives in the space of flat outputs
  - Minimum jerk: minimize angular velocity, good for visual tracking
  - Minimum snap: minimize differential thrust, saves energy

Derivative	Translation	Rotation	Thrust
0	Position		
1	Velocity		
2	Acceleration	Rotation	
3	Jerk	Angular Velocity	
4	Snap	Angular Acceleration	Differential Thrust
5	Crackle	Angular Jerk	Change in Thrust

# Minimum Snap Trajectory Generation

- Multi-segment minimum snap trajectory



# Minimum Snap Trajectory Generation

- Formulation – segment durations must be known!

$$f(t) = \begin{cases} f_1(t) \doteq \sum_{i=0}^N p_{1,i}(t - T_0)^i & T_0 \leq t \leq T_1 \\ f_2(t) \doteq \sum_{i=0}^N p_{2,i}(t - T_1)^i & T_1 \leq t \leq T_2 \\ \vdots & \\ f_M(t) \doteq \sum_{i=0}^N p_{M,i}(t - T_{M-1})^i & T_{M-1} \leq t \leq T_M \end{cases}$$

Subject to:

$$\text{Derivative constraints: } \begin{cases} f_j^{(k)}(T_{j-1}) & = x_{0,j}^{(k)} \\ f_j^{(k)}(T_j) & = x_{T,j}^{(k)} \end{cases}$$

$$\text{Continuity constraints: } f_j^{(k)}(T_j) = f_{j+1}^{(k)}(T_j)$$

- Minimum degree polynomial:
  - Minimum jerk:  $N = 2 * 3(\text{jerk}) - 1 = 5$
  - Minimum snap:  $N = 2 * 4(\text{snap}) - 1 = 7$

# Minimum Snap Trajectory Generation

- Cost function for one polynomial segment:

$$f(t) = \sum_i p_i t^i$$

$$\Rightarrow f^{(4)}(t) = \sum_{i \geq 4} i(i-1)(i-2)(i-3)t^{i-4}p_i$$

$$\Rightarrow (f^{(4)}(t))^2 = \sum_{i \geq 4, j \geq 4} i(i-1)(i-2)(i-3)j(j-1)(j-2)(j-3)t^{i+j-8}p_i p_j$$

$$\Rightarrow J(T) = \int_0^T (f^{(4)}(t))^2 dt = \sum_{i \geq 4, j \geq 4} \frac{i(i-1)(i-2)(i-3)j(j-1)(j-2)(j-3)}{i+j-7} T^{i+j-7} p_i p_j$$

$$\Rightarrow J(T) = \int_0^T (f^{(4)}(t))^2 dt = \begin{bmatrix} \vdots \\ p_i \\ \vdots \end{bmatrix}^T \begin{bmatrix} \vdots & & \\ \dots & \frac{i(i-1)(i-2)(i-3)j(j-1)(j-2)(j-3)}{i+j-7} T^{i+j-7} & \dots \\ \vdots & & \end{bmatrix} \begin{bmatrix} \vdots \\ p_j \\ \vdots \end{bmatrix}$$

$$\Rightarrow J_k(T) = \mathbf{p}_k^T \mathbf{Q}_k \mathbf{p}_k$$

Minimize this!

# Minimum Snap Trajectory Generation

- Derivative constraint for one polynomial segment
  - Also models waypoint constraint ( $0^{th}$  order derivative)

$$\begin{aligned}
 f_j^{(k)}(T_j) &= x_j^{(k)} \\
 \Rightarrow \sum_{i \geq k} \frac{i!}{(i-k)!} T_j^{i-k} p_{j,i} &= x_{T,j}^{(k)} \\
 \Rightarrow \begin{bmatrix} \cdots & \frac{i!}{(i-k)!} T_j^{i-k} & \cdots \end{bmatrix} \begin{bmatrix} \vdots \\ p_{j,i} \\ \vdots \end{bmatrix} &= x_{T,j}^{(k)} \\
 \Rightarrow \begin{bmatrix} \cdots & \frac{i!}{(i-k)!} T_{j-1}^{i-k} & \cdots \\ \cdots & \frac{i!}{(i-k)!} T_j^{i-k} & \cdots \end{bmatrix} \begin{bmatrix} \vdots \\ p_{j,i} \\ \vdots \end{bmatrix} &= \begin{bmatrix} x_{0,j}^{(k)} \\ x_{T,j}^{(k)} \end{bmatrix} \\
 \Rightarrow \mathbf{A}_j \mathbf{p}_j &= \mathbf{d}_j
 \end{aligned}$$



# Minimum Snap Trajectory Generation

- Continuity constraint between two segments:
  - Ensures continuity between trajectory segments when no specific derivatives are given

$$\begin{aligned}
 f_j^{(k)}(T_j) &= f_{j+1}^{(k)}(T_j) \\
 \Rightarrow \sum_{i \geq k} \frac{i!}{(i-k)!} T_j^{i-k} p_{j,i} - \sum_{l \geq k} \frac{l!}{(l-k)!} T_j^{l-k} p_{j+1,l} &= 0 \\
 \Rightarrow \left[ \cdots \quad \frac{i!}{(i-k)!} T_j^{i-k} \quad \cdots \quad -\frac{l!}{(l-k)!} T_j^{l-k} \quad \cdots \right] &\begin{bmatrix} \vdots \\ p_{j,i} \\ \vdots \\ p_{j+1,l} \\ \vdots \end{bmatrix} = 0 \\
 \Rightarrow [A_j \quad -A_{j+1}] \begin{bmatrix} p_j \\ p_{j+1} \end{bmatrix} &= 0
 \end{aligned}$$

# Minimum Snap Trajectory Generation

- Constrained quadratic programming (QP) formulation:

$$\begin{aligned}
 \min \quad & \begin{bmatrix} \mathbf{p}_1 \\ \vdots \\ \mathbf{p}_M \end{bmatrix}^T \begin{bmatrix} \mathbf{Q}_1 & & \\ & \ddots & \\ & & \mathbf{Q}_M \end{bmatrix} \begin{bmatrix} \mathbf{p}_1 \\ \vdots \\ \mathbf{p}_M \end{bmatrix} \\
 \text{s.t.} \quad & \mathbf{A} \begin{bmatrix} \mathbf{p}_1 \\ \vdots \\ \mathbf{p}_M \end{bmatrix} = \begin{bmatrix} \mathbf{d}_1 \\ \vdots \\ \mathbf{d}_M \end{bmatrix}
 \end{aligned}$$

# Minimum Snap Trajectory Generation

- Direct optimization of polynomial trajectories is numerically unstable
- A change of variable that instead optimizes segment endpoint derivatives is preferred

$$J = \begin{bmatrix} \mathbf{d}_1 \\ \vdots \\ \mathbf{d}_M \end{bmatrix}^T \begin{bmatrix} A_1 & & \\ & \ddots & \\ & & A_M \end{bmatrix}^{-T} \begin{bmatrix} Q_1 & & \\ & \ddots & \\ & & Q_M \end{bmatrix} \begin{bmatrix} A_1 & & \\ & \ddots & \\ & & A_M \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{d}_1 \\ \vdots \\ \mathbf{d}_M \end{bmatrix}$$

# Minimum Snap Trajectory Generation

- Use a selection matrix to separate free and constrained variables
  - Free variables: derivatives unspecified, only enforced by continuity constraints

$$J = \begin{bmatrix} \mathbf{d}_F \\ \mathbf{d}_P \end{bmatrix}^T \underbrace{CA^{-T}QA^{-1}C^T}_R \begin{bmatrix} \mathbf{d}_F \\ \mathbf{d}_P \end{bmatrix} = \begin{bmatrix} \mathbf{d}_F \\ \mathbf{d}_P \end{bmatrix}^T \begin{bmatrix} R_{FF} & R_{FP} \\ R_{PF} & R_{PP} \end{bmatrix} \begin{bmatrix} \mathbf{d}_F \\ \mathbf{d}_P \end{bmatrix}$$

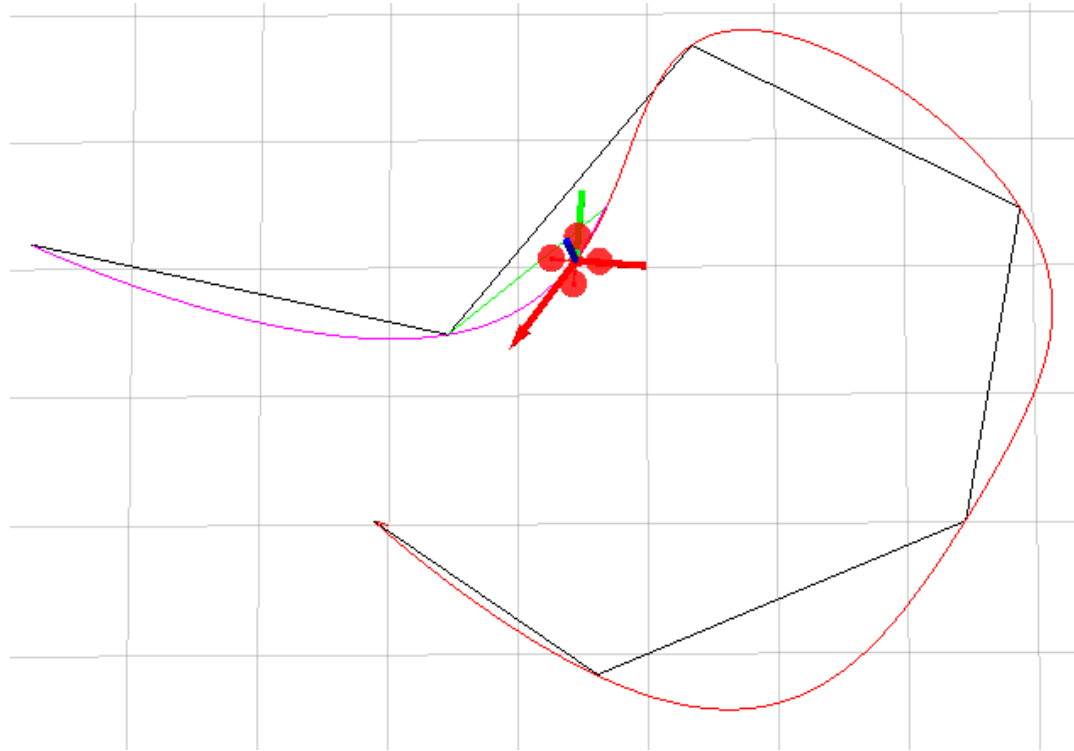
- Turned into an unconstrained quadratic programming that can be solved in closed form:

$$J = \mathbf{d}_F^T R_{FF} \mathbf{d}_F + \mathbf{d}_F^T R_{FP} \mathbf{d}_P + \mathbf{d}_P^T R_{PF} \mathbf{d}_F + \mathbf{d}_P^T R_{PP} \mathbf{d}_P$$

$$\mathbf{d}_P^* = -R_{PP}^{-1} R_{FP}^T \mathbf{d}_F$$

# Minimum Snap Trajectory Generation

- Final trajectory



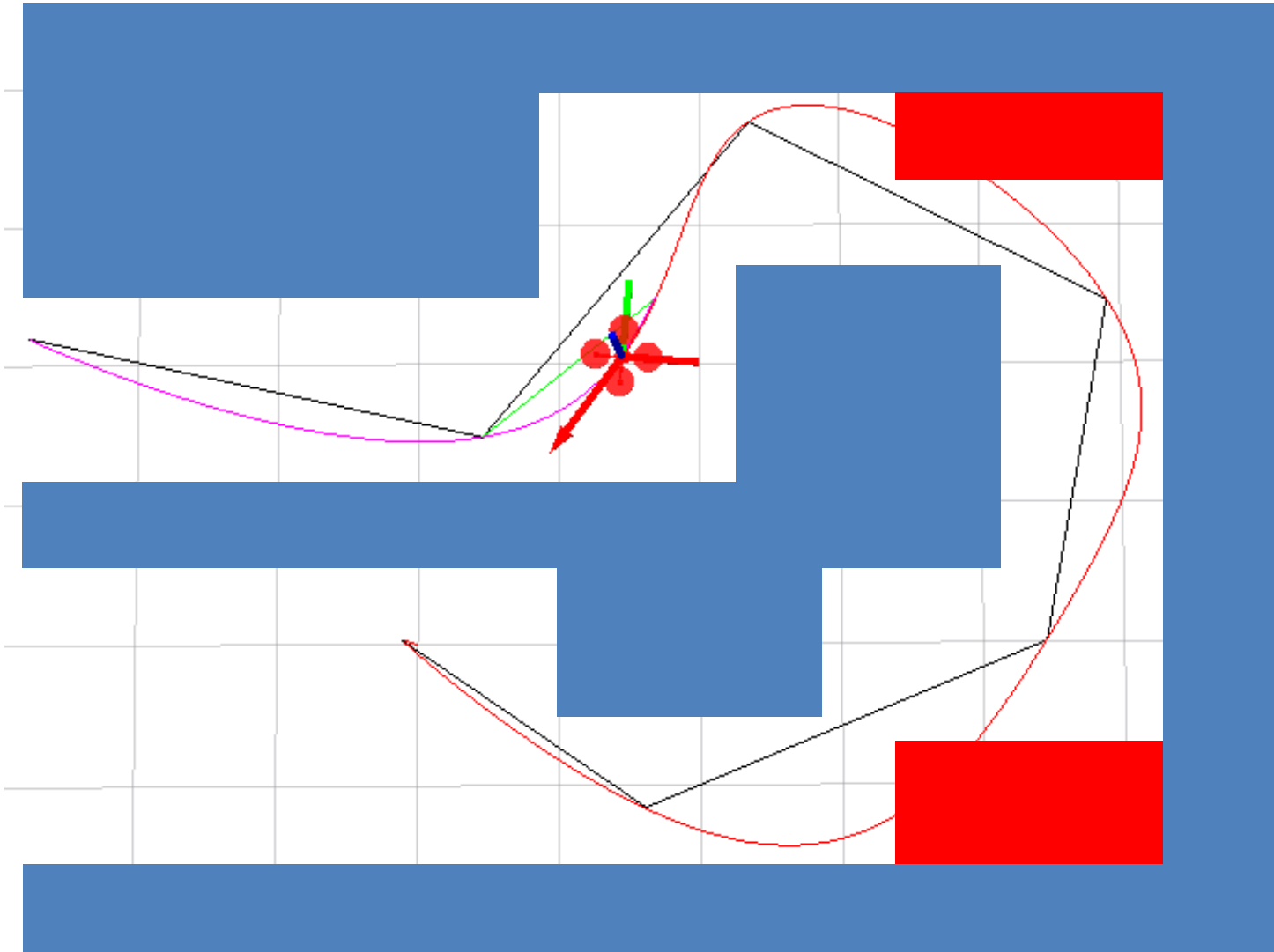
# Minimum Snap Trajectory Generation

## **Aggressive Quadrotor Part II**

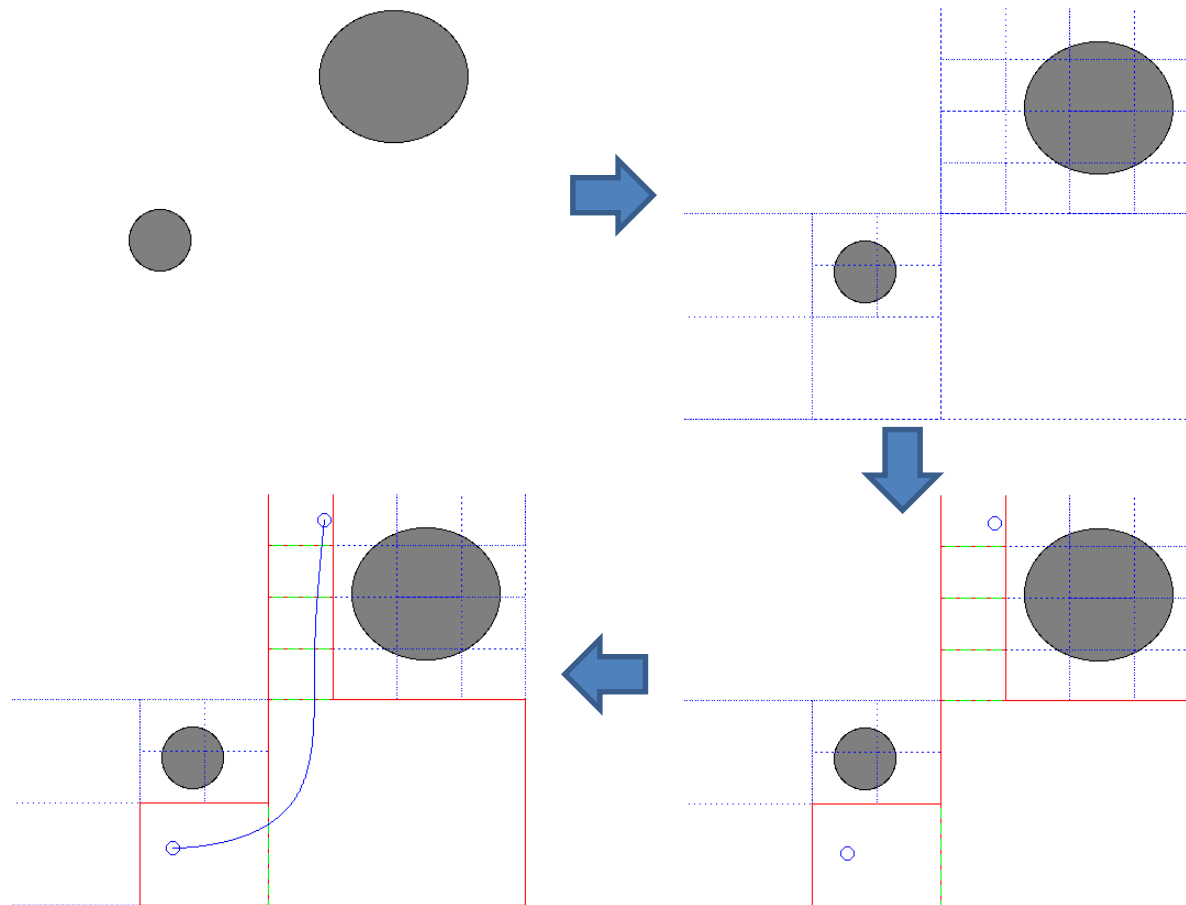
**Daniel Mellinger and Vijay Kumar**

**GRASP Lab, University of Pennsylvania**

# Minimum Snap Safe Trajectory Generation

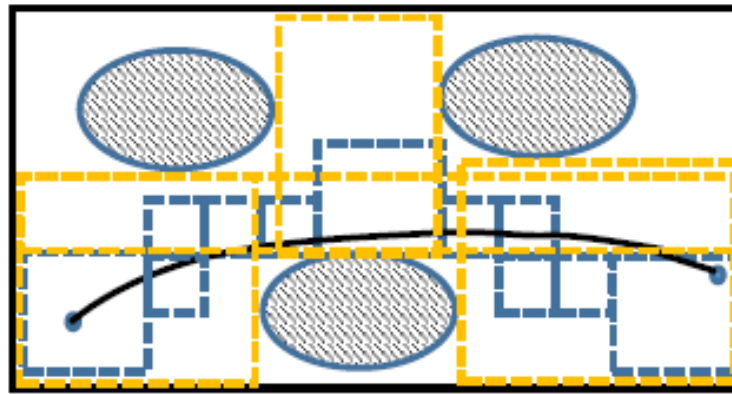


# Smooth Trajectory Generation with Guaranteed Obstacle Avoidance

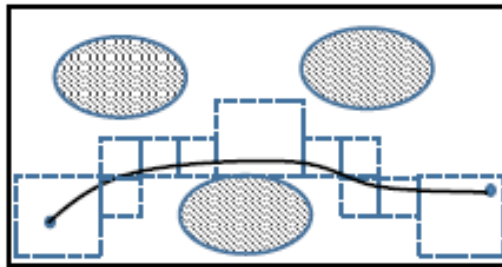




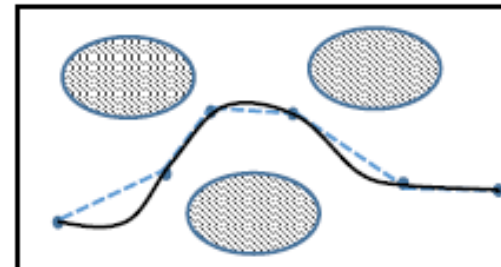
# Smooth Trajectory Generation with Guaranteed Obstacle Avoidance



(a) The proposed method



(b) Our previous method [17]

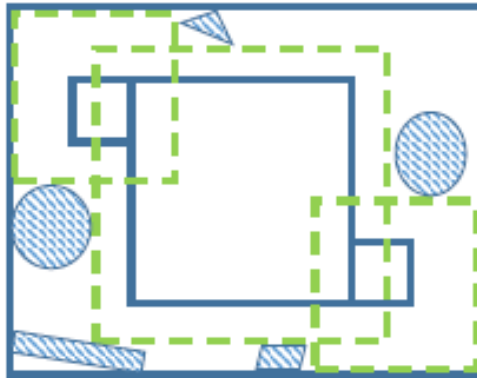


(c) Waypoint-based method [2]

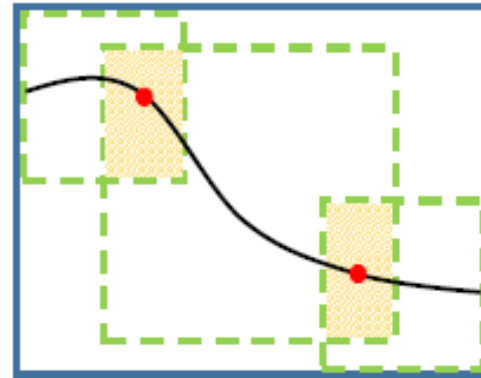
# Smooth Trajectory Generation with Guaranteed Obstacle Avoidance

## Quadratic programming formulation

$$\begin{aligned}
 \min \quad & \mathbf{p}^T \mathbf{H} \mathbf{p} \\
 \text{s.t.} \quad & \mathbf{A}_{eq} \mathbf{p} = \mathbf{b}_{eq} \\
 & \mathbf{A}_{lq} \mathbf{p} \leq \mathbf{b}_{lq}
 \end{aligned}$$

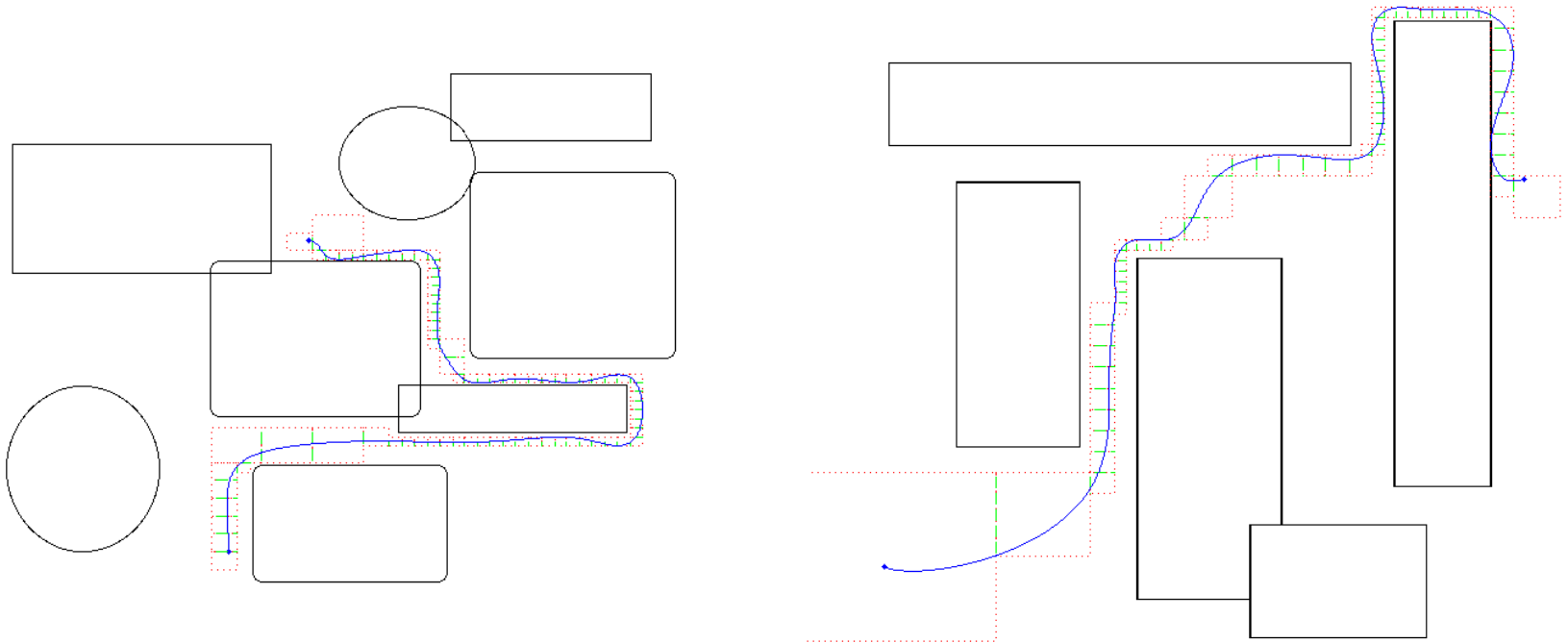


(a) Maximal volume flight corridor shown as green dashed boxes



(b) Adjustment of trajectory transition points within overlapping regions

# Results - Trajectory Generation with Obstacle Avoidance



# Results - Trajectory Generation with Obstacle Avoidance

## Online Generation of Collision-Free Trajectories for Quadrotor Flight In Unknown Environments

Jing Chen, Tianbo Liu, and Shaojie Shen



High resolution video available at:  
<http://www.ece.ust.hk/~eeshaojie/icra2016jing.mp4>

# Reading

- Paper Reading: “The GRASP Multiple Micro-UAV Test Bed”, Nathan Michael, Daniel Mellinger, Quentin Lindsey, and Vijay Kumar.
- Paper Reading: “Minimum Snap Trajectory Generation and Control for Quadrotors”, Daniel Mellinger and Vijay Kumar.

# Logistics

- Project 1, phase 2 will be released tomorrow (30/9)
  - Tentative due: 7/10