香港科技大學
THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY
電子及計算機工程學系
DEPARTMENT OF ELECTRONIC & COMPUTER ENGINEERING

Introduction to Aerial Robotics
Lecture 3

Shaojie Shen
Assistant Professor
Dept. of ECE, HKUST

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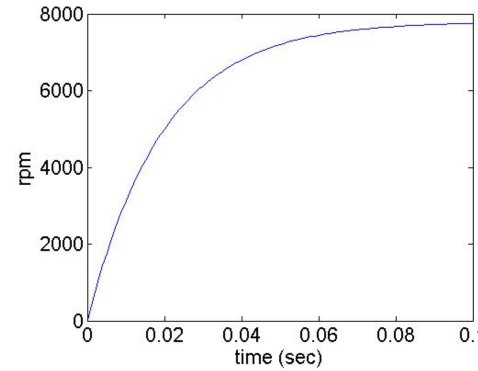
Outline

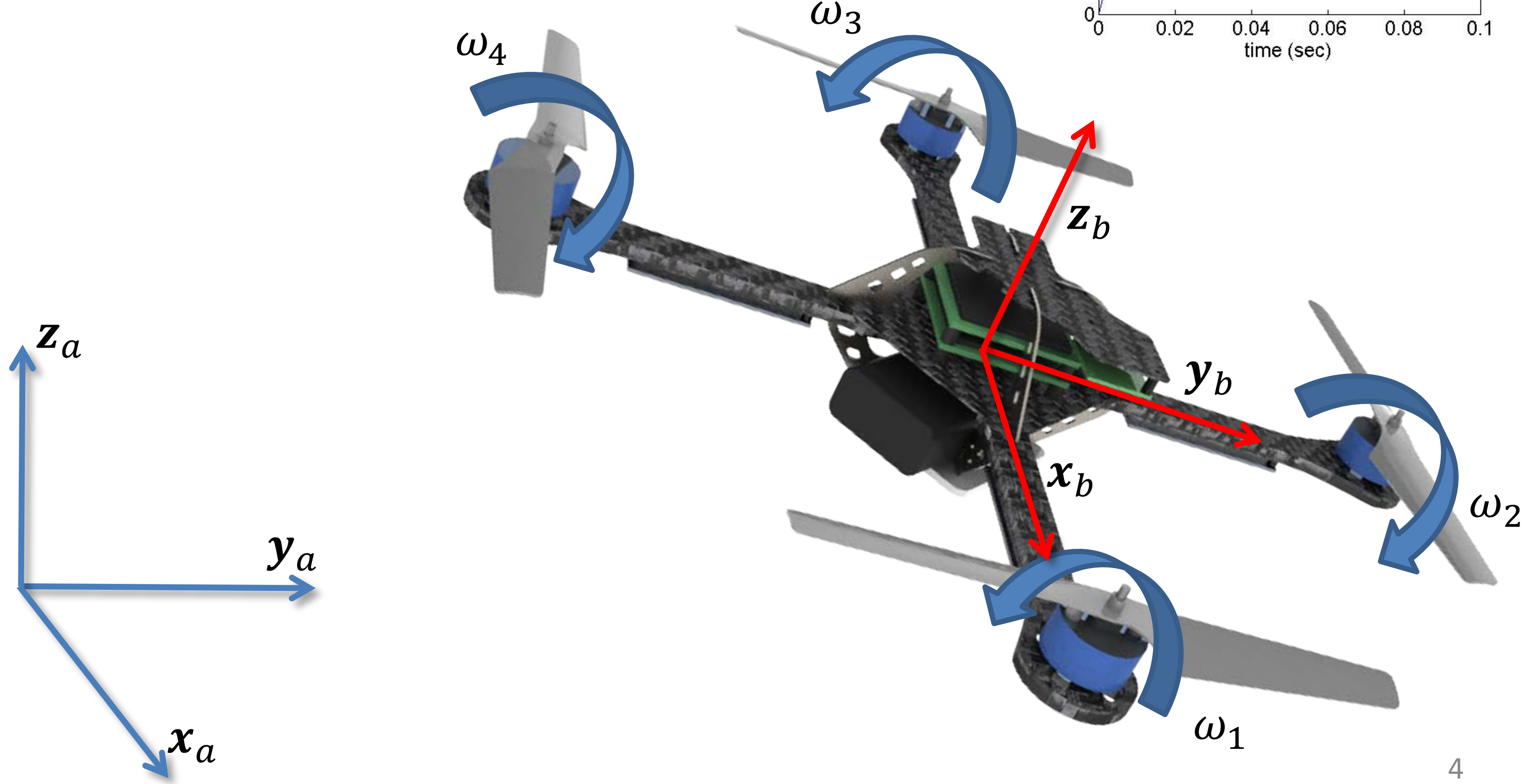
- Review: Quadrotor Dynamics
- Control System Design
- Quadrotor Control

Review: Quadrotor Dynamics

Quadrotor Dynamics

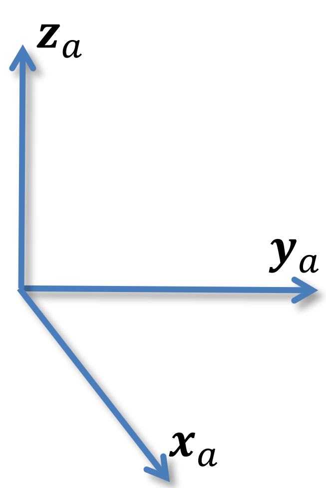
- Motor model: $\dot{\omega}_i = k_m(\omega_i^{des} - \omega_i)$
- Thrust from individual motor: $F_i = k_F \omega_i^2$
- Moment from individual motor: $M_i = k_M \omega_i^2$

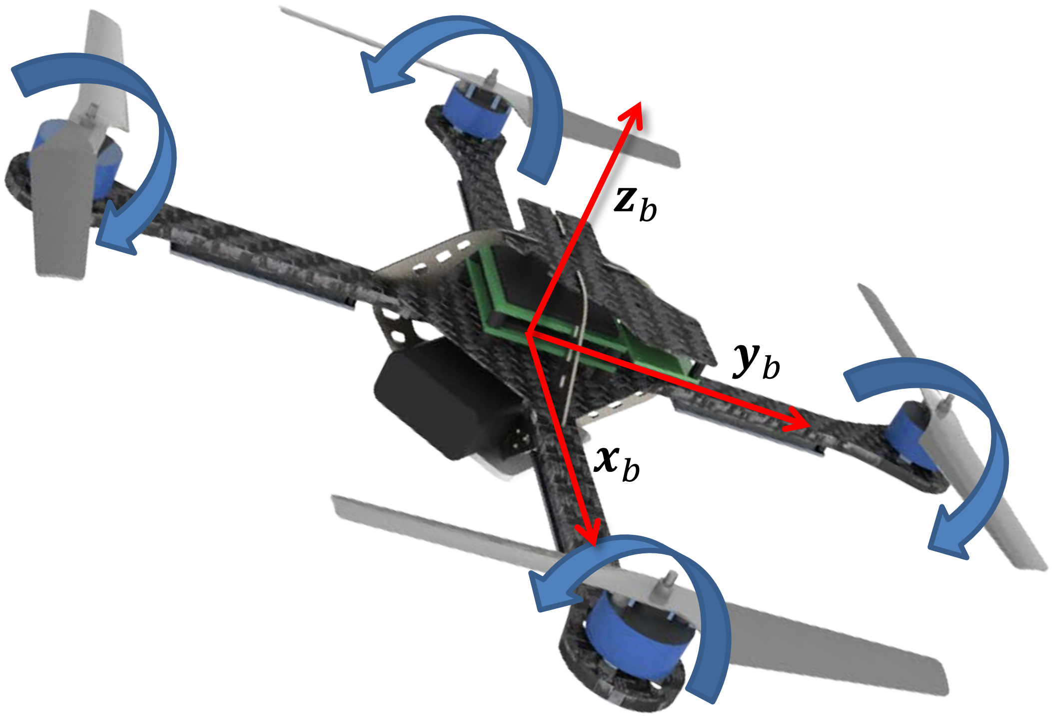




Quadrotor Dynamics

- Z-X-Y Euler Angles: $\boldsymbol{R}_{ab} = \boldsymbol{R}_z(\psi) \cdot \boldsymbol{R}_x(\phi) \cdot \boldsymbol{R}_y(\theta)$
- Sequence of three rotations about body-fixed axes
- What are the singularities?

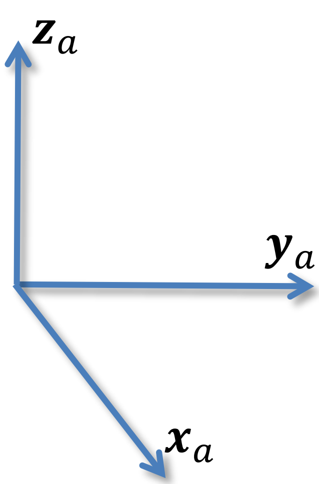


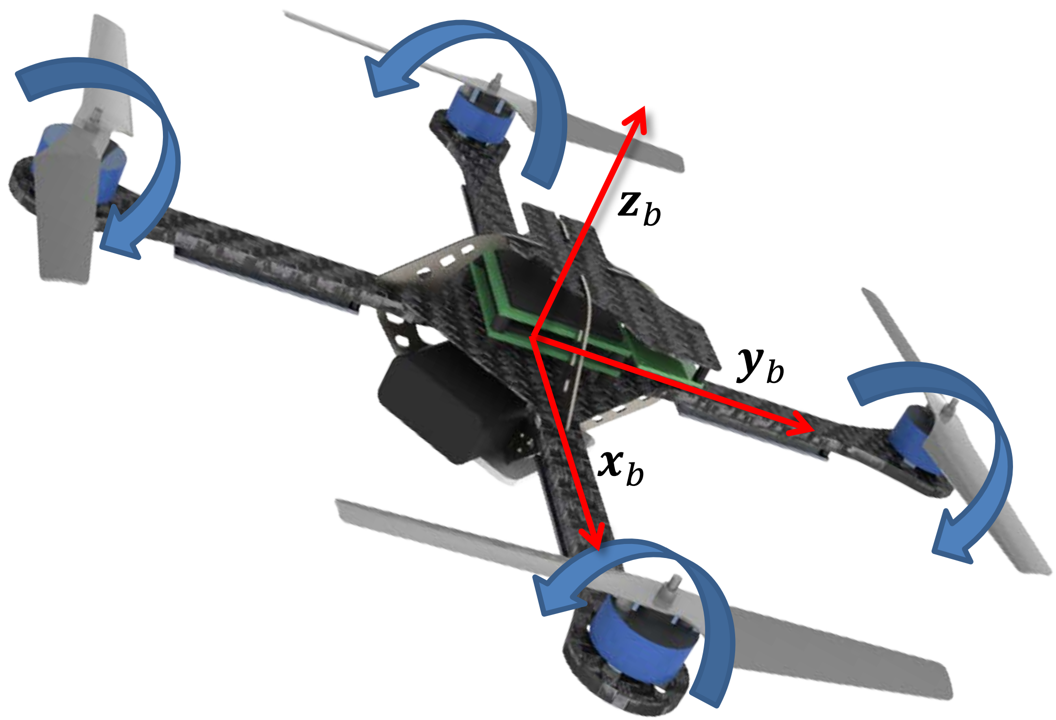


Quadrotor Dynamics

- $\boldsymbol{R}_{ab} = \begin{bmatrix} c\psi c\theta - s\phi s\psi s\theta & -c\phi s\psi & c\psi s\theta + c\theta s\phi s\psi \\ c\theta s\psi + c\psi s\phi s\theta & c\phi c\psi & s\psi s\theta - c\psi c\theta s\phi \\ -c\phi s\theta & s\phi & c\phi c\theta \end{bmatrix}$
- $\boxed{\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}} = \begin{bmatrix} c\theta & 0 & -c\phi s\theta \\ 0 & 1 & s\phi \\ s\theta & 0 & c\phi c\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$

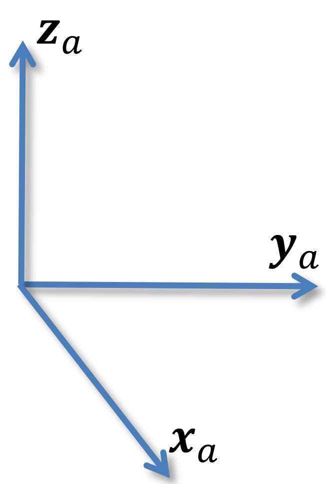
Instantaneous body angular velocity.

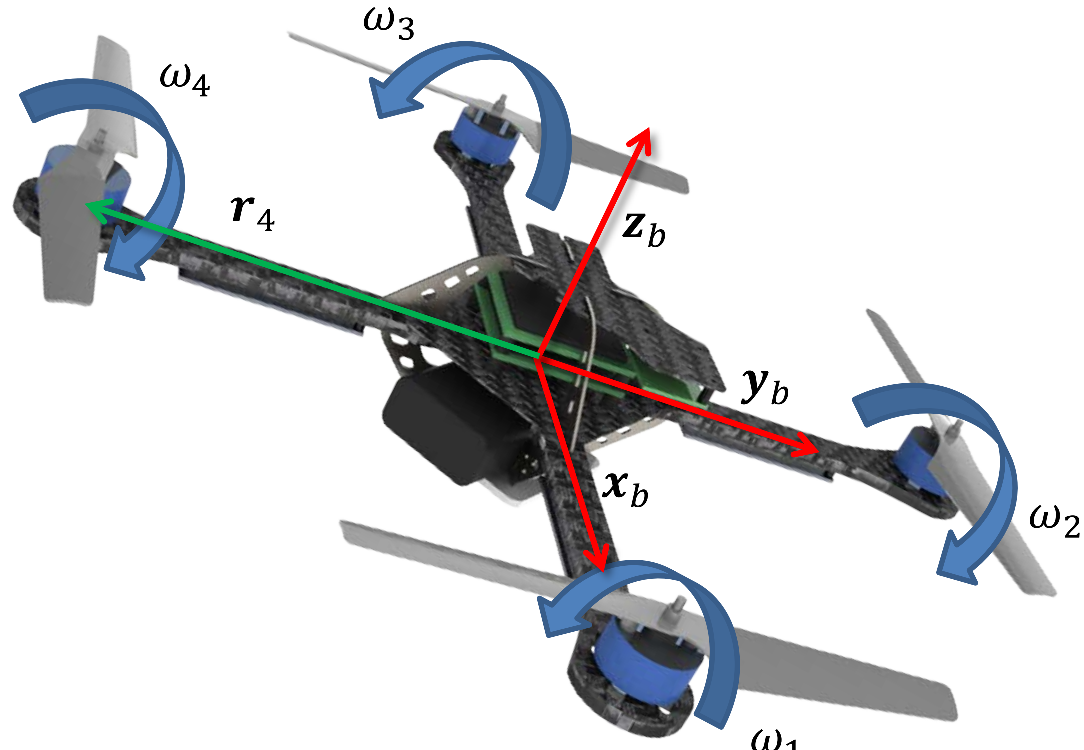




Quadrotor Dynamics

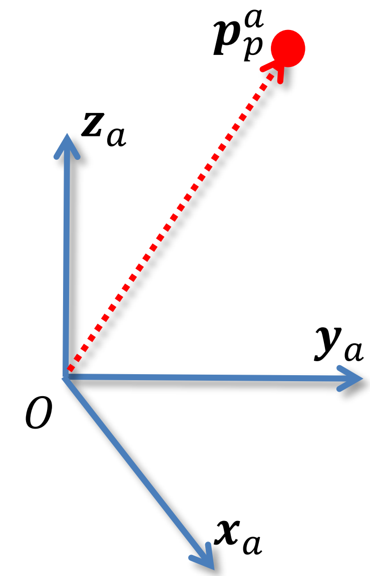
- $\boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 + \boldsymbol{F}_4 - mg\boldsymbol{z}_a$
- $\boldsymbol{M} = \boldsymbol{r}_1 \times \boldsymbol{F}_1 + \boldsymbol{r}_2 \times \boldsymbol{F}_2 + \boldsymbol{r}_3 \times \boldsymbol{F}_3 + \boldsymbol{r}_4 \times \boldsymbol{F}_4 + \boldsymbol{M}_1 + \boldsymbol{M}_2 + \boldsymbol{M}_3 + \boldsymbol{M}_4$
- $\boldsymbol{F}_i = [0, 0, F_i]^T$
- $\boldsymbol{M}_i = [0, 0, \pm M_i]^T$





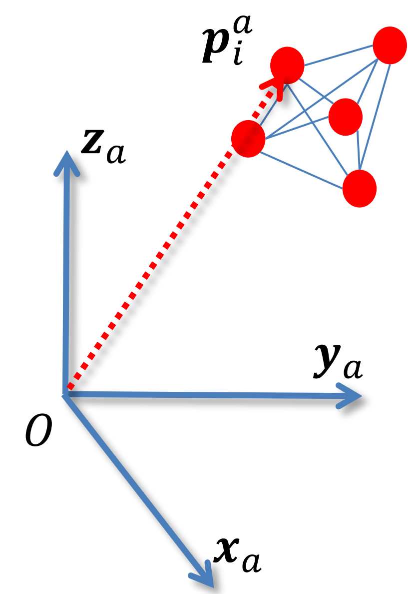
Newton-Euler Equations

- Newton's Second Law for a particle in the inertial frame A:
  - Position vector: $\boldsymbol{p}_p^a$
  - Velocity: $\boldsymbol{v}_p^a = \frac{d\,\boldsymbol{p}_p^a}{dt}$
  - Force acting on the particle with mass m: $\boldsymbol{F} = m \cdot \frac{d\,\boldsymbol{v}_p^a}{dt}$
  - Linear momentum: $\boldsymbol{L}_p^a = m\boldsymbol{v}_p^a$
  - Angular momentum relative to O: $\boldsymbol{H}_p^{ao} = \boldsymbol{p}_p^a \times \boldsymbol{L}_p^a$
- We are interested in the rate of change of linear and angular momentums in A:
  - $\frac{d\,\boldsymbol{L}_p^a}{dt} = \boldsymbol{F}$
  - $\frac{d\,\boldsymbol{H}_p^{ao}}{dt} = \boldsymbol{M}$



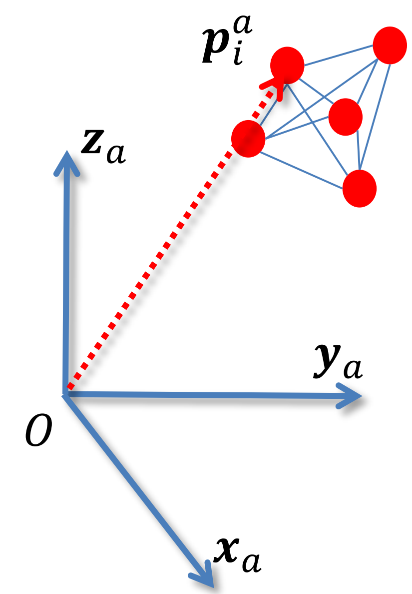
Newton-Euler Equations

- Newton's Second Law for a system of particles in the inertial frame A:
  - Mass m_i at $\boldsymbol{p}_i^a$
  - $\boldsymbol{F}_i = \boldsymbol{F}_{ik}^{int} + \boldsymbol{F}_i^{ext}$ is the net internal and external forces acting on m_i
  - Total mass $m = \sum m_i$
  - Center of mass $\boldsymbol{r}_c = \frac{1}{m}\sum m_i \boldsymbol{p}_i^a$
  - The center of mass of a system of particles S, accelerates in an inertial frame A as if it is a single particle with mass m, acted upon by a force equal to the net external force $\boldsymbol{F} = \sum \boldsymbol{F}_i^{ext}$



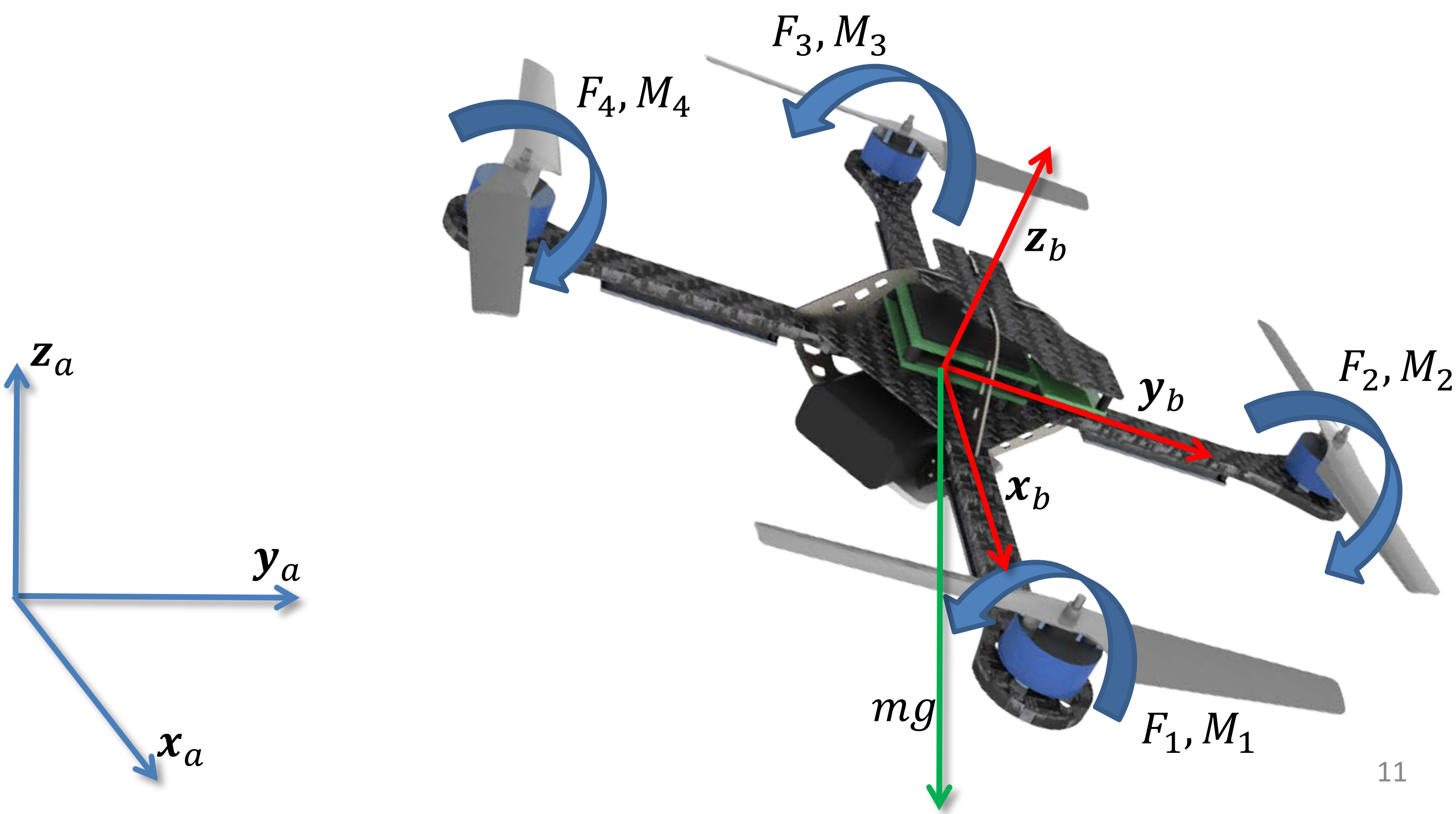
Newton-Euler Equations

- Linear momentum of the center of mass in frame A:
  - $\boldsymbol{L}_c^a = m \cdot \boldsymbol{v}_c^a$
- Rate of change of linear momentum:
  - $\boldsymbol{F} = m \cdot \frac{d\,\boldsymbol{v}_c^a}{dt} = \frac{d\,\boldsymbol{L}_c^a}{dt}$



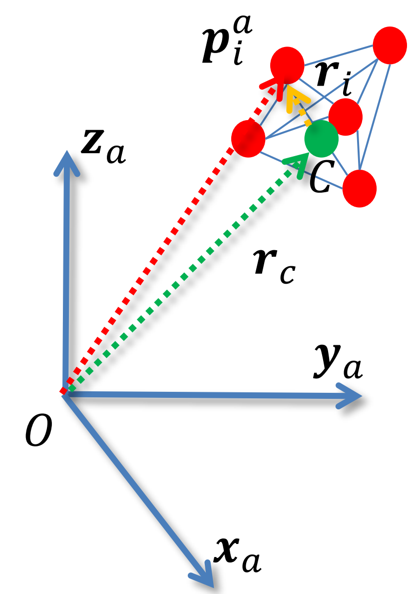
Quadrotor Dynamics

- Newton Equation: $m\ddot{\boldsymbol{r}}^a = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + \boldsymbol{R}_{ab} \begin{bmatrix} 0 \\ 0 \\ F_1 + F_2 + F_3 + F_4 \end{bmatrix}$



Newton-Euler Equations

- Angular momentum of a particle in the inertial frame A relative to O:
  - $\boldsymbol{H}_i^{ao} = \boldsymbol{p}_i^a \times m_i \boldsymbol{v}_i^a$
- Angular momentum of a particle in the inertial frame A Relative to C:
  - $\boldsymbol{H}_i^{ac} = \boldsymbol{r}_i \times m_i \boldsymbol{v}_i^a$
- Angular momentum of the system S related to the center of mass C in frame A:
  - $\boldsymbol{I}_S^a$: Moment of inertia tensor calculated in the inertial frame
  - $\boldsymbol{\omega}_S^a$: angular velocity of the system viewed in the inertial frame
  - $\boldsymbol{H}_S^{ac} = \sum \boldsymbol{r}_i \times m_i \, \boldsymbol{v}_i^a = \boldsymbol{I}_S^a \boldsymbol{\omega}_S^a$

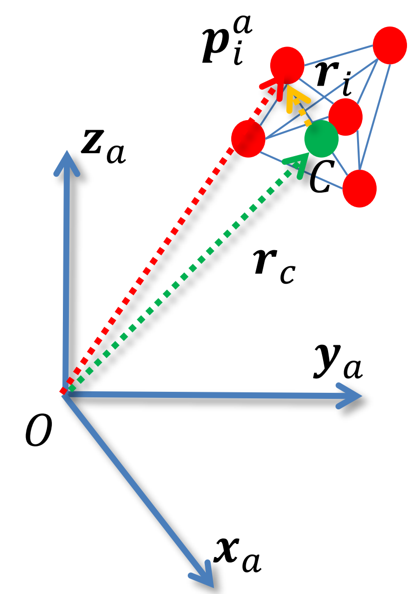


Further Reading: http://ocw.mit.edu/courses/aeronautics-and-astronautics/16-07-dynamics-fall-2009/lecture-notes/MIT16_07F09_Lec26.pdf

Newton-Euler Equations

- Angular momentum of the system S:
  - $\boldsymbol{H}_S^{ac} = \boldsymbol{I}_S^a \cdot \boldsymbol{\omega}_S^a$
- Rate of change of angular momentum is equal to the resultant moment of all external forces and torques acting on the system S related to C:
  - $\frac{d\boldsymbol{H}_S^{ac}}{dt} = \frac{d}{dt}\left(\boxed{\boldsymbol{I}_S^a \cdot \boldsymbol{\omega}_S^a}\right) = \boldsymbol{M}_S^c$

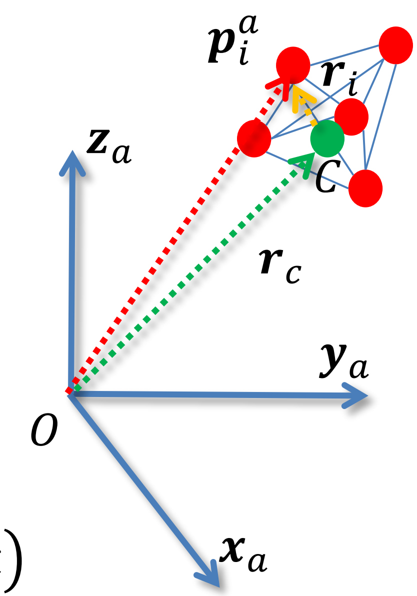
Not very useful. Both $\boldsymbol{I}_S^a$ and $\boldsymbol{\omega}_S^a$ changes during motion.



Switch to an ROTATING reference frame!

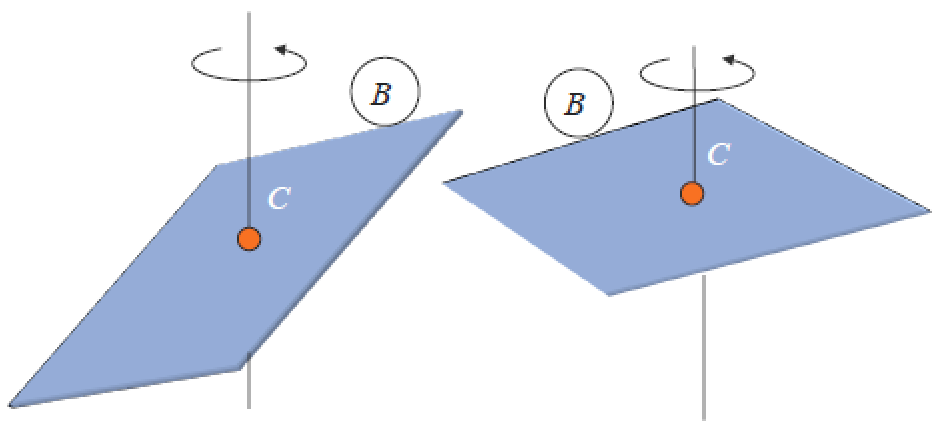
Newton-Euler Equations

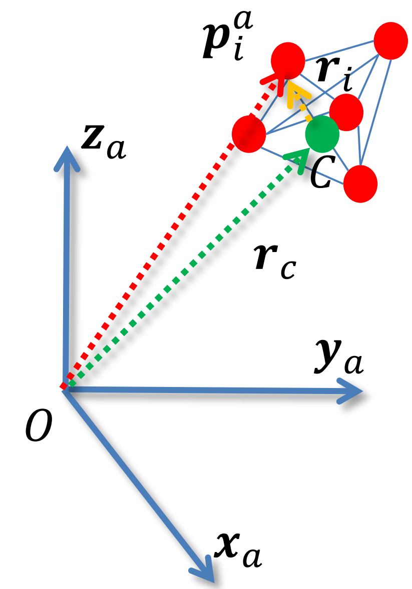
- Let $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ be the unit basis vectors of the rotating reference frame. The derivative of a unit vector in the rotating frame about the axis $\boldsymbol{\omega}$:
  - $\dot{\boldsymbol{u}} = \boldsymbol{\omega} \times \boldsymbol{u}$
- Consider the vector function:
  - $\boldsymbol{f}(t) = f_x(t) \cdot \boldsymbol{i} + f_y(t) \cdot \boldsymbol{j} + f_z(t) \cdot \boldsymbol{k}$
- Time derivative in rotating reference frame:
  - $\dot{\boldsymbol{f}} = \dot{f}_x \cdot \boldsymbol{i} + \dot{f}_y \cdot \boldsymbol{j} + \dot{f}_z \cdot \boldsymbol{k} + \dot{\boldsymbol{i}} \cdot f_x + \dot{\boldsymbol{j}} \cdot f_y + \dot{\boldsymbol{k}} \cdot f_z = \left(\dot{f}_x \cdot \boldsymbol{i} + \dot{f}_y \cdot \boldsymbol{j} + \dot{f}_z \cdot \boldsymbol{k}\right) + \boldsymbol{\omega} \times \left(f_x \cdot \boldsymbol{i} + f_y \cdot \boldsymbol{j} + f_z \cdot \boldsymbol{k}\right)$
  - $\dot{\boldsymbol{f}} = \dot{\boldsymbol{f}}|_{rot} + \boldsymbol{\omega} \times \boldsymbol{f}$



Newton-Euler Equations

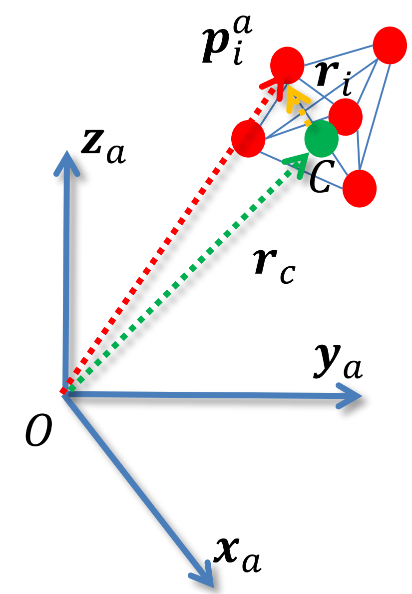
- Align the moment of inertia tensor with the rotating reference frame and obtain the constant and diagonal $\boldsymbol{I}_s^b$
  - $\boldsymbol{I}_s^a = \boldsymbol{R} \cdot \boldsymbol{I}_s^b \cdot \boldsymbol{R}^{\mathrm{T}}$
  - $\boldsymbol{\omega}_s^a = \boldsymbol{R} \cdot \boldsymbol{\omega}_s^b$





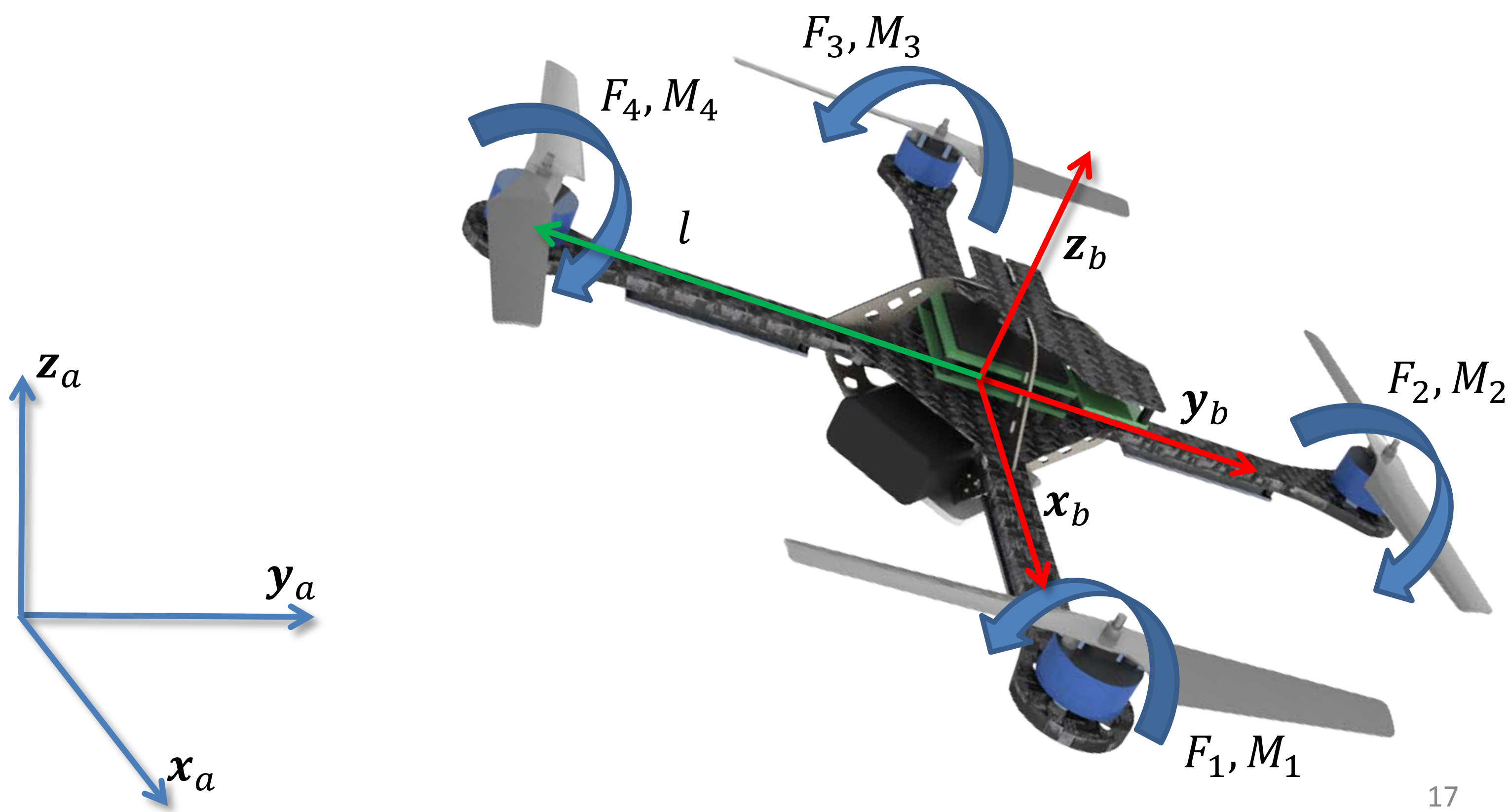
Newton-Euler Equations

- Angular momentum of the system S:
  - $\boldsymbol{H}_S^{ac} = \boldsymbol{I}_S^a \cdot \boldsymbol{\omega}_S^a$
- Rate of change of angular momentum in the inertial frame:
  - $\frac{d\boldsymbol{H}_S^{ac}}{dt} = \boldsymbol{M}_S^c$
- Rate of change of angular momentum in the rotating reference frame, where $\boldsymbol{I}_S^b$ is a constant:
  - $\boldsymbol{H}_S^{bc} = \boldsymbol{I}_S^b \boldsymbol{\omega}_S^b$
  - $\frac{d\boldsymbol{H}_S^{bc}}{dt} + \boldsymbol{\omega}_S^b \times \boldsymbol{H}_S^{bc} = \boldsymbol{M}_S^c$
- The Euler equation of motion:
  - $\boldsymbol{I}_S^b \dot{\boldsymbol{\omega}}_S^b + \boldsymbol{\omega}_S^b \times \boldsymbol{I}_S^b \boldsymbol{\omega}_S^b = \boldsymbol{M}_S^c$



Quadrotor Dynamics

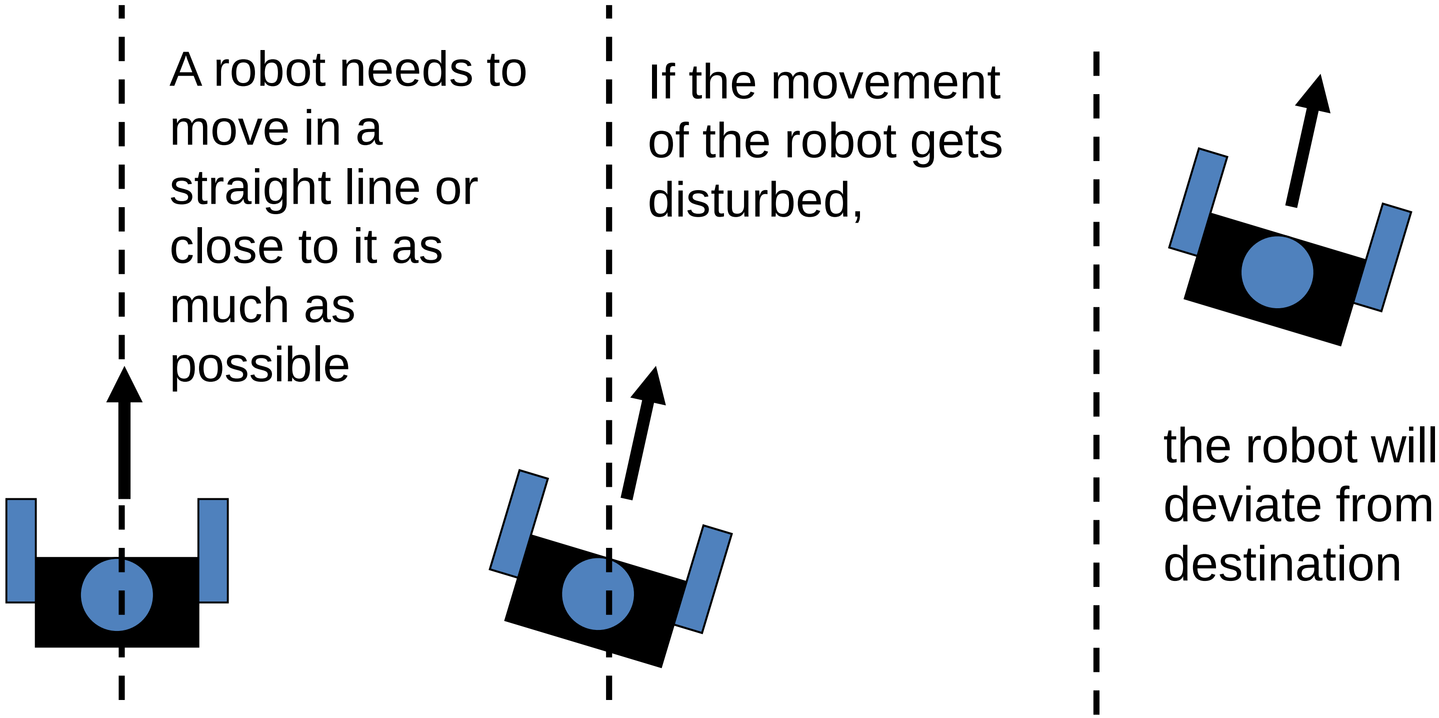
- Euler Equation: $\boldsymbol{I} \cdot \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} + \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \times \boldsymbol{I} \cdot \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} l(F_2 - F_4) \\ l(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix}$



Quadrotor Dynamics

- Motor model: $\dot{\omega}_i = k_m(\omega_i^{des} - \omega_i)$
- Thrust from individual motor: $F_i = k_F \omega_i^2$
- Moment from individual motor: $M_i = k_M \omega_i^2$
- Newton Equation: $m\ddot{\boldsymbol{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + \boldsymbol{R} \begin{bmatrix} 0 \\ 0 \\ F_1 + F_2 + F_3 + F_4 \end{bmatrix}$
- Euler Equation: $\boldsymbol{I} \cdot \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} + \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \times \boldsymbol{I} \cdot \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} l(F_2 - F_4) \\ l(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix}$

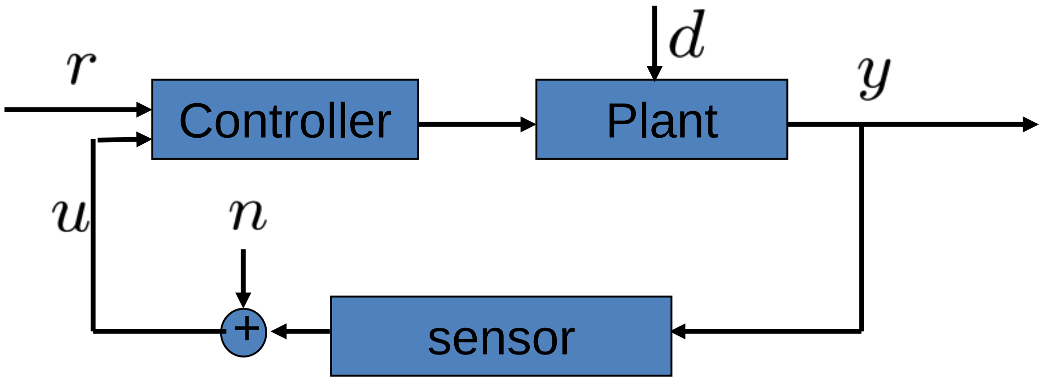
Control System Design



- Therefore we need to have a controller to control its movement in real time based on its movement and the destination

- Open loop control
  - Move the robot in a pre-determined way
  - Example: walking with your eyes closed
- Closed loop (feedback) control
  - Use the output (i.e. the location of the robot) to adjust the input (i.e. the direction and may be speed) to the movement of the robot
  - We also call it feedback control, since we make the control decision based on the output feedback
  - Example: walking with your eyes open
- We want to stabilize a system with closed loop control

- One objective of control is to make the plant stable and track a given reference signal as precise as possible



r: reference input

u: controller input

d: plant disturbance

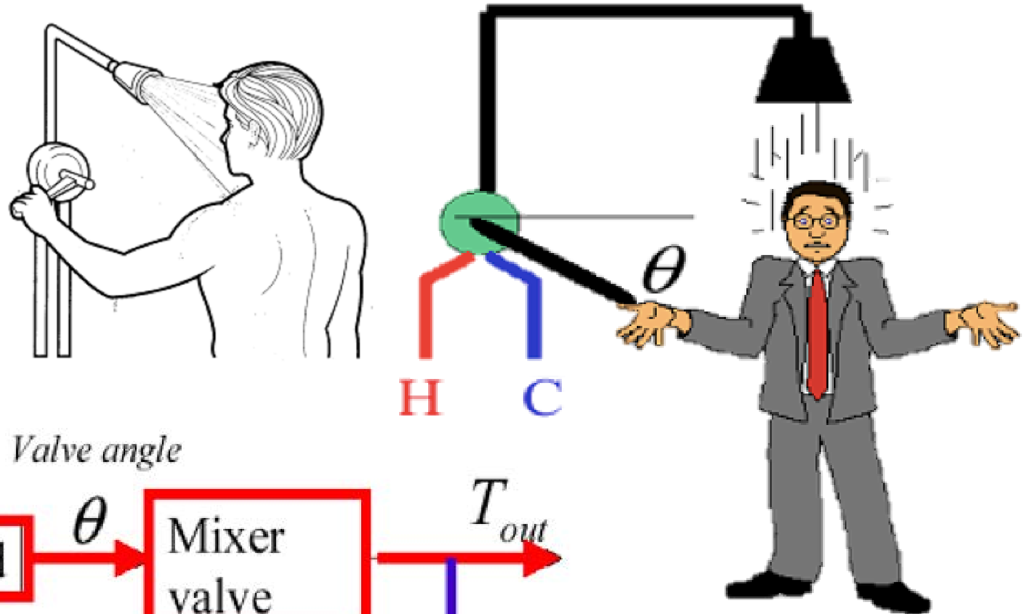
y: output

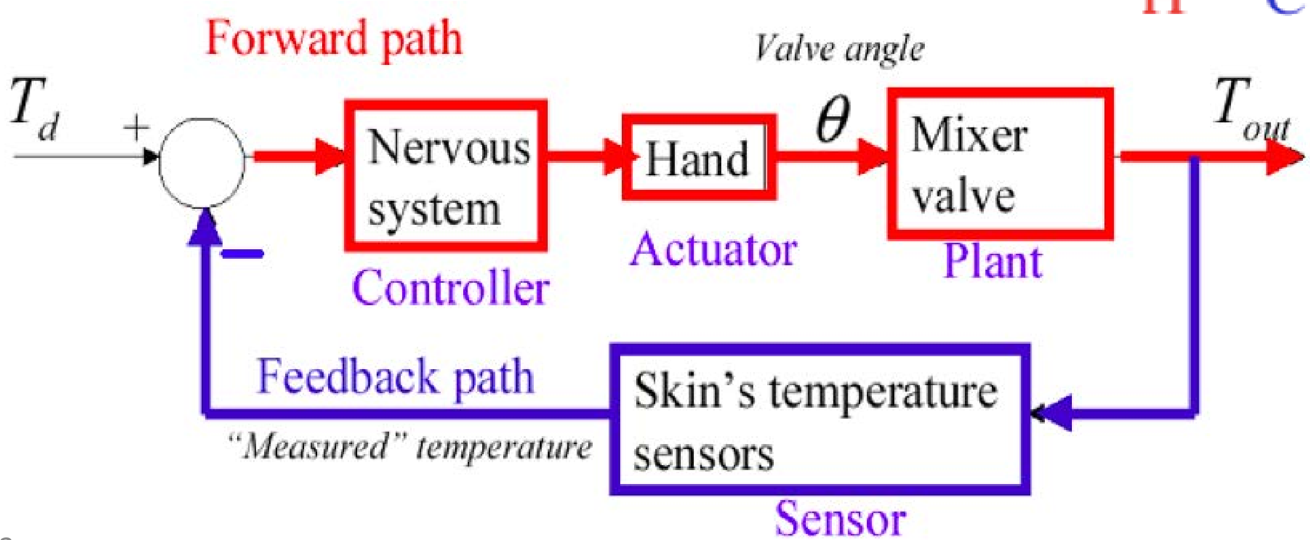
n: communication noise

- A controller is simply a computation unit that computes the "optimal" or "desired" input to the plant

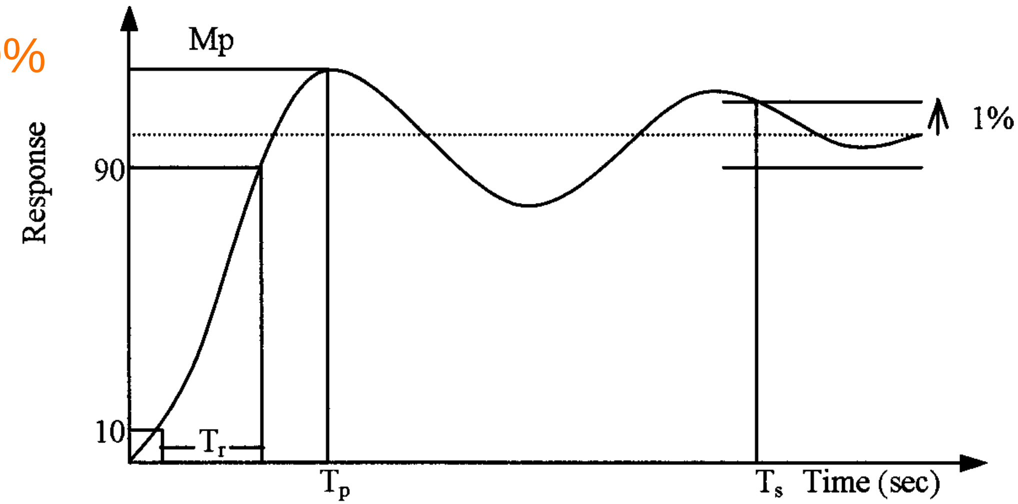
"Feedback is a method of controlling a system by inserting into it the result of its past performance"

- When taking a shower



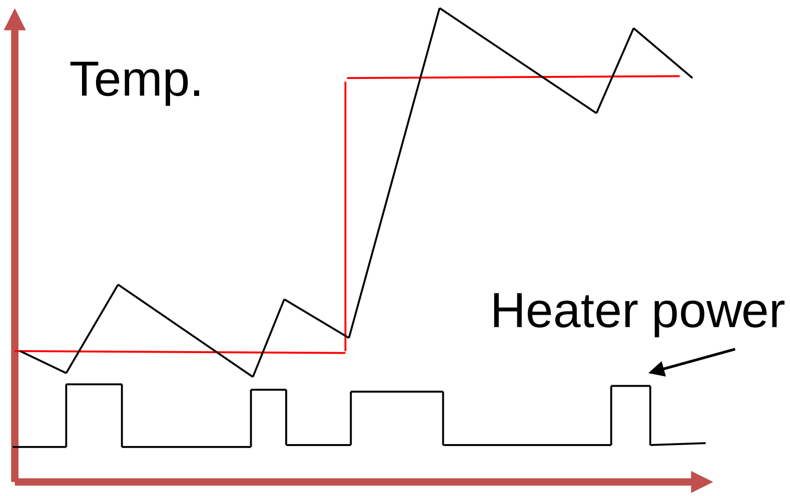


- Rise time:
  - Time it takes from 10% to 90%
- Steady-state error
- Overshoot
  - Percentage by which peak exceeds final value



- Settling time
  - Time it takes to reach 1% of final value
- A good control system has small rise time, overshoot, settling time and steady-state error

- Example: shower water temperature control
  - Turn the heater on if T_{water} is below certain value
  - Turn the heater off if T_{water} is above certain value
- Simple
- Transition is not smooth



Cont. Penn Slides

Next Lecture…

- Time & Trajectory
- Trajectory Generation

Logistics

- Project 1, phase 1 will be released tomorrow (23/9)
  - Tentative due: 30/9
- Schedule changes – check website for details