香港科技大學
THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY
電子及計算機工程學系
DEPARTMENT OF ELECTRONIC & COMPUTER ENGINEERING

Introduction to Aerial Robotics
Lecture 10

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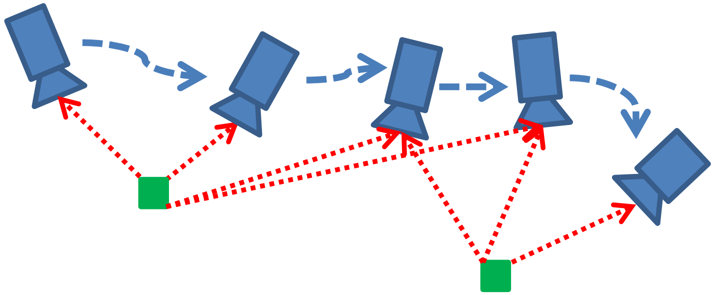
Outline

- The Basics

$$P(x \mid y, z) = \frac{P(y \mid x, z)\ P(x \mid z)}{P(y \mid z)}$$

- 2D Pose Graph SLAM

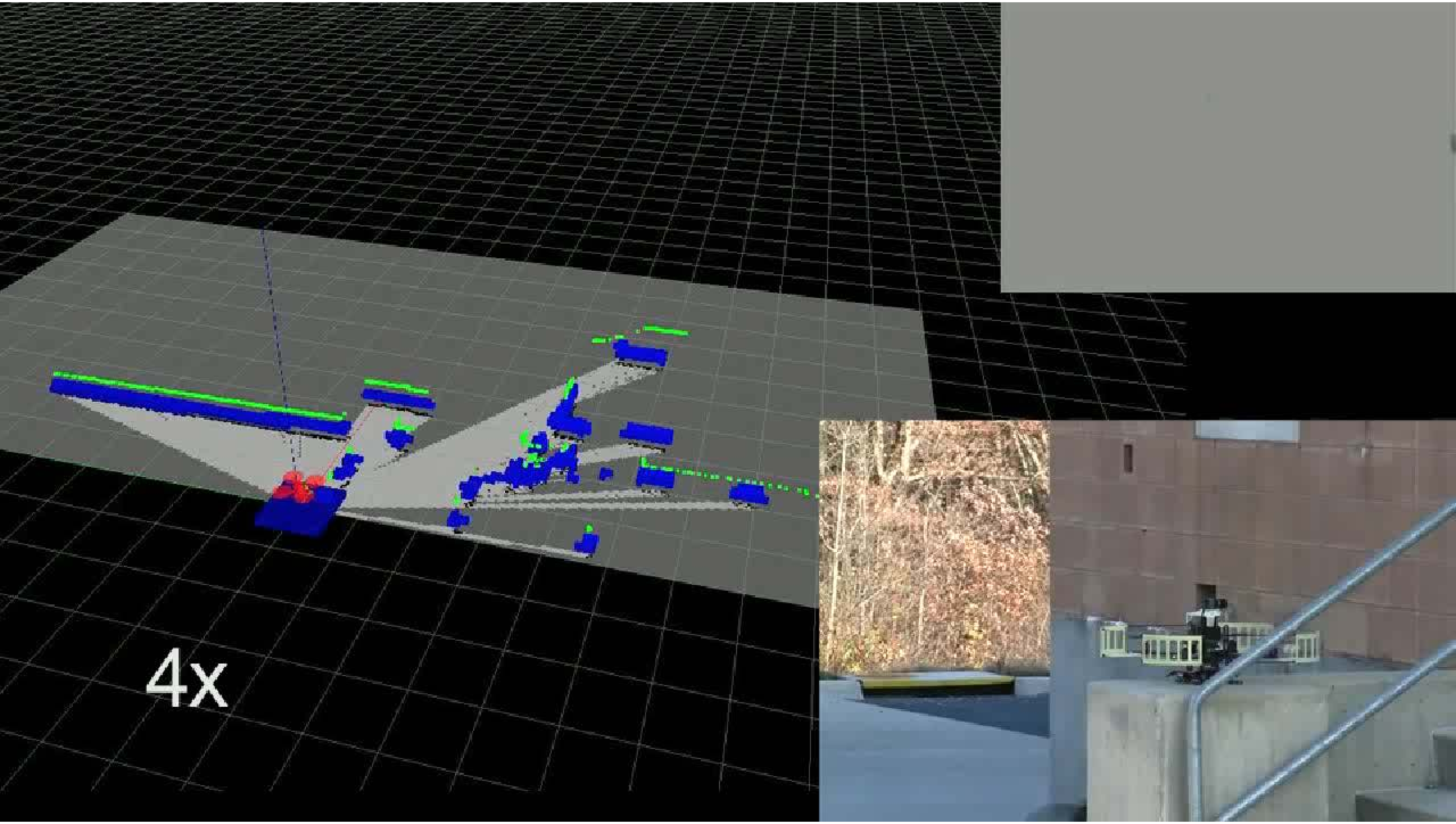
- Monocular Visual-Inertial SLAM



This is Science Fiction…

From Movie “Prometheus”

This is Real…



Shen, et al, 2013

The SLAM Problem

A robot is exploring an unknown, static environment.

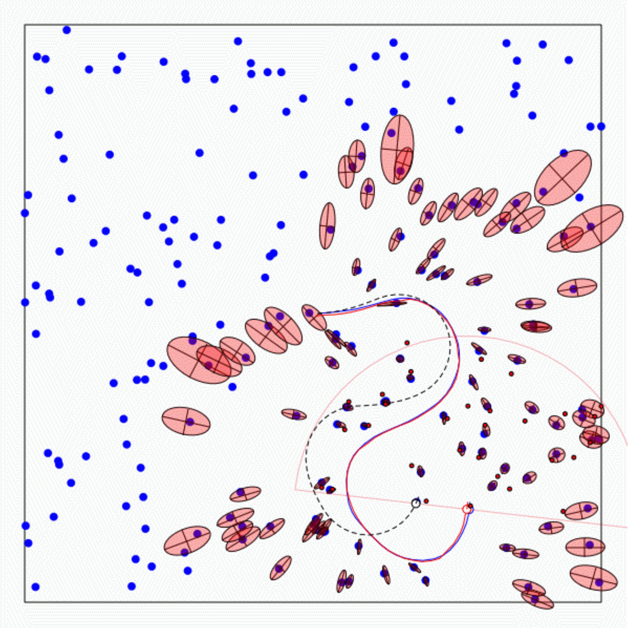
Given:

– Measurements from sensor(s)

Estimate:

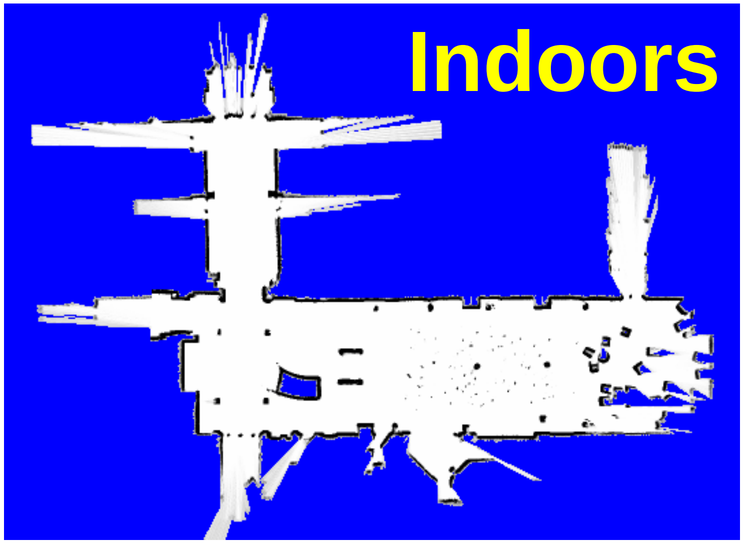
– The map

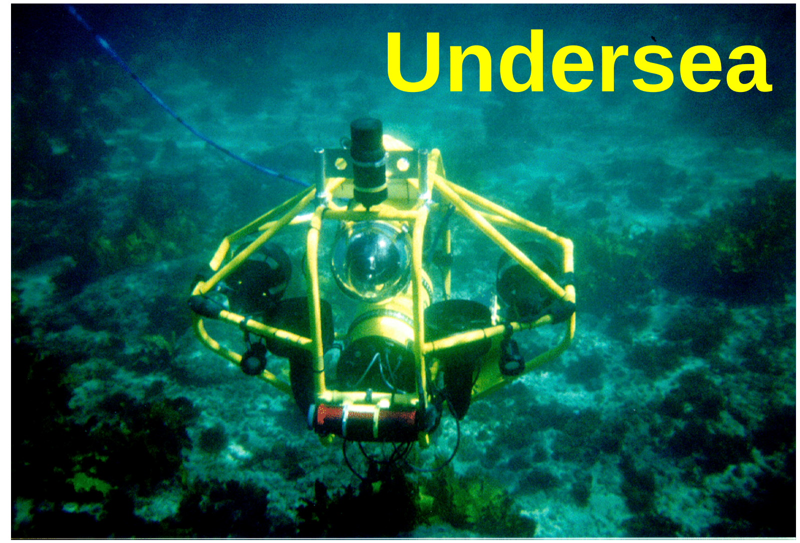
– Path/trajectory of the robot

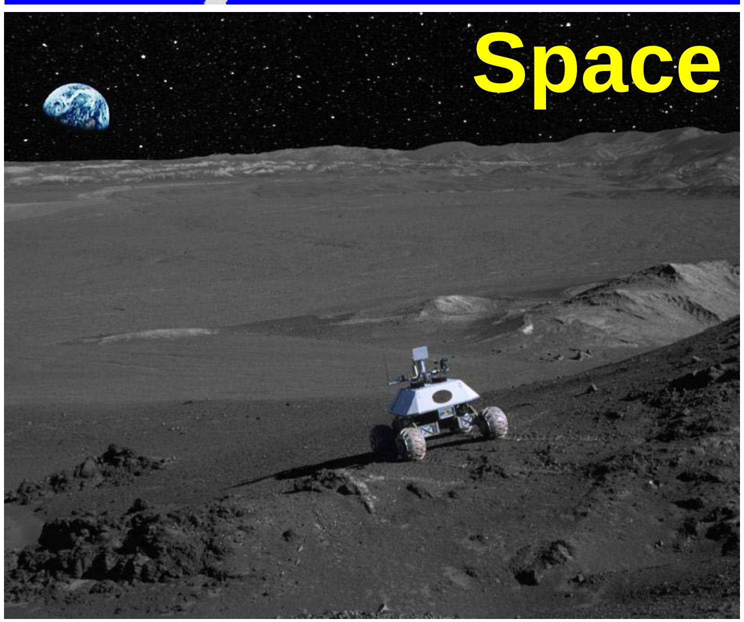


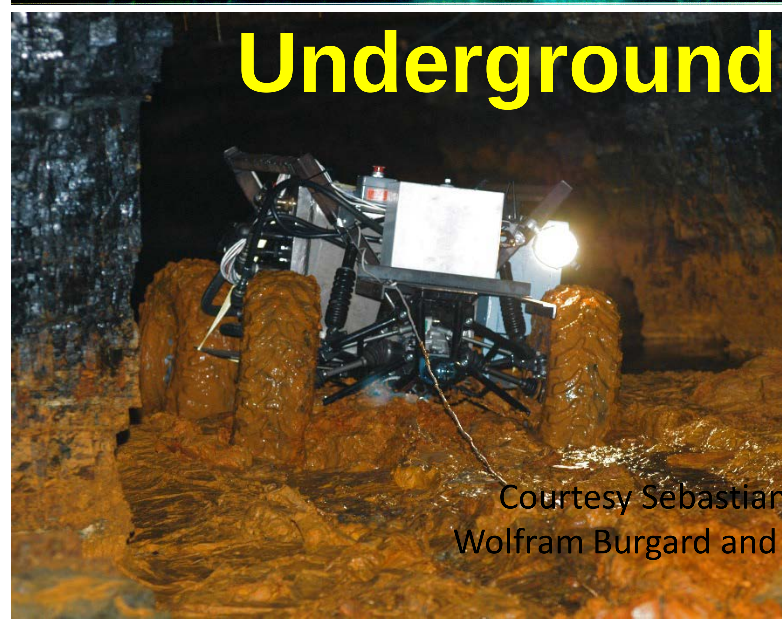
Courtesy Sebastian Thrun, Wolfram Burgard and Dieter Fox

SLAM Applications





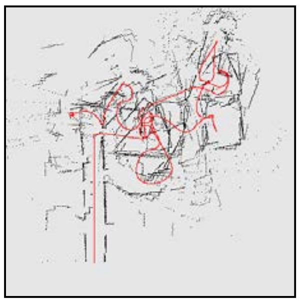


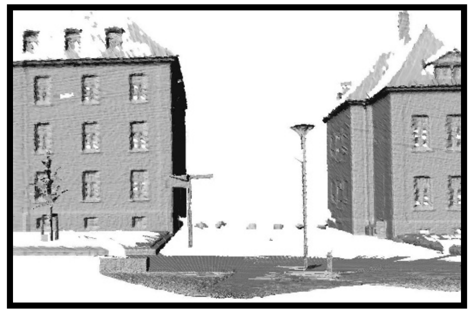


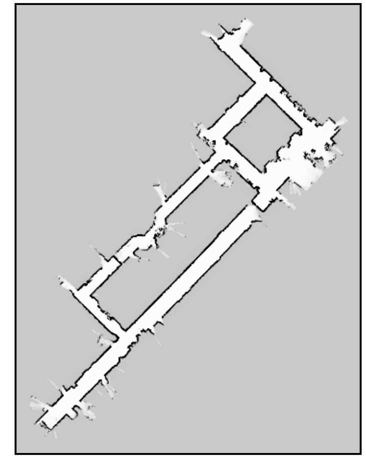
Courtesy Sebastian Thrun,
Wolfram Burgard and Dieter Fox

Representations

- Grid maps or scans

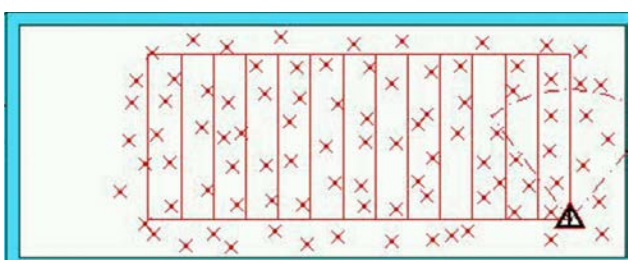


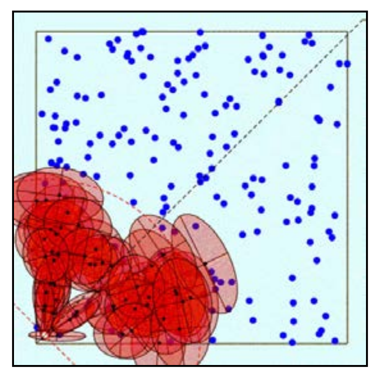


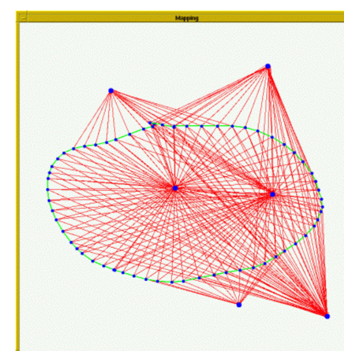


[Lu & Milios, 97; Gutmann, 98: Thrun 98; Burgard, 99; Konolige & Gutmann, 00; Thrun, 00; Arras, 99; Haehnel, 01;…]

- Landmark-based





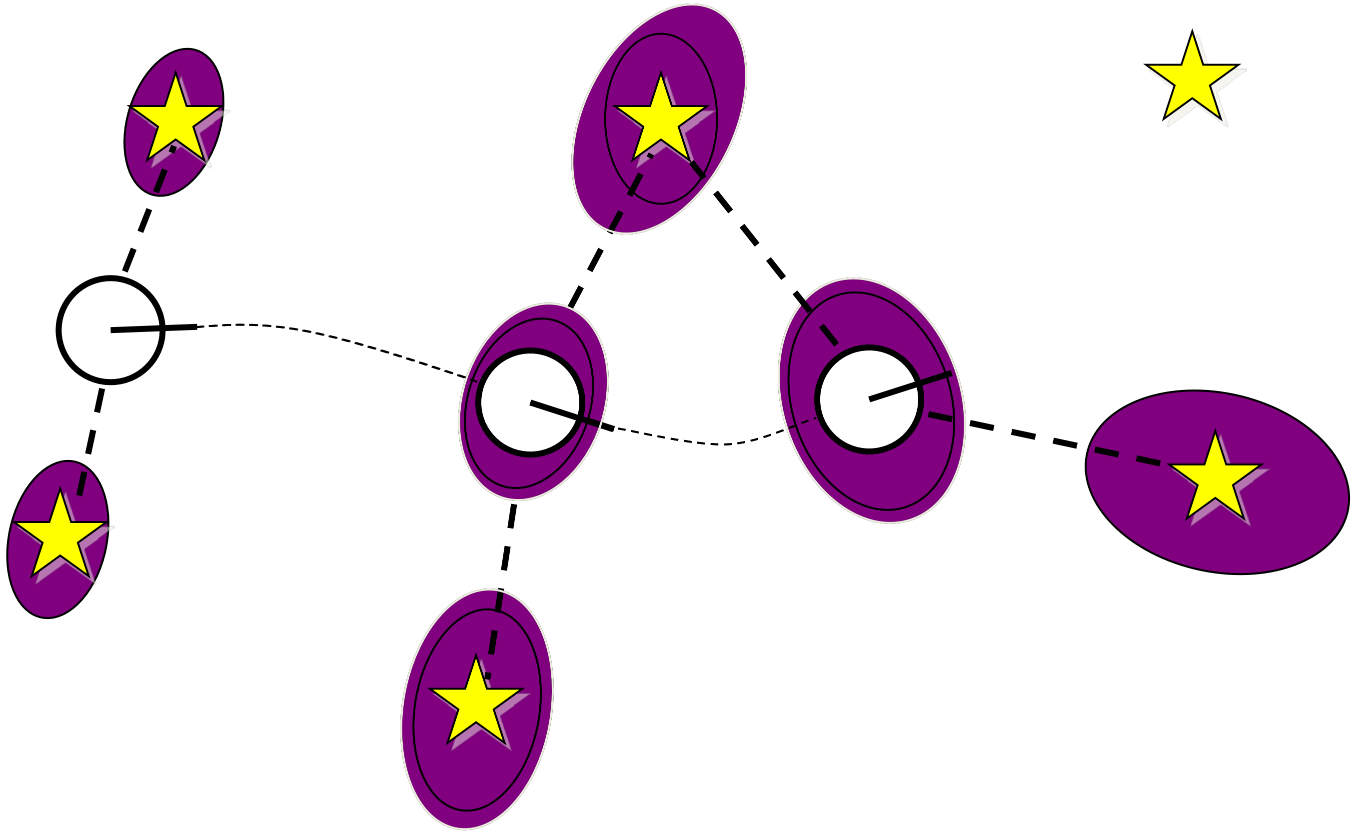


Courtesy Sebastian Thrun, Wolfram Burgard and Dieter Fox

[Leonard et al., 98; Castelanos et al., 99: Dissanayake et al., 2001; Montemerlo et al., 2002;…

Why is SLAM a hard problem?

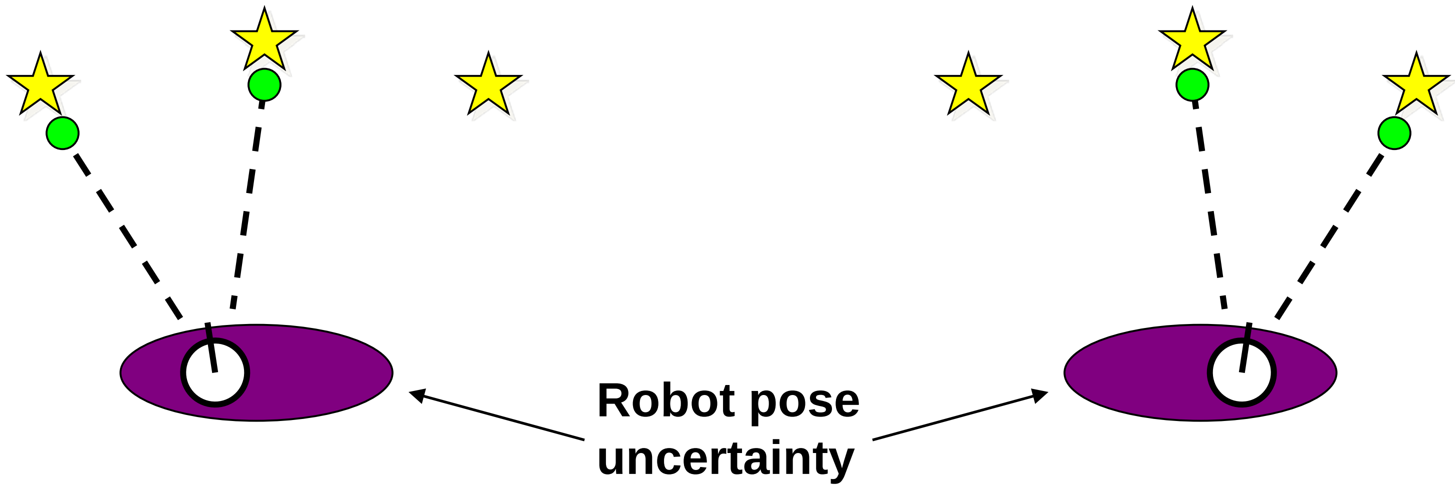
SLAM: robot path and map are both **unknown**



Robot path error correlates errors in the map

Courtesy Sebastian Thrun, Wolfram Burgard and Dieter Fox

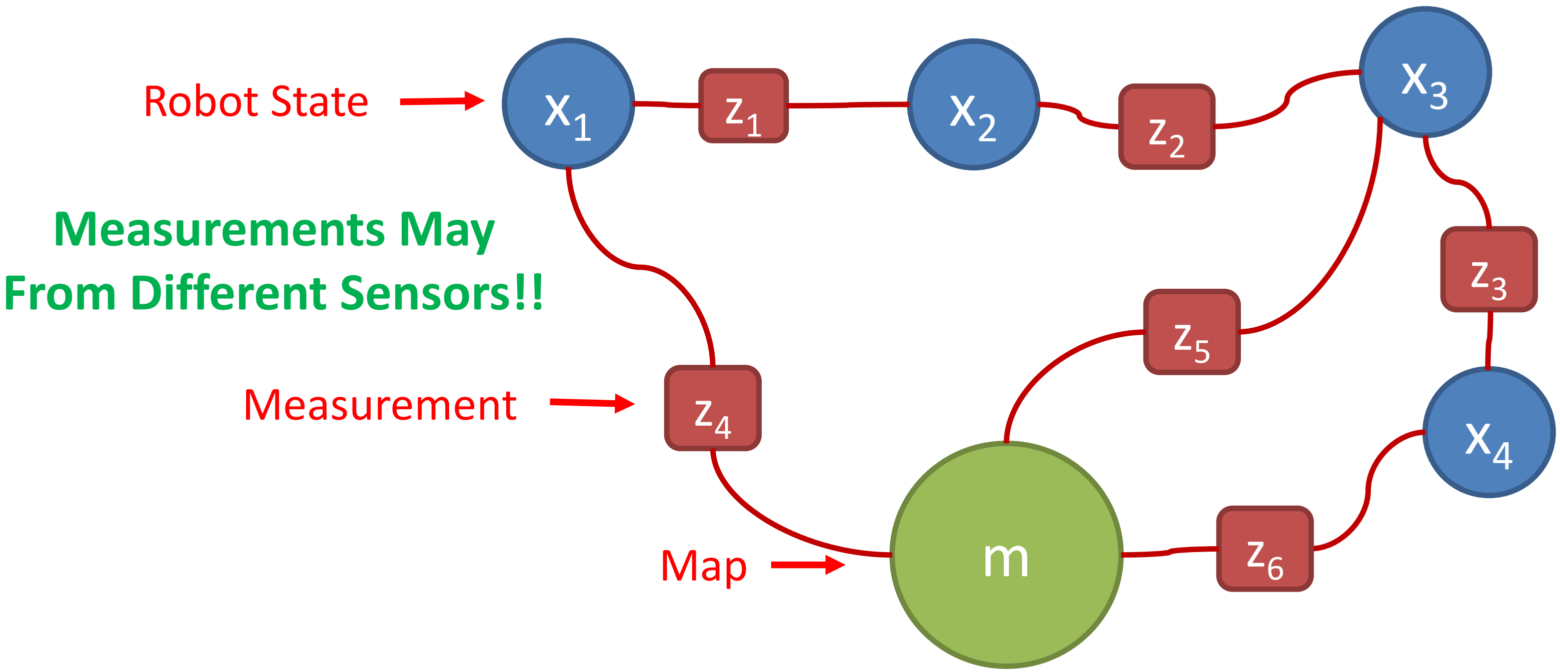
Why is SLAM a hard problem?



- In the real world, the mapping between observations and landmarks is unknown
- Picking wrong data associations can have catastrophic consequences
- Pose error correlates data associations

Courtesy Sebastian Thrun, Wolfram Burgard and Dieter Fox

Graphical Model of SLAM

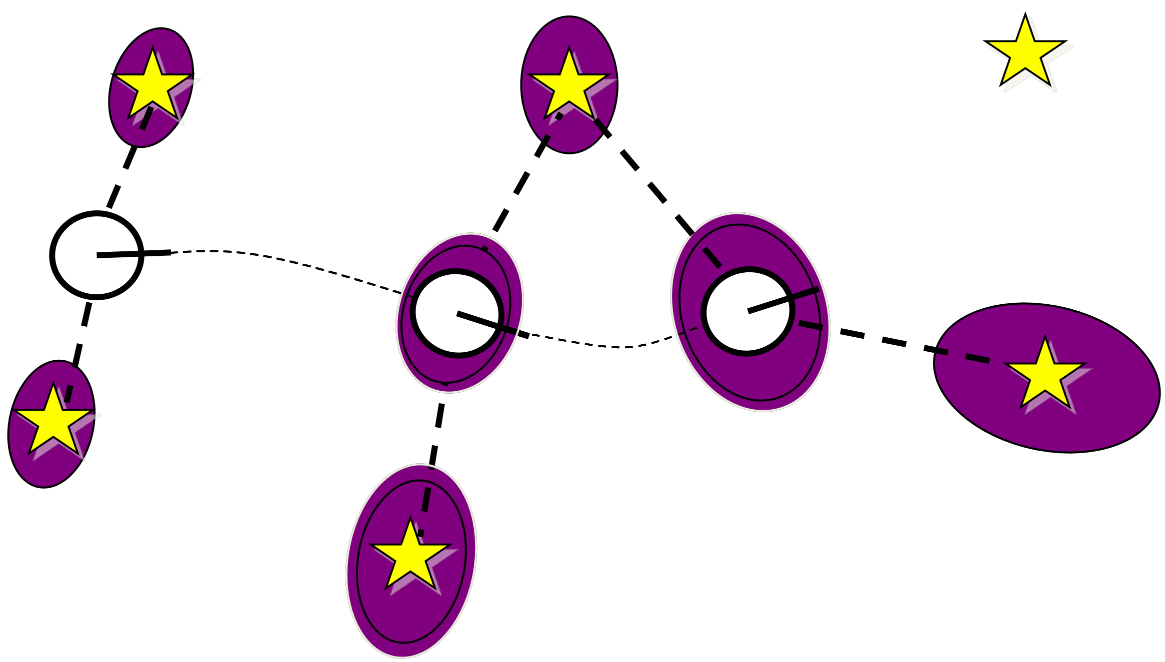


$$(\hat{x}_{1:t}, \hat{m}) = \arg\max_{x_{1:t},\, m} p(x_{1:t}, m \mid z_{1:N})$$

??????

Probabilistic Robotics

Key idea: Explicit representation of uncertainty using the calculus of probability theory



Courtesy Sebastian Thrun, Wolfram Burgard and Dieter Fox

Bayes' Theorem

$$P(x, y) = P(x \mid y)P(y) = P(y \mid x)P(x)$$

$$\Rightarrow$$

$$P(x \mid y) = \frac{P(y \mid x)\ P(x)}{P(y)} = \frac{\text{likelihood} \cdot \text{prior}}{\text{evidence}}$$

$$P(x \mid y, z) = \frac{P(y \mid x, z)\ P(x \mid z)}{P(y \mid z)}$$

Courtesy Sebastian Thrun, Wolfram Burgard and Dieter Fox

Conditional Independence

$$P(x, y \mid z) = P(x \mid z) P(y \mid z)$$

equivalent to

$$P(x \mid z) = P(x \mid z, y)$$

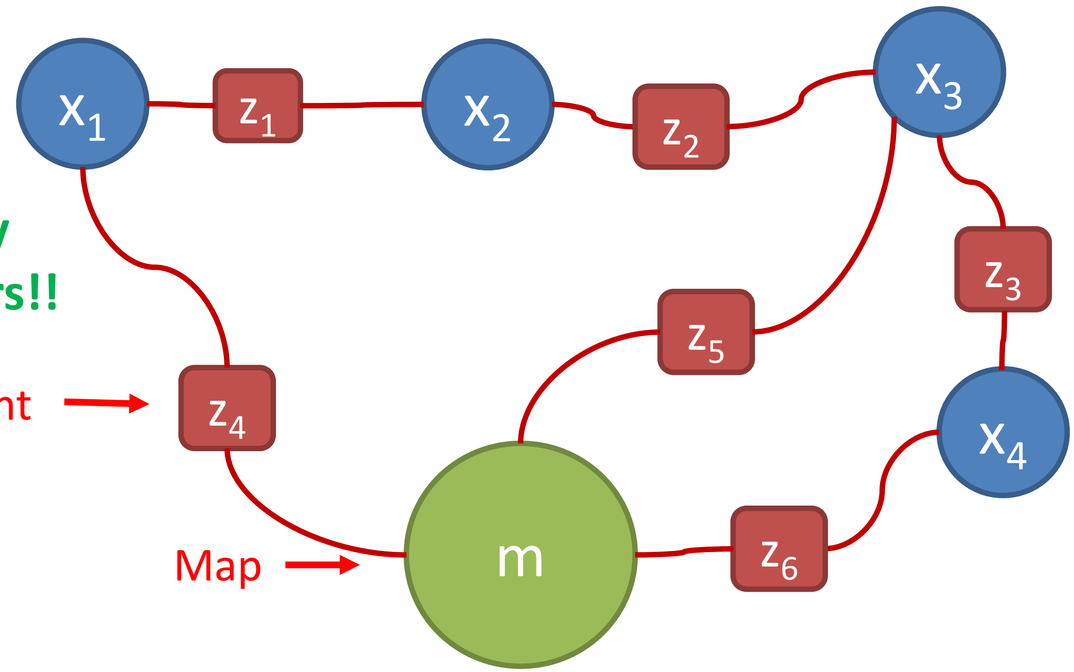
and

$$P(y \mid z) = P(y \mid z, x)$$

Courtesy Sebastian Thrun, Wolfram Burgard and Dieter Fox

SLAM as Maximum A Posteriori Estimation

Robot State



Measurements May From Different Sensors!!

Measurement

Map

Bayes' Rule & Independent Measurements

$$(\hat{x}_{1:t}, \hat{m}) = \arg\max_{x_{1:t},\, m} p(x_{1:t}, m \mid z_{1:N})$$

$$= \arg\max_{x_{1:t},\, m} \prod_{i=1\ldots N} \boxed{p(z_i \mid x_{1:t}, m)} \cdot \boxed{p(x_{1:t}, m)}$$

Measurement Models

Prior

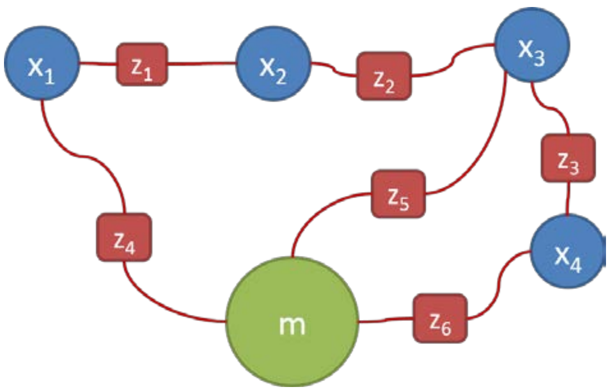
The Gaussian Case

- The multivariate Gaussian distribution

$$\mathbf{x} \sim \mathcal{N}_k(\boldsymbol{\mu}, \boldsymbol{\Sigma}).$$

$$f_{\mathbf{x}}(x_1, \ldots, x_k) = \frac{1}{\sqrt{(2\pi)^k |\boldsymbol{\Sigma}|}} \exp\left(-\frac{1}{2}\boxed{(\mathbf{x}-\boldsymbol{\mu})}^{\mathrm{T}} \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right),$$

Measurement Residual



- The Gaussian case:

$$(\hat{x}_{1:t}, \hat{m}) = \arg\max_{x_{1:t}\ \ m} p(x_{1:t}, m \mid z_{1:N})$$

-log

$$= \arg\max_{x_{1:t}\ \ m} \prod_{i=1\ldots N} p(z_i \mid x_{1:t}, m) \cdot p(x_{1:t}, m)$$

Measurement Residual

$$= \arg\min_{x_{1:t}\ \ m} \sum_{i=1\ldots N} \boxed{r_{z_i}^T} \Sigma_{z_i}^{-1} r_{z_i} + r_{xm}^T \Sigma_{xm}^{-1} r_{xm}$$

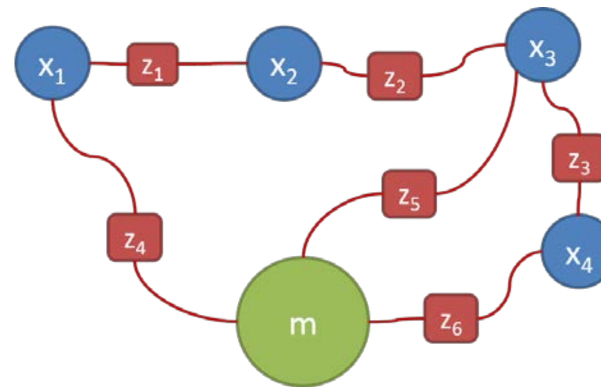
$$\boxed{= \arg\min_{x_{1:t}\ \ m} \sum_{i=1\ldots N} \left\| r_{z_i} \right\|_{\Sigma_{z_i}}^2 + \left\| r_{xm} \right\|_{\Sigma_{xm}}^2}$$

Nonlinear Least Square!

SLAM as Nonlinear Least Square

- Nonlinear least square

$$\min_{x_{1:t}\ \ m} \sum_{i=1\ldots N} r_{z_i}^T \Sigma_{z_i}^{-1} r_{z_i} + r_{xm}^T \Sigma_{xm}^{-1} r_{xm}$$



$$X = [x_{1:t}, m]$$

- Linearization

$$r_{z_i} \approx r_{z_i,\hat{X}} + \frac{\partial r_{z_i,\hat{X}}}{\partial X}\Big|_{\hat{X}} \delta X = r_{z_i,\hat{X}} + \boxed{J_{z_i,\hat{X}}}\,\delta X$$

Measurement Jacobians

Iterate

- Gauss-Newton Method

$$\boxed{J^T \Sigma^{-1} J}\,\delta X = -\boxed{J^T \Sigma^{-1} r}$$

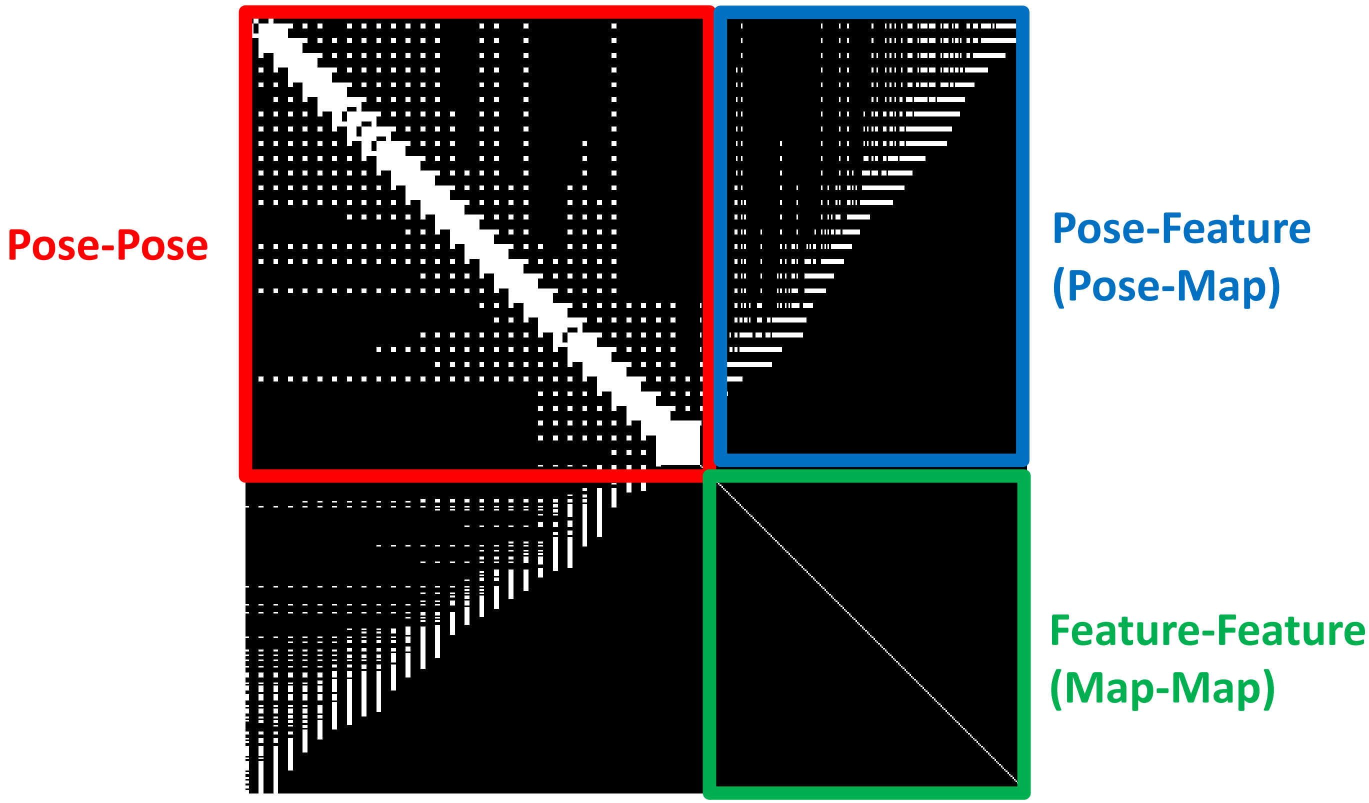
$$\hat{X}^{(k+1)} = \hat{X}^{(k)} + \delta X$$

Information matrix

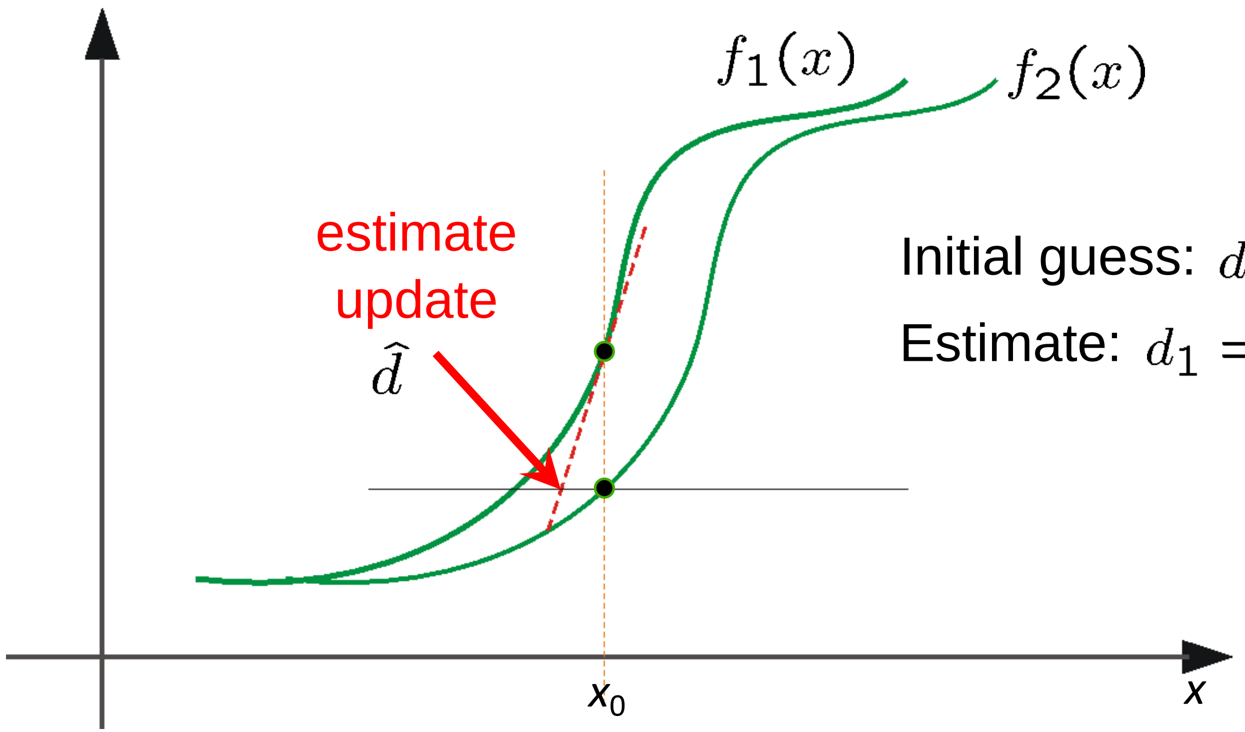
Stacked Jacobian, covariance, and residual

SLAM as Nonlinear Least Square

- Linear system - the information matrix
  - Efficient solution via sparse linear equation solver



Gauss-Newton Method

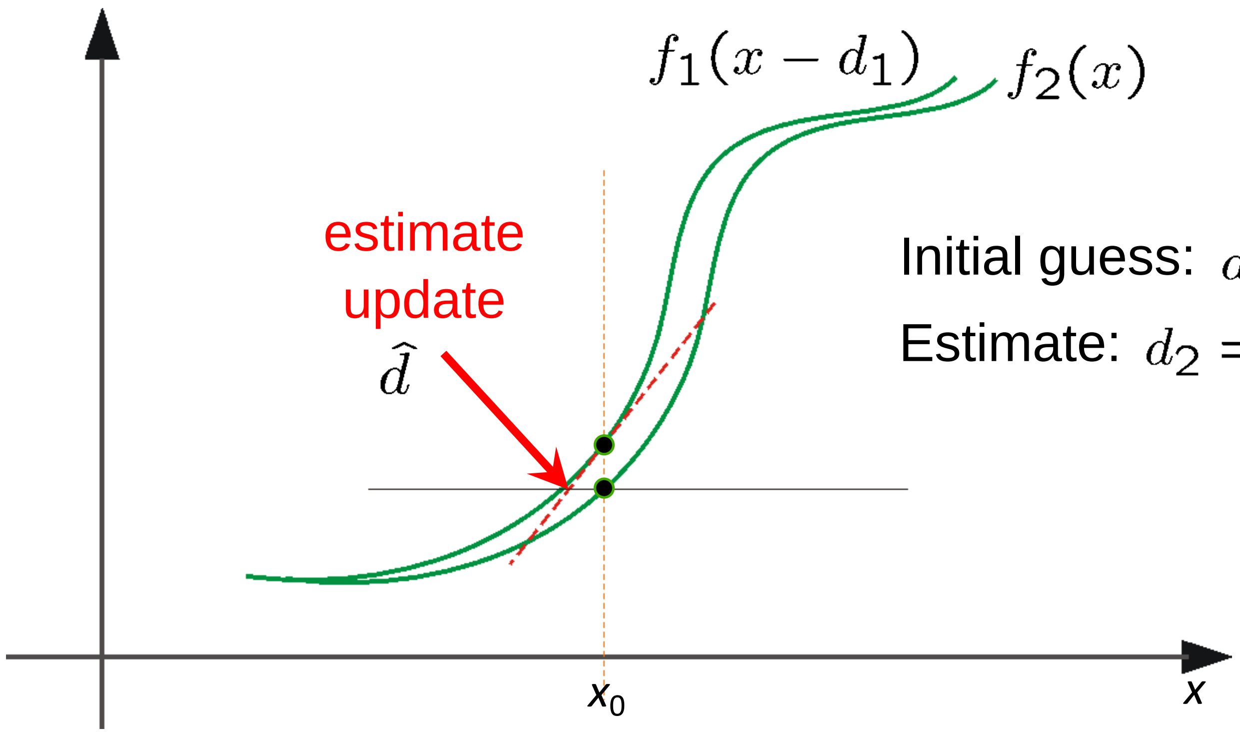


Initial guess: $d_0 = 0$

Estimate: $d_1 = d_0 + \hat{d}$

Adapted from slides from Richard Szeliski

Gauss-Newton Method

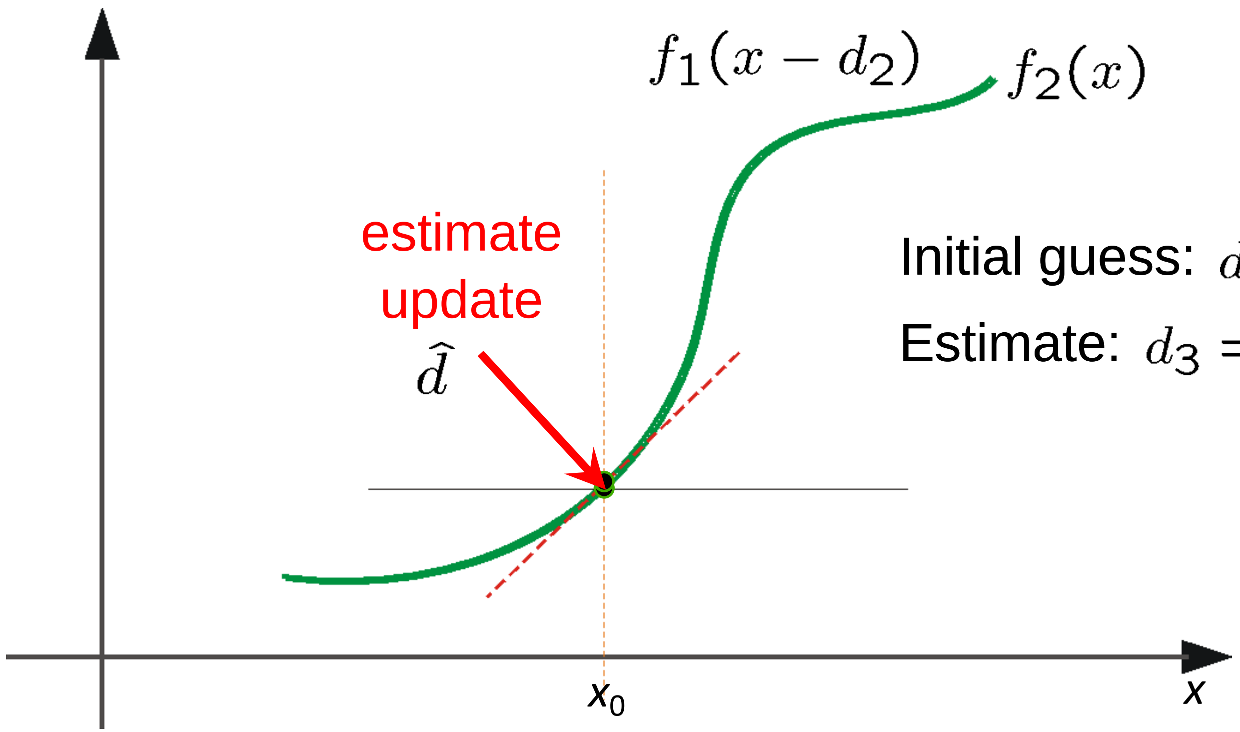


Initial guess: d_1

Estimate: $d_2 = d_1 + \hat{d}$

Adapted from slides from Richard Szeliski

Gauss-Newton Method

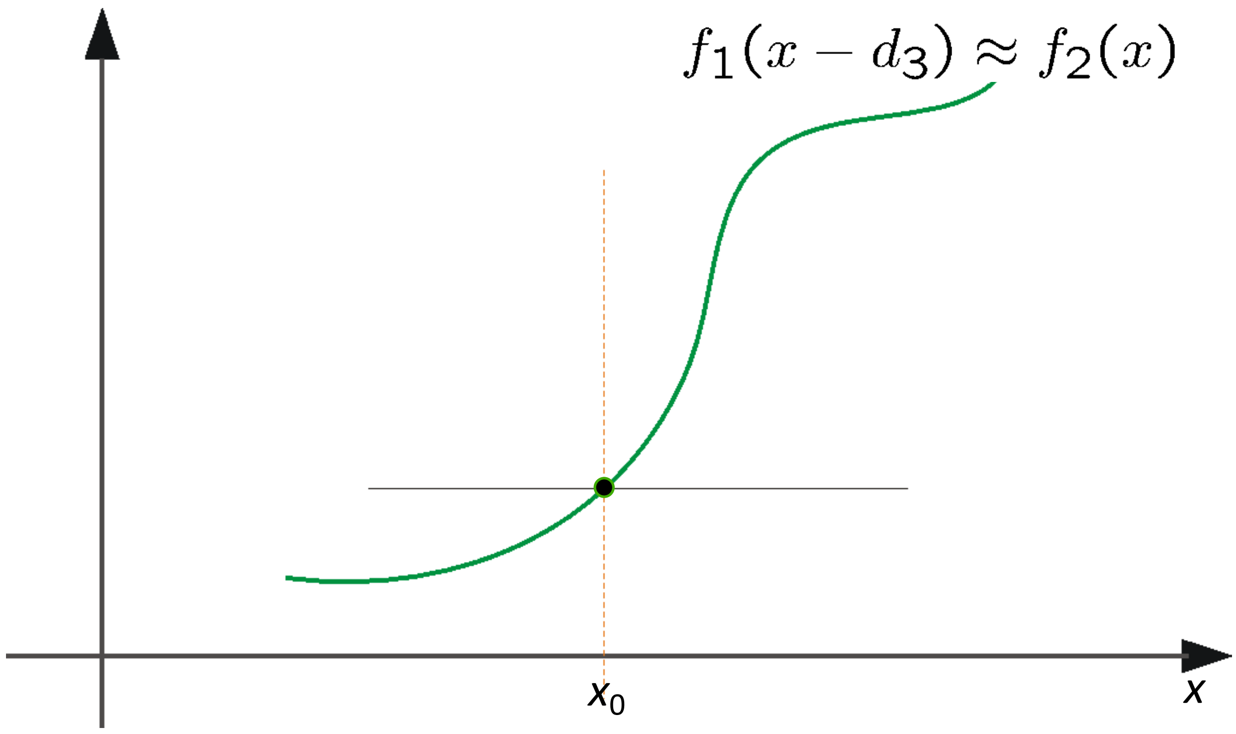


Initial guess: d_2

Estimate: $d_3 = d_2 + \hat{d}$

Adapted from slides from Richard Szeliski

Gauss-Newton Method



Adapted from slides from Richard Szeliski

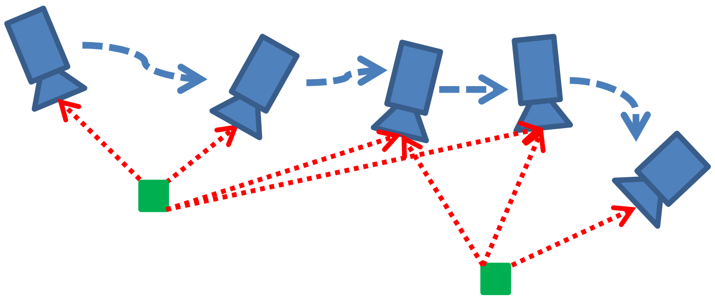
Outline

- The Basics

$$P(x \mid y, z) = \frac{P(y \mid x, z)\ P(x \mid z)}{P(y \mid z)}$$

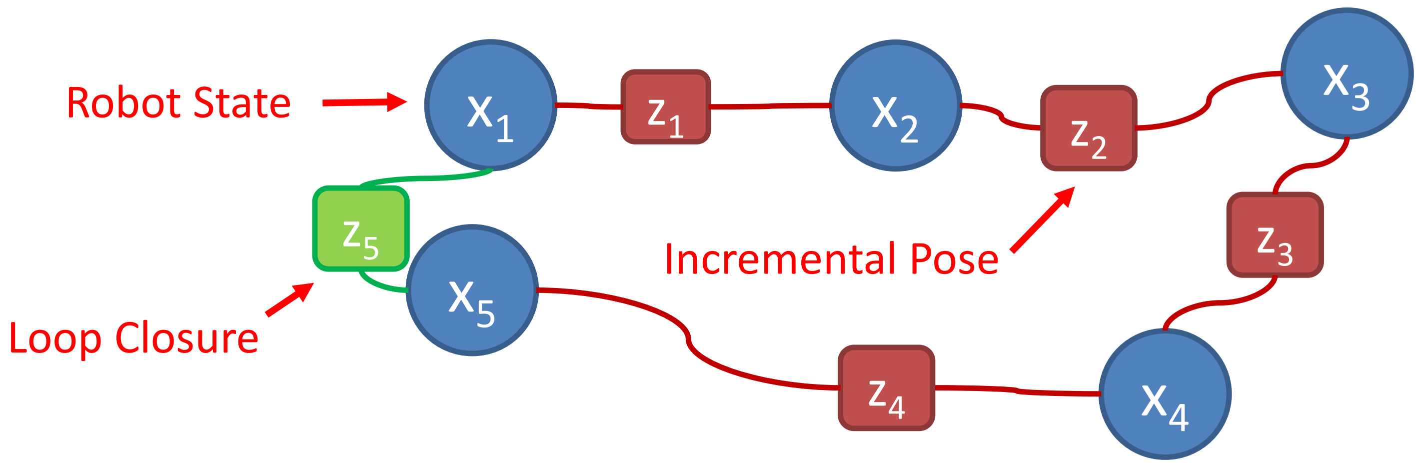
- 2D Pose Graph SLAM

- Monocular Visual-Inertial SLAM



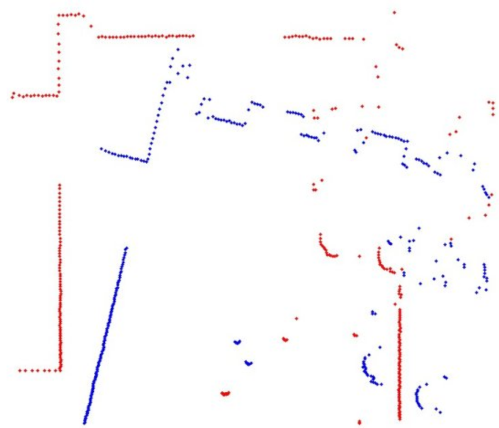
2D Pose Graph SLAM

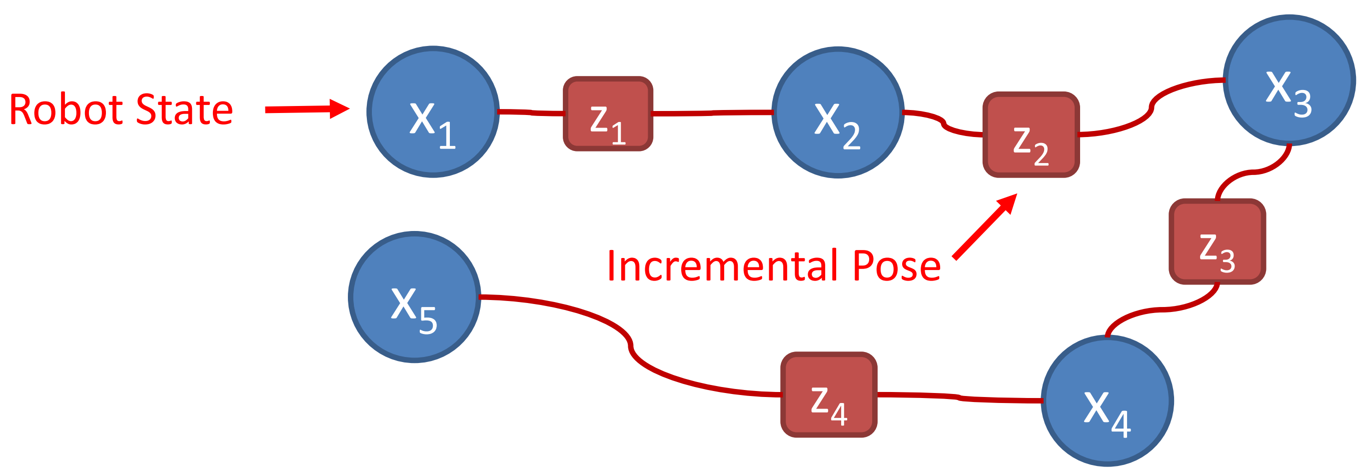
- The only type of measurement is 2D rigid body transformations between 2D robot poses
- No Map



2D Pose Graph SLAM

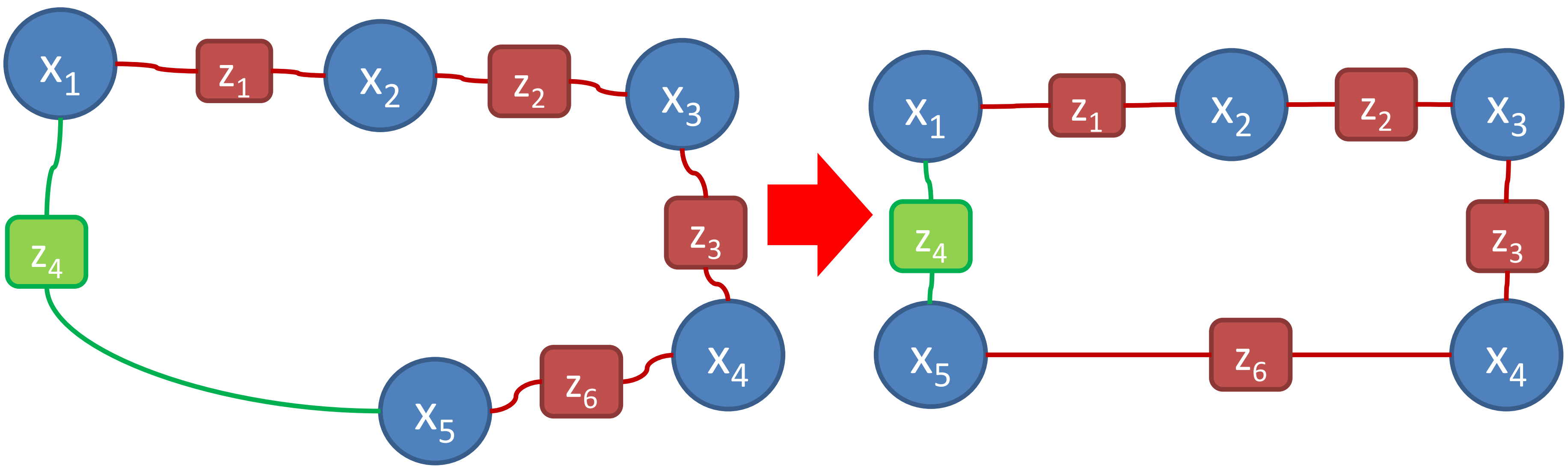
- Incremental pose transformation from scan matching





2D Pose Graph SLAM

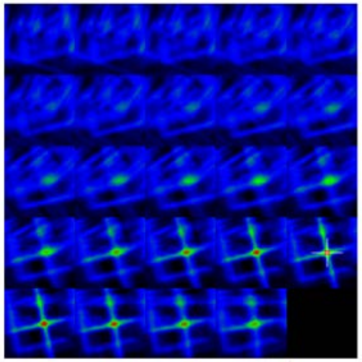
- Global consistency enforced by loop closure

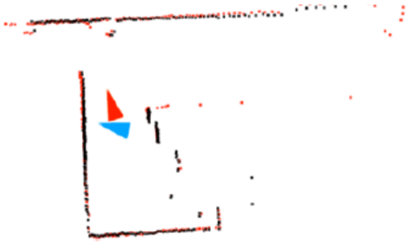


Loop Closure

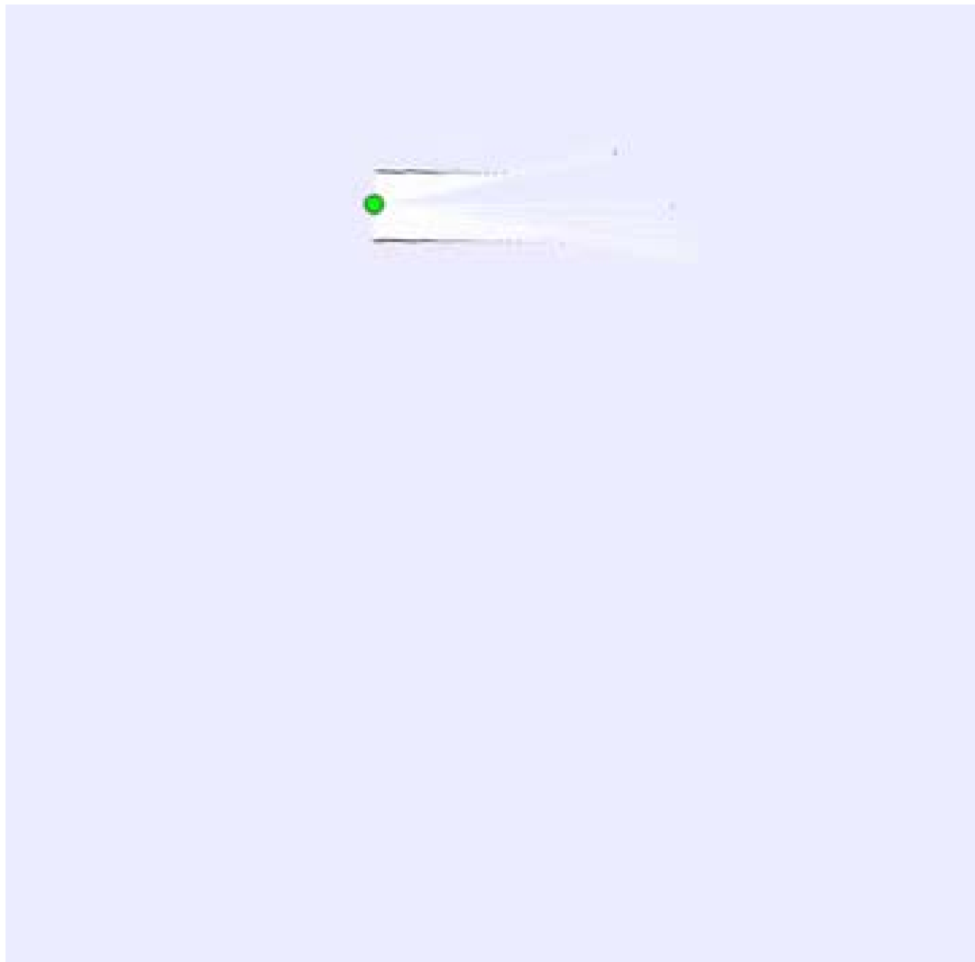
- Question: Have I been to this place before?
  - Appearance-based: FABMAP (M. Cummins and P. Newman, 2008)

  - Scan matching (E. Olson, 2009)





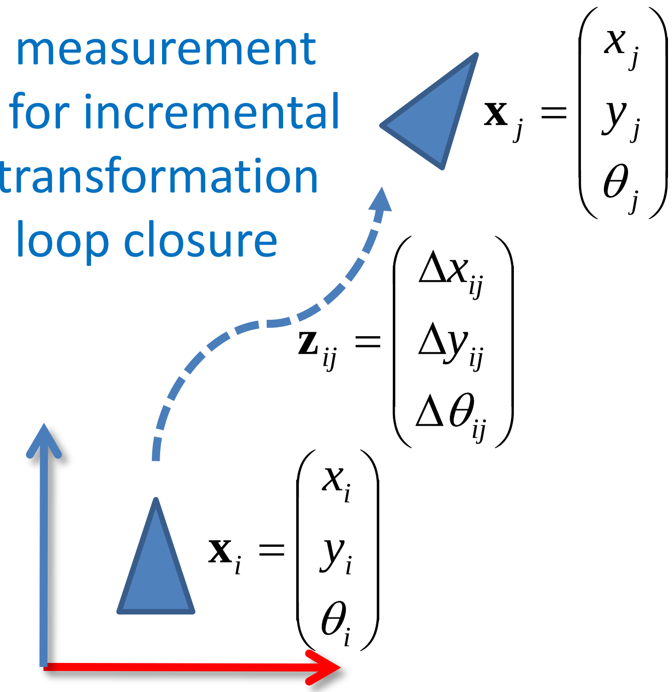
Example



2D Pose Graph SLAM

$$
\begin{aligned}
\hat{\mathbf{x}}_{1:t} &= \arg\max_{\mathbf{x}_{1:t}} p(\mathbf{x}_{1:t} \mid \mathbf{z}_{1:N}) \\
&= \arg\max_{\mathbf{x}_{1:t}} \prod_{i,j} p(\mathbf{z}_{ij} \mid \mathbf{x}_{1:t}) \cdot p(\mathbf{x}_1) \\
&= \arg\min_{\mathbf{x}_{1:t}} \sum_{i,j} \boxed{\mathbf{r}_{z_{ij}}^T} \Sigma_{z_{ij}}^{-1} \mathbf{r}_{z_{ij}} + \mathbf{r}_{x_1}^T \Sigma_{x_1}^{-1} \mathbf{r}_{x_1}
\end{aligned}
$$

Same measurement model for incremental pose transformation and loop closure



Measurement Residual

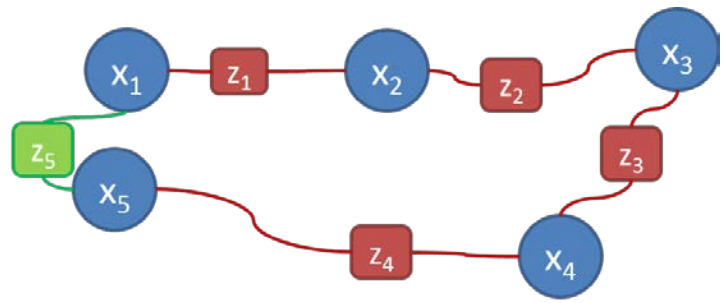
$$
\mathbf{r}_{z_{ij}} = \begin{pmatrix} \Delta x_{ij} \\ \Delta y_{ij} \\ \Delta \theta_{ij} \end{pmatrix} - \begin{bmatrix} \begin{pmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{pmatrix}^{-1} \begin{pmatrix} x_j - x_i \\ y_j - y_i \end{pmatrix} \\ \theta_j - \theta_i \end{bmatrix} = \mathbf{z}_{ij} - h(\mathbf{x}_i, \mathbf{x}_j)
$$

$$
\mathbf{r}_{z_{ij}} \sim N(0, \Sigma_{z_{ij}})
$$

SLAM as Nonlinear Least Square

- Nonlinear least square

$$\hat{\mathbf{x}}_{1:t} = \arg\min_{\mathbf{x}_{1:t}} \sum_{i,j} \mathbf{r}_{z_{ij}}^T \Sigma_{z_{ij}}^{-1} \mathbf{r}_{z_{ij}} + \cancel{\mathbf{r}_{x_1}^T \Sigma_{x_1}^{-1} \mathbf{r}_{x_1}}$$



- Linearization

$$\mathbf{r}_{z_{ij}} \approx \mathbf{r}_{z_{ij},\hat{X}} + \frac{\partial \mathbf{r}_{z_{ij},\hat{X}}}{\partial X}\Big|_{\hat{X}} \delta X = \mathbf{r}_{z_{ij},\hat{X}} + \mathbf{J}_{z_{ij},\hat{X}} \delta X \qquad X = [\mathbf{x}_{1:t}]$$

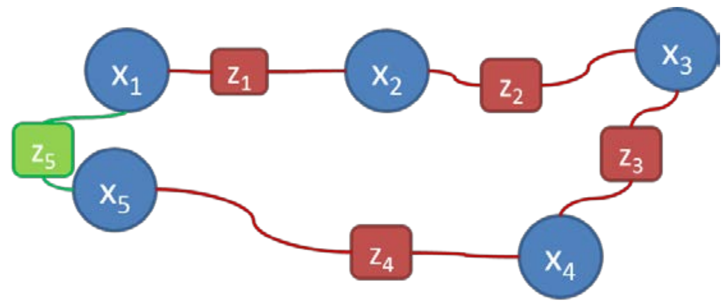
$$\mathbf{J}_{z_{ij},\hat{X}} = \begin{bmatrix} 0 & \cdots & \underbrace{\boxed{\begin{pmatrix} \hat{\mathbf{R}}_{\theta_i}^{-1} \begin{pmatrix} \sin\hat{\theta}_i & -\cos\hat{\theta}_i \\ \cos\hat{\theta}_i & \sin\hat{\theta}_i \end{pmatrix} \begin{pmatrix} \hat{x}_j - \hat{x}_i \\ \hat{y}_j - \hat{y}_i \end{pmatrix} \\ 0 \quad -1 \end{pmatrix}}}_{\text{i Block}} & \cdots & \underbrace{\boxed{\begin{pmatrix} -\hat{\mathbf{R}}_{\theta_i}^{-1} & 0 \\ 0 & 1 \end{pmatrix}}}_{\text{j Block}} & \cdots \end{bmatrix}$$

3x3N

SLAM as Nonlinear Least Square

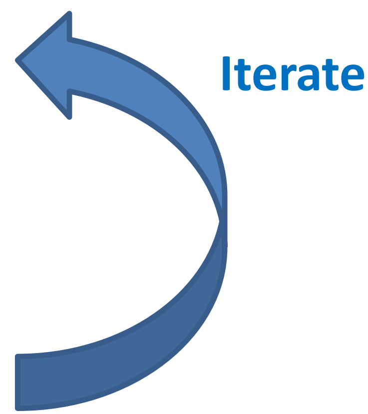
- Nonlinear least square

$$\hat{\mathbf{x}}_{1:t} = \arg\min_{\mathbf{x}_{1:t}} \sum_{i,j} \mathbf{r}_{z_{ij}}^T \Sigma_{z_{ij}}^{-1} \mathbf{r}_{z_{ij}} + \cancel{\mathbf{r}_{x_1}^T \Sigma_{x_1}^{-1} \mathbf{r}_{x_1}}$$



- Linearization

$$\mathbf{r}_{z_{ij}} \approx \mathbf{r}_{z_{ij},\hat{X}} + \frac{\partial \mathbf{r}_{z_{ij},\hat{X}}}{\partial X}\Big|_{\hat{X}} \delta X = \mathbf{r}_{z_{ij},\hat{X}} + \mathbf{J}_{z_{ij},\hat{X}} \delta X$$



- Gauss-Newton method

$$\sum_{i,j} \mathbf{J}_{z_{ij},\hat{X}}^T \Sigma_{z_{ij}}^{-1} \mathbf{J}_{z_{ij},\hat{X}} \cdot \delta X = -\sum_{i,j} \mathbf{J}_{z_{ij},\hat{X}}^T \Sigma_{z_{ij}}^{-1} \mathbf{r}_{z_{ij},\hat{X}}$$

$$\hat{X}^{(k+1)} = \hat{X}^{(k)} + \delta X$$

More Example

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- The Basics

$$P(x \mid y, z) = \frac{P(y \mid x, z)\ P(x \mid z)}{P(y \mid z)}$$

- 2D Pose Graph SLAM

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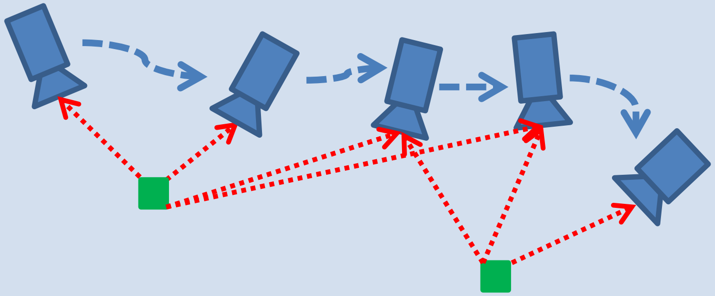
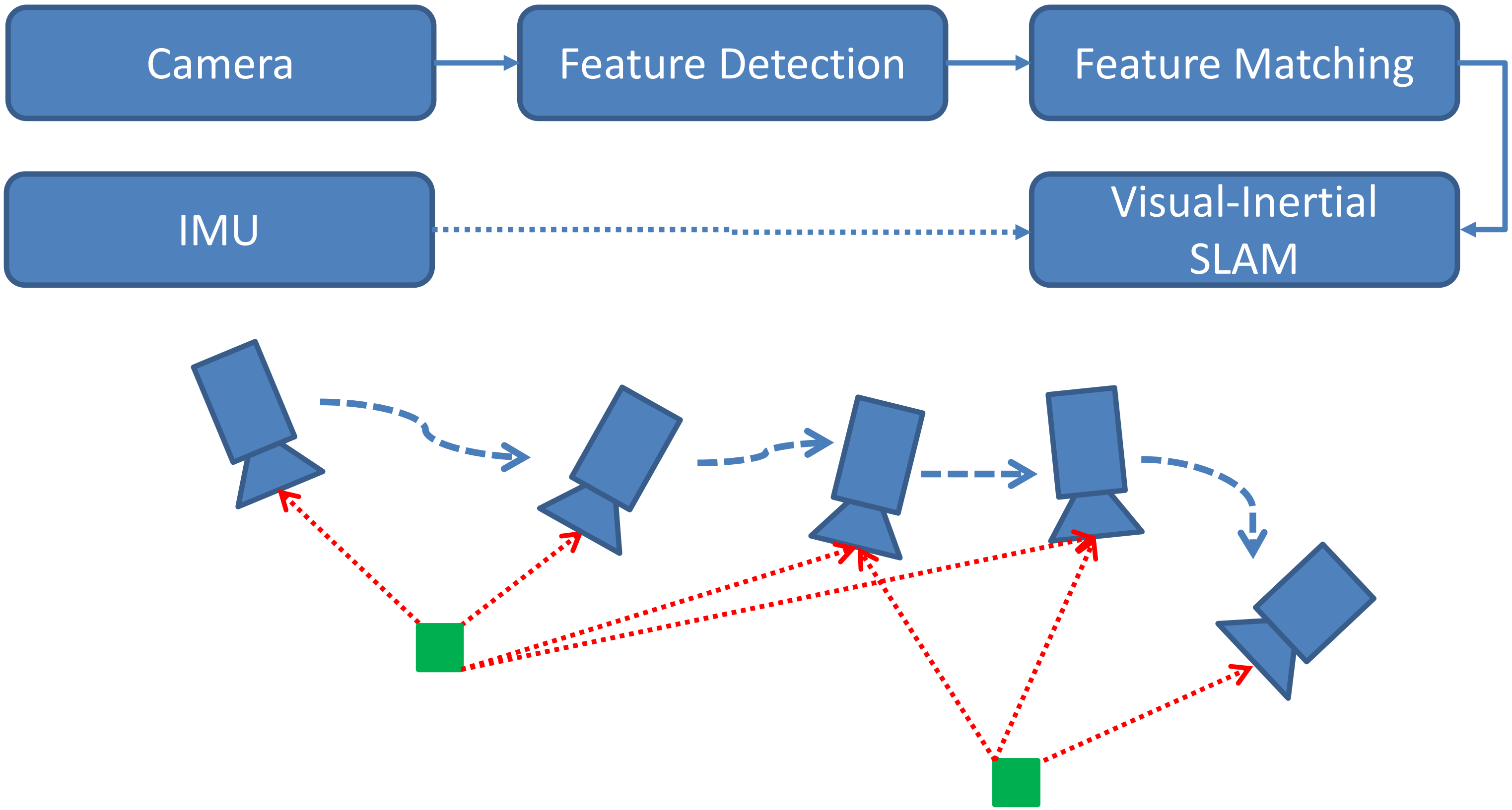


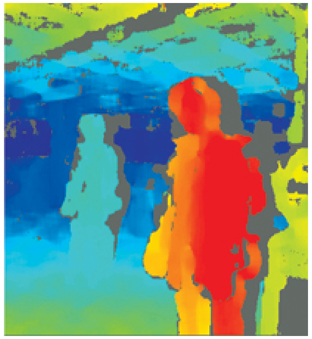
Visual-Inertial SLAM Pipeline



Camera Setup

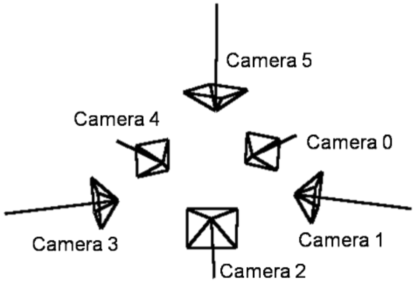
- Monocular
  - Depth Unknown

- Stereo
  - Able to compute depth
  - Depth accuracy affect by baseline, resolution, and calibration



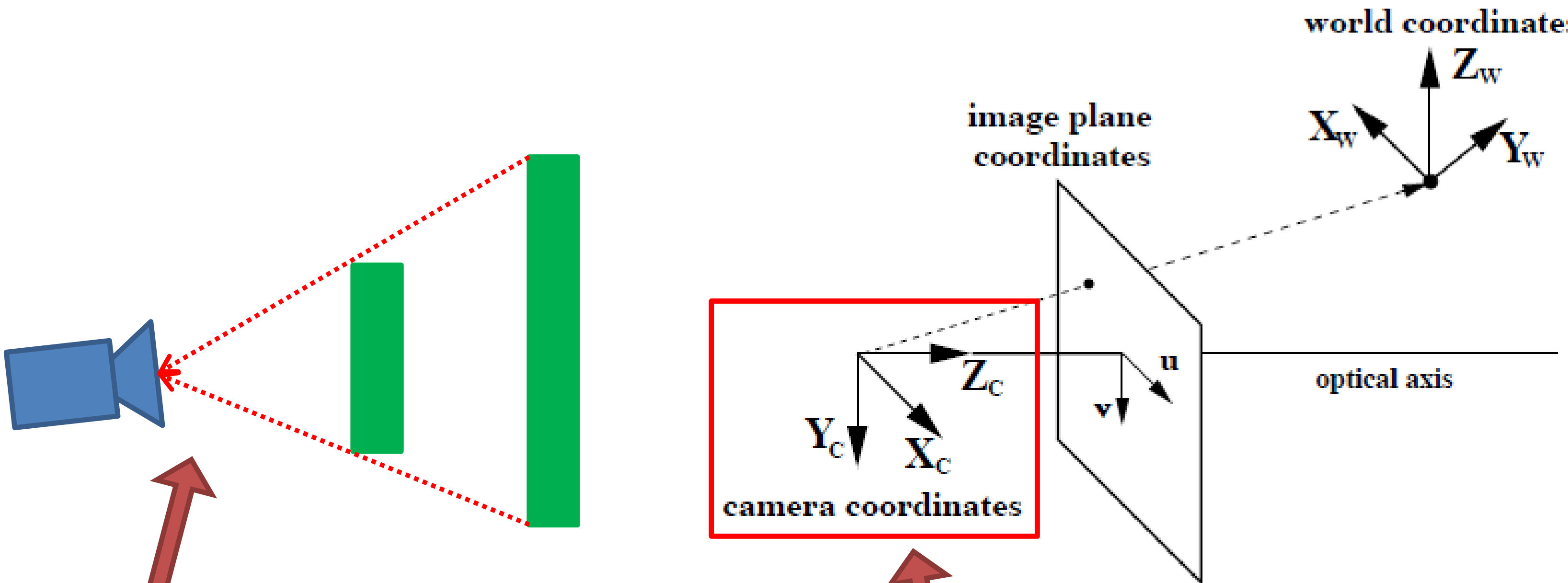
- Multi-Camera
  - Overlapping / Non-overlapping field-of-view

Omnidirectional multi-camera system Ladybug



Relative position and posture of each camera

Pin-hole Camera Model



$$\boxed{\lambda}\boxed{\begin{pmatrix}u\\v\\1\end{pmatrix}} = \boxed{\begin{pmatrix} f & 0 & u_o\\ 0 & f & v_o\\ 0 & 0 & 1\end{pmatrix}}\boxed{\begin{pmatrix} R & t\end{pmatrix}}\boxed{\begin{pmatrix}X_w\\Y_w\\Z_w\\1\end{pmatrix}} = P\begin{pmatrix}X_w\\Y_w\\Z_w\\1\end{pmatrix}$$

Unknown Depth

Pixel Values

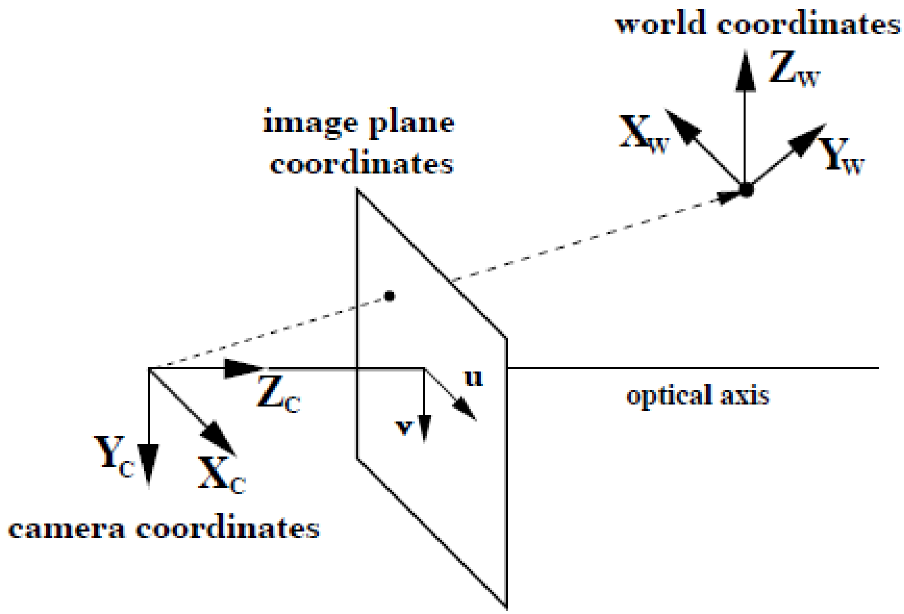
Intrinsic Calibration Matrix

Camera Pose

World Point

Courtesy Kostas Daniilidis

Problem Formulation



Known

Measurement

To be Estimated

$$\lambda \boxed{\begin{pmatrix} u \\ v \\ 1 \end{pmatrix}} = \boxed{\begin{pmatrix} f & 0 & u_o \\ 0 & f & v_o \\ 0 & 0 & 1 \end{pmatrix}} \boxed{\begin{pmatrix} R & t \end{pmatrix}} \boxed{\begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}} = P \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

Pixel Values

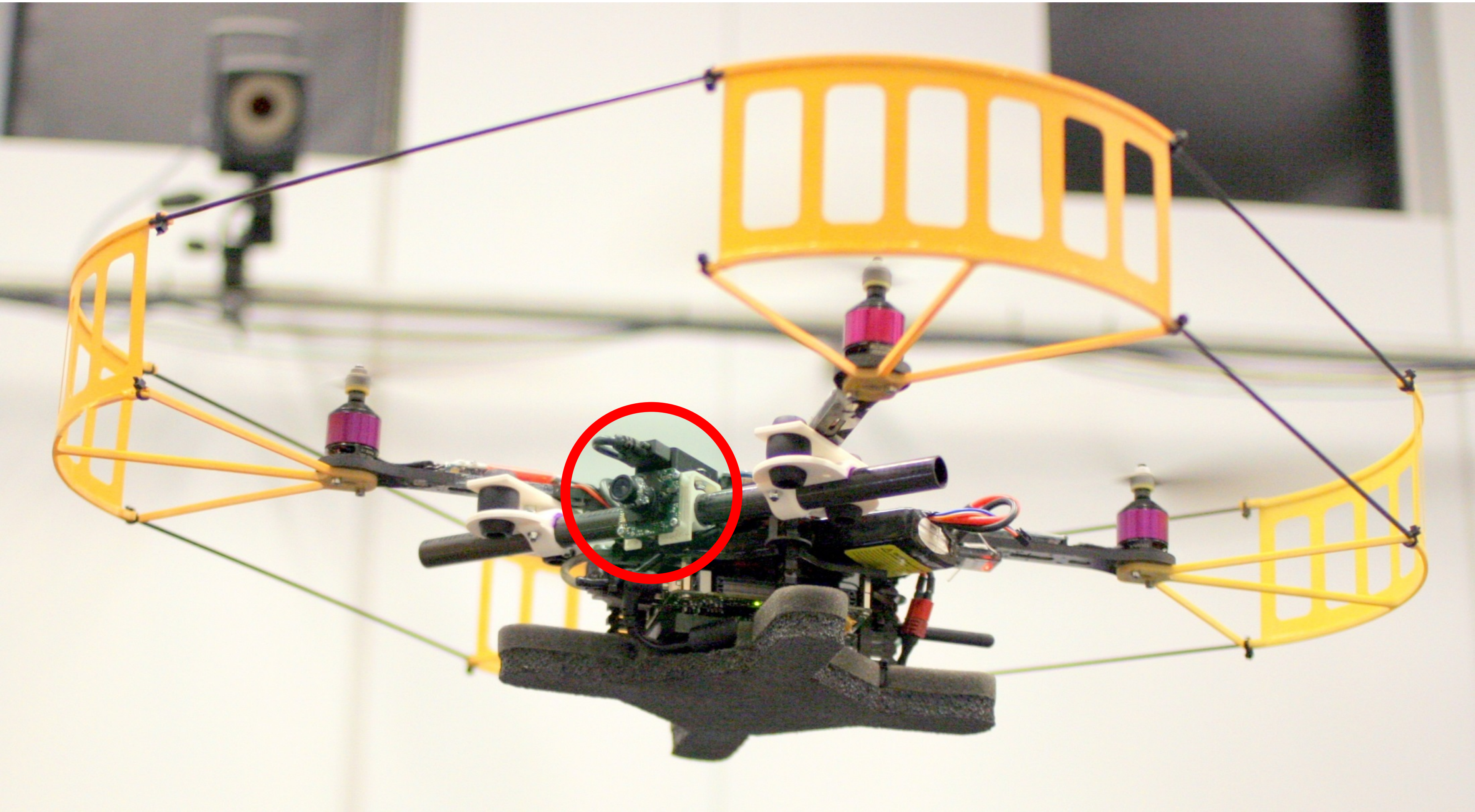
Intrinsic Calibration Matrix

Camera Pose

World Point

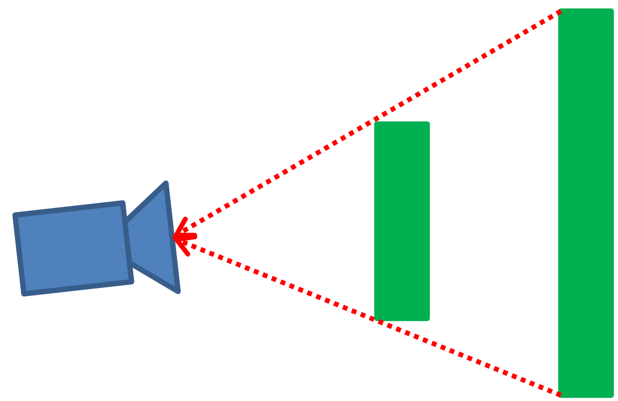
Courtesy Kostas Daniilidis

Fly with 1 Camera + 1 IMU

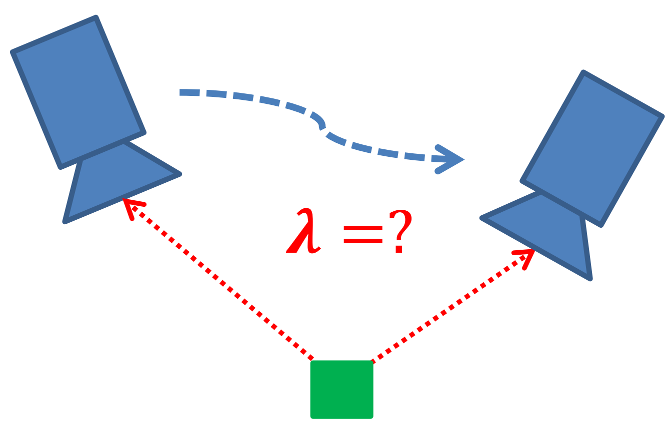


Challenges

- Scale ambiguity

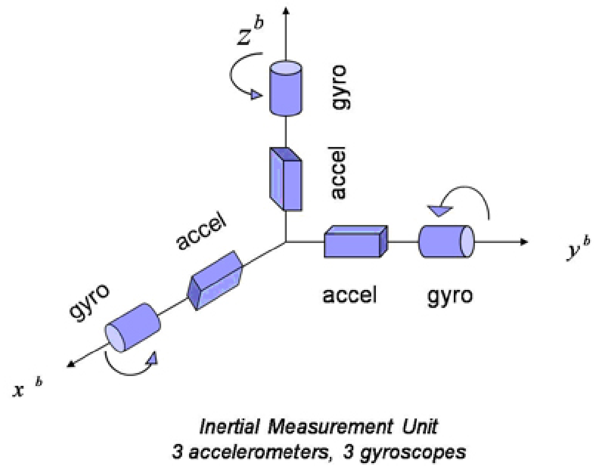


- Up-to-scale motion estimation and 3D reconstruction (Structure from Motion)



Challenges

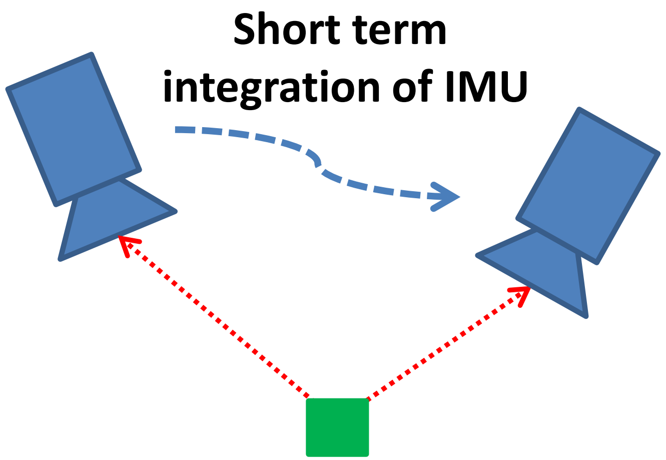
- Inertial Measurement Unit (IMU)
  - 3D linear acceleration
  - 3D angular velocity
  - Widely used in aviation singe decades ago
  - Low-cost but inaccurate MEMS IMUs in our phones and consumer UAVs



Challenges

- With IMU, scale is observable (via double integration)
- But…
  - High-rate IMU data – hard to construct a graph
  - Requires initial velocity and attitude (gravity)
  - Highly nonlinear system – requires initial values to converge

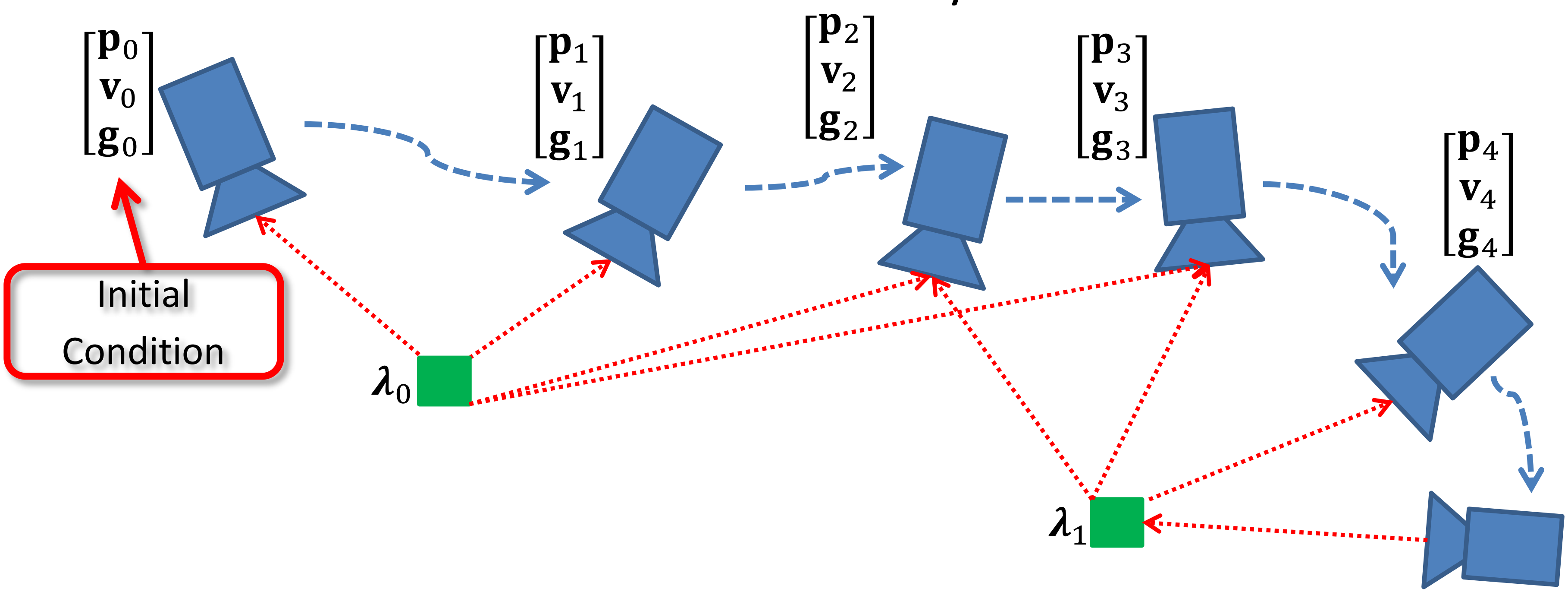
$v_0 = ?$
$g_0 = ?$



Can we operate without initialization?

Linear Monocular Visual-Inertial Estimator

- Estimates position, velocity, gravity, and feature depth
- Depth-based representation of features
- Linear formulation enables recovery of initial condition

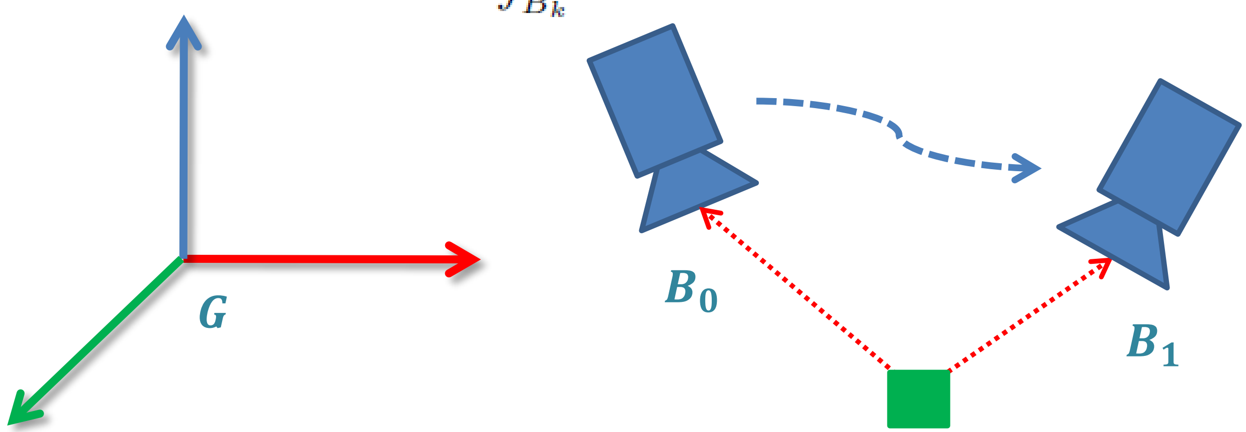


IMU Model

- IMU integration in global frame
  - IMU has higher rate than camera
  - Nonlinearity from global rotation
  - Requires global rotation at the time of integration

$$\mathbf{p}^G_{B_{k+1}} = \mathbf{p}^G_{B_k} + \mathbf{v}^G_{B_k}\Delta t + \iint_{B_k}^{B_{k+1}} \left(\mathbf{R}^G_B \mathbf{a}^B - \mathbf{g}^G\right) dt^2$$

$$\mathbf{v}^G_{B_{k+1}} = \mathbf{v}^G_{B_k} + \int_{B_k}^{B_{k+1}} \left(\mathbf{R}^G_B \mathbf{a}^B - \mathbf{g}^G\right) dt$$



IMU Pre-Integration

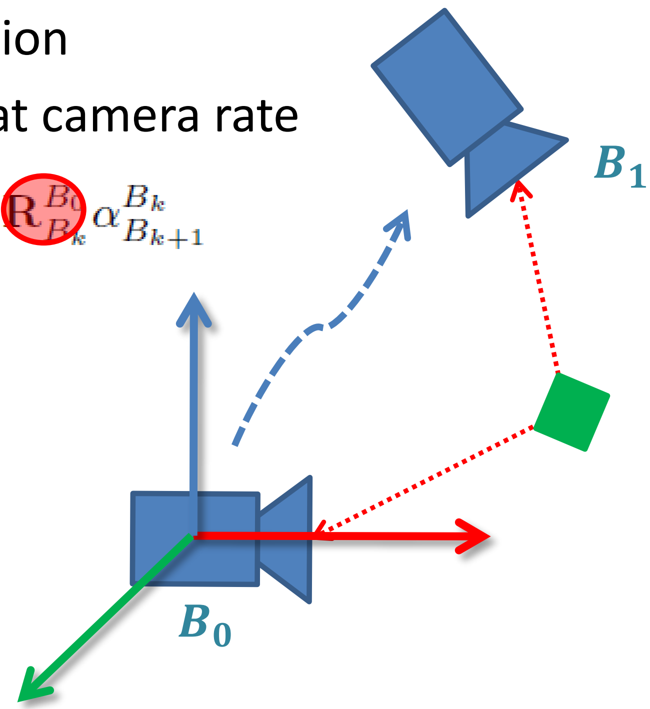
- IMU integration in the body frame of the first pose
  - Nonlinearity from relative rotation only
  - Relative rotation from gyroscope
  - Linear update equations for position, velocity, and gravity
  - IMU Integration without initialization
  - Only need to create graph nodes at camera rate

$$\mathbf{p}^{B_0}_{B_{k+1}} = \mathbf{p}^{B_0}_{B_k} + \mathbf{v}^{B_0}_{B_k}\Delta t - \mathbf{g}^{B_0}\Delta t^2/2 + \mathbf{R}^{B_0}_{B_k}\alpha^{B_k}_{B_{k+1}}$$

$$\mathbf{v}^{B_0}_{B_{k+1}} = \mathbf{v}^{B_0}_{B_k} - \mathbf{g}^{B_0}\Delta t + \mathbf{R}^{B_0}_{B_k}\beta^{B_k}_{B_{k+1}}$$

$$\alpha^{B_k}_{B_{k+1}} = \iint_{B_k}^{B_{k+1}} \mathbf{R}^{B_k}_{B}\mathbf{a}^{B}\,dt^2$$

$$\beta^{B_k}_{B_{k+1}} = \int_{B_k}^{B_{k+1}} \mathbf{R}^{B_k}_{B}\mathbf{a}^{B}\,dt$$



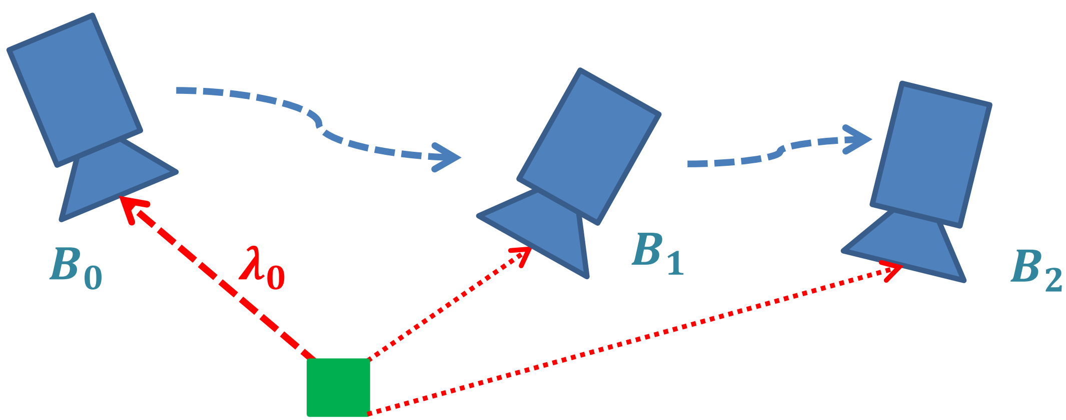
Linear Camera Model

- Linear in position and feature depth
- Nonlinear in feature observation

$$\mathbf{0} = \begin{bmatrix} -1 & 0 & u_l^{B_j} \\ 0 & -1 & v_l^{B_j} \end{bmatrix} \mathbf{R}_{B_0}^{B_j} \left(\mathbf{p}_{B_i}^{B_0} - \mathbf{p}_{B_j}^{B_0} + \lambda_l \mathbf{R}_{B_i}^{B_0} \begin{bmatrix} u_l^{B_i} \\ v_l^{B_i} \\ 1 \end{bmatrix} \right) = \mathbf{H}_l^{B_j} \mathcal{X}$$

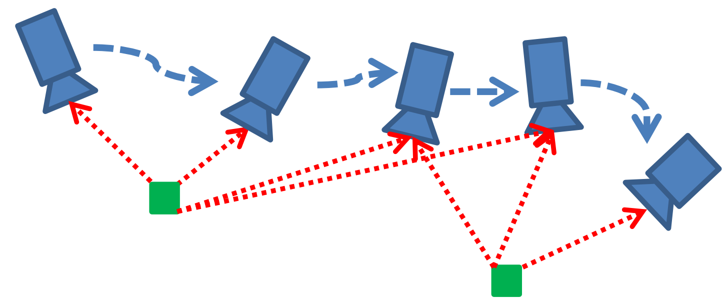
  - Unknown scaling factor in observation covariance

$$\mathbf{z}_l^{B_j} \sim \mathcal{N}\left(\mathbf{H}_l^{B_j} \mathcal{X}, {\lambda_l^{B_j}}^2 \bar{\mathbf{P}}_l^{B_j} \right)$$



Linear Monocular Visual-Inertial Estimator

- Linear system
  - Prior is not needed
  - Initial condition recoverable
  - Recoverable from failure



$$\min_{\mathcal{X}} \left\{ \underbrace{\cancel{(\mathbf{b}_p - \Lambda_p \mathcal{X})}}_{\text{Prior}} + \underbrace{\sum_{k\in\mathcal{D}} \left\| \hat{\mathbf{z}}^{B_k}_{B_{k+1}} - \mathbf{H}^{B_k}_{B_{k+1}} \mathcal{X} \right\|^2_{\mathbf{P}^{B_k}_{B_{k+1}}}}_{\text{IMU Constraints}} + \underbrace{\sum_{(l,j)\in\mathcal{C}} \left\| \hat{\mathbf{z}}^{B_j}_{l} - \mathbf{H}^{B_j}_{l} \mathcal{X} \right\|^2_{\mathbf{P}^{B_j}_{l}}}_{\text{Camera Constraints}} \right\}$$

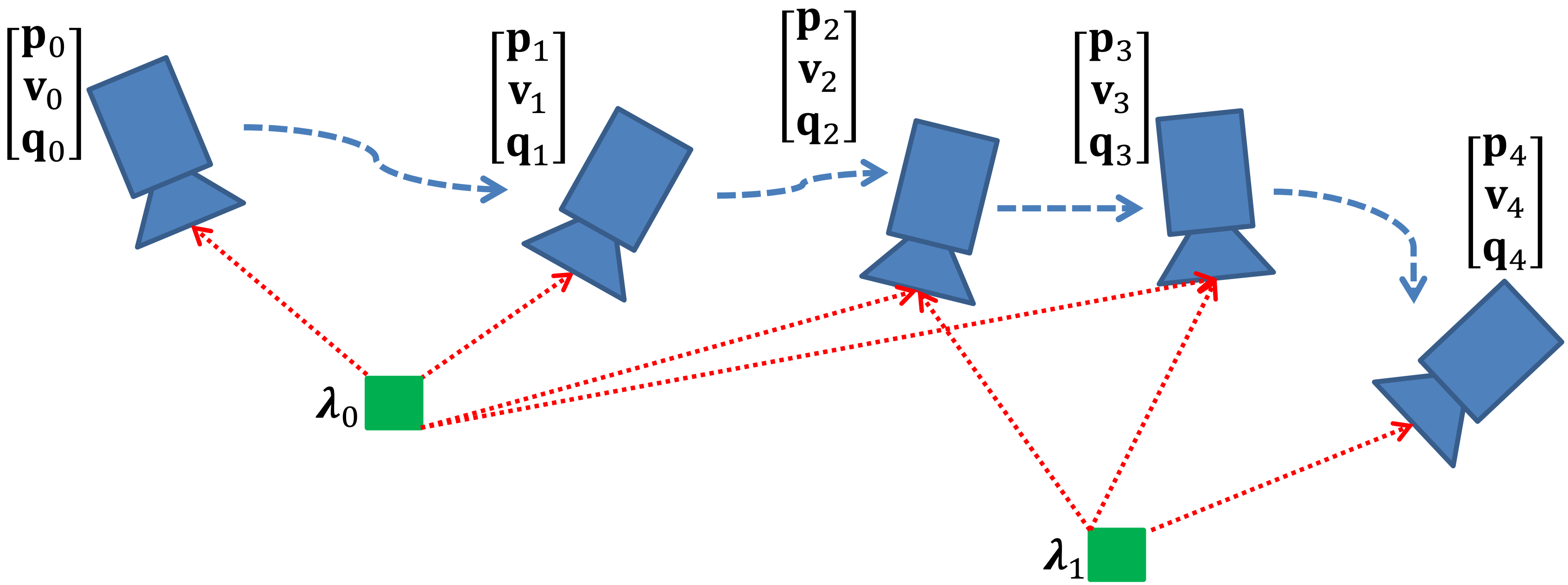
$$\left(\cancel{\Lambda_p} + \Lambda_{imu} + \Lambda_{cam}\right)\mathcal{X} = \left(\cancel{\mathbf{b}_p} + \mathbf{b}_{imu} + \mathbf{b}_{cam}\right)$$

Tightly-Coupled Nonlinear Optimization

- Nonlinear optimization based on linear initialization
  - Optimize position, velocity, rotation, and feature depth simultaneously:

$$\mathcal{X} = [\mathbf{x}_0^0, \mathbf{x}_1^0, \cdots \mathbf{x}_N^0, \lambda_0, \lambda_1, \cdots \lambda_M]$$

$$\mathbf{x}_k^0 = [\mathbf{p}_k^0, \mathbf{v}_k^k, \mathbf{q}_k^0] \quad \text{for } k = 1, ..., N$$



Tightly-Coupled Nonlinear Optimization

- Nonlinear optimization based on linear initialization
  - Iteratively minimize residuals from all sensors

Residual Measurement

$$\min_{\mathcal{X}} \left\{ \left(\mathbf{b}_p - \mathbf{\Lambda}_p \mathcal{X}\right) + \sum_{k\in\mathcal{D}} \left\| r_{\mathcal{D}}(\hat{\mathbf{z}}^k_{k+1}, \mathcal{X}) \right\|^2_{\mathbf{P}^k_{k+1}} + \sum_{(l,j)\in\mathcal{C}} \left\| r_{\mathcal{C}}(\hat{\mathbf{z}}^j_l, \mathcal{X}) \right\|^2_{\mathbf{P}^j_l} \right\}$$

Linearization Point **Linearization**

$$\min_{\delta\mathcal{X}} \left\{ \left(\mathbf{b}_p - \mathbf{\Lambda}_p \hat{\mathcal{X}}\right) + \sum_{k\in\mathcal{D}} \left\| r_{\mathcal{D}}(\hat{\mathbf{z}}^k_{k+1}, \hat{\mathcal{X}}) + \mathbf{H}^k_{k+1}\delta\mathcal{X} \right\|^2_{\mathbf{P}^k_{k+1}} + \sum_{(l,j)\in\mathcal{C}} \left\| r_{\mathcal{C}}(\hat{\mathbf{z}}^j_l, \hat{\mathcal{X}}) + \mathbf{H}^j_l\delta\mathcal{X} \right\|^2_{\mathbf{P}^j_l} \right\}$$

Prior IMU residual Camera residual

  - Solve via Guass-Newton method:

$$\left(\mathbf{\Lambda}_p + \mathbf{\Lambda}_{\mathcal{D}} + \mathbf{\Lambda}_{\mathcal{C}}\right)\delta\mathcal{X} = \left(\mathbf{b}_p + \mathbf{b}_{\mathcal{D}} + \mathbf{b}_{\mathcal{C}}\right)$$

$$\hat{\mathcal{X}} = \hat{\mathcal{X}} + \delta\mathcal{X}$$

$$\mathbf{p} = \hat{\mathbf{p}} + \delta\mathbf{p}$$
$$\mathbf{v} = \hat{\mathbf{v}} + \delta\mathbf{v}$$
$$\lambda = \hat{\lambda} + \delta\lambda$$
$$\mathbf{q} = \hat{\mathbf{q}} + \delta\mathbf{q}$$

??????

Tightly-Coupled Nonlinear Optimization

- Nonlinear optimization based on linear initialization
  - Optimize position, velocity, rotation, and feature depth simultaneously:

SO(3) -> so(3)

$$\mathcal{X} = \left[\mathbf{x}_0^0,\, \mathbf{x}_1^0,\, \cdots\, \mathbf{x}_N^0,\, \lambda_0,\, \lambda_1,\, \cdots\, \lambda_M\right]$$

$$\mathbf{x}_k^0 = \left[\mathbf{p}_k^0,\, \mathbf{v}_k^k,\, \mathbf{q}_k^0\right] \quad \text{for } k = 1, \ldots, N$$

  - Error state formulation by modelling the rotation error on the tangent space of the rotation manifold:

$$\delta\mathcal{X} = \left[\delta\mathbf{x}_n^0,\, \delta\mathbf{x}_{n+1}^0,\, \cdots\, \delta\mathbf{x}_{n+N}^0,\, \delta\lambda_m,\, \delta\lambda_{m+1},\, \cdots\, \delta\lambda_{m+M}\right]$$

$$\delta\mathbf{x}_k^0 = \left[\delta\mathbf{p}_k^0,\, \delta\mathbf{v}_k^k,\, \delta\boldsymbol{\theta}_k^0\right]$$

$$\mathbf{p} = \hat{\mathbf{p}} + \delta\mathbf{p}$$

$$\mathbf{v} = \hat{\mathbf{v}} + \delta\mathbf{v}$$

$$\lambda = \hat{\lambda} + \delta\lambda$$

$$\mathbf{q} = \hat{\mathbf{q}} \otimes \delta\mathbf{q}, \quad \delta\mathbf{q} \approx \begin{bmatrix} \frac{1}{2}\delta\boldsymbol{\theta} \\ 1 \end{bmatrix}$$

so(3) -> SO(3)

  - Solve via Guass-Newton method:

$$\left(\boldsymbol{\Lambda}_p + \boldsymbol{\Lambda}_{\mathcal{D}} + \boldsymbol{\Lambda}_{\mathcal{C}}\right)\delta\mathcal{X} = \left(\mathbf{b}_p + \mathbf{b}_{\mathcal{D}} + \mathbf{b}_{\mathcal{C}}\right)$$

$$\hat{\mathcal{X}} = \hat{\mathcal{X}} \oplus \delta\mathcal{X}$$

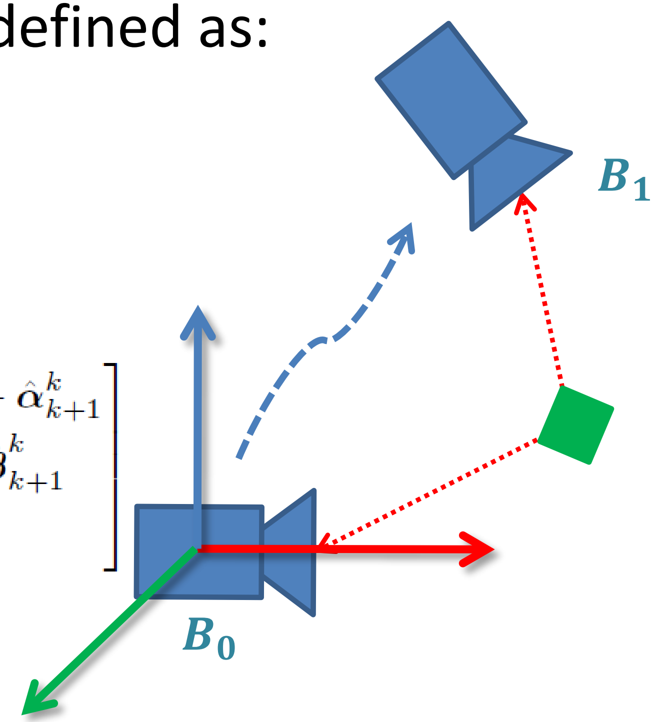
IMU Pre-Integration on Manifold

- IMU integration in the body frame of the first pose
  - Take rotation uncertainty into account
  - Uncertainty propagation on manifold
  - Measurement and residual defined as:

$$\hat{\mathbf{z}}^k_{k+1} = \begin{bmatrix}\hat{\boldsymbol\alpha}^k_{k+1}\\ \hat{\boldsymbol\beta}^k_{k+1}\\ \hat{\mathbf{q}}^k_{k+1}\end{bmatrix} = \begin{bmatrix}\iint_{t\in[k,k+1]} \hat{\mathbf{R}}^k_t \hat{\mathbf{a}}^b_t dt^2 \\ \int_{t\in[k,k+1]} \hat{\mathbf{R}}^k_t \hat{\mathbf{a}}^b_t dt \\ \int_{t\in[k,k+1]} \boldsymbol\Omega(\hat{\boldsymbol\omega}^b_t)\hat{\mathbf{q}}^k_t dt\end{bmatrix}$$

$$r_{\mathcal{D}}(\hat{\mathbf{z}}^k_{k+1}, \mathcal{X}) =$$

$$\begin{bmatrix}\delta\boldsymbol\alpha^k_{k+1}\\ \delta\boldsymbol\beta^k_{k+1}\\ \delta\boldsymbol\theta^k_{k+1}\end{bmatrix} = \begin{bmatrix} \mathbf{R}^k_0\left(\mathbf{p}^0_{k+1} - \mathbf{p}^0_k + \mathbf{g}^0\frac{\Delta t^2}{2}\right) - \mathbf{v}^k_k\Delta t - \hat{\boldsymbol\alpha}^k_{k+1}\\ \mathbf{R}^k_0\left(\mathbf{R}^0_{k+1}\mathbf{v}^{k+1}_{k+1} + \mathbf{g}^0\Delta t\right) - \mathbf{v}^k_k - \hat{\boldsymbol\beta}^k_{k+1} \\ 2\left[\hat{\mathbf{q}}^{k^{-1}}_{k+1}\otimes \mathbf{q}^{0^{-1}}_k \otimes \mathbf{q}^0_{k+1}\right]_{xyz}\end{bmatrix}$$

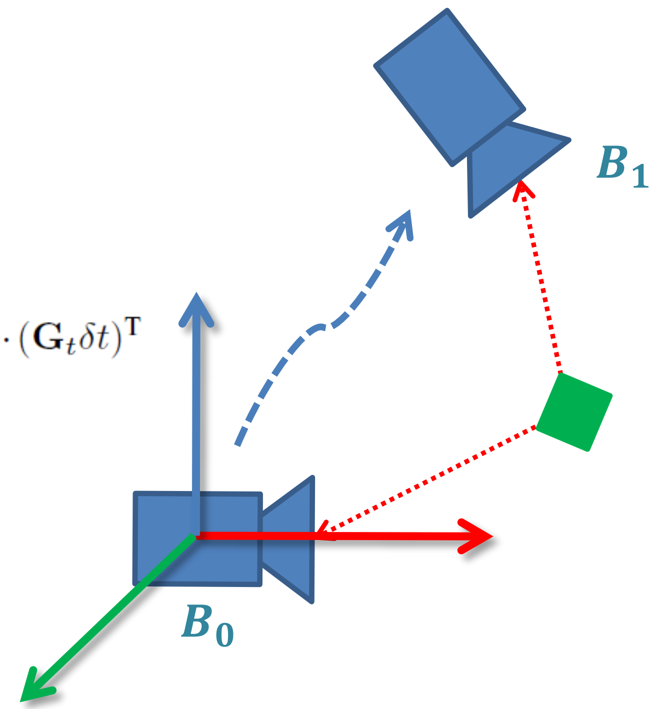


IMU Pre-Integration on Manifold

- IMU integration in the body frame of the first pose
  - Take rotation uncertainty into account
  - Uncertainty propagation on manifold

$$\begin{bmatrix} \delta\dot{\boldsymbol{\alpha}} \\ \delta\dot{\boldsymbol{\beta}} \\ \delta\dot{\boldsymbol{\theta}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbb{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\hat{\mathbf{R}}_t^k \lfloor \hat{\mathbf{a}}_t^b \times \rfloor \\ \mathbf{0} & \mathbf{0} & -\lfloor \hat{\boldsymbol{\omega}}_t^b \times \rfloor \end{bmatrix} \begin{bmatrix} \delta\boldsymbol{\alpha}_t^k \\ \delta\boldsymbol{\beta}_t^k \\ \delta\boldsymbol{\theta}_t^k \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -\hat{\mathbf{R}}_t^k & \mathbf{0} \\ \mathbf{0} & -\mathbb{I} \end{bmatrix} \begin{bmatrix} {}^a\mathbf{n}_t \\ {}^\omega\mathbf{n}_t \end{bmatrix} = \mathbf{F}_t \delta\mathbf{z}_t^k + \mathbf{G}_t \mathbf{n}_t$$

$$\mathbf{P}_{t+\delta t}^k = (\mathbb{I} + \mathbf{F}_t \delta t) \cdot \mathbf{P}_t^k \cdot (\mathbb{I} + \mathbf{F}_t \delta t)^{\mathrm{T}} + (\mathbf{G}_t \delta t) \cdot \mathbf{Q}_t \cdot (\mathbf{G}_t \delta t)^{\mathrm{T}}$$



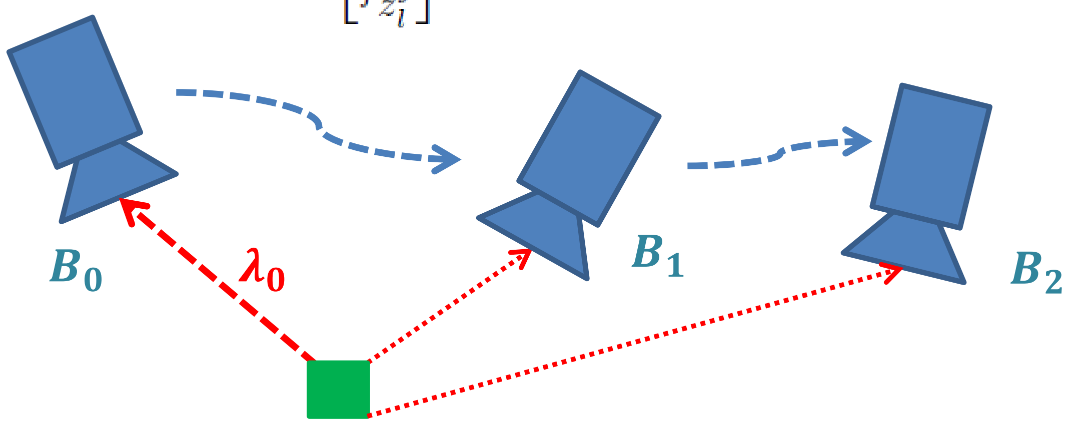
Projective Camera Model

- Camera residual the reprojection error
  - Measurement and residual defined as:

$$\hat{\mathbf{z}}_l^j = \left[\hat{u}_l^j,\ \hat{v}_l^j\right]^T$$

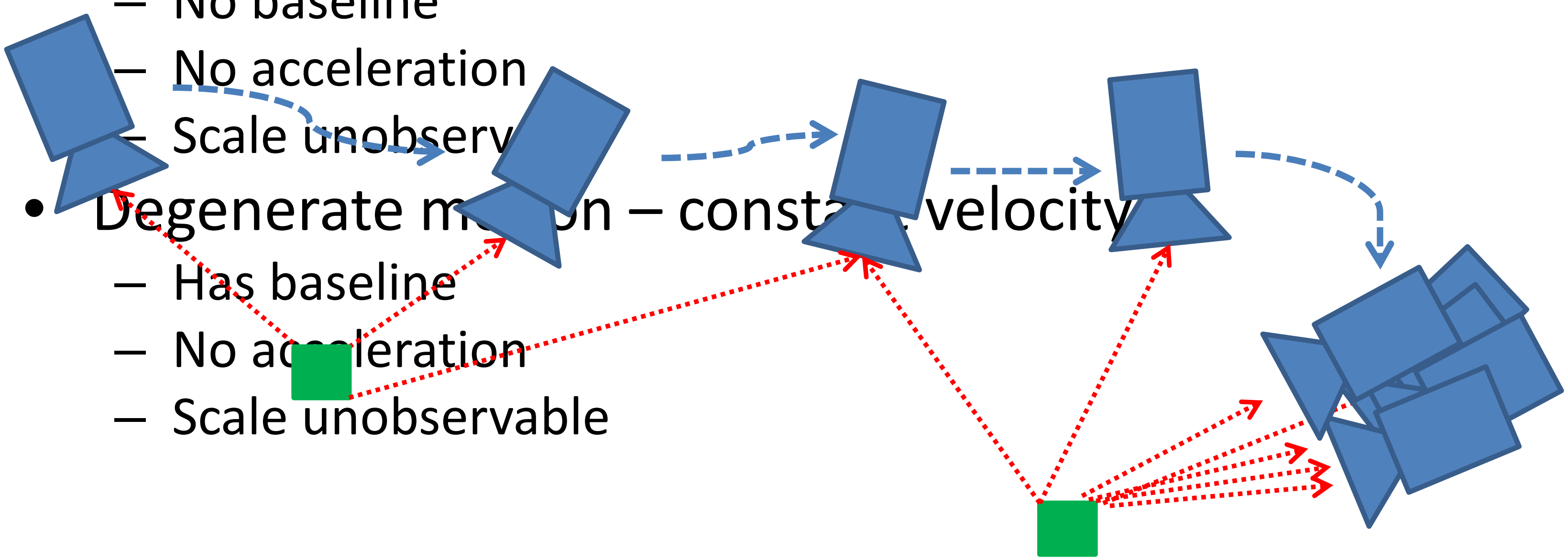
$$r_{\mathcal{C}}(\hat{\mathbf{z}}_l^j, \mathcal{X}) = \begin{bmatrix} \frac{{}^f x_l^j}{{}^f z_l^j} - \hat{u}_l^j \\ \frac{{}^f y_l^j}{{}^f z_l^j} - \hat{v}_l^j \end{bmatrix}$$

$$\mathbf{f}_l^j = \begin{bmatrix} {}^f x_l^j \\ {}^f y_l^j \\ {}^f z_l^j \end{bmatrix} = \mathbf{R}_0^j \left(\mathbf{p}_i^0 - \mathbf{p}_j^0 + \lambda_l \mathbf{R}_i^0 \mathbf{u}_l^i\right)$$



Marginalization

- Remove old poses to bound computation complexity
  - Convert removed measurements into prior
- General motion:
  - Linear acceleration is required for scale observability
- Degenerate motion – hover
  - No baseline
  - No acceleration
  - Scale unobservable
- Degenerate motion – constant velocity
  - Has baseline
  - No acceleration
  - Scale unobservable



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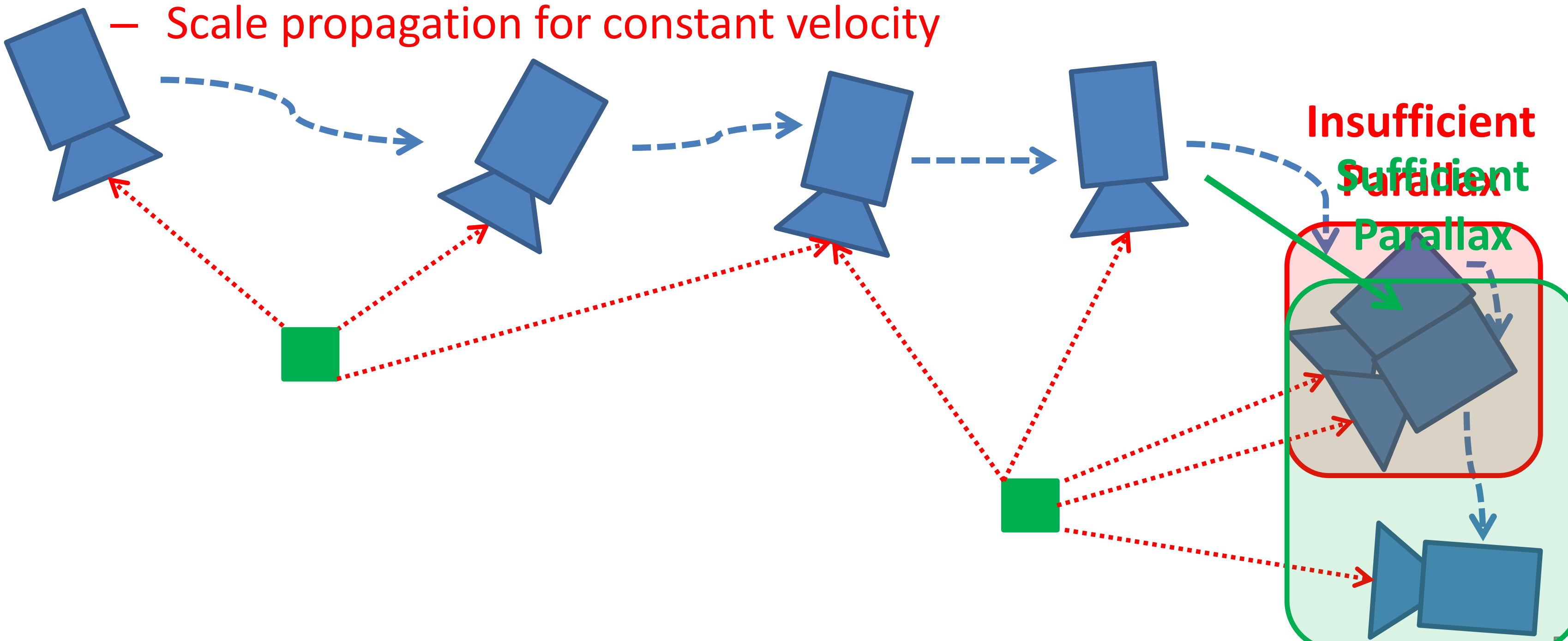
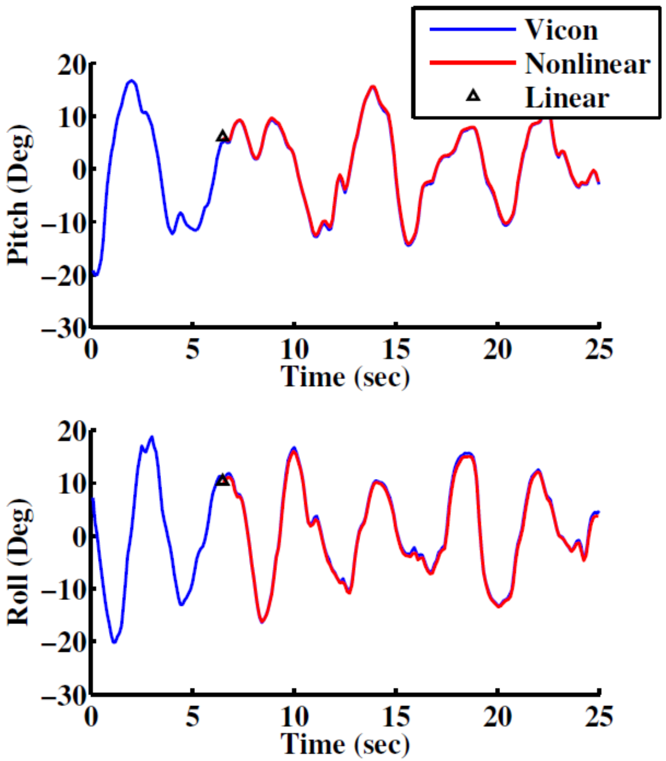
電子及計算機工程學系
DEPARTMENT OF ELECTRONIC & COMPUTER ENGINEERING

Marginalization

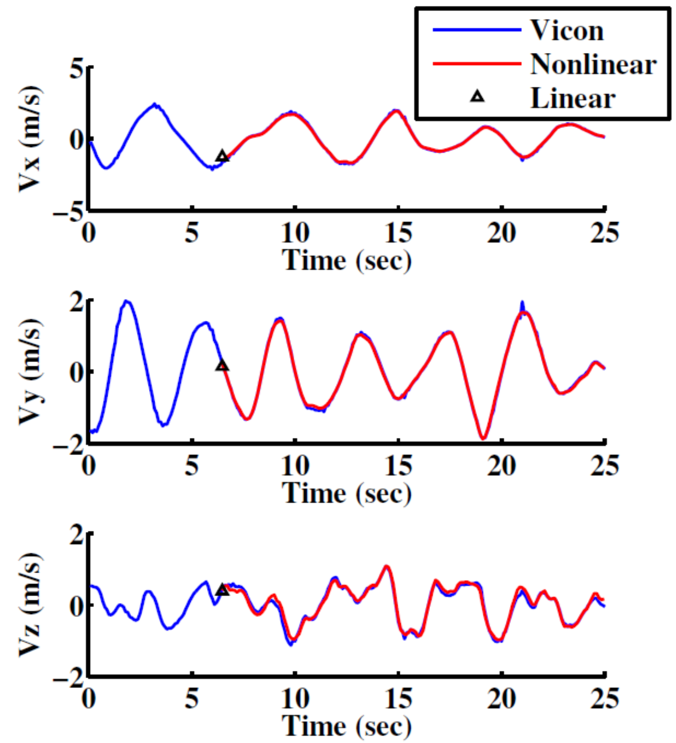
- Two-way marginalization
  - Preserve acceleration and baseline within the sliding window
  - Marginalize either recent or old pose based on parallax heuristic
  - Scale observable for hover
  - Scale propagation for constant velocity



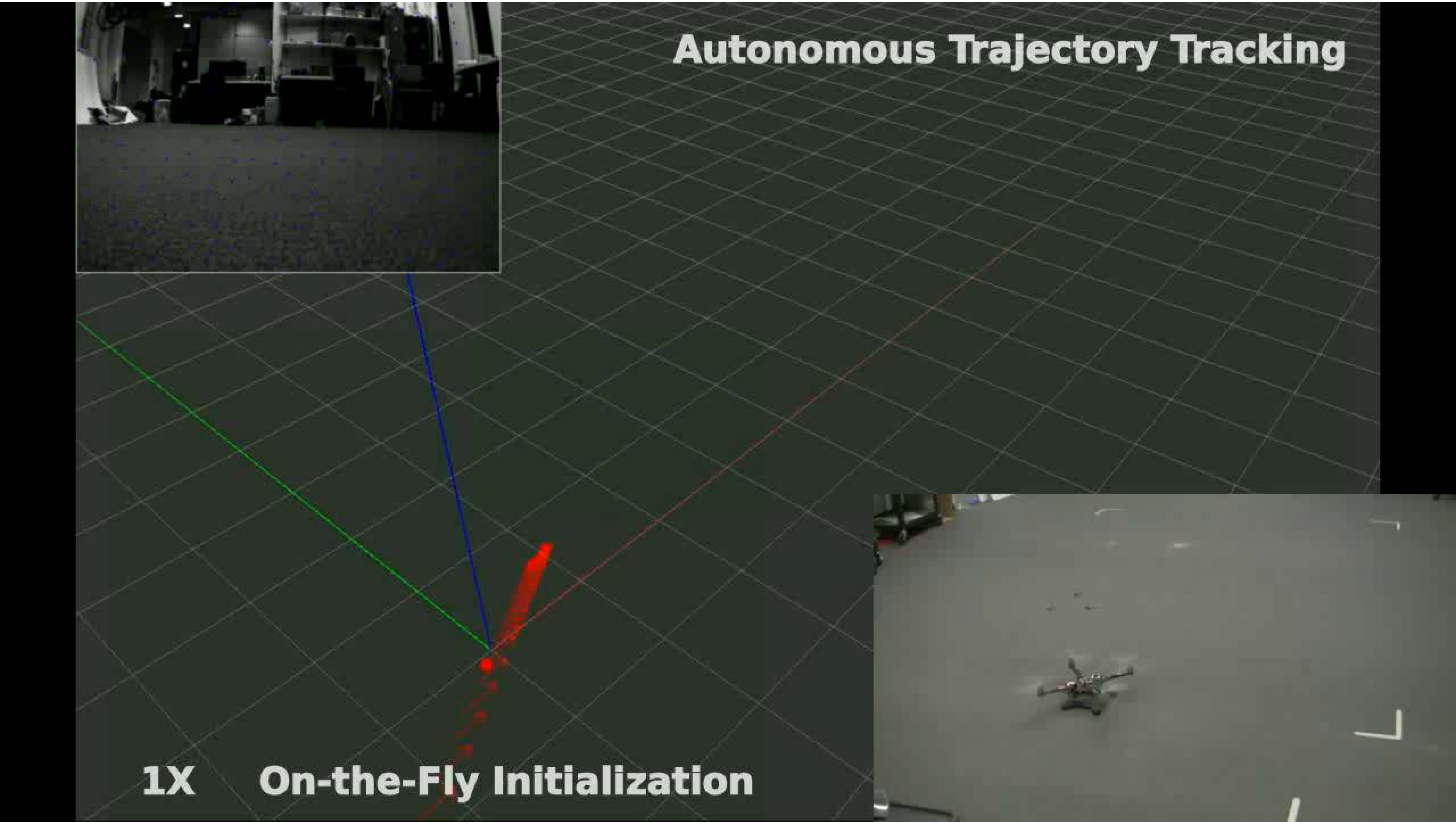
Initialization and State Estimation



Attitude

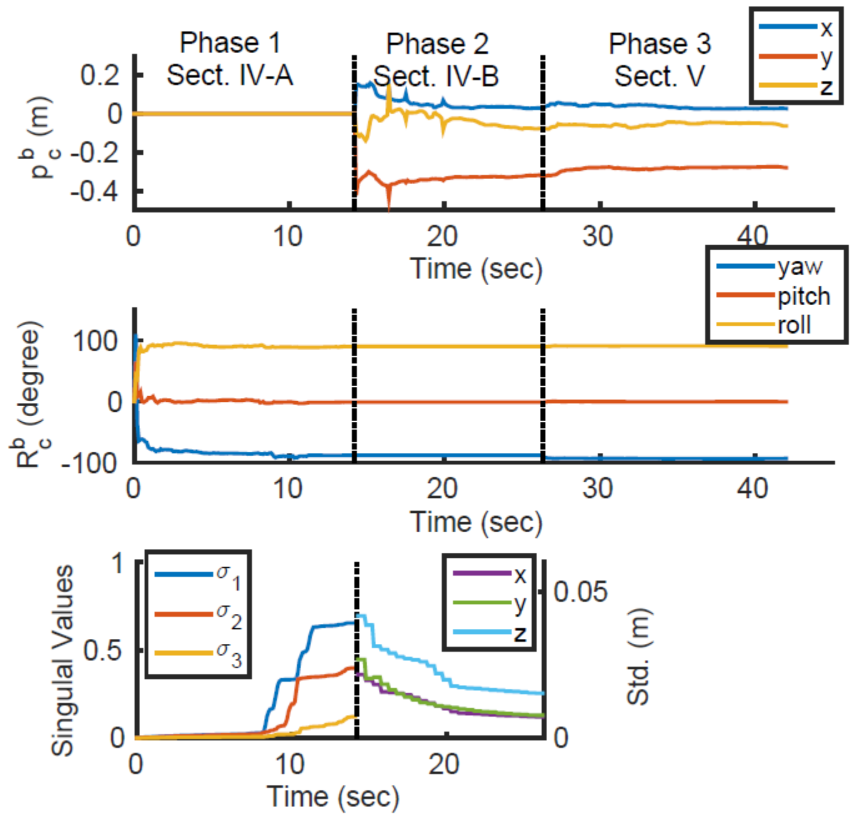


Velocity

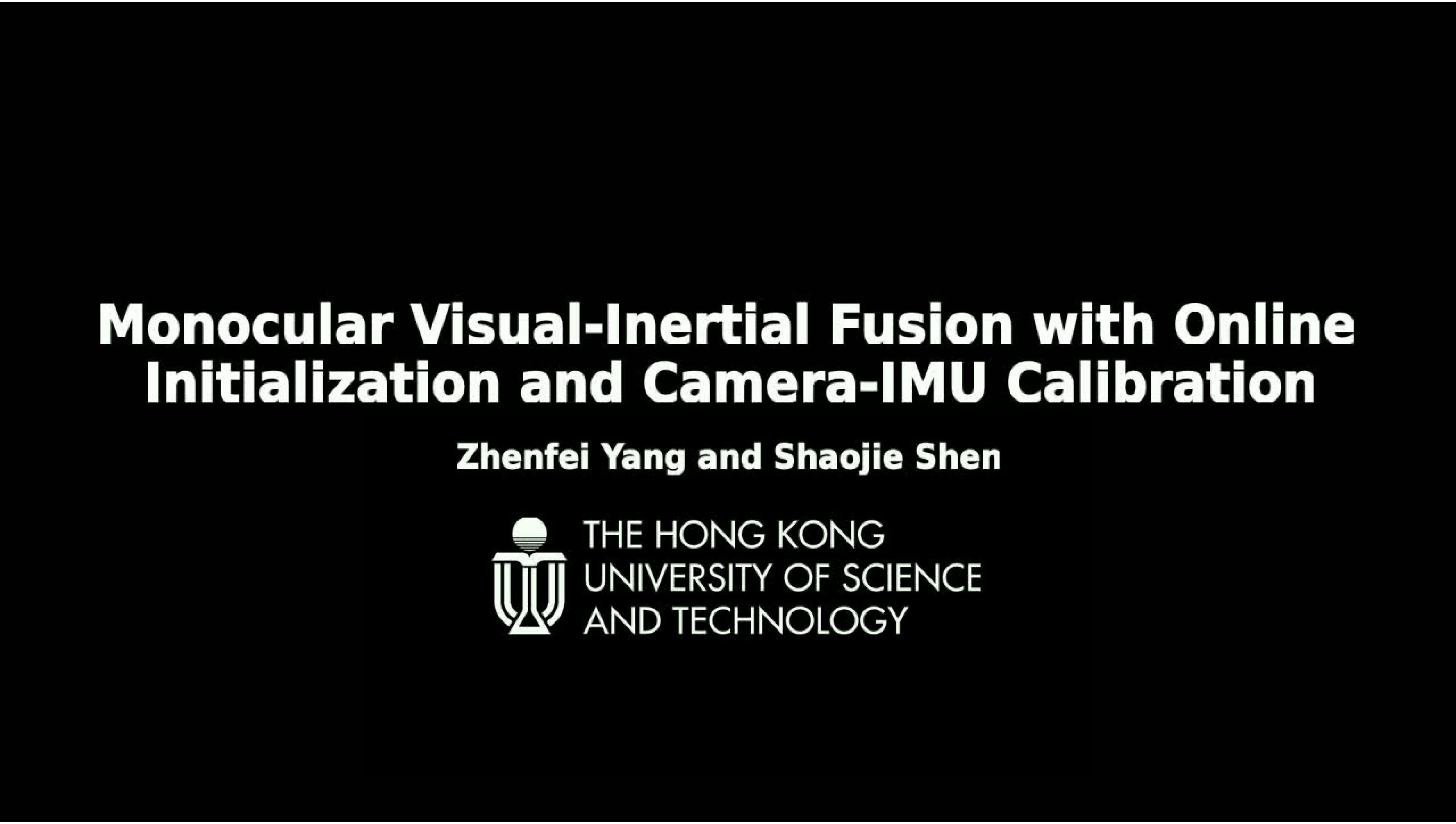


Shen, et al, 2014

Monocular Visual-Inertial Fusion with Self-Calibration



Monocular Visual-Inertial Fusion with Self-Calibration

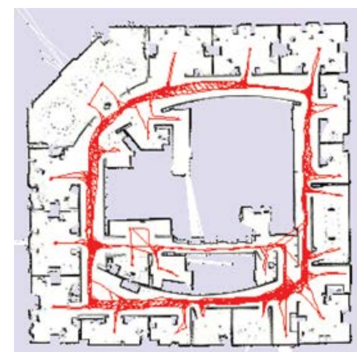


Summary

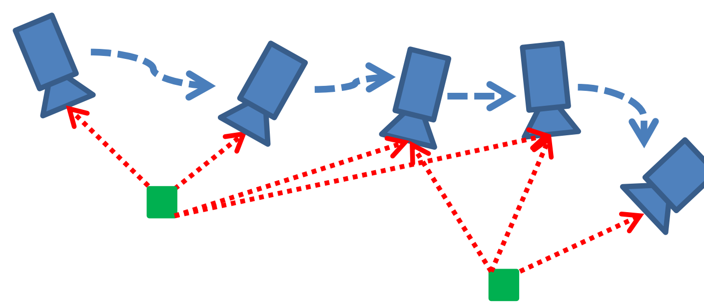
- The Basics

$$P(x \mid y, z) = \frac{P(y \mid x, z)\; P(x \mid z)}{P(y \mid z)}$$

- 2D Graph-Based SLAM



- Monocular Visual-Inertial SLAM



Todo…

- Dynamic environments
- Better convergence property
- Real-time Dense mapping
- Multi-robot SLAM

That's all about this course…

Week	Lecture Date	Topic	Lab	Assignment
1	1/9	Introduction Rigid Body Transformation slides	No Lab	
2	8/9	Rigid Body Transformation Quadrotor Modeling slides	No Lab	
*	15/9	*	No Lab	
3	22/9	Control Basics Quadrotor Control slides 1 slides 2 L3_pid_setpoint	No Lab	Project 1: Phase 1 Out code
4	29/9	Time & Motion Trajectory Generation slides	No Lab	Project 1: Phase 1 Due Project 1: Phase 2 Out code
5	6/10	Camera Modeling & Calibration Feature Detection & Matching slides	ROS Tutorial 1	Project 1: Phase 2 Due Project 1: Phase 3 Out assignment code
6	13/10	Optical Flow Dense Stereo slides	Project 1 Phase 3	
7	20/10	Multi-View Geometry 2D-2D, 3D-2D, 3D-3D slides	Public Holiday	
8	27/10	Midterm Exam	Cancelled	
9	3/11	Probability Basics Bayesian Inferencing Maximum Likelihood Estimation slides	Project 1: Phase 3	Project1: Phase 3 Due Project2: Phase 1 Out assignment code
10	10/11	Kalman Filter Sensor Fusion slides	No Lab	Project 2: Phase 1 Due Project 2: Phase 2 Out
11	17/11	SLAM	No Lab	Project 2: Phase 2 Due Project 3 Out
12	24/11	No Lecture	Project 3	
13	1/12	No Lecture	Project 3 (Optional)	Project 3 Due

Logistics

- No lab this week
- Project 2, Phase 2 due next Wednesday (25 Nov.)
- Project 3 (lab) starting from next week
  - May take 1-2 weeks depending on your progress

- This is the last lecture
- Please do the course evaluation survey

Thank You!!!