香港科技大學
THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

電子及計算機工程學系
DEPARTMENT OF ELECTRONIC & COMPUTER ENGINEERING

Introduction to Aerial Robotics
Lecture 7

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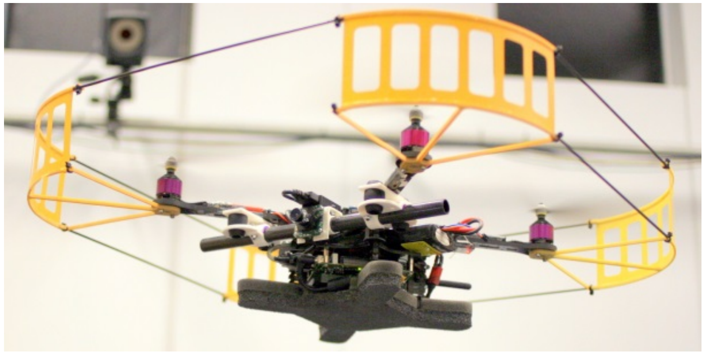
Assistant Professor

Dept. of ECE, HKUST



24 October 2015

Outline

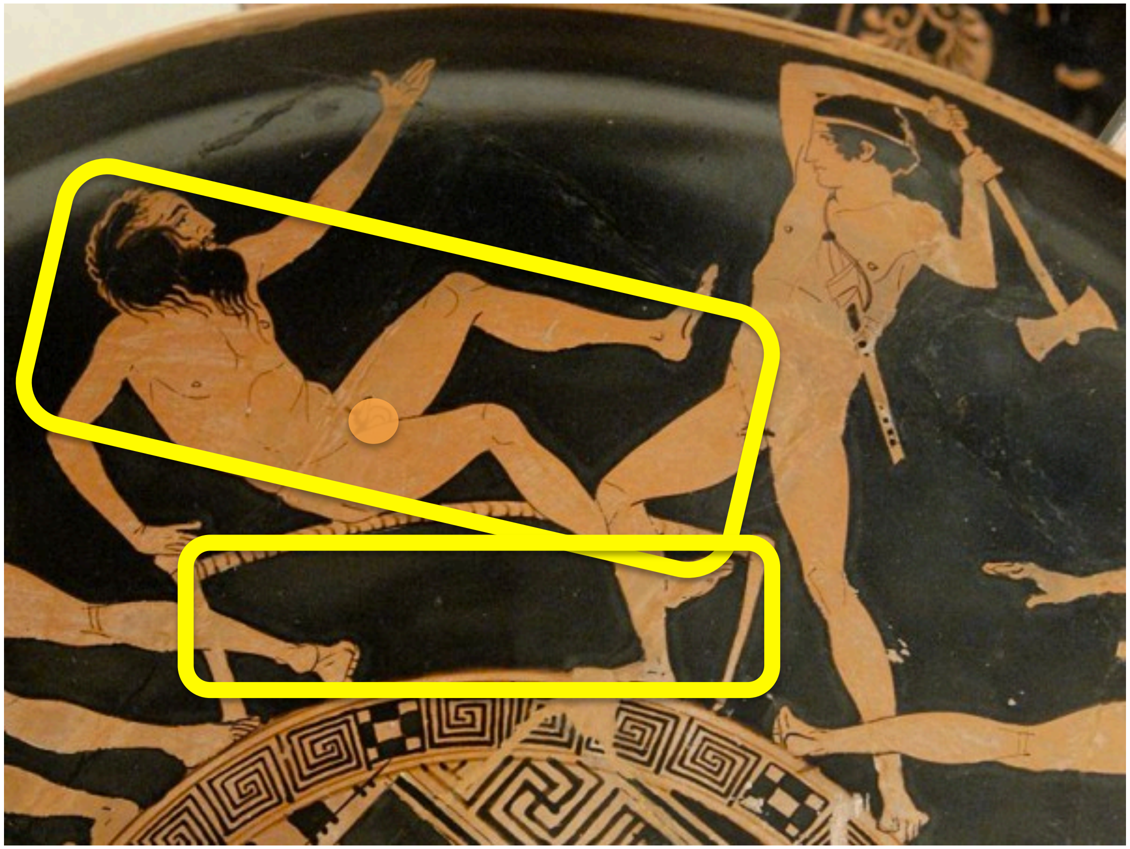
- 3D-3D Pose Estimation
- 2D-3D Pose Estimation

Robot Perception: Pose from 3D Point Correspondences or the Procrustes Problem

Advanced Robotics

Kostas Daniilidis

Procrustes Problem



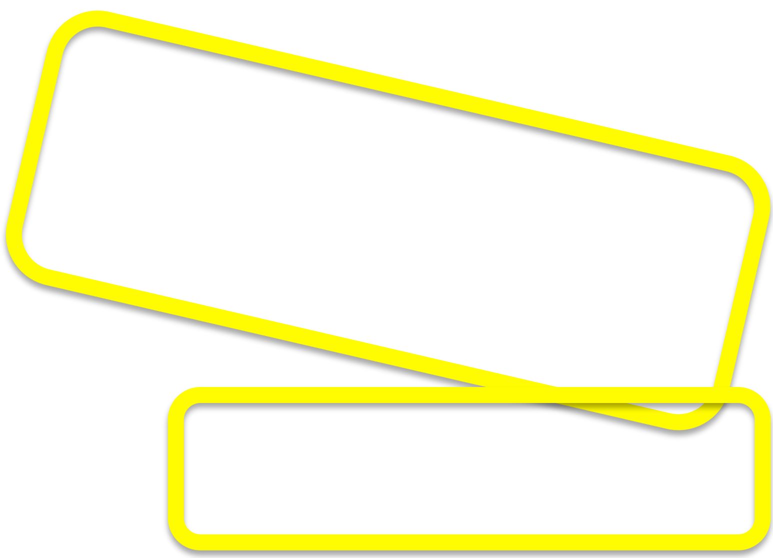
Given two shapes find the scaling, rotation, and translation that fits one into the other.

3D-3D Pose or Procrustes Problem

Given correspondences of points $A_i \in \mathbb{R}^3$ and $B_i \in \mathbb{R}^3$ find the scaling, rotation, and translation transformation, called *similitude* transformation, that satisfies

$$A_i = sRB_i + T$$

for $R \in SO(3)$, $T \in \mathbb{R}$, and $s \in \mathbb{R}^+$.

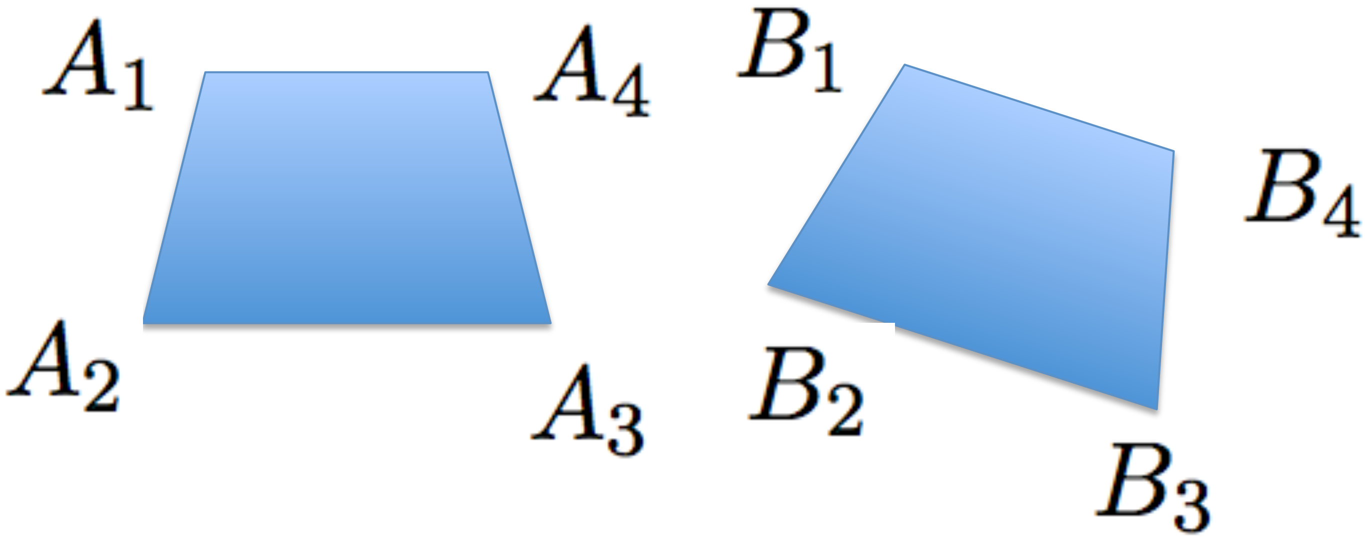


3D-3D Pose or Procrustes Problem

In the camera rigid pose problem scale $s = 1$ is known:

$$Z_i p_i^{cam} = R P_i^{obj} + T$$

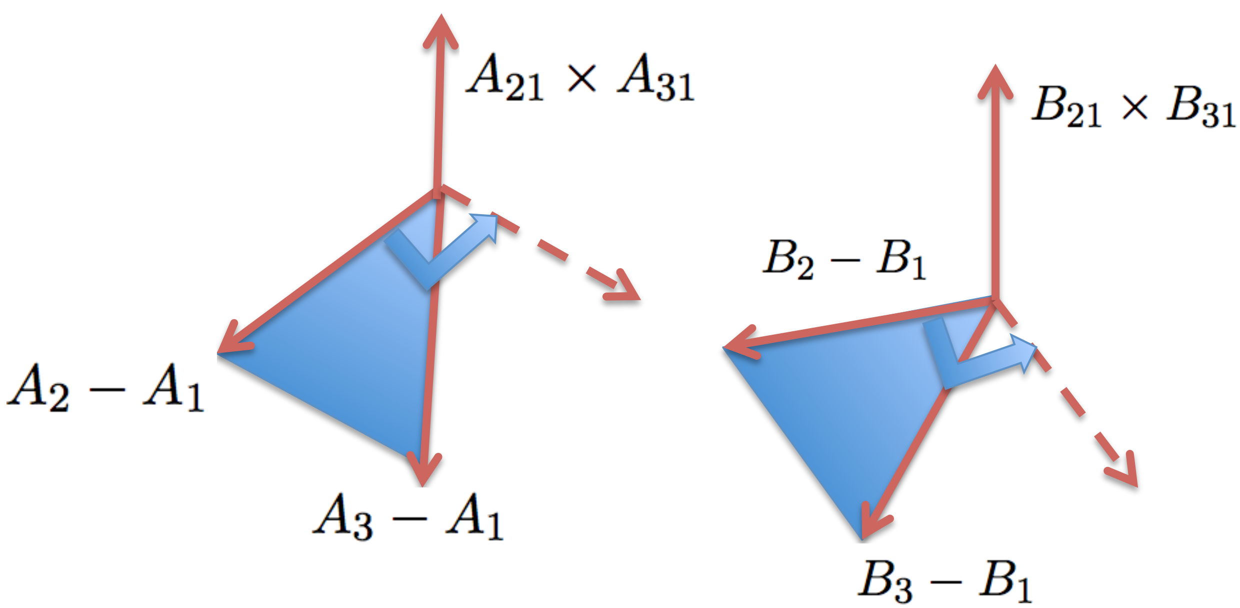
This is the last step of the P3P problem or the entire problem of finding rigid pose when we know the depth at every point (e.g., in am RGB-D sensor).

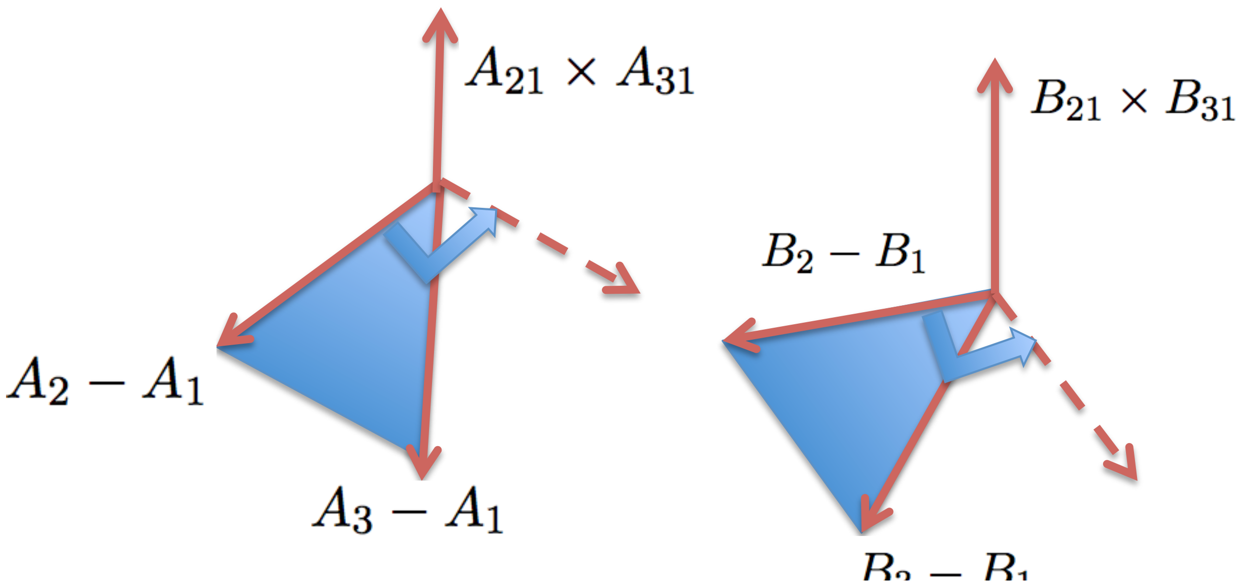


How do we solve for R, T from n point correspondences?

$$A_i = RB_i + T$$

What is the minimal number of points needed?





Three non-collinear points suffice: each triangle $A_{i=1\ldots3}$ and $B_{i=1\ldots3}$ make an orthogonal basis

$$\begin{pmatrix} A_{21} & (A_{21} \times A_{31}) \times A_{21} & A_{21} \times A_{31} \end{pmatrix}$$

and

$$\begin{pmatrix} B_{21} & (B_{21} \times B_{31}) \times B_{21} & B_{21} \times B_{31} \end{pmatrix}$$

Rotation between two orthogonal bases is unique.

We solve a minimization problem for $N > 3$ point correspondences:

$$\min_{R,T} \sum_{i}^{N} \|A_i - RB_i + T\|^2$$

After differentiating with respect to T we observe that the translation is the difference between the centroids:

$$T = \frac{1}{N}\sum_{i}^{N} A_i - R\frac{1}{N}\sum_{i}^{N} B_i = \bar{A} - R\bar{B}$$

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We subtract the centroids $\bar{A}$ and $\bar{B}$ and rewrite the objective function as

$$\min_R \|A - RB\|_F^2$$

where

$$A = \begin{pmatrix} A_1 - \bar{A} & \dots & A_N - \bar{A} \end{pmatrix}$$

and

$$B = \begin{pmatrix} B_1 - \bar{B} & \dots & B_N - \bar{B} \end{pmatrix}$$

We rewrite the Frobenius norm using the trace of the matrix

$$\|A - RB\|_F^2 = tr(A^T A) + tr(B^T B) - tr(A^T RB) - tr(B^T R^T A)$$

and observe that only the two last terms depend on the unknown R yielding a maximization problem.

Even without using the properties of the trace we can see that both last terms are equal to

$$\sum_i^N R(B_i - \bar{B})(A_i - \bar{A})^T = tr(RBA^T)$$

The 3D-3D pose problem reduced to

$$\max_R tr(RBA^T)$$

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The 3D-3D pose problem reduced to

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If the SVD of BA^T is USV^T and $Z = V^T RU$

$$tr(RBA^T) = tr(RUSV^T) = tr(ZS) = \sum_{1}^{3} z_{ii}\sigma_i \leq \sum_{1}^{3} \sigma_i$$

and, hence, the upper bound is obtained by setting

$$R = UV^T$$

To guarantee that it has determinant 1

$$R = U \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \det(UV^T) \end{pmatrix} V^T$$

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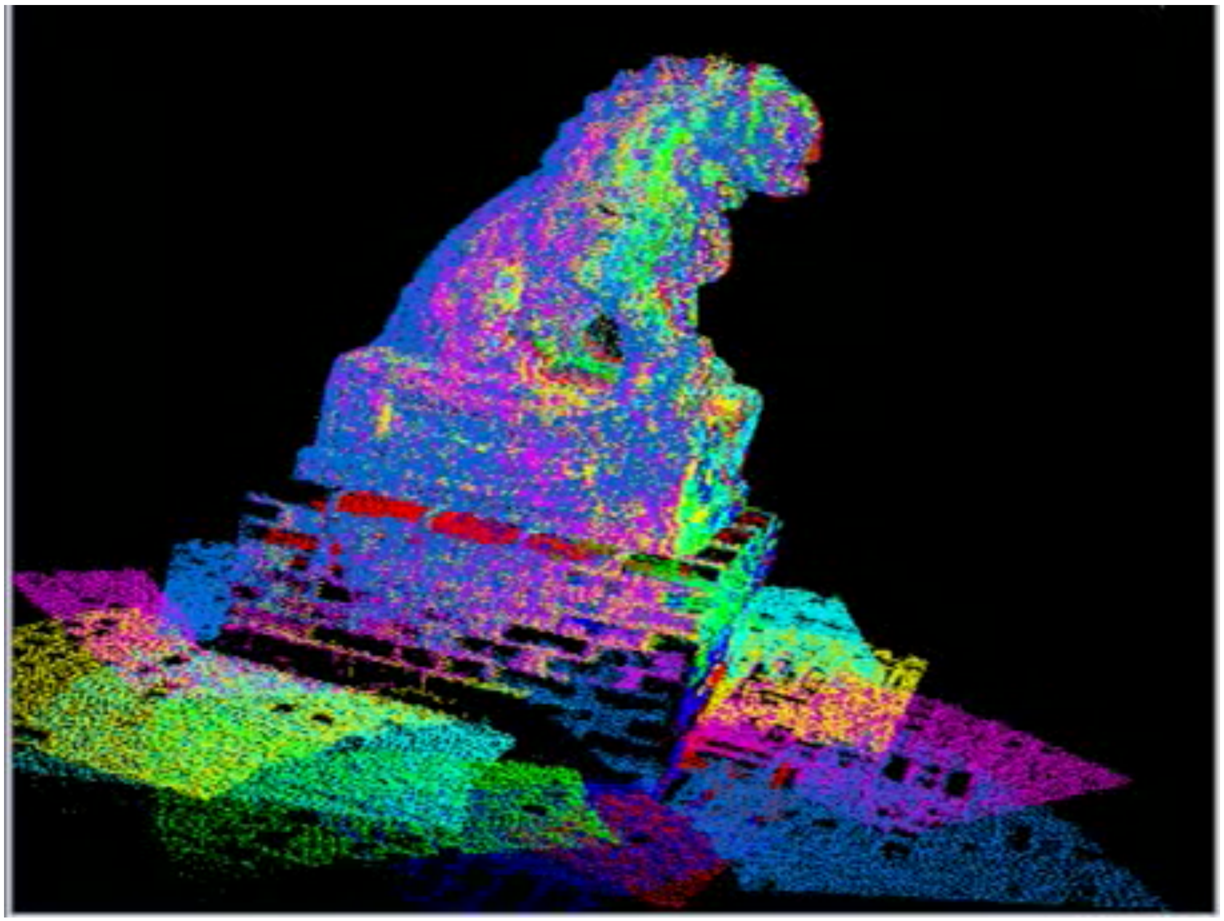
3D-3D Registration enables the creation of 3D models from multiple point clouds:



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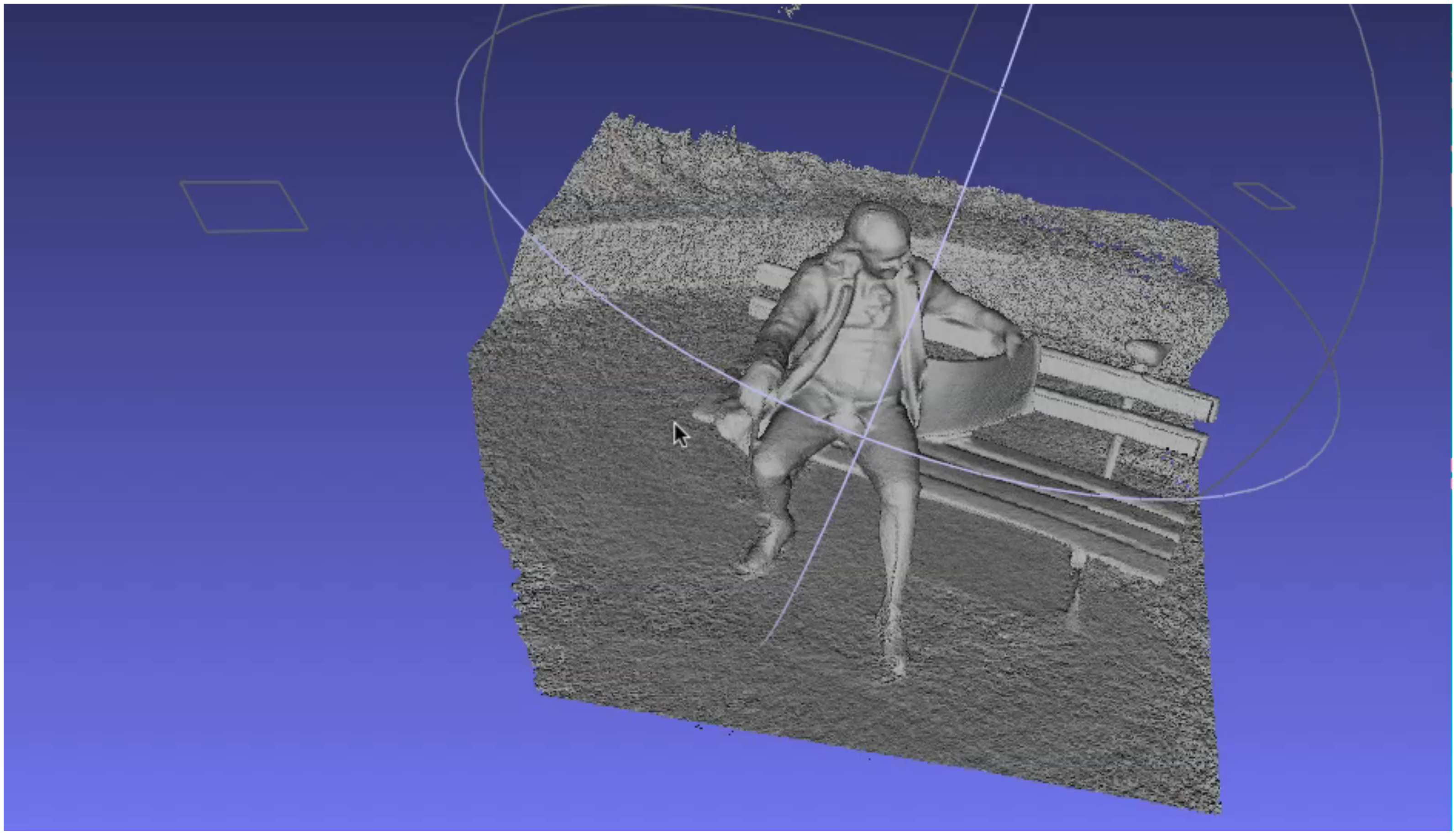
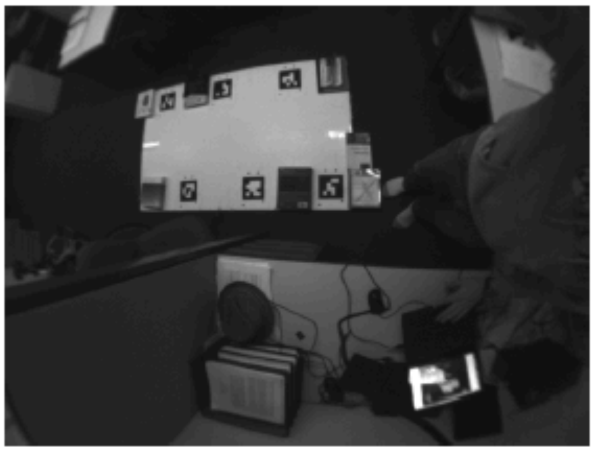


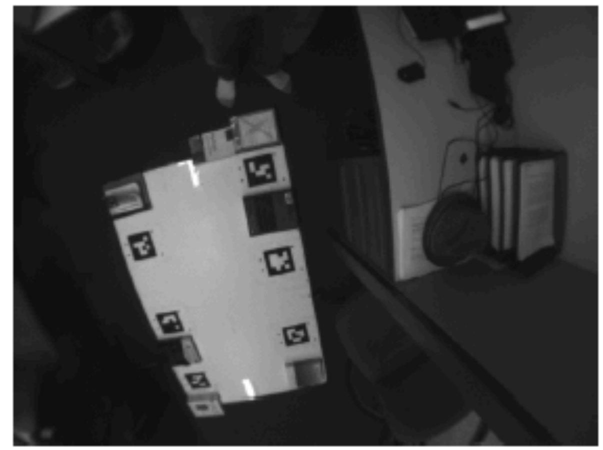
Robot Perception: Pose from Projective Transformations

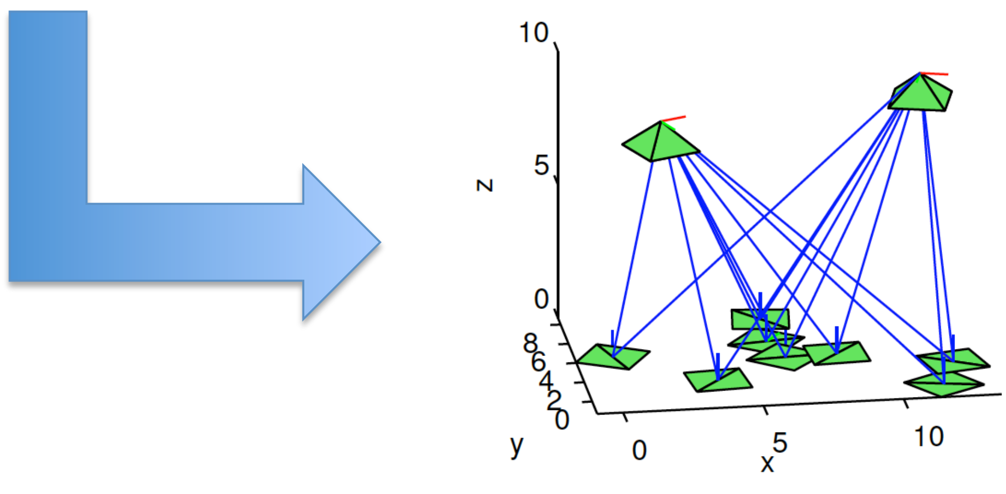
Advanced Robotics

Kostas Daniilidis

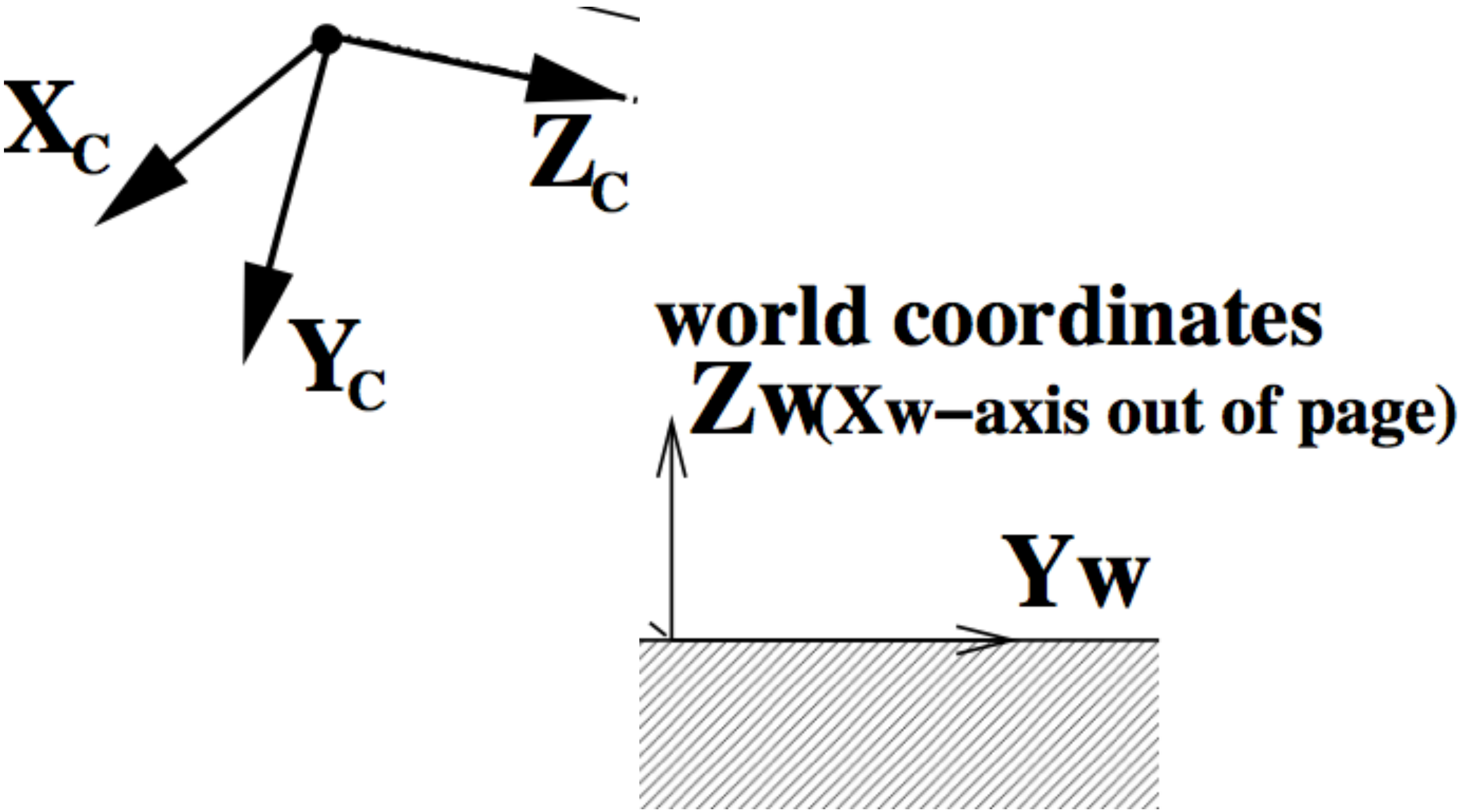
Using the projective transformation the pose of a robot with respect to a planar pattern:







Pose from reference points on plane Z_w=0



Recall the projection from world to camera

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = K \begin{pmatrix} r_1 & r_2 & r_3 & T \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix}$$

and assume that all points in the world lie in the ground plane $Z = 0$.

Then the transformation reads

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Suppose we estimate an H from $N \geq 4$ correspondences.

Let us assume that we know the intrinsic parameters K.

Pose estimation means finding R, T given H and intrinsics K.

We observe that

$$K^{-1}H = \begin{pmatrix} r_1 & r_2 & T \end{pmatrix}$$

has specific properties: its first two columns are orthogonal unit vectors.

Nothing guarantees that that the H we computed will satisfy this condition.

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Let us name the columns of $K^{-1}H$:

$$K^{-1}H = \begin{pmatrix} h_1' & h_2' & h_3' \end{pmatrix}$$

We seek orthogonal r_1 and r_2 that are the closest to h_1' and h_2'. The solution to this problem is given by the Singular Value Decomposition.

We find the orthogonal matrix R that is the closest to $\begin{pmatrix} h_1' & h_2' & h_1' \times h_2' \end{pmatrix}$:

$$\arg\min_{R \in SO(3)} \| R - \begin{pmatrix} h_1' & h_2' & h_1' \times h_2' \end{pmatrix} \|_F^2$$

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If the SVD of

$$\begin{pmatrix} h_1' & h_2' & h_1' \times h_2' \end{pmatrix} = USV^T$$

then the solution is

$$R = U \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \det(UV^T) \end{pmatrix} V^T$$

The diagonal matrix is inserted to guarantee that $\det(R) = 1$.

To find the translation : $T = h_3' / \|h_1'\|$

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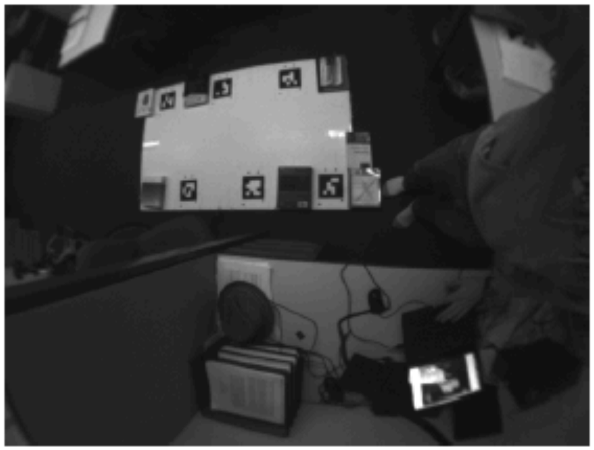

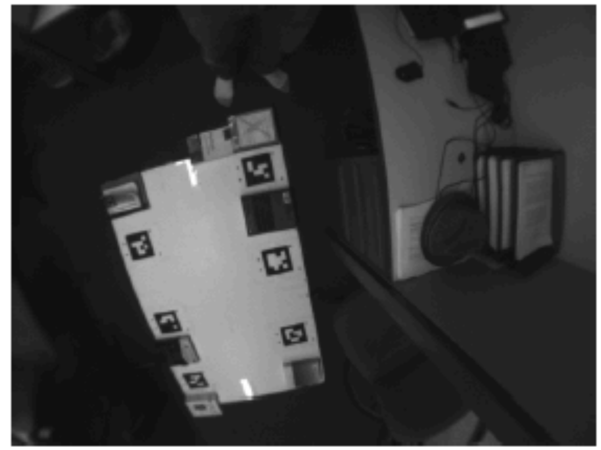

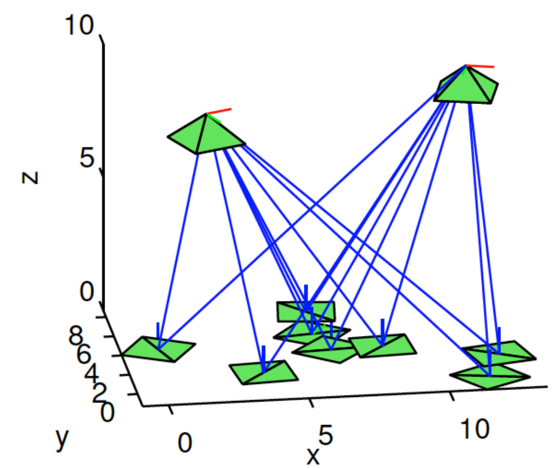

Logistics

- Midterm on Tuesday (27/10)
  - Closed book, closed notes
  - Bring your own calculator
  - Covers lectures 1-5
  - 2 hours exam
- Lab session this week
  - This week: Straight line & multi-waypoint trajectory tracking