

Introduction to Aerial Robotics

Lecture 6

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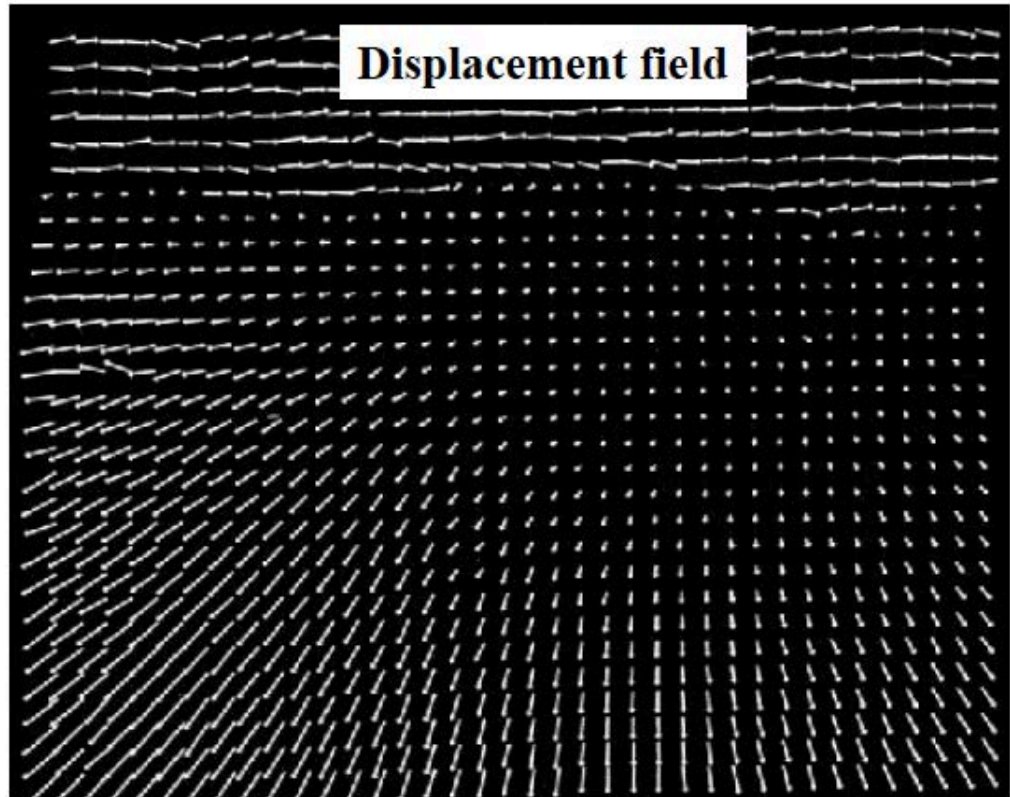
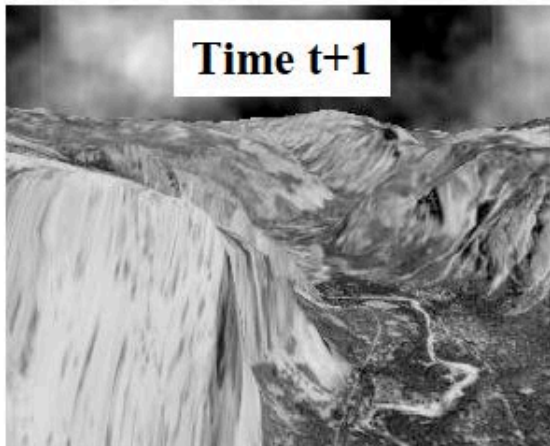
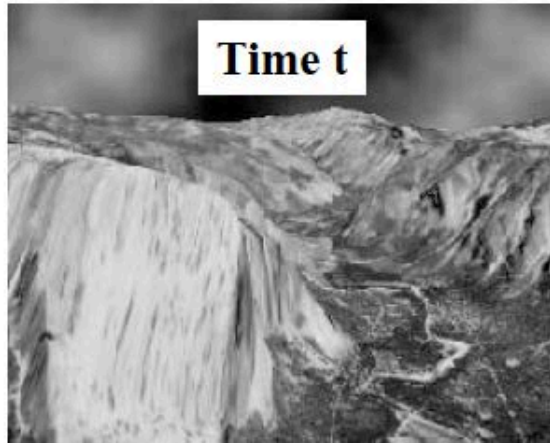
13 October 2015

Outline

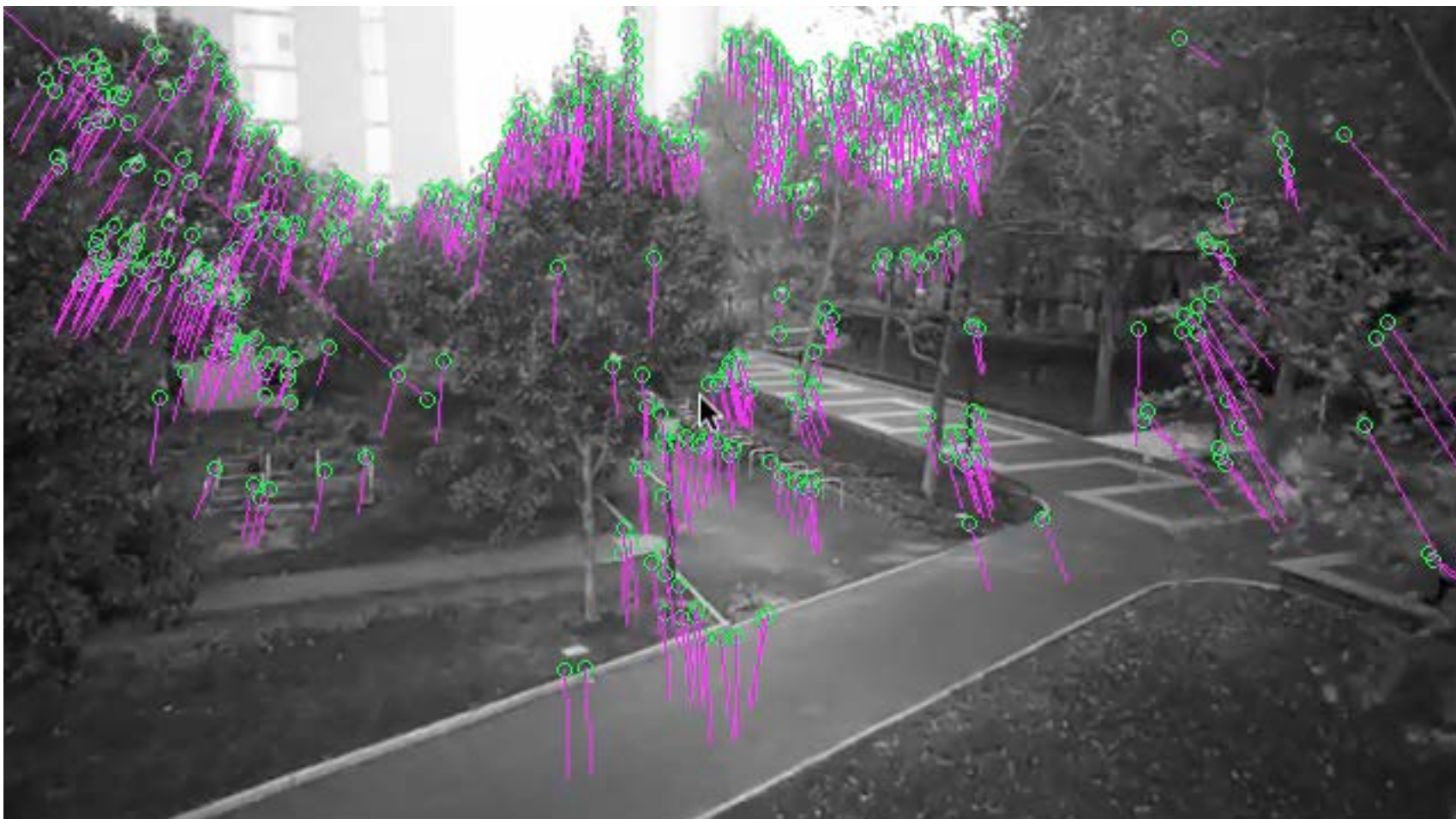
- Optical Flow
- Dense Stereo

Optical Flow

Optical Flow



Optical Flow







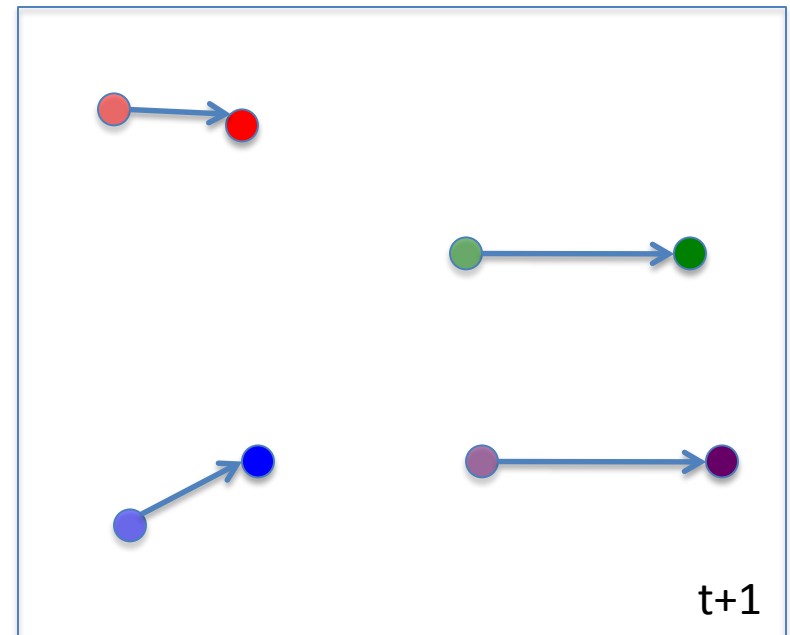
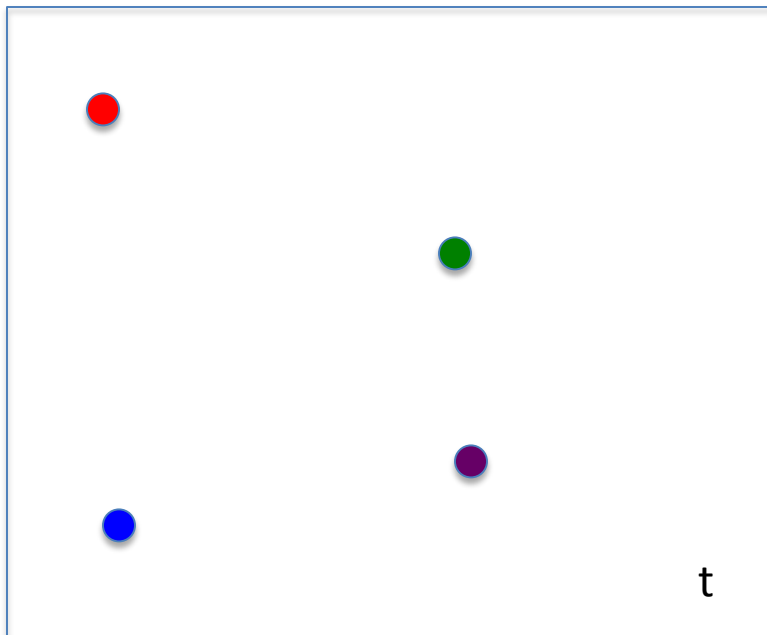
Forward motion, 1- \rightarrow 2

Backward motion: $1 \leftarrow 2$

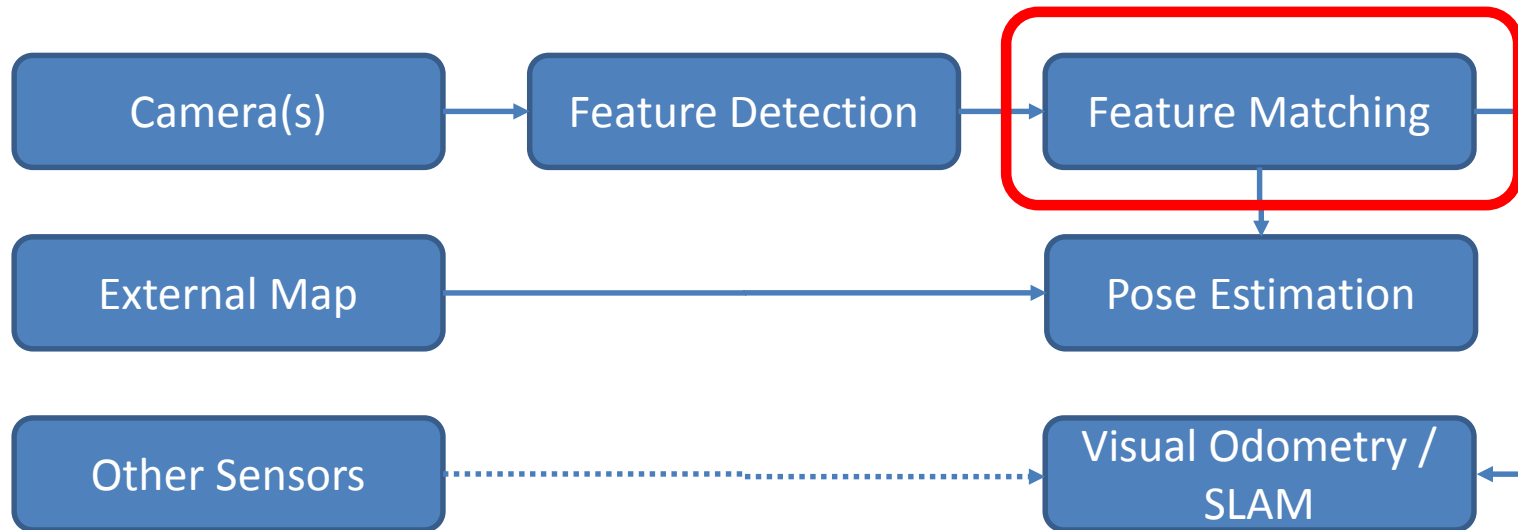


Problem Definition

- Define regions of interests, or points of interests in the first image at time t
- Search for correspondences in the second image at time $t + 1$

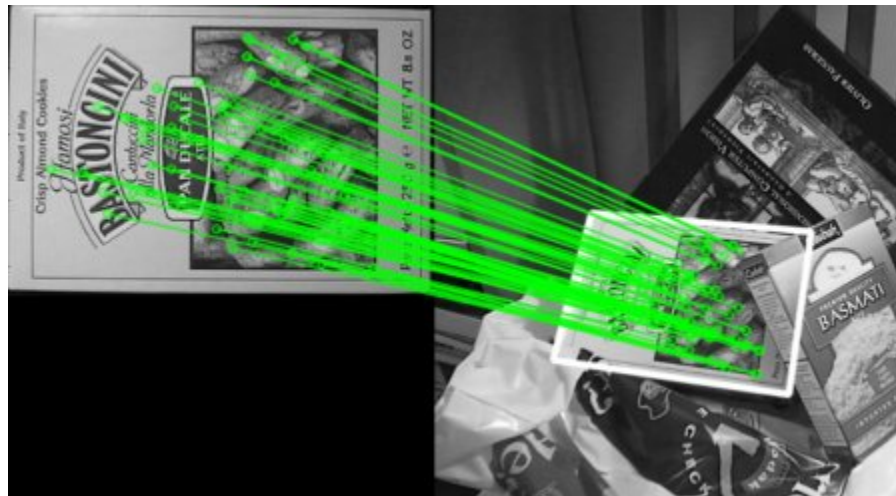


Vision-based State Estimation Pipeline



Discrete Matching Approach

- Detect corners features in both images
- Use image patch as feature description
 - Could be extended to color, texture, SIFT/HOG descriptor
- Find correspondences using descriptor matching

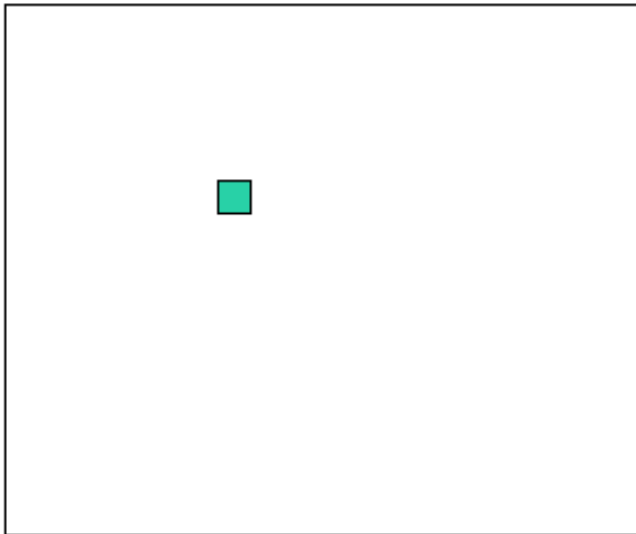




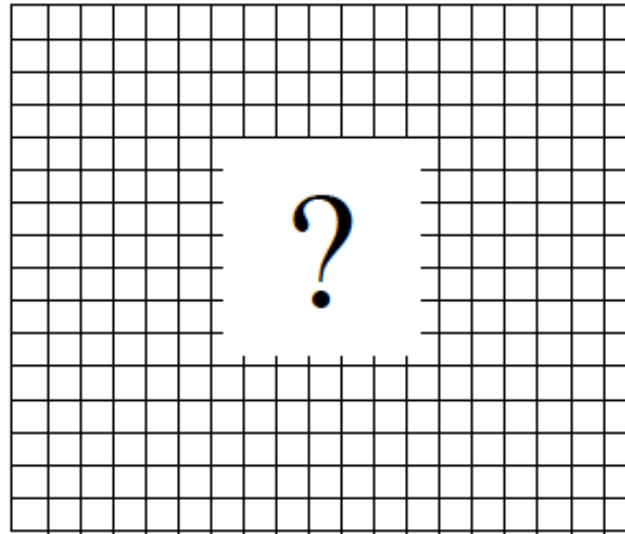
Differential Approach: KLT Tracker

- Detect corners features in first image
- Use image patch as feature description
 - Could be extended to color and texture descriptors
- Use Lucas-Kanade algorithm to compute displacement of the pixels in the patch
 - Motion model could be translation (2-DoF), affine (6-DoF), or more general 3D models
- Subpixel accuracy
- Do not need repeated detection

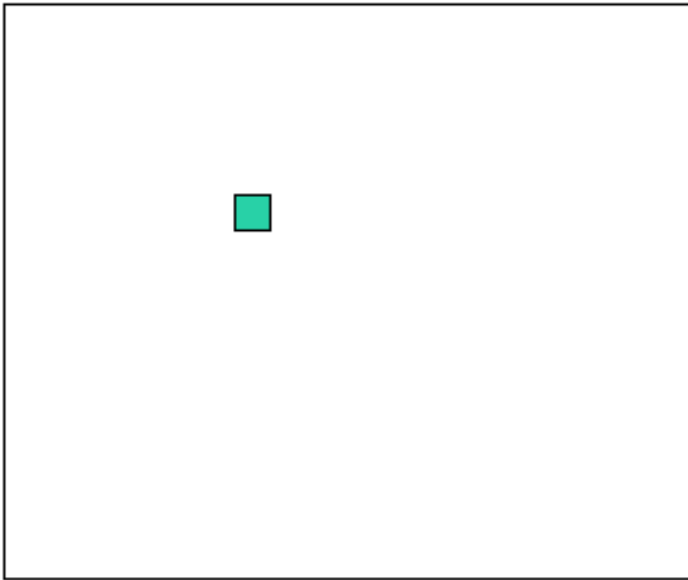
Given image patch in one image



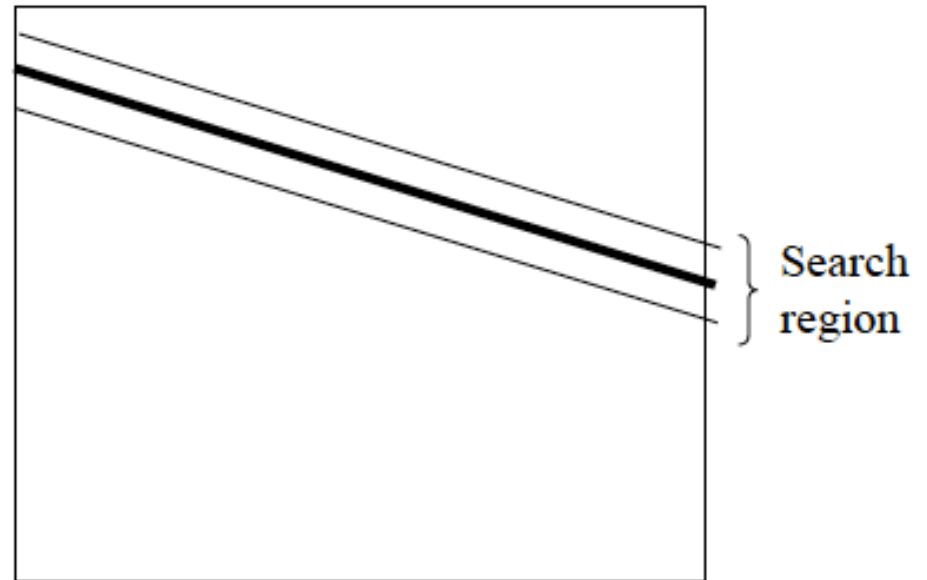
We don't want to search everywhere
in the second image for a match.



Given image patch in one image

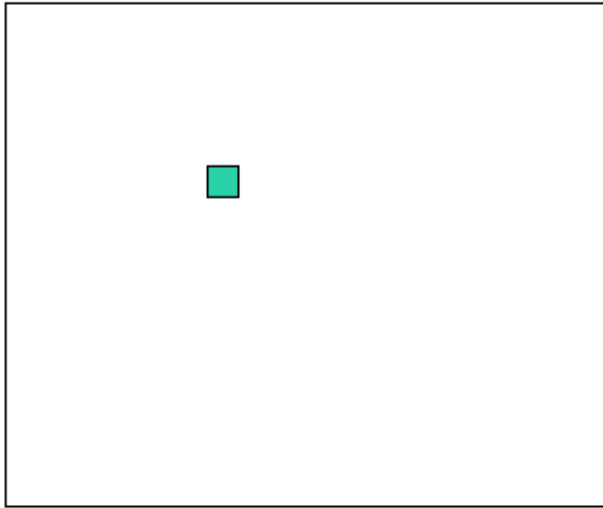


We don't want to search everywhere in the second image for a match.

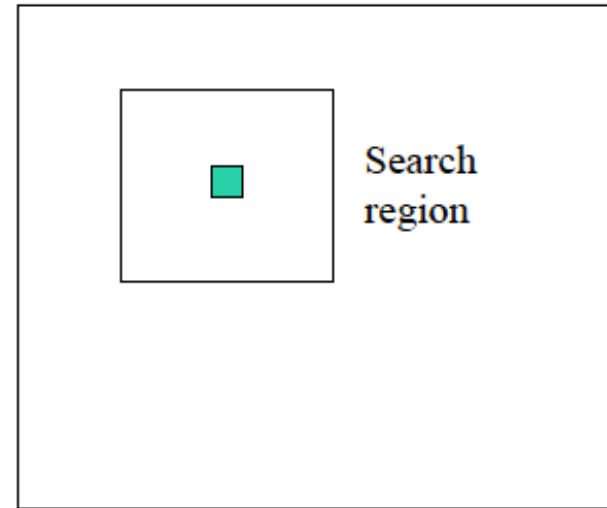


With stereo, we had an epipolar line constraint.

Given image patch in one image



We don't want to search everywhere in the second image for a match.

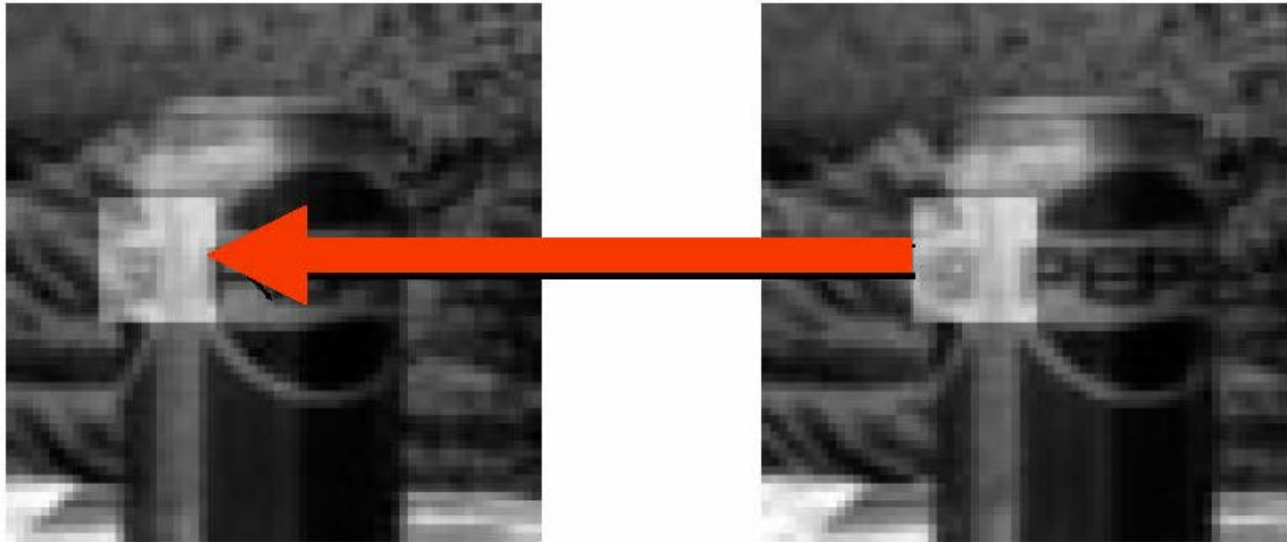


We don't know the relative R, T here, so no epipolar constraint.

But... motion is known to be “small”, so can still bound the search region.

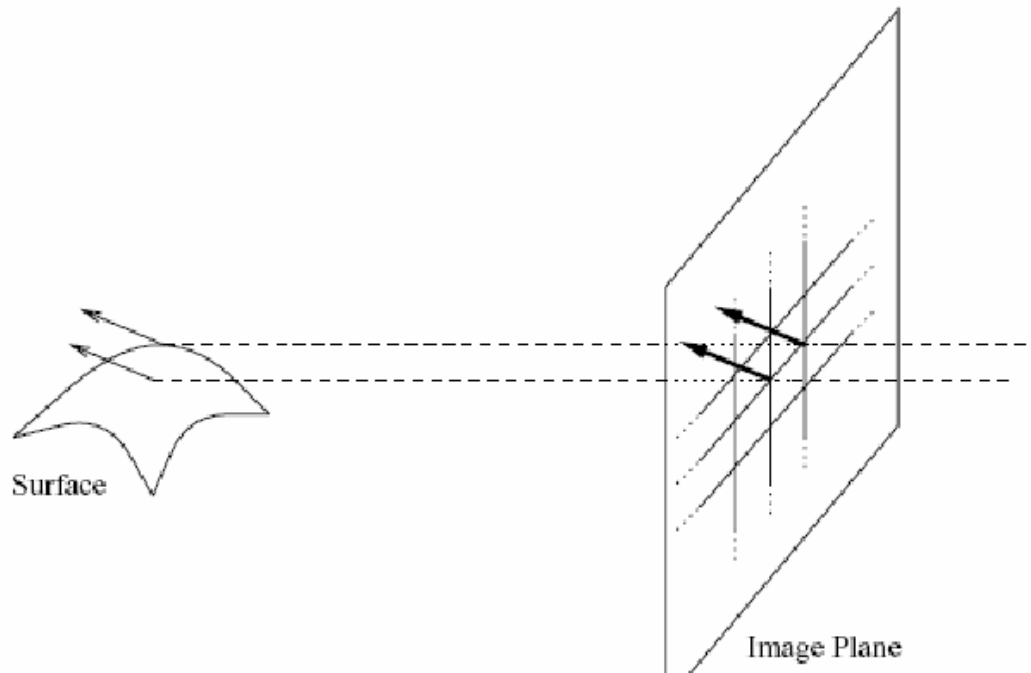
Optical Flow Assumption: Brightness Constancy

- Image measurements (brightness) in a small region remains the same even though their location may change



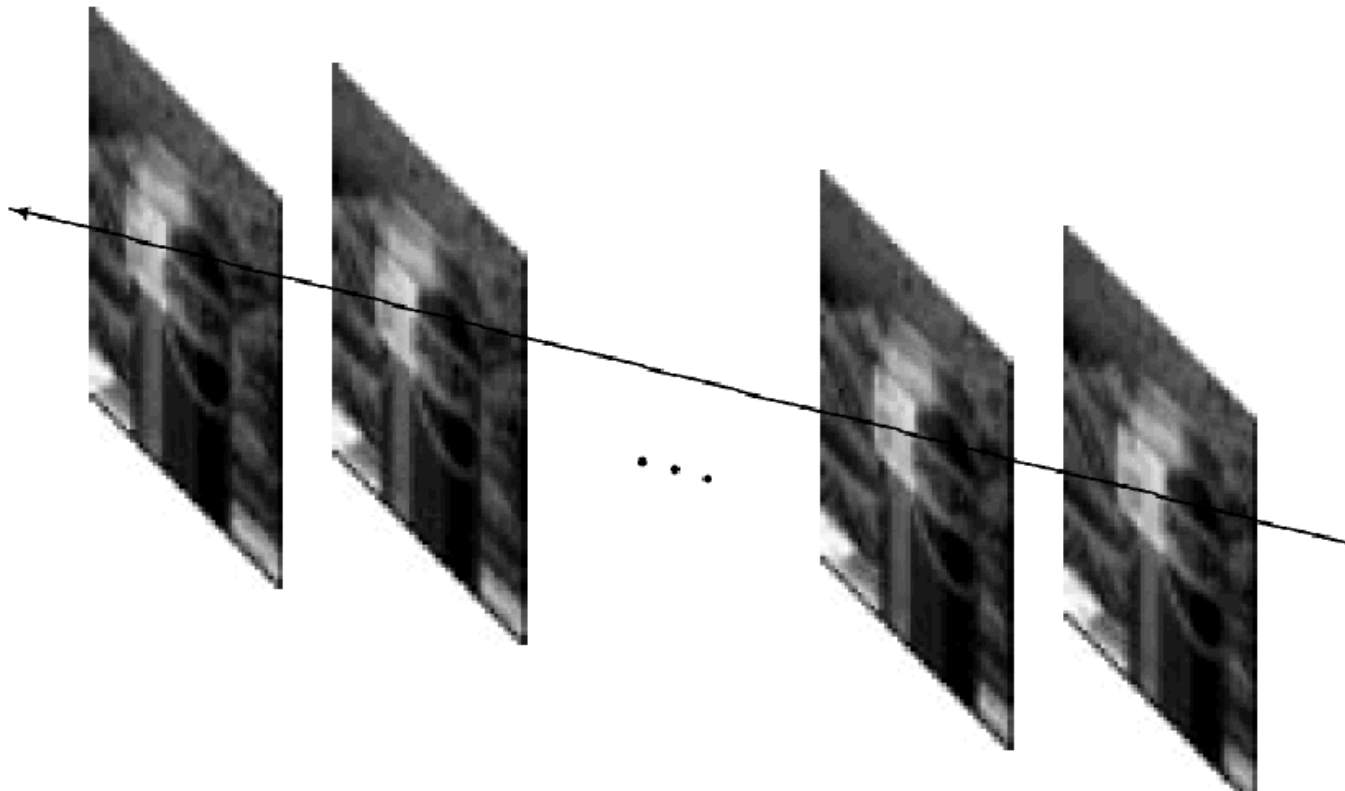
Optical Flow Assumption: Spatial Coherence

- Neighboring points in the scene typically belong to the same surface and have similar motions



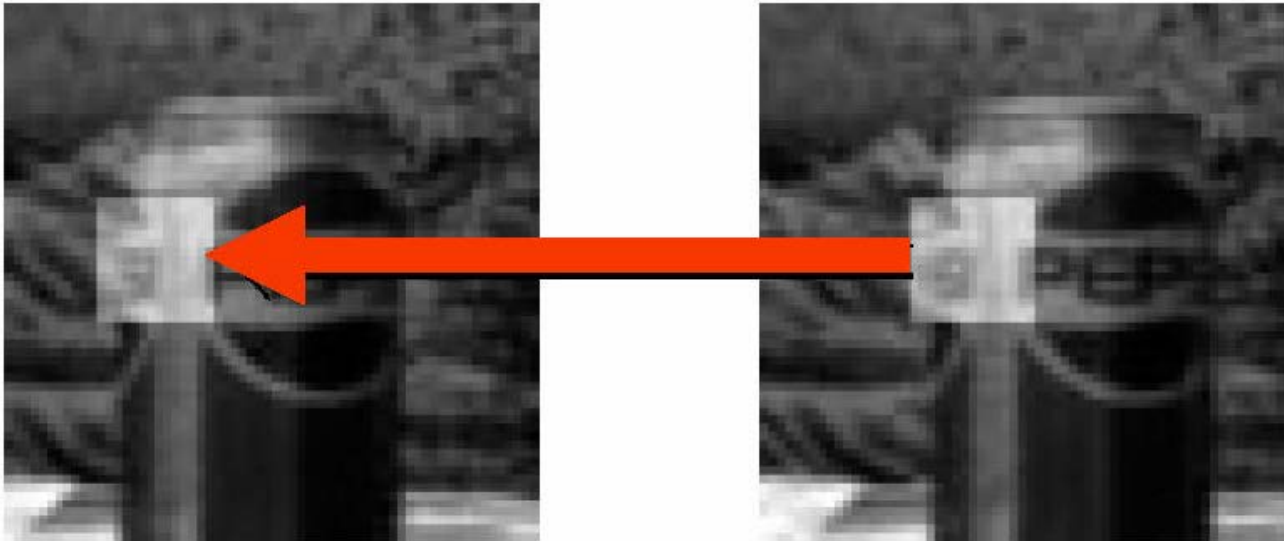
Optical Flow Assumption: Temporal Persistence

- Image motion of a surface patch changes smoothly over time



Lucas-Kanade (KLT) Tracking

- Intensity constancy constraint: $J(x + d) = I(x)$
 - $J(x) = I(x, t + 1)$
 - $I(x) = I(x, t)$




Lucas-Kanade (KLT) Tracking

- Define Sum of Squared Difference (SSD) error as:
 - $\epsilon = \int_W [J(x + d) - I(x)]^2 \omega(x) dx$
 - $\omega(x)$ is the smoothing term
 - Minimize ϵ with respect to $d \in R^{2 \times 1}$
- 4 steps for solving this problem:
 - Set $\frac{\partial \epsilon}{\partial d}$ to 0
 - Linearization by Taylor expansion on $J(x + d)$ with respect to d
 - Solve the resulting linearized system
 - Iterative refinement

Step 1: Set Derivative to 0

Differentiate SSD with respect to d and set to 0:


$$\frac{1}{2} \frac{\partial \epsilon}{\partial d} = \int_W [J(x + d) - I(x)] g w dx = 0$$


$$g = \left(\frac{\partial J}{\partial x}, \frac{\partial J}{\partial y} \right)^T$$

Step 1: Set Derivative to 0

Differentiate SSD with respect to d and set to 0:

$$\frac{1}{2} \frac{\partial \epsilon}{\partial d} = \int_W [J(x + d) - I(x)] g w \, dx = 0$$



$$g = \left(\frac{\partial J}{\partial x}, \frac{\partial J}{\partial y} \right)^T$$

Step 2: Linearization

$$[J(x + d) - I(x)]$$

Assume small motion, Taylor expansion of $J(x + d)$ is:

$$J(x + d) = J(x) + g^T d$$


$$g = \left(\frac{\partial J}{\partial x}, \frac{\partial J}{\partial y} \right)^T$$

Step 2: Linearization

Combining previous equations:

$$\frac{1}{2} \frac{\partial \epsilon}{\partial d} = \int_W [J(x + d) - I(x)] g w dx = 0$$

$$J(x + d) = J(x) + g^T d$$



$$\int_W g (g^T d) w dx = \int_W [I(x) - J(x)] g w dx$$

Step 3: Solve Linear System

$$\int_W g (g^T d) w \, dx = \int_W [I(x) - J(x)] g \, w \, dx$$



$$\int_W g (g^T) w \, dx = \sum_{i,j} \begin{bmatrix} g_x(i,j)g_x(i,j) & g_y(i,j)g_x(i,j) \\ g_x(i,j)g_y(i,j) & g_y(i,j)g_y(i,j) \end{bmatrix}$$



A: second moment matrix

Step 3: Solve Linear System

$$\int_W g (g^T d) w \, dx = \int_W [I(x) - J(x)] g \, w \, dx$$



$$\int_W g (g^T) w \, dx = \sum_{i,j} \begin{bmatrix} g_x(i, j) [I(i, j) - J(i, j)] \\ g_y(i, j) [I(i, j) - J(i, j)] \end{bmatrix}$$



Error vector b

Step 3: Solve Linear System

$$\int_W g (g^T d) w \, dx = \int_W [I(x) - J(x)] g \, w \, dx$$

$$A = \sum_{i,j} \begin{bmatrix} g_x(i,j)g_x(i,j) & g_y(i,j)g_x(i,j) \\ g_x(i,j)g_y(i,j) & g_y(i,j)g_y(i,j) \end{bmatrix}$$

$$b = \sum_{i,j} \begin{bmatrix} g_x(i,j)[I(i,j) - J(i,j)] \\ g_y(i,j)[I(i,j) - J(i,j)] \end{bmatrix}$$

$$A \, d = b$$

$$d = A^{-1} b$$

Step 3: Solve Linear System

$$\int_W g (g^T d) w \, dx = \int_W [I(x) - J(x)] g \, w \, dx$$

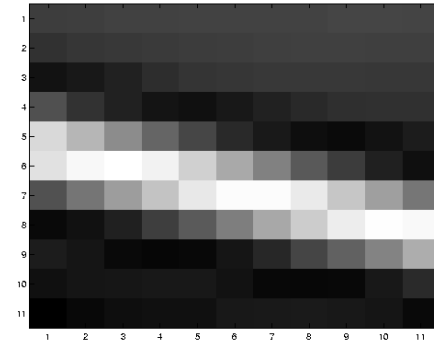
$$A = \sum_{i,j} \begin{bmatrix} g_x(i,j)g_x(i,j) & g_y(i,j)g_x(i,j) \\ g_x(i,j)g_y(i,j) & g_y(i,j)g_y(i,j) \end{bmatrix}$$

$$b = \sum_{i,j} \begin{bmatrix} g_x(i,j)[I(i,j) - J(i,j)] \\ g_y(i,j)[I(i,j) - J(i,j)] \end{bmatrix}$$

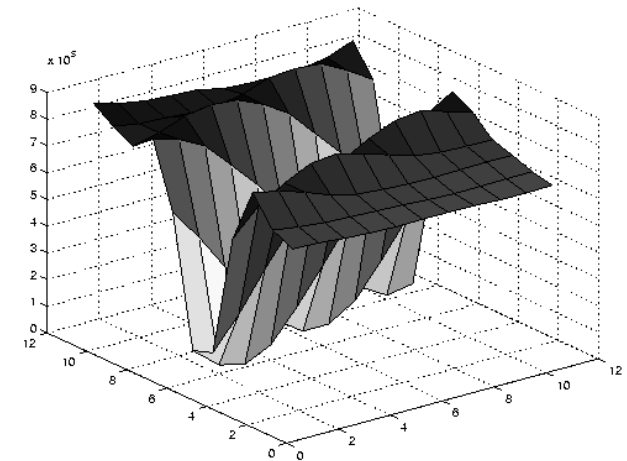
$$d = A^{-1} b$$

- What if A is not full rank? Recall the structure of A :
 - Same as the one for corner detection
 - Eigenvalues and eigenvectors of A tells whether we are tracking a corner

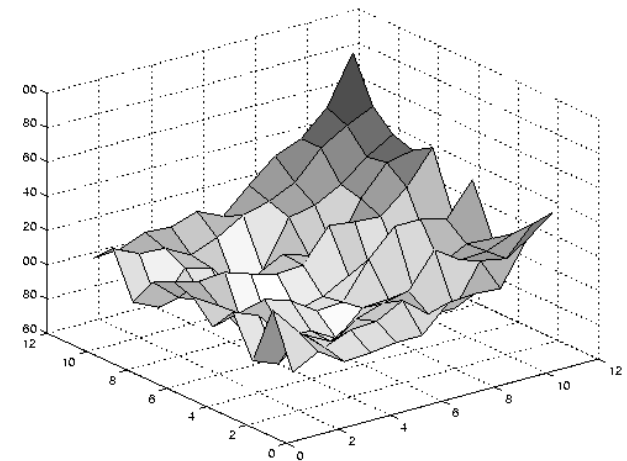
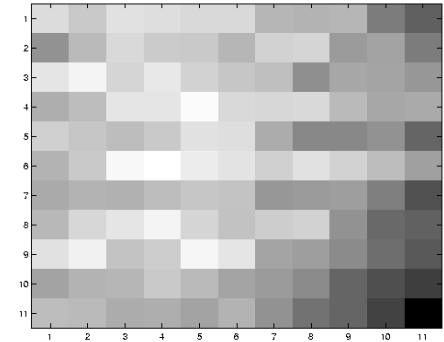
Edge



- large gradients, all the same direction
- large λ_1 , small λ_2

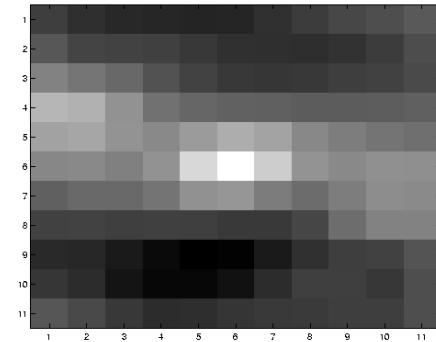


Low Texture Region

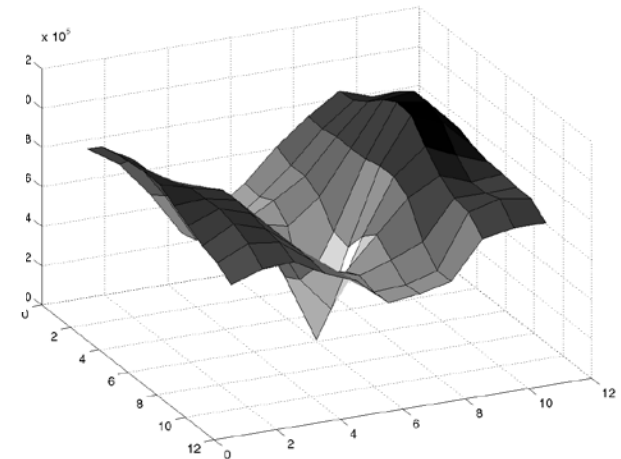


- gradients have small magnitude
- small λ_1 , small λ_2

High Texture Region



- gradients are different, large magnitudes
- large λ_1 , large λ_2



Step 4: Iterative Refinement

- Iterative refinement
 - Estimate velocity at pixels of interests using one iteration of Lucas-Kanade algorithm
 - Transform pixels using the estimated flow field
 - Refine estimate by repeating the process

Step 4: Iterative Refinement

$$\boxed{\int_W g (g^T d) w \, dx = \int_W [I(x) - J(x)] g \, w \, dx}$$

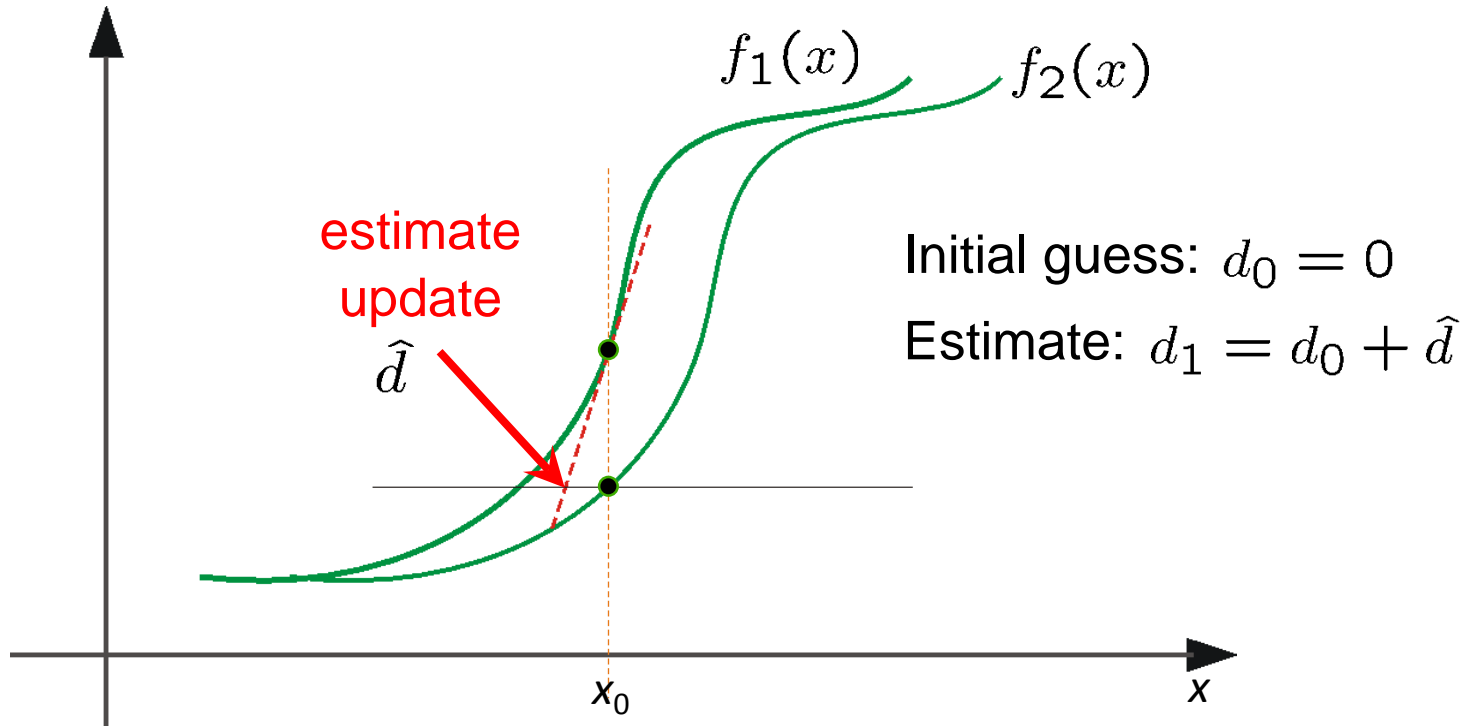
$$A = \sum_{i,j} \begin{bmatrix} g_x(i,j)g_x(i,j) & g_y(i,j)g_x(i,j) \\ g_x(i,j)g_y(i,j) & g_y(i,j)g_y(i,j) \end{bmatrix}$$

$$b = \sum_{i,j} \begin{bmatrix} g_x(i,j)[I(i,j) - J(i,j)] \\ g_y(i,j)[I(i,j) - J(i,j)] \end{bmatrix}$$

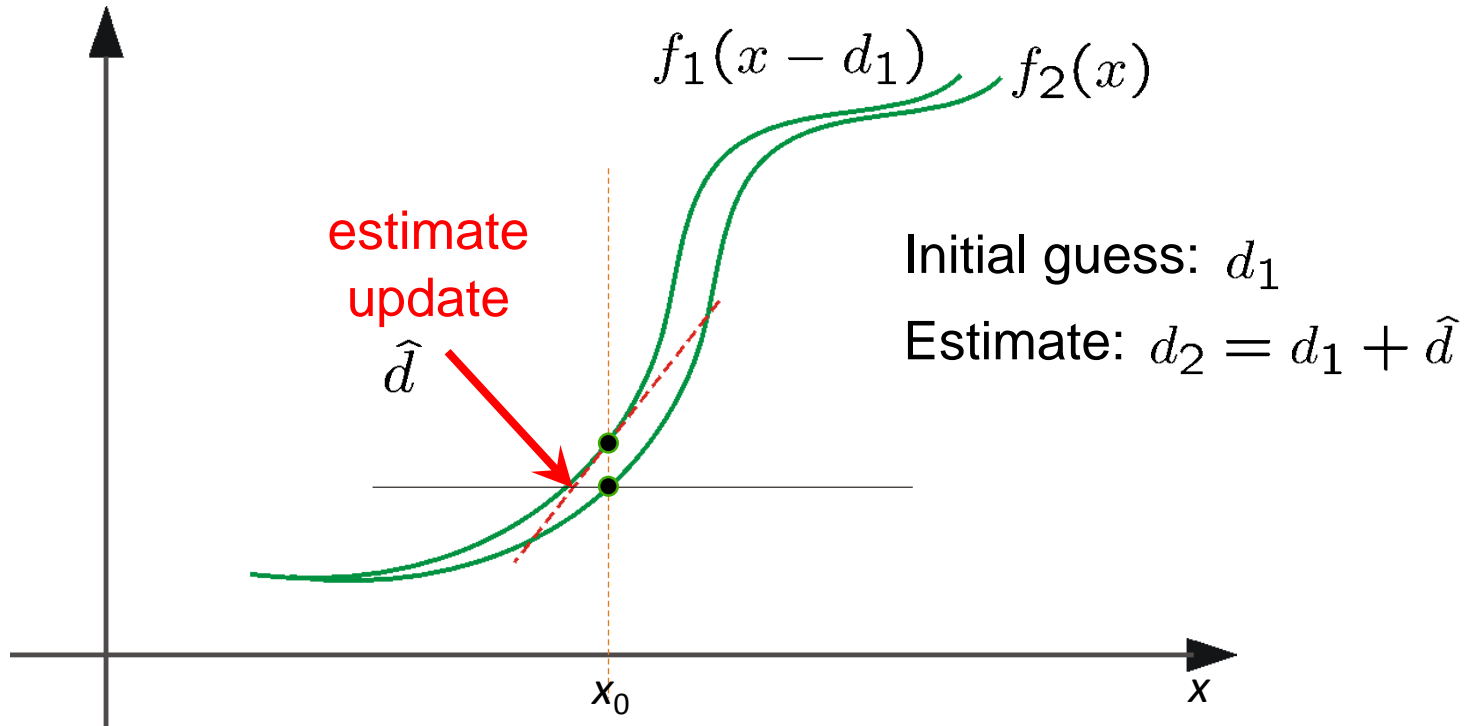
$$d = A^{-1} b$$

- Iterate:
 - Update $J_{i+1}(x) \rightarrow J_i(x + d)$
 - Recompute d between $J_{i+1}(x)$ and $I(x)$

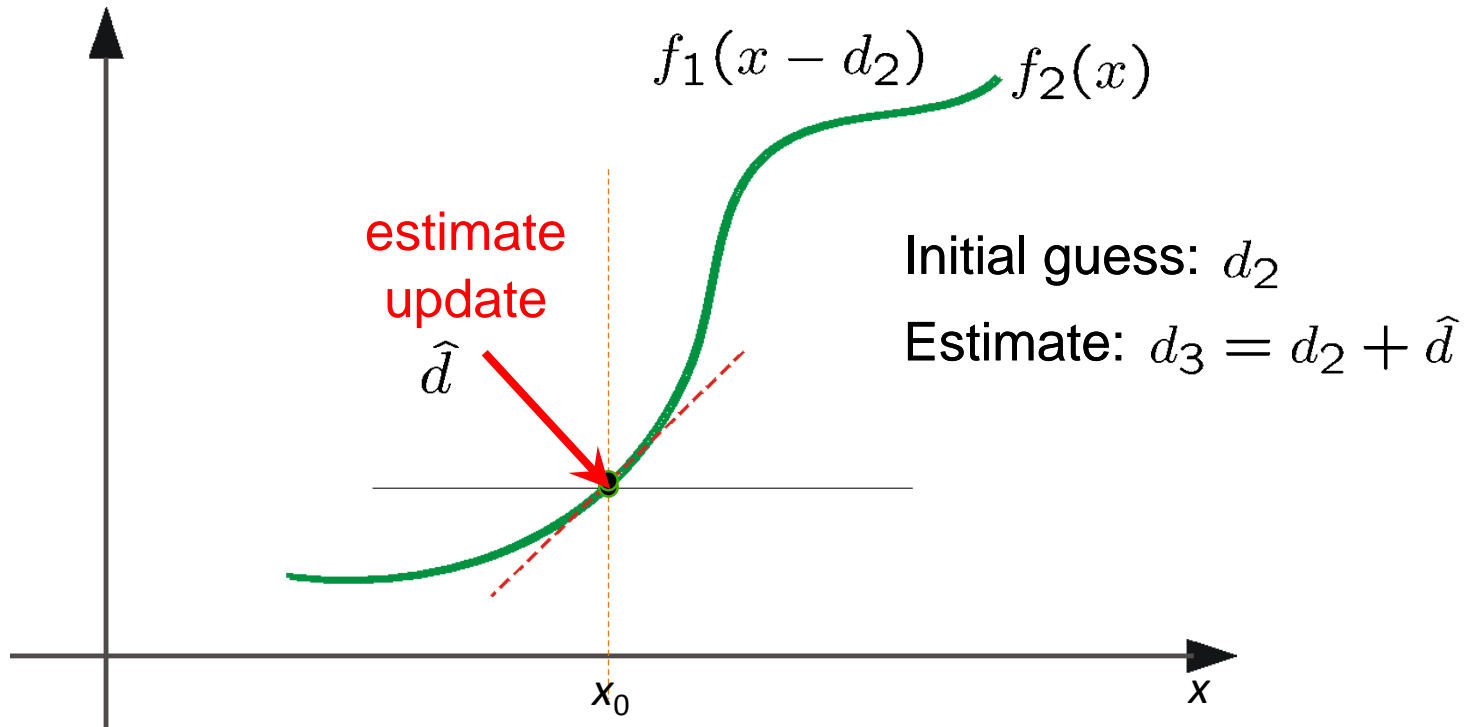
Step 4: Iterative Refinement



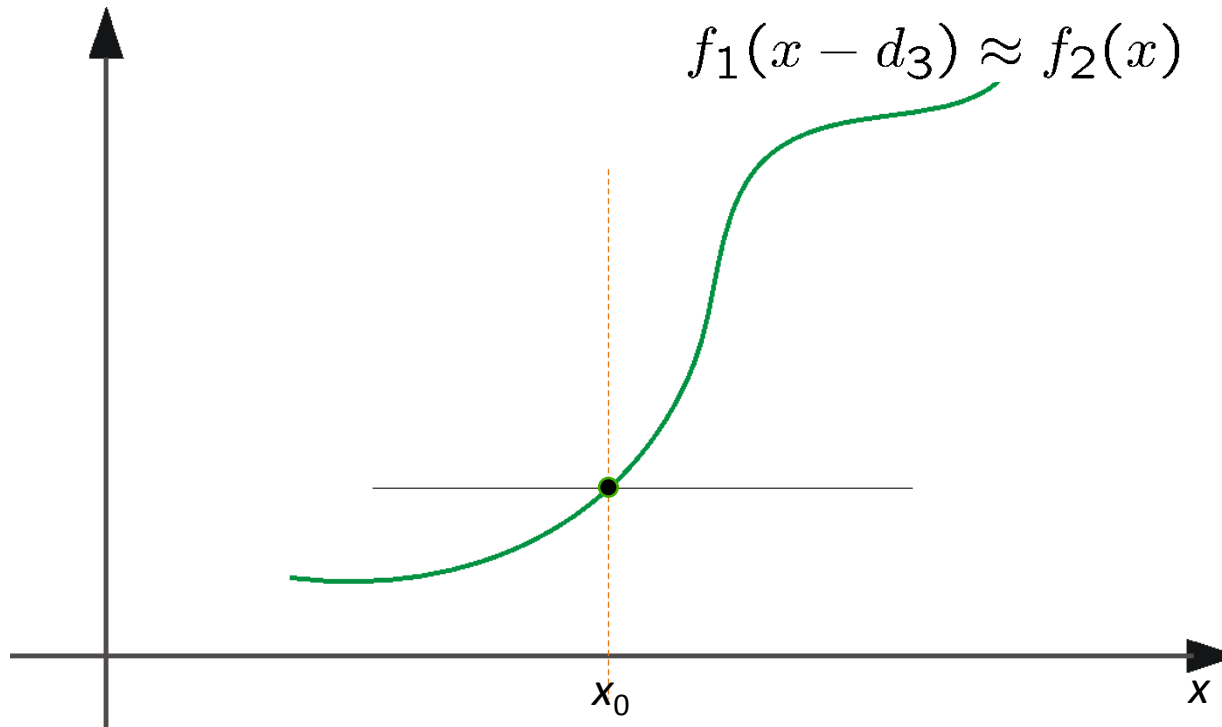
Step 4: Iterative Refinement



Step 4: Iterative Refinement



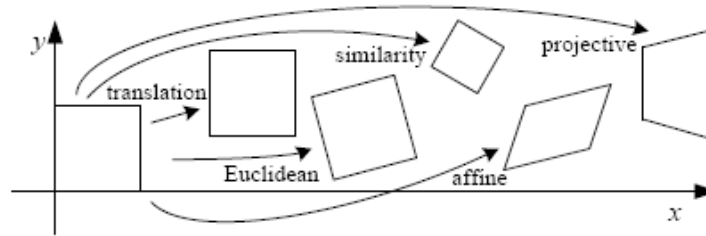
Step 4: Iterative Refinement



Motion Models

- 2D Models:
 - Affine
 - Quadratic
 - Planar projective transform (Homography)
- 3D Models:
 - Instantaneous camera motion models
 - Homography+epipole
 - Plane+Parallax

Motion Models

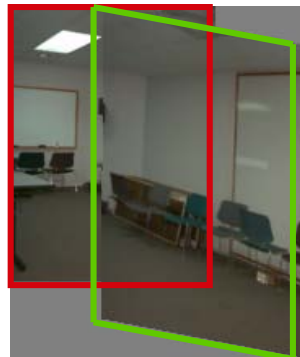


Translation



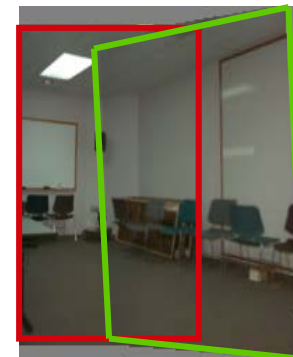
2 unknowns

Affine



6 unknowns:
 $x' = Ax + d$

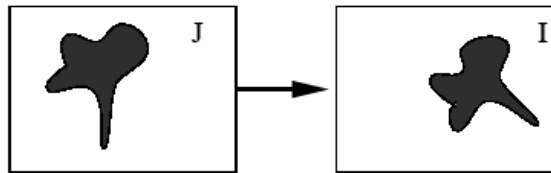
3D rotation



3 unknowns

Compute Affine Motion

- Intensity constancy constraint: $J(Ax + d) = I(x)$



- Define Sum of Squared Difference, SSD, error as:

$$\epsilon = \int_W [J(Ax + d) - I(x)]^2 w(x) dx \quad (1)$$

Let $A = I + D$, min. ϵ with respect to $D \in R^{2 \times 2}$, and $d \in R^{2 \times 1}$.

- Three steps for solving this problem:
 - Set $\frac{\partial \epsilon}{\partial D}$, $\frac{\partial \epsilon}{\partial d}$ to 0;
 - Taylor expansion on $J(Ax+d)$ respect to x ;
 - Solve for A (D) and d , iterate with Newton Raphson.

$$\epsilon = \int_W [J(Ax + d) - I(x)]^2 w(x) dx$$

- Differentiate ϵ with respect to D and d ,

$$\frac{1}{2} \frac{\partial \epsilon}{\partial D} = \int_W [J(Ax + d) - I(x)] g x^T w dx = 0 \quad (2)$$

$$\frac{1}{2} \frac{\partial \epsilon}{\partial d} = \int_W [J(Ax + d) - I(x)] g w dx = 0 \quad (3)$$

where $g = (\frac{\partial J}{\partial x}, \frac{\partial J}{\partial y})^T$.

- Assume small motion, $Ax + d = x + (Dx + d) = x + u$,

– Taylor expansion of $J(Ax+d)$ is

$$J(Ax + d) = J(x) + g^T u \quad (4)$$

$$\epsilon = \int_W [J(Ax + d) - I(x)]^2 w(x) dx$$

- From previous slide, we have:

$$\begin{aligned} \int_W [J(Ax + d) - I(x)] g x^T w dx &= 0 \\ \int_W [J(Ax + d) - I(x)] g w dx &= 0 \\ J(Ax + d) &= J(x) + g^T u \end{aligned}$$

where $g = (\frac{\partial J}{\partial x}, \frac{\partial J}{\partial y})^T$.

- Combining them, we have:

$$\int_W g x^T (g^T u) w dx = \int_W [I(x) - J(x)] g x^T dx \quad (5)$$

$$\int_W g (g^T u) w dx = \int_W [I(x) - J(x)] g w dx \quad (6)$$

- Can rewrite (5) and (6) as a linear system of 6 equations and 6 unknowns.

$$\epsilon = \int_W [J(Ax + d) - I(x)]^2 w(x) dx$$

• Tz = a:

$$T = \int_W \begin{bmatrix} g_x^2 x^2 & g_x g_y xy & g_x^2 xy & g_x g_y x^2 & g_x^2 x & g_x g_y x \\ g_x g_y xy & g_y^2 y^2 & g_x g_y y^2 & g_y^2 xy & g_x g_y y & g_y^2 y \\ g_x^2 xy & g_x g_y y^2 & g_x^2 y^2 & g_x g_y xy & g_x^2 y & g_x g_y y \\ g_x g_y x^2 & g_y^2 xy & g_x g_y xy & g_y^2 x^2 & g_x g_y x & g_y^2 x \\ g_x^2 x & g_x g_y y & g_x^2 y & g_x g_y x & g_x^2 & g_x g_y \\ g_x g_y x & g_y^2 y & g_x g_y y & g_y^2 x & g_x g_y & g_y^2 \end{bmatrix} w \, dx \quad (7)$$

and

$$z = [D(1, 1), D(2, 2), D(1, 2), D(2, 1), d(1), d(2)]^T \quad (8)$$

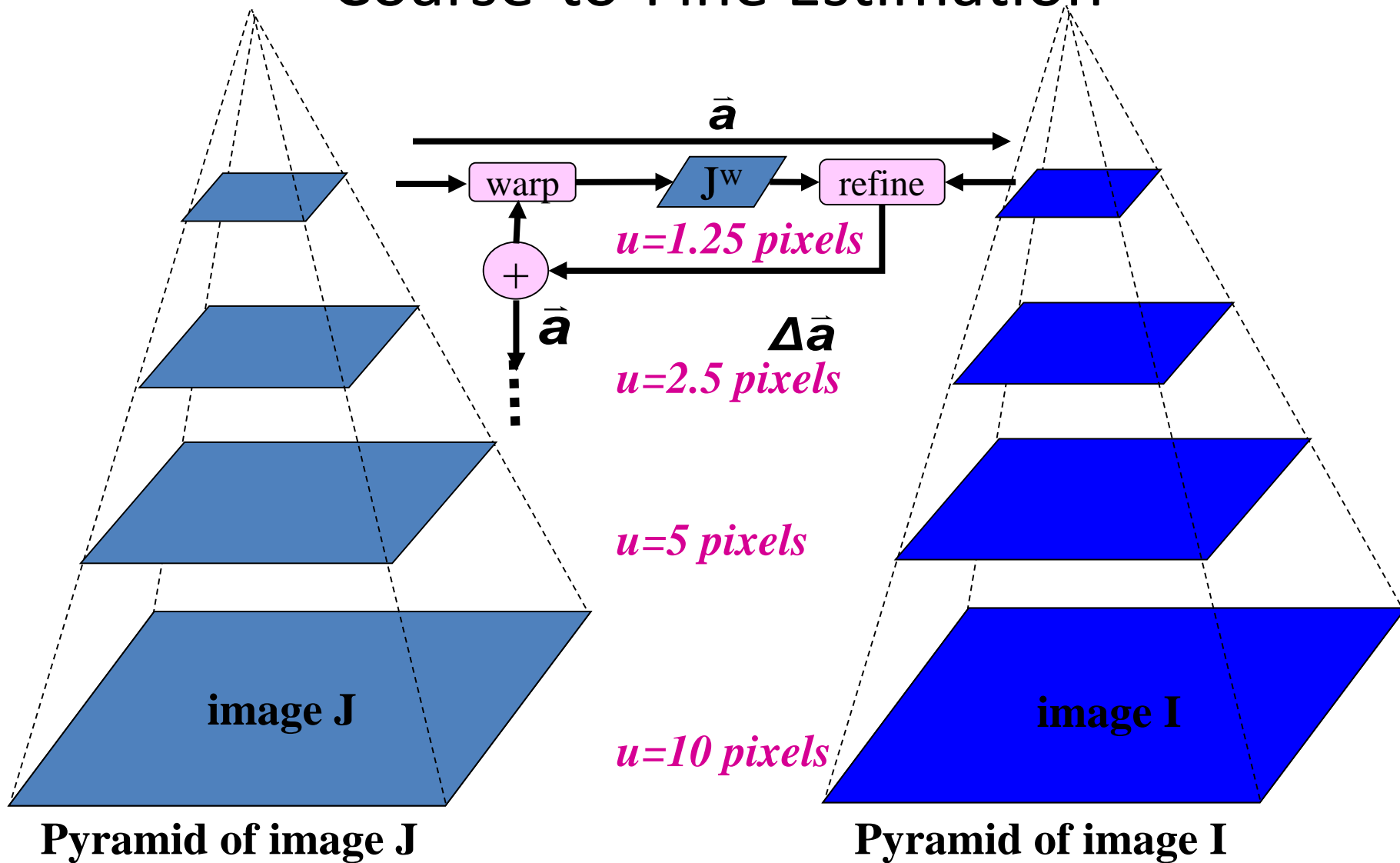
$$a = \int_W (I(x) - J(x)) \begin{bmatrix} g_x x \\ g_y y \\ g_x y \\ g_y x \\ g_x \\ g_y \end{bmatrix} dx \quad (9)$$



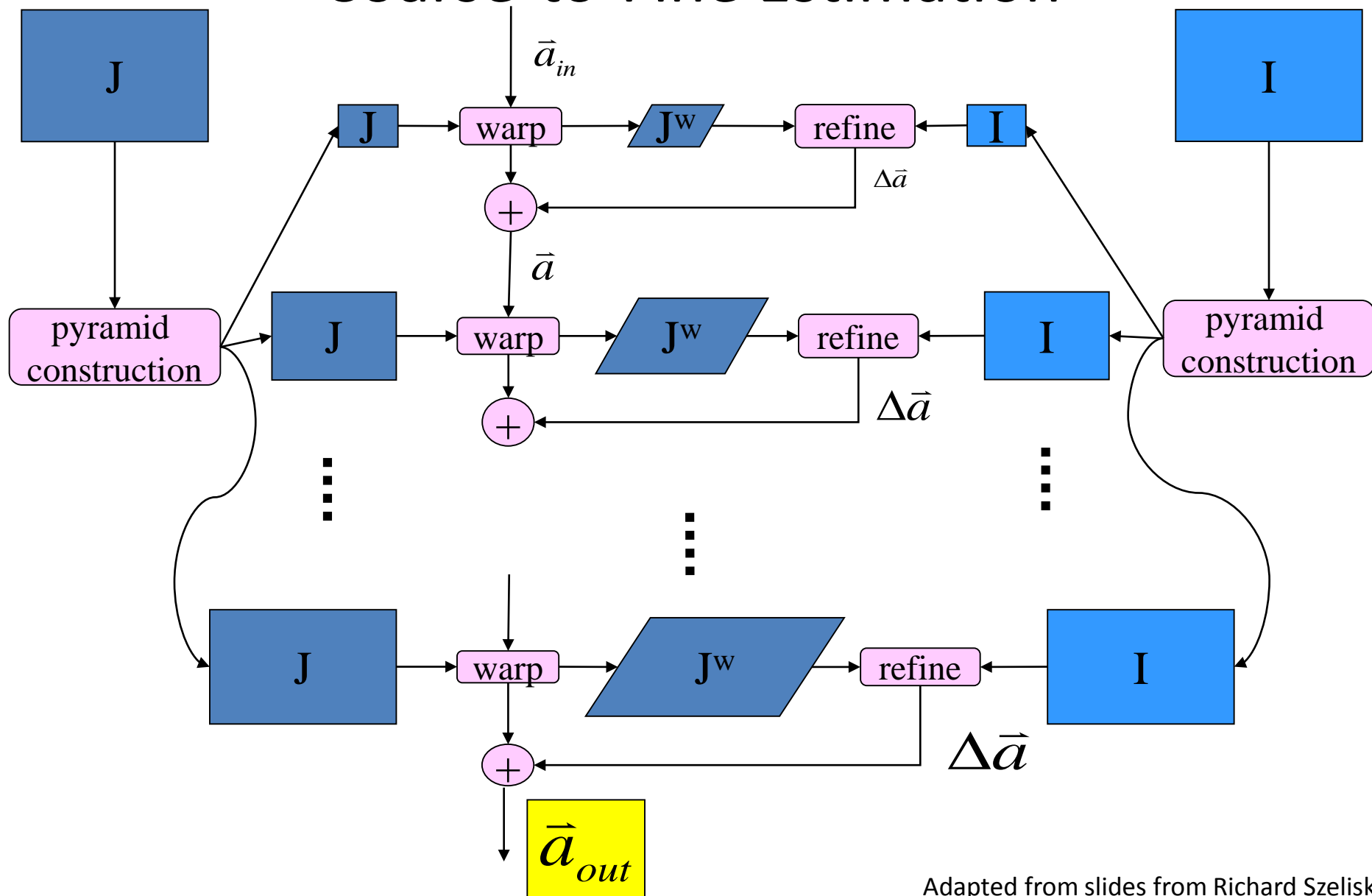
Limits of the KLT Tracker

- Fails when intensity structure in window is poor
- Fails when the displacement is large (typical operating range is motion of 1 pixel)
 - Linearization of brightness is suitable only for small displacement
- Brightness is not strictly constant in images
 - Actually less problematic than it appears, since we can filter images to make them look similar

Coarse-to-Fine Estimation



Coarse-to-Fine Estimation

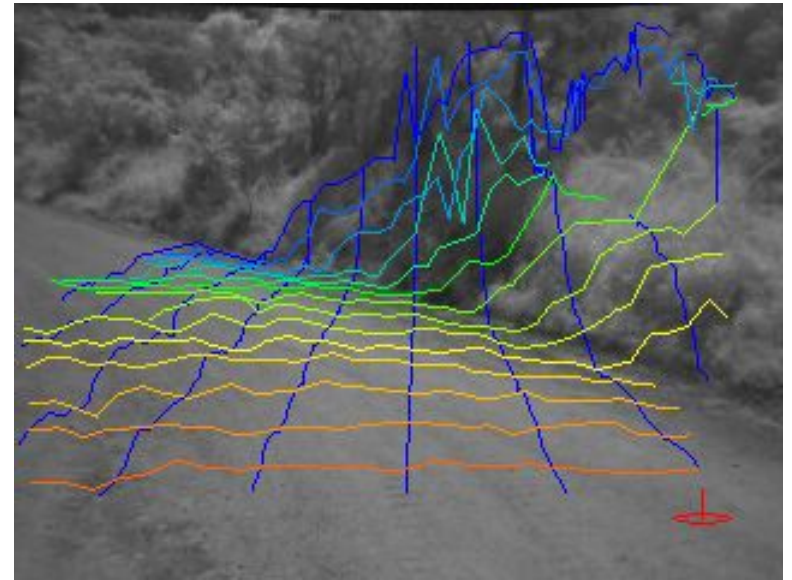
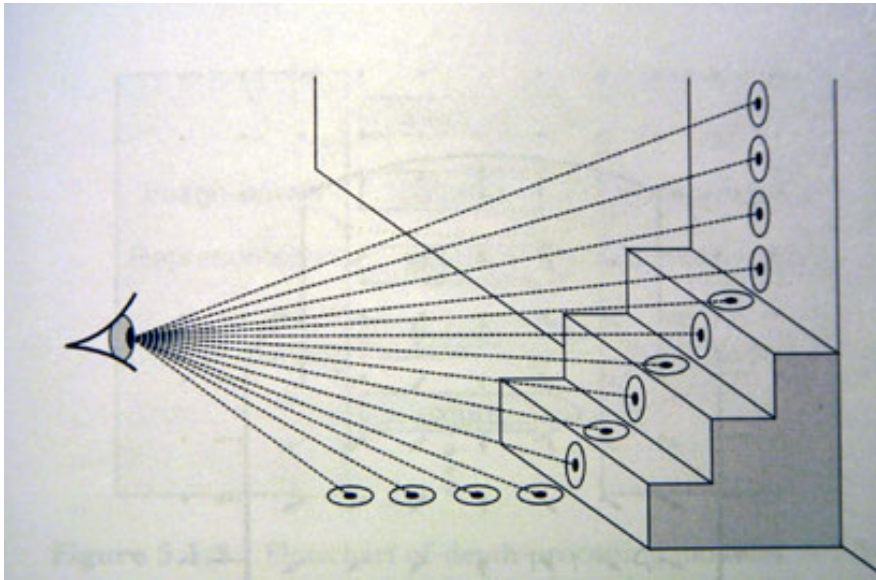


Optical Flow-based Velocity Estimator

Dense Stereo

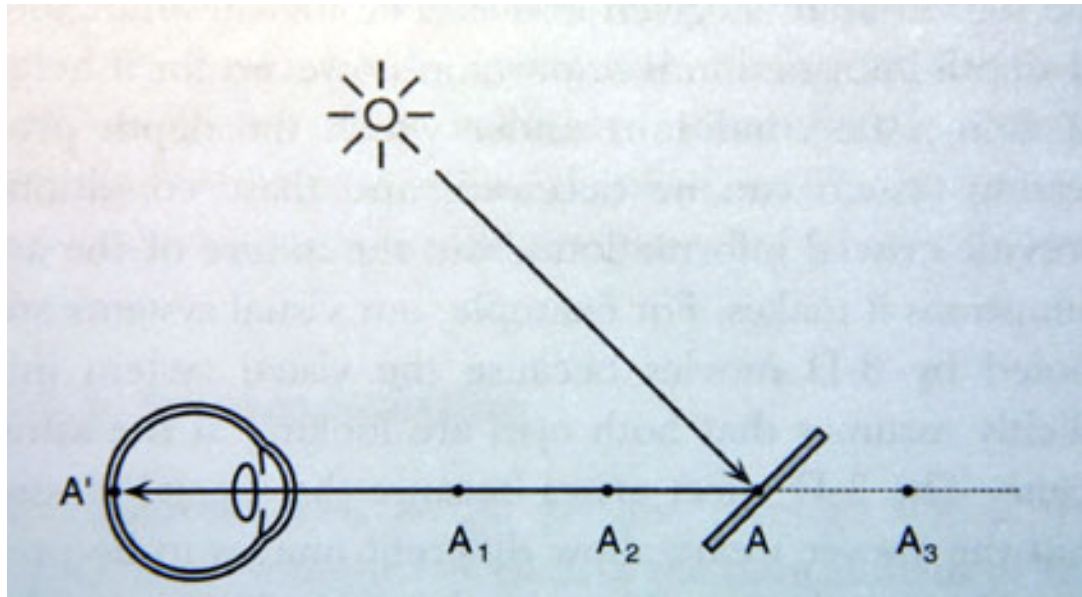
3D Shape perception

- 1) **Depth:** *the distance of the surface from the observer*
- 2) **Surface orientation:** *the slant and tilt of the surface with respect to observers' sight*



Depth ambiguity

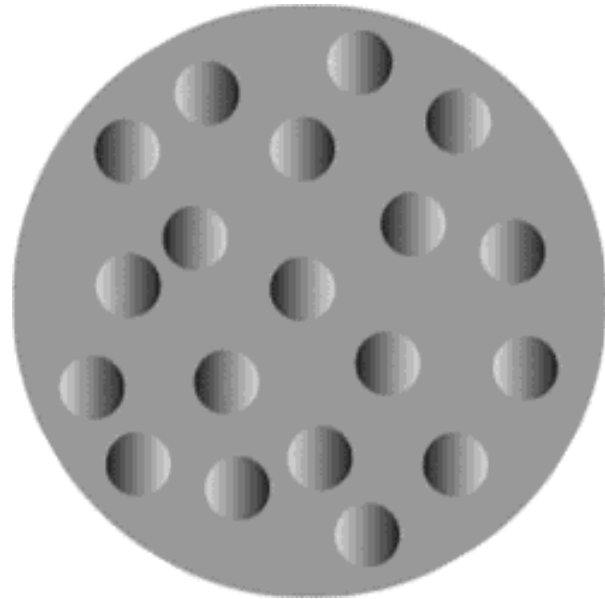
Inverse problem: multiple solution exists

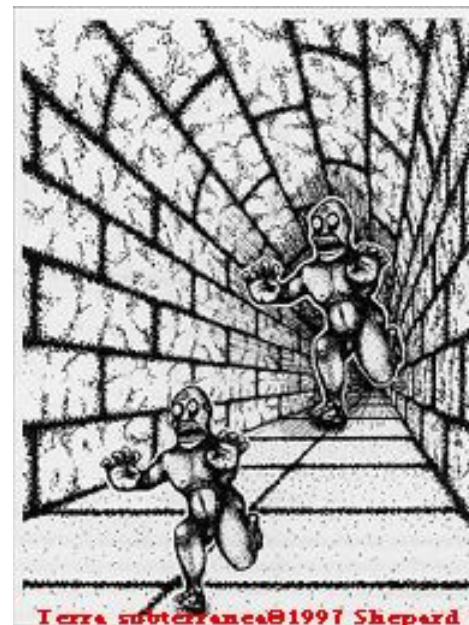


Pictorial cues for 3D shape

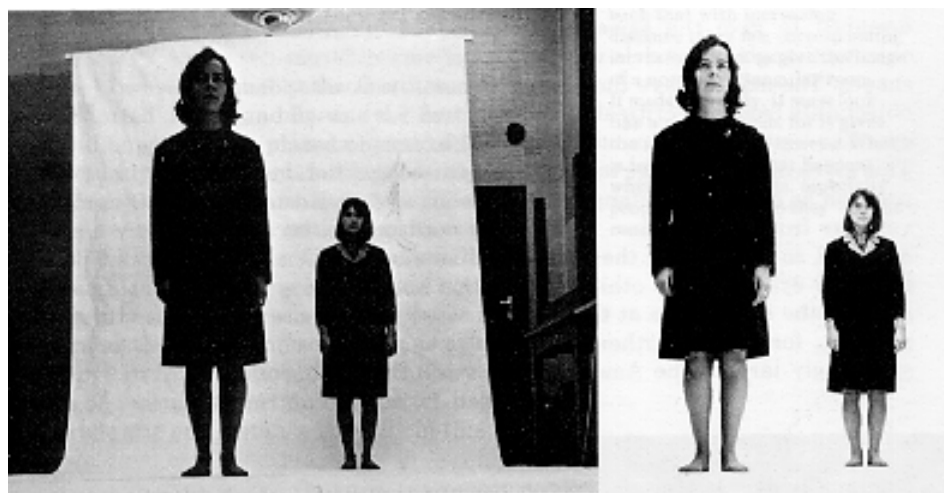
Perspective projection gives us the relative position to horizon, therefore we can deduce its physical size.

Shading also reveal shape using illumination model



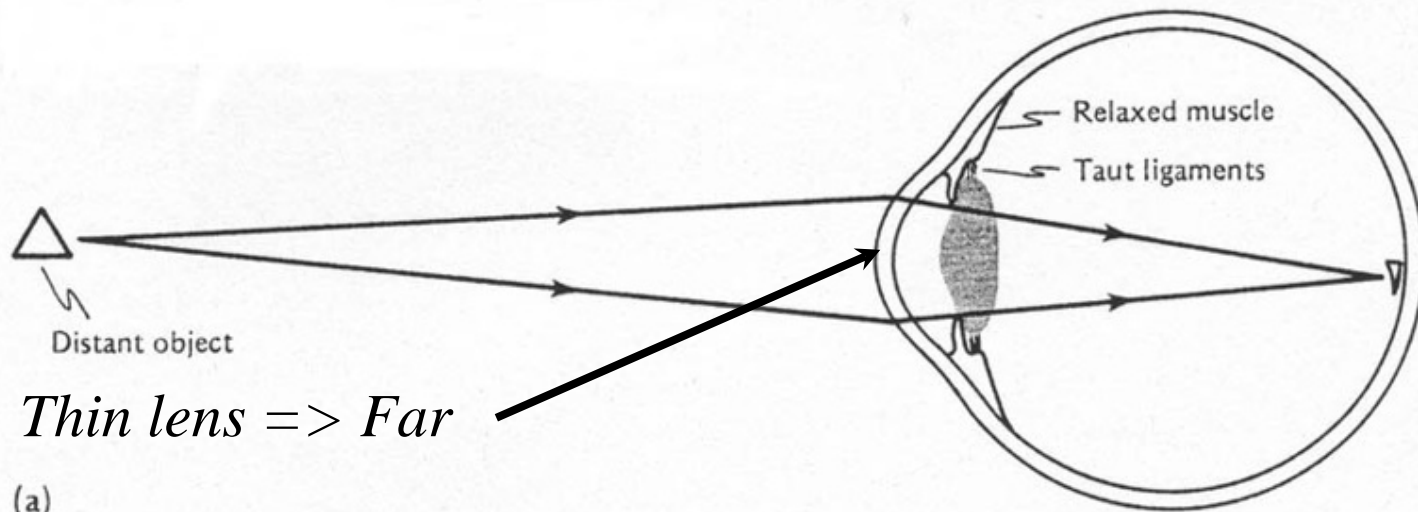


How?



Adapted from slides from Kostas Daniilidis

Shape from Focus, Accommodation

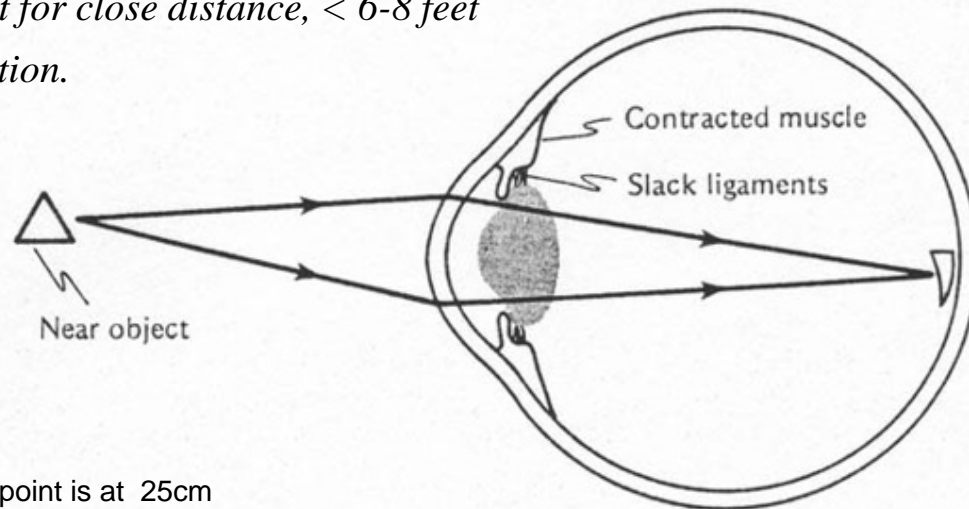


Thin lens => Far

(a)

Human performance: use it for close distance, < 6-8 feet

Use it more for size perception.

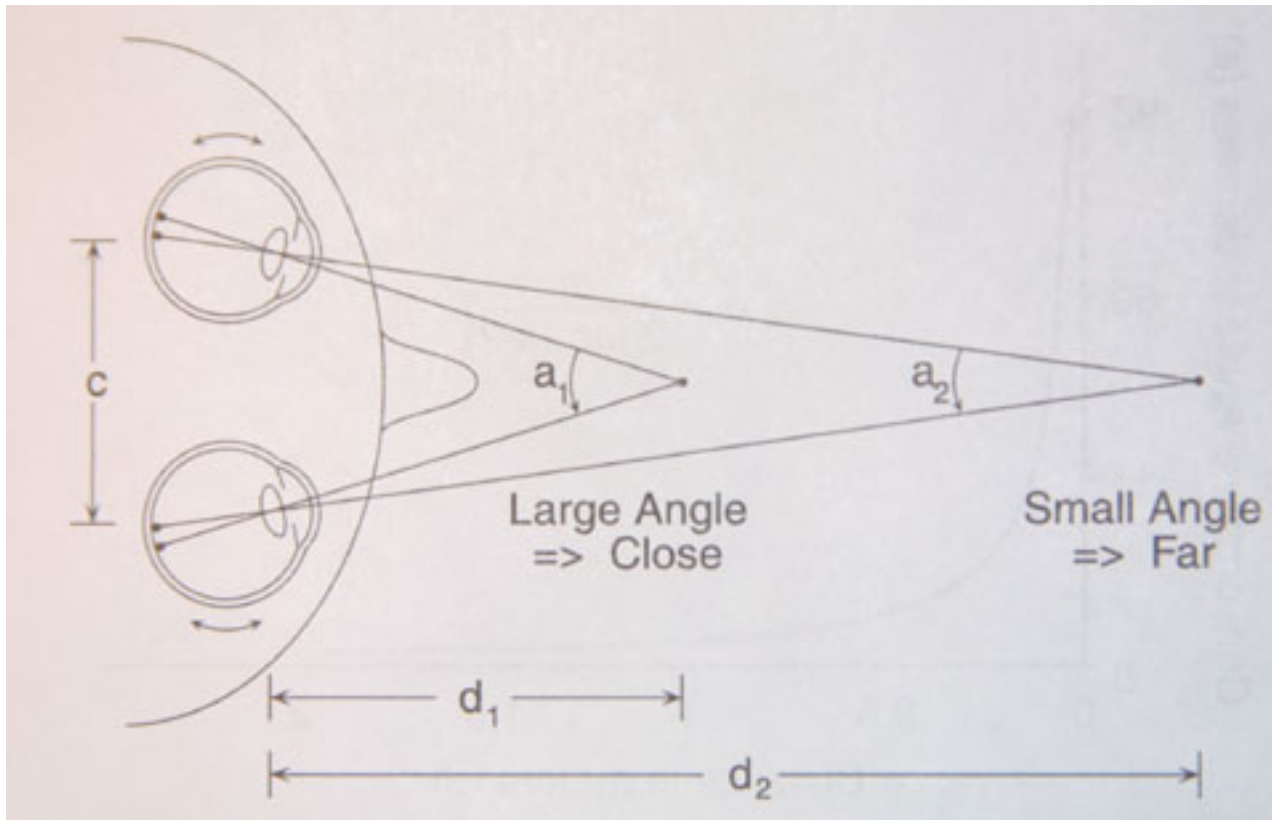


(b) For a normal eye, the near point is at 25cm



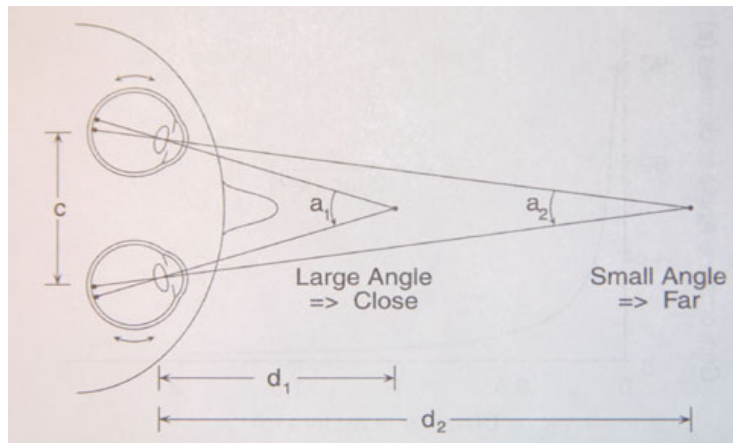
- 1) Because of its restricted range (6-8 feet), accommodation is rarely a crucial source of depth in humans.
- 2) In the chameleon, it is of paramount importance, for it controls this organism's ability to feed itself. A chameleon catches its prey by slicking its sticky tongue out just the right distance to catch an insect.
- 3) When chameleons were outfitted with prisms and spectacles that manipulated the accommodation and convergence of their eyes, the distance they flicked their tongues was changed.

Convergence



Convergence

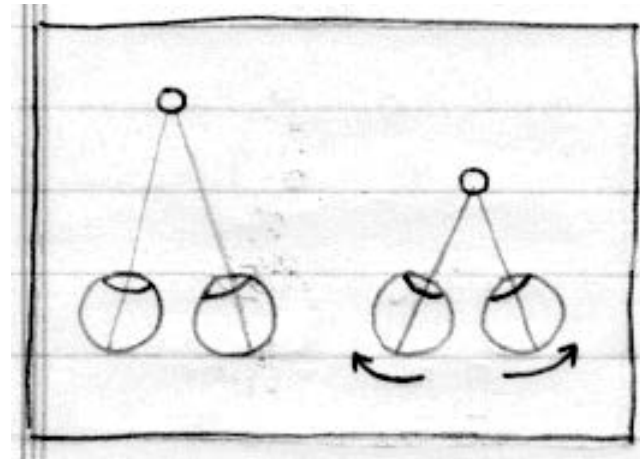
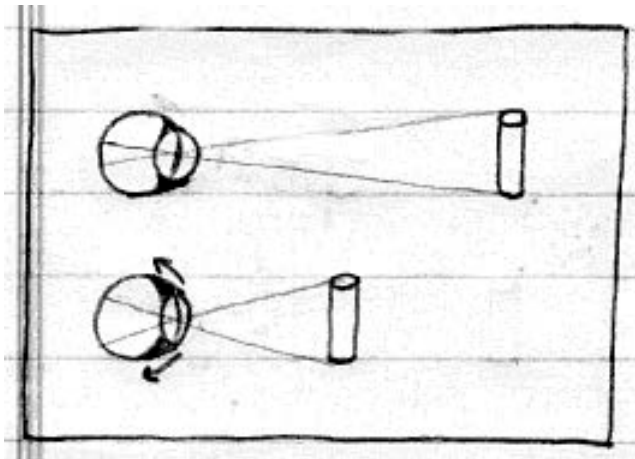
- The eyes fixate a given point in external space when both of them, are aimed directly at the point so that light coming from it falls on the centers of both foveae simultaneously.
- The crucial fact about the convergence that provides information about fixation depth is that the angle formed by the two lines of sight varies systematically with distance between the observer and the fixated point.



$$d = \frac{c}{2 \tan(a/2)}$$

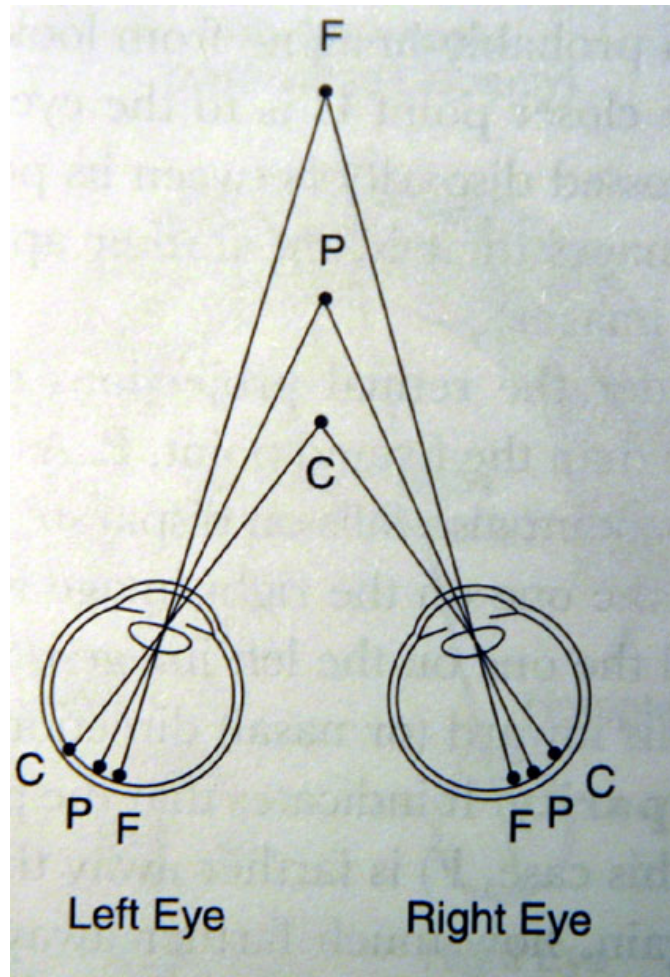
Accommodation and Convergence

- Accommodation and convergence normally change in lock steps. For human, they are important sources of depth information at close distance.



Human performance: upto a few meters

Stereoscopic : binocular disparity



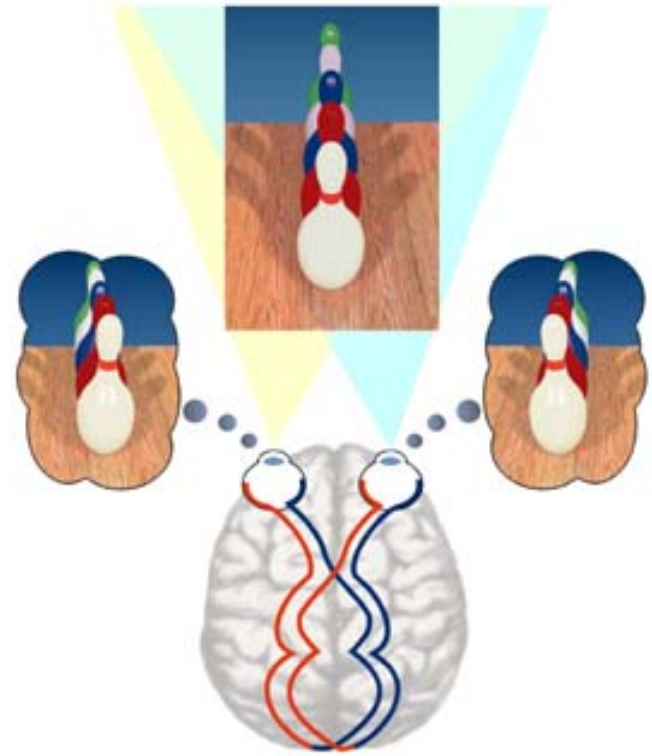
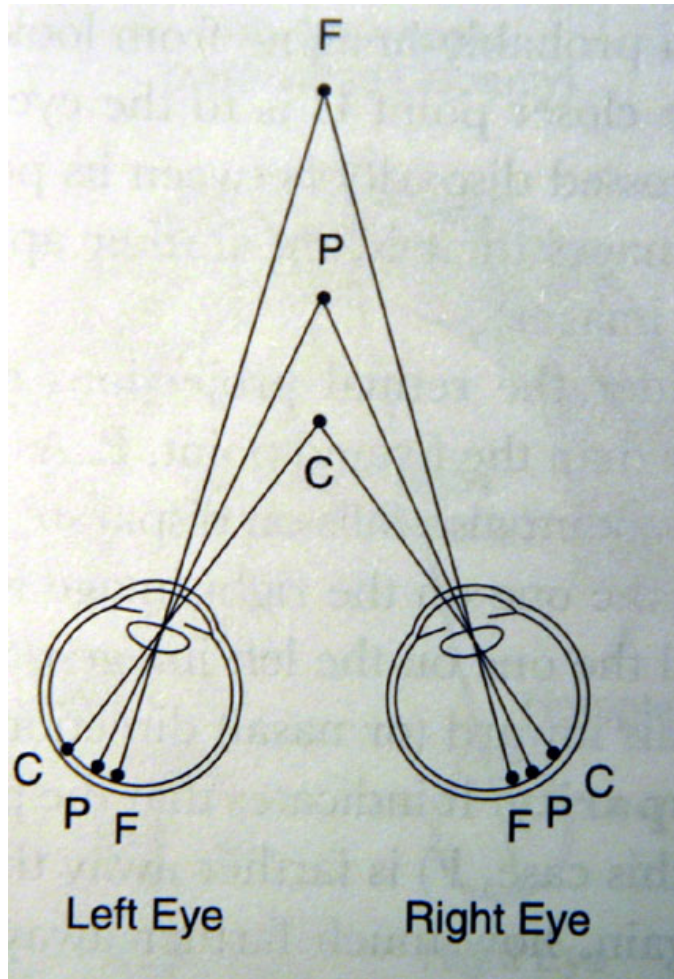
P: converging point

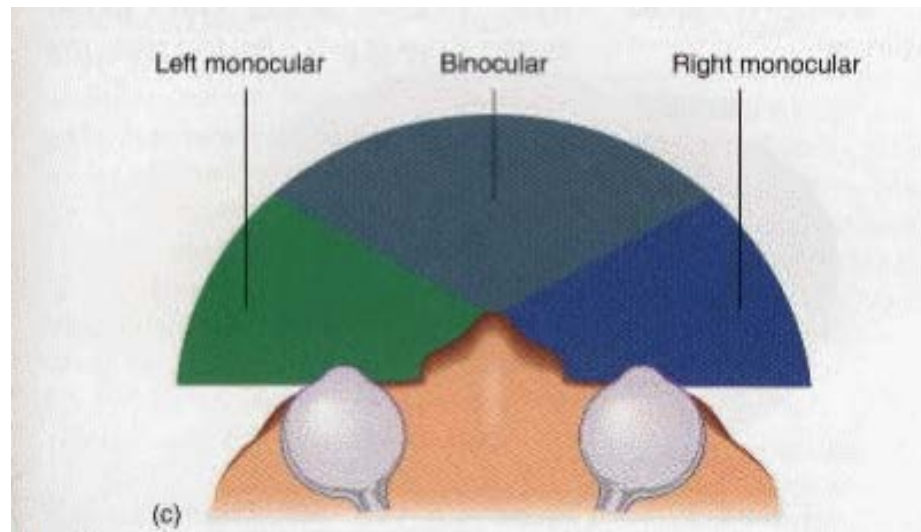
*C: object nearer
projects to the
outside of the P,
disparity = +*

*F: object farther
projects to the
inside of the P,
disparity = -*

Sign and magnitude of disparity

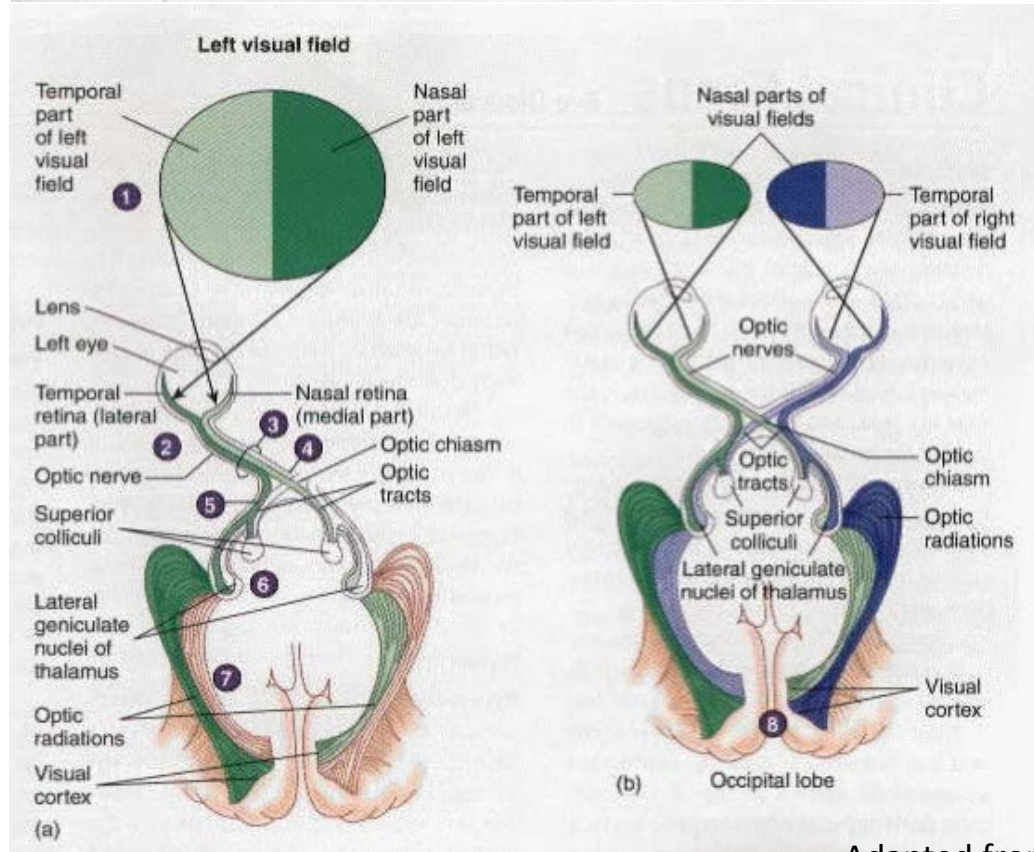
Stereo Vision





Our visual angle is 104d,
and it is facing forward.

What happened to rabbit's
vision?

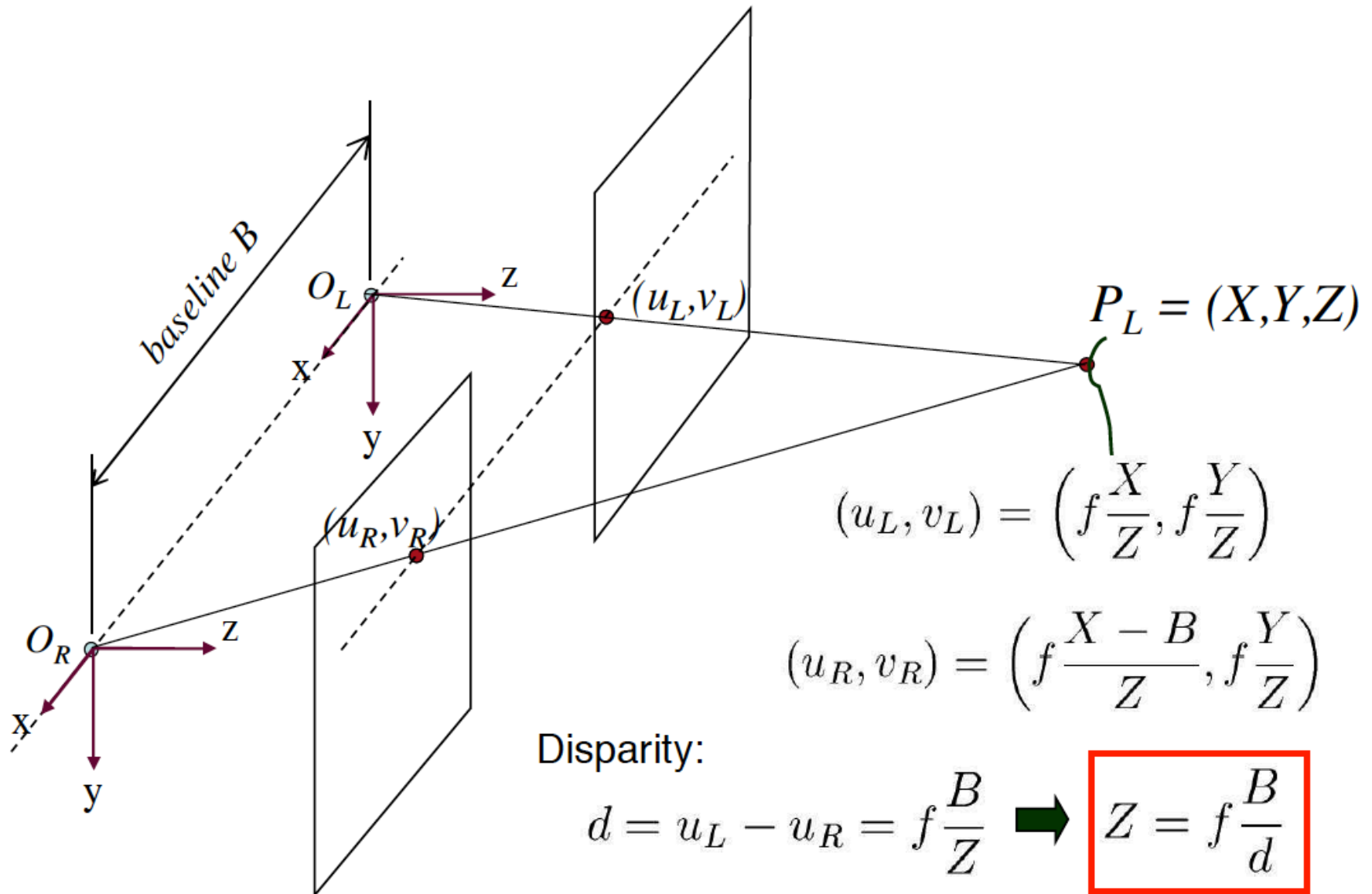


Adapted from slides from Kostas Daniilidis

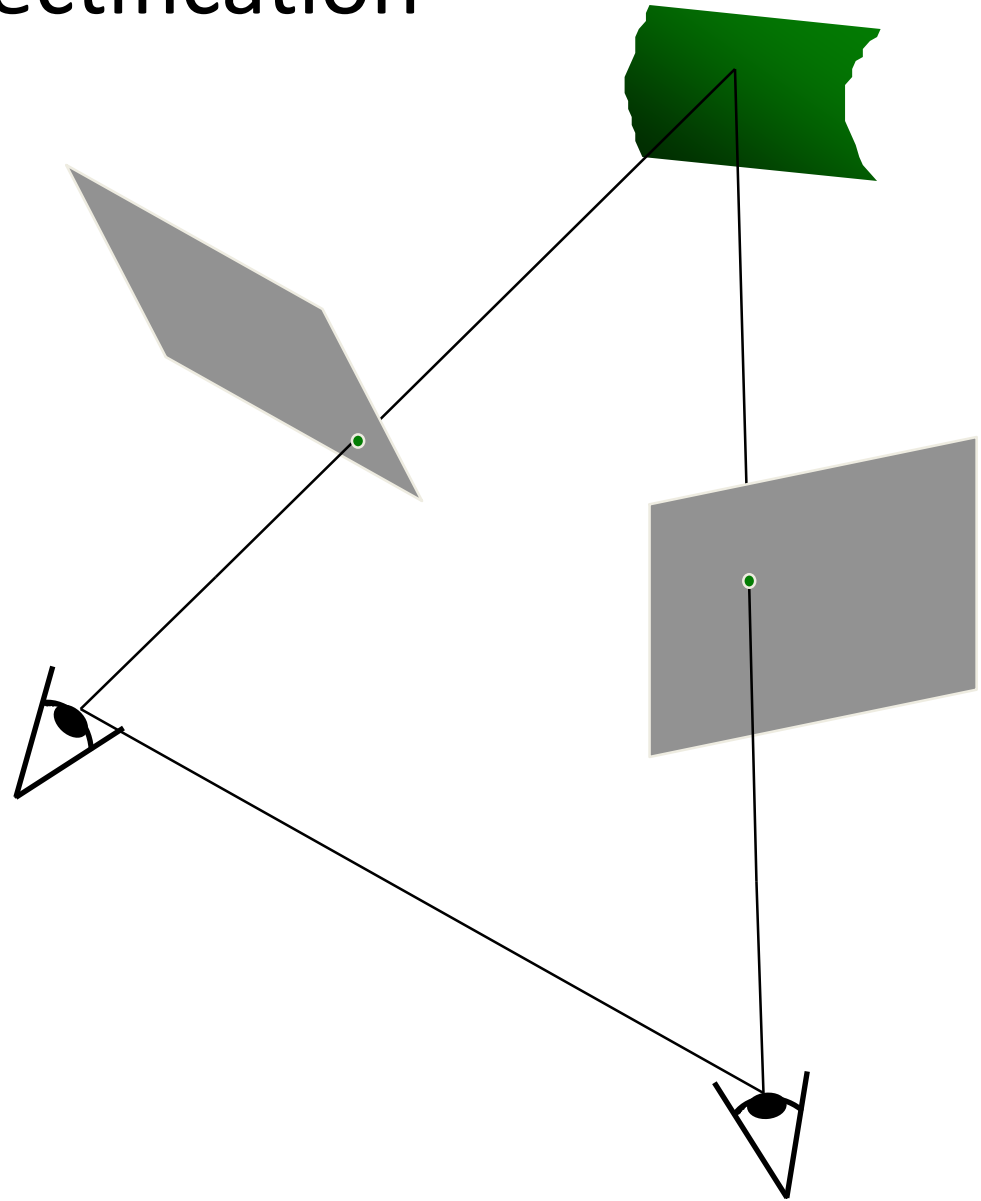
Computational Stereo Vision Outline:

- Basic geometrical setup of the stereo vision
- Computational model of stereo vision,
Correspondence problem.

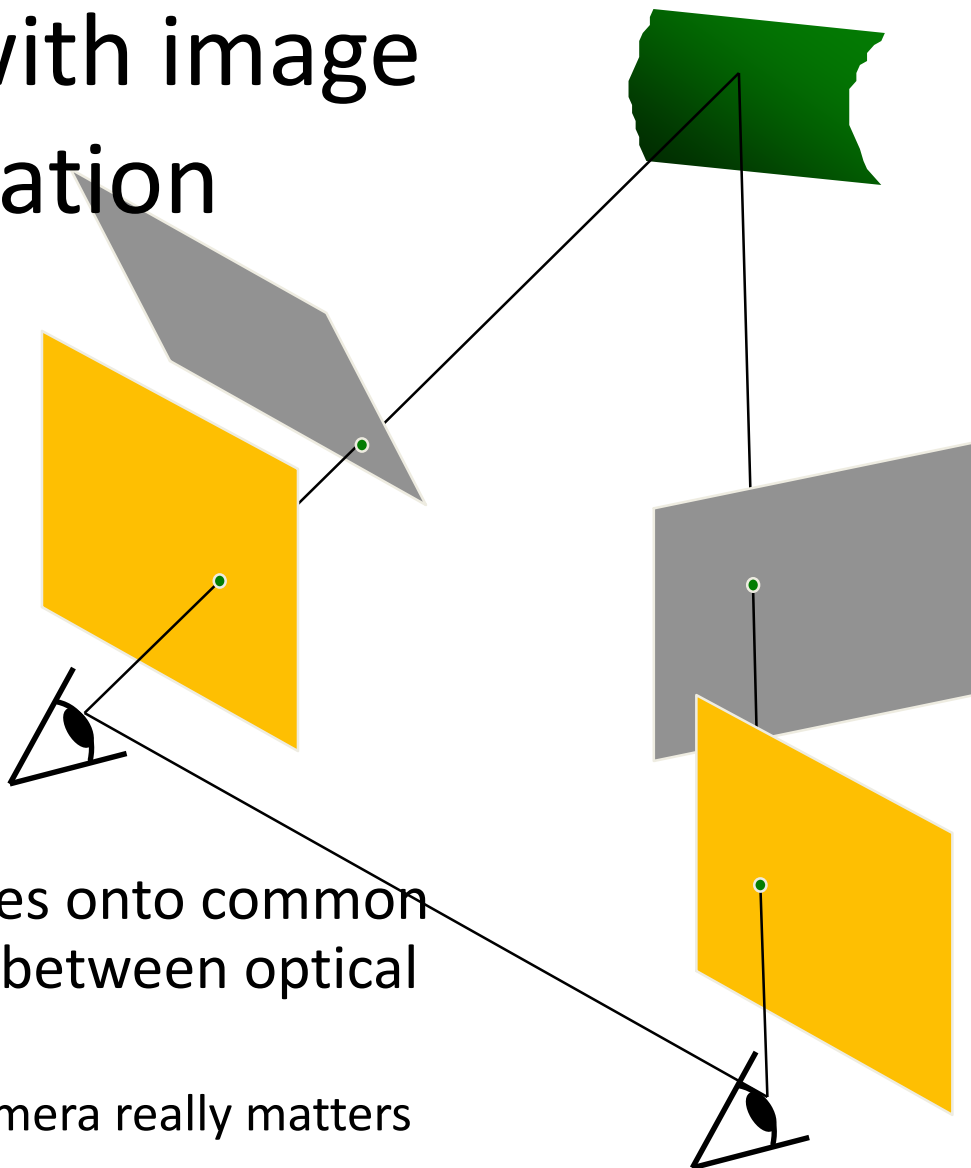
Basic Stereo Derivations



Stereo Image Rectification



We can always achieve this geometry with image rectification



- Image Reprojection
 - reproject image planes onto common plane parallel to line between optical centers
- Notice, only focal point of camera really matters

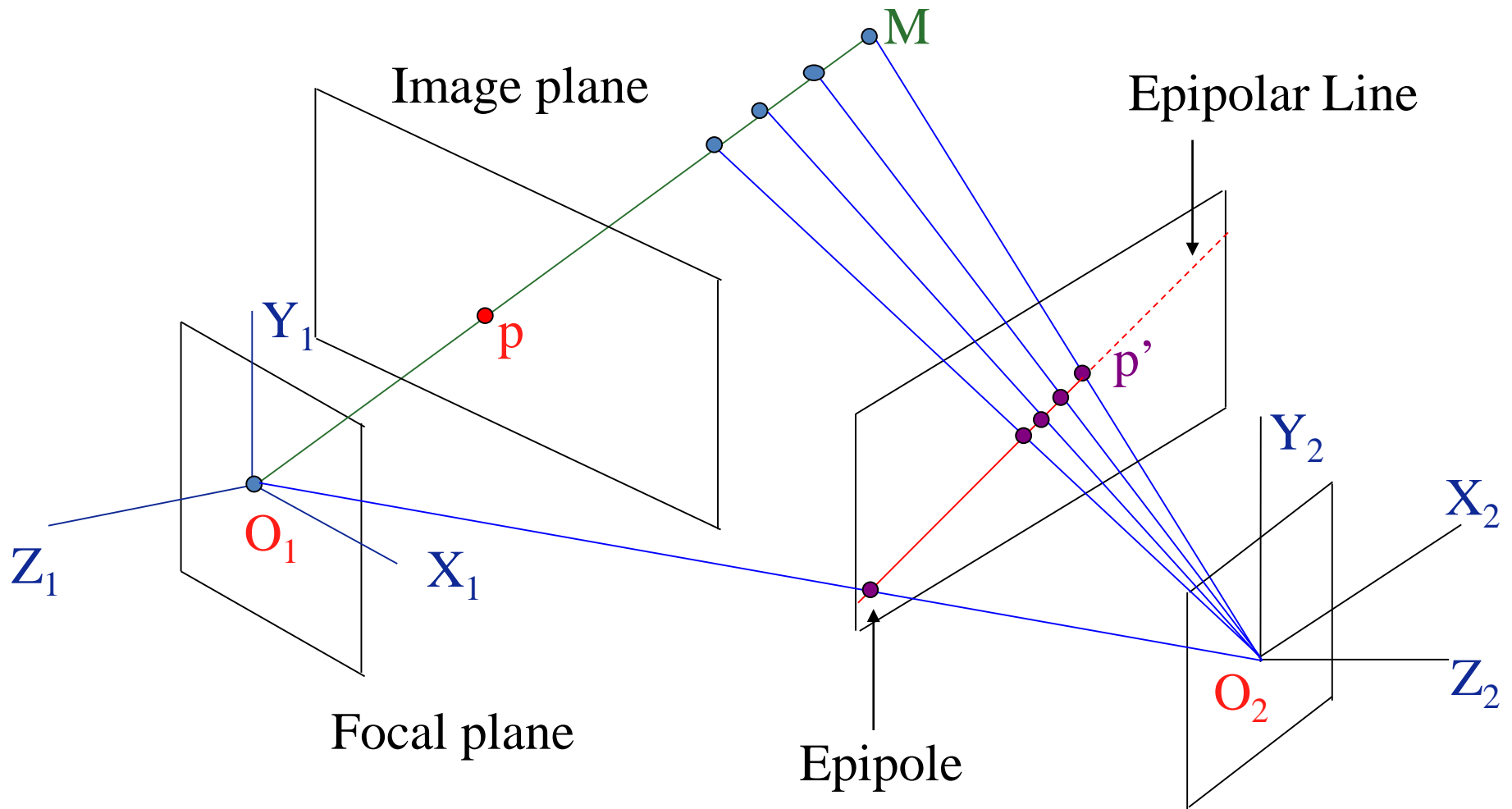
(Seitz)

Adapted from slides from Kostas Daniilidis

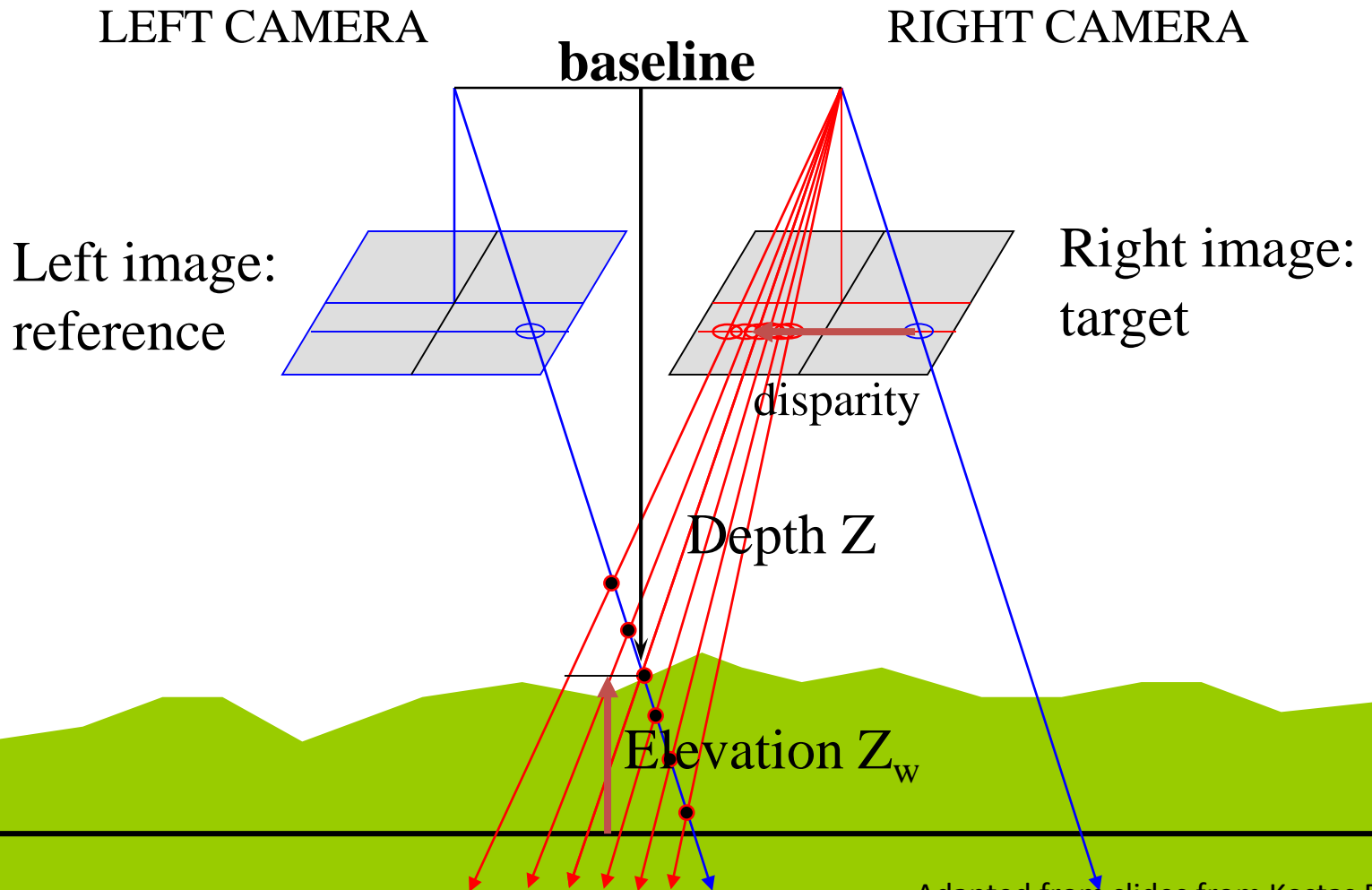
Stereo Rectification



Stereo Constraints

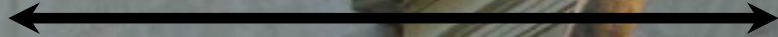
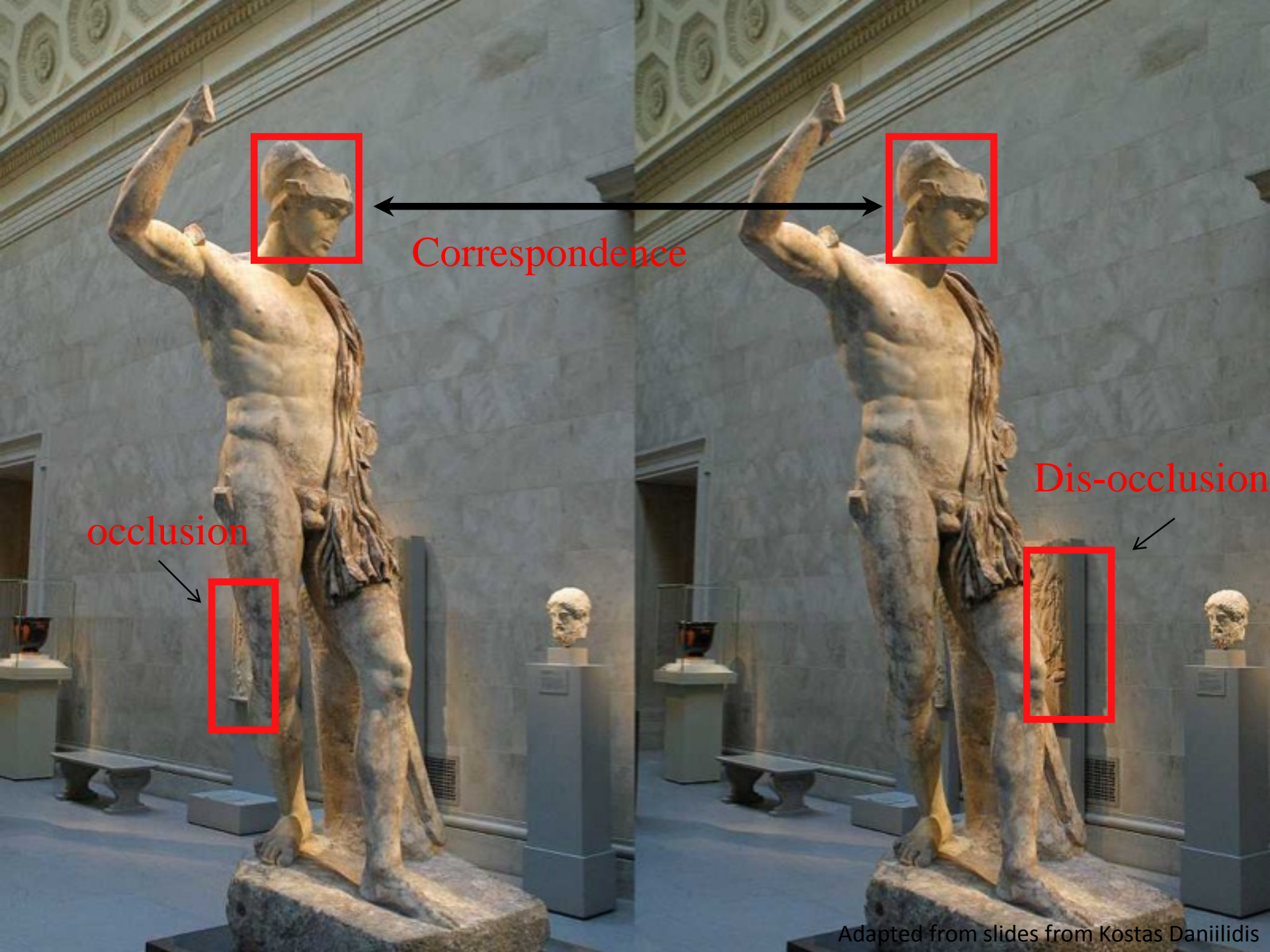


A Simple Stereo System









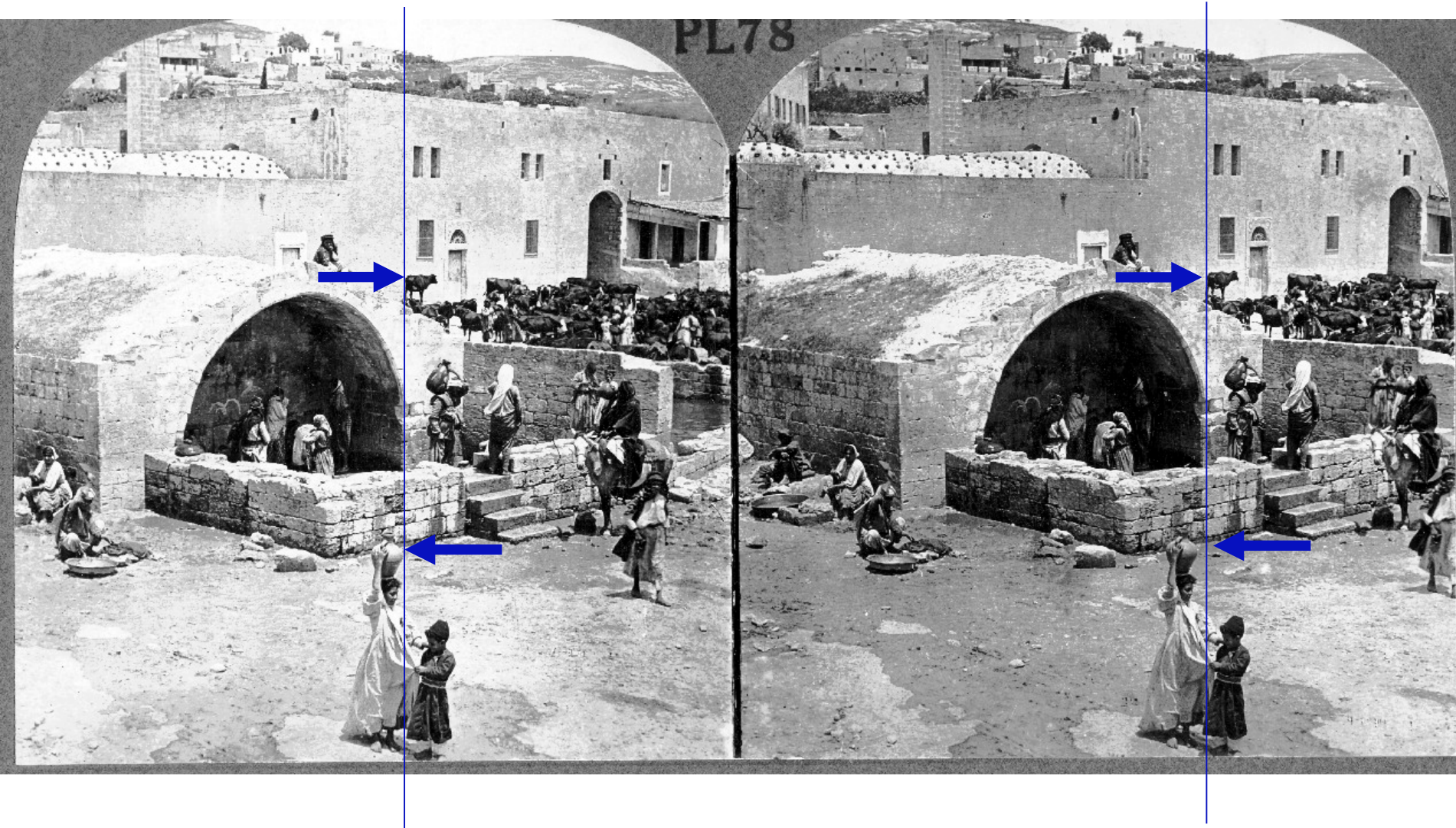
Correspondence

occlusion



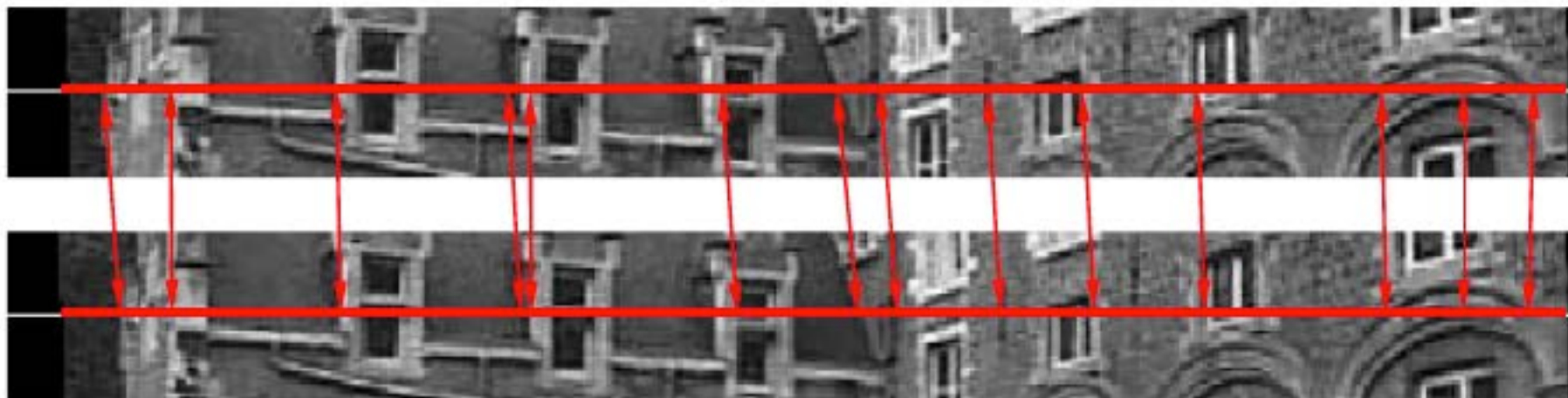
Dis-occlusion



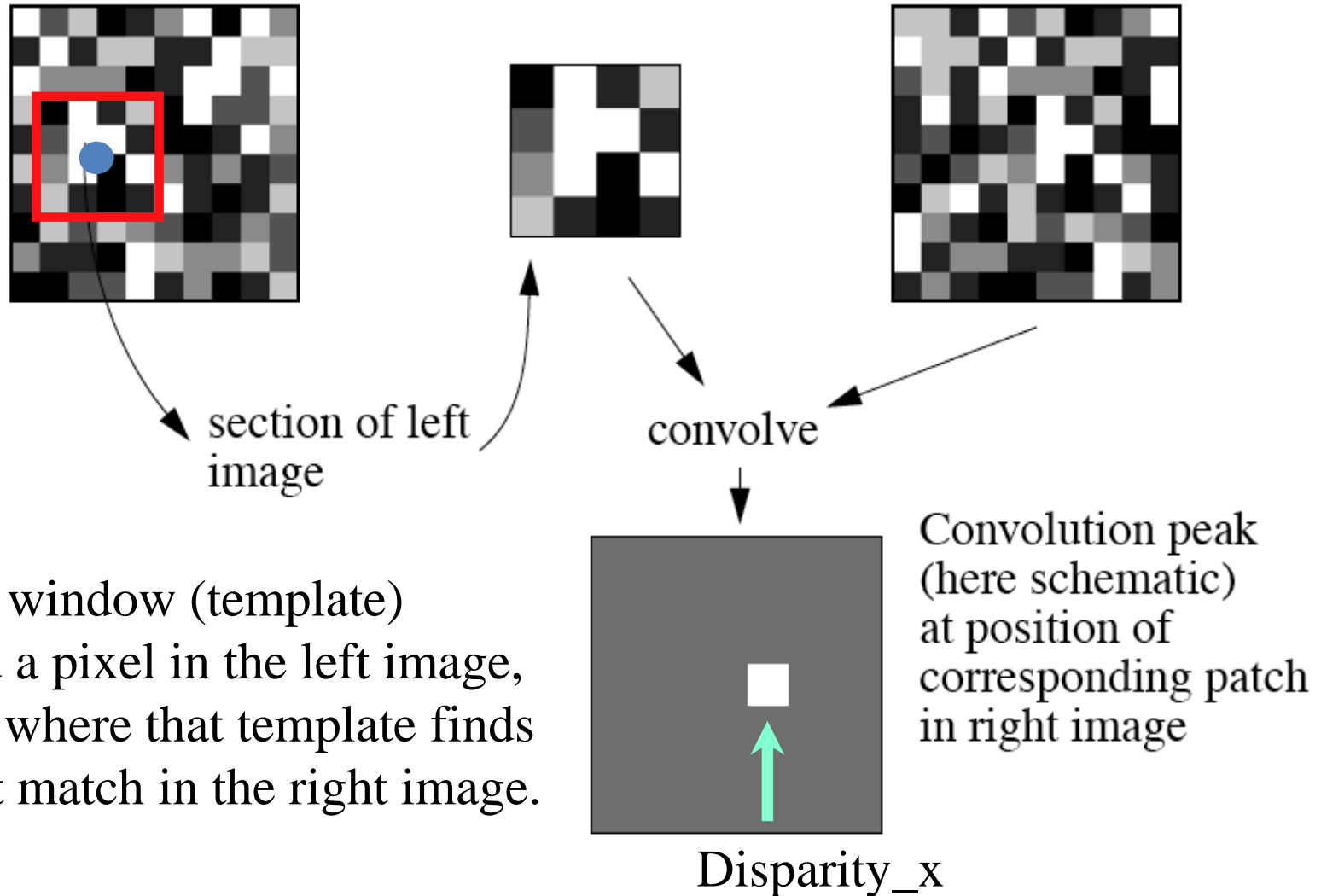


Notice the disparity difference

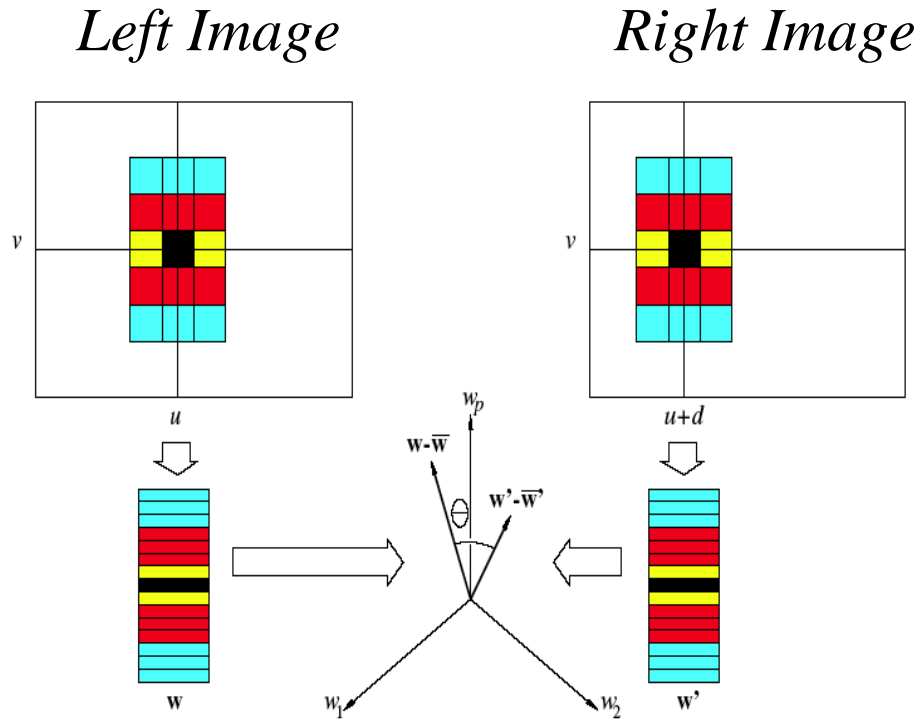
Correspondence



Computing Correspondence



Correspondence using convolution as image patch similarity function



Convolution distance:

$$d(f, g) = \sum_{i,j} f(i, j)g(i, j)$$

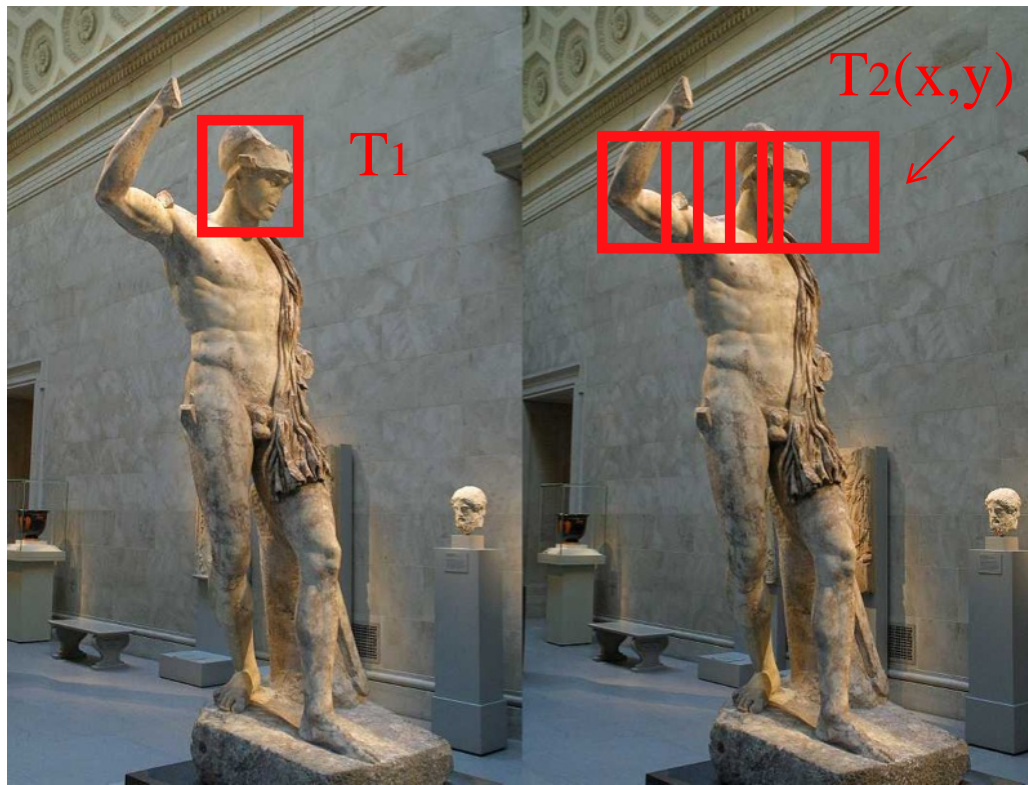
Choice of similarity function for image patches



Sum of squared differences

$$SSD(f, g) = \sum_{i,j} (f(i, j) - g(i, j))^2$$

We want similarity function to be resistant to image noise, illumination changes.



To find the corresponding image patch of T_1 , in the second image, we need to do a search:

For each location (x,y) in the second image J ,

$$\text{Err}(x,y) = \text{SSD}(\mathbf{T}_1, \mathbf{T}_2(x,y));$$

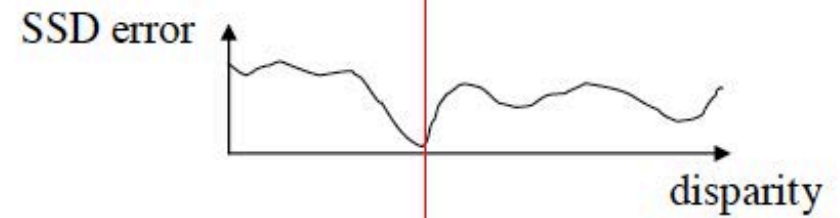
End

Disparity of $\mathbf{T}_1 = \arg.\min. \text{Err}(x,y)$

Correspondence Using Correlation

Left

Right

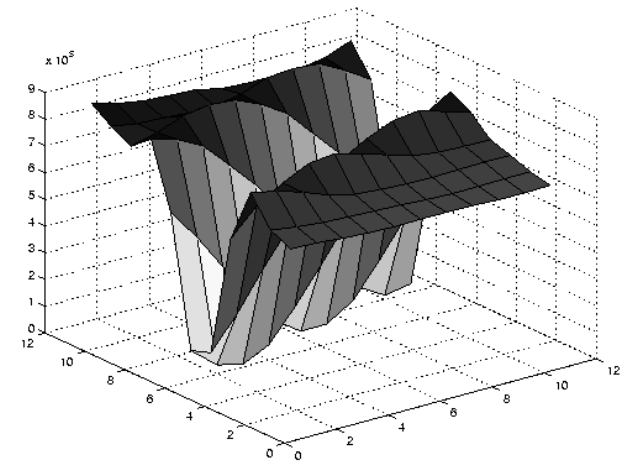
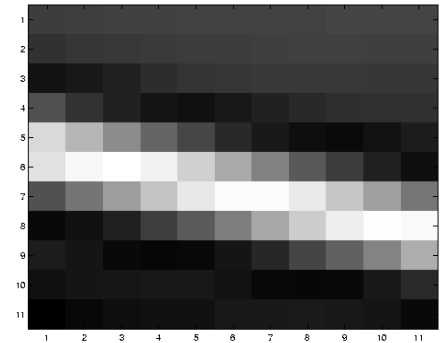


Edge

Sum of squared differences



$\text{Err}(x,y)$

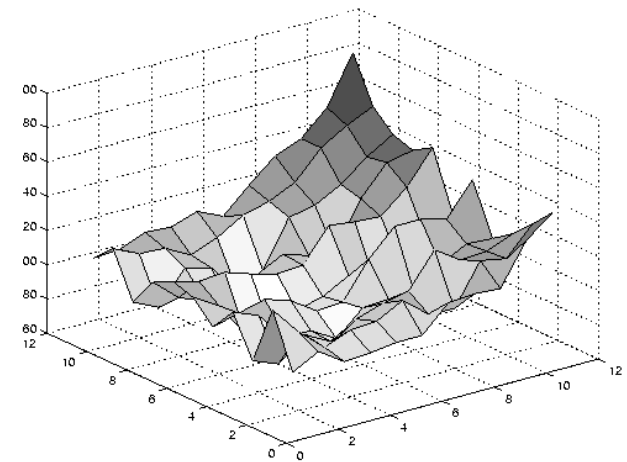
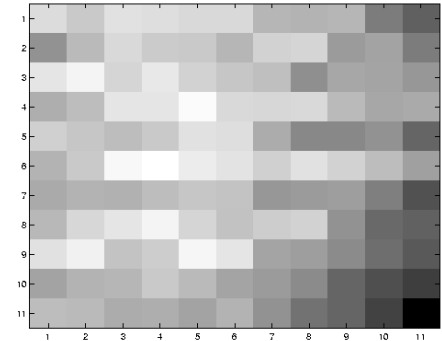


Low texture region

Sum of squared differences



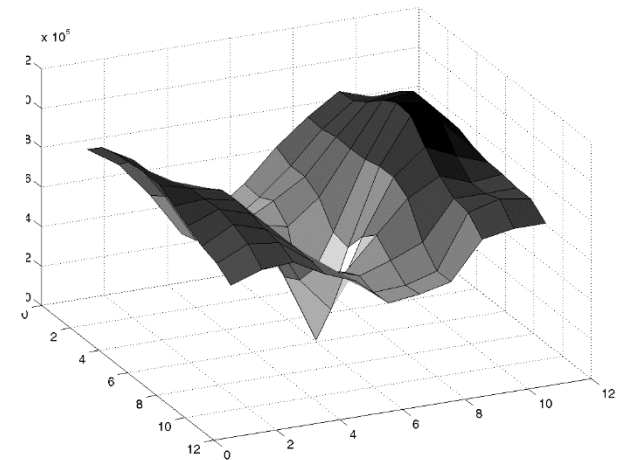
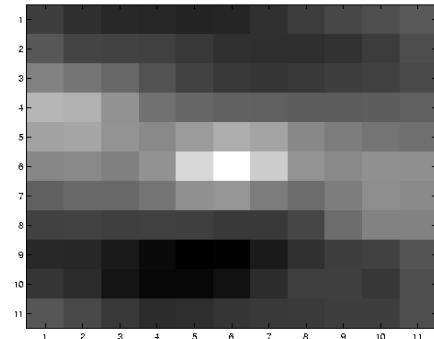
$\text{Err}(x,y)$



High textured region *Sum of squared differences*



Err(x,y)



| MATCH METRIC | DEFINITION |
|------------------------------------|---|
| Normalized Cross-Correlation (NCC) | $\frac{\sum_{u,v} (I_1(u,v) - \bar{I}_1) \cdot (I_2(u+d,v) - \bar{I}_2)}{\sqrt{\sum_{u,v} (I_1(u,v) - \bar{I}_1)^2 \cdot \sum_{u,v} (I_2(u+d,v) - \bar{I}_2)^2}}$ |
| Sum of Squared Differences (SSD) | $\sum_{u,v} (I_1(u,v) - I_2(u+d,v))^2$ |
| Normalized SSD | $\sum_{u,v} \left(\frac{(I_1(u,v) - \bar{I}_1)}{\sqrt{\sum_{u,v} (I_1(u,v) - \bar{I}_1)^2}} - \frac{(I_2(u+d,v) - \bar{I}_2)}{\sqrt{\sum_{u,v} (I_2(u+d,v) - \bar{I}_2)^2}} \right)^2$ |
| Sum of Absolute Differences (SAD) | $\sum_{u,v} I_1(u,v) - I_2(u+d,v) $ |
| Zero Mean SAD | $\sum_{u,v} (I_1(u,v) - \bar{I}_1) - (I_2(u+d,v) - \bar{I}_2) $ |
| Rank | $I'_k(u,v) = \sum_{m,n} I_k(m,n) < I_k(u,v)$ $\sum_{u,v} (I'_1(u,v) - I'_2(u+d,v))$ |
| Census | $I'_k(u,v) = \text{BITSTRING}_{m,n}(I_k(m,n) < I_k(u,v))$ $\sum_{u,v} \text{HAMMING}(I'_1(u,v), I'_2(u+d,v))$ |

These two
are actually
the same

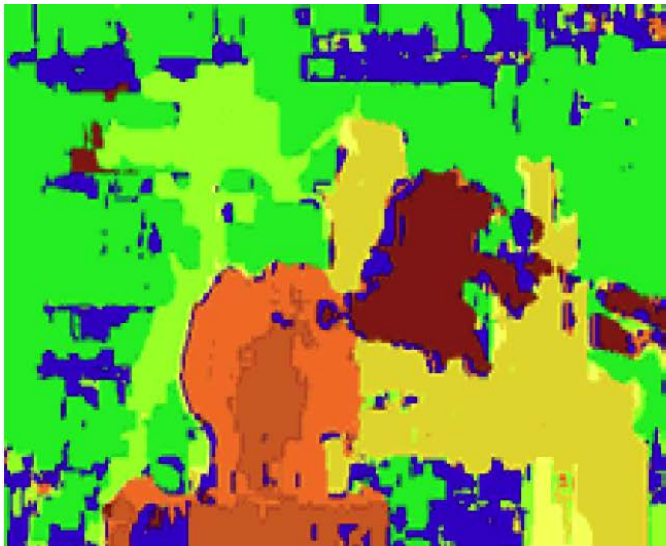
Disparity computation using SSD



Scene



Ground truth



Alternative Dissimilarity Measures

- Rank and Census transforms [Zabih ECCV94]
- Rank transform:
 - Define window containing R pixels around each pixel
 - Count the number of pixels with lower intensities than center pixel in the window
 - Replace intensity with rank ($0..R-1$)
 - Compute SAD on rank-transformed images
- Census transform:
 - Use bit string, defined by neighbors, instead of scalar rank
- Robust against illumination changes

Census measure

$$\begin{array}{ccc}
 127 & 127 & 129 \\
 126 & 128 & 129 \\
 127 & 131 & A
 \end{array}
 \longrightarrow
 \begin{array}{ccc}
 1 & 1 & 0 \\
 1 & 0 & \\
 1 & 0 & a
 \end{array}
 \longrightarrow
 \{1, 1, 0, 1, 0, 1, 0, a\}$$

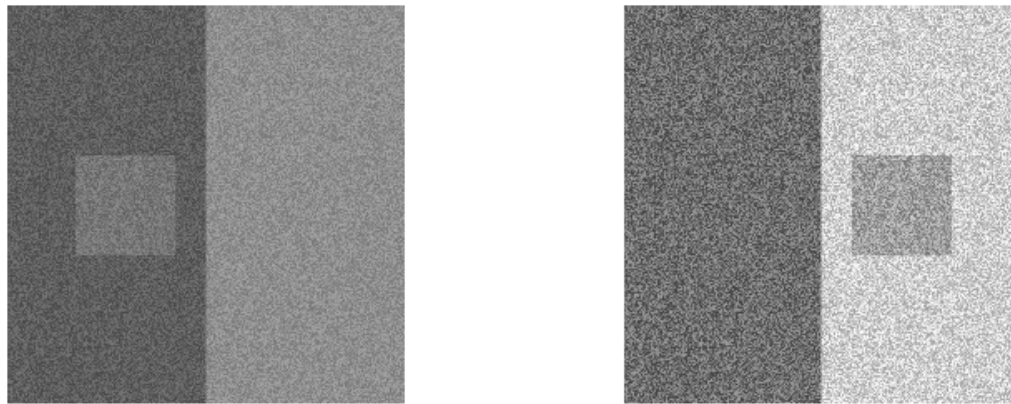


Fig.2. Right and left random-dot stereograms



Fig.3. Disparities from normalized correlation, rank and census transforms

Adapted from slides from Kostas Daniilidis

Correspondence



Vision-Based Autonomous Navigation in Complex Environments with a Quadrotor

Shaojie Shen, Nathan Michael, and Vijay Kumar



Reading

- “Computer Vision: Algorithms and Applications”, Richard Szeliski, Chapter 8.4, Chapter 11
 - Download at: <http://szeliski.org/Book/>

Logistics

- Lab session this week - fly a robot!
 - Come to your dedicated lab session only
 - You have two weeks to complete Project 1, phase 3
- Tips
 - Maximum 2 groups fly simultaneously in the field
 - Use your hand to hold the robot while testing
 - Write controller in C++ following the code structure we provide to you
 - You do not have to write the trajectory generator in C++. You may generate the trajectory in MATLAB and move the coefficients onboard.
 - No need to worry about system setup. We have done it for you.
- Expectations:
 - This week: Get familiar with the setup; complete controller; hovering;
 - Next week: Straight line & multi-waypoint trajectory tracking

Logistics

- Proposed change of lecture time for next week
 - 24 Oct (Saturday), 10am