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SHAPE INTERPOLATION USING FOURIER DESCRIPTORS WITH APPLICATION TO ANIMATION GRAPHICS

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Abstract. We propose a way to generate a sequence of closed curves leading from an initial curve to a final one, making use of Fourier descriptors. In order to ensure a good closure of intermediary shapes, we have to pay attention to the definition of these descriptors. The change between successive shapes can be fully controlled through the frequency content of these shapes; many parameters are at the disposal of the user providing a great flexibility in the use of this method. We illustrate the variety of possibilities with three examples of interpolation algorithms.

Zusammenfassung. Wir stellen eine neue Möglichkeit vor, eine Folge von geschlossenen Kurven zwischen einer Start-und einer Zielkurve zu interpolieren. Dabei benützen wir Fourier-Deskriptoren. Um zu gewährleisten, daß die interpolierten Kurven geschlossen bleiben, müssen wir diese Deskriptoren sehr sorgfältig definieren. Die Verformung aufeinanderfolgender Kurven wird durch die Veränderung des Frequenzinhalts der Formen kontrolliert. Zur Anpassung der Methode an ein konkretes Problem bleiben noch zahlreiche Freiheitsgrade übrig. Diese Flexibilität wird durch 3 verschiedene Beispiele für Interpolationsalgorithmen erlaütert.

Résumé. Nous proposons une nouvelle méthode de génération d'une séquence de figures interpolées entre figures extrémales. Cette méthode s'appuie sur les descripteurs de Fourier. Afin d'assurer la fermeture de toutes les courbes intermédiaires, nous devons choisir une définition judicieuse des descripteurs de Fourier. Les déformations successives sont controlées par les modifications du contenu fréquentiel de la forme. Nous disposons de nombreux degrés de liberté afin d'adapter l'évolution à un problème précis. Trois exemples d'algorithmes d'interpolation sont présentés à titre d'illustrations.

Keywords. Animation graphics, shape interpolation, Fourier descriptors.

1. Introduction

The interpolation of a family of curves knowing two extremal positions is of great importance in the field of animation graphics. Classical methods rely on vectorial or angular deformations of the initial shape [1], more sophisticated solutions take advantage of special effect techniques, like rubber sheet deformations [2–3] or dislocation in elementary parts, or reduction to some prespecified shapes. We propose here to make use of the Fourier descriptors to generate a new family of methods to solve this problem.

Let us describe a planar closed curve in the complex plane as a complex function $z(s) = x(s) + iy(s)$ of a real variable s. s, its arc length, is supposed to be normalized so that its sum over the whole curve is equal to 2π, thus $z(s+2\pi) = z(s)$. $\phi(s)$ denotes the cumulated angular variation between the origin of the curve ($s = 0$) and the point at position s.

Two methods have been proposed to describe the curve with a finite set of parameters called Fourier descriptors. They differ with respect to the choice of developed functions:

– Zahn and Roskies [4] have proposed to use the real periodical continuous function $\phi(s) - s$;

– Granlund [5] made directly use of the complex function $z(s)$. Many applications of both these techniques are available in the literature, used in

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pattern recognition, shape approximation, or data compression.

For our purpose of animation graphics, the representation using the angular function $\phi(s)$ presents a major drawback: if a modification is introduced in the set of Fourier descriptors, the new shape so described is usually no longer a closed curve. Some conditions to ensure a correct closure have been proposed [4], but they often appear too restrictive [6]. For this reason we followed [5] in the choice of $z(s)$ to derive Fourier descriptors; this ensures that a closed curve will correspond to any set of descriptors.

We also used a discrete polygonal representation of the shape, the only sampled points being the vertices. This choice is convenient for animation graphics, it leads without major problem to a non uniform sampling. The extension of the method to other kinds of representation (Freeman code, binary picture, etc) is theoretically straightforward.

Given a set of N vertices $\{z(i): i=1,\dots N\}$ the Fourier descriptors are defined as the classical coefficients of discrete Fourier transform:

$$c(k)=\frac{1}{N}\sum_{i=1}^{N} z(i)\exp\left(-2\pi \mathrm{j}\frac{ik}{N}\right) \tag{1}$$

for the N values of $k: k=-\frac{1}{2}N+1,\dots 0,\dots,\frac{1}{2}N$. (For the sake of computational speed, N will often be taken as a power of 2.) The inverse relationship exists between $c(k)$ and $z(i)$:

$$z(i)=\sum_{k=-N/2+1}^{N/2} c(k)\exp\left(2\pi \mathrm{j}\frac{ki}{N}\right). \tag{2}$$

2. Some useful properties of Fourier descriptors

In eq. (1), we denote frequency k, and frequency content $c(k)$. In agreement with Shannon's theorem, the highest frequency is obtained for $k=\frac{1}{2}N$. Due to the discrete representation of $z(i)$, for k greater than $\frac{1}{2}N$, we have $c(k)=c(k-pN)$, for any integer value p. Thus we will now restrict k to the set $\{-\frac{1}{2}N+1, \frac{1}{2}N\}$.

For $k=0$, we obtain:

$$c(0)=\frac{1}{N}\sum z(i)$$

which represents the position of the center of gravity of the shape in the complex plane. This term is often of no interest for our applications and will be kept unmodified.

The first frequency component $c(k=1)$ is related to the size of the shape. This can be shown with a circle of radius R centered at $z=0$. The circle is approximated by an N-sided polygon; i.e.:

$$z(i)=R\exp\left(2\pi \mathrm{j}\frac{i}{N}\right)$$

and

$$c(1)=\frac{1}{N}R\sum\exp\left(2\pi \mathrm{j}\frac{i}{N}\right)\exp\left(-2\pi \mathrm{j}\frac{i}{N}\right)=R$$

Due to the orthogonality of complex exponential functions, all the higher frequency harmonics of a circle ($k\neq 1$) will be equal to zero. This is in agreement with our intuition of a circle being the simplest shape (only described by its fundamental frequency).

Now let us describe a shape by $c(1)$ and a second frequency component $c(k)=a\exp(\mathrm{j}\theta)$, all other being set to zero. We obtain the following rules:

– For positive values of k, the effect of $c(k)$ is to push on the circle at $k-1$ points. These points are periodically placed on the circle at positions

$$i=\frac{N}{2(k-1)}+p\frac{N}{k-1}+\frac{\theta}{2\pi}N,$$

$$p=1,2,\dots,k-1.$$

– For negative values of k, the effect of $c(k)$ is to pull on the circle at $1-k$ points, at positions:

$$i=\frac{N}{2(1-k)}+p\frac{N}{1-k}+\frac{\theta}{2\pi}N,$$

$$p=1,2,\dots,1-k.$$

The effect of a phase factor θ is clearly a rotation of the perturbation due to $c(k)$.

If we combine several frequency components $c(k)$, we obtain a summation of the individual effects. In Fig. 1, we present some shapes with a constant $c(1)$ and different values of c for other frequencies.

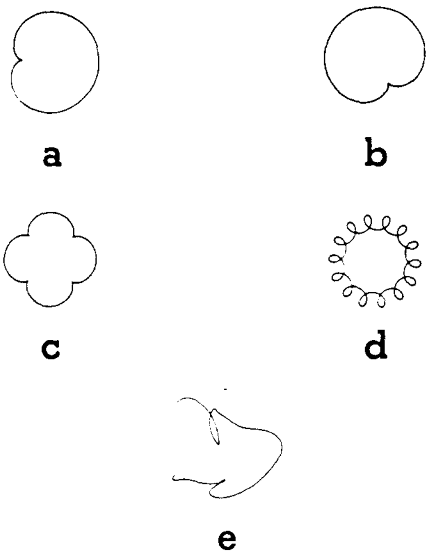

Fig. 1. Influence of frequency content on the shape of a curve. All these curves have the same value $c(1)$ but different values $c(k) = a(k)\exp(j\theta(k))$. (a) $a(2) = 1$, $\theta(2) = 0$; (b) $a(2) = 1$, $\theta(2) = \frac{1}{3}\pi$; (c) $a(5) = 1$, $\theta(5) = 0$; (d) $a(-13) = 1$, $\theta(-13) = 0$; (e) only for $k = 2, 3, -4, 5$ and 11 $c(k)$ is different from zero.

These simple properties allow an easy manipulation of the shape and a short experimental practice leads to a fairly good intuitive understanding of the structure of the resulting curve. We will now take advantage of this frequency modifications to interpolate two shapes in order to obtain animation graphics.

3. Fourier descriptors used for animation graphics

The goal of interpolation in animation graphics is to create a series of drawings leading from an initial shape z_0 to a final one z_Q. An intermediary curve is represented by z_q, $q = 1, 2, \ldots Q-1$. For the sake of simplicity, we define both extremal shapes z_0 and z_Q with the same number of samples N (in case of different values N_0 and N_Q, the greater value must be taken, and the other curve resampled or additional pseudo-vertices introduced). The distance between two curves z and z' is taken to be the Euclidean distance and will be measured either on the curve representation or on the Fourier descriptors according to Parseval's theorem:

$$\left.\begin{array}{l} d(z, z') = \sum\limits_{i=1}^{N} |z(i) - z'(i)|^2 \\[2ex] d(c, c') = \sum\limits_{k=-N/2+1}^{N/2} |c(k) - c'(k)|^2 \end{array}\right\} \Rightarrow$$

$$d(z, z') = d(c, c').$$

We denote the distance between successive pictures by $d(q) = d(z_q, z_{q+1})$. Any possible law $d(q)$ can be applied to the series of pictures, allowing accelerated or deccelerated transformations from z_0 to z_Q. In the discussion to follow, we assume a uniform movement without any loss of generality. We have also a wide choice of methods to change z_q into z_{q+1} by modification of its Fourier descriptors c_q. We present here three possible ways.

3.1. Linear interpolation

Starting from z_0 and z_Q, we define:

$$D = d(z_0, z_Q) = d(c_0, c_Q),$$

and the distance increment δ for the series: $\delta = D/Q$.

The rule for linear interpolation on Fourier Descriptors is then:

$$c_q(k) = \frac{q\delta}{D} c_Q(k) + \left(1 - \frac{q\delta}{D}\right) c_0(k). \qquad (3)$$

Using definition (2), and the linearity of Fourier transformation, we see that (3) can be reduced to:

$$z_q(i) = \frac{q\delta}{D} z_Q(i) + \left(1 - \frac{q\delta}{D}\right) z_0(i). \qquad (4)$$

This last relation is well known in animation graphics: it expresses a linear interpolation between shapes. But direct linear interpolation (4) compared to its Fourier counter-part (3) presents the advantage of a lighter computation. Thus, if linear interpolation is indeed possible with Fourier descriptors, it is not an original and economical way to make it.

3.2. High frequency substitution

High frequencies are less sensitive features in the shape description. Therefore we propose to substitute the frequency components of c_Q to those of c_0, starting with highest frequencies. The generating equation is then:

$$c_q(k) = c_0(k), \qquad \forall |k| < K(q),$$

$$c_q(k) = c_Q(k), \qquad \forall |k| \geq K(q).$$

The decision frequencies $K(q)$ are such that:

$$\sum_{K(q-1)}^{K(q)} |c_{q-1}(k) - c_Q(k)|^2 = \delta.$$

Usually we cannot determine exact values for $K(q)$, which satisfy the above relation. So, we start with $q = 1$ and the highest frequency $\frac{1}{2}N$; if $|c_0(\frac{1}{2}N) - c_Q(\frac{1}{2}N)|^2$ is greater than δ, then we only change a part α of $c_0(\frac{1}{2}N)$ into $c_Q(\frac{1}{2}N)$, i.e., we make a linear combination between $c_0(\frac{1}{2}N)$ and $c_Q(\frac{1}{2}N)$ with α and $1-\alpha$ as coefficients. If it is smaller, we substitute $c_0(\frac{1}{2}N)$ with $c_Q(\frac{1}{2}N)$ and go to the next frequencies $-(\frac{1}{2}N - 1)$. The same procedure is iterated, comparing $|c_0(-\frac{1}{2}N+1) - c_Q(-\frac{1}{2}N+1)|^2$ to the corrected value $\delta - |c_0(\frac{1}{2}N) - c_Q(\frac{1}{2}N)|^2$.

At the qth iteration, it is easy to prove that the complex fractional factor α verifies the two relations:

$$|\alpha|^2 + |1-\alpha|^2 = 1,$$

$$|1-\alpha|^2 = \frac{\delta - \sum_{K(q-1)}^{K(q)+1} |c_{q-1}(k) - c_Q(k)|^2}{|c_0(k) - c_Q(k)|^2} = \lambda,$$

thus leading to the solution

$$\alpha = (1-\lambda) + \mathrm{j}\sqrt{\lambda(1-\lambda)}.$$

This procedure is then iterated to highest orders of q.

We present in Fig. 2 the result of such an interpolation using high frequency substitution, with $N = 64$ and $Q = 19$.

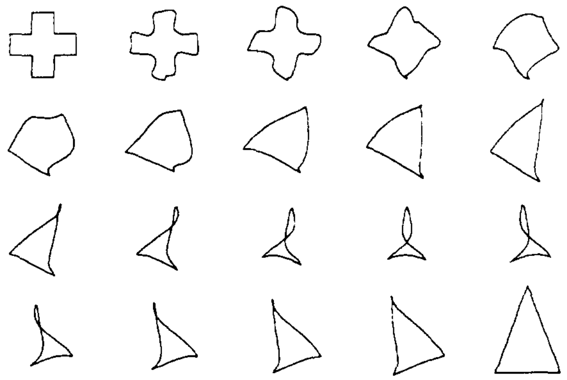

Fig. 2. A sequence of interpolated curves using high frequency substitution. The high frequencies of z_Q are progressively introduced into z_0. $N = 64$; $Q = 19$.

Experiments indeed indicate that a constant step $\delta = D/Q$ will not be perceived as a regular modification of the shape, because we are much more sensitive to high frequencies. In the result of Fig. 2 we have chosen a linear law so that $\delta(q) = \delta_0 + q\delta'$, with $\sum_{q=1}^{Q} \delta(q) = D$.

3.3. A casting method

The method here presented is to reduce the winding of the initial shape by suppression of high frequencies down to $\pm\nu$. Therefore the resulting shape looks like a "cast" replication $z'_0(s)$ of the initial shape $z_0(s)$. At the same moment we calculate a cast replication of the final shape $z'_Q(s)$. The Fourier descriptors c'_0 and c'_Q verify:

$$c'_0(k) = c_0(k), \qquad \forall |k| \leq \nu,$$

$$c'_Q(k) = c_Q(k), \qquad \forall |k| \leq \nu,$$

$$c'_0(k) = c'_Q(k) = 0, \quad \forall |k| > \nu,$$

The interpolations between the three points (z_0, z'_0), (z'_0, z'_Q) and (z'_Q, z_Q) can be made either by high frequency substitution or linear interpolation in respectively Q_1, Q_2 and Q_3 steps so that $Q_1 + Q_2 + Q_3 = Q$. Another parameter to be determined is the cut off frequency $\pm\nu$. Its influence is clearly involved in the complexity of the intermediary shapes z'_0 and z'_Q; the lower ν, the coarser the cast replications.

We present in Fig. 3 a sequence of interpolations for $Q_1 = Q_3 = 8$, $Q_2 = 3$, $N = 64$ and $\nu = 6$.

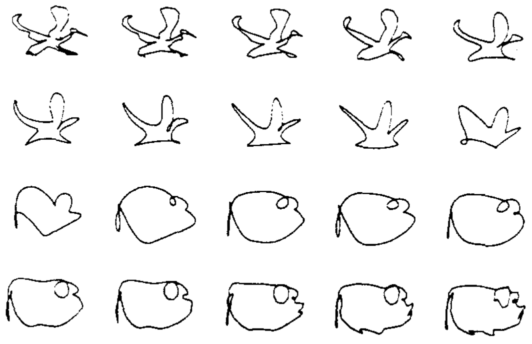

Fig. 3. A sequence of interpolated curves using cast replications. Between z_0 and z'_0, we suppress high frequencies in z_0. Between z'_Q and z_Q we introduce high frequencies in the cast replication z'_Q of z_Q. In between the two shapes z'_0 and z'_Q, the curves are interpolated by classical linear interpolation ($N = 64$; $Q_1 = 8$; $Q_2 = 3$; $Q_3 = 8$; $\nu = 6$).

The intermediary drawings are indeed very different from those obtained with high frequency substitution. We get now smooth curves when the previous ones were jarred and entangled (Fig. 4 is given as a comparison of high frequency substitution with $N = 64$ and $Q = 19$).

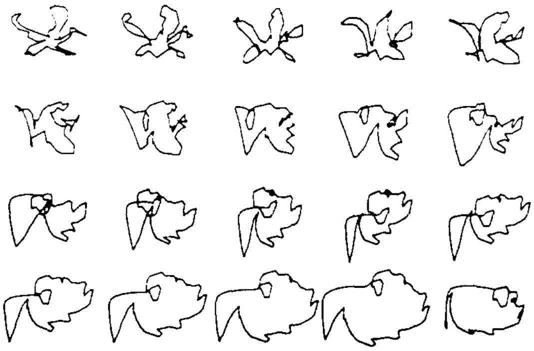

Fig. 4. For comparison with Fig. 3 the two shapes z_0 and z_Q are interpolated using high frequency substitution.

3.4. Computational complexity

Compared to classical interpolations using either segment or angle deformations, the above methods seem more complex and time consuming, but the additional load due to Fourier transformation can be drastically reduced. To achieve this, we will take advantage of the small step δ between two successive pictures in practical applications: only a small number of frequency components are modified at each step, often no more than one. Thus, making use of the linearity of Fourier transformation, we only introduce a few corrective terms to the position of each vertex of the drawing. We only need to compute the Fourier transforms of initial and final curves once and this can be done with fast algorithms. These remarks lead to a computational load very similar to existing methods.

4. Conclusion

A new family of methods has been presented to interpolate closed curves. It includes the classical linear interpolation method often used for animation drawings, but many other possibilities are free to obtain special effects. These procedures are attractive because they introduce a global modification of the shape, acting upon the frequency content, as opposed to more traditional methods inducing local transformations. Furthermore, with minimal modifications, they become compatible with interactive techniques like for instance skeleton control [7]. The computational load of these methods is definitely equivalent to the classical methods. Their range of application is mainly for animated graphics, computer aided design, interactive computer drawing and manufacturing, and movie industry.

Acknowledgement

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