Qn 30.1-1

$$
\begin{array}{rrrrrrr}
 & & & 7x^3 & -x^2 & +x & -10 \\
 & & & 8x^3 & & -6x & +3 \\
\hline
 & & & 21x^3 & -3x^2 & +3x & -30 \\
 & & -42x^4 & +6x^3 & -6x^2 & +60x & \\
56x^6 & -8x^5 & +8x^4 & -80x^3 & & & \\
\hline
56x^6 & -8x^5 & -34x^4 & -53x^3 & -9x^2 & +63x & -30
\end{array}
$$

Qn 30.1-2

$$A(x) = q(x)\,(x - x_0) + r$$

$$\Rightarrow \quad A(x) = [(x_0, y_0), (x_1, y_1) \cdots, (x_n, y_n)] \quad \text{PVR.}$$

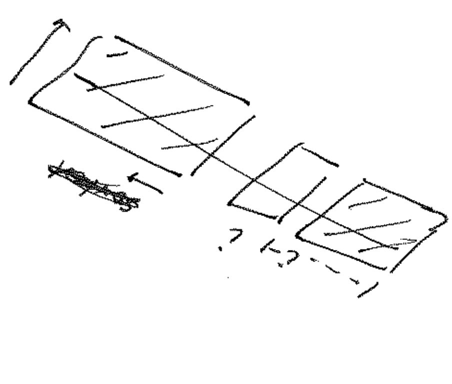
$$q(x) = [(x_0, z_0), (x_1, z_1) \cdots, (x_{n-1}, z_{n-1})] \quad \text{PVR.}$$

$$(x - x_0) = [(x_0, 0), (x_1, a_1)]$$

$$r = [\ ].$$

$y_1 = z_1 \cdot a_1$

$$\left[\qquad \right] = \left[\qquad \right]$$



Qn 30.1-4

n-1 point value pairs $\Rightarrow$ n-1 degree bound.

$\downarrow$ add one more arbitrarily chosen point value pair.

n point value pairs $\Rightarrow$ n degree bound & unique.

Unique to the arbitrary chosen number.

(*) The arbitrary chosen number may represent

$$y_n = A(x_n)$$

which makes only 1 unique (x_n, y_n) pair.

(*) Another choice could differ from this making some other unique pair.

Prove that n describe $A(x)$. uniquely and less than n will not be able to.

$A(x) =$ ~~a_0, ...~~ $a_0 x_0 + a_1 x_1 + \dots + a_{n-1} \cdot x_{n-1}$.

n - degree bound.

For $A(x)$ to be described in point-value representation.

every degree-bounding term must be represented OR

there must be a point representing all of $x_0, x_1, \dots x_{n-1}$

else they will not be able to define

$$a_0 x_0 + a_1 x_1 + \dots + a_{n-1} \cdot x_{n-1}$$

Qn $\underline{30.1-5}$

Eqn. $$A(x) = \sum_{k=0}^{n-1} y_k \frac{\prod_{j\neq k}(x - x_j)}{\prod_{j \neq k}(x_k - x_j)}$$

```
for k = 0 to n-1
  xxj = 1 ; xkxj = 1 ;
  for j = 0 to n-1
      if j ≠ k :
          x xj *=  x - x[j]
          xkxj *=  x[k] - x[j]
  A x = y[k] * (xxj / xkxj) ;
```

Qn $\underline{30.1-6}$

"Obvious" approach:

$A_1(x) = [(x_0, y_0), (x_1, y_1), \ldots, (x_{n-1}, y_{n-1})]$

$A_2(x) = [(x_0, z_0), (x_1, z_1), \ldots, (x_{n-1}, z_{n-1})]$

(A_1/A_2) $\mathrm{Div}(x) = [(x_0, y_0/z_0), (x_1, y_1/z_1) \ldots, (x_{n-1}, y_{n-1}/z_{n-1})]$

Qn 30.1-7

$A = [(x_0, a_0), (x_1, a_1) \ldots, (x_{n-1}, a_{n-1})]$

$B = [(x_0, b_0), (x_1, b_1) \ldots, (x_{n-1}, b_{n-1})]$

$C = [(x_0, a_0 + b_0), (x_1, (a_1 + b_1)), \ldots, (x_{n-1}, (a_{n-1} + b_{n-1}))]$

$0 \to 10n \to$ range of integers Yes!

$A = \{1, 3, 5, 7\}$. $\{1 \ 3 \ 5\}$

$B = \{2, 9, 11, 20\}$. $\{1 \ 2 \ 4.\}$.

2, 3, ~~5~~, 4, 7, 6, 9

2
3
4
5

1 1 1 / 3 1 2.

$1+1$, $3+2$

A_L A_R.

B_L B_R.

$(A_L + B_L)$, $(A_L + B_R)$, $(A_R + B_L)$, $(A_R + B_R)$. $(3+1)-(2+1)$

$(3+1) - 2$

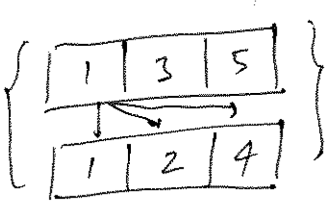
$(3+1) - 1$

$A(x) = 1 + 3x$ ~~$+ 5x^2$~~

$B(x) = 1 + 2x$ ~~$+ 4x^2$~~

Mult:
```
for i=1 to n
    for j=1 to n
        C[i] = C[i] + A[i] * B[j]
```

Naive:
```
k = 0
for i=1 to n
    for j=1 to n
        if A[i] + B[j] not in C[k]:
            C[k] = A[i] + B[j]
            k++;
```



3 5
2 4

$(1+1)$, $(3+2)$

2, 5, 3, $(3+2) - (1+1)$

~~$(3+2) - (1+2)$~~

$(3+2) - 1$

5, 7, 9 $(5+4)$, $(3+2)$

$(5+4) - 3$

$(5+4) - 2$

$\boxed{n=4}$

$k = 0$ to $n/2 - 1 = 1$

$c_k = a_k + \omega b_k$.

$\cancel{c_{k+n/2} = a_{k+n/2} - \omega b_{k+n/2}}$

$\omega = \omega . \omega_n$

1 3

1 2

$\begin{bmatrix} 0 : 2 \\ 1 : 5 \end{bmatrix}$

$\begin{bmatrix} 2 : 0 \\ 3 : 1 \end{bmatrix}$

2, 3, 4, 5.

1 3

1 2. $\rightarrow \sum_{k=0}^{n-1} a_k . \omega_n^{k}$ $n=4$.

$1 . \omega_2^1$, $2 . \omega_2^2 + 1 . \omega_2^2$

nlogn ? $1 e^{2\pi i/4}$, 2 and 3.

3, 4.

-1 -1

2, 3, 4, 5.

4+3 -1 -1

5.

$2n \times 2n^2 - 1$ 2n.   2n.1, 3

$$\begin{bmatrix} \boxed{1\,0\,0\,1\,0\,0} \\ 1\,0\,0\,0\,1\,0 \\ 1\,0\,0\,0\,0\,1 \\ 0\,1\,0\,0\,0\,0 \\ 0\,1\,0\,0\,1\,0 \\ 0\,1\,0\,0\,0\,1 \\ 0\,0\,1\,0\,0\,0 \\ 0\,0\,1\,0\,1\,0 \\ 0\,0\,1\,0\,0\,1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 4 \\ 2 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 7 \\ 5 \\ 6 \\ 9 \\ 6 \\ 4 \\ 7 \end{bmatrix}$$

ω_n^1, $3\omega_2^2 + 1\omega_2^2$.

3 + 4

-1, 4

3 5 → 1, 9.

2 . 4 . → $1\omega_3^{10}$, $1\omega_3^2 + 2\omega_3^2$, $1\omega_3^3 + 2\omega_3^3 + 4\omega_3^3$

2, 3, 5, 4, 8, 7, 6, 7, 9 = 1, 3, ~~1+2+4~~ 6

1, 3, 6

| 6 |
| 5 |
| 4 |
| 2 |

| 1 | 2 | 3 | 4 | 5 | 6 |

| 1 | 2 | 3 | 4 | 5 | 6 |

| 2 |

6 - 3   3

9 - 4   5

3 - 1   2

4 - 1   3

2, 3, 5, 6, 9, 8, 4.

(6+9) - (3+4)

8 ?

(6+9) - (3+4+1+1)

(3+4) - (1+1) = 5.

| 1 | 2 | 3 | → | 1 | → | 2 | 3 |

| 1 | 2 | 3 | | 1 | | 2 | 3 | → 4, 5, 6

2, 1

Qn 5.1-1

If we know which candidate is better of two we know their relative ranks. If we hire the first and fire him/her only when we find a better one, we know other candidates relative to the hired.

For finding the best → $\underline{n-1 \text{ comparisons}}$ are required.

⇒ The above goes on for $n-1$ times and we know which one is best which means we know their ranks relative to someone else. So, we ultimately know a total order on the ranks.

Qn 5.1-3

```
Random (0,1)
  k ← 1/p
  while (k--) {
        Randomp (0,1)
  k ← ⌈1/(p-1)⌉
  while (k++)
        Randomp (0,1)

    return Randomp (0,1)
```

Qn 5.2-1

$E[X_1] = Pr\{ 1 \text{ is the best} \}$.

1 is better than $(n-1)$ others.

$= Pr\{\text{hire all}\} - Pr\{\text{hire } (n-1)\}$.

Qn 5.2-5

$$E[X] = \cancel{E\left[\sum_{i=1}^{n} i!\right]} = E\left[\sum_{i=1}^{n} (n-i)\right]$$

$$\cancel{= \sum_{i=1}^{n} i!} = \sum_{i=1}^{n} (n-i)$$

$$= \cancel{\log(i!)} = n^2 - \frac{n(n+1)}{2}.$$

$(1! + 2! + \cancel{3!} + 4! + \cdots + i!)$

$$= n^2 - \frac{n^2}{2} - \frac{n}{2}$$

$$= \frac{n(n-1)}{2}. = \underline{O(n^2)}.$$

Qn 30.2-1

Corollary : $\omega_n^{n/2} = \omega_2 = -1$ $\quad n > 0$.

$$\omega_n^{n/2} = e^{\frac{2\pi i (n/2)}{n}} = e^{\frac{2\pi i}{2}} = \omega_2 = -1$$

Qn 30.2-2

a = Vector (0, 1, 2, 3)

$$DFT_4(a) = \sum_{j=0}^{n-1} a_j \omega_n^{kj}$$

$$y_0 = 0 \cdot \omega_n^{0.0} = 0$$

$$y_1 = 0 + 1 \cdot \omega_4^{1.1} = 1 \cdot e^{2\pi i/4} = i$$

$$y_2 = 1 \cdot \omega_4^{1.2} + 2 \cdot \omega_4^{2.2} = \omega_4^2 + 2 \cdot \omega_4^4 = \omega_4^2 + 2.1 = \omega_4^2 + 2 = \boxed{1}$$

$$y_3 = 0 + \omega_4^3 + 2 \cdot \cancel{\omega_4^6}^{\,\omega_4^2} + 3 \cdot \omega_4^{3.3} = \omega_4^3 - 2 + 3\omega_4^1$$

$$= i - 2 - 3i$$

$$= \boxed{-2 - 2i}$$

$$= \boxed{-2(1+i)}$$

$$DFT_4(a) = \text{Vector}\left(0, i, 1, -2(1+i)\right)$$

Qn <u>30. 2-3</u>

Qn 30.2-4

Inverse Transform (vector)

$w = 1$

$w_n^{(n-1)} = \left(e^{2\pi i/n}\right)^{n-1}$

$w_n = e^{2\pi i/n}$

$a^{[0]} = (y_0, y_2, \ldots, y_{n-2})$

$a^{[1]} = (y_1, y_3, \ldots, y_{n-1})$

$y^{[0]} =$ ~~rec~~ Inverse Transform $(a^{[0]})$

$y^{[1]} =$ Inverse Transform $(a^{[1]})$

for $k = 0$ to $n-1$

$y_k = w_n^{(n-1)} \cdot w_n / n \cdot$ ~~vector~~ $(y_k^{[0]} + w\, y_k^{[1]})$

$a_k = y_k^{[0]} \cdot w_n^{n-1} + w y_k^{[1]} \cdot w_n^{n-1}$

$w_n^{n-1} = w_n^{n-1} / w$

$w_n = w \cdot w_n$

return a.

Qn 30.2-6

n is a power of 3 $\Rightarrow n = 3^1, 3^2, 3^3 \dots 3^n \dots \infty$

How is it gonna make a difference?

$\frac{n}{2}, \frac{n+1}{2}$: Since, n = odd now

So, $T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n+1}{2}\right) + O(n).$

$= T\left(\frac{n}{4}\right) + T\left(\frac{n+2}{4}\right) + O(n) + O(n).$

$= T\left(\frac{n}{8}\right) + T\left(\frac{n+4}{8}\right) + 3n.$

$\vdots$

$= T\left(\frac{n}{2^k}\right) + T\left(\frac{n+2^{k-1}}{2^k}\right) + kn.$

$T(3) = T(1) + T(2) + n.$
$\downarrow \quad + \quad \downarrow$
$1 \qquad 1$

$\frac{n}{2^k} = 1 \Rightarrow k = \log_2 n.$ $= 2 + n.$

$\frac{n+2^{k-1}}{2^k} = 2 \Rightarrow n + 2^{k-1} = 2^{k+1}$

$\Rightarrow n = (2^{k-1}) \cdot 2^2$

$\Rightarrow \log\left(\frac{n}{4}\right) = k-1$

$\Rightarrow \log n - 2\log 2 = k-1$

$\Rightarrow \log n - 1 = k.$

$\Rightarrow k = O(\log n).$

$= T(1) + T(2) + O(\log n) \cdot n = \Theta(n \log n)$

Qn 30.2-6

Z_m of integers modulo m. ?

$m = 2^{tn/2} + 1$ $\quad t \rightarrow$ arbitrary +ve integer

$\omega = 2^t$ instead of ω_n

$\cancel{\omega_n = (m-1) = 2^{tn/2}}$. $\omega_n = 2^{t/n}$. $(m-1) = 2^{tn/2} = \omega_n^{n/2}$

$= \omega^{n/2}$

$DFT = \sum_{j=0}^{n-1} a_j \, \omega_n^{kj}$ $\Rightarrow$ $DFT = \sum_{j=0}^{n-1} a_j \cdot \omega^{kj}$ $\quad a_j \rightarrow Z_m$ of integers?

$y_k = A(\omega_n^k)$. $\quad y_k = A(\omega^k)$

FFT :

$A_{odd}(x) : (a_1, a_3, a_5, \ldots, a_{n-1})$

$A_{even}(x) = (a_0, a_1, \ldots \ldots a_{n-2})$

} Z_m of integers?

Qn 30. 2-7

$z_0, z_1, \ldots, z_{n-1}$ → find the coefficients of polynomials $P(x)$ of degree-bound $(n+1)$

time = $O(n \log^2 n)$

$P(x)$ = multiple of $(x - z_j)$ ⇒ 0 at z_j.

Qn 30.2-8

$a = (a_0, a_1 \cdots a_{n-1})$

$y = (y_0, y_1 \cdots y_{n-1})$

$$y_k = \sum_{j=0}^{n-1} a_j \cdot z^{kj}$$ $z \to$ complex no.

DFT $\to$ chirp transform spl case. when $z = \omega_n$.

$Z_1 = z^{1/2}$

$Z_2 = z^{2^2/2} = z^{4/2}$

$Z_k = (\sqrt{z})^{k^2}$

$= \prod_{j=\phi}^{k} (\sqrt{z})^j$

Recursive - Chirp (a)

n = a.length

if n == 1

  return a.

$z = ?$

$Z_k \to z^{k^2/2}$] what is this ?

$a_{odd} = [a_1, a_3, a_5 \cdots, a_{n-1}]$

$a_{even} = [a_0, a_2, a_4 \cdots, a_{n-2}]$

y_{even} = Recursive - Chirp (a_{even})

y_{odd} = Recursive - Chirp (a_{odd})

~~$z_k = z^{k^2/2}$~~

for k = 0 to $n/2 - 1$

  $t = z \cdot y_{odd}$

  $y_k = y_{even} + t$

  $y_{k+n/2} = y_{even} - t$.

  $z = z \cdot z_n$.

$t = z_\phi \cdot y_{odd} / z_k$

$y_k = (y_{even} + t) \cdot Z_k$.

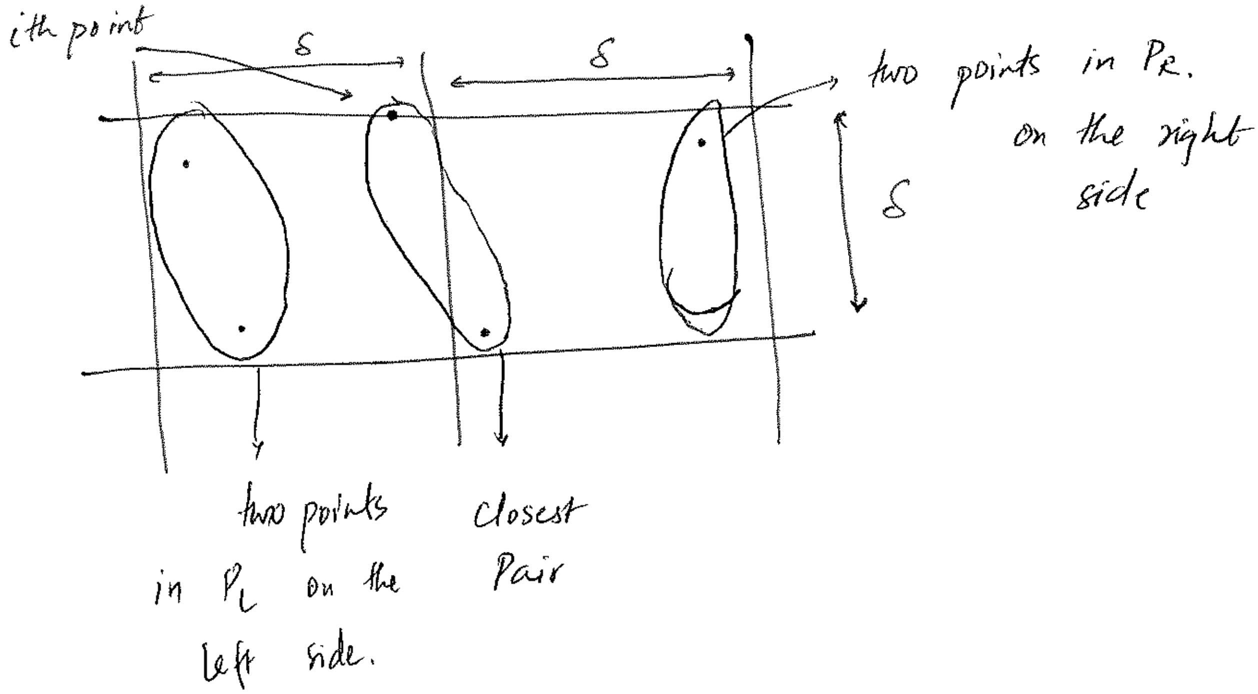
$y_{k+n/2} = \left(y_{even} - \frac{Z}{Z_{k+n/2}} \cdot y_{odd}\right)$

$Z_{k+n/2}$

$Z_k =$

$$Z_{k+n/2} = z^{(k+n/2)^2/2}$$

Qn 33.4-2


ith point
δ
δ
two points in P_R. on the right side
δ
two points in P_L on the left side.
closest Pair

Qn 33.4-3

Euclidean Distance → Manhattan Distance.

```
Closest Pair( A, B )

if |A| ≤ 3 :
    min ← ∞
    for i = 1 to 3
        for j = 1 to 3
            if Manhattan-dist( A[i], A[j] ) < min :
                min = Manhattan-dist (A[i], A[j]).

    return min.
m = A[|A|/2]

A_L = A[1...m]
A_R = A[m+1...|A|]

for i = 1 to |B| :
    ~~if A[i] < m~~
    if B[i].x < m :
        B_L ← B[i]
    else B_R ← B[i]

δ_L ← ClosestPair (A_L, B_L)
δ_R ← ClosestPair (A_R, B_R)
δ ← min(δ_L, δ_R)

min ← δ

for i = 1 to |B| :
    for k = 1 to 5 :
        if manhattan-dist( B[i], B[k] ) < min :
            min = Manhattan-dist (B[i], B[k])


return min.
```