Qn 4.1-5

CHAPTER 4

```
Maximum Subarray Problem (A, n)
P[0] = 0
for i = 1 to n
    P[i] = P[i-1] + A[i]
suffix = 0 , prefix = 0
for i = 1 to n
    if P[suffix] - P[prefix] > maxSum then
        maxsum = P[suffix] - P[prefix]
    if P[suffix] - P[prefix] ≤ 0 then
        prefix = suffix
    suffix ++;
```

Qn 4.2-1

$$A = \begin{bmatrix} 1 & 3 \\ 7 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 6 & 8 \\ 4 & 2 \end{bmatrix}$$

$S_1 = B_{12} - B_{22} = 8 - 2 = 6$

$S_2 = A_{11} + A_{12} = 1 + 3 = 4$

$S_3 = A_{21} + A_{22} = 7 + 5 = 12$

$S_4 = B_{21} - B_{11} = 4 - 6 = -2$

$S_5 = A_{11} + A_{22} = 1 + 5 = 6.$

$S_6 = B_{11} + B_{22} = 6 + 2 = 8$

$S_7 = A_{12} - A_{22} = 3 - 5 = -2$

$S_8 = B_{21} + B_{22} = 4 + 2 = 6$

$S_9 = A_{11} - A_{21} = 1 - 7 = -6$

$S_{10} = B_{11} + B_{12} = 6 + 8 = 14$

$P_1 = A_{11} \cdot S_1 = 6$

$P_2 = S_2 \cdot B_{22} = 8$

$P_3 = S_3 \cdot B_{11} = 72$

$P_4 = A_{22} \cdot S_4 = -10$

$P_5 = S_5 \cdot S_6 = 48$

$P_6 = S_7 \cdot S_8 = -12$

$P_7 = S_9 \cdot S_{10} = -84.$

$C_{11} = P_5 + P_4 - P_2 + P_6 = 48 - 10 - 8 - 12 = 18$

$C_{12} = P_1 + P_2 = 14$

$C_{21} = P_3 + P_4 = 72 - 10 = 62$

$C_{22} = P_5 + P_1 - P_3 - P_7 = 48 + 6 - 72 + 84 = 56$

$$\boxed{C = \begin{bmatrix} 18 & 14 \\ 62 & 56 \end{bmatrix}}$$

Qn 4.2-6

If k is a multiple of n.

then $O(k\, n^{\log 7})$

for multiplying $kn \times n$ matrix by a $n \times kn$ matrix.

runs k times {
while $(k \neq 0)$
partition $A(kn, n) \rightarrow A(n, n)$ and $A((k-1)n \times n)$ {same with B}
Strassen(n, A, B)

Qn 4.2-7

$a + bi$ and $c + di$

mult. $= ac + cbi + adi - bd$

(1) $(a+b)\cdot(c+d)$
(2) ac
(3) bd
(4) $\cancel{(a+b)}$ (1) − (2) − (3)

} 3 mult.
3 additions
3 subtractions.

mult = (2) − (3) + i((1) − (2) − (3)) {karatsuba}.

Qn 4.3-1

$$T(n) = T(n-1) + n \qquad T(1) = 1.$$

$$= T(n-2) + n + n.$$

$$= T(n-3) + n + 2n$$

$$\vdots$$

$$= T(n-k) + kn.$$

for $n-k = 1$
$\Rightarrow k = (n-1).$

$$= T(n-(n-1)) + (n-1)n.$$

$$= T(1) + (n-1).n$$

$$= O(n^2).$$

Qn 4.3-2

$$T(n) = T(\lceil n/2 \rceil) + 1 \qquad T(1) = 1$$

$$= T(n/4) + 1 + 1$$

$$= T\left(\frac{n}{8}\right) + 1 + 2$$

$$\vdots$$

$$= T\left(\frac{n}{2^k}\right) + k.$$

for $\frac{n}{2^k} = 1$
$\Rightarrow 2^k = n$
$\Rightarrow k = \log n$

$$= T(1) + \log n$$

$$= O(\log n).$$

Qn 4.3-3

$$T(n) = 2\,T(n/2) + n = O(n\log n)$$

$$\geq \frac{1}{2} n \log n \geq c\,(n\log n).$$

$(c = 1/2) > 0$

$$\Rightarrow T(n) = \Omega(n\log n).$$

and $T(n) = \Theta(n\log n)$.

Qn <u>4.5-2</u>

$\log_2 7$ v/s $\log_4 a$

we want a such that

$\log_4 a < \log_2 7$.

or $\log_4 a < \dfrac{\log_4 7}{\log_2 4}$.

$\Rightarrow 2 \cdot \log_4 a < \log_4 7$.

$\Rightarrow \log_4 a < \dfrac{\log_4 7}{2}$.

$\Rightarrow a < 4^{\frac{\log_4 7}{2}}$

$\Rightarrow \boxed{a < 2^{\log_4 7}}$

Qn <u>4.5-4</u> $T(n) = 4\,T(n/2) + n^2 \log n$

$a = 4,\ b = 2,\ k = 2$ MM cannot be applied.

$T(n) = 4\,T(n/2) + n^2 \log n$

$\le 4\,T(n/4) + 4n^2 \log n + n^2 \log n$

$\le 4\,T(n/8) + 4n^2 \log n + 5n^2 \log n$.

$\le 4\,T(n/2^k) + (4k-3)\, n^2 \log n$.

$n/2^k = 1$

$\Rightarrow k = \log n$

$\le 4\,T(1) + (4\log n - 3)\, n^2 \log n$

$\le 4 + 4n^2 \log^2 n - 3n^2 \log n$.

$\le O(n^2 \log^2 n) \quad \geq \Omega(n^2 \log^2 n) \Rightarrow \Theta(n^2 \log^2 n)$

4-1

(a) $T(n) = 2T(n/2) + n^4$

$a=2,\ b=2,\ k=4,\ \log_b a = 1$

$k > \log_b a$ ⇒ decreasing sequence.

$\Rightarrow\ O(n^4).$

(b). $T(n) = T(7n/10) + n$

$a=1,\ b=\frac{10}{7}.\ k=1\ \ \log_b a = \cancel{\log}\ 0$

$k > \log_b a.$

$\Rightarrow\ O(n).$

(c) $T(n) = 16\ T(n/4) + n^2$

$a=16,\ b=4,\ k=2,\ \log_b a = 2$

Case 2

$O(n^2 \log n)$

(d) $T(n) = 7\ T(n/3) + n^2$

$a=7,\ b=3,\ k=2,\ \log_b a = \log_3 7 < k.$

$\Rightarrow\ O(n^2)$

(f) $T(n) = 2\ T(n/4) + \sqrt{n}$

$a=2,\ b=4,\ k=\frac{1}{2}\ \ \log_b a = 1/2$

Case 2

$\Rightarrow\ O(\sqrt{n}\log n)$

(e) $T(n) = 7\ T(n/2) + n^2$

$a=7,\ b=2,\ k=2,\ \log_b a = \log_2 7 > k.$

$\Rightarrow\ O(n^{\log_2 7})$

Qn 4-1 (g) $T(n) = T(n-2) + n^2$

$$\Rightarrow \quad T(n) = T(n-4) + (n-2)^2 + n^2$$
$$\leq T(n-4) + 2n^2$$
$$\leq T(n-6) + 3(n^2)$$
$$\leq T(n-8) + 4(n^2)$$
$$\vdots$$
$$\leq T(n-2k) + k(n^2)$$

$n - 2k = 1$
when $2k = (n-1)$
or $k = \frac{(n-1)}{2}$.

$$\leq O(n^3)$$

Qn 4-5   n chips.

accumulates 2 chips at a time.

for i = 1 to n
    for j = 1 to n
        if i != j
        test i and j

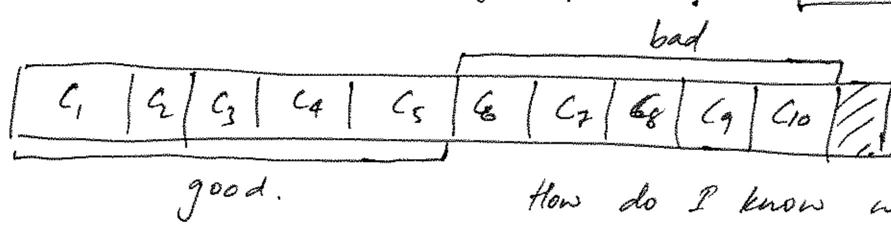
$O(n^2)$.

1 chip tests with all others in one run of inner for loop.

I: $n/2$ chips are bad. or $A[n/2 \ldots n]$ are bad.

=> for $i < n/2$ and $j < n/2$ & for $i > n/2$ and $j > n/2$ → Answer = both good or both bad. $\boxed{\text{Accept}}$

& for $i > n/2$ and $j < n/2$ ~~or~~ & for $i < n/2$ and $j > n/2$ → Answer = one of them is bad. $\boxed{\text{Reject}}$



How do I know which is good & which is bad?

logic: Enough no. of chips remain to move to the next level.

=> $\frac{n}{2}$ good & $\frac{n}{2}$ bad means all get rejected!

& more than $\frac{n}{2}$ good means we still have some good chips.