Appendix

A.1-1.

$$\sum_{k=1}^{n} (2k-1)$$

$$= \sum_{k=1}^{n} 2k - \sum_{k=1}^{n} 1$$

$$= 2\,\frac{k(k+1)}{2} - k.$$

$$= O(k^2).$$

A.1-2

$$\sum_{k=1}^{n} \frac{1}{(2k-1)} = \ln(\sqrt{n}) + O(1) \quad ?$$

$$\sum_{k=1}^{n} \frac{1}{(2k-1)} = 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \frac{1}{13} + \cdots$$

$$= 1 + \boxed{\frac{1}{3} + \frac{1}{5}} + \boxed{\frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \frac{1}{13}} + \cdots$$

$$\approx \underbrace{1}_{1} + \underbrace{\boxed{\frac{1}{4} + \frac{1}{4}}}_{2} + \underbrace{\boxed{\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}}}_{4} + \underbrace{\cdots}_{8\ \cdots}$$

$$\simeq \frac{1}{2}\log(n) + O(1).$$

$$= \log(n^{1/2}) + O(1).$$

$$= \log(\sqrt{n}) + O(1).$$

A.1-3

$$\sum_{k=0}^{\infty} k^2 \cdot x^k = \frac{x(1+x)}{(1-x)^3} \quad \text{for} \quad 0 < |x| < 1$$

$$\sum_{k=0}^{\infty} k^2 \cdot x^k = \lim_{n \to \infty} \sum_{k=0}^{n} k^2 \cdot x^k .$$

$$=$$

Qn 4.1-4

$$\sum_{k=0}^{\infty} \frac{(k-1)}{2^k} = 0$$

A.1-6

$$\sum_{k=1}^{n} O(f_k(i)) = O\left(\sum_{k=1}^{n} f_k(i)\right) \ ?$$

$$\sum_{k=1}^{n} O(f_k(i)) = \sum_{k=1}^{n} O(f_1(i)) + O(f_2(i)) + \dots + O(f_n$$

$$= O\left(f_1(i) + f_2(i) + \dots + f_n(i)\right) \quad \{\text{linearity}\}$$

$$= O\left(\sum_{k=1}^{n} f_k(i)\right).$$

A.1-7

$$\prod_{k=1}^{n} 2.4^k = \ ?$$

$$= \log\left(\prod_{k=1}^{n} 2.4^k\right) = \sum_{k=1}^{n} \log(2.4^k)$$

$$= \sum_{k=1}^{n} \left(\log(2) + \log(2^{2k})\right) = \sum_{k=1}^{n} \left((1) + (2k)\right)$$

$$= k + O(k^2).$$

$$= \underline{O(k^2)}.$$

A.1-8

$$\prod_{k=2}^{n} \left(1 - \frac{1}{k^2}\right) = \ ?$$

$$= \sum_{k=2}^{n} \log\left(1 - \frac{1}{k^2}\right) = \sum_{k=2}^{n} \left(\log(k^2-1) - \log(k^2)\right)$$

$$= \sum_{k\geq 2}^{n} \left(\log(2^2-1) - \log(2^2)\right) + \left(\log(3^2-1) - \log(3^2)\right) + \dots \left(\log(n^2-1) - \log\right.$$

A.2-1

$$\sum_{k=1}^{n} \frac{1}{k^2}$$

integrable

$$= \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \frac{1}{49} + \cdots$$

$$\leq \boxed{1} + \boxed{\frac{1}{4} + \frac{1}{4}} + \boxed{\frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}} + \cdots$$

1 2 4 8 ...

$$= \frac{1}{4} \log(n) + O(1).$$

$$= \log(n^{1/4}) + O(1).$$

A.2-2

$$\sum_{k=0}^{\lfloor \log n \rfloor} \lceil n/2^k \rceil$$

$$= \cancel{\sum_{k=}} \left\lceil \frac{n}{1} \right\rceil + \left\lceil \frac{n}{2} \right\rceil + \left\lceil \frac{n}{4} \right\rceil + \left\lceil \frac{n}{8} \right\rceil + \cdots$$

$$= (n)\left[\lceil 1 \rceil + \left\lceil \frac{1}{2} \right\rceil + \left\lceil \frac{1}{4} \right\rceil + \left\lceil \frac{1}{8} \right\rceil + \left\lceil \frac{1}{16} \right\rceil + \left\lceil \frac{1}{32} \right\rceil + \left\lceil \frac{1}{64} \right\rceil + \cdots \right]$$

$$= n\left[\underset{1}{1} + \underset{2}{\boxed{\frac{1}{2} + \frac{1}{2}}} + \underset{4}{\boxed{\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}}} + \underset{8}{\cdots} \right] \quad \cdots$$

$$= n \cdot \frac{1}{2^{\log n}} \, ? \, \log(\log(n)) + \varepsilon_1$$

$$= \cancel{n} \cdot \frac{1}{\cancel{n}} \cdot \log(\log(n)) + \varepsilon_1$$

$$= O(\log(\log(n)))$$

A.2-3

$$\sum_{k=1}^{n} \frac{1}{k} = \sum_{k=1}^{n/2} \left(\frac{1}{k}\right) + \sum_{k=n/2+1}^{n} \left(\frac{1}{k}\right)$$

$$\geq \sum_{k=\frac{n}{2}+1}^{n} \left(\frac{1}{k}\right)$$

$$\geq \cancel{\log \frac{n}{2}} \sum_{i=0}^{\lfloor \log(\frac{n}{2}) \rfloor} \sum_{j=0}^{2^i-1} \frac{1}{2^{i+j}}$$

$$\geq \sum_{i=0}^{\lfloor \log(\frac{n}{2}) \rfloor} \sum_{j=0}^{2^i-1} \frac{1}{2^i}$$

$$= \sum_{i=0}^{\lfloor \log(\frac{n}{2}) \rfloor} 1$$

$$= \log\left(\frac{n}{2}\right) = \cancel{\Theta}\ \Omega(\log(n)). \quad \{\log(\tfrac{n}{2}) = \log \ldots = O(\log(n$$

A.2-4

$$\sum_{k=1}^{n} k^3$$ monotonically increasing function

$$= \int_{k=1}^{n} k^3 \leq \int_{k=\emptyset}^{n+1} k^3$$

$$\leq \int_{k=1}^{n+1} n^3$$

$$\leq O(n^3).$$

$$\sum_{k=1}^{n} k^3 = \cancel{\sum_{k=1}^{n/2} k^3} + \cancel{\sum_{k=\frac{n}{2}+1}^{n} k^3} \int_{k=1}^{n} k^3$$

$$\geq \int_{k=0}^{n} k^3 \quad \Rightarrow \Theta(n^3).$$

A - 1

(a) $\sum_{k=1}^{n} k^r = \sum_{k=1}^{n} k^r$

$\leq \sum_{k=1}^{n} n^r$

$\leq O(n \cdot n^r) = \underline{O(n^{r+1})}.$

$\sum_{k=1}^{n} k^r = \sum_{k=1}^{n/2} k^r + \sum_{k=\frac{n}{2}+1}^{n} k^r$

$\geq \sum_{k=\frac{n}{2}+1}^{n} k^r$

$\geq \sum_{k=\frac{n}{2}+1}^{n} \left(\frac{n}{2}\right)^r$

$\geq \Omega\left(n \cdot \left(\frac{n}{2}\right)^r\right) = \Omega(n^{r+1})$

$\Rightarrow \underline{\Theta(n^{r+1})}.$

(b). $\sum_{k=1}^{n} \log^s k = \sum_{k=1}^{n} \log^s k$

$\leq \sum_{k=1}^{n} \log^s(n).$ $\therefore \Theta(n \cdot \log^s(n))$

$\leq O(n \cdot \log^s(n))$

$\sum_{k=1}^{n} \log^s k = \sum_{k=1}^{n/2} \log^s(k) + \sum_{k=\frac{n}{2}+1}^{n} \log^s(k)$

$\geq \sum_{k=\frac{n}{2}+1}^{n} \log^s(k) \geq \sum_{k=\frac{n}{2}+1}^{n} \log^s\left(\frac{n}{2}\right) \geq \Omega\left(n(\log^s n - 1)\right)$

$= \Omega(n \cdot \log^s n)$

(c) $$\sum_{k=1}^{n} k^r \cdot \log^s k = \sum_{k=1}^{n} k^r \cdot \log^s k$$

$$\leq \sum_{k=1}^{n} n^r \cdot \log^s n$$

$$\leq O(n^{r+1} \cdot \log^s n)$$

$$\sum_{k=1}^{n} k^r \cdot \log^s k = \sum_{k=1}^{n/2} k^r \cdot \log^s k + \sum_{k=\frac{n}{2}+1}^{n} k^r \cdot \log^s k.$$

$$\geq \sum_{k=\frac{n}{2}+1}^{n} k^r \cdot \log^s k.$$

$$\geq \sum_{k=\frac{n}{2}+1}^{n} n^r \cdot \log^s(n).$$

$$= \cancel{\Theta}\ \Omega(n^{r+1} \cdot \log^s(n))$$

$$\Rightarrow \underline{\Theta(n^{r+1} \cdot \log^s(n))}$$