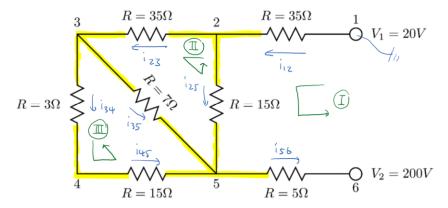
MTE 204 Project 1: Gauss-Seidel Method

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Circuit #1

The annotated circuit as follows, with assumed direction of currents marked in blue, loops in green, and nodes in yellow. A ground is assumed to be at node one to allow for Loop I.



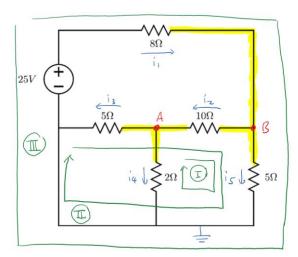
The partially pivoted matrix corresponding to this circuit, and the derivation of its row, is as follows.

$$A_{1}\{\vec{b}_{1}\} = \begin{bmatrix} -35 & 0 & -15 & 0 & 0 & 0 & -5 \\ 0 & -35 & 15 & 0 & -7 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 7 & -15 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_{12} \\ i_{23} \\ i_{25} \\ i_{34} \\ i_{35} \\ i_{45} \\ i_{56} \end{bmatrix} = \begin{bmatrix} 180 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Row	1	2	3	4	5	6	7
Derived from	Loop I	Loop II	Node 2	Node 4	Node 3	Loop III	Node 5

Circuit #2

The annotated circuit as follows, with assumed direction of currents marked in blue, loops in green, and nodes in yellow/red.



The partially pivoted matrix corresponding to this circuit, and the derivation of its row, is as follows.

$$A_{2}\{\vec{b}_{2}\} = \begin{bmatrix} 8 & 0 & 0 & 0 & 5 \\ 0 & 10 & 5 & 0 & -5 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 10 & 0 & 2 & -5 \end{bmatrix} \begin{bmatrix} i_{1} \\ i_{2} \\ i_{3} \\ i_{4} \\ i_{5} \end{bmatrix} = \begin{bmatrix} 25 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Row	1	2	3	4	5
Derived from	Loop III	Loop II	Node A	Loop I	Node B

Convergence and Relaxation

A relaxation factor is used to speed up convergence for values above 1, and slow down and increase the likelihood of convergence occurring for values between 0 and 1.

For Circuit #1, the relaxation factor and their corresponding iterations to convergence are as follows.

Relaxation	0.25	0.50	0.75	1.00	1.25	1.50
Iterations	124	55	30	318	854	Did not converge

For Circuit #2, the relaxation factor and their corresponding iterations to convergence are as follows.

Relaxation	0.10	0.25	0.50	0.75
Iterations	371	187	201	Did not converge

Evidently, higher values of the relaxation factor tend to result in the algorithm not reaching convergence, and conversely lower values resulting in large numbers of iterations. Of the relaxation factors tested for Circuit #1, a value of 0.75 is most optimal, and for Circuit #2 it is a value of 0.25.

Of note, Circuit #2 required lower values of the relaxation factor as its matrix is not diagonally dominant, as had been the case for Circuit #1. Then, Circuit #2 can be described as not being well conditioned.