T10, T11, T22 Sols.

MATA33S – Quiz 4

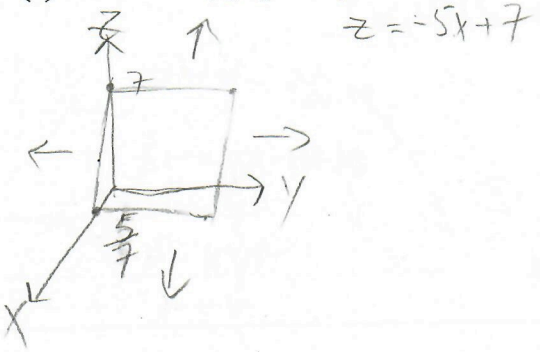
First Name: _____ **Score:** _____/30

Last Name: _____ **Student number:** _____

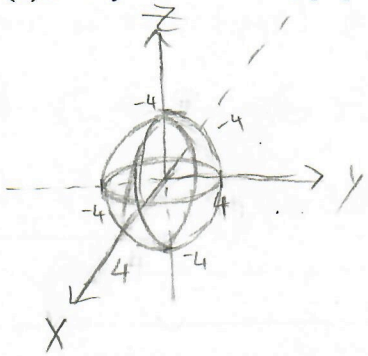
1. Sketch the following surfaces.

(a) $5x + z = 7$, [3 points]

$z = -5x + 7$

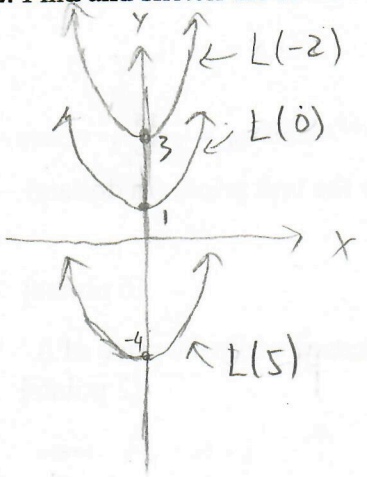


(b) $x^2 + y^2 + z^2 = 16$, [3 points]



2. Find and sketch the level curves $L(0)$, $L(-2)$, and $L(5)$ for the function:

$z = f(x, y) = x^2 - y + 1.$ [5 points]



$L(-2) = -2 = x^2 - y + 1$

$y = x^2 + 3$

$L(0) = 0 = x^2 - y + 1$

$y = x^2 + 1$

$L(5) = 5 = x^2 - y + 1$

$y = x^2 - 4$

3. Find the first partial derivatives $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ of the function $z = x^{-y}$. [4 points]

$\frac{\partial z}{\partial x} = (-y)x^{-y-1}$

$\frac{\partial z}{\partial y} = \ln(x) \cdot x^{-y} \cdot (-1)$

$= -\ln(x)\, x^{-y}$

4. Find the equation of the horizontal plane that is tangent to the graph of the function:

$$z = G(x, y) = x^2 - 4xy - 2y^2 + 12x - 12y - 1$$ [7 points]

find partial derivatives with respect to x, y and let it equal to 0 to find pts of (x, y) that is tangent to $\mathcal{D}(x, y)$

$\mathcal{D}_x = 2x - 4y + 12$ let $\mathcal{D}_x = 0 \Rightarrow 2x - 4y = -12$

$\mathcal{D}_y = 4x - 4y = 12$ let $\mathcal{D}_y = 0 \Rightarrow -4x - 4y = 12$ ①

$6x = -24$

$x = -4$

$2(-4) - 4y = -12$

$-4y = -12 + 8$

$-4y = -4$

$y = 1$

$\mathcal{D}(-4, 1) = (-4)^2 - 4(-4)(1) - 2(1)^2 + 12(-4) - 12(1) - 1$

$= 16 + 16 - 2 - 48 - 12 - 1$

$= 32 - 63$

$= -31$

5. The demand equations for related products A and B are $q_a = e^{-(P_a + P_b)}$ and $q_b = \frac{16}{P_a^2 P_b^2}$ where q_a and q_b are the number of units of products A and B demanded when the unit prices (in dollars) are P_a and P_b, respectively.

(i) Classify A and B as competitive, complementary, or neither. [6 points]

(ii) What would happen to the demand for A if the price of B was increased while the price of A was held constant? [2 points]

(i) $q_a = e^{-(P_A + P_B)}$ $q_b = 16 P_A^{-2} P_B^{-2}$

$\frac{\partial q_a}{\partial P_B} = -e^{-(P_A + P_B)}$ $\frac{\partial q_B}{\partial P_A} = -32 P_A^{-3} P_B^{-2}$

< 0 < 0

∴ A & B are complementary

(ii) Price B ↑, price A ~

i.e. reaction of $\frac{q_A}{P_B}$ if P_B ↑

$\frac{\partial q_a}{\partial P_B} = -e^{-(P_A + P_B)}$

< 0

∴ if P_B ↓ then q_A ↑

∴ P_B ↑ then q_A ↓