MATA33 Winter 2020 - Quiz 1 **Name:** **/30**

- 45 Minutes **Utorid:**

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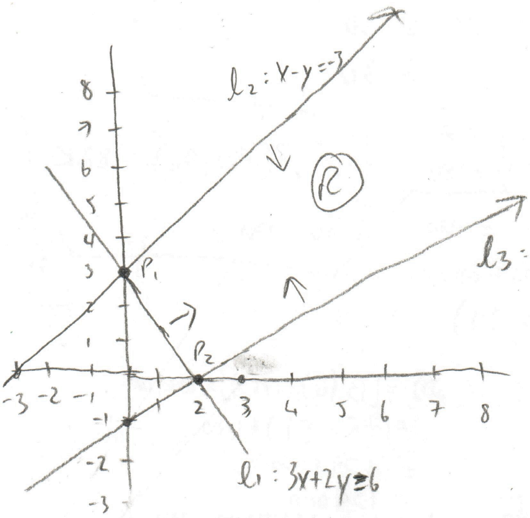
x	y
0	3
2	0

x	y
0	3
-3	0

x	y
0	-1
2	0

(1) Consider the objective function of the form Z = -ax-by, where a and b are positive constants. It is subject to the following constraints: 3x + 2y $\geq$ 6, x - y $\geq$ -3 and x - 2y $\leq$ 2. Your solution should show the feasible region, and labeled corner points.

(a) (7 marks) Draw a diagram with the feasible region, labeled lines, corner points.



$P_1 : (0,3) \longrightarrow l_2 : x-y=-3$

$P_2 : (2,0)$

$l_1 : 3x+2y=6$

$3\cdot l_2 - l_1 : 0 - 5y = -15$

$y=3$

$x-(3)=-3$

$x=0$

$(0,3)$

$l_3 : x-2y=2$

$l_1 : 3x+2y=6$

$3\cdot l_3 - l_1 : 0 - 4y = 0$

$y = 0$

$x-2(0)=2$

$x=2$

$(2,0)$

(b) (3 marks) Find the value of the constants a and b such that Z has a maximum value of 25 at every point on the line segment joining the corner(extrema) points.

$Z = 25 = -ax - by \Rightarrow Z(0,3) = -a(0) - b(3) = -3b \Rightarrow 25 = -3b,\ b = -\frac{25}{3}$

$Z(2,0) = -a(2) - b(0) = -2a \Rightarrow 25 = -2a,\ a = -\frac{25}{2}$

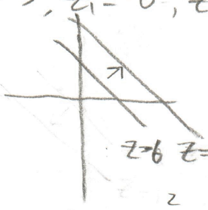
(c) (3 marks) Explain why no positive constants of a and b will result in a minimum value for Z on the feasible region R. Provide mathematical or graphical reasoning.

- the objective function $Z = -ax-by$ shifts away from the origin if $Z \downarrow$

eg. let $a, b = 3$, $Z_1 = -6$, $Z_2 = -12$

$-6 = -3x-3y$

$-12 = -3x-3y$



as $Z \downarrow$,

$\therefore$ no min, b/c as $Z \downarrow$, the function shifts to infinite without bound

Alternatively, the band as $x \to \infty$ challenges $x-y=-3$, a boundary of R

$y = x+3$

$Z(x,y) = Z(x, x+3)$

$= -ax - b(x+3)$

$= -ax - bx - 3b$ assuming $a, b > 0$

$= -\infty$

$\therefore$ no bounded minimum for all $a, b > 0$