MATA33 Winter 2020 - Quiz 1 Name: Tutorial #22 /30

- 45 Minutes Utorid:

x	y
2	0
0	3

x	y
-3	0
0	3

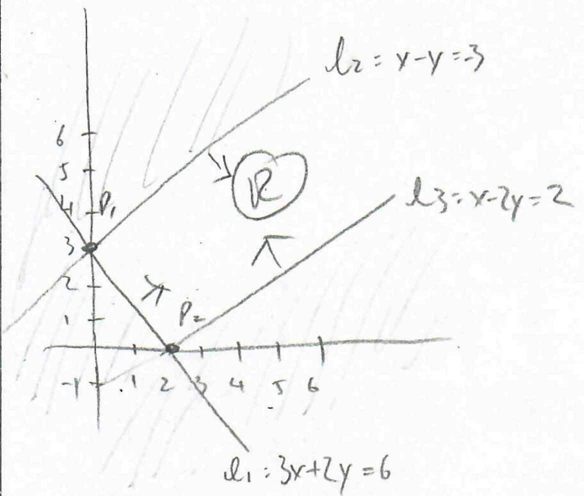
x	y
2	0
0	-1

(1) Consider the objective function of the form Z = -ax-by, where a and b are positive constants. It is subject to the following constraints: $3x + 2y \geq 6$, $x - y \geq -3$ and $x - 2y \leq 2$. Your solution should show the feasible region, and labeled corner points.

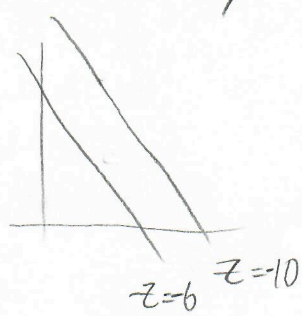
(a) (7 marks) Draw a diagram with the feasible region, labeled lines, corner points.

$P_1 : l_1 \cap l_2 \ (0,3)$

$P_2 : l_1 \cap l_3 \ (2,0)$



$Z = -ax - by$



typo, -25

(b) (3 marks) Find the value of the constants a and b such that Z has a maximum value of 25 at every point on the line segment joining the corner(extrema) points.

$25 = -ax-by \Rightarrow$ ① $25 = -a(0)-b(3)$ ② $25 = -a(2)-b(0)$

$25 = -3b$ $\qquad 25 = -2a$

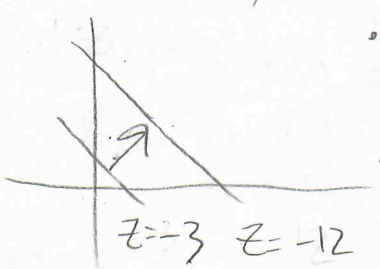
$b = -\frac{25}{3}$ $\qquad a = -\frac{25}{2}$

(c) (3 marks) Explain why no positive constants of a and b will result in a minimum value for Z on the feasible region R. Provide mathematical or graphical reasoning.

as an example, let a, b be positive, specifically a = 3, b = 3

$Z = -(3)x - (3)y$

$= -3x - 3y$



- to achieve a minimum, ie, Z ↓ (from -3 to -12) we shift away from the origin (0,0)
- However, the graph of constraints is unbounded as we shift away from origin ∴ no min is possible

the same applies for all positive a and b, because they maintain the same pattern

(2) A University produces two courses for its Management students this year. MATA32 and MATA33. Each unit of MATA32 requires 6 hours of teaching, 2 hours of administration, with a revenue of \$1300. Each unit of MATA33 requires 7 hours of teaching, 1 hour of administration with a revenue of \$1700. Due to the University's resources, there is a maximum of 2010 teaching hours and 150 administration hours. The University must also admit at least 30 MATA32 and at least 30 MATA33 students to qualify for the Ontario Government Grant.

Revenue : $1300x + 1700y = Z$

(a) (8 marks) What is the maximum revenue?

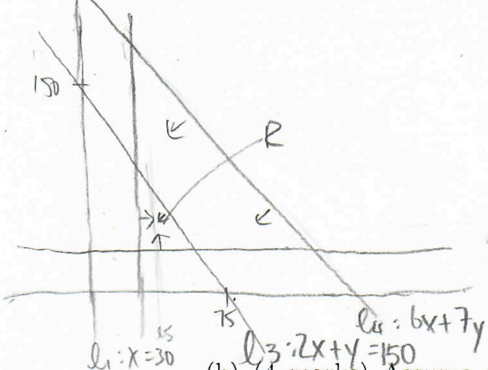
	teaching	admin	revenue	min	max
MATA32(x)	6	2	1700	30	/
MATA33(y)	7	1	1300	30	/
min	/	/	/	/	/
max	2010	150	/	/	/

Constraints: $6x + 7y \le 2010$

$2x + y \le 150$

$x \ge 30$

$y \ge 30$



$P_1 : l_1 \cap l_2 : (30, 30)$

$P_2 : l_1 \cap l_3 : (30, 90)$

$P_3 : l_2 \cap l_3 : (60, 30)$

$x = 30$

$2 ① - ②$ $2x + y = 150$

$0 - y = -90$

$y = 90$

$y = 30$

$⊖$ $2x + y = 150$

$-2x = -120$

$x = 60$

$Z(30,30) = 100*10\,[13\cdot(3) + 17(3)]$

$= 1000\,[39 + 51]$

$= 90\,000$

$Z(60,30) = 100*10\,[13(6) + 17(3)]$

$= 1000\,[78 + 51]$

$= 129000$

$Z(60,90) = 100*10\,[13\cdot(3) + 17(9)] = 1000\,[39 + 153] = 192\,000$

$\therefore Z(60,90) = 192000$ is max

(b) (4 marks) Assume everything from part a, but with an additional 10 people taking MATA32. Determine the new corner points only.

new constraint : $x \ge 40$ (l_5)

$P_{N1} : l_2 \cap l_5$ $(40, 30)$

$x = 40, y = 30$

$(l_5)\ (l_2)$

$P_{N2} : l_3 \cap l_5$ $(40, 70)$

$x = 40$ ①

$2x + y = 150$ ②

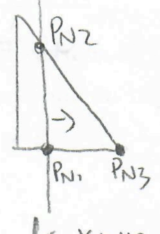
sub $x = 40$ into ②

$80 + y = 150$

$y = 70$

$P_{N3} : l_3 \cap l_2$ $(60, 30)$

same as before

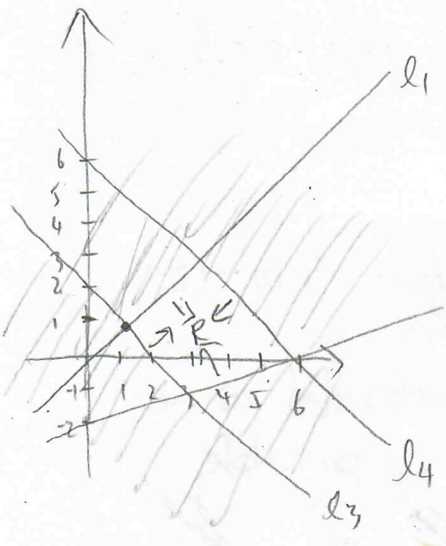


(3) (5 marks) Let Z = ax-by where x and y are variables and a,b > 0 are constants.
The constraints are: $x \ge y$ (l_1), $x-3y \le 6$ (l_2), $x+y \ge 2$ (l_3), $x+y \le 6$ (l_4). Can a minimum occur on (1,1)?
If so, provide a possible a,b value and justification regardless.

x	y
6	0
0	-2

x	y
2	0
0	2

x	y
6	0
0	6



By LP theorem,

the feasible region is bounded & non-empty

$\therefore$ a minimum & maximum must exist

$\therefore$ it is possible for a minimum to occur on (1,1)

let us choose an a, b value such that the isoprofit line will touch (1,1) last via graphical method

$a = 10,\ b = 1 \Rightarrow Z = 10x - y$