MATA33 Winter 2020 - Quiz 1 - 45 Minutes

Name: Tutorial # 10

Utorid:

/30

x	y
0	5
5	0

x	y
3	0
0	6

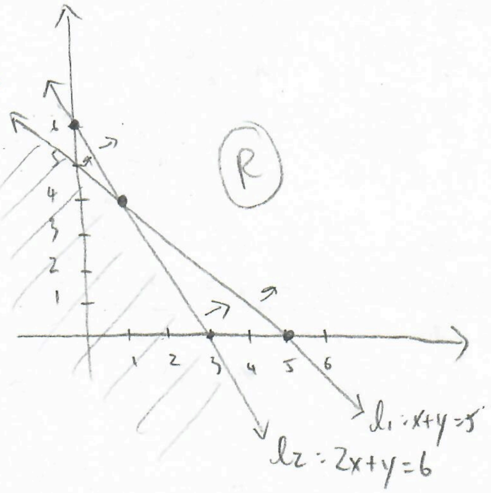
(1) Given the constraints x+y≥5, 2x+y≥6, x≥0, y≥0, with corner points of (0,6), (1,4), (5,0)

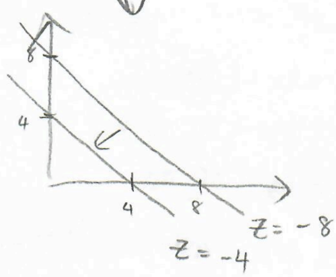
(a) (3 marks) Find an objective function that would make ~~the~~ any corner point a **minimum**, if it exists

(b) (3 marks) Find an objective function that would make ~~the~~ any corner point a **maximum**, if it exists

a) eg. $Z = x + y$

b) eg. $Z = -x - y$





as Z ↑ (ie. going to a max), the isoprofit line shifts towards origin and will eventually touch a corner point last

(2) (7 marks) Consider the objective function of the form Z = 3x + 3y. It is subject to the following constraints: 2x + y ≤ 4, -x + y ≤ 1, 2x + 3y ≥ 12, and negative x & y values cannot occur. Determine the maximum and minimum values of Z and where they occur for the feasible region R, or determine that Z has no optimal value on R with justification. Your solution should show labelled lines, the feasible region, and corner points if they exist.

$2x+y=4$

x	y
2	0
0	4

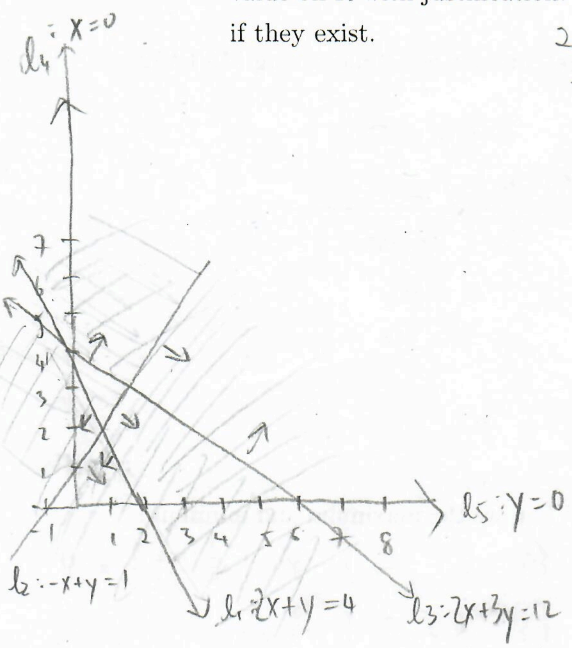
$-x+y=1$

x	y
-1	0
0	1

$2x+3y=12$

x	y
6	0
0	4

$x=0$, $y=0$



- there is no point in the graph that is feasible
  ↳ satisfies emptiness condition

by LP theorem,

b/c it is empty, there is no max/min

∴ no optimal values exist

(3) A University produces two courses for its Management students this year. MATA32 and MATA33. Each unit of MATA32 requires 6 hours of teaching, 2 hours of administration, with a revenue of \$1300. Each unit of MATA33 requires 7 hours of teaching, 1 hour of administration with a revenue of \$1700. Due to the University's resources, there is a maximum of 2010 teaching hours and 150 administration hours. The University must also admit at least 30 MATA32 and at least 30 MATA33 students to qualify for the Ontario Government Grant.

(a) (8 marks) What is the maximum revenue?

	teaching	admin	Revenue	min	max
MATA32 (x)	6	2	1700	30	/
MATA33 (y)	7	1	1300	30	/
min	/	/	/	/	/
max	2010	150	/	/	/

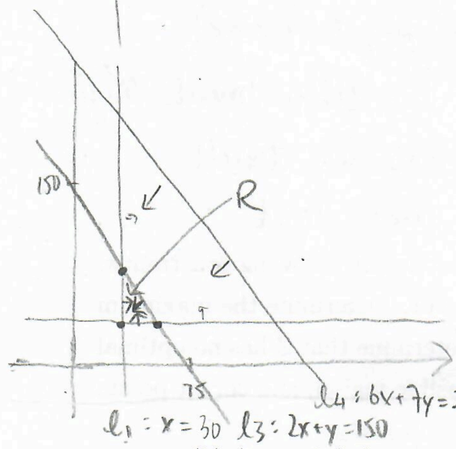
Revenue : $1300x + 1700y = Z$

Constraints : $6x + 7y \le 2010$

$2x + y \le 150$

$x \ge 30$

$y \ge 30$



$l_2: y=30$

$x = 30$

$2① - ②$ $\quad 2x + y = 150$

$0 - y = -90$

$y = 90$

$y = 30$

$⊖\ 2x + y = 150$

$-2x = -120$

$x = 60$

$P_1 : l_1 \cap l_2\ (30, 30)$

$P_2 : l_1 \cap l_3\ (30, 90)$

$P_3 : l_2 \cap l_3\ (60, 30)$

$Z(30, 30) = 100*10\,[13(3) + 17(3)]$

$= 1000\,(39 + 51)$

$= 90\,000$

$Z(60, 30) = 100*10\,[13(6) + 17(3)]$

$= 1000\,[78 + 51]$

$= 129\,000$

$Z(30, 90) = 100*10\,[13(3) + 17(9)]$ $\therefore$ 192000 is max revenue

$= 1000\,(39 + 153)$

$= 192000$

(b) (4 marks) Assume everything from part a, but now there is an equal amount of people taking MATA32 and MATA33. Determine the new corner points only.

new constraint : $x = y$ (l_5)

$P_{N1} : l_1 \cap l_5\ (30, 30)$

$P_{N2} : l_3 \cap l_5\ (50, 50)$

$x = 30$

$⊖\ x - y = 0$

$y = 30$

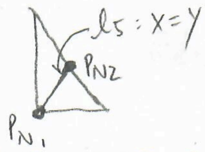
$2x + y = 150$

$⊕\ x - y = 0$

$3x = 150$

$x = 50$

$y = 50$



(4) (5 marks) Let Z = x-3y subject to 1≤x≤5 and $\frac{1}{2}x - \frac{1}{2} \le y \le -\frac{1}{2}x + \frac{9}{2}$. What are the maximum and minimum values?

$Z = x - 3y = 3$

x	y
3	0
0	-1

$x - 3y = 6$

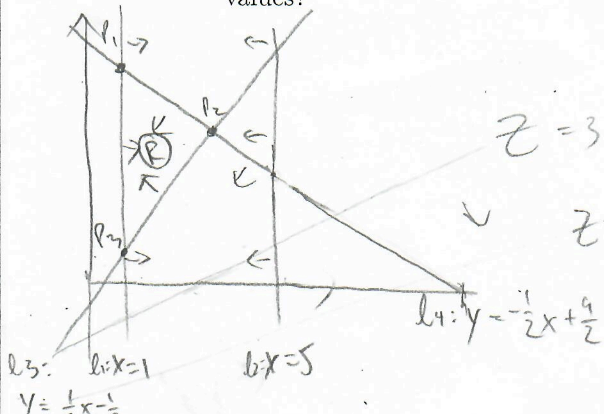
x	y
6	0
0	-2

$P_1 = (1, 4)$ $\quad y = -\frac{1}{2}(1) + \frac{9}{2} = 4$

$P_2 = (5, 2)$ $\quad \frac{1}{2}x - \frac{1}{2} = -\frac{1}{2}x + \frac{9}{2}$

$P_3 = (1, 0)$ $\quad x = 5$

$y = 2$



$Z = 3$

$Z = 6$

$Z(1, 4) = -11 \Rightarrow$ min

$Z(5, 2) = -1$

$Z(1, 0) = 1 \Rightarrow$ max

see notes on graph on next page!

Remark on Q4: the graph actually looks like this(see below).
Notice that y=1/2x-1/2 and y=-1/2x+9/2 and x=5 intersect simultaneously

This questions shows that we should try to draw the graph with accuracy, when we are able to.

For this particular example, a badly sketched graph will still work, but you may not get so lucky in other cases.
Eg. what if I add in a new constraint x >= 4? If your graph is misaligned, you may think there is no feasible regio