MATA33 Winter 2020 - Quiz 1 Name: /30
- 45 Minutes Utorid:

(1) (8 marks) Find all values of x in which det(A) = 0

$$A = \begin{bmatrix} 0 & 0 & 0 \\ x & -6-x & -2 \\ 0 & 5 & 1-x \end{bmatrix}$$

$$\det\begin{pmatrix} 0 & 0 & 0 \\ x & -6-x & -2 \\ 0 & 5 & 1-x \end{pmatrix} = 0 + (-1)^{2+1}\cdot x \cdot \begin{vmatrix} 0 & 0 \\ 5 & 1-x \end{vmatrix}$$

$$\det(A) = 0 + (-1)^3(x)((0)(1-x)-(0)(5))$$

$$\det(A) = 0 - x(0)$$

no matter what the value of x is,
$x \cdot 0$ is always 0

$\therefore\ x \in \mathbb{R}$ for $\det(A) = 0$

(2) (9 marks) For the following, recall that $A^*A^{-1} = I$.

(a) (3 marks) Given $3B^2 - 4B = -I$, what is the inverse of B?

$$-3B^2 + 4B = I$$
$$(-3B^2+4B)B^{-1} = IB^{-1}$$
$$-3BB^{-1} + 4BB^{-1} = B^{-1}$$
$$-3B + 4I = B^{-1}$$

(b) (3 marks) Show that $(AB)^{-1} = B^{-1}A^{-1}$.

$$(AB)^{-1}(AB) = B^{-1}A^{-1}(AB)$$
$$\cancel{(AB)^{-1}(AB)} = B^{-1}\cancel{A^{-1}A}B$$
$$I = \cancel{B^{-1}B}$$
$$I = I$$

(c) (3 marks) Show that $\det(A^{-1}) = \frac{1}{det(A)}$.

$$AA^{-1} = I$$
$$\det(AA^{-1}) = \det(I)$$
$$\det(A)\det(A^{-1}) = \det(I)$$
$$\det(A^{-1}) = \frac{\det(I)}{\det(A)} = \frac{1}{\det(A)}$$

$\det(P^{-1})$

(3) (6 marks) Let P and Q be ~~5 x 5~~ 4x4 matrices such that det(P) = 2 and det(Q) = -3.

Find $\det(-2(P^{-2})(3Q)^{-1})$. Show your steps when using determinant properties for part marks.

$$= \det(-2(P^{-2}))\det((3Q)^{-1})$$
$$= XY$$
$$= (4)\left(-\frac{1}{243}\right)$$
$$= -\frac{4}{243}$$

$$X = \det(-2(P^{-2}))$$
$$= (-2)^4 \det(P^{-2})$$
$$= 16 \frac{1}{\det(P^2)}$$
$$= 16 \frac{1}{2^2}$$
$$= \frac{16}{4}$$
$$= 4$$

$$Y = \det((3Q)^{-1})$$
$$= \frac{1}{\det(3Q)}$$
$$= \frac{1}{(3)^4 \det(Q)}$$
$$= \frac{1}{(81)(-3)}$$
$$= \frac{1}{-243}$$

(4) (7 marks) The different parts of this question are somewhat related, and may help you solve other parts of the question.

(a) (2 marks) What is the relationship between A^{-1} and det(A) = 0?

$A^{-1} \iff \det(A) = 0$

(b) (2 marks) Give an example of a matrix whose determinant is 0 and has infinitely many solutions.

$$\det(A) = \det\begin{pmatrix} 7 & 6 \\ 0 & 0 \end{pmatrix}$$

(c) (3 marks) Given Ax = 0. A is a non-zero 2x2 matrix. $X = x_1, x_2$ have non-zero values. Show that A is not invertible.

$$Ax = 0$$
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \longrightarrow \begin{array}{l} ① \; a_{11}x_1 + a_{12}x_2 = 0 \\ ② \; a_{21}x_1 + a_{22}x_2 = 0 \end{array}$$

$\downarrow$

① & ② are the same equation

$\therefore a_{11} = a_{21}$ & $a_{12} = a_{22}$

$\downarrow$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{11} & a_{12} \end{bmatrix}$$

$\downarrow$

$$\det(A) = \det\begin{pmatrix} a_{11} & a_{12} \\ a_{11} & a_{12} \end{pmatrix} = a_{11}a_{12} - a_{12}a_{11} = 0$$

b/c $\det(A) = 0 \Rightarrow A^{-1}$ DNE