T11

MATA33H3S
Calculus for Management II
Winter 2020 Quiz 2

Last Name: ______
First Name: ______
Student Number: ______
Mark: ______/30

1. (12 points) Let $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -27 \\ -18 & 33 \end{bmatrix}$

(a) Find $(A^2 - 3I)^T$

(b) Find all 2×2 diagonal matrices D s.t. $D^2 - A^2 = B$

a) $A \cdot A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 16+6 & 12+15 \\ 8+10 & 6+25 \end{bmatrix} = \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix}$

$3I = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$A^2 - 3I = \begin{bmatrix} 22 & 37 \\ 18 & 31 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 19 & 27 \\ 18 & 28 \end{bmatrix}$

$(A^2 - 3I)^T = \begin{bmatrix} 19 & 18 \\ 27 & 28 \end{bmatrix}$

b) $D^2 - A^2 = B$

$D^2 = B + A^2$

$= \begin{bmatrix} 3 & -27 \\ -18 & 33 \end{bmatrix} + \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix}$

$= \begin{bmatrix} 25 & 0 \\ 0 & 64 \end{bmatrix} = \begin{bmatrix} (5)^2 & 0 \\ 0 & (8)^2 \end{bmatrix}$

$D = \begin{bmatrix} 5 & 0 \\ 0 & 8 \end{bmatrix}$, $D = \begin{bmatrix} 5 & 0 \\ 0 & -8 \end{bmatrix}$, $D = \begin{bmatrix} -5 & 0 \\ 0 & 8 \end{bmatrix}$, $D = \begin{bmatrix} -5 & 0 \\ 0 & 8 \end{bmatrix}$

2. (6 points) Consider the following the system of equations:

$$x + 2y - z = 2$$

$$-3x + y = 2$$

$$4x + y - 3z = 3$$

Solve this system using matrix reduction.

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ -3 & 1 & 0 & 2 \\ 4 & 1 & -3 & 3 \end{array}\right] \xrightarrow{\substack{R_2: R_2+3R_1 \\ R_3: R_3-4R_1}} \sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 7 & -3 & 8 \\ 0 & -7 & 1 & -5 \end{array}\right] \xrightarrow{R_3: R_3+R_2} \sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 7 & -3 & 8 \\ 0 & 0 & -2 & 3 \end{array}\right] \xrightarrow{R_3: -\frac{1}{2}R_3} \sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 7 & -3 & 8 \\ 0 & 0 & 1 & -\frac{3}{2} \end{array}\right]$$

$$\xrightarrow{\substack{R_1: R_1+R_3 \\ R_2: R_2+3R_3}} \sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & \frac{1}{2} \\ 0 & 7 & 0 & \frac{7}{2} \\ 0 & 0 & 1 & -\frac{3}{2} \end{array}\right] \xrightarrow{R_2 \to \frac{1}{7}R_2} \sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{3}{2} \end{array}\right] \xrightarrow{R_1: R_1-2R_2} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{3}{2} \end{array}\right]$$

$x = -\frac{1}{2}$

$y = \frac{1}{2}$

$z = -\frac{3}{2}$

$-\frac{9}{2}$

3. (12 points) Determine the value(s) of real numbers a and b so that the system of equations:

$$x - y + 2z = 4$$
$$3x - 2y + 9z = 14$$
$$2x - 4y + az = b$$

has (i) no solution; (ii) a unique solution; (iii) infinitely many solutions.

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 3 & -2 & 9 & 14 \\ 2 & -4 & a & b \end{array}\right] \xrightarrow[R_3 : R_3 - 2R_1]{R_2 : R_2 - 3R_1} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & -2 & a-4 & b-8 \end{array}\right] \xrightarrow[R_3 : R_3 + 2R_2]{R_1 : R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 5 & 6 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & a+2 & b-4 \end{array}\right]$$

(i) no solution

to achieve no solution, must be inconisistent (ie. $0x + 0y + 0z = -4$)

let $a+2 = 0 \Rightarrow a = -2$

then $b-4$ cannot be 0 ie. $b-4 \neq 0 \Rightarrow b \neq 4$ (or else infinite solutions)

(ii) unique solution

pick anything such that is consistent, and not infinite solutions

let $a+2 \neq 0$ ie. $a \neq -2$

then b can be anything, ie. $b \in \mathbb{R}$

(iii) infinite many solutions

want more variables than rows, so make last row $0x + 0y + 0z = 0$

let $a+2 = 0 \Rightarrow a = -2$

then $b-4 = 0 \Rightarrow b = 4$