T22

MATA33W-Quiz2

Score _____/30

Tutorial Number_____

Student's Name _____ Student's id _____

Question #1 (a) Construct the matrix $A = [A_{ij}]$ if A is 2*3 and $A_{ij} = -i+2j$

(b) Construct the 2*4 matrix $C = [(i+j)^2]$

(8 marks)

a) $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \end{bmatrix}$

b) $C = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} = \begin{bmatrix} 4 & 9 & 16 & 25 \\ 9 & 16 & 25 & 36 \end{bmatrix}$

Question #2 Express the matrix equation $x\begin{bmatrix}3\\2\end{bmatrix} - y\begin{bmatrix}-4\\7\end{bmatrix} = 3\begin{bmatrix}2\\4\end{bmatrix}$ as a system of linear equations and solve.

(6 marks)

$3x + 4y = 6$

$2x - 7y = 12$

$2\times① \quad 6x + 8y = 12$

$-\ 3\times② \quad 6x - 21y = 36$

$29y = -24$

$y = -\frac{24}{29}$

$3x + 4\left(-\frac{24}{29}\right) = 6$

$3x - \frac{96}{29} = 6$

$3x = \frac{174}{29} + \frac{96}{29}$

$3x = \frac{270}{29} \qquad x = \frac{90}{29}$

180 - 6

Question #3 A stockbroker sold a customer 200 shares of stock A, 300 shares of stock B, 500 shares of stock C, and 250 shares of stock D. The prices per share of A, B, C, and D are \$100, \$150, \$200, and \$300, respectively. Write a row vector representing the number of shares of each stock bought. Write a column vector representing the price per share of each stock. Using matrix multiplication, find the total cost of the stocks. (8 marks)

20K
45K
10K
75K
240K

# of stocks bought = $\begin{bmatrix} 200 & 300 & 500 & 250 \end{bmatrix}$

price per share of each stock = $\begin{bmatrix} 100 \\ 150 \\ 200 \\ 300 \end{bmatrix}$

Total cost = $\begin{bmatrix} 200 & 300 & 500 & 250 \end{bmatrix} \begin{bmatrix} 100 \\ 150 \\ 200 \\ 300 \end{bmatrix}$

$= \begin{bmatrix} 20000 + 45000 + 100000 + 75000 \end{bmatrix}$

$= \begin{bmatrix} 240\,000 \end{bmatrix}$

Question #4 A company has taxable income of \$312,000. The federal tax is 25% of that portion that is left after the state tax has been paid. The state tax is 10% of that portion that is left after the federal tax has been paid. Find the company's federal and state taxes.(8 marks)

let x be federal tax in \$

let y be state tax in \$

$x = 0{,}25(312K - y) = 78000 - 0{,}25y \Rightarrow x + 0.25y = 78000$

$y = 0.10(312K - x) = 31200 - 0.10x \Rightarrow 0.10x + y = 31200$

$$\begin{bmatrix} 1 & 0{,}25 & 78\,000 \\ 0{,}1 & 1 & 31200 \end{bmatrix} \xrightarrow{R_2 : 10R_2} \sim \begin{bmatrix} 1 & 0{,}25 & 78000 \\ 1 & 10 & 312\,000 \end{bmatrix} \xrightarrow{R_2 : R_2 - R_1} \sim \begin{bmatrix} 1 & 0{,}25 & 78\,000 \\ 0 & 9{,}75 & 234\,000 \end{bmatrix} \xrightarrow{R_2 : \frac{1}{10}R_2} \sim \begin{bmatrix} 1 & \frac{10}{40} & 78\,000 \\ 0 & \frac{39}{40} & 23400 \end{bmatrix}$$

$\frac{39}{4}$

$$\xrightarrow{R_2 : \frac{40}{39}R_2} \sim \begin{bmatrix} 1 & \frac{10}{40} & 78\,000 \\ 0 & 1 & 24\,000 \end{bmatrix} \xrightarrow{R_1 : R_1 - \frac{1}{4}R_2} \sim \begin{bmatrix} 1 & 0 & 72000 \\ 0 & 1 & 24000 \end{bmatrix}$$

$\therefore x = 72K$

$y = 24K$

$\frac{40}{39}(23400) = 23400 + \frac{1}{39}(23400)$

$= 23400 + 600$

$= 24000$

$\frac{234}{39} = 6 \qquad \frac{240}{40} = 6$