T10

MATA33
QUIZ 2

Tutorial #:
Student #:

First Name:
Last Name:

Question 1 (10 points)

3×4 4×3

75+9

50
2

Construct a matrix $A=[A_{ij}]$ with size 4×3 such that $A_{ij}=2i-j$. Compute A^TA.

a) $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 5 & 4 & 3 \\ 7 & 6 & 5 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 2 & 4 & 6 \\ -1 & 1 & 3 & 5 \end{bmatrix}$

$$A^TA = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 2 & 4 & 6 \\ -1 & 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 5 & 4 & 3 \\ 7 & 6 & 5 \end{bmatrix} = \begin{bmatrix} 1+9+25+49 & 0+6+20+42 & -1+3+15+35 \\ 0+6+20+42 & 0+4+16+36 & 0+2+12+30 \\ -1+3+15+35 & 0+2+12+30 & 1+1+9+25 \end{bmatrix}$$

$$= \begin{bmatrix} 84 & 68 & 52 \\ 68 & 56 & 44 \\ 52 & 44 & 36 \end{bmatrix}$$

notice, A^TA is a symmetric matrix

Also, diagonals are sums are $a_{i1}^2 + a_{i2}^2 + a_{i3}^2 + a_{i4}^2$

Question 2 (10 points)

Discuss the scenarios for the following system of equations to have no solution, unique solution, and infinitely many solutions.

$$\begin{cases} x + ay = 2 \\ 3x + 2y = b \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & a & 2 \\ 3 & 2 & b \end{array}\right] \xrightarrow{R_2 : R_2 - 3R_1} \sim \left[\begin{array}{cc|c} 1 & a & 2 \\ 0 & 2-3a & b-6 \end{array}\right]$$

(i) no solution

- to achieve no solution, must be inconsistent (i.e. $0x+0y+0z=5$)

let $2-3a=0 \Rightarrow a=\frac{2}{3}$

then $b-6 \neq 0 \Rightarrow b \neq 6$

(ii) unique solution

- anything that is not inconsistent & not infinite solution

let $2-3a \neq 0 \Rightarrow a \neq \frac{2}{3}$

then b can be anything i.e. $b \in \mathbb{R}$

(iii) infinite many solutions

- want more variables than rows, so make last row $0x+0y+0z=0$

let $a-3a=0 \Rightarrow a=\frac{2}{3}$

then $b-6=0 \Rightarrow b=6$

Question 3 (10 points)

Using matrix reduction, solve the following equation set and express your solution in a parametric form.

$-3+4$

$$x + 2y + 5z + 5w = -3$$
$$x + y + 3z + 4w = -1$$
$$x - y - z + 2w = 3$$

$R_2: R_2 - R_1$
$R_3: R_3 - R_1$

$R_3: R_3 - 3R_2$

$R_2 \to -R_2$

$$\left[\begin{array}{cccc|c} 1 & 2 & 5 & 5 & -3 \\ 1 & 1 & 3 & 4 & -1 \\ 1 & -1 & -1 & 2 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 5 & 5 & -3 \\ 0 & -1 & -2 & -1 & 2 \\ 0 & -3 & -6 & -3 & 6 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 5 & 5 & -3 \\ 0 & -1 & -2 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 5 & 5 & -3 \\ 0 & 1 & 2 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1: R_1 - 2R_2$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 3 & 1 \\ 0 & 1 & 2 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$X = -z - 3w + 1 = -r - 3s + 1$

$Y = -2z - w - 2 = -2r - s - 2$

z is free var, let z be $r \in \mathbb{R}$

w is free var, let w be $s \in \mathbb{R}$

$$\begin{bmatrix} X \\ Y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -r - 3s + 1 \\ -2r - s - 2 \\ r \\ s \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix} + r \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -3 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$