MATA33	Tutorial #:	First Name:
QUIZ 3	Student #:	Last Name:

1.(8 points) Let a, b and c be fixed real numbers and let x, y and z be variables. Use Cramer's rule to solve for x in the linear systems:

$$M = \begin{matrix} 2x + y + z = a \\ -x + 0y + 2z = b \\ 3x + y + 3z = c \end{matrix}$$

$$D_x = \begin{vmatrix} a & 1 & 1 \\ b & 0 & 2 \\ c & 1 & 3 \end{vmatrix} = (-1)^3 b \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} + (-1)^4(0)\begin{vmatrix} & \\ & \end{vmatrix} + (-1)^5(2)\begin{vmatrix} a & 1 \\ c & 1 \end{vmatrix}$$

$$= -b(3-1) + 0 - 2(a-c)$$

$$= -2b - 2a + 2c$$

$$D = \begin{vmatrix} 2 & 1 & 1 \\ -1 & 0 & 2 \\ 3 & 1 & 3 \end{vmatrix} = (-1)^3(-1)\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} + (-1)^4(0)\begin{vmatrix} & \\ & \end{vmatrix} + (-1)^5(2)\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix}$$

$$= (3-1) + 0 - 2(2-3)$$

$$= (2) - 2(-1)$$

$$= 4$$

$$X = \frac{D_x}{D} = \frac{-2b-2a+2c}{4}$$

$$= -\frac{b}{2} - \frac{a}{2} + \frac{c}{2}$$

2. a) (4 points) Let P and Q be 4 *4 matrices such that det(P) = 3, det(Q) = -2
Find: det(-2(P^{-1})) and det(($(3P)^{-1}Q^2$)

$$= (-2)^4 \det(P^{-1})$$

$$= 16 \cdot \frac{1}{\det(P)}$$

$$= 16 \cdot \frac{1}{3}$$

$$= \frac{16}{3}$$

$$= \det\left(\tfrac{1}{3}P^{-1}Q^2\right)$$

$$= \left(\tfrac{1}{3}\right)^4 \det(P^{-1}) \det(Q^2)$$

$$= \left(\tfrac{1}{3}\right)^4 \cdot \frac{1}{\det(P)} \cdot \det(Q)^2$$

$$= \left(\tfrac{1}{3}\right)^4 \cdot \frac{1}{3} \cdot (-2)^2$$

$$= \frac{4}{243}$$

b) (6 points) Suppose A is 3*3 matrix such that $A^4 - 2A^3 + 5A^2 - 2I = 0$. Show that A is invertible.

def of invertibility : $AA^{-1} = I$

$$A^4 - 2A^3 + 5A^2 - 2I = 0$$

$$A^4 - 2A^3 + 5A^2 = 2I \qquad \therefore A^{-1} = \frac{1}{2}A^3 - A^2 + \frac{5}{2}A$$

$$A(A^3 - 2A^2 + 5A) = 2I$$

$$\cancel{2}(A)\left(\tfrac{1}{2}A^3 - A^2 + \tfrac{5}{2}A\right) = \cancel{2}I$$

$$A\underbrace{\left(\tfrac{1}{2}A^3 - A^2 + \tfrac{5}{2}A\right)}_{A^{-1}} = I$$

$36-5(7)$
$= 36-35$

$3-8$ $\quad$ $a-2(-17)$ $= 2+34$ $\quad$ $-13-5(-\frac{5}{2})$ $= -13+\frac{25}{2} = -\frac{1}{2}$

3.(12 points) Let A= $\begin{bmatrix} 2 & 5 & 3 \\ 1 & 2 & 3 \\ 1 & 0 & 8 \end{bmatrix}$, B= $\begin{bmatrix} 2 \\ -3 \\ -17 \end{bmatrix}$, express the matrix B as the sum of scalar multiples of the columns of A.

$$AX = B \iff \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} a_{11}X_1 + a_{12}X_2 \\ a_{21}X_1 + a_{22}X_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \iff \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}X_1 + \begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix}X_2 = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & 5 & 3 & 2 \\ 1 & 2 & 3 & -3 \\ 1 & 0 & 8 & -17 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 0 & 8 & -17 \\ 1 & 2 & 3 & -3 \\ 2 & 5 & 3 & 2 \end{array}\right] \xrightarrow[R_3 : R_3 - 2R_1]{R_2 : R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 0 & 8 & -17 \\ 0 & 2 & -5 & 14 \\ 0 & 5 & -13 & 36 \end{array}\right] \xrightarrow[R_3 : R_3 - 5R_2]{R_2 : \frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 0 & 8 & -17 \\ 0 & 1 & -\frac{5}{2} & 7 \\ 0 & 0 & -\frac{1}{2} & 1 \end{array}\right] \xrightarrow{R_3 : -2R_3} \left[\begin{array}{ccc|c} 1 & 0 & 8 & -17 \\ 0 & 1 & -\frac{5}{2} & 7 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\xrightarrow[R_2 : R_2 + \frac{5}{2}R_3]{R_1 : R_1 - 8R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}X_1 + \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}X_2 + \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix}X_3 = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$X_1 = -1$
$X_2 = 2$
$X_3 = -2$

$$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}(-1) + \begin{bmatrix} 5 \\ 2 \\ 0 \end{bmatrix}(2) + \begin{bmatrix} 3 \\ 3 \\ 8 \end{bmatrix}(-2) = \begin{bmatrix} 2 \\ -3 \\ -17 \end{bmatrix}$$