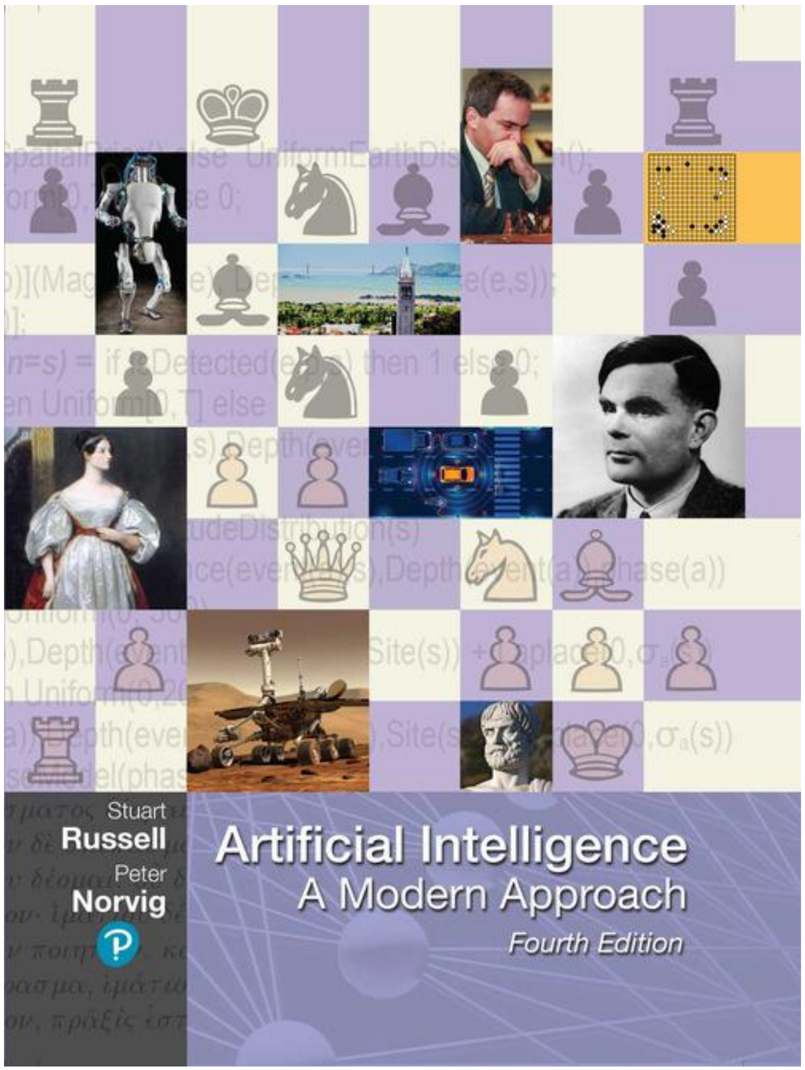
Artificial Intelligence: A Modern Approach

Fourth Edition



Chapter 3

Solving Problems By Searching

Pearson

Outline

- Problem-solving agents
- Example Problems
- Problem formulation
- Search Algorithms
- Uninformed Search Strategies
- Informed (Heuristic) Search Strategies
- Heuristic Functions

Introduction

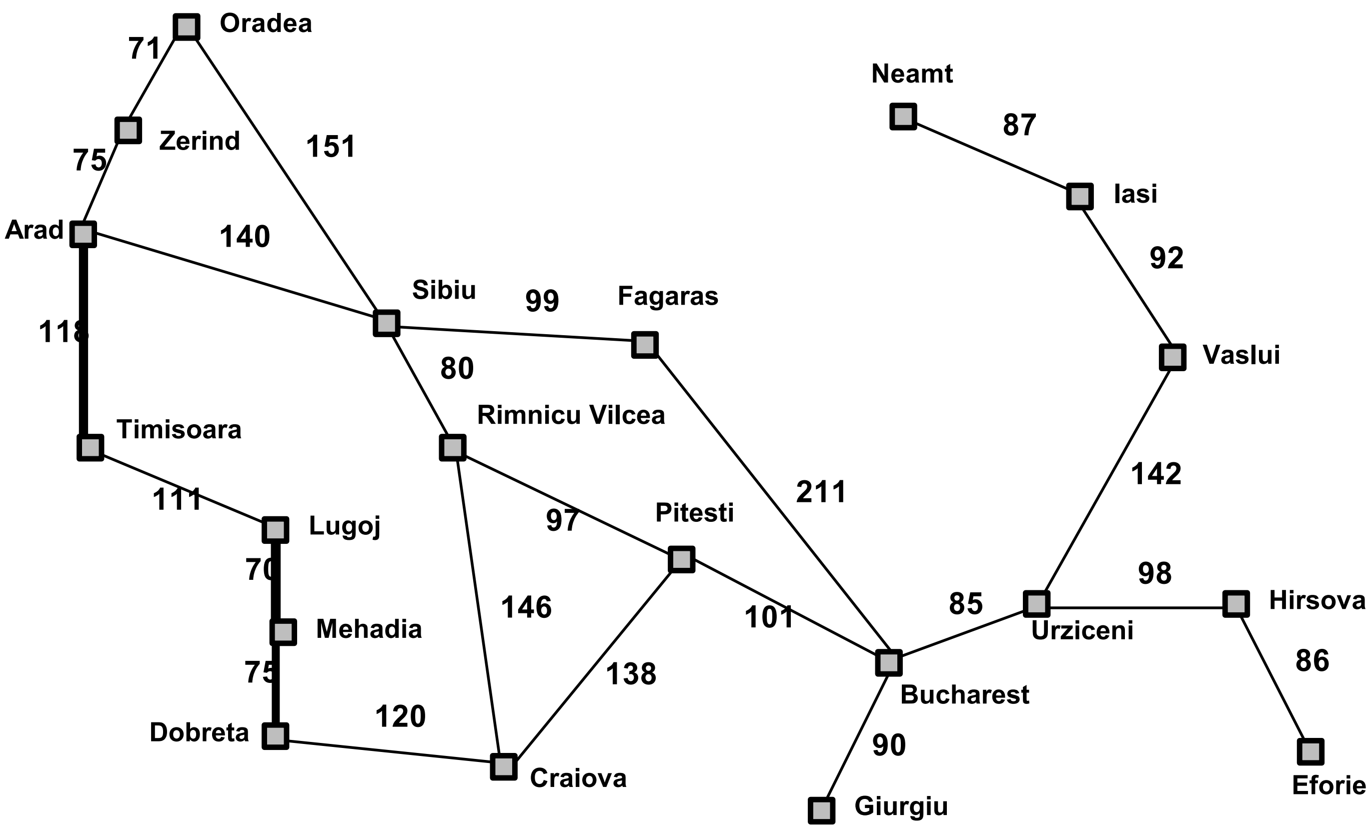
- This section shows how an informed search strategy—one that uses domain-specific hints about the location of goals—can find solutions more efficiently than an uninformed strategy.

- The hints come in the form of a heuristic function, denoted h(n)

h(n) = estimated cost of the cheapest path from the state at node n to the goal

- For example, in route-finding problems, we can estimate the distance from the current state to a goal by computing the straight-line distance on the map between the two points.

Romania with step costs in km



Straight−line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
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Zerind	374

Greedy search

- **Greedy best-first search** is a form of best-first search that expands first the node with the lowest value—the node that appears to be closest to the goal—on the grounds that this is likely to lead to a solution quickly.

- So, the evaluation function f(n) = h(n)

- Notice that the values of h_{sld} cannot be computed from the problem description itself (that is, the ACTIONS and RESULT functions).

- It takes a certain amount of **world knowledge** to know that h_{sld} is **correlated** with actual road distances and is, therefore, a useful **heuristic**.

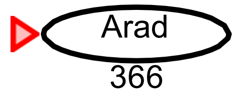
Greedy search

Evaluation function $h(n)$ (heuristic)
= estimate of cost from n to the closest goal

E.g., $h_{\mathrm{SLD}}(n)$ = straight-line distance from n to Bucharest

Greedy search expands the node that appears to be closest to goal

Greedy search example



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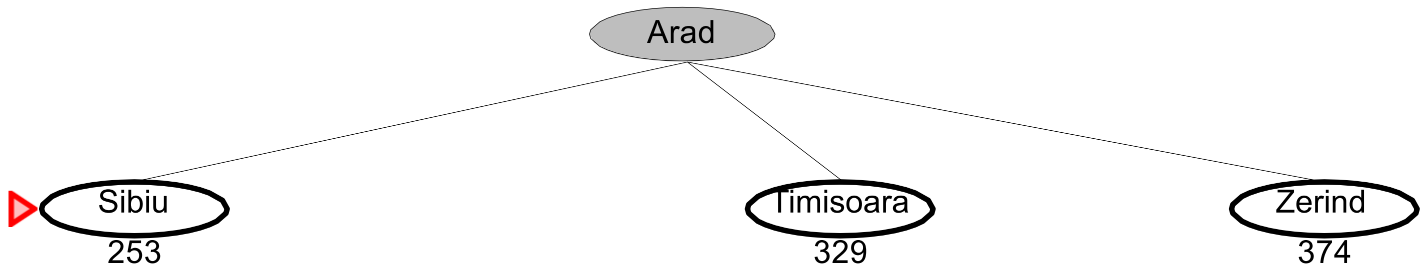
Greedy search example



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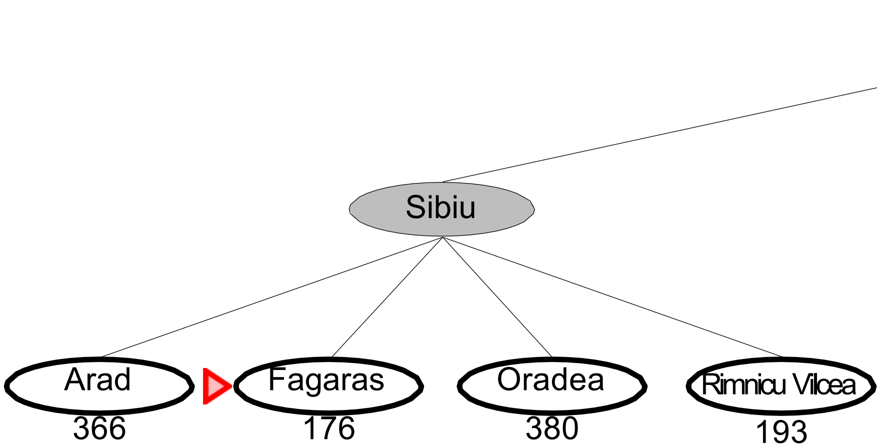
Greedy search example



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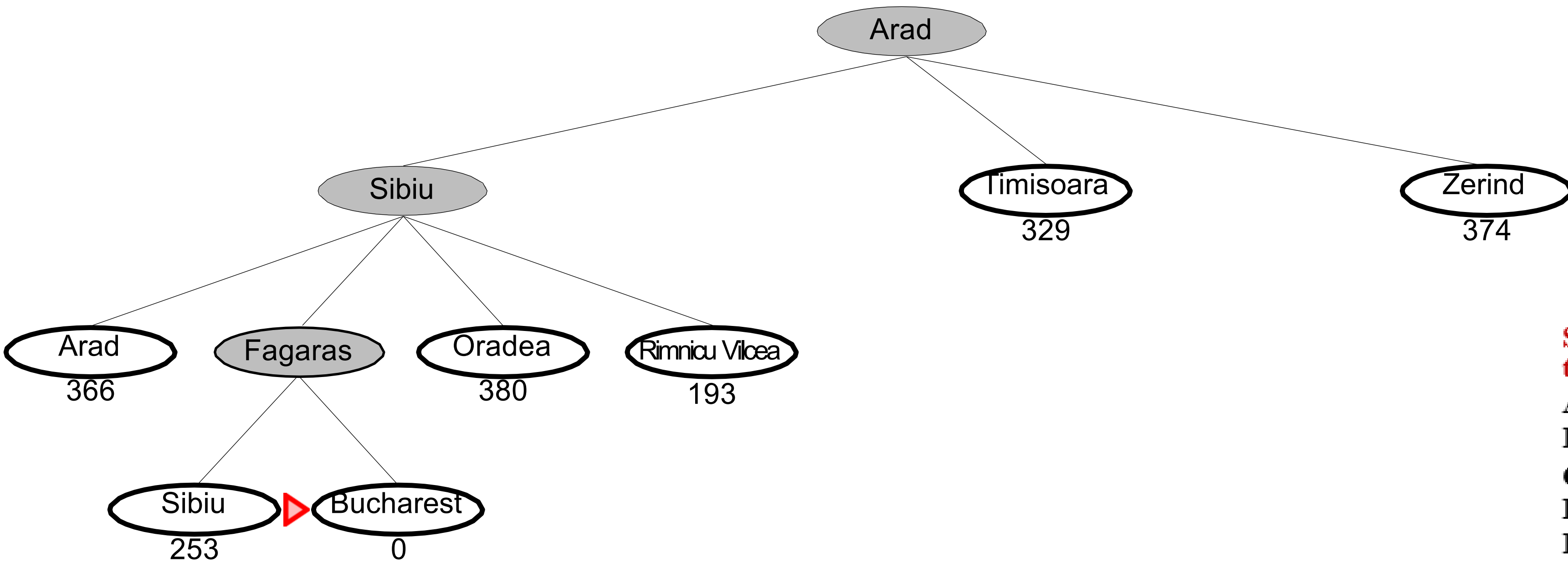
Greedy search example



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Greedy search

- For this particular problem, greedy best-first search using h_{sld} finds a solution without ever expanding a node that is not on the solution path.

- The solution it found does not have optimal cost, however ☹

- The path via Sibiu and Fagaras to Bucharest is 32 miles longer than the path through Rimnicu Vilcea and Pitesti.

- This is why the algorithm is called **"greedy"**—on each iteration it tries to get as close to a goal as it can, but greediness can lead to worse results than being careful.

Greedy search

- Greedy best-first graph search is complete in finite state spaces, but not in infinite ones.

- With a good heuristic function, the complexity can be reduced substantially, on certain problems reaching **O(bm)**

Properties of greedy search

Complete??

Properties of greedy search

Complete?? No–can get stuck in loops, e.g., with Oradea as goal,

Iasi → Neamt → Iasi → Neamt →

Complete in finite space with repeated-state checking

Time??

Properties of greedy search

Complete?? No–can get stuck in loops, e.g.,

Iasi → Neamt → Iasi → Neamt →

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space??

Properties of greedy search

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Iasi → Neamt → Iasi → Neamt →

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$—keeps all nodes in memory

Optimal??

Properties of greedy search

Complete?? No–can get stuck in loops, e.g., Iasi
→ Neamt → Iasi → Neamt →
Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$—keeps all nodes in memory

Optimal?? No

A* search

- The most common informed search algorithm is A* search (pronounced “A-star search”),

- It is a **best-first search** that uses the evaluation function

  $f(n) = g(n) + h(n)$

  where:
  $g(n)$ is the path cost from the initial state to node n and
  $h(n)$ is the estimated cost of the shortest path from n to a goal state,
  so we have

  f(n) = estimated cost of the best path that <u>continues</u> from n to goal

A* search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

$g(n)$ = real/actual cost so far to reach n

$h(n)$ = estimated cost to goal from n

$f(n)$ = estimated total cost of path through n to goal

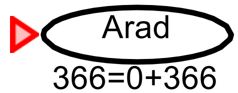
A* search uses an **admissible** heuristic

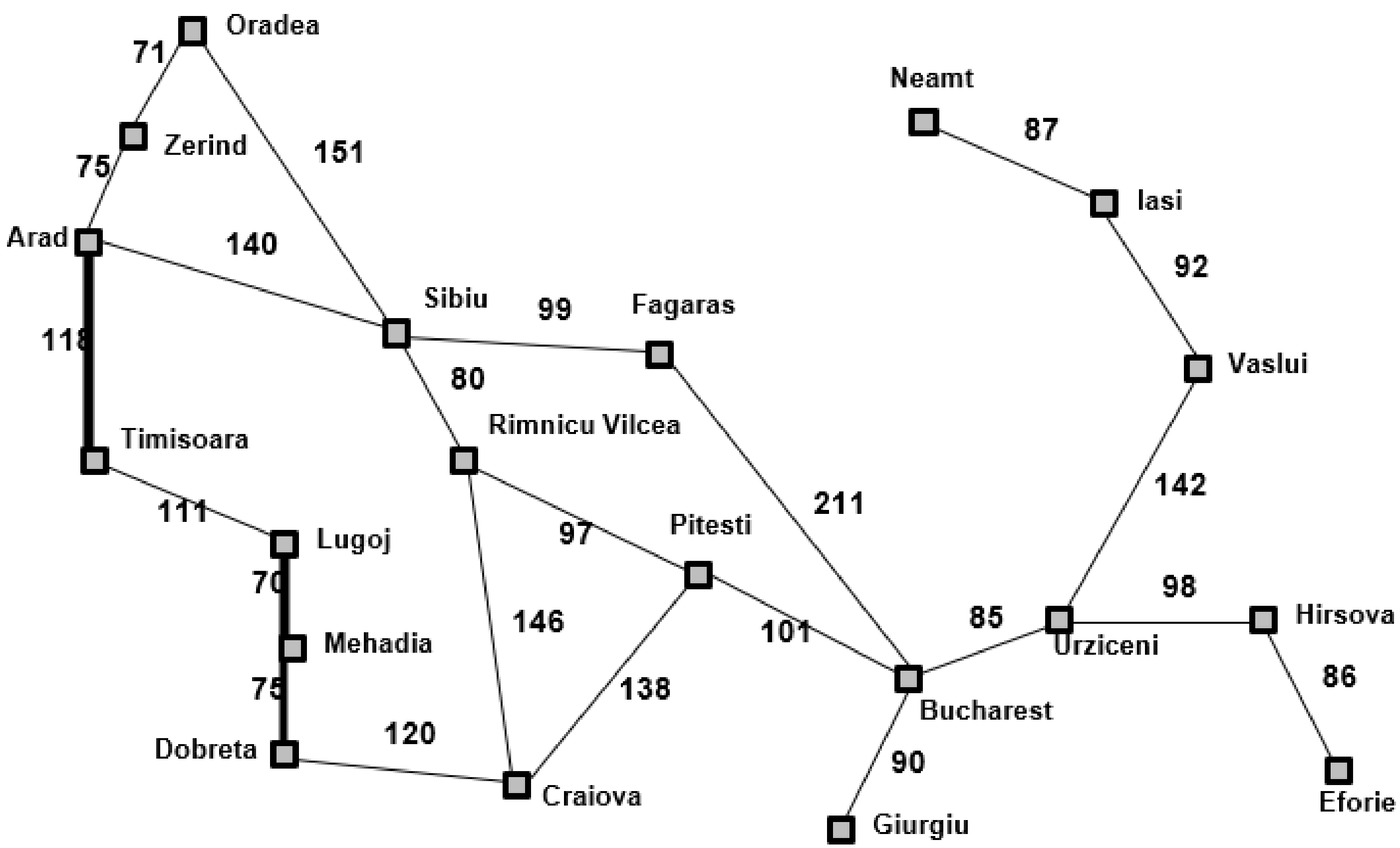
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the true cost from n.
(Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G.)

E.g., $h_{\mathrm{SLD}}(n)$ **never overestimates the actual road distance**

Theorem: A* search is optimal

A^* search example





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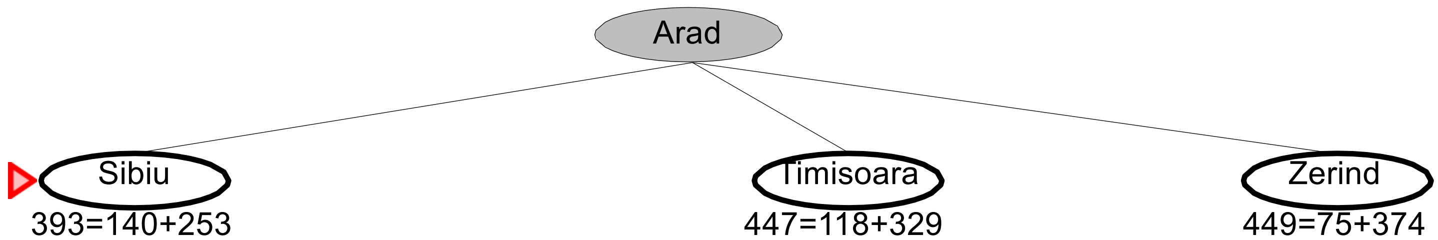
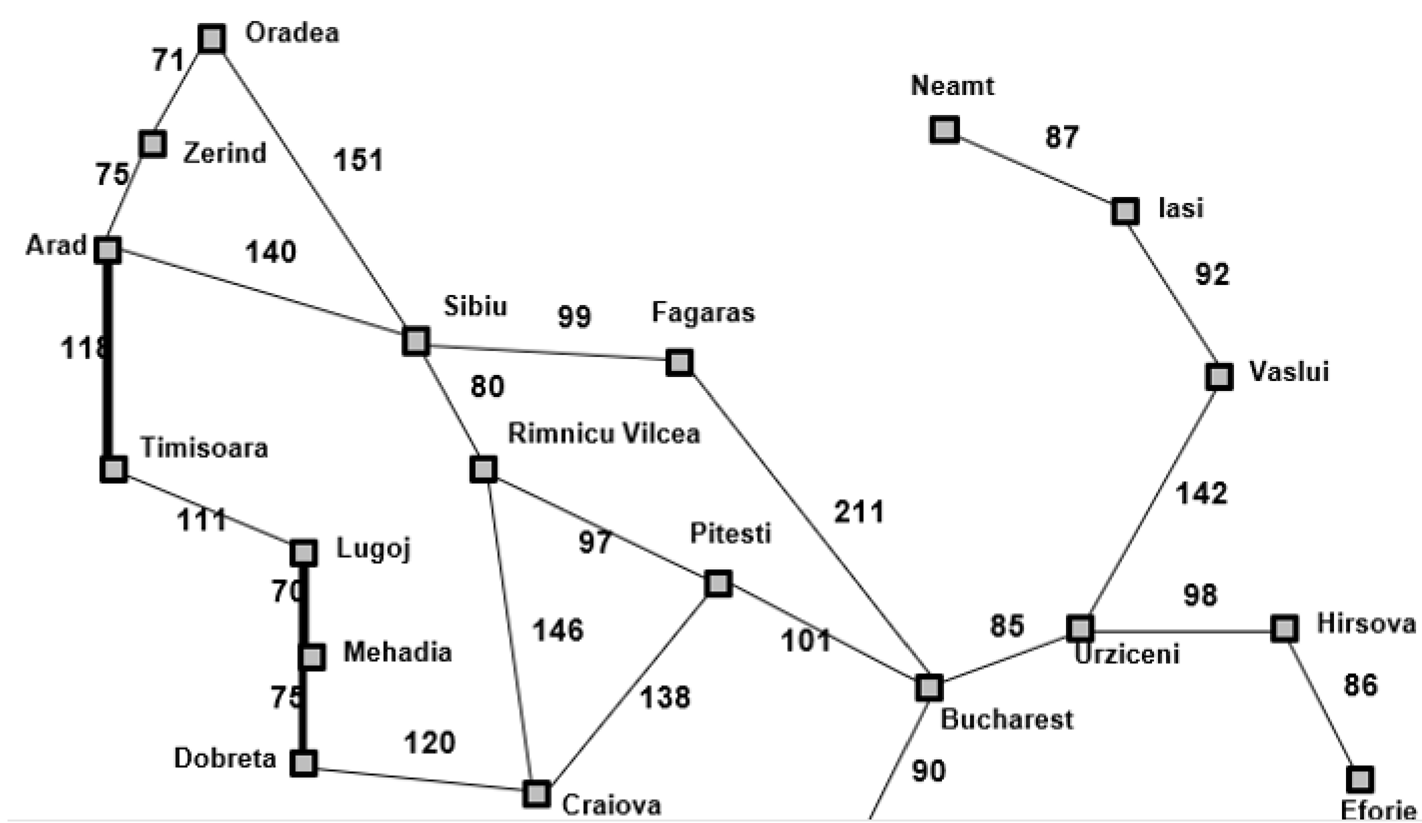
A* search example





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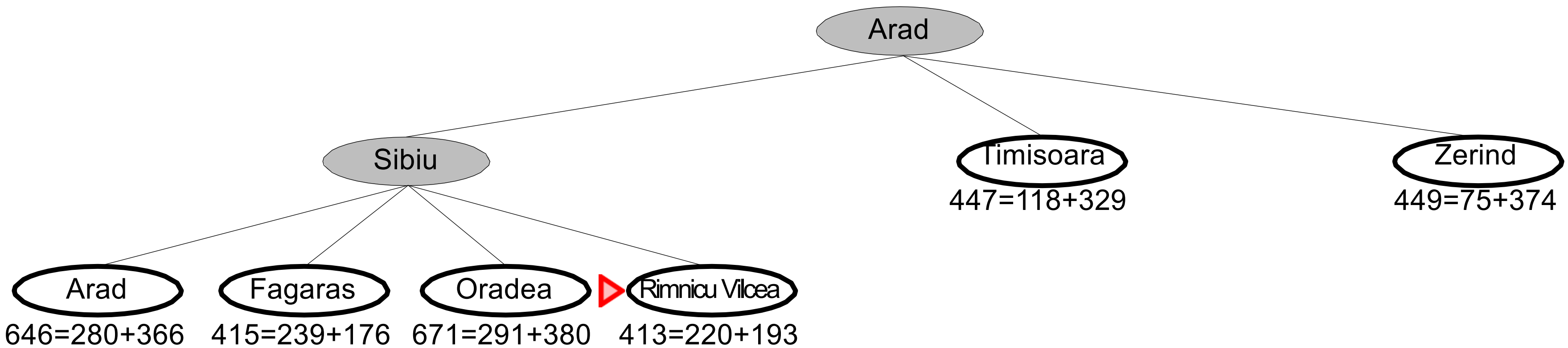
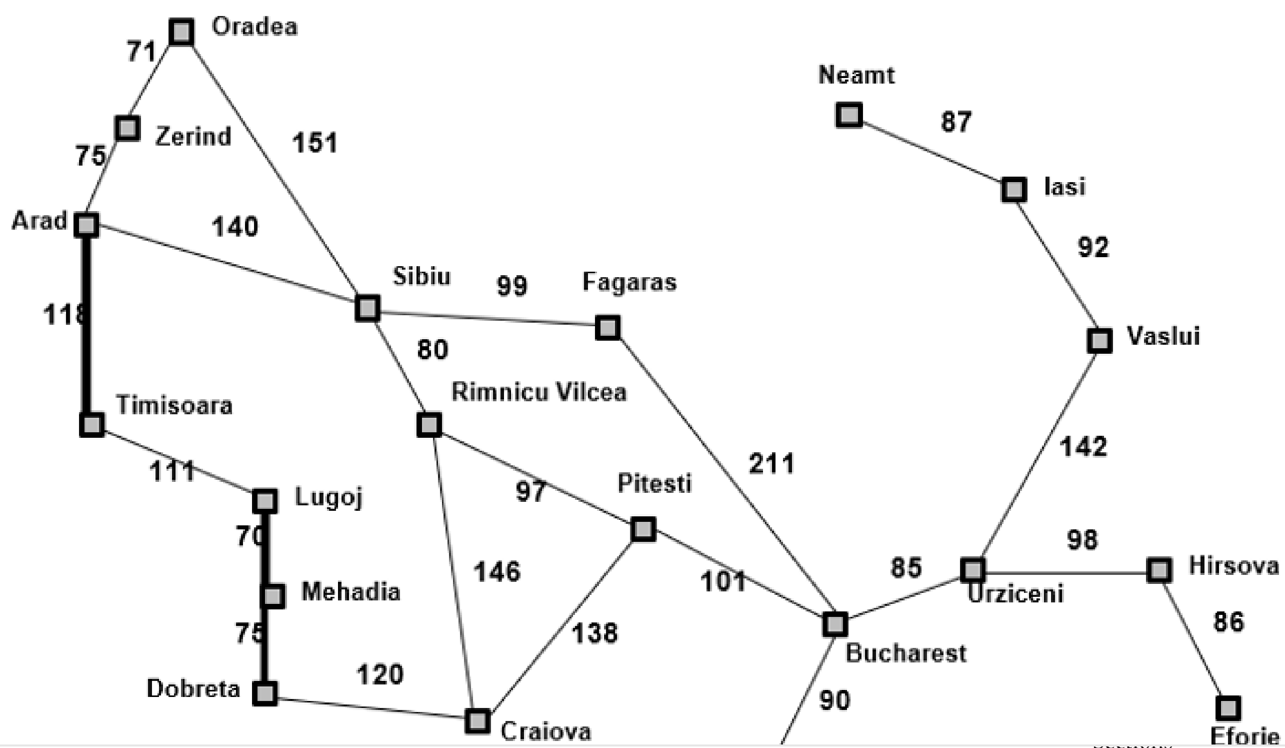
A* search example





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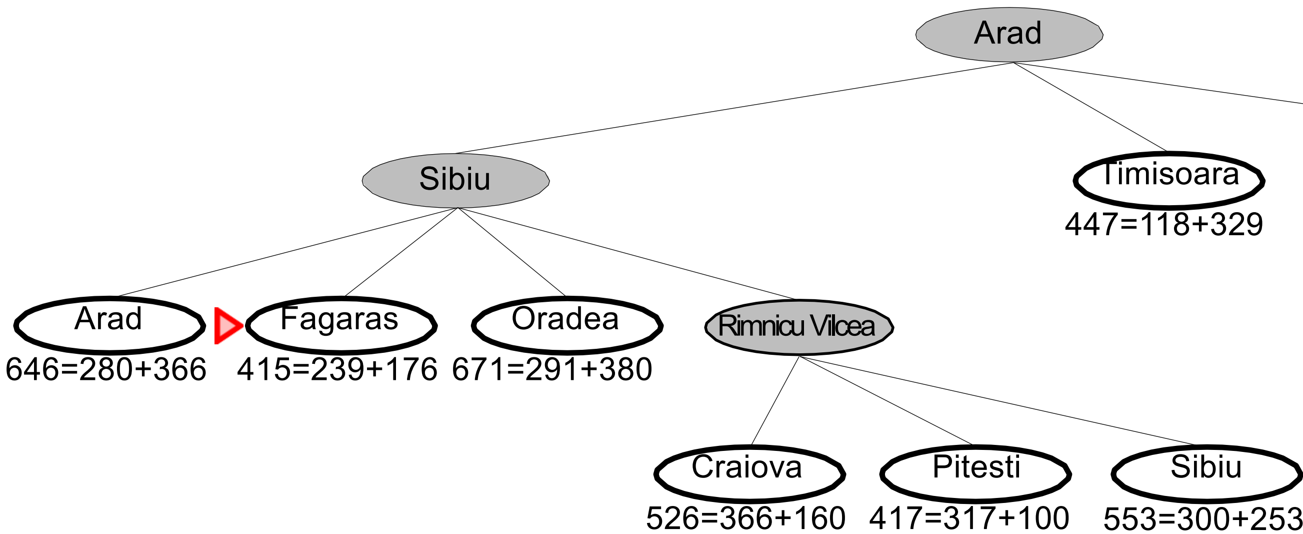
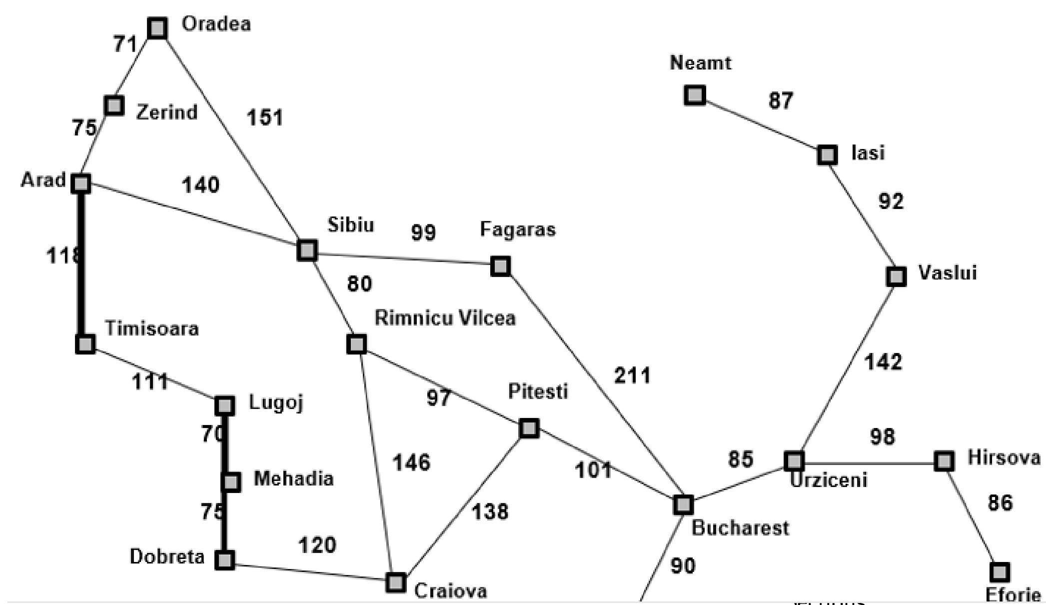
A* search example





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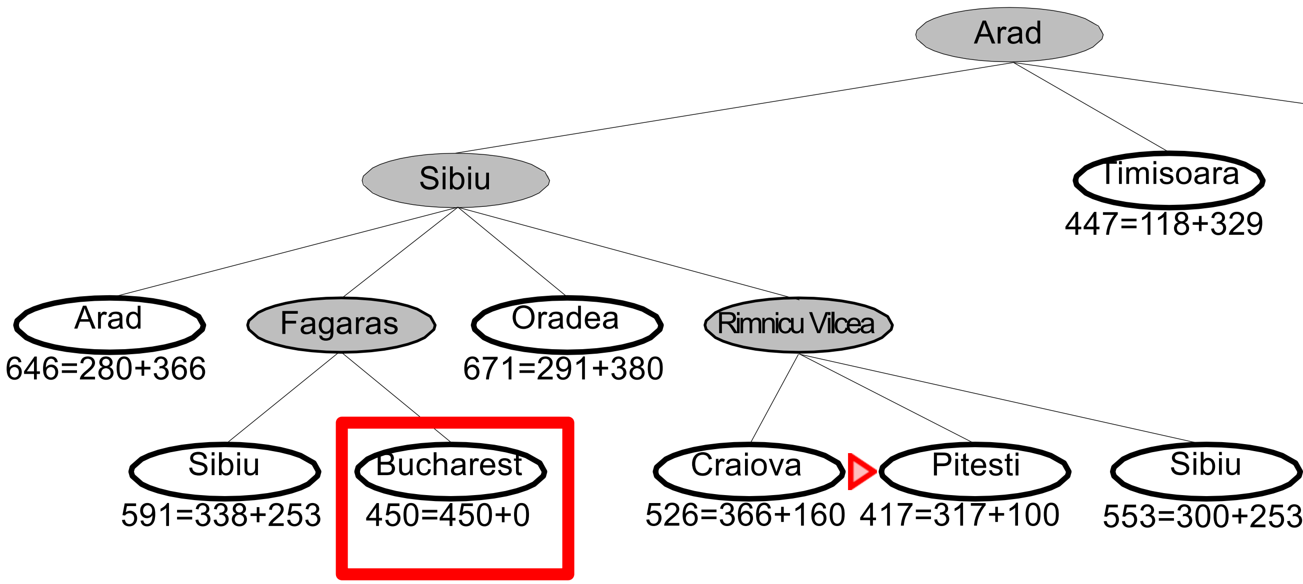
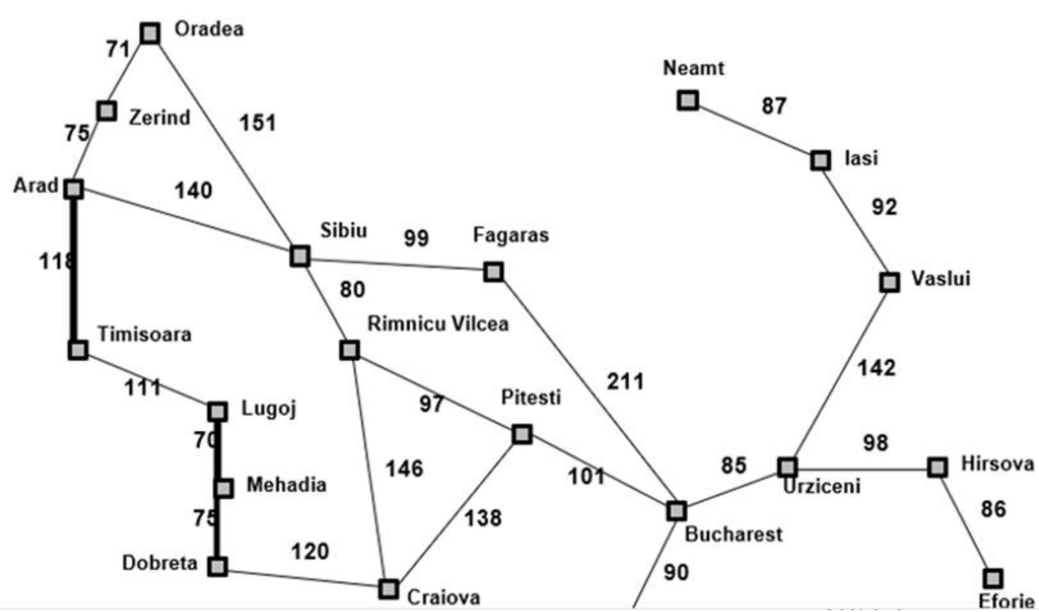
A* search example





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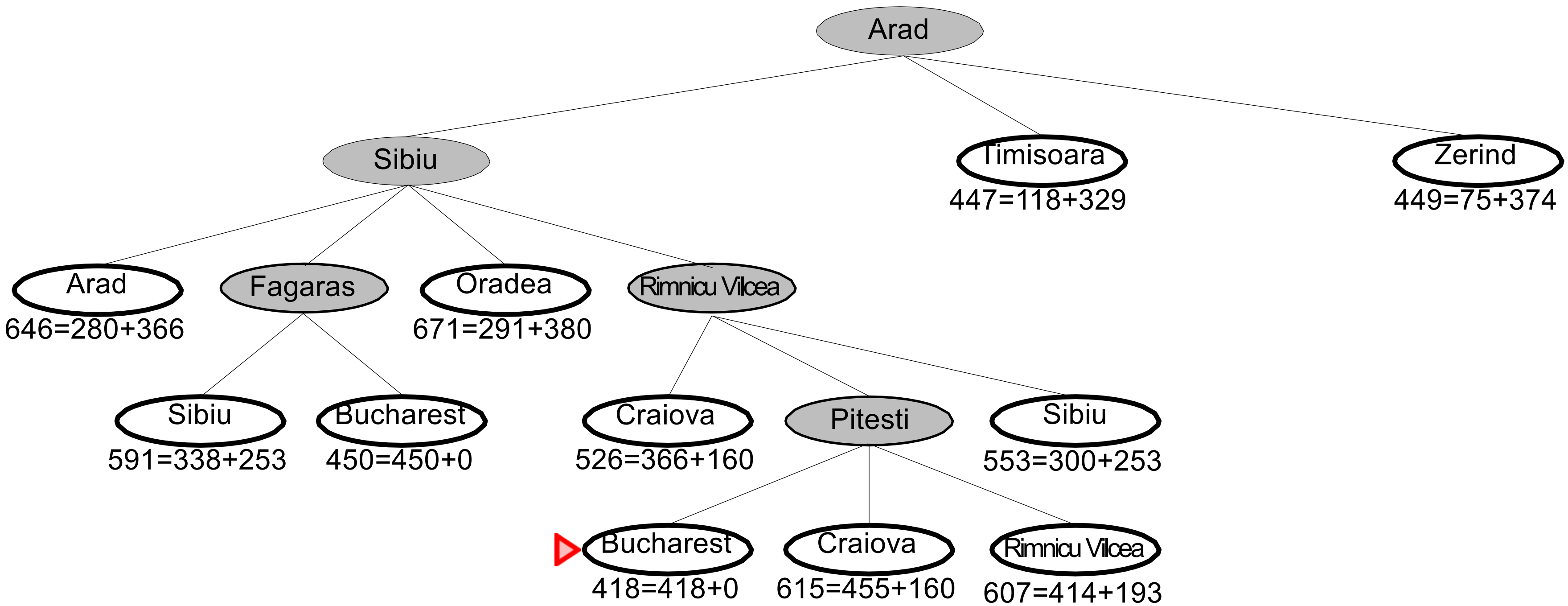
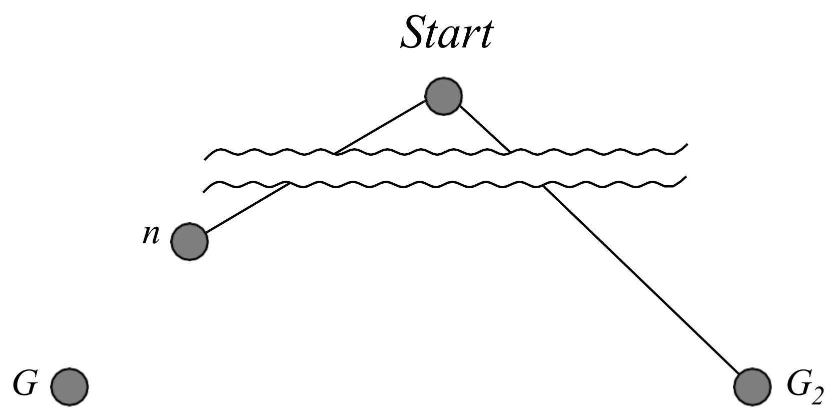
A* search example



Optimality of A* (standard proof)

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G.



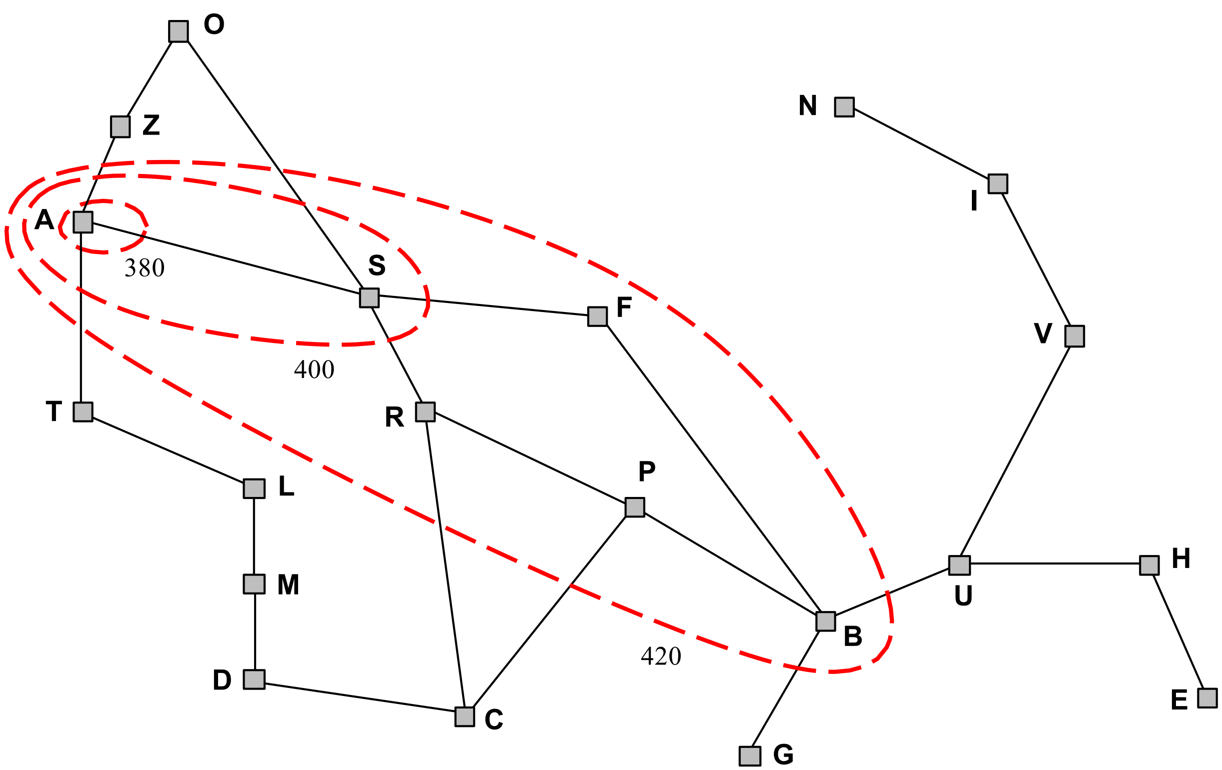
$$
\begin{aligned}
f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\
&> g(G) && \text{since } G_2 \text{ is suboptimal} \\
&\geq f(n) && \text{since } h \text{ is admissible}
\end{aligned}
$$

Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Optimality of A* (more useful)

Lemma: A* expands nodes in order of increasing f value*

Gradually adds "f-contours" of nodes (cf. breadth-first adds layers)
Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Properties of A^*

Complete??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in h $\times$ length of soln.]

Space??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in $h \times$ length of soln.]

Space?? Keeps all nodes in memory

Optimal??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in h $\times$ length of soln.]

Space?? Keeps all nodes in memory

Optimal?? Yes—cannot expand f_{i+1} until f_i is finished

A$*$ expands all nodes with $f(n) < C*$

A$*$expands some nodes with $f(n) = C*$

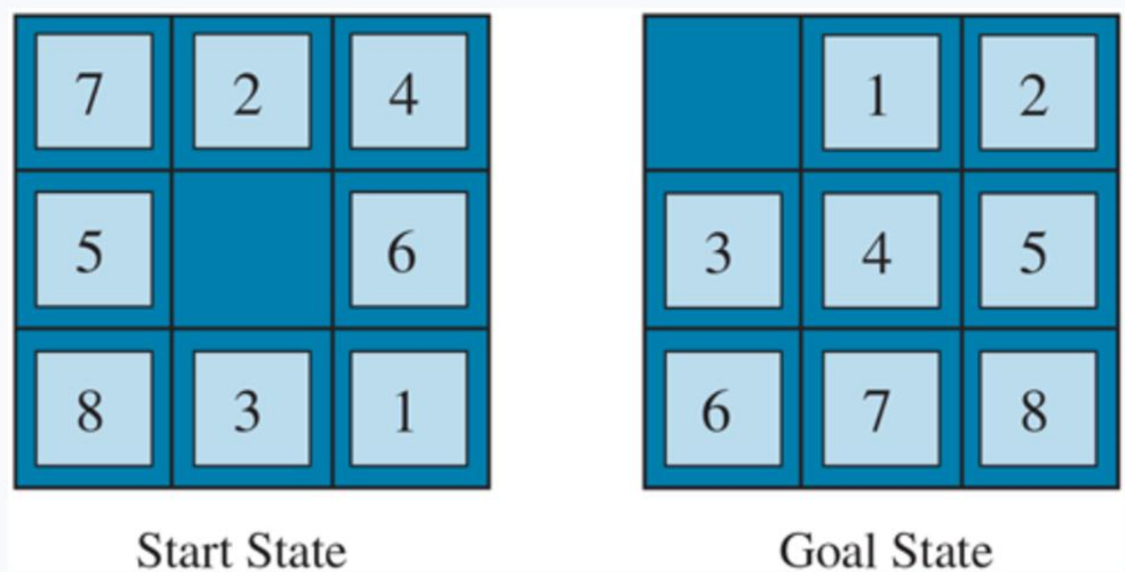
A$*$expands no nodes with $f(n) > C*$

Admissible heuristics

- There are 9!/2 reachable states in an 8-puzzle, so a search could easily keep them all in memory.

- But for the 15-puzzle, there are 16!/2 states—over 10 trillion—so to search that space we will need the help of a good **admissible heuristic function**.

- There is a long history of such heuristics for the 15-puzzle; here are two commonly used candidates:



A typical instance of the 8-puzzle. The shortest solution is 26 actions long.

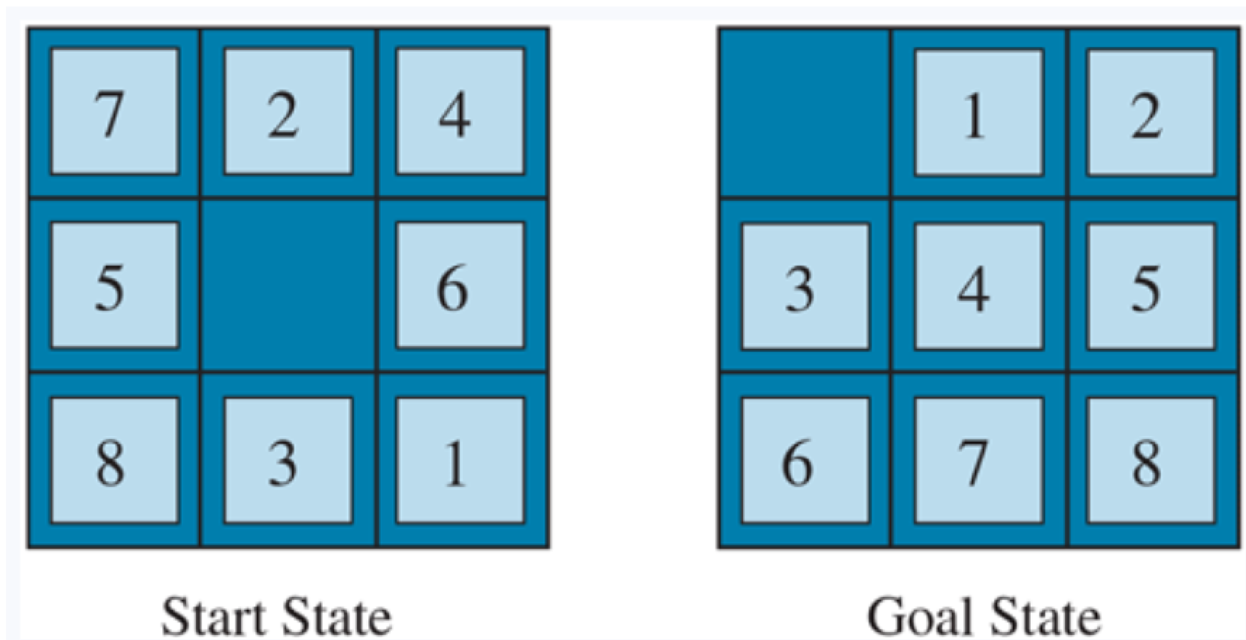
Pearson

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles (blank included)
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)

$h_1(S)$ =??
$h_2(S)$ =??



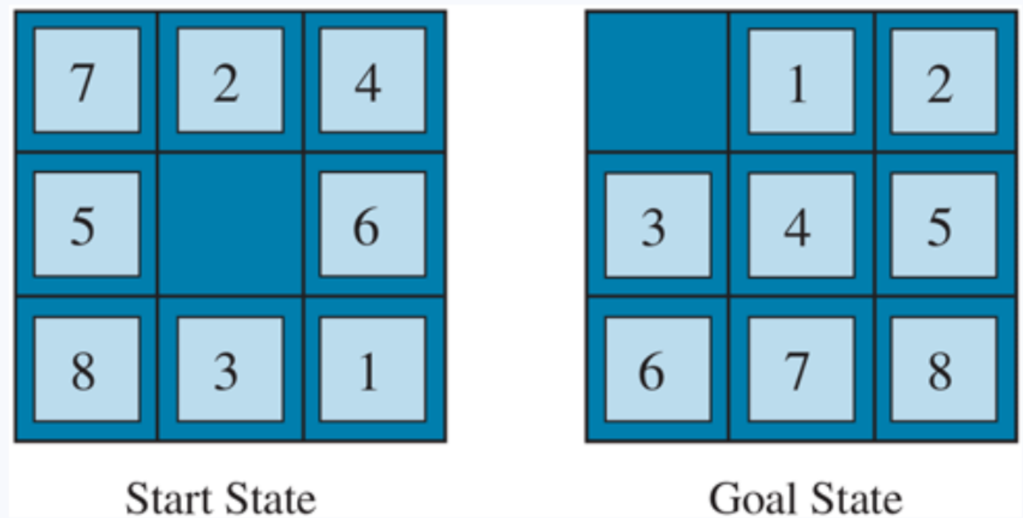
A typical instance of the 8-puzzle. The shortest solution is 26 actions long.

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)

$h_1(S)$ =?? 8
$h_2(S)$ =?? 4+0+3+4+1+0+2+1= 15



A typical instance of the 8-puzzle. The shortest solution is 26 actions long.

Admissible heuristics

- H_1 is an admissible heuristic because any tile that is out of place will require at least one move to get it to the right place.

- H_2 is also admissible because all any move can do is move one tile one step closer to the goal.

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible), then
h_2 **dominates** h_1 and is better for search

Typical search costs:

$d = 14$ IDS = 3,473,941 nodes
A$*(h_1)$ = 539 nodes
A$*(h_2)$ = 113 nodes
$d = 24$ IDS $\approx$ 54,000,000,000 nodes
A$*(h_1)$ = 39,135 nodes
A$*(h_2)$ = 1,641 nodes

Given any admissible heuristics h_a, h_b,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates h_a, h_b

Relaxed problems

Admissible heuristics can be derived from the exact solution cost of a relaxed version of the problem

If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

Summary

A problem consists of five parts: the **initial state**, a set of **actions**, a **transition model** describing the results of those actions, a set of **goal states**, and an **action cost function**.

Uninformed search methods have access only to the **problem definition**. Algorithms build a search tree in an attempt to find a solution.

Informed search methods have access to a **heuristic** function h(n) that estimates the cost of a solution from n.