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REVIEW

IAN HACKING
Why Is There Philosophy of Mathematics At All?
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The philosophy of mathematics occupies a rather peripheral place in the practice of current philosophy. In this thought-provoking book, Ian Hacking argues that mathematics exhibits several striking features that ought to make a properly conceived philosophical investigation of mathematics 'a matter of general philosophy' (p. 81). These elements are the existence of surveyable proofs that convey unshakable conviction in their theorems and the many ways that mathematics is applied outside of mathematics. Hacking combines his enthusiasm for the problems raised by proof and application with a strong reluctance to propose philosophical solutions to these problems. Instead, he tries to stay focused on particular cases from the history and current practice of mathematics. This preoccupation with examples serves to illustrate the many ways that proof and application have developed, and also to highlight the historical contingency of the development of mathematics. Hacking seems to think that the allure of traditional philosophical positions like Platonism and nominalism can be countered by his case-driven approach. He concludes the book by claiming that any philosophical consensus on Platonism 'is unlikely to make much difference to mathematical activity or to what mathematicians think they are doing' (pp. 223–4). So, there are good reasons for philosophers to be preoccupied with mathematics, but philosophers should not expect that their investigations will help to clarify genuinely mathematical problems.

The book begins with a 'Cartesian introduction' that sets out the two themes of proof and application, and their role in the philosophy of mathematics. Descartes was preoccupied with a special sort of proof: 'a certain type of philosophical mind is deeply impressed by *experiencing* a Cartesian proof, of

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seeing why such-and-such *must* be true' (p. 28). Classic examples of such proofs are the doubling of the square, the irrationality of the square root of two and the infinity of primes (p. 32). Hacking adds a proof from Littlewood concerning the impossibility of decomposing a cube into cubes of unequal sides (p. 33). Philosophical reflection on Cartesian proofs has a long history, and the ongoing quest for new Cartesian proofs is a central engine in the growth of mathematics. Hacking mentions, for example, Grothendieck's remark that mathematical creation involves 'bringing to light the "obvious" thing that no one had seen' (p. 29). It remains unclear exactly why this phenomenon should be so interesting to philosophers. One suggestion considered in this chapter is that it supports the conviction that 'reason is its own self-authenticating guarantor' (p. 38). Here philosophers encounter what Wittgenstein dubbed '"bedrock", within which all discourse makes sense' (p. 38). In characteristic style, Hacking does not endorse this picture or investigate it further.

Chapter 2 considers what, if anything, might unify all of mathematics and render it a single object of philosophical investigation. Hacking contrasts several 'grand narratives' that seek to distil the essence of all of mathematics throughout history with Wittgenstein's remark on the 'motley' of mathematics (p. 57). Dictionary entries show certain common patterns across languages, but this is merely a preliminary to investigating how mathematics has actually been done. Not all proofs are Cartesian proofs, for example, and the open-ended character of proof does not settle where mathematics ends and something else begins. Ultimately, Hacking endorses a historical 'Latin' model akin to the growth of a language over time. Even if Latin had in fact yielded only one descendent language, the speakers of that language could recognize the viability of other descendants. Similarly, it appears that even though mathematics has evolved over time in one particular way, we should not conclude that it must have evolved in that way or that this contingent evolution reveals anything deep about the nature of mathematics.

Chapter 3 examines the perennial attraction of philosophy of mathematics. Hacking traces the conviction that the subject matter of mathematics is 'out there' to the experience of proof. He notes the remarks of many mathematicians along these lines, including Alain Connes on the finite simple groups.[1] The finite simple groups are said to have been exhaustively characterized through a heroic process of discovery (p. 87). This proto-Platonism is partially rejected: 'what Connes *says* is right but what he *means* is wrong' (p. 93). Strictly speaking, he is right in what he says because these discoveries did

[1] Many of Hacking's remarks about mathematics and mathematicians are independently confirmed by Frenkel ([2013]). Frenkel emphasizes both the search for special sorts of proofs, especially with respect to the Langlands programme, and the surprising ways that such 'deep' mathematics is applied in physics.

happen as he reports. However, he is wrong in positing 'the series of whole numbers, archaic and primordial, whose structures have nothing to do with the brain, except that our brains enable us to apprehend them' (p. 93). The alternative is not some kind of materialism or empiricism about numbers, but only an open-ended research project into the 'cognitive plateau of capacities that humans have discovered how to use in order to engage in mathematical thinking' (p. 94). Additional cases of proof and the contingent historical origins of proofs are investigated in Chapter 4.

Chapter 3 also introduces the 'Enlightenment' problem of applications, which is explored further in Chapter 5. For Hacking, the crucial turning point came when the ancient notion of mixed mathematics was transformed into the modern distinction between pure and applied mathematics. The 'sheer contingency' (p. 146) of our current separation is meant to highlight the contingency of the corresponding network of philosophical problems. Hacking distinguishes a variety of ways in which mathematics is applied (pp. 165–8) and uses this to undermine what he calls the 'representational-deductive picture' (p. 173). On this approach, there is an initial step where one provides a simplified mathematical representation of some natural phenomenon. Mathematical reasoning is then applied to this representation via a deductive proof. The application ends when the resulting representation is translated back into a claim about the phenomenon of interest (p. 172).

One attraction of this picture of applications is that it can preserve a rigid distinction between pure and applied mathematics by insisting that 'applied maths is not maths' (p. 173). The problem with this position is that it is forced to ignore the diversity of modes of applications that Hacking emphasizes. For example, a lot of scientific effort is focused on 'constantly remoulding and reorganizing the theory so that one *can* derive ever more precise consequences' (p. 174). Hacking backs up these remarks with three cases from the history of science: accounts of rigidity (especially in structural engineering), aircraft wings, and the role of pure geometry in physics.

The book concludes with two chapters that consider various Platonisms and counter-Platonisms. Hacking focuses on statements and debates involving actual mathematicians. He expresses a surprisingly tolerant attitude towards what these mathematicians say: 'Each mathematician's philosophy makes sense, for the individual, but what makes sense of the mathematical life for one person may be the opposite of what makes sense for another person' (p. 196). Alain Connes is used to clarify a Platonist position, while Timothy Gowers supports some kind of non-Platonism. Hacking is not impressed with the arguments that these two mathematicians deploy to support their claims about the nature of mathematics. He instead tries to trace the positions back to each mathematician's practice *qua* mathematician. That is, how do they aim to prove their theorems or what theorems do they judge to be important?

While patient and sympathetic to the remarks of mathematicians, Hacking is decidedly uninterested in engaging with philosophers who embrace Platonism or nominalism. In Hacking's eyes, these current debates turn on tacitly assuming 'denotational semantics' (pp. 218, 238). This position, as found in Benacerraf's 'Mathematical Truth', starts with the view that mathematical language should be treated in line with our best account of natural language. It adds a particular approach to natural language that posits a uniform notion of reference and word–world relations as the key to semantics. Hacking opposes this 'semantic uniformity' thesis to Wittgenstein's maxim (also endorsed by Gowers): 'don't ask for the meaning, ask for the use' (pp. 217–8). Thomas Hofweber is praised for proposing an account of number-talk that considers the possibility that number-words might function differently from ordinary names (p. 229).

Hacking, ultimately, does not endorse any metaphysical interpretation of mathematical language. At one point he insists that 'my personal location is of zero philosophical significance' (p. 238). However, the concluding sections of the book suggest a more ambitious project. Hacking mentions Peirce on hypostatization: 'The true precept is not to abstain from hypostatization, but to do it intelligently' (p. 255). One interpretation of this claim is that it is inevitable that we take some names to stand for exotic, 'abstract' objects, but that we should carefully and reflectively choose which objects to countenance. The right choices will support a flourishing mathematics and science, while the wrong choices will reinforce a preoccupation with metaphysical mysteries or hobble innovation.

The cases and positions considered in this book make it a useful supplement to philosophical discussions of mathematics that assume that the most important philosophical problems concern arithmetic or set-theoretic foundations. Hacking's book would serve as a wonderful antidote to these more standard introductions, and should direct students (and philosophers from other areas of philosophy) to some of the most interesting issues that arise when we reflect on how mathematics is actually done. Despite these positive aspects, many readers will be dismayed by Hacking's attitude towards these philosophical problems. Again and again we are told that a given problem has a contingent origin and that its solution will do little to the practice of mathematics.

For my part, it was not clear if Hacking would now take the same attitude towards the perennial philosophical problems raised in the course of scientific practice. Earlier in his career, Hacking endorsed a kind of limited 'entity' realism for those unobservable entities that scientists could manipulate: 'if you can spray them then they are real' (Hacking [1983], p. 23). Entity realism emphasized the significance of successful interventions in the world, as opposed to more theoretical sorts of successes. Entity realism is rejected by many, including those like Fine who insist that scientists themselves make do with a certain 'natural ontological attitude' that eschews scientific realism of any form. One common criticism of scientific realism is that it goes beyond the attitude of the working scientist. As a result, scientific realism is unlikely to make much difference to scientific activity or to what scientists think they are doing. One response to this criticism is to agree that scientific realism is not an urgent issue for working scientists to resolve, and yet still insist that it is a coherent position that addresses a genuine philosophical problem. The most popular forms of scientific realism today eschew a global acceptance of our best scientific theories in favour of a more limited acceptance of parts of those theories in special circumstances. These limited realisms are the descendants of Hacking's own entity realism.

Hacking may no longer wish to defend his earlier entity realism, but I doubt he would dismiss the scientific realism debate as idle or philosophically unimportant. It would also be implausible to tie the scientific realism debate to a tacit semantic uniformity thesis. I would argue that the same attitude is appropriate for contemporary debates about Platonism and nominalism. There is a genuine philosophical problem here that concerns the place of mathematical objects in our best metaphysical account of the world. This problem may not be an urgent problem for mathematicians, but there is little reason to expect philosophical problems to be the focus of scholars in other domains.

A more spirited response to critics of both realism and Platonism is to insist that these issues do matter to scientists and mathematicians. Scientists and mathematicians regularly maintain that they are merely discovering mind-independent features of the world. These claims need not be dismissed as merely expressing an 'attitude' that is conducive to their research; they may be well-informed conclusions drawn by experts in the field. Of course, not all scientists and mathematicians agree on their realism or Platonism. But this ongoing disagreement does not show that the claims are idle to the practice. It may merely show that the issues are difficult to resolve.

Hacking may endorse some sort of Wittgensteinian picture of philosophical problems. On this picture, philosophical problems reflect a confusion that has arisen in the course of ordinary reasoning. If this is indeed the source of the problem, then it would be misguided to try to solve the problem along the lines pursued by traditional philosophers. Instead, one should try to point out the confusion and direct one who is preoccupied with the problem back to the original source. Hacking certainly proceeds in line with this way of thinking about philosophical problems. He is eager to find the source for our philosophical fascination with mathematics, and he offers a plausible diagnosis in terms of Cartesian proofs and myriad applications. Still, I do not think Hacking has done enough to show that these problems are based on confusion, if that was his goal. Instead, he seems to have put his finger on two genuinely important issues about mathematics. We can appreciate these problems and their source, recognize their historical contingency, and be motivated further to solve them.

Elsewhere Hacking has written that 'philosophical problems are created when the space of possibilities in which we organize our thoughts has mutated' ([2002], p. 14). In this book Hacking aligns his discussion of mathematics with a broader 'study of styles of scientific reasoning' (p. 142).[2] A particular style can be isolated when we find an approach that acts as 'our standard of good reason in a particular sphere of inquiry. It does not answer to some higher standard of truth' (p. 142). Hacking is clear that he takes mathematics to involve one or more such style.[3] This entails that there is no external justification for mathematics as this 'would be an independent way of showing that the style gets at the truth, but there is no characterization of the truth over and above what is reached by the styles of reason itself' ([2002], pp. 175–6). Also, in mathematics there would be no 'independently identified objects to be correct about, prior to the development of a style of reasoning' ([2002], pp. 188–9). If these ambitious philosophical claims could be elaborated and defended, then we would have a new sort of answer to our perennial questions about mathematics. This philosophical position would render both Platonism and most forms of nominalism redundant. Until this styles project is completed, though, I believe that philosophers should continue to investigate and to try to solve the philosophical problems raised by mathematics.

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[2] See (Hacking [2002], Chapters 11 and 12; Hacking [2012]).

[3] See (pp. 189–90) where the many modes of application are said to 'illustrate the ways in which these styles fruitfully interact'.