BDA – Practical Sessions

Session 2

Introduction – Shingling, Min-Hashing, LSH

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Agenda for today

1. Quick recap

2. Exercises from the book (From Session 3 - exercises)

3. LSH exercise (From session 3 – practice)

4. Word counting in PySpark (From Session 2 - exercises)
(Not in class, I will provide solutions)

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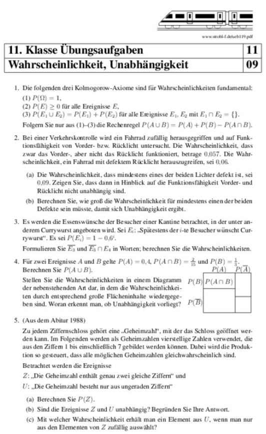
Recap

Docu mnet $\xrightarrow{\text{Shingling}}$ Vector representation $\xrightarrow{\text{Min Hashing}}$ Signature $\xrightarrow{\text{LSH}}$ Candidate pairs



11. Klasse Übungsaufgaben 11
Wahrscheinlichkeit, Unabhängigkeit 09

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ \vdots \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ 1 \\ 0 \\ 2 \\ 4 \end{bmatrix}$$

{Document1, Document 5}
{Document3, Document 7}
{Document8, Document 2}

Jaccard Similarity

Jaccard Similarity of Sets:

- *Consider only unique elements*
- *E.g. SIM({1,2,3,4}, {2,3,5,7}) = 2/6*

Jaccard Similarity of Bags:

- *Include repeating items*
- *E.g. SIM({1,1,1,2}, {1,1,2,2,3}) = 3/9 = 1/3*
- *Maximum Jaccard similarity of bags is 1/2 (when the bags are equivalent)*

Shingling

- By character
  - E.g. ‘Hello! How are you?’ with k=3:
    [‘hel’, ‘ell’, ‘llo’, ‘lo!’, ‘o! ‘, ‘! h’, ‘ ho’, ‘how’, …]
- By word
  - E.g. ‘Hello! How are you?’ with k=2:
    [‘hello! how’, ‘how are’, ‘are you’, …]

- How to chose k?
- k should be picked large enough that the probability of any given shingle appearing in any given document is low
- Rule of thumb: k=5 for small documents, k=9 for large.
- You need to ensure that (n possible characters)^k >> characters in average document

Min Hashing

Element	S_1	S_2	S_3	S_4
0	0	1	0	1
1	0	1	0	0
2	1	0	0	1
3	0	0	1	0
4	0	0	1	1
5	1	0	0	0

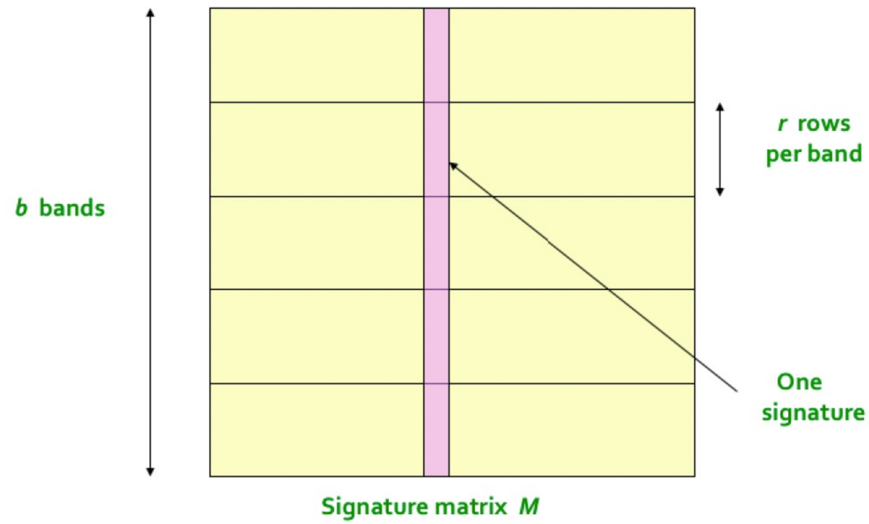
Few ways to do it:

- Clustering
  - Use random hash function to reduce size of the characteristic matrix
- Using permutations *(Min Hashing)*
  - *Permute rows and grab first '1' from top*
  - *Slow in practice, permuting rows on large datasets is computationally expensive*
- Using hash functions *(Fast Min Hashing)*
  - *Apply hash functions to imitate permutations*
  - *Faster than permutations*
  - *We will use this method in class*

Locality Sensitive Hashing

1. Apply hash function to each band
2. If two columns hash to the same bucket make them a candidate pair

Two columns will hash to the same bucket if they are identical in one of the bands, therefore:
Identical in one of the bands = candidate pair



length of signature = b*r

s = true similarity between 2 documents

b = number of bands

r = number of rows in each band

P(candidate pair) = $1 - (1 - s^r)^b$

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HW3

1. (**Exercise 3.2.3 MMDS book**) What is the largest number of k-shingles a document of n bytes can have? You may assume that the size of the alphabet is large enough that the number of possible strings of length k is at least as n. (In UTF-8 encoding each letter occupies 1 byte(8 bits).)

$n = \#\text{char}$

$k = 3$

$$\overbrace{\underbrace{abcde\ \ldots}_{n-k}\ \underbrace{\overbrace{xyz}^{k}}_{1}}^{n}$$

$n - k + 1$

HW3

3. (**Exercise 3.3.3 MMDS book**) In Fig. 3.5 is a matrix with six rows.

Element	S_1	S_2	S_3	S_4
0	0	1	0	1
1	0	1	0	0
2	1	0	0	1
3	0	0	1	0
4	0	0	1	1
5	1	0	0	0

Figure 3.5: Matrix for Exercise 3.3.3

- Compute the minhash signature for each column if we use the following three hash functions: $h_1(x) = 2x + 1 \bmod 6$; $h_2(x) = 3x + 2 \bmod 6$; $h_3(x) = 5x + 2 \bmod 6$.
- Which of these hash functions are true permutations?
- How close are the estimated Jaccard similarities for the six pairs of columns to the true Jaccard similarities?

HW3

3. (**Exercise 3.3.3 MMDS book**) In Fig. 3.5 is a matrix with six rows.

- Compute the minhash signature for each column if we use the following three hash functions: $h_1(x) = 2x + 1 \bmod 6$; $h_2(x) = 3x + 2 \bmod 6$; $h_3(x) = 5x + 2 \bmod 6$.

Rows	2x+1 mod 6	3x+2 mod 6	5x + 2 mod 6
0	1	2	2
1	3	5	1
2	5	2	0
3	1	5	5
4	3	2	4
5	5	5	3

HW3

3. (**Exercise 3.3.3 MMDS book**) In Fig. 3.5 is a matrix with six rows.

- Compute the minhash signature for each column if we use the following three hash functions: $h_1(x) = 2x + 1 \bmod 6$; $h_2(x) = 3x + 2 \bmod 6$; $h_3(x) = 5x + 2 \bmod 6$.

Element	S_1	S_2	S_3	S_4
0	0	1	0	1
1	0	1	0	0
2	1	0	0	1
3	0	0	1	0
4	0	0	1	1
5	1	0	0	0

(h1)	(h2)	(h3)
2x+1 mod 6	**3x+2 mod 6**	**5x + 2 mod 6**
1	2	2
3	5	1
5	2	0
1	5	5
3	2	4
5	5	3

$\Longrightarrow$

	S_1	S_2	S_3	S_4
h1	5	1	1	1
h2	2	2	2	2
h3	0	1	4	0

min(5,5) min(2,5) min(0,3)

HW3

3. (**Exercise 3.3.3 MMDS book**) In Fig. 3.5 is a matrix with six rows.

- Which of these hash functions are true permutations?

(h1) 2x+1 mod 6	(h2) 3x+2 mod 6	(h3) 5x + 2 mod 6
1	2	2
3	5	1
5	2	0
1	5	5
3	2	4
5	5	3

h1 – not a true permutation
h2 – not a true permutation
h3 – true permutation

HW3

3. (**Exercise 3.3.3 MMDS book**) In Fig. 3.5 is a matrix with six rows.

- How close are the estimated Jaccard similarities for the six pairs of columns to the true Jaccard similarities?

Sim(S_1, S_2) = 0/4 = 0, estimated 1/3
Sim(S_1, S_3) = 0/4 = 0, estimated 1/3
Sim(S_1, S_4) = 1/4, estimated 2/3
Sim(S_2, S_3) = 0/4 = 0, estimated 2/3
Sim(S_2, S_4) = 1/4, estimated 2/3
Sim(S_3, S_4) = 1/4, estimated 2/3

Characteristic matrix

Element	S_1	S_2	S_3	S_4
0	0	1	0	1
1	0	1	0	0
2	1	0	0	1
3	0	0	1	0
4	0	0	1	1
5	1	0	0	0

Signature

	S_1	S_2	S_3	S_4
h1	5	1	1	1
h2	2	2	2	2
h3	0	1	4	0

Notes:
For similarity between the shingle representations (from characteristic matrix) we use the Jaccard Similarity of sets. E.g:
Sim(S_1, S_4) = SIM({2,5}, {0,2,4})

For estimated similarity (from signatures) we use the formula:
Sim(S_1, S_4) = $\frac{1}{K}\sum_{k=1}^{K} I(h_k(S_1) = h_k(S_2))$
where K=length of signature and I is identity function that is 1 if $h_k(S_1) = h_k(S_2)$ else 0. We baiscally sum up the rows where singatures have equal elements and divide by length of signature.

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