Universidad Centroamericana
“José Simeón Cañas”

Proyecto

Estudiante: Roberto Ernesto Hernandez Contreras

Catedrático: Enmanuel Araujo

Carnet: 00162317

Indicaciones

Cada estudiante estará a cargo de su propio proyecto, cuyo desarrollo consistirá en lo siguiente:

1. La entrega de un documento, en Word y en PDF, donde se realiza el MEF en 3 dimensiones del modelo seleccionado y aprobado. Deberá contener el desarrollo detallado paso a paso del MEF con conclusiones sobre los resultados. Esta parte constituye un 25% de la nota del Proyecto.

Desarrollo

Modelo

Modelo

$$(x+y)z\nabla^2\vec{u} - (2x+y)z\nabla^2\vec{u} + yz\nabla^2 p = g$$

$$3xz\nabla^2\vec{u} = 0$$

Paso 1

Paso 1

$$(x+y)z\nabla^2\vec{u} - (2x+y)z\nabla^2\vec{u} + yz\nabla^2 p = \vec{g}$$

$$3xz\nabla^2\vec{u} = 0$$

$$\vec{u} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$N_1 = 1 - \varepsilon - \eta - \kappa$

$N_2 = \varepsilon$

$N_3 = \eta$

$N_4 = \kappa$

Paso 2

Paso 2

$$u \approx N_1 U_1 + N_2 U_2 + N_3 U_3 + N_4 U_4 \qquad \vec{u} = \begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} \underline{\underline{N}}\,\underline{\underline{U}} \\ \underline{\underline{N}}\,\underline{\underline{V}} \\ \underline{\underline{N}}\,\underline{\underline{W}} \end{bmatrix}$$

$$= \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \underline{\underline{N}}\,\underline{\underline{U}}$$

$$V = N_1 V_1 + N_2 V_2 + N_3 V_3 + N_4 V_4 = \underline{\underline{N}}\,\underline{\underline{V}}$$

$$W = N_1 W_1 + N_2 W_2 + N_3 W_3 + N_4 W_4 = \underline{\underline{N}}\,\underline{\underline{W}}$$

$$P = N_1 P_1 + N_2 P_2 + N_3 P_3 + N_4 P_4 = \underline{\underline{N}}\,\underline{\underline{P}}$$

Paso 3

Paso 3

$$(x+y) Z \nabla \underline{\underline{N}}\,\underline{\underline{\vec{U}}} - (2x+y) Z \nabla^2 \underline{\underline{N}}\,\underline{\underline{\bar{U}}} + yZ\,\underline{\underline{N}}\,\underline{\underline{P}} = \vec{g}$$

$$3XZ\nabla^2 \underline{\underline{N}}\,\underline{\underline{U}} = 0$$

$$R_1 = g - \left[(x+y) Z \nabla^2 \underline{\underline{N}}\,\underline{\underline{U}}\right] - \left[-(2x+y) Z \nabla^2 \underline{\underline{N}}\,\underline{\underline{U}}\right] - \left[yz\nabla^2 \underline{\underline{N}}\,\underline{\underline{P}}\right]$$

$$R_2 = -3XZ\nabla^2 \underline{\underline{N}}\,\underline{\underline{\bar{U}}}$$

Paso 4

Paso 4:

$$\int_V W_1 R_1 dv = 0 \quad \wedge \quad \int_V W_2 R_2 dv = 0$$

$$\bullet \int_V W_1 \left[g - [(x+y) z \underline{N}\,\underline{U}] + [+(2x+y) z \nabla^2 \underline{N}\,\underline{U}] - [y z \nabla^2 \underline{N}\,\underline{P}] \right] dv = 0$$

$$\bullet \int_V W_2 \left[-3xz \nabla^2 \underline{N}\,\underline{U} \right] dv = 0$$

Paso 5

Paso 5:

$$W_1 = \underline{N}^T, \quad W_2 = \underline{N}^T, \quad \int_V \underline{N}^T R_1 dv = 0, \quad \int_V N^T R_2 dv = 0$$

$$\bullet \int_V N^T \left[g - [(x+y) z \underline{N}\,\underline{U}] + [+(2x+y) z \nabla^2 \underline{N}\,\underline{U}] - [y z \nabla^2 \underline{N}\,\underline{P}] \right] dv = 0$$

$$\bullet \int_V N^T (-3xz \nabla^2 \underline{N}\,\underline{U}) dv = 0$$

$$\int_V N^T \left[g - [(x+y) z \underline{N}\,\underline{U}] + [+(2x+y) z \nabla^2 \underline{N}\,\underline{U}] - [y z \nabla^2 \underline{N}\,\underline{P}] \right] dv + \int_V N^T (-3xz \nabla^2 \underline{N}\,\underline{U}) dv = 0$$

$$\int_V \left[\underline{N}^T g - N^T [(x+y) \nabla^2 \underline{N}\,\underline{U} z] + N^T [(2x+y) \nabla^2 \underline{N}\,\underline{U} z] - N^T y z \nabla^2 \underline{N}\,\underline{P} + N^T (-3xz \nabla^2 \underline{N}\,\underline{U}) \right] dv = 0$$

$$\left(\int_V N^T g\, dv \right) - \left(\int_V N^T (x+y) z \nabla^2 \underline{N} \right) \underline{U} + \left(\int_V N^T (2x+y) \nabla^2 z \underline{N} dv \right) \underline{U} - \left(\int_V N^T y z \nabla^2 \underline{N} dv \right) \underline{P} + \ldots$$

$$\ldots \left(\int_V N^T (-3xz \nabla^2 \underline{N} dv \right) U = 0$$

$$\underline{A} - \underline{B}\,\underline{U} + \underline{C}\,\underline{U} + \underline{D}\,\underline{P} - \underline{E}\,\underline{U}$$

$$\left. \begin{array}{l} -\underline{B}\,\underline{U} + \underline{C}\,\underline{U} - \underline{D}\,\underline{P} = \underline{A} \\ -\underline{E}\,\underline{U} = \underline{0} \end{array} \right] \rightarrow \text{Forma Fuerte}$$

Paso 6

$\int u\,dv = uv - \int v\,du$

Paso 6 $\underline{\underline{B}}, \underline{\underline{C}}, \underline{\underline{D}}, \underline{\underline{E}}$

$\rightarrow$ $\underline{\underline{B}} = \int_V N^T (x+y) \nabla^2 \underline{\underline{N}} z\, dv \rightarrow \underline{\underline{B}} = \int_V N^T (x+y) z \nabla(\nabla \underline{\underline{N}})\, dv$

$u = \underline{\underline{N}}^T \qquad dv = (x+y) z \nabla(\nabla \underline{\underline{N}})$

$du = \nabla \underline{\underline{N}}^T \qquad v = (x+y) z \nabla \underline{\underline{N}}$

$[\underline{\underline{N}}^T \cancel{(x+y) z \nabla \underline{\underline{N}}}]\big|_\Gamma - \int_V (x+y) z \nabla \underline{\underline{N}} \nabla \underline{\underline{N}}^T dv$

$0 - K$

$\underline{\underline{B}} = -\underline{\underline{K}}$

$\rightarrow$ $\underline{\underline{C}} = \int_V \underline{\underline{N}}^T (2x+y) z \nabla^2 N\, dv \rightarrow \underline{\underline{C}} = \int_V N^T (2x+y) z \nabla(\nabla N)\, dv$

$u = N^T \qquad dv = (2x+y) z \nabla(\nabla \underline{\underline{N}})$

$du = \nabla N^T \qquad v = (2x+y) z \nabla \underline{\underline{N}}$

$[\underline{\underline{N}}^T \cancel{(2x+y) z \nabla \underline{\underline{N}}}]\big|_\Gamma - \int_V (2x+y) z \nabla \underline{\underline{N}} \nabla \underline{\underline{N}}^T dv$

$0 - \underline{\underline{m}}$

$\underline{\underline{C}} = -\underline{\underline{m}}$

$\rightarrow$ $\underline{\underline{D}} = \int_V \underline{\underline{N}}^T y z \nabla^2 \underline{\underline{N}}\, dv \rightarrow \underline{\underline{D}} = \int_V \underline{\underline{N}}^T y z \nabla(\nabla \underline{\underline{N}})\, dv$

$u = \underline{\underline{N}}^T \qquad dv = y z \nabla(\nabla \underline{\underline{N}})$

$du = \nabla \underline{\underline{N}}^T \qquad v = y z \nabla \underline{\underline{N}}$

$[N^T \cancel{(y z \nabla \underline{\underline{N}})}]\big|_\Gamma - \int_V y z \nabla \underline{\underline{N}} \nabla \underline{\underline{N}}^T dv$

$0 - L$

$\underline{\underline{D}} = -\underline{\underline{L}}$

$$\rightarrow \underline{\underline{E}} = \int_v \underline{\underline{N}}^T (3xz\nabla^2 \underline{\underline{N}})dv \rightarrow \underline{\underline{E}} = \int_v \underline{\underline{N}}^T (3xz\nabla(\nabla \underline{\underline{N}}))dv$$

$u = \underline{\underline{N}}^T \qquad dv = (3xz\nabla(\nabla \underline{\underline{N}}))$

$du = \nabla \underline{\underline{N}}^T \qquad v = 3xz\nabla\underline{\underline{N}}$

$$[\underline{\underline{N}}^T \cancel{(3xz\nabla\underline{\underline{N}})}^{0}]\Big|_\Gamma - \int_v (3xz\nabla\underline{\underline{N}})\nabla\underline{\underline{N}}^T$$

$0 \qquad -T$

$\underline{\underline{E}} = -\underline{\underline{T}}$

$$-(-\underline{\underline{K}})\underline{U} + (-\underline{\underline{m}})\underline{U} - (-\underline{\underline{L}})\underline{P} = \underline{\underline{A}}$$

$$-(-T)\underline{U} = 0$$

$$\underline{\underline{K}}\underline{U} - \underline{\underline{m}}\underline{U} + \underline{\underline{L}}\underline{P} = \underline{\underline{A}}$$

$$\underline{\underline{T}}\underline{U} = 0$$

$$\left[\begin{bmatrix} K+m & L \\ T & 0 \end{bmatrix} \begin{bmatrix} U \\ P \end{bmatrix} = \begin{bmatrix} A \\ 0 \end{bmatrix}\right] \rightarrow \text{Forma Débil}$$

<u>Componentes</u>

Recordando

$$\underline{x} = x_{21}\varepsilon + x_{31}\eta + x_{41}\zeta + x_1$$
$$y = y_{21}\varepsilon + y_{31}\eta + y_{41}\zeta + y_1$$
$$z = z_{21}\varepsilon + z_{31}\eta + z_{41}\zeta + z_1$$

Variables

$x_{21} = a$	$y_{21} = F$	$z_{21} = J$
$x_{31} = b$	$y_{31} = g$	$z_{31} = K$
$x_{41} = c$	$y_{41} = h$	$z_{41} = L$
$x_1 = d$	$y_1 = I$	$z_1 = m$

Componente A

Componente $\underline{A}$

$$\underline{\underline{A}} = \int_v N^T g\, dv$$

$$\underline{\underline{A}} = \int_v \begin{bmatrix} \underline{N}^T & 0 & 0 \\ 0 & \underline{N}^T & 0 \\ 0 & 0 & \underline{N}^T \end{bmatrix} dv \rightarrow \underline{\underline{A}} = \begin{bmatrix} \int_v N^T dv & 0 & 0 \\ 0 & \int_v \underline{N}^T dv & 0 \\ 0 & 0 & \int_v \underline{N}^T dv \end{bmatrix}$$

$$\underline{\underline{A}} = J \underbrace{\begin{bmatrix} 1/24 & 0 & 0 \\ 1/24 & 0 & 0 \\ 1/24 & 0 & 0 \\ 1/24 & 0 & 0 \\ 0 & 1/24 & 0 \\ 0 & 1/24 & 0 \\ 0 & 1/24 & 0 \\ 0 & 0 & 1/24 \\ 0 & 0 & 1/24 \\ 0 & 0 & 1/24 \\ 0 & 0 & 1/24 \end{bmatrix}_{12\times3}}_{\underline{\underline{\gamma}}} \rightarrow \underline{\underline{A}} = J\, \underline{\underline{\gamma}}_{12\times3}\, g_{3\times1}$$

$$\underline{\underline{A}} = (\ )_{12\times1}$$

Componente B

Componente B

$$\underline{\underline{B}} = \int_v (x+y) z \nabla \underline{\underline{N}} \nabla \underline{\underline{N}}^T dv$$

$$\nabla_x \underline{\underline{N}}^T = \frac{1}{D} \underset{12\times 3}{\underline{\underline{B}}^T} \underset{3\times 3}{\underline{\underline{\alpha}}^T} \quad \wedge \quad \nabla_x \underline{\underline{N}} = 1/D \underset{3\times 3}{\underline{\underline{\alpha}}} \underset{3\times 12}{\underline{\underline{B}}}$$

$$\underline{\underline{B}} = 1/D^2 \underline{\underline{B}}^T \underline{\underline{\alpha}}^T \underline{\underline{B}} \underline{\underline{\alpha}} \int_v (x+y) z dv$$

Tomando la integral

$$\underline{\underline{B}} = \int_v (x+y) z dv$$

$$1) \int_0^1 \int_0^{1-\varepsilon} \int_0^{1-\varepsilon-\eta} (1-\varepsilon-\eta-\kappa)((a\varepsilon + b\eta + c\kappa + d) + (F\varepsilon + g\eta + h\kappa + I))$$
$$(J\varepsilon + K\eta + L\kappa + m) d\kappa \, d\eta \, d\varepsilon$$

$$E_F = (a(3K+l+6m) + b(3K+l+6m) + 2c(K+l+3m) + 6d(2K+l+5m) + f$$
$$(3K+l+6m) + g(3K+l+6m) + 2(h(K+l+3m) + 3i(2K+l+5m)))/720$$

Resultado final

$$\underline{B} = \frac{P}{D^2} \underset{12\times 3}{\underline{B}^T} \underset{3\times 3}{\underline{\alpha}^T} \underset{3\times 2}{\underline{B}} \underset{3\times 3}{\underline{\alpha}}$$

Componente C

Componente C

$$\underline{\underline{C}} = \int_v (2x+y)\, z\, \nabla \underline{\underline{N}}\, \nabla \underline{\underline{N}}^T dv$$

$$\nabla_x \underline{\underline{N}}^T = 1/D \underset{12\times3}{\underline{\underline{B}}^T} \underset{3\times3}{\underline{\underline{\alpha}}^T} \quad \wedge \quad \nabla_x \underline{\underline{N}} = 1/D \underset{3\times3}{\underline{\underline{\alpha}}} \underset{3\times12}{\underline{\underline{B}}}$$

$$\underline{\underline{C}} = 1/D^2\, \underline{\underline{B}}^T \underline{\underline{\alpha}}^T \underline{\underline{\alpha}}\, \underline{\underline{B}} \int_v (2x+y)\, z\, dv$$

Tomando lo integral

$$\underline{\underline{C}} = \int_v (2x+y)\, z\, dv$$

Compo...

$$1)\ \int_0^1 \int_0^{1-\varepsilon} \int_0^{1-\varepsilon-\eta} (1-\varepsilon-\eta-\kappa)(z(a\varepsilon+b\eta+c\kappa+d)+(f\varepsilon+g\eta+h\kappa+I))(J\varepsilon+k\eta+L\kappa+m)\, d\kappa\, d\eta\, d\varepsilon$$

$$E_1 = [(a(2j+k+l+6m)+b(j+2k+l+6m)+c(j+k+2(l+3m))+6d(j+k+l+5m)) + f(2j+k+l+6m)+g(j+2k+l+6m)+h(j+k+2(l+3m))+6I(j+k+l+5m))/360]$$

...m) dκ dη dE

Resultado final

$$\underline{\underline{C}} = \frac{B'}{D^2} \underset{12\times3}{\underline{\underline{B}}^T} \underset{3\times3}{\underline{\underline{\alpha}}^T} \underset{3\times12}{\underline{\underline{B}}} \underset{3\times3}{\underline{\underline{\alpha}}}$$

Componente D

Componente D

$$\underline{\underline{D}} = \int_v yz \nabla \underline{\underline{N}} \nabla \underline{\underline{N}}^T dv$$

$$\nabla_x \underline{\underline{N}}^T = \underset{3\times3}{\underline{\underline{\alpha}}^T} \underset{12\times3}{\underline{\underline{B}}^T} \quad \wedge \quad \nabla_x \underline{\underline{N}} = \underset{3\times3}{\underline{\underline{\alpha}}} \underset{3\times12}{\underline{\underline{B}}}$$

$$\underline{\underline{D}} = 1/D^2 \, \underline{\underline{B}}^T \underline{\underline{\alpha}}^T \underline{\underline{B}} \, \underline{\underline{\alpha}} \int_v yz \, dv$$

Tomando la integral

$$\underline{\underline{D}} = \int_v yz dv$$

Componente D

$$1) \int_0^1 \int_0^{1-\varepsilon} \int_0^{1-\varepsilon-\eta} (1-\varepsilon-\eta-\kappa)(F\varepsilon+g\eta+h\kappa+I)(J\varepsilon+K\eta+L\kappa+m)\, d\kappa\, d\eta\, d\varepsilon$$

$$E_1 = (f(2j+k+l+6m)+g(j+2k+l+6m)+h(j+k+2(l+3m))+6i(j+k+l+5m))/720$$

Resultado final

$$\underline{D} = \frac{\alpha'}{D^2} \underset{12\times3}{\underline{\underline{B}}^T} \underset{3\times3}{\underline{\alpha}^T} \underset{3\times12}{\underline{B}} \underset{3\times3}{\underline{\underline{\alpha}}}$$

Componente E

Componente $\underline{\underline{E}}$

$$\underline{\underline{E}} = \int_V (3xz \nabla \underline{\underline{N}}) \nabla \underline{\underline{N}}^T$$

$$\nabla_x \underline{\underline{N}}^T = \underset{3\times3}{\underline{\underline{\alpha}}^T} \; \underset{12\times3}{\underline{\underline{\beta}}^T} \quad \wedge \quad \nabla_x \underline{\underline{N}} = \underset{3\times3}{\underline{\underline{\alpha}}} \; \underset{3\times12}{\underline{\underline{B}}}$$

$$\underline{\underline{E}} = 1/D^2 \, \underline{\underline{\beta}}^T \underline{\underline{\alpha}}^T \underline{\underline{B}} \, \underline{\underline{\alpha}} \int_V 3xz \, dv$$

Tomando la integral

$$\underline{\underline{E}} = \int_V 3xz \, dv$$

Componente E

1) $\int_0^1 \int_0^{1-\varepsilon} \int_0^{1-\varepsilon-\eta} (1-\varepsilon-\eta-\kappa)(3(a\varepsilon+b\eta+c\kappa+d)(j\varepsilon+k\eta+l\kappa+m)\,d\kappa\,d\eta\,d\varepsilon$

$E_1 = (a(2j+k+l+6m)+b(j+2k+l+6m)+c(j+k+2l+3m)+6d(j+k+l+5m))/_{2}40$

Resultado final

$$\underline{E} = \frac{\gamma}{D^2} \; \underset{12\times3}{\underline{B}^T} \; \underset{3\times3}{\underline{\alpha}^T} \; \underset{3\times12}{\underline{B}} \; \underset{3\times3}{\underline{\alpha}}$$

Conclusión

Al momento de estar resolviendo los componentes, el resultado del primer elemento de una matriz generaba una expresión matemática muy grande. Eso en manera de código, es necesario manejar las triples integrales triples en forma matricial, para que la computadora pueda manejar esas dichas ecuaciones. Si se manejara el resultado como la respuesta que dio en la calculadora, se estarían manejando integrales infinitas lo cual se vuelve complicado a resolver para la computadora.

Cuando los términos de Navier Stokes se vuelven complejos. La respuesta de las integrales llega a tener una extensa longitud dejando como resultado final también extenso.