Support Vector Machines

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In this lesson we'll learn the theory behind using Linear Discriminant Analysis (LDA) as a supervised classification technique. We'll then use LDA to classify the UCI wine dataset in R.

Additional packages needed

To run the code you may need additional packages.

- If necessary install the followings packages.

```
install.packages("ggplot2");
install.packages("e1071");
install.packages("kernlab");
```

```
require(ggplot2)

## Loading required package: ggplot2

require(e1071)

## Loading required package: e1071

require(kernlab)

## Loading required package: kernlab

##
## Attaching package: 'kernlab'

## The following object is masked from 'package:ggplot2':
##
##     alpha
```

Data

We will be using the dataset 'svm_regression.csv' which is randomly generated. svm_regression.csv. T

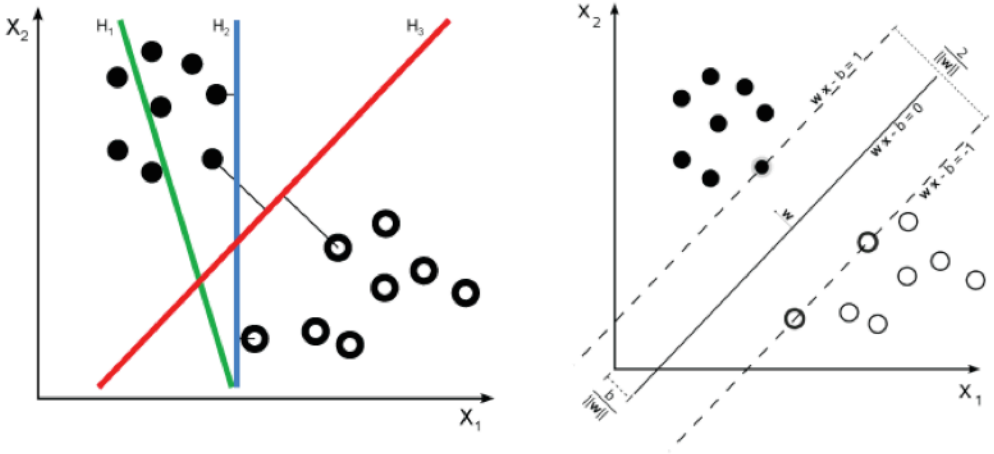
```
data_url <-
'http://nikbearbrown.com/YouTube/MachineLearning/M08/svm_regression.csv
'
svm.data <- read.csv(url(data_url), sep=",", header = TRUE)
head(svm.data)
```

```
##   X Y
## 1 1 3
## 2 2 4
## 3 3 8
## 4 4 4
## 5 5 6
## 6 6 9
```

Support Vector Machines

Support vector machine constructs a hyperplane or set of hyperplanes in a high- or infinite-dimensional space, which can be used for classification, regression, or other tasks. Given a set of data points that belong to either of two classes, an SVM finds the hyperplane that:

- Leaves the largest possible fraction of points of the same class on the same side.
- Maximizes the distance of either class from the hyperplane.
- Find the optimal separating hyperplane that minimizes the risk of misclassifying the training samples and unseen test samples.



SVM

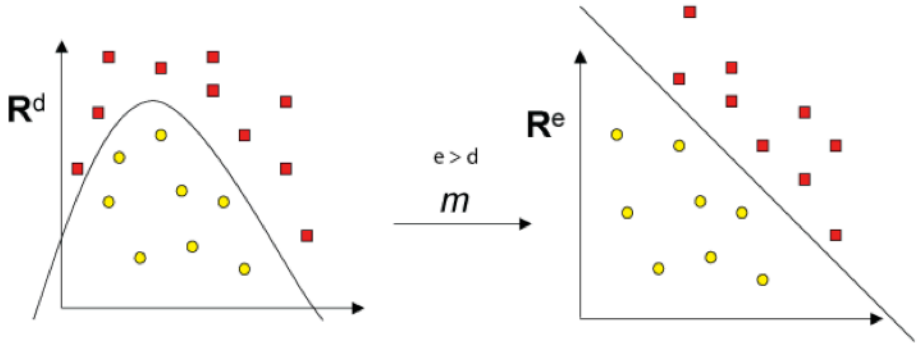
Pseudocode

Given a guess of width and bias we can:

- Compute whether all data points are in the correct half-planes.
- Compute the width of the margin.
- Search the space of width's and bias to find the widest margin that matches all the datapoints.

Kernels

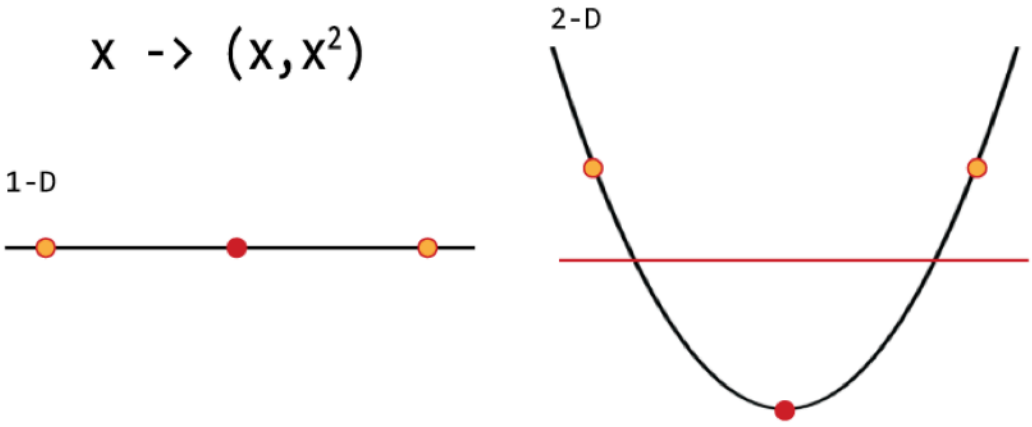
It is much easier and efficient to find Separating boundaries which are in the form of a straight lines as oppossed to curvy Separating boundaries. Kernels help us turn a linear classifier into a non-linear one. That is, they transform a curvy separating boundary that is only non-linearly separable to a linearly separable discriminant in a higher dimensional space.



Kernels

Kernels (1-D Example)

For example, squaring the data may take some points that are not linearly separable in 1-D space to be linearly separable in 2D space.



Kernels

Kernel Trick

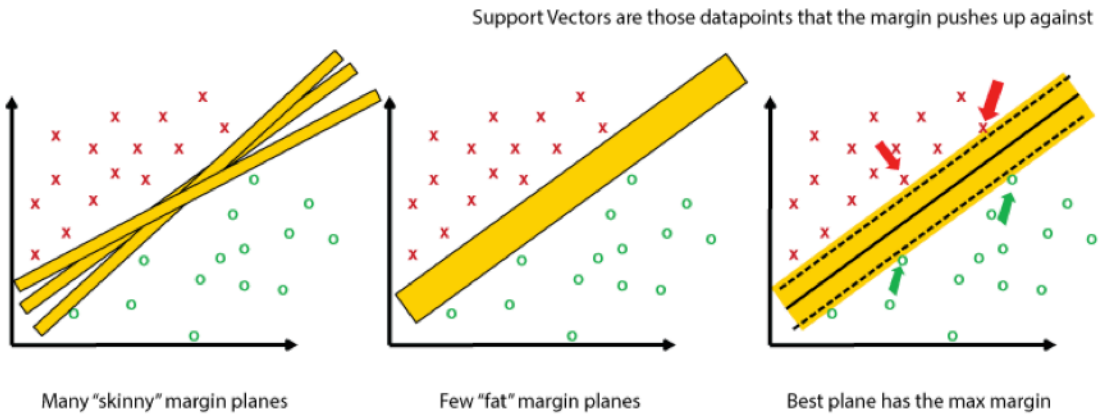
While coming up with functions that map linearly separable sets to SVM, we only need to know the inner product of vectors in the linearly separable sets in higher dimensional spaces arbitrarily coordinate space.

Computing the kernel of x and y gives the same result as the dot product in the mapped (high-dimensional) space. The mappings used by SVM schemes are designed to ensure that dot products may be computed easily in terms of the variables in the original space, by defining them in terms of a kernel function k(x,y) selected to suit the problem. This "Kernel trick" is" essentially is to define a similarity function in terms of original space itself without even defining (or even knowing), what the transformation function K will be.

Linear SVM

Support Vectors are those datapoints that the margin pushes up against

Given some training data $\mathcal{D}$, a set of n points of the form $\mathcal{D} = \{(\mathbf{x}_i, y_i) \mid \mathbf{x}_i \in \mathbb{R}^p,\ y_i \in \{-1,1\}\}_{i=1}^n$ where the y_i is either 1 or −1, indicating the class to which the point $\mathbf{x}_i$ belongs. Each $\mathbf{x}_i$ is a p-dimensional real vector. We want to find the maximum-margin hyperplane that divides the points having $y_i = 1$ from those having $y_i = -1$. Any hyperplane can be written as the set of points $\mathbf{x}$ satisfying $\mathbf{w} \cdot \mathbf{x} - b = 0$, where $\mathbf{w} \cdot \mathbf{x}$,denotes the dot product of $\mathbf{w}$ and $\mathbf{x}$, The variable $\mathbf{w}$ is the (not necessarily normalized) normal vector to the hyperplane. The parameter $b/\| \mathbf{w} \|$ determines the offset of the hyperplane from the origin along the normal vector $\mathbf{w}$.



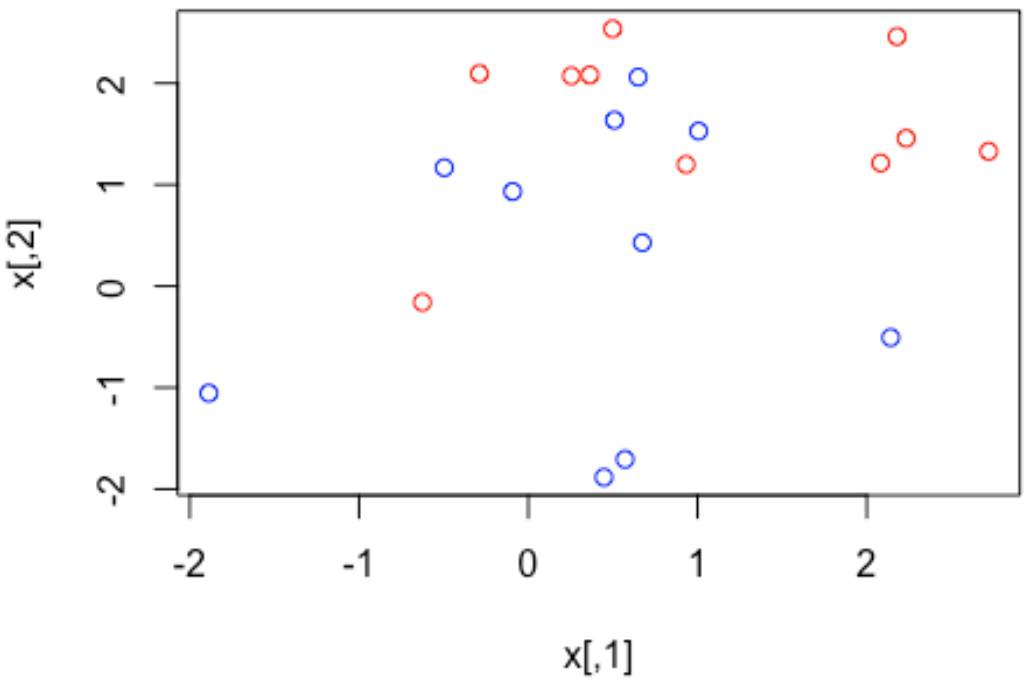
Linear SVM

Non-Linear SVM

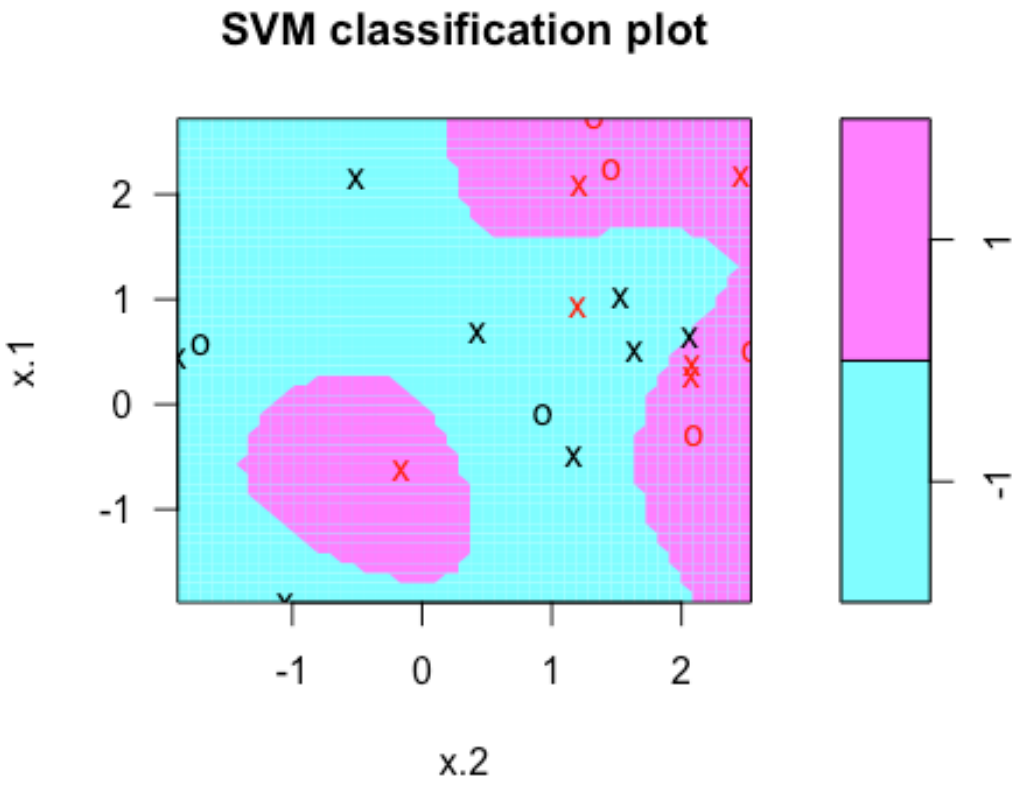
Non-linear classifiers are created by applying the kernel trick then generating maximum-margin hyperplanes.

Support Vector Machines in R

```
#Generate random numbers
x<-matrix(rnorm(40),ncol=2)
set.seed(333)

#First 10 elements of Y are -1 and the rest are 1
y<-c(rep(-1,10),rep(1,10))

#Add 1 to the last 10 rows of x
x[y==1,]<- x[y==1,]+1

#Checking if classes are linearly separable
plot(x,col=(3-y))   #col= 2 means blue, col= 4 means red
```

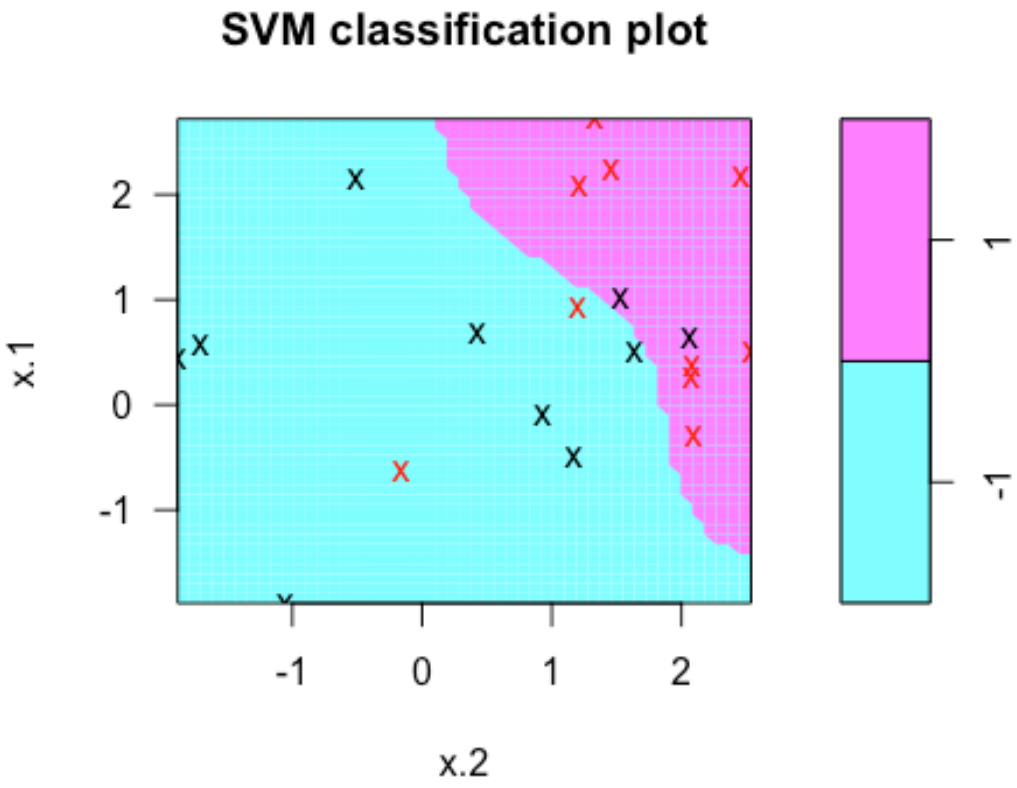



```
dataframe<-data.frame(x=x,y=as.factor(y))



#----------------------Training a model on the data ----

svm.fit<- svm(y~.,data=dataframe, kernal="linear",cost=10,scale=FALSE)
plot(svm.fit,dataframe)
```




```
# List my Support Vectors
svm.fit$index

##  [1]  1  3  5  6  7  8  9 10 13 14 15 16 19 20

summary(svm.fit)

##
## Call:
## svm(formula = y ~ ., data = dataframe, kernal = "linear", cost = 10,
##     scale = FALSE)
##
##
## Parameters:
##    SVM-Type:  C-classification
##  SVM-Kernel:  radial
##        cost:  10
##       gamma:  0.5
##
## Number of Support Vectors:  14
##
##  ( 8 6 )
##
##
```

```
## Number of Classes:  2
##
## Levels:
##  -1 1

# Let's chage Cost
svm.fit1<- svm(y~.,data=dataframe,
kernal="linear",cost=0.1,scale=FALSE)
plot(svm.fit1,dataframe)  # Lower Cost - Wider Margin
```




```
#Cross-Validation -Find Best Model for prediction
tune.out<-
tune(svm,y~.,data=dataframe,kernal="linear",ranges=list(cost=c(0.001,0.
01,0.1,1,5,10,100)))
bestmodel=tune.out$best.model
summary(bestmodel)

##
## Call:
## best.tune(method = svm, train.x = y ~ ., data = dataframe, ranges =
list(cost = c(0.001,
##     0.01, 0.1, 1, 5, 10, 100)), kernal = "linear")
##
##
```

```
## Parameters:
##    SVM-Type:  C-classification
##  SVM-Kernel:  radial
##        cost:  5
##       gamma:  0.5
##
## Number of Support Vectors:  15
##
##  ( 8 7 )
##
##
## Number of Classes:  2
##
## Levels:
##  -1 1

##----------------------Let's focus on Predicting the Function ---

## Generate Test data
xtest = matrix(rnorm(40),ncol=2)
ytest = sample(c(-1,1),20,rep=TRUE)
xtest[ytest==1,]<- xtest[ytest==1,]+1

testdf<-data.frame(x=xtest,y=as.factor(ytest))
testdf

##            x.1         x.2  y
## 1  -1.12490028 -0.70278946 -1
## 2  -0.87430362  0.29986416 -1
## 3   1.04386162  2.14946968  1
## 4  -0.58397514 -0.30164149 -1
## 5   0.17670142  0.45266065  1
## 6   0.11059280 -0.38540415 -1
## 7   0.36472778  0.27637203 -1
## 8   0.05370100  0.15186081 -1
## 9   2.36867235  0.56861921  1
## 10  0.61954276 -0.87050006  1
## 11 -0.70632036  0.07633326 -1
## 12  1.96687656  0.76964405  1
## 13 -0.71016648  0.31746484 -1
## 14 -0.05917365 -1.59268440 -1
## 15 -0.23094342 -0.18194062 -1
## 16  1.69200045  2.75614174  1
## 17  2.35682290  1.11394932  1
## 18  2.78872284  0.70783492  1
## 19  1.41071582  2.75409316  1
## 20 -2.18785470  1.80958455 -1

#Predict testdata(i.e. testdf ) based on bestmodel
```

```
ypred=predict(bestmodel,testdf)
ypred
```

```
##  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20
##  1 -1  1  1 -1 -1 -1 -1  1 -1 -1  1 -1 -1 -1  1  1  1  1 -1
## Levels: -1 1
```

```
table(pred=ypred,truth=testdf$y)
```

```
##     truth
## pred -1 1
##   -1  9 2
##   1   2 7
```

```
# look only at agreement vs. non-agreement
# construct a vector of TRUE/FALSE indicating correct/incorrect
predictions
agreement <- ypred == testdf$y
table(agreement)
```

```
## agreement
## FALSE  TRUE
##     4    16
```

```
prop.table(table(agreement))
```

```
## agreement
## FALSE  TRUE
##   0.2   0.8
```

```
##-------------- Improving model performance -----

classifier_rbf <- ksvm(y ~ ., data = testdf, kernel = "rbfdot")
predictions_rbf <- predict(classifier_rbf, testdf)
predictions_rbf
```

```
##  [1] -1 -1 1  -1 -1 -1 -1 -1 1  -1 -1 1  -1 -1 -1 1  1  1  1  -1
## Levels: -1 1
```

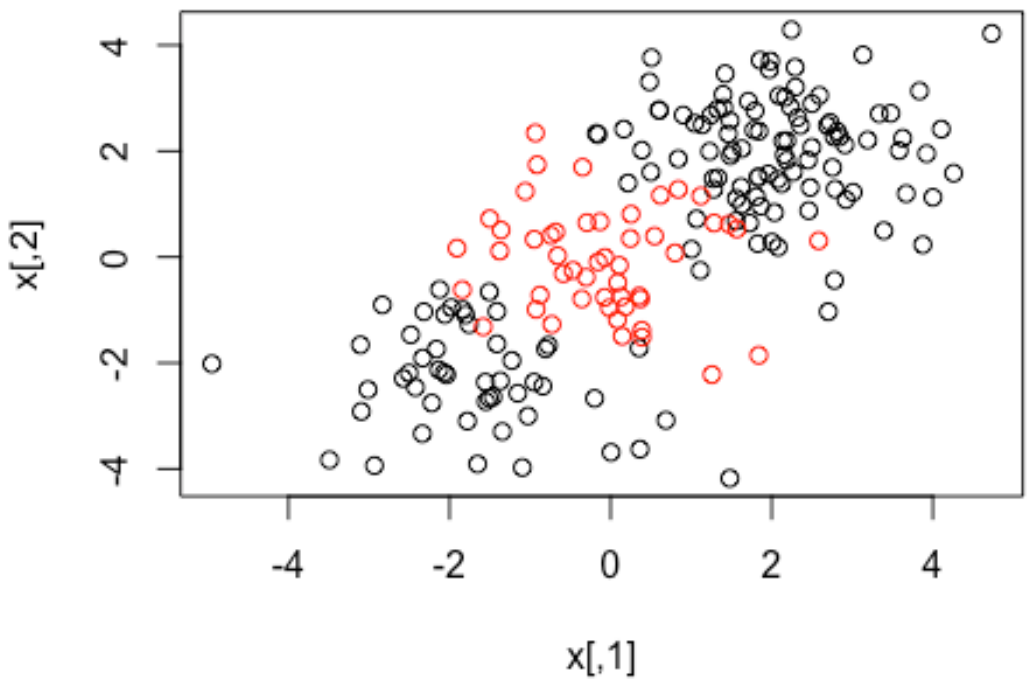
```
agreement_rbf <- predictions_rbf == testdf$y
table(agreement_rbf)
```

```
## agreement_rbf
## FALSE  TRUE
##     2    18
```

```
prop.table(table(agreement_rbf))
```

```
## agreement_rbf
## FALSE  TRUE
##   0.1   0.9
```

```
####------------------------- RADIAL KERNAL ( play with gamma and
cost)  ------------------

# -----------Generate random Data----------

set.seed(33)
x<-matrix(rnorm(400),ncol=2)
x[1:100,]=x[1:100,]+2
x[101:150,]=x[101:150,]-2

y=c(rep(1,150),rep(2,50))
dat= data.frame(x=x,y=as.factor(y))

plot(x,col=y)
```



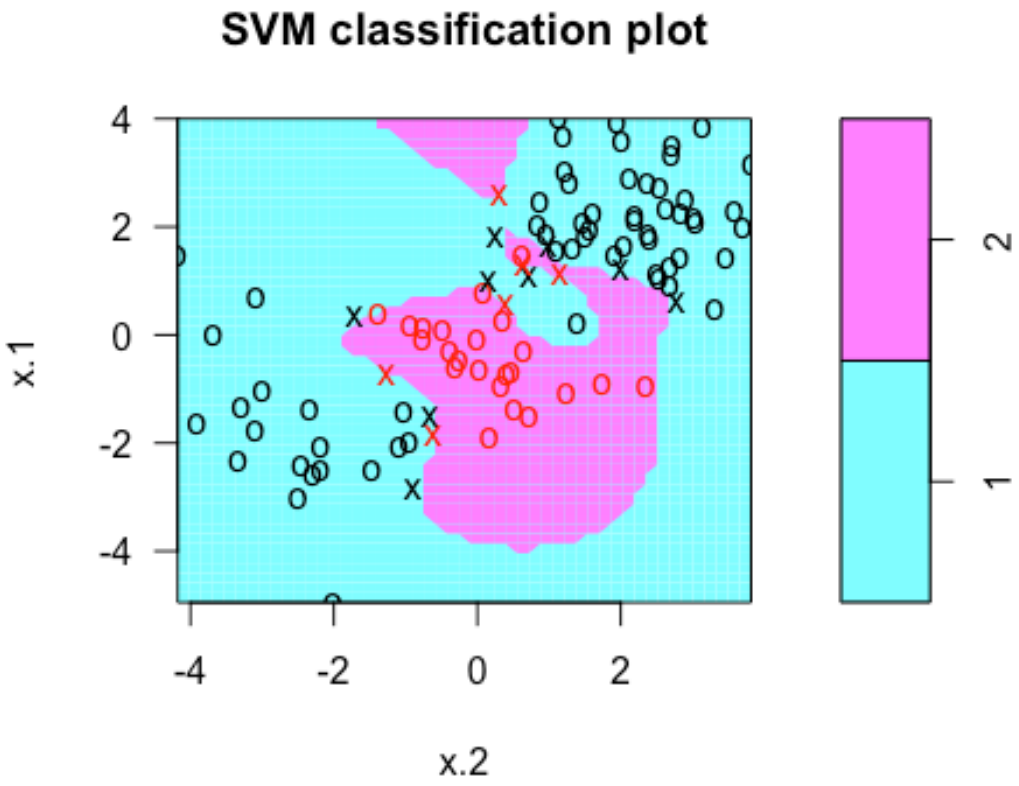

```
# ----------- Training Model on data --------

train = sample(200,100)
train

##   [1]  61   1 130  56  43  20 158 160  68 122 100  26  55  74  22
3 116
##  [18] 188 174  91 121 153  23 187 142 192   4 185 127  67 155 157
```

```
179 125
##  [35]  69 108 200 106 133  49 171  87 135  11  18  76  57  95 190
77  38
##  [52]  72 129 169 175 162  80 196  60  30 177  92  35 176 152   7
126 134
##  [69]  28 199 128 191 173   2  34  58 165 167  86  54 181 145  24
16 156
##  [86]  10  66 112 103  78  31  90 111 123 184  65  53 117 150  89
```

```
svmfit1<-svm(y~.,data=dat[train,],kernal="radial",gamma=1,cost=100000)
svmfit1
```

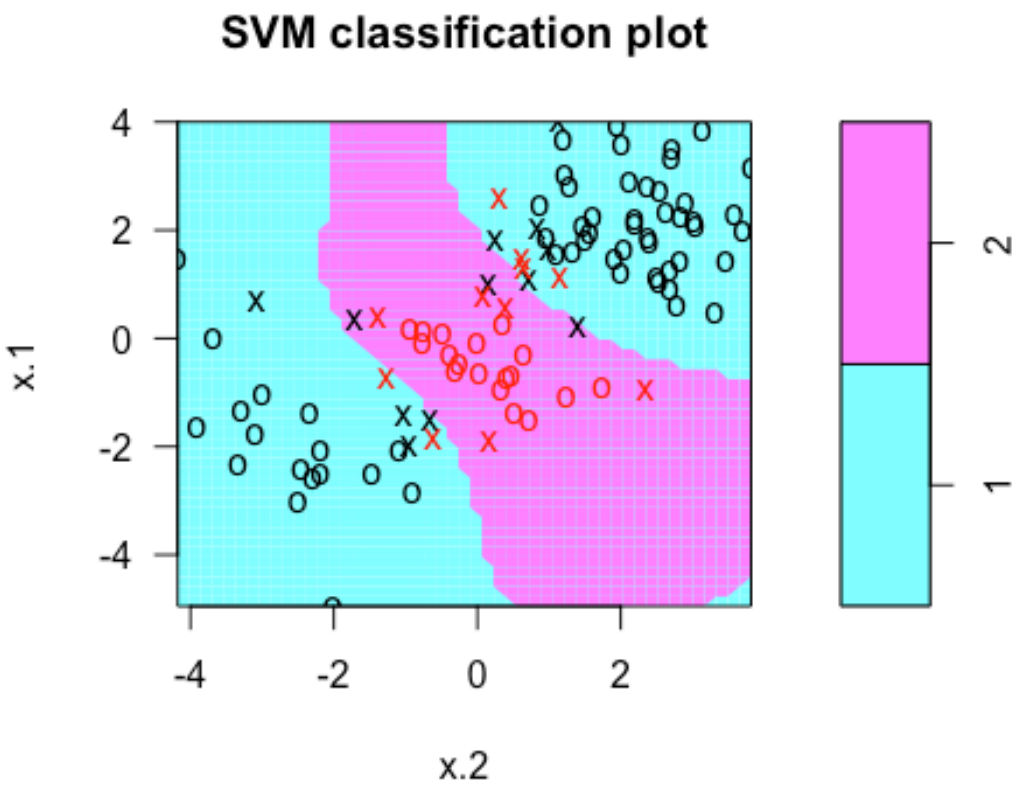
```
##
## Call:
## svm(formula = y ~ ., data = dat[train, ], kernal = "radial",
##     gamma = 1, cost = 1e+05)
##
##
## Parameters:
##    SVM-Type:  C-classification
##  SVM-Kernel:  radial
##        cost:  1e+05
##       gamma:  1
##
## Number of Support Vectors:  15
```

```
plot(svmfit1,dat[train,])
```




```
#------------- Cross Validation to set best choice of gamma and cost

tune.out=
tune(svm,y~.,data=dat[train,],kernal="radial",ranges=list(cost=c(0.1,10
,100,1000)),gamma=c(0.5,1,2,3,4))
summary(tune.out)

##
## Parameter tuning of 'svm':
##
## - sampling method: 10-fold cross validation
##
## - best parameters:
##  cost
##    10
##
## - best performance: 0.12
##
## - Detailed performance results:
##    cost error dispersion
## 1 1e-01  0.19 0.15951315
## 2 1e+01  0.12 0.07888106
```

```
## 3 1e+02  0.14 0.10749677
## 4 1e+03  0.17 0.10593499

bestmodel1<-tune.out$best.model
bestmodel1

##
## Call:
## best.tune(method = svm, train.x = y ~ ., data = dat[train, ],
##     ranges = list(cost = c(0.1, 10, 100, 1000)), kernal = "radial",
##     gamma = c(0.5, 1, 2, 3, 4))
##
##
## Parameters:
##    SVM-Type:  C-classification
##  SVM-Kernel:  radial
##        cost:  10
##       gamma:  0.5 1 2 3 4
##
## Number of Support Vectors:  23

plot(bestmodel1,dat[train,])
```



```
test=-train
train
```

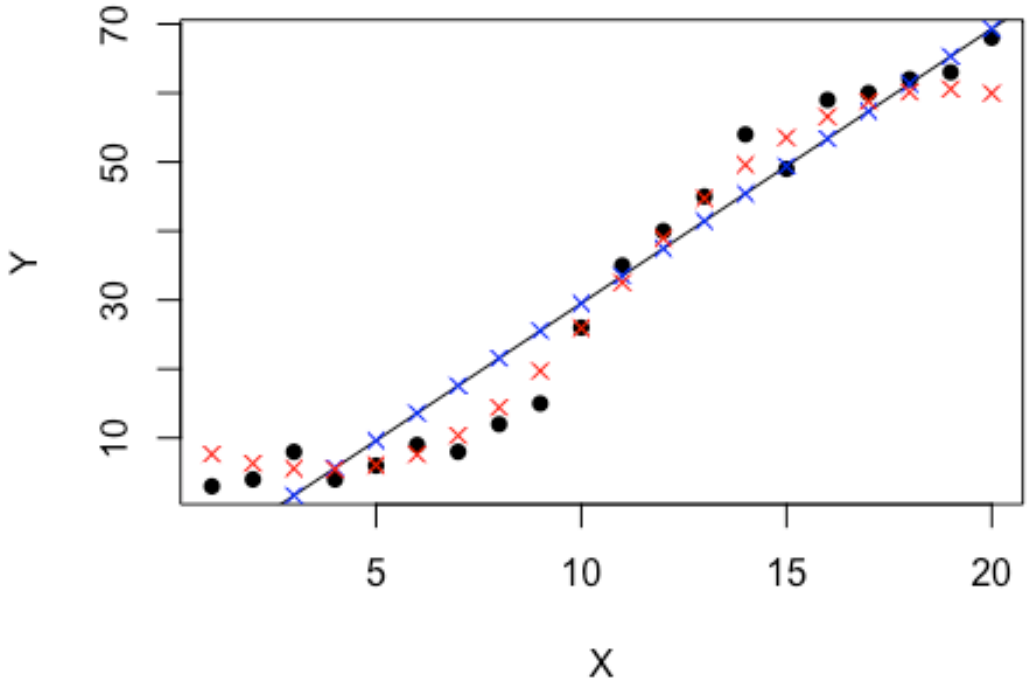
```
##   [1]  61   1 130  56  43  20 158 160  68 122 100  26  55  74  22
3 116
##  [18] 188 174  91 121 153  23 187 142 192   4 185 127  67 155 157
179 125
##  [35]  69 108 200 106 133  49 171  87 135  11  18  76  57  95 190
77  38
##  [52]  72 129 169 175 162  80 196  60  30 177  92  35 176 152   7
126 134
##  [69]  28 199 128 191 173   2  34  58 165 167  86  54 181 145  24
16 156
##  [86]  10  66 112 103  78  31  90 111 123 184  65  53 117 150  89
```

```
 #  Prediction Error Comparison between Linear Regression and SVM  #


# Plot the data
plot(svm.data, pch=16)

# Create a linear regression model
lr_model <- lm(Y ~ X, svm.data)

# Add the fitted line
abline(lr_model)

# make a prediction for each X
predictedY <- predict(lr_model, svm.data)

# display the predictions
points(svm.data$X, predictedY, col = "blue", pch=4)

# Function to find the Error
rmse <- function(error)
{
  sqrt(mean(error^2))
}
error <- lr_model$residuals         # same as data$Y - predictedY
predictionRMSE <- rmse(error)
predictionRMSE
```

```
## [1] 5.703778
```

```
                            # Support Vector Machine(Finding root mean
square error)



# Create Support Vector Model
```

```
svm_model <- svm(Y ~ X , svm.data)

# make a prediction for each X
predictedY1 <- predict(svm_model, svm.data)

# display the predictions
points(svm.data$X, predictedY1, col = "red", pch=4)
```




```
# Function to find the Error
error <- svm.data$Y - predictedY1
svrPredictionRMSE <- rmse(error)

tuneResult <- tune(svm, Y ~ X,  data = svm.data,ranges = list(epsilon =
seq(0,1,0.1), cost = 2^(2:9)))
print(tuneResult)

##
## Parameter tuning of 'svm':
##
## - sampling method: 10-fold cross validation
##
## - best parameters:
##  epsilon cost
```

```
##        0  128
##
## - best performance: 8.702864

svrPredictionRMSE

## [1] 3.157061
```

Resources

- Support Vector Regression with R
- Computing and visualizing LDA in R
- SVM example with Iris Data in R