

# K.C.S.E. MATHEMATICS PAPER 121/2 2005

## SECTION I

Answer all the questions in this section

1. Without using Mathematical Tables, simplify  $\frac{\sqrt{63} + \sqrt{72}}{\sqrt{32} + \sqrt{28}}$  (3 marks)
2. The first three consecutive terms of a geometrical progression are  $3, x$  and  $5\frac{1}{3}$ . Find the value of  $x$ . (2 marks)
3. Find, without using Mathematical Tables, the values of  $x$  which satisfy the equation  $\log_2(x^2 - 9) = 3\log_2 2 + 1$  (4 marks)
4. Given that  $\cos 2x^\circ = 0.8070$ , find  $x$  when  $0^\circ \leq x \leq 360^\circ$ . (4 marks)
5. Successive moving averages of order 5 for the numbers 9, 8.2, 6.7, 5.4, 4.7 and  $k$  are A and B. Given that  $A - B = 0.6$  find the value of  $k$ . (3 marks)
6. Expand and simplify  $(3x - y)^4$ . Hence use the first three terms of the expansion to approximate the value of  $(6 - 0.2)^4$ . (4 marks)
7. The density of a solid spherical ball varies directly as its mass and inversely as the cube of its radius. When the mass of the ball is 500g and the radius is 5cm, its density is 2g per cm<sup>3</sup>. Calculate the radius of a solid spherical ball of mass 540g and density of 10g per cm<sup>3</sup>. (3 marks)
8. A stone is thrown vertically upwards from a point O. After  $t$  seconds, the stone is  $S$  metres from O. Given that  $S = 29.4t - 4.9t^2$ , find the maximum height reached by the stone (3 marks)
9. A point R divides a line PQ internally in the ratio 3 : 4. Another point S, divides the line PR externally in the ratio 5 : 2. Given that PQ = 8cm, calculate the length of RS, correct to 2 decimal places (3 marks)
10. Given that  $\sin(90 - x)^\circ = 0.8$ , where  $x$  is an acute angle, find without using Mathematical Tables the value of  $\tan x^\circ$ . (2 marks)
11. Two teachers are chosen randomly from a staff consisting of 3 women and 2 men to attend a HIV/AIDS seminar. Calculate the probability that the two teachers chosen are:
  - (a) of the same sex (2 marks)
  - (b) of opposite sex (2 marks)

12. In this question Mathematical Tables should not be used.  
The base and perpendicular height of a triangle measured to the nearest centimetre are 6 cm and 4 cm respectively.

Find:

- (a) the absolute error in calculating the area of the triangle (2 marks)
- (b) the percentage error in the area, giving the answer to 1 decimal place (2 marks)

13. Make P the subject of the formula

$$P^2 = (P - q)(P - r) \quad (3 \text{ marks})$$

14. Point T is the midpoint of a straight line AB. Given that the position vectors of A and T are  $\mathbf{i} - \mathbf{j} + \mathbf{k}$  and  $2\mathbf{i} + 1\frac{1}{2}\mathbf{k}$  respectively, find the position vector of B in terms of  $\mathbf{i}, \mathbf{j}$  and  $\mathbf{k}$ . (3 marks)

15. A bank either pays simple interest at 5% p.a. or compound interest at 5% p.a. on deposits. Nekesa deposited sh P in the bank for two years on simple interest terms. If she had deposited the same amount for two years on compound interest terms, she would have earned sh 210 more.

Calculate, without using Mathematical Tables, the value of P (4 marks)

16. The acceleration,  $a \text{ ms}^{-2}$ , of a particle is given by  $a = 25 - 9t^2$ , where  $t$  is time in seconds after the particle passes a fixed point O.

## SECTION II

Answer any six questions in this section

17. A curve is represented by the function  $y = \frac{1}{3}x^3 + x^2 - 3x + 2$ .

(a) Find  $\frac{dy}{dx}$  (1 mark)

(b) Determine the values of  $y$  at the turning points of the curve  $y = \frac{1}{3}x^3 + x^2 - 3x + 2$  (4 marks)

(c) In the space provided below, sketch the curve of  $y = \frac{1}{3}x^3 + x^2 - 3x + 2$ . (3 marks)

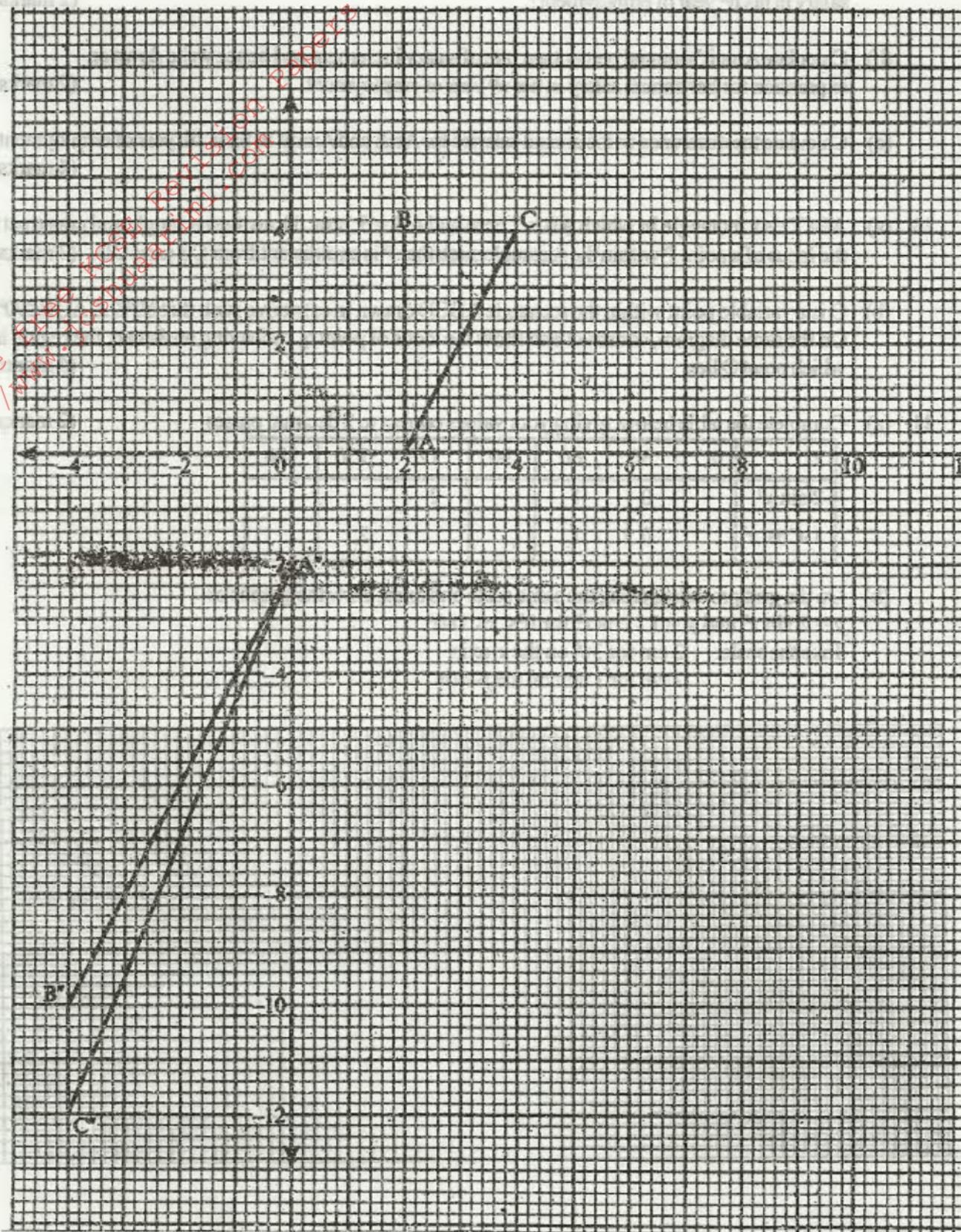
18. Triangles ABC and A'B'C' are drawn on the cartesian plane provided.

Triangles ABC is mapped onto A'B'C' by two successive transformations

$$\mathbf{R} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ followed by } \mathbf{P} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

(a) Find R (4 marks)

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- (b) Using the same scale and axes, draw triangle ABC, the image of triangle A'B'C', under transformation R. (2 marks)
- (c) Describe fully, the transformation represented by matrix R. (2 marks)



19. Abdi and Amoit were employed at the beginning of the same year. Their annual salaries, in shillings, progressed as follows:

Abdi: 60 000, 64 800, 69 600, ...

Amit: 60 000, 64 800, 69 984, ...

- (a) Calculate Abdi's annual salary increment and hence write down an expression for his annual salary in his  $n^{\text{th}}$  year of employment? (2 marks)

- (b) Calculate Amoit's annual percentage rate of salary increment and hence write down an expression for her annual salary in her  $n^{\text{th}}$  year of employment. (2 marks)

- (c) Calculate the difference in the annual salaries for Abdi and Amoit in their 7th year of employment (4 marks)

20. (a) ABCD is a rectangle in which  $AB = 7.6\text{cm}$  and  $AD = 5.2\text{cm}$ . Draw the rectangle and construct the locus of a point P within the rectangle such that P is equidistant from CB and CD. (3 marks)

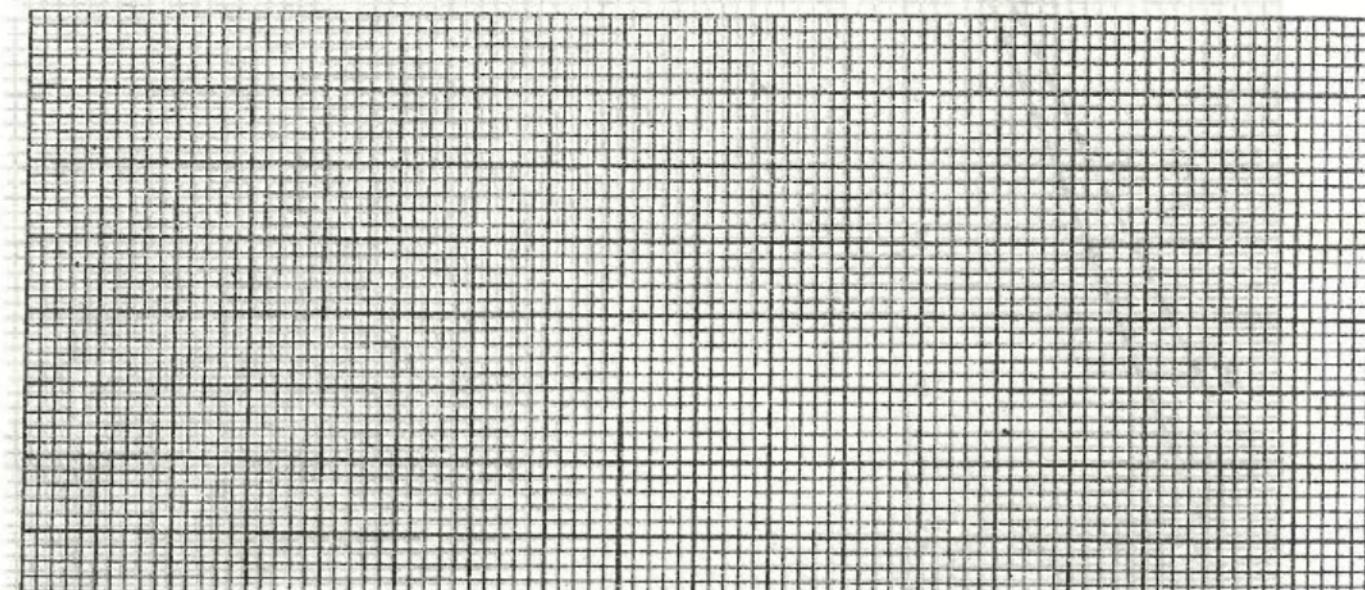
- (b) Q is a variable point within the rectangle ABCD drawn in (a) above such that  $60^{\circ} \leq \angle AQB \leq 90^{\circ}$ . On the same diagram, construct and show the locus of point Q, by leaving unshaded, the region in which point Q lies. (5 marks)

21. (a) Complete the table below, giving your values correct to 2 decimal places (2 marks)

$x^{\circ}$	0	30	60	90	120	150	180
$2 \sin x^{\circ}$		0	1	.	2		
$1 - \cos x^{\circ}$		0.5	1			2	

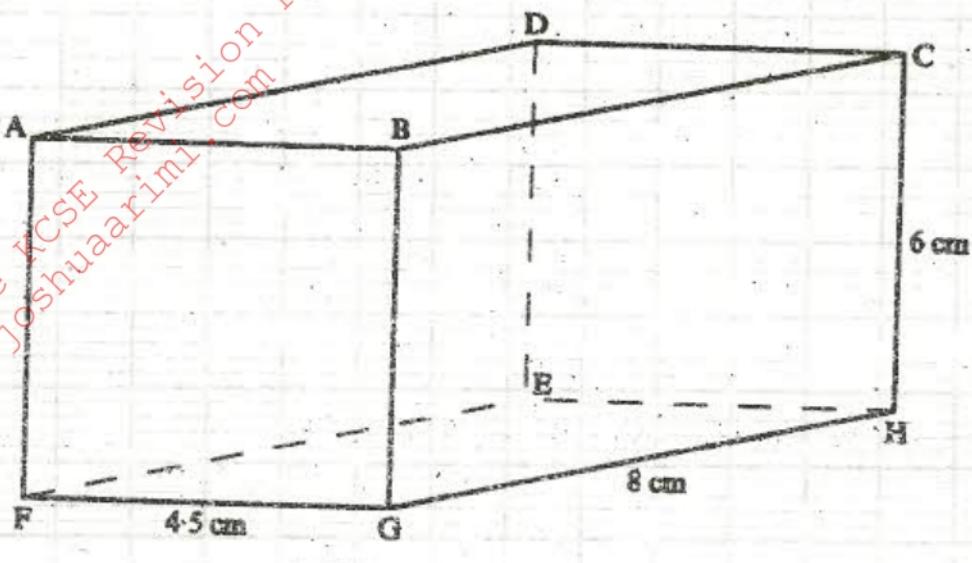
- (b) On the grid provided, using the same scale and axes, draw the graphs of  $y = 2 \sin x^{\circ}$  and  $y = 1 - \cos x^{\circ}$  for  $0^{\circ} \leq x \leq 180^{\circ}$ .

Take the scale: 2 cm for  $30^{\circ}$  on the x-axis  
2 cm for 1 unit on the y-axis



- (c) Use the graph in (b) above to
- Solve the equation  
 $2 \sin x^\circ + \cos x^\circ = 1$  (1 mark)
  - determine the range of values of  $x$  for which  $2 \sin x^\circ \geq 1 - \cos x^\circ$ . (1 mark)

22. The diagram below represents a cuboid ABCDEFGH in which  $FG = 4.5\text{cm}$ ,  $GH = 8\text{cm}$  and  $HC = 6\text{ cm}$



Calculate

- the length of  $FC$  (2 marks)
- (i) the size of the angle between the lines  $FC$  and  $FH$  (2 marks)
- (ii) the size of the angle between the lines  $AB$  and  $FH$  (2 marks)
- the size of the angle between the planes  $ABHE$  and the plane  $FGHE$  (2 marks)

23. A diet expert makes up a food product for sale by mixing two ingredients N and S. One kilogram of N contains 25 units of protein and 30 units of vitamins. One kilogram of S contains 50 units of protein and 45 units of vitamins.

The food is sold in small bags each containing at least 175 units of proteins and at least 180 units of vitamins. The mass of the food product in each bag must not exceed 6 kg.

If one bag of the mixture contains  $x$  kg of N and  $y$  kg of S;

- Write down all the inequalities, in terms of  $x$  and  $y$ , representing the information above (3 marks)
- On the grid provided draw the inequalities by shading the unwanted regions. (2 marks)
- If one kilogram of N costs sh 20 and one kilogram of S costs sh 50, use the graph to determine the lowest cost of one bag of the mixture (3 marks)

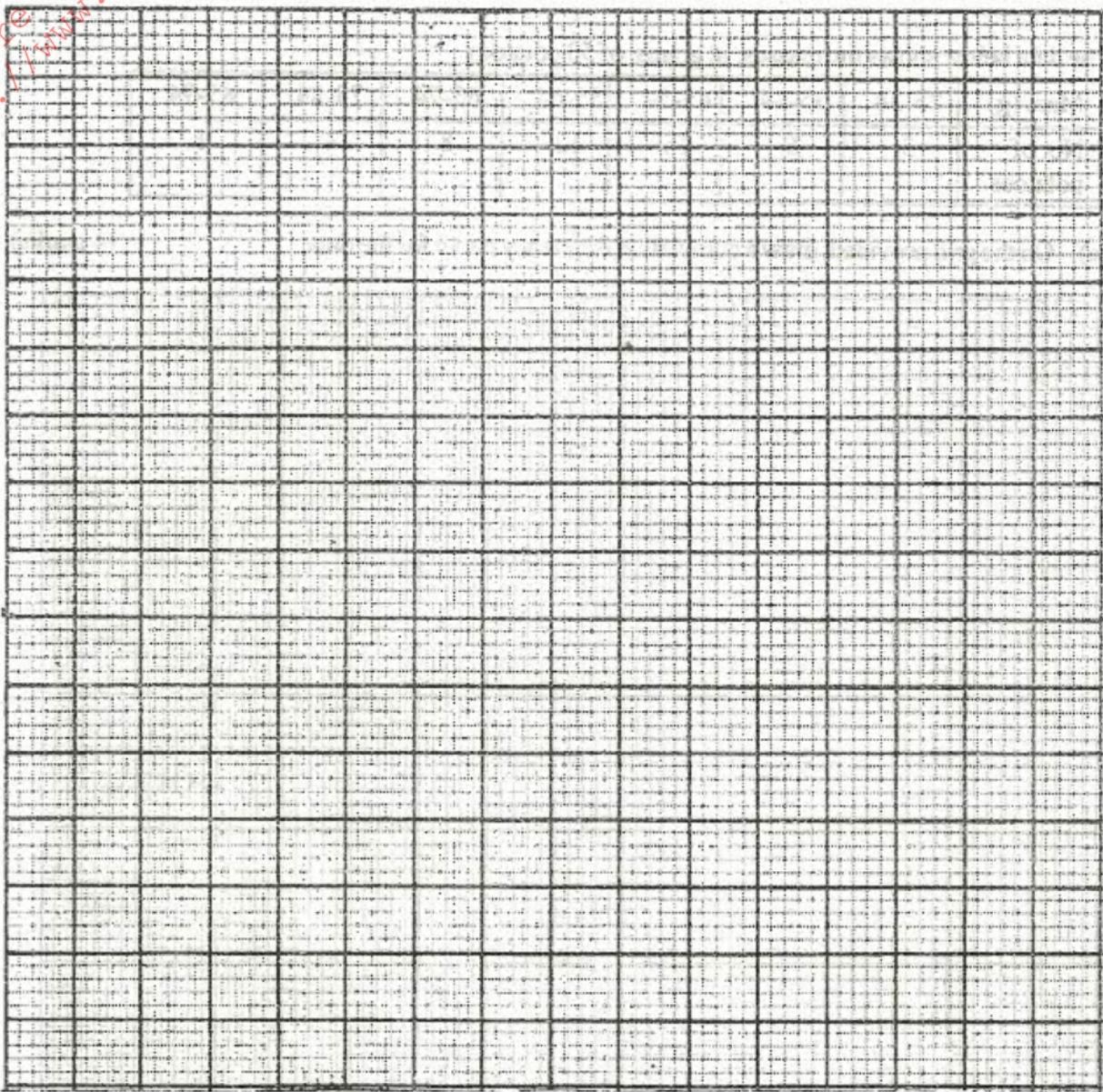


24. The points P, Q, R and S have position vectors  $2\mathbf{p}$ ,  $3\mathbf{p} + \mathbf{r}$  and  $3\mathbf{r}$  respectively, relative to an origin O. A point T divides PS internally in the ratio 1:6
- (a) Find, in the simplest form, the vectors OT and QT in terms of P and r (4 marks)
- (b) (i) Find, that the points Q, T and R lie on a straight line (3 marks)
- (ii) Determine the ratio in which T divides QR (1 mark)

25. The table below gives some of the values of x and y for the function  $y = \frac{1}{2}x^2 + 2x + 1$  in the interval  $0 \leq x \leq 6$ .

x	0	1	2	3	4	5	6
y	1	3.5	7	11.5	17	23.5	31

- (a) Use the values in the table to draw the graph of the function on the grid provided (2 marks)



- (b) (i) Using the graph and the mid-ordinate rule with six (6) strips, estimate the area bounded by the curve, the x-axis, the y-axis and the line  $x = 6$  (4 marks)
- (ii) If the exact area of the region described in (b) (i) above is  $78 \text{ cm}^2$ , calculate the percentage error made when the mid-ordinate rule is used (2 marks)

Give the answer correct to two decimal places (2 marks)

26. The gradient of a curve at point  $(x, y)$  is  $4x - 3$ . The curve has a minimum value of  $-\frac{1}{8}$ .

(a) Find:

(i) the value of  $x$  at the minimum point (1 mark)

(ii) the equation of the curve (4 marks)

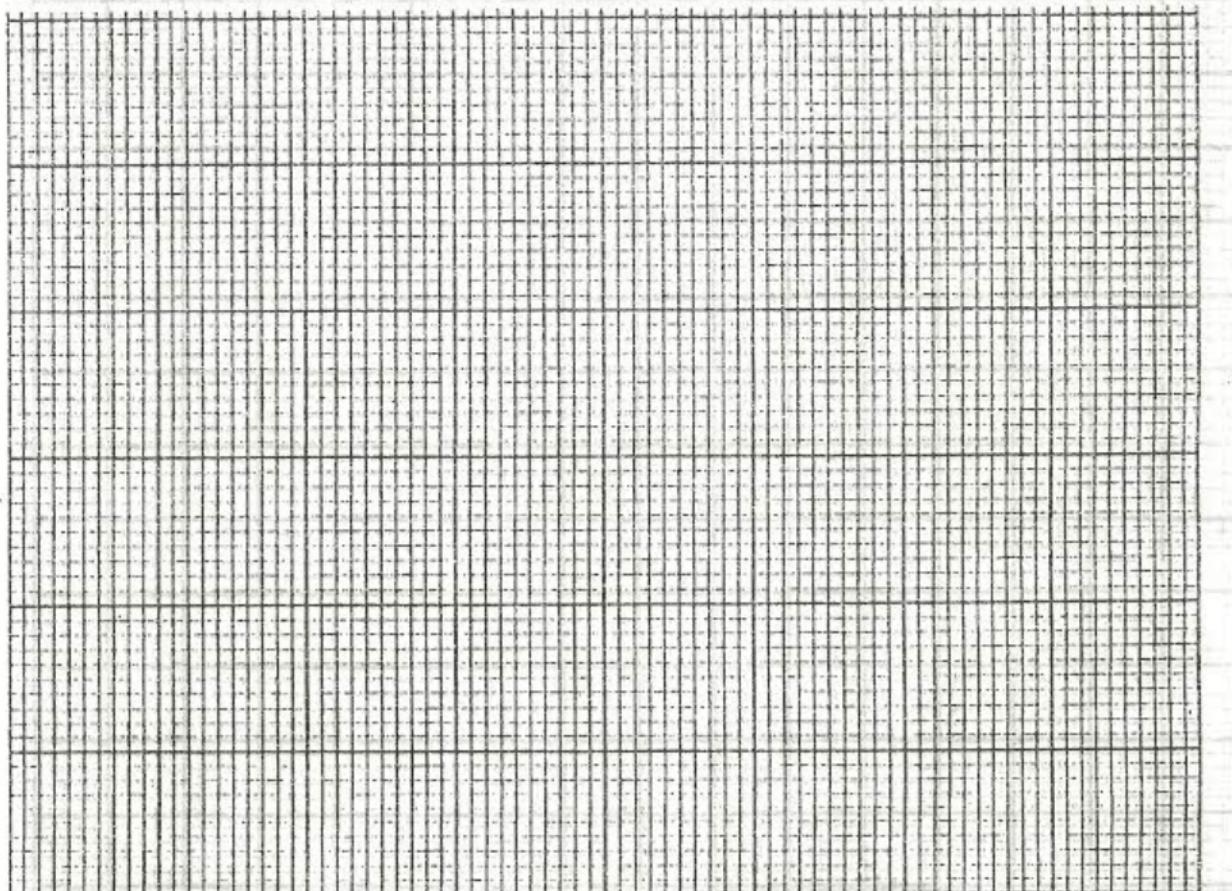
2	6	8	5	1	0	x
6	25	41	21	5	25	
8	36	52	32	8	36	
5	25	40	25	5	25	
1	6	8	5	1	6	

(b) P is a point on the curve in part (a) (ii) above. If the gradient of the curve at P is -7, find the coordinates of P (3 marks)

27. The data below shows the masses, in grams, of 50 potatoes

Mass (g)	25-34	35-44	45-54	55-64	65-74	75-84	85-94
No. of potatoes	3	6	16	12	8	4	1

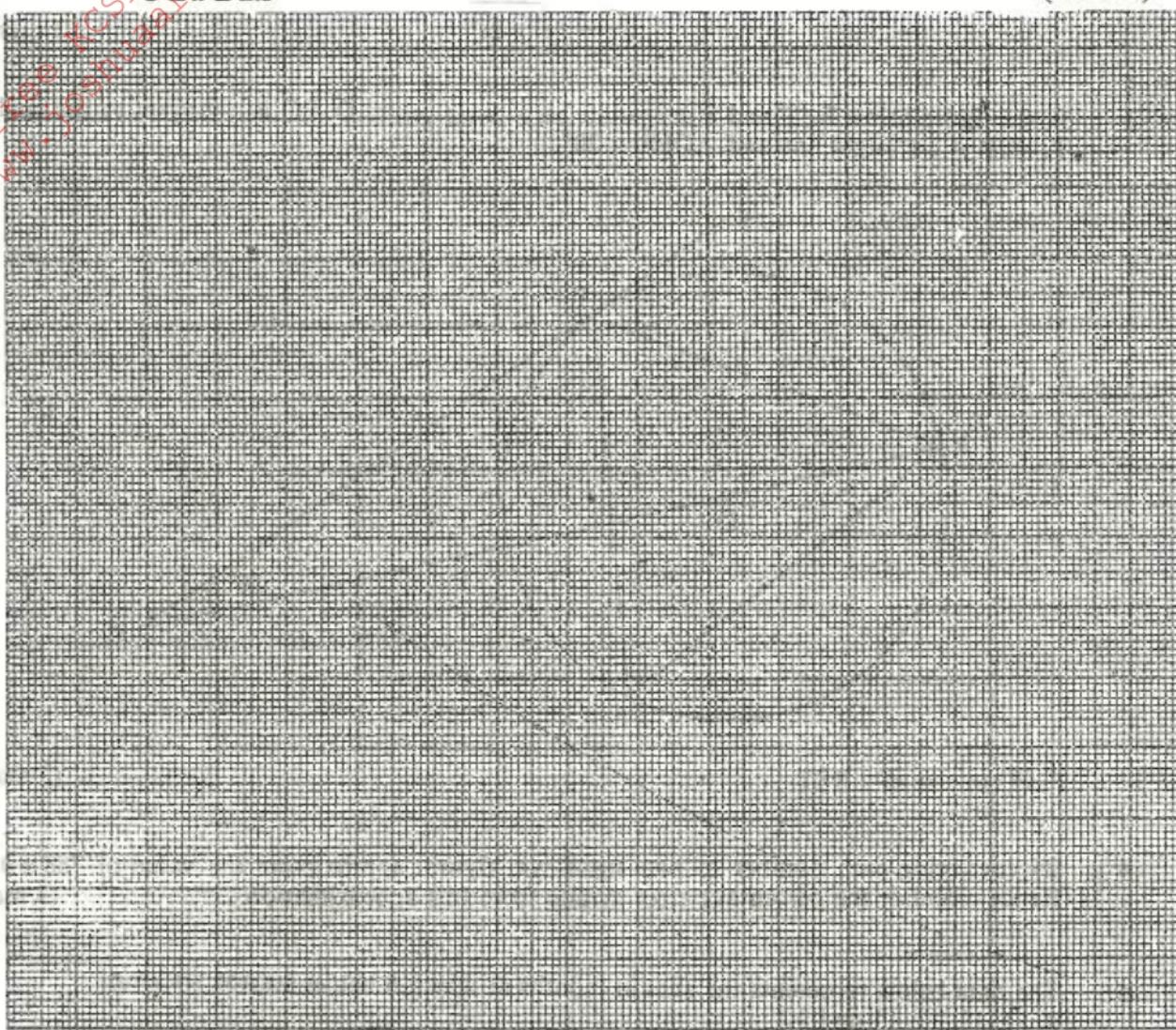
(a) On the grid provided, draw a cumulative frequency curve for the data (4 marks)



- (b) use the graph in (a) above to determine
- the 60th percentile mass (1 mark)
  - the percentage of potatoes whose masses lie in the range 53g to 68g (3 marks)
28. (a) (i) Complete the table below for the function  $y = x^3 + x^2 - 2x$  (2 marks)

$x$	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5
$y$		-4.4		1.9		1.1		-0.6		2.6		16.9

- (ii) On grid provided, draw the graph of  $y = x^3 + x^2 - 2x$  for the values of  $x$  in the interval  $-3 \leq x \leq 2.5$  (2 marks)



- (iii) State the range of negative values of  $x$  for which  $y$  is also negative (1 mark)
- (b) Find the coordinates of two points on the curve other than  $(0,0)$  at which  $x$ -coordinate and  $y$ -coordinate are equal (3 marks)

$$\begin{aligned}y &= x^3 + x^2 - 2x \\y &= x\end{aligned}$$