

K.C.S.E. MATHEMATICS PAPER 121/2 2003

SECTION I

Answer all the questions in this section

1. Solve the equation

$$\log_{10}(6x - 2) - 1 = \log_{10}(x - 3)$$

(3 marks)

2. Find the coordinates of the turning point of the curve whose equation is $y = 6 + 2x - 4x^2$.

(3 marks)

3. Given that $a = \frac{1}{\sqrt{3}}$ and $b = \sqrt{13}$ express $2\sqrt{3} - 6\sqrt{39}$ in terms of a and b and simplify the answer.

(3 marks)

4. a) Expand and simplify the binomial expression $(2 - x)^6$.

(2 marks)

- b) Use the expansion up to the term in x^2 to estimate 1.99^6 .

(2 marks)

5. A mixed school can accommodate a maximum of 440 students. The number of girls must be at least 120 while the number of boys must exceed 150.

Taking x to represent the number of boys and y the number of girls, write down all the inequalities representing the information above.

(3 marks)

6. A colony of insects was found to have 250 insects at the beginning. Thereafter the number of insects doubled every 2 days.

Find how many insects there were after 16 days.

(3 marks)

7. The distance s metres of an object varies partly with time t seconds and partly with the square root of time.

Given that $s = 14$ when $t = 4$ and $s = 27$ when $t = 9$, write an equation connecting s and t .

(4 marks)

8. Make c the subject of the formula:

$$T = x\sqrt{c^2 + d^2}$$

(3 marks)

9. A water pump costs Ksh 21 600 when new. At the end of first year its value depreciates by 25%. The depreciation at the end of the second year is 20% and thereafter the rate of depreciation is 15% yearly. Calculate the exact value of the water pump at the end of the fourth year. (3 marks)

10. Given that $x = 2i + j - 2k$, $y = -3i + 4j - k$ and $z = 5i + 3j + 2k$ and that $p = 3x - y + 2z$, find the magnitude of vector p to 3 significant figures (4 marks)

11. Solve the equation $3 \tan^2 x - 4 \tan x - 4 = 0$ for (4 marks)

12. There are three cars A, B, C in a race. A is twice as likely to win as B while B is twice as likely to win as C. Find the probability that (4 marks)

a) A wins the race (2 marks)

b) Either B and C wins the race (1 mark)

13. The velocity $V \text{ ms}^{-1}$ of a particle in motion is given by $V = 3t^2 - t + 4$, where t is time in seconds. Calculate the distance travelled by the particle between the time $t = 1$ second and $t = 5$ seconds. (3 marks)

SECTION II

Answer any six questions from this section

14. Given the simultaneous equations

$$5x + y = 19$$

$$-x + 3y = 0$$

a) Write the equations in matrix form. Hence solve simultaneous equations. (5 marks)

b) Find the distance of the point of intersection of the line $5x + y = 19$ and $-x + 3y = 0$ from the point $(11, -2)$. (3 marks)

15. A dealer has three grades of coffee X, Y, and Z. Grade X costs sh 140 per kg, grade Y costs sh 160 per kg and grade Z costs sh 256 per kg.

a) The dealer mixes grade X and Y in the ratio 5 : 3 to make a brand of coffee which he sells at sh 180 per kg.

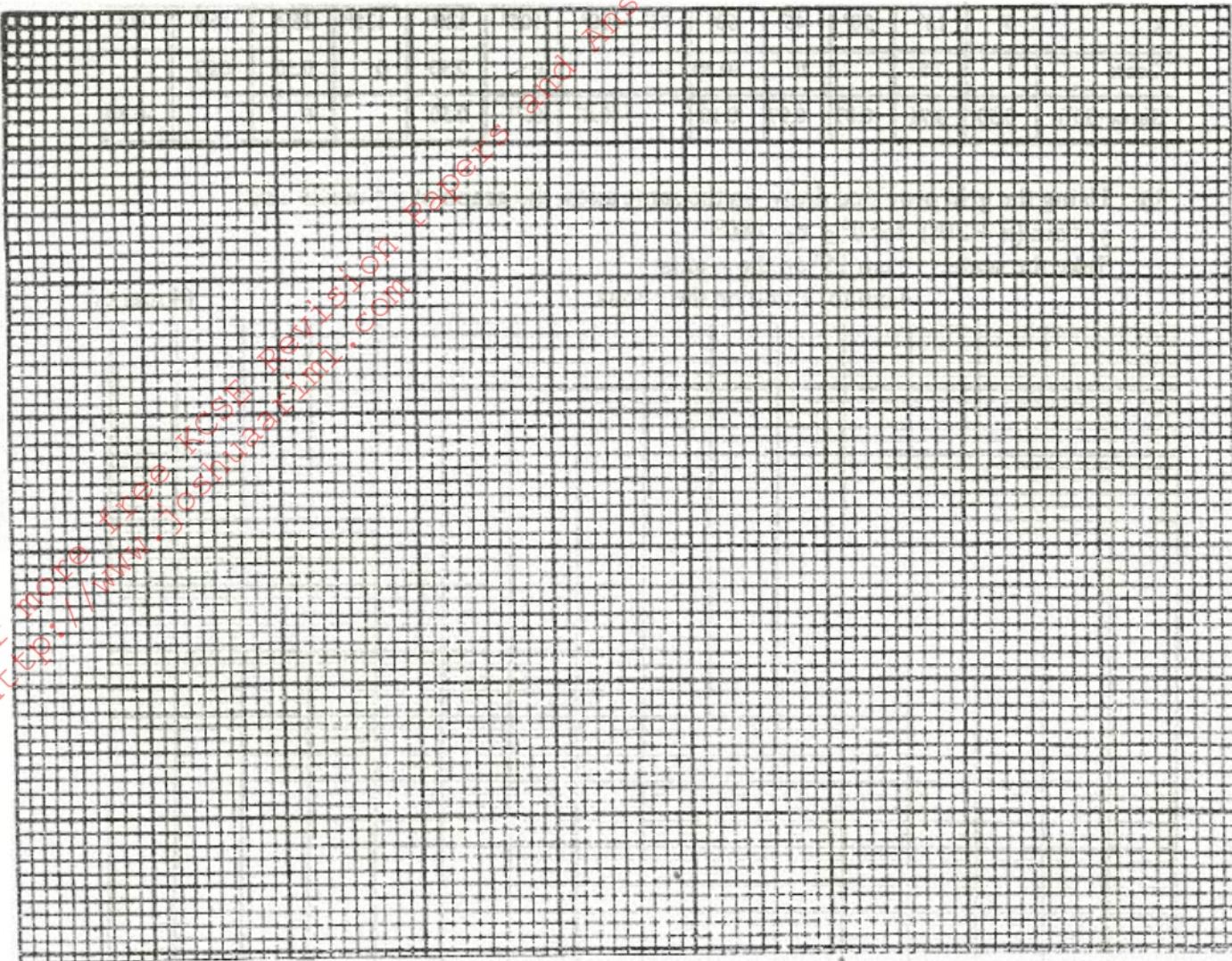
Calculate the percentage profit he makes. (3 marks)

- b) The dealer makes a new brand by mixing the three grades of coffee, in the ratios X:Y=5:3 and Y:Z=2:5.
i) the ratio X:Y:Z in its simplest form (2 marks)
ii) the selling price of the new brand if he has to make a 30% profit. (3 marks)
16. A ship leaves P for port R through port Q. Q is 200km on a bearing of 220° from P. R is 420km on a bearing of 140° from Q.
- a) Using the scale 1:4 000 000, draw a diagram showing the relative positions of the three ports P, Q and R. (3 marks)
- b) By further drawing, on the same diagram, determine how far R is to the east of P. (2 marks)
- c) If the ship had sailed directly from P to R at an average speed of 40 knots, find how long it would have taken to arrive at R. (Take 1 nautical mile = 1.853km) (3 marks)
17. Omondi makes two types of shoes: A and B. He takes 3 hours to make one pair of type A and 4 hours to make one pair of type B. He works for a maximum of 120 hours to make x pairs of type A and y pairs of type B. It costs him sh 400 to make a pair of type A and sh 150 to make a pair of type B.

His total cost does not exceed sh 9000. He must make 8 pairs of type A and more than 12 pairs of type B.

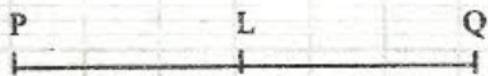
- a) Write down four inequalities representing the information above. (2 marks)
- b) On the grid provided, draw the inequalities and shade the unwanted regions. (3 marks)
- c) Omondi makes a profit of sh 40 on each pair of type A and sh 70 on each pair of type B shoes.

Use the graph in part (b) above to determine the maximum possible profit he makes.
(2 marks)



18. a) i) Find the coordinates of the stationary points on the curve $y = x^3 - 3x + 2$ (2 marks)
ii) For each stationary point determine whether it is minimum or maximum. (4 marks)
- (b) In the space provided below, sketch the graph of the function $y = x^3 - 3x + 2$. (2 marks)

19. The line PQ below, is 8cm long and L is its midpoint.



- a) i) Draw the locus of point R above line PQ such that the area of triangle PQR is 12cm^2 .
(2 marks)
- ii) Given that point R is equidistant from P and Q, show the position of point R. (2 marks)
- b) Draw all the possible locii of a point T such that $\angle RQL = \angle RTL$ (4 marks)

20. a) Complete the table below, giving your values correct to 2 decimal places. (2 marks)

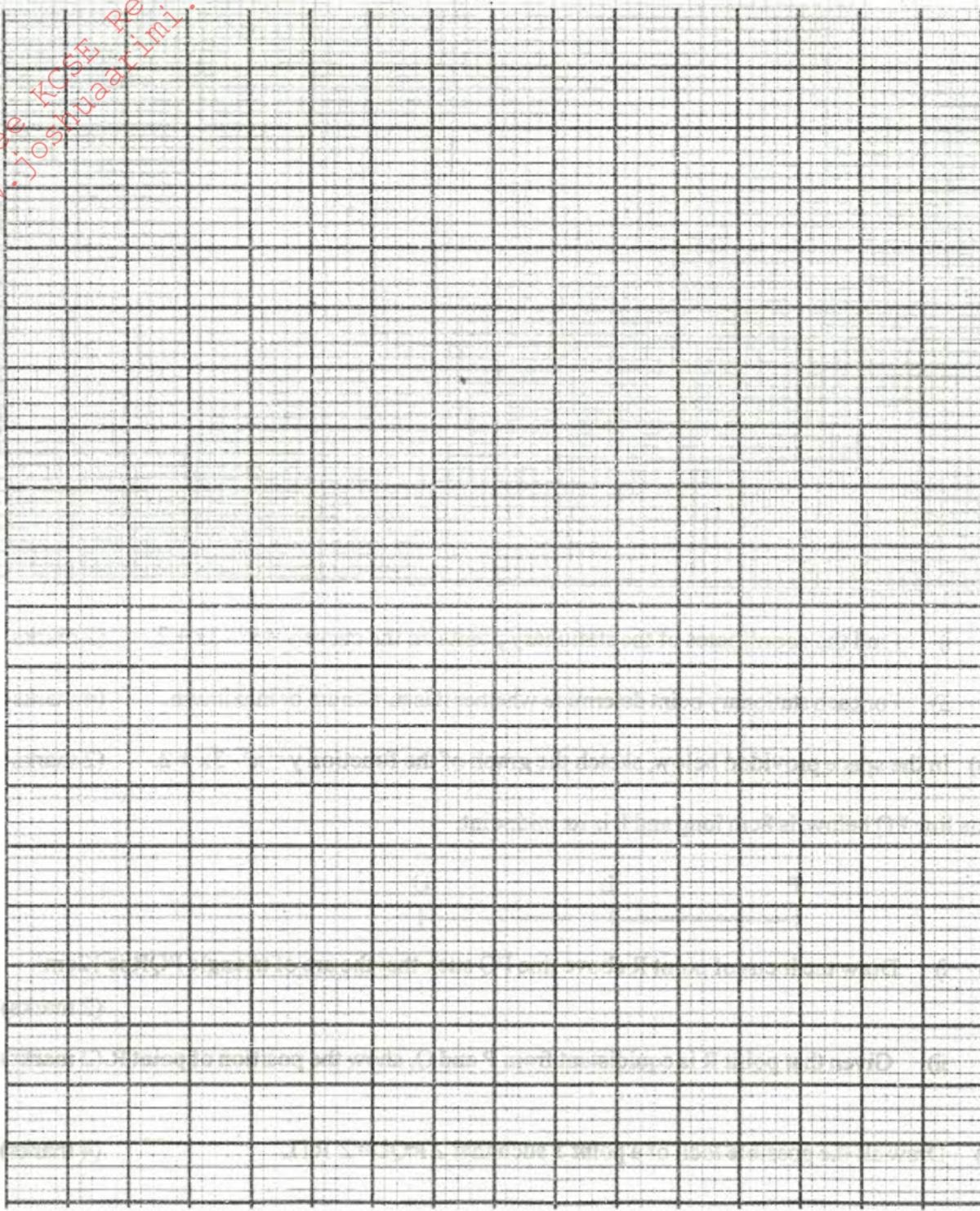
x	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
Cos x	1	0.87		0	-0.50	-1		0	0.50	0	0.50	0.87	1
sin(x+30°)	0.50	0.71	0.87	0.97	0.10		0.87	0.71	0.50		0		-0.50

b) Using the grid provided draw, on the same axes, the graph of $y = \cos 2x$ and $y = \sin(x + 30^\circ)$ for $0^\circ \leq x \leq 180^\circ$.

Take the scale: 1 cm 15° on the x-axis

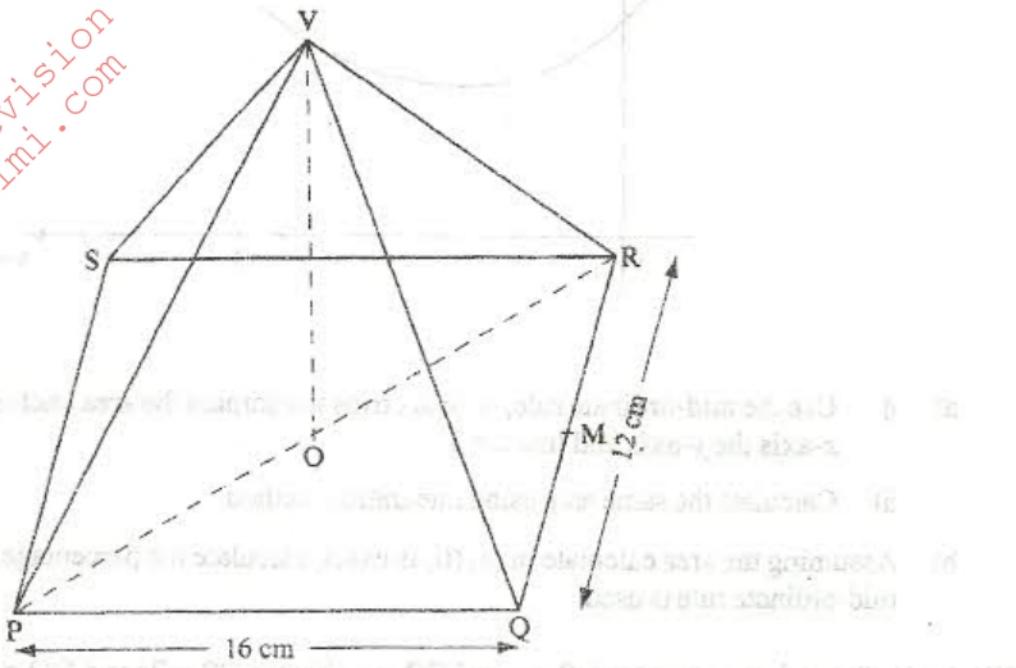
4 cm for 1 unit on the y-axis.

(4 marks)



- c) Find the period of the curve $y = \cos 2x$ (1 mark)
- d) Using the graphs in part (b) above, estimate the solutions to the equation $\sin(x + 30^\circ) = \cos 2x$ (2 marks)

21. The figure below represents a right pyramid with vertex V and a rectangular base PQRS. $VP = VQ = VR = S = 18\text{cm}$. $PQ = 16\text{cm}$ and $QR = 12\text{cm}$. M and O are the midpoints of QR and PR respectively.



Find:

- a) the length of the projection of line VP on the plane PQRS (2 marks)
- b) the size of the angle between line VP and the plane PQRS. (2 marks)
- c) the size of the angle between the planes VQR and PQRS. (4 marks)
22. The masses of 40 babies in a certain clinic were recorded as follows:

<u>Mass in Kg</u>	<u>No. of babies</u>
1.0 - 1.9	6
2.0 - 2.9	14
3.0 - 3.9	10
4.0 - 4.9	7
5.0 - 5.9	2
6.0 - 6.9	1

Calculate:

- a) the interquartile range of the data (4 marks)
- b) the standard deviation of the data using 3.45 as the assumed mean (4 marks)

23. The diagram below is a sketch of the curve $y = x^2 + 5$.

(a) Sketch

y-axis

0

$y = x^2 + 5$

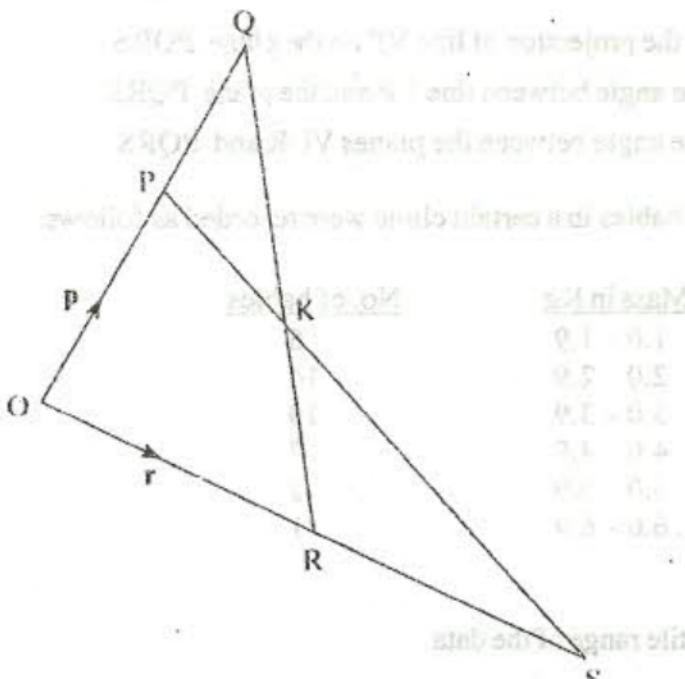
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x-axis

(b) Calculate the area enclosed by the curve, the x-axis, the y-axis and line $x = 3$.

- a) i) Use the mid-ordinate rule, with six strips to estimate the area enclosed by the curve, the x-axis, the y-axis and line $x = 3$ (4 marks)
- ii) Calculate the same area using integration method (2 marks)
- b) Assuming the area calculated in (a) (ii) is exact, calculate the percentage error made when the mid-ordinate rule is used (2 marks)

24. In the figure below, vector $OP = p$ and $OR = r$. Vector $OS = 2r$ and $OQ = \frac{3}{2} p$



- a) Express in terms of P and r

- i) QR
ii) PS

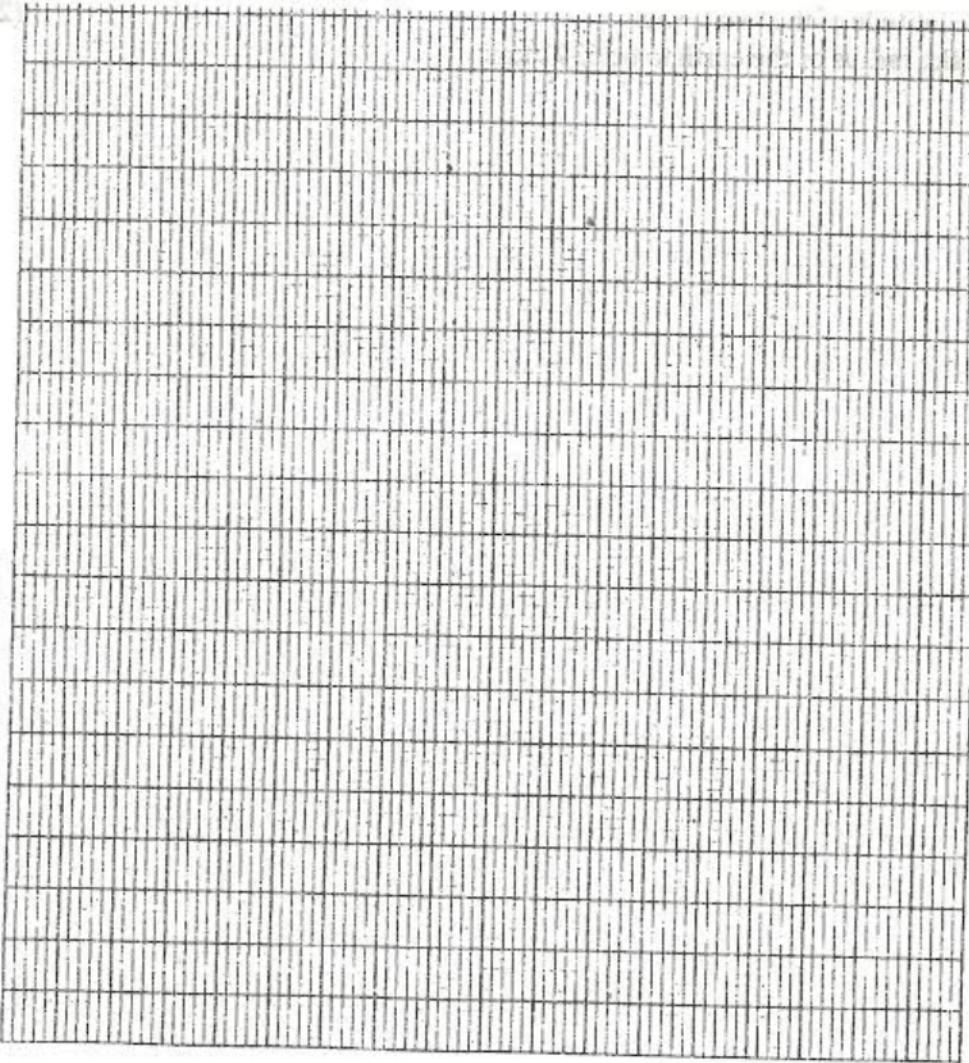
(1 mark)
(1 mark)

- b) The line QR and PS intersect at K such that $QK = mQR$ and $PK = nPS$, where m and n are scalars. Find two distinct expressions for OK in terms of P, r, m and n. Hence find the values of m and n. (5 marks)
- c) State the ratio PK : KS. (1 mark)

25. a) Complete the table below, for the function $y = 2x^2 + 4x - 3$ (2 marks)

x	-4	-3	-2	-1	0	1	2
$2x^2$	32		8	2	0	2	
$4x - 3$			-11		-3		5
y			-3		3	13	

- b) On the grid provided, draw the graph of the function $y = 2x^2 + 4x - 3$ for $-4 \leq x \leq 2$ and use the graph to estimate the roots of the equation $2x^2 + 4x - 3 = 0$ to 1 decimal place (3 marks)
- c) In order to solve graphically the equation $2x^2 + x - 5 = 0$, a straight line must be drawn to intersect the curve $y = 2x^2 + 4x - 3$. Determine the equation of this straight line, draw the straight line and hence obtain the roots of the equation. $2x^2 + x - 5 = 0$ to 1 decimal place (3 marks)



26. A businessman obtained a loan of sh 450 000 from a bank to buy a matatu valued at the same amount. The bank charges interest at 24% per annum compounded quarterly.

- a) Calculate the total amount of money the businessman paid to clear the loan in $1\frac{1}{2}$ years
(3 marks)

- b) The average income realised from the matatu per day was sh 1500. The matatu worked for 3 years at an average of 280 days per year.
Calculate the total income from the matatu
(2 marks)

- c) During the 3 years, the value of the matatu depreciated at the rate of 16% per annum. If the businessman sold the matatu at its new value, calculate the total profit he realised by the end of the 3 years.
(3 marks)

27. Two town A and B lie on the same latitude in the northern hemisphere. When it is 8.00am at A, the time at B is 11.00am.

- a) Given that the longitude of A is 15°E , find the longitude of B
(2 marks)

- b) A plane leaves A for B and takes $3\frac{1}{2}$ hours to arrive at B travelling along a parallel of latitude at 850km/h

Find:

- i) the radius of the circle of latitude on which towns A and B lie
(3 marks)

- ii) the latitude of the two towns
(2 marks)

(Take radius of the earth to be 6371 km)