

# K.C.S.E. MATHEMATICS PAPER 121/2 2001

## SECTION I

*Answer all the questions in this section*

- Two variables A and B are such that A varies partly as B and partly as the square of B. Given that A = 30, when B = 9 and A = 16 when B = 14, find A when B = 36. (4 marks)
  - A telephone bill includes Kshs. 4,320 for local calls, Kshs 3,260 for trunk calls and a rental charge of Kshs 2,080. A value added tax (V.A.T.) is then charged at 15%. Find the total bill. (3 marks)
  - Solve the equation:  $\log(x + 24) - 2 \log 3 = \log(9 - 2x)$  (3 marks)
  - The table below shows the number of bags of sugar sold per week and their moving averages.
- |                      |     |     |   |     |     |     |
|----------------------|-----|-----|---|-----|-----|-----|
| No. of bags per week | 340 | 330 | x | 343 | 350 | 345 |
| Moving averages      | 331 | 332 | y | 346 |     |     |
- State the order of moving average. (1 mark)
  - Find the values of x and y. (2 marks)
- Expand  $(2 + x)^5$  in ascending powers of x up to the term in  $x^3$ . Hence, approximate the value of  $(2.03)^5$  to 4 s.f. (4 marks)
  - A curve is given by the equation:  $y = 5x^3 - 7x^2 + 3x + 2$   
Find the:
    - gradient of the curve at  $x = 1$ . (2 marks)
    - equation of the tangent to the curve at the point (1, 3). (2 marks)
  - Given that  $\tan 75^\circ = 2 + \sqrt{3}$ , find without using tables  $\tan 15^\circ$  in the form  $P + q\sqrt{m}$ , where p, q and m are integers. (3 marks)
  - The diagram below represents a field PQR.
- 
- Draw the locus of points equidistant from sides PQ and PR. (1 mark)
  - Solve the equation  $4 \sin^2 \theta + 4 \cos \theta = 5$  for  $0^\circ \leq \theta \leq 360^\circ$  Give the answer in degrees. (3 marks)

10. A poultry farmer vaccinated 540 of his 720 chickens against a disease. Two months later, 5% of the vaccinated and 80% of the unvaccinated chicken contracted the disease. Calculate the probability that a chicken chose at random contracted the disease. (3 marks)

11. Make  $x$  the subject of the formula:

$$S = W\sqrt{a^2 - x^2} \quad (3 \text{ marks})$$

12. A particle is projected from the origin. Its speed was recorded as shown in the table below

|             |   |     |     |     |     |     |     |     |
|-------------|---|-----|-----|-----|-----|-----|-----|-----|
| Time (sec)  | 0 | 5   | 10  | 15  | 20  | 25  | 30  | 35  |
| Speed (m/s) | 0 | 2.1 | 5.3 | 5.1 | 6.8 | 6.7 | 4.7 | 2.6 |

Use the trapezoidal rule to estimate the distance covered by the particle within the 35 seconds.

(3 marks)

13. Given that  $\sin(x + 30)^\circ = \cos 2x^\circ$  for  $0^\circ < x < 90^\circ$  find the value of  $x$ . Hence find the value of  $\cos^2 3x^\circ$  (3 marks)

14. Given that  $P = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$  and  $Q = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$  find the matrix product  $PQ$  (1 mark)

Hence, solve the simultaneous equations below:

$$\begin{aligned} 2x - 3y &= 5 \\ -x + 2y &= -3 \end{aligned} \quad (3 \text{ marks})$$

15. The position vectors for points P and Q are  $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$  and  $3\mathbf{i} - 6\mathbf{j} + 6\mathbf{k}$  respectively. Express vector  $\mathbf{PQ}$  in terms of unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$ . Hence find the length of  $\mathbf{PQ}$ , leaving your answer in simplified surd form (3 marks)

## SECTION II

*Answer any six questions in this section*

16. The marks obtained by 10 pupils in an English test were 15, 14, 12, 13, P, 16, 11, 13, 12 and 17.

The sum of the squares of the marks,  $\sum x^2 = 1794$

- a) Calculate the:

i) Value of P (2 marks)

ii) Standard deviation. (4 marks)

- b) If each mark is increased by 3, write down the:

i) new mean (1 mark)

ii) new standard deviation (1 mark)

17. The  $n^{\text{th}}$  term of a sequence is given by  $2n + 3$ .

a) Write down the first four items of the sequence. (2 marks)

b) Find  $S_{50}$ , the sum of the first fifty terms of the sequence. (2 marks)

c) Show that the sum of the first  $n$  terms of the sequence is given by

$$S_n = n^2 + 4n$$

Hence or otherwise find the largest integral value of  $n$  such that

$$S_n < 725.$$

(2 marks)

18. a) Complete the table given below by filling in the blank spaces.

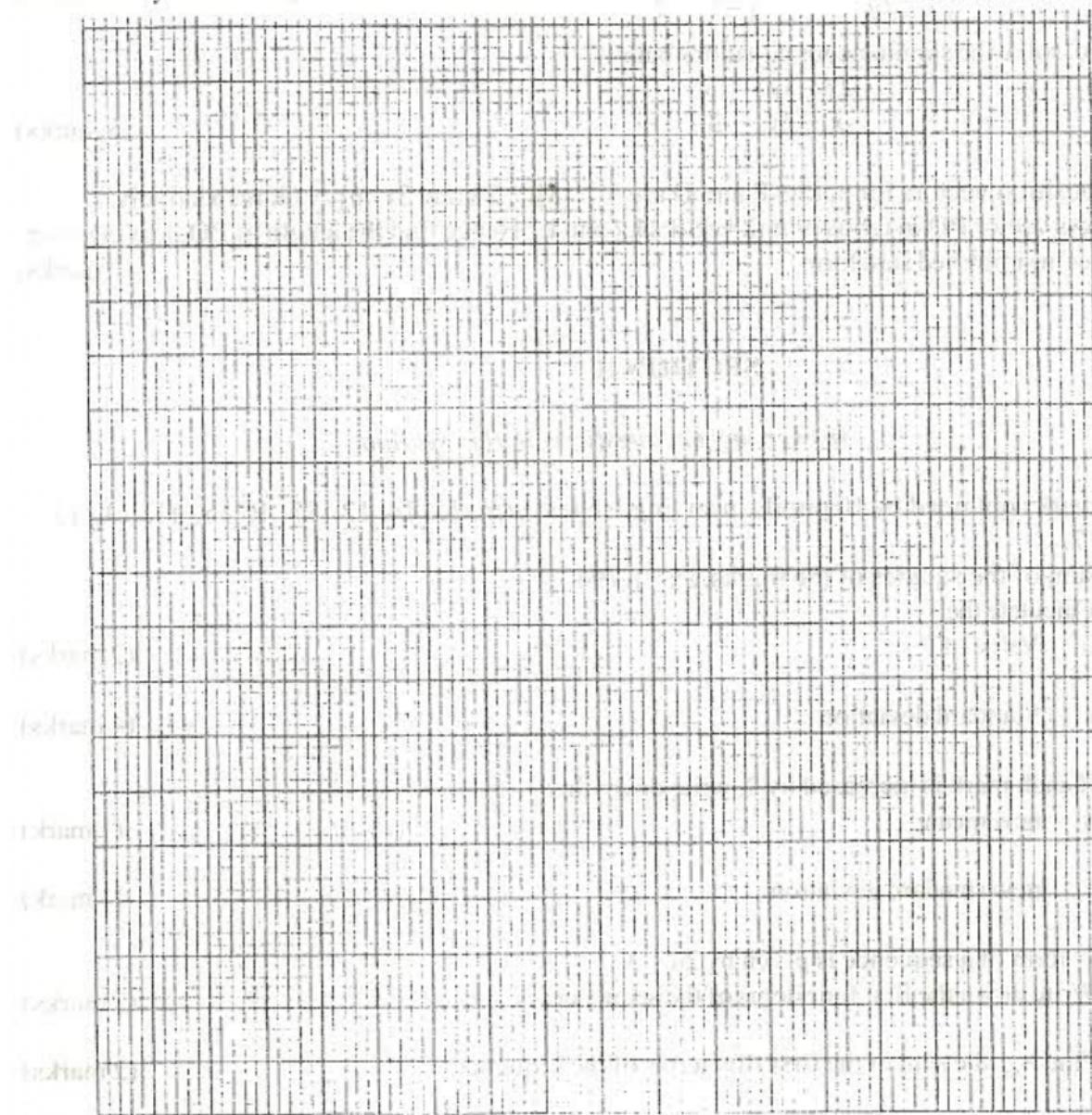
| x                     | 0° | 15°    | 30° | 45°   | 60°  | 75° | 90° | 105°   | 120°   | 135° | 150°   | 165° | 180° |
|-----------------------|----|--------|-----|-------|------|-----|-----|--------|--------|------|--------|------|------|
| $3 \cos 2x$           | 3  | -2.598 | 1.5 | 0     | -1.5 |     | -3  | -2.598 | -1.5   | 0    | 2.598  | 3    |      |
| $2 \sin(2x+30^\circ)$ | 1  |        | 2   | 1.732 | 1    | 0   |     | -1     | -1.732 | -2   | -1.732 | -1   | 1    |

(2 marks)

b) On the grid provided draw, on the same axes, the graph of  $y = 3 \cos 2x$  and  $y = \sin(2x+30^\circ)$  for  $0^\circ \leq x \leq 180^\circ$ .

Take the scale: 1 cm for  $15^\circ$  on the x-axis and 2 cm for 1 unit on the y-axis. (4 marks)

e) Use your graph to estimate the range of values of  $x$  for which  $3 \cos 2x \leq 2 \sin(2x + 30^\circ)$ . Give your answer to the nearest degree. (2 marks)



19. The displacement  $x$  metres a particle after  $t$  seconds given by

$$x = t^3 - 2t^2 + 6, \quad t > 0.$$

- a) Calculate the velocity of the particle in m/s when  $t = 2$  seconds. (3 marks)
- b) When the velocity of the particle is zero, calculate its:
- i) displacement (3 marks)
  - ii) acceleration. (2 marks)

20. Bot Juice Company has two types of machines, A and B, for juice production.

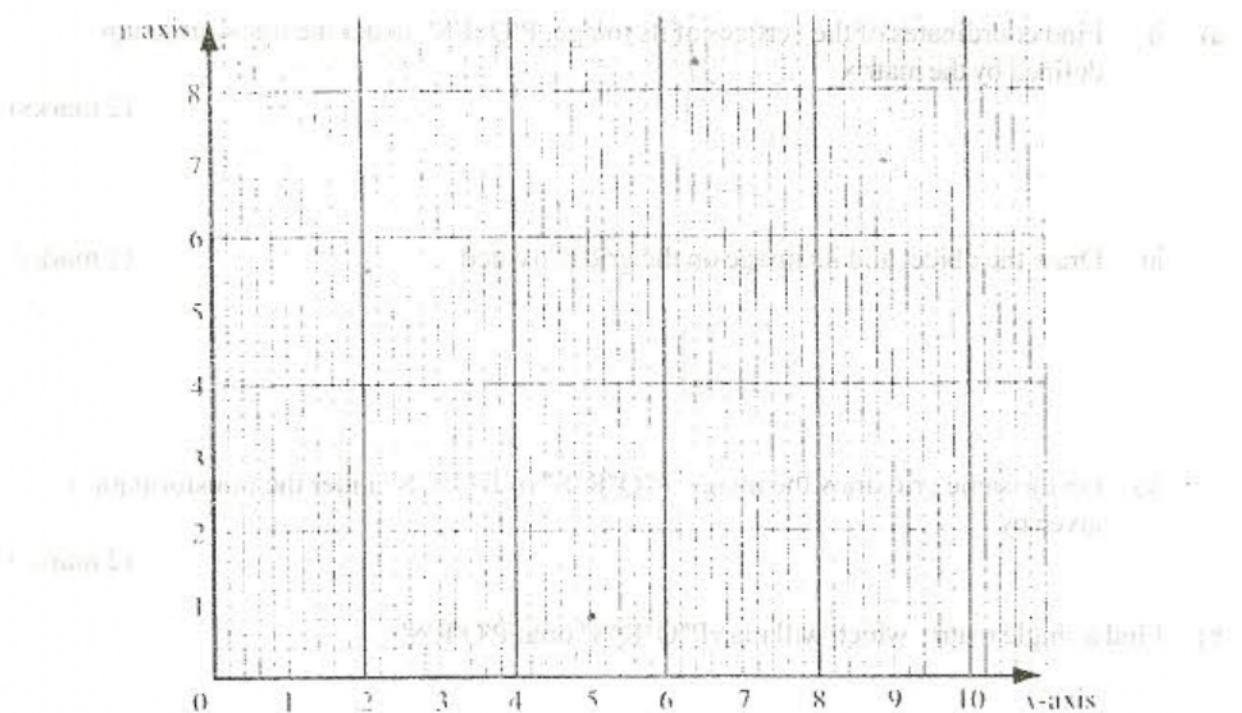
Type A machine can produce 800 litres per day while type B machine produces 1,600 litres per day.

Type A machine needs 4 operators and type B machine needs 7 operators.

At least 8,000 litres must be produced daily and the total number of operators should not exceed 41. There should be 2 or more machines of each type.

Let  $x$  be the number of machines of type A and  $y$  the number of machines for type B.

- a) Form all inequalities in  $x$  and  $y$  to represent the above information. (3 marks)
- b) On the grid provided below, draw the inequalities and shade the unwanted regions. (3 marks)



- c) Use the graph in (b) to determine the least number of operators required for the maximum possible production. (2 marks)

21. The table below shows income tax rates.

| Monthly taxable pay<br>K£ | Rate of tax<br>Ksh in 1K£ |
|---------------------------|---------------------------|
| 1 - 435                   | 2                         |
| 436 - 870                 | 3                         |
| 871 - 1305                | 4                         |
| 1306 - 1740               | 5                         |
| Excess over 1740          | 6                         |

$$1\text{K£} = \text{sh } 20$$

A company employee earns monthly basic salary of Ksh 30,000 and is also given taxable allowances amounting to Ksh 10,480.

- Calculate the total income tax (4 marks)
- The employee is entitled to a personal tax relief of Ksh 800 per month. Determine the net tax (1 mark)
- If the employee received a 50% increase in his total income, calculate the corresponding percentage increase on the income tax. (3 marks)

22. The coordinates of the vertices of rectangle PQRS are P(1,1), Q(6,1), R(6,4) and S(1,4).

- i Find coordinates of the vertices of its image, P'Q'R'S', under the transformation defined by the matrix

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$$

(2 marks)

- ii Draw the object and its image on the grid provided (2 marks)

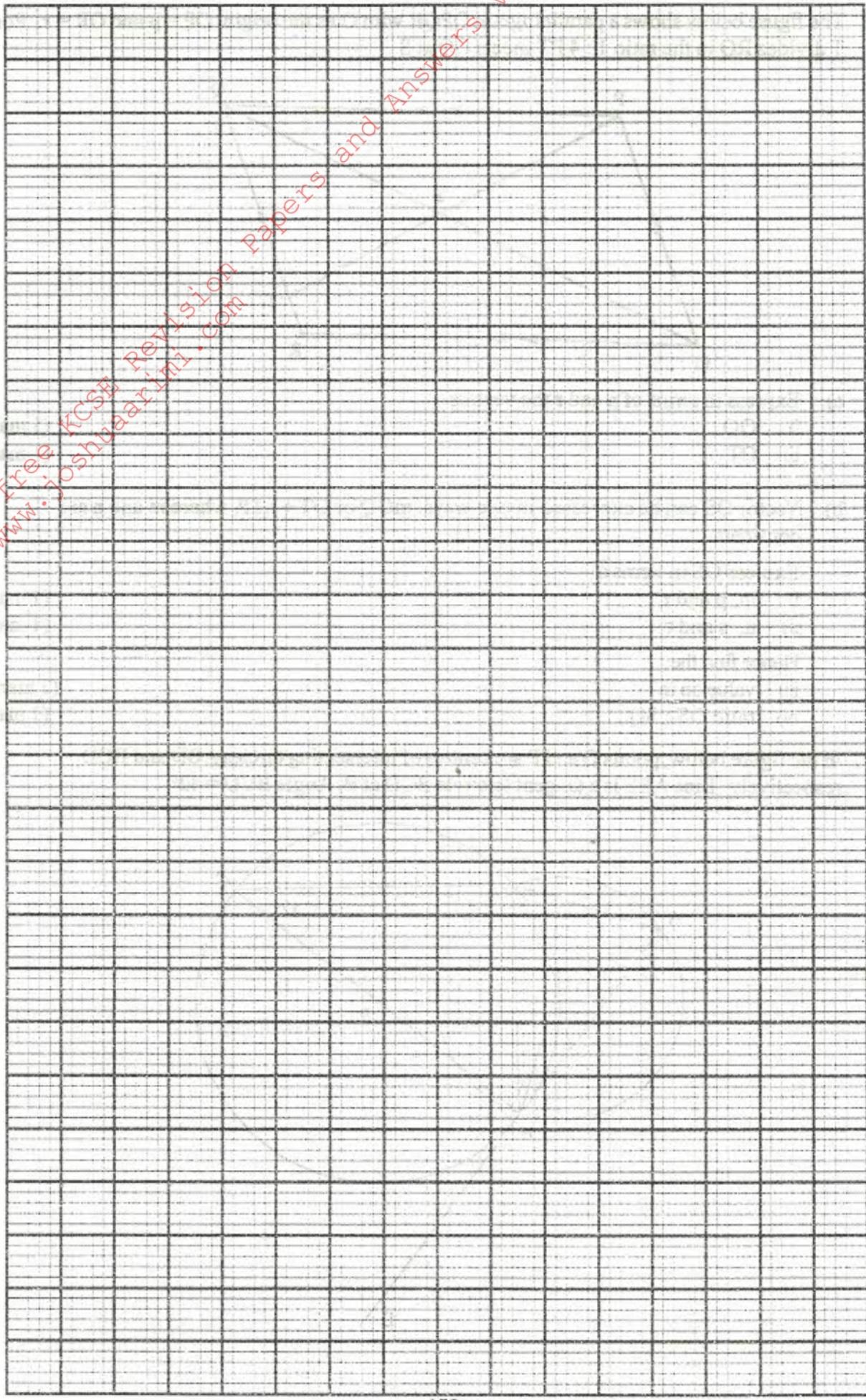
$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- iii On the same grid draw the image, P''Q''R''S'' of P'Q'R'S' under the transformation given by

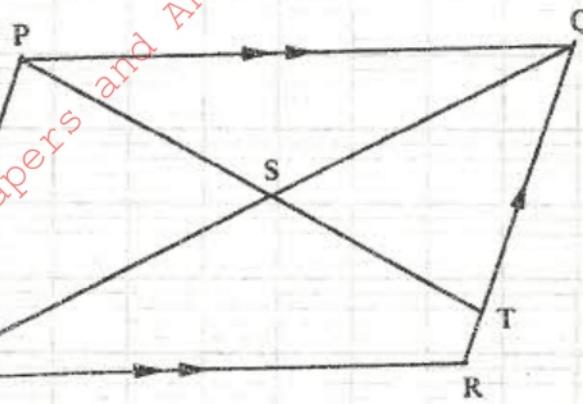
(2 marks)

- b Find a single matrix which will map P''Q''R''S'' onto P'Q'R'S'

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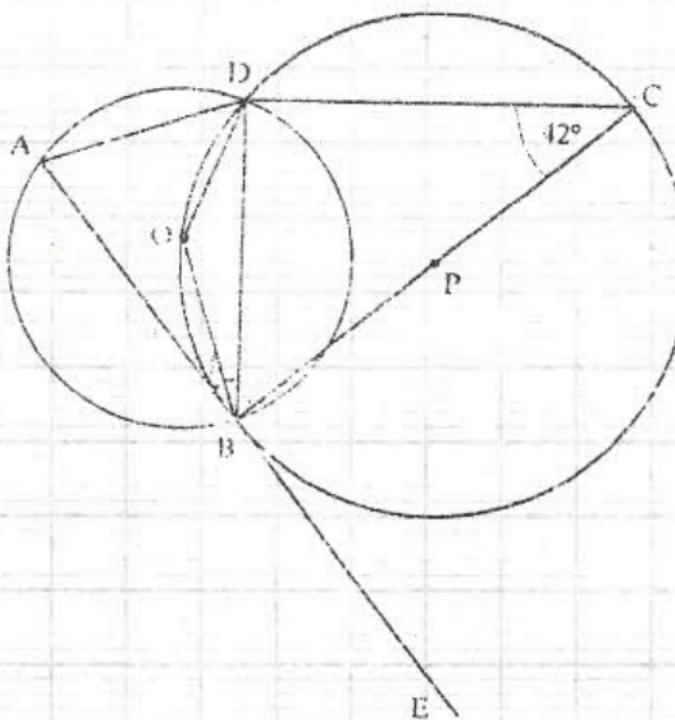


23. The figure below shows a parallelogram  $OPQR$  with  $O$  as the origin,  $OP = \mathbf{p}$  and  $OR = \mathbf{r}$ . Point  $T$  divides  $RQ$  in the ratio  $1 : 4$ .  $PT$  meets  $OQ$  at  $S$ .



- a) Express in terms of  $\mathbf{p}$  and  $\mathbf{r}$  the vectors
- $OQ$
  - $OT$
- (1 mark)  
(1 mark)
- b) Vector  $OS$  can be expressed in two ways:  $mOQ$  or  $OT + nTP$ , where  $m$  and  $n$  are constants.
- Express  $OS$  in terms of
- $m, p$  and  $r$
  - $n, s$  and  $r$
- (1 mark)  
(1 mark)
- Hence find the:
- value of  $m$
  - ratio  $OS : SQ$
- (3 marks)  
(1 mark)

24. In the figure below, points  $O$  and  $P$  are centres of intersecting circles  $ABD$  and  $BCD$  respectively. Line  $ABE$  is a tangent to circle  $BCD$  at  $B$ . Angle  $BCD = 42^\circ$ .



- a) Stating reasons, determine the sizes of  
 i)  $\angle CBD$  (2 marks)  
 ii) Reflex  $\angle BOD$  (3 marks)
- b) Show that  $\Delta ABD$  is isosceles  $\frac{dy}{dx} = 2x^2 - 5$  (3 marks)

25. a) The gradient function of a curve is given by

Find the equation of the curve, given that  $y = 3$ , when  $x = 2$  (4 marks)

- b) The velocity,  $v$  m/s of a moving particle after  $t$  seconds is given by:  $v = 2t^3 + t^2 - 1$   
 Find the distance covered by the particle in the interval  $1 \leq t \leq 3$  (4 marks)

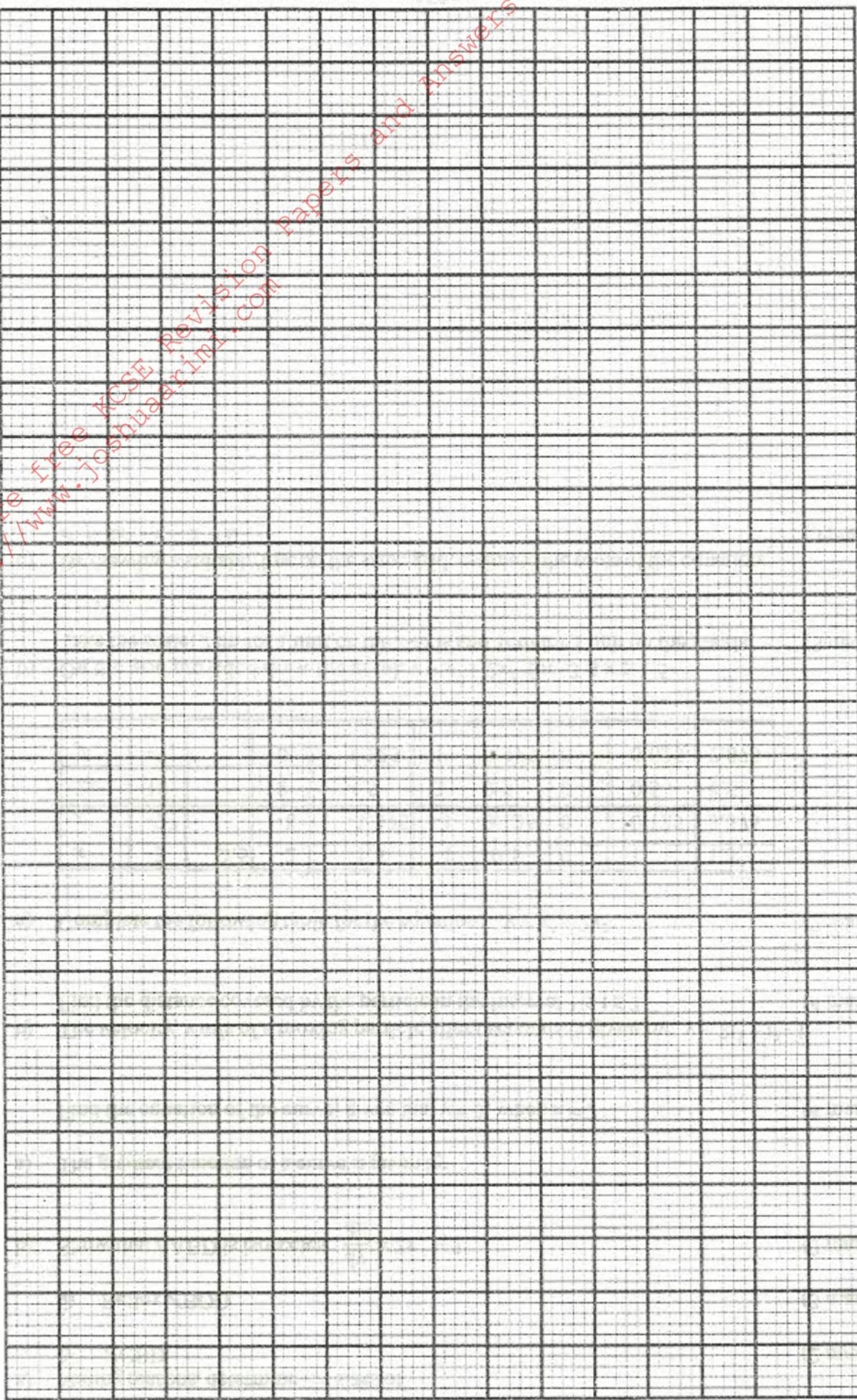
26. a) Complete the following table for the equation:  $y = x^3 + 2x^2$  (2 marks)

|        |     |      |    |        |    |       |   |       |       |
|--------|-----|------|----|--------|----|-------|---|-------|-------|
| $x$    | -3  | -2.5 | -2 | -1.5   | -1 | -0.5  | 0 | 1     | 1.5   |
| $x^3$  | -27 |      | -8 | -3.375 | -1 | 0.125 | 0 | 0.125 | 3.375 |
| $2x^2$ | 18  |      | 8  | 4.5    | 2  | 0.5   | 0 | 0.5   | 4.5   |
| $y$    | -9  |      | 0  | 1.125  | 1  | 0.375 | 0 | 0.625 | 7.875 |

- b) On the grid provided draw the graph  $y = x^3 + 2x^2$  for  $-3 \leq x \leq 1.5$   
 Take the scale: 2cm for 1 unit on the  $x$ -axis and 1cm for 1 unit on the  $y$ -axis (3 marks)

- c) By drawing a suitable line on the same grid, estimate the roots of the equation:  
 $x^3 + 2x^2 - x - 2 = 0$  (3 marks)

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27. The probabilities of three darts players Akinyi, Kamau and Juma hitting the bull's eye are 0.2, 0.3 and 0.15 respectively

- a) Draw a probability tree diagram to show the possible outcomes (2 marks)
- b) Find the probability that:  
i) all hit the bull's eye (2 marks)  
ii) only one of them hit the bull's eye (2 marks)  
iii) at most one misses the bull's eye (2 marks)

28. A plane flying at 200 knots left an airport A ( $30^{\circ}\text{S}$ ,  $31^{\circ}\text{E}$ ) and flew due North to an airport B ( $30^{\circ}\text{N}$ ,  $31^{\circ}\text{E}$ )

- a) Calculate the distance covered by the plane, in nautical miles (2 marks)
- b) After a 15 minutes stopover at B, the plane flew due West to an airport C ( $30^{\circ}\text{N}$ ,  $130^{\circ}\text{E}$ ) at the same speed.
- Calculate the total time taken to complete the journey from airport A to airport C, through airport B. (6 marks)

