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(DAA)
Tutorial Sheet-1.

Ans 1 (3) O(N+M) time O(1) space.

2. $T(n) = O(n)$; space $O(1)$

3. $T(n) = O(\log_2 n)$, space $O(1)$

4.
```
int sum=0, i;
for (i=0; i*i<n; i++)
{
    sum+=i;
}
```
$n+(n-1)+(n-4)+(n-9)+ --- (n-k)$

$n+(n*k) - (1^2+2^2+3^2+ --- k^2)$

$\sqrt{n}$

$i^2 < n$

$i^2 < \sqrt{n}$

$T(n) = O(\sqrt{n})$, space $O(1)$

5-
```
int j=1, i=0
while (i<n)
{
    i=i+j;
    j++;
}
```
$(0,1,3,6,10,15,21, --- n)$

k^{th} term $= \left(\frac{k*(k+1)}{2}\right)$

$n = \frac{k^2+k}{2}$

$k^2+k = 2n$

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$K^2 + K = 2n$

$K^2 + k - 2n = 0$

~~K = -1 + 1~~ $K = \frac{\sqrt{8n}}{2} = \sqrt{n}$

$T(n) = \sqrt{n}$, Space $\rightarrow O(1)$

6- Void Recursion (int n) $\rightarrow T(n)$

```
{
  if (n==1) return;
  recursion (n-1)  ----> T(n-1)
  print(n);
  recursion (n-1);  ---> T(n-1)
}
```

$$T(n) = \begin{cases} 1 & n=1 \\ 2T(n-1)+1 & n>1 \end{cases}$$

$T(n) = 2T(n-1) + 1 \rightarrow$ eq 1

$T(n-1) = 2T(n-2) + 1$

$T(n) = 4T(n-2) + (1+2)$ — eq 2,

$T(n-2) = 2(T(n-3)) + 1$

$T(n) = 4(2T(n-3)+1) + (1+2)$

$T(n) = 8T(n-3) + (1+2+4)$ — eq 3

$T(n) = 8[2(T(n-4)+1)] + [1+2+4]$

$T(n) = 16\,T(n-4) + (1+2+4+8)$ — eq 4

$T(n) = 2^k\, T(n-k) + (1+2+4+8+ \ldots)$

$T(n-k) = T(1)$

$k = n-1$

$T(n) = 2^{n-1}\, T(1) + (1+2+4+8+ \ldots)$
n-1 times.

$T(n) = \frac{2^n}{2} + \left(\frac{2^{n-1}-1}{1}\right)$

$T(n) = \frac{2^n}{2} + \frac{2^n}{2} - 1$

$T(n) = 2\left(\frac{2^n}{2}\right) - 1$

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$T(n) = 2^n - 1$

$T(n) = O(2^n)$

7 It is a Binary search algorithm.

$T(n) = \log_2 n$

$T(n) = T(\frac{n}{2}) + 1$

by using Master method (can't be solved)

$a = 1$

$b = 2$

$f(n) = 1$

$c = \log_b a = \log_2 1 = 0$

$0 \simeq 1$

$n^0 = f(n) = 1$

$n^c = f(n)$

$T(n) = O(\log_2 n)$

8 $T(1) = 1$

1. $T(n) = T(n-1) + 1$ — eq 1

$T(n) = T(n-2) + 2$ — eq 2

$T(n) = T(n-3) + 3$ — eq 3

$n - k = 1$

$k = n - 1$

$T(n) = T(1) + n - 1$

$T(n) = n$

$T(n) = O(n)$

2. $T(n) = T(n-1) + n$ — eq 1

$T(n-1) = T(n-2) + (n-1)$

$T(n) = T(n-2) + (n + (n-1))$ — eq 2

$T(n) = T(n-3) + (n + (n-1) + (n-2))$ — eq 3

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$T(n) = T(n-k) + (n + (n-1) + (n-2) - - - (n-k-1))$

$T(n-k) = T(1)$

$n = k+1$

$k = n-1$

$T(n) = T(1) + (n + (n-1) + (n-2) - - - (n - n + n - 1))$

$T(n) = 1 + (n + (n-1) + (n-2) + - - - 1)$

$T(n) = 1 + \frac{n(n+1)}{2} = \frac{n^2+1+1}{2}$

$T(n) = \frac{n^2+2}{2}$

$T(n) = O(n^2)$ //

Q. $T(n) = T(n/2) + 1$ — eq ①

$T(n/2) = T(n/4) + 1$

$T(n) = T(n/4) + 2$ — eq 2

$T(n/4) = T(n/8) + 1$

$T(n) = T(n/8) + 3$ — eq 3

$T(n) = T\left(\frac{n}{2^k}\right) + k$ — eq 4

$\frac{n}{2^k} = 1$

$2^k = n$

$k = \log_2 n$

~~Ans~~ 8 (Ans 4) → $T(n) = 2T(n/2) + 1$

$c = 1$

$n^c = n$

$f(n) = 1$

$n^c > f(n)$, $T(n) = \Theta(n)$

~~Ans 8~~ ~~Ans 5.~~ $T(n) = 2T(n-1)+1$

$T(n) = O(2^n)$

~~Ans 8~~ ~~Ans 6~~ - $T(n) = 3T(n-1),\ T(0)=1$

$T(n) = 3(T(n-1))$ — eq 1

$T(n) = 9T(n-2)$

$T(n) = 3^3\ T(n-3)$

$T(n) = 3^k\ T(n-k)$

for $n-k=0$

$n=k$

$T(n) = 3^n T(0)$

$T(n) = 3^n$

$T(n) = O(3^n)$

~~Ans 8~~ ~~Ans 7~~ - $T(n) = T(\sqrt{n})+1$ — eq 1

$T\sqrt{n} = T(n^{1/4})+1$

$T(n) = T(n^{1/4})+2$ — eq 2.

$T(n) = T(n^{1/8})+3$ — eq 3.

$T(n) = T(n^{1/k})+k$

for $T((\sqrt{n})^{1/k}) = T(2)$

$n^{1/k} = 2.$

$n^{\frac{1}{2k}} = 1.$

$\frac{1}{2^k} \log n = 1$

$2^k = \log n$

$k = \log_2(\log n)$

$T(n) = O(\log(\log(n)))$

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Ans 8

$T(n) = T(\sqrt{n}) + n$

$T(\sqrt{n}) = T(n^{1/4}) + \sqrt{n}$

$T(n) = T(n^{1/2k}) + (n + n^{1/2} + n^{1/4} + \ldots)$

for $n^{1/2k} = 2$.

$\frac{1}{2^k} = \frac{1}{\log(n)}$

$2^k = \log(n)$

$k = \log(\log(n))$

$T(n) = 1 + (n + \sqrt{n} + \sqrt{n} \cdot \sqrt{n} + \ldots)$

$T(n) = 1 + ($ G.P $a = n$, $r = \sqrt{n}$, No. of terms $= k)$

$T(n) = 1 + \left(\frac{n(\sqrt{n})^k - 1}{(k-1)}\right)$

$T(n) = 1 + n\left(\frac{(\sqrt{n})^{\log\log(n)} - 1}{\log\log(n) - 1}\right)$

$T(n) = n \log\log(n)$

$T(n) = O(n \log(\log(n)))$

Ans 9.

```
int sum = 0, i;
for (i = 0; i < n; i++)
{
    sum += i;
}
```

So, $T(n) = O(n)$, space $O(1)$

10. $O(N * (N, N-1, \ldots 1))$

$O(N * (\frac{N+1}{2}))$

$O(N * N)$

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(11) - $O\left(\frac{n}{2} * (\log_2 N)\right)$

$O(n \log n)$

(12) (2) x will always be a better choice for large input.

(13) (4) $O(\log N)$

(14) $T(n) = 7\left(T\left(\frac{n}{2}\right)\right) + (3n^2+2)$

$f(n) = 3n^2+2$

$a = 7$

$b = 2$

$c = \log_b a = \log_2 7 = 2.807$

$n^c = n^{2.8} \approx n^{2.8}$

$f(n) = 3n^2 + 2$

so, $n^c > f(n)$

so. $T(n) = \theta(n^{2.8})$

(15)

$f_1(n) = n^{\sqrt{n}}$

$f_2(n) = 2^n$

$f_3(n) = (1.00001)^n$

$f_4(n) = n(10 * 2^{n/2})$

(a) - $f_2(n) > f_4(n) > f_3(n) > f_1(n)$

(16) - $f(n) = 2^{2n}$

$\log f(n) = 2n \log_2 2$

$\log f(n) = 2n$ $\quad$ or $f(n) = 2^n \cdot 2^n$

$\Omega(2^n)$

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17. $T(n) \leq 2T\left(\frac{n}{2}\right) + n^2$

$c = 1$

$n^c = n$

$n^2 > n$

$f(n) > n^c$

$T(n) = \theta(n^2)$

A. (18). $O(\log N)$

A. (19). $T(n) = O(N^2 + N)$

$T(n) = O(N^2)$