1 Expected Utility and Risk Aversion

This chapter is a first step toward understanding how an individual's preferences toward risk affect his portfolio behavior. It was shown that if an individual's risk preferences satisfied specific plausible conditions, then her behavior could be represented by a **von Neumann-Morgenstern expected utility function**. In turn, the shape of the individual's utility function determines a measure of risk aversion that is linked to two concepts of a risk premium.

- The first one is the **monetary payment** that the individual is willing to pay to avoid a risk, an example being a premium paid to insure against a property/casualty loss.
- The second is the rate of return in excess of a riskless rate that the individual requires to hold a risky asset, which is the common definition of a **security risk premium** used in the finance literature.

Finally, it was shown how an individual's absolute and relative risk aversion affect his choice between a risky and risk-free asset. In particular, individuals with decreasing (increasing) relative risk aversion invest proportionally more (less) in the risky asset as their wealth increases.

2 Mean-Variance Analysis

When the returns on individual assets are multivariate normally distributed, a risk-averse investor optimally chooses among a set of **mean-variance efficient portfolios**.
Such portfolios make best use of the **benefits of diversification** by providing the highest mean portfolio return for a given portfolio variance.

The particular efficient portfolio chosen by a given investor depends on her level of risk aversion. However, the ability to trade in only two efficient portfolios is sufficient to satisfy all investors, because any efficient portfolio can be created from any other two. When a riskless asset exists, the set of efficient portfolios has the characteristic that the portfolios' mean returns are linear in their portfolio variances.

- In such a case, a more risk-averse investor optimally holds a positive amount of the riskless asset and a positive amount of a particular risky-asset portfolio,
- while a less risk-averse investor optimally borrows at the riskless rate to purchase the same risky-asset portfolio in an amount exceeding his wealth.

3 CAPM, Arbitrage, and Linear Factor Models

In this chapter we took a first step in understanding the equilibrium determinants of individual assets' prices and returns. The Capital Asset Pricing Model(**CAPM**) was shown to be a natural extension of Markowitz's mean-variance portfolio analysis.

However, in addition to deriving CAPM from investor mean-variance risk-preferences, we showed that CAPM and its multifactor generalization Arbitrage Pricing Theory(**APT**), could result from assumptions of a linear model of asset returns and an absence of arbitrage opportunities.

4 Consumption-Savings and State Pricing

This chapter began by extending an individual's portfolio choice problem to include an initial consumption-savings decision. With this modification, we showed that an optimal portfolio is one where asset's expected marginal utility-weighted returns are equalized.

- Also, the individual's optimal level of savings involves an **inter-temporal trade-off** where the marginal utility of future consumption.

The individual's optimal decision rules can be reinterpreted as an asset pricing formula. This formula values assets' returns using a **stochastic discount factor** equal to the marginal rate of substitution between present and future consumption. Importantly, the stochastic discount factor is independent of the asset being priced and determines the asset's risk premium based on the covariance of the asset's return with the marginal utility of consumption.

- Moreover, this consumption-based stochastic discount factor approach places restrictions on assets' risk premia relative to the volatility of consumption. However, these restrictions appear to be violated when empirical evidence is interpreted using standard utility specifications.

This contrary empirical evidence does not automatically invalidate the stochastic discount factor approach to pricing assets. Rather than deriving discount factors as the marginal rate of substituting present for future consumption, we showed that they can be derived based on the alternative assumptions of market completeness and an absence of arbitrage. When assets' returns spanned the economy's states of nature, state prices for valuing any derivative asset could be derived. Finally, we showed how an alternative risk-neutral pricing formula could be derived by transforming the states' physical probabilities to reflect an adjustment for risk.

5 A Multiperiod Discrete-Time Model for Consumption and Portfolio choice

An individual's **optimal strategy for making lifetime** consumption-savings and portfolio allocation decisions is a topic having practical importance to financial planners. This chapter's analysis represents a first step in formulating and deriving a lifetime financial plan. We showed that an individual could approach this problem by a backward dynamic programming technique that first considered how decisions would be made when he reached the end of his planning horizon.

- For prior periods, consumption and portfolio decisions were derived using **the recursive Bellman equation** which is based on the concept of a derived utility of wealth function. The multiperiod planning problem was transformed into a series of easier-to-solve one-period problems.
  - While the consumption-portfolio choice problem in this chapter assumed that lifetime utility was time separable, in future chapters we show that the Bellman equation solution technique often can apply to cases of time-inseparable lifetime utility.

Our general solution technique was illustrated for the special case of an individual having logarithmic utility and no wage income. It turned out that this individual's optimal consumption decision was to consume a proportion of wealth each period, where the proportion was a function of the remaining periods in the individual's planning horizon but not of the current or future distributions of asset returns.

- In other words, future investment opportunities did not affect the individual's current consumption-savings decision. Optimal portfolio allocations were also relatively simple because they depended only on the current period's distribution of asset returns.

6 Multiperiod Market Equilibrium

When individuals choose lifetime consumption and portfolio holdings in an optimal fashion, a multiperiod stochastic discount factor can be used to price assets. This is an important generalization of our earlier single-period pricing result. We also demonstrated that if an asset's dividends (cashflows) are modeled explicitly, the asset's price satisfies a discounted dividend formula.

- **The Lucas endowment economy model** took this discounted dividend formula a step further by equating aggregate dividends to aggregate consumption.

    - This simplified valuing a claim on aggregate dividends, since now the value of this market portfolio could be expressed as an expectation of a function of only the future dividend (output) process.

In an infinite horizon model, the possibility of rational asset price bubbles needs to be considered. In general, there are multiple solutions for the price of a risky asset. **Bubble solutions** represent nonstationary alternatives to the asset's fundamental value.

- However, when additional aspects of the economic environment are considered, the conditions that would give rise to rational bubbles appear to be rare.

7 Basics of Derivative Pricing

In an environment where there is an absence of arbitrage opportunities, the price of a contingent claim is restricted by the price of its underlying asset. For some derivative securities, such as forward contracts, the contract's payoff can be replicated by the underlying asset and a riskless asset using a static trading strategy.

- In such a situation, the absence of arbitrage leads to a unique link between the derivative's price and that of its underlying asset without the need for additional assumptions regarding the asset's return distribution.
    - For other types of derivatives, including options, static replication may not be possible. An additional assumption regarding the underlying asset's return distribution is necessary for valuing such derivative contracts.

An example is the assumption that the underlying asset's returns are **binomially distributed**. In this case, an option's payoff can be dynamically replicated by repeated trading in a portfolio consisting of its underlying asset and a risk-free asset.

- Consistent with our earlier analysis, this situation of a dynamically complete market allows us to value derivatives using the risk-neutral approach.
- We also illustrated the flexibility of this binomial model by applying it to value options having an early exercise feature as well as options written on a dividend-paying asset.