Lab 7 grammar 3 proof

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There exists a language $L = \{ww | w \in \{0,1\}^*\}$. The grammar that produced this language is not a context-free grammar; I will show this by demonstrating that this language is not a context-free language.

Proof by using the Pumping Lemma for Context-Free Grammars

If a language L is context-free, then there exists some integer $p \geq 1$ such that every string s in L with $|s| \geq p$ can be written as

$s = uvxyz$

with substring u, v, x, y and z, such that

1. $|vxy| \leq p$
2. $|vy| \geq 1$, and
3. $uv^n xy^n z \in L \forall n \geq 0$.

Let $s = 0^{p/2}1^{p/2}0^{p/2}1^{p/2}$. For $w = 0^n 1^n$, this string is in L. Because $|vxy| \leq p$, a limited number of possibilities exist:

1. vxy is contained within the first w.

   If case 1 is correct, then $uv2xy^2z$) cannot be within the language, as the first and second ws must be identical, and the second w remains unchanged between $uvxyz$ and $uv2xy^2z$; it is contained entirely within the substring z.

2. vxy is contained within the second w.

   This argument, stating that $uv2xy^2z$ cannot be within the language, is symmetric with the first. In this case, the first w remains entirely unchanged as it is contained entirely within the substring u.

3. vxy is spread across both ws.

   In this case, vxy being spread across both ws means that it is of the form

   (a) $0^k 1^{p/2} 0^j$,
   (b) $1^k 0^j$, or
   (c) $1^k 0^{p/2} 1^j$.

   In each of these cases, pumping n to any length other than 1 will result in the first string not matching the second string. This is obvious by inspection in case (b). Cases (a) and (c) create mismatch by choosing u and z to have non-zero length in cases (a) and (c), respectively. u must be entirely 0s, z must be entirely 1s, and in each case, the lengths of the 0s and 1s will not be the same (one will be of length n, the other of $|u| + n$ or $n + |z|$.)

   Thus, in none of these cases is $uv^n xy^n z \in L \forall n \geq 0$.

   $\therefore L$ is not context-free.

   $\therefore$ Any grammar that produced L is not a context-free grammar.