Lecture Note 4

Testing the CAPM [1]

I. Introduction

II. The Early Evidence on the CAPM

- Black, Jensen and Scholes

III. Some evidence against the CAPM

IV. The Roll Critique

V. Stronger Evidence against the CAPM

- Fama and French (1992)

VI. Implications for Portfolio Risk & Return

VII. Implications for Rational Asset Pricing Models

VIII. Conclusions

I. Introduction

A. *Why should care about understanding this evidence?*

1. The CAPM is still widely used

a. Corporate Project Valuation

b. Evaluating Portfolio Managers

2. *To be able to understand when you can and cannot use it you need to know why and how it fails.*

3. Finally, the failure of the CAPM implies that the market portfolio is not efficient. To understand what portfolio is efficient, you need to understand how it fails

B. How to test the CAPM

1. What does the CAPM tell us?

2. The CAPM states that:

$$E(\tilde{r}_i) = r_f + \beta_i \cdot [E(\tilde{r}_m) - r_f]$$

or, in words:

a. The relation of expected return and β_i is linear

b. *Only* β_i is necessary to explain differences in returns among securities.

c. The expected return of an asset with a β of zero is r_f.

d. The expected return of an asset with a β of one is the same as the expected return on the market.

3. These are (typically) the aspects of the CAPM that are tested.

4. So, how do we test it?

C. A simple test of the CAPM (Lintner (1965))

1. First gather the data:

a. Monthly returns for 100 firms for five years

b. Returns on the S&P 500 Index for the same period.

c. The risk-free rate each month

2. Then run a regression based on the security characteristic line (SCL) for **each firm** (the *first-pass regression*):

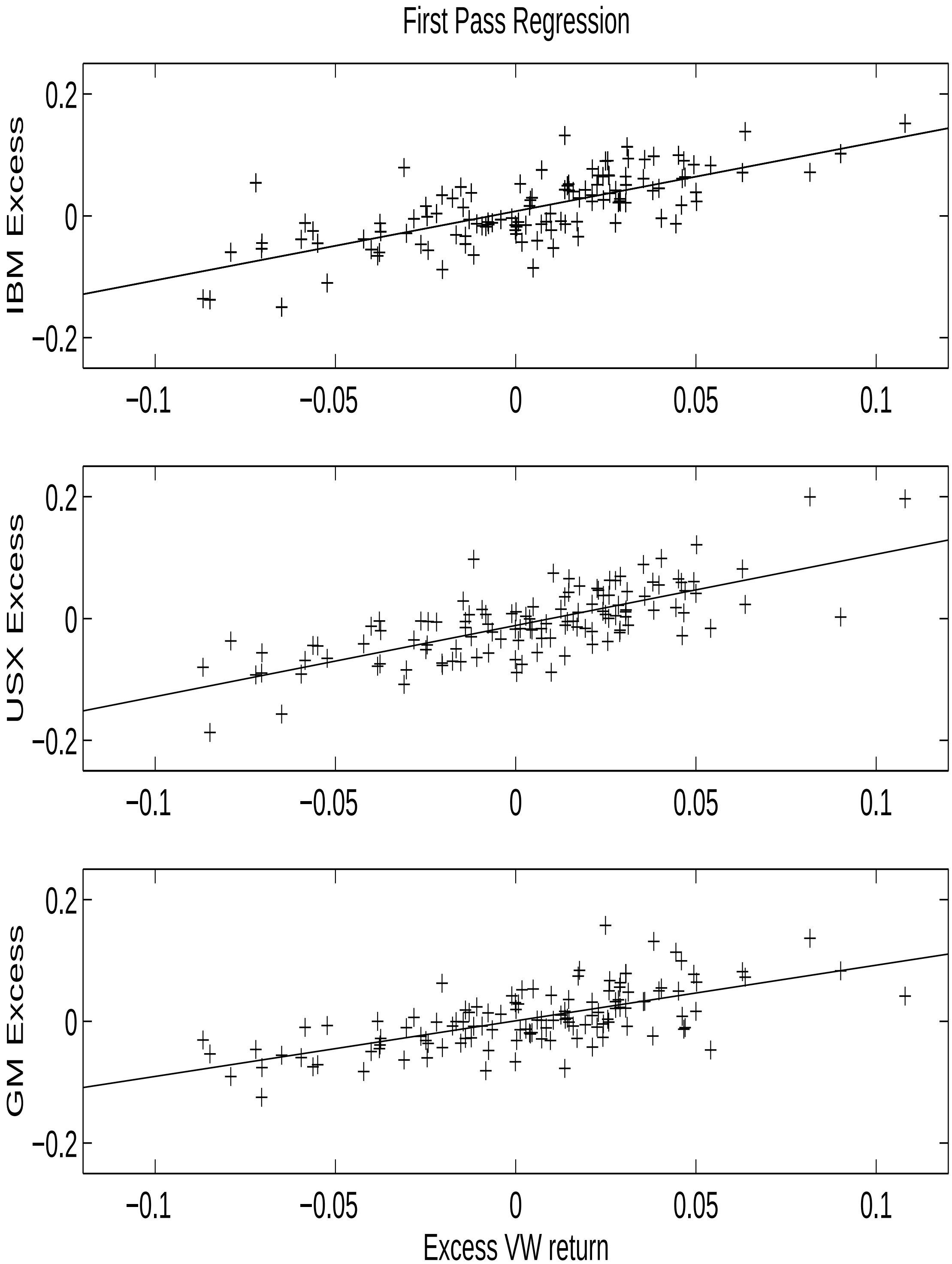
$$\tilde{r}_{i,t} - r_{f,t} = \alpha_i + \beta_i \cdot (\tilde{r}_{m,t} - r_{f,t}) + \epsilon_{i,t}$$

- This is also often called the *time-series regression*.

3. For example, for IBM, USX and GM over the period 1960-1970 (120 months)you would get the following *first-pass* or *time-series* regression results:

statistic	IBM	USX	GM
α	0.0077	-0.0114	0.0008
$T(\alpha)$	2.1200	-3.0317	0.2207
β	1.1355	1.1674	0.9126
$T(\beta)$	11.4671	11.3611	9.6283
$R^2(\%)$	52.7491	56.0148	44.8229

4. Graphically, the three regressions look like this:



5. Now, for each of these regressions, collect the following:

a. The average return, $\overline{r_i - r_f}$,

b. The estimated β_i, called $\hat{\beta}_i$

c. The estimated variance of the residuals, called $\hat{\sigma}^2(\epsilon_i)$

Also, you will need the average $\overline{\tilde{r}_m - r_f}$ over the period.

6. Now run a *second-pass regression* (or *cross-sectional* regression) to see whether the β's are related to the average return in the way predicted by the CAPM:

$$\overline{r_i - r_f} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{\sigma}^2(\epsilon_i) + \tilde{u}_i$$

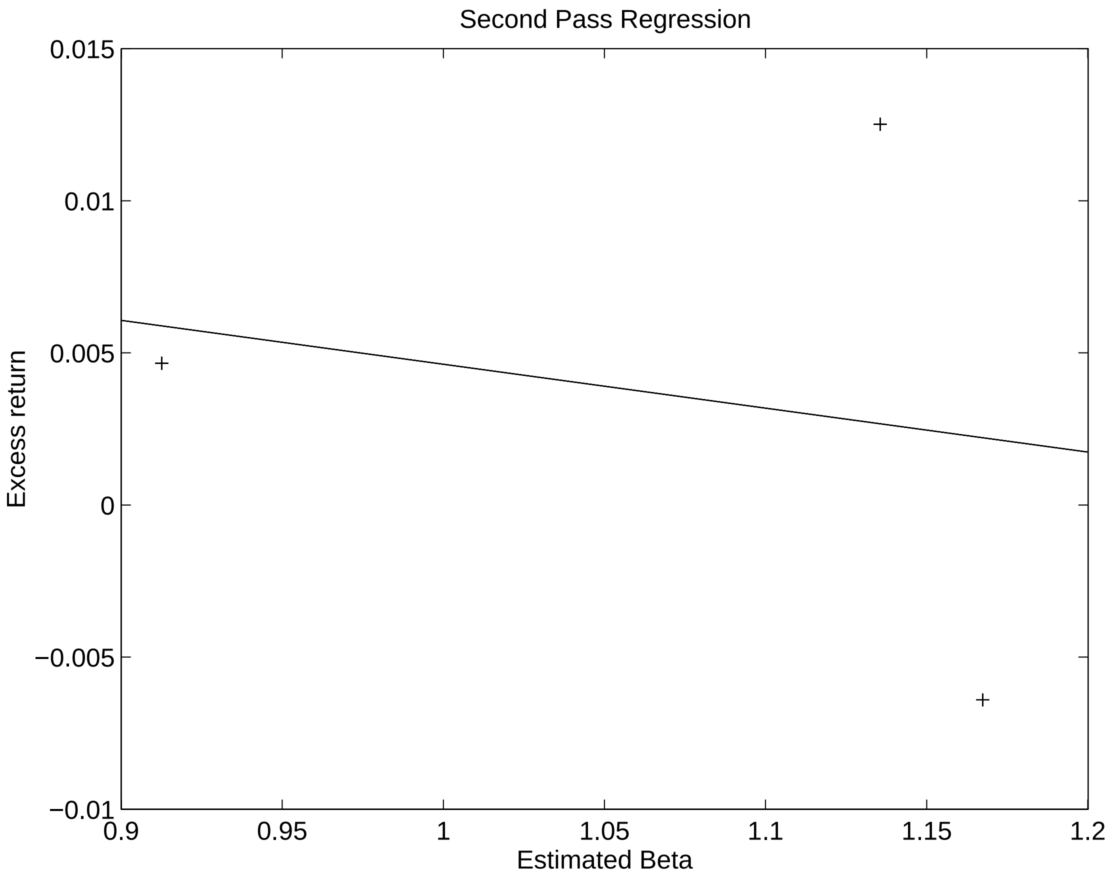
- What is the point of the γ_2 term?

7. If the CAPM is correct the estimated regression coefficients should be equal to:

$$\begin{aligned} \hat{\gamma}_0 &= 0 \\ \hat{\gamma}_1 &= \overline{\tilde{r}_m - r_f} \\ \hat{\gamma}_2 &= 0 \end{aligned}$$

- Why is this?

8. For our three stocks, the cross-sectional regression looks like this:



and the regression results are:

γ_0	γ_1	$R^2(\%)$
0.019 (0.264)	-0.014 (-0.216)	4.4

9. For a larger test, you get the following results (annualized results):[2]

coef- ficient	estimated value	standard error
$\hat{\gamma}_0$	0.127	(0.006)
$\hat{\gamma}_1$	0.042	(0.006)
$\hat{\gamma}_2$	0.310	(0.026)

Note: The average excess return on the market over this period was 0.165/year

10. How does this test fail?

11. Why does this test fail?

- Errors-in-Variables problem
- Correlation of Residual variance with true β.

D. Can these problems be fixed?

[2]See BKM, page 372.

II. The Black-Jensen-Scholes (1972) and Fama-McBeth (1973) tests:

A. The *errors-in-variables* problem

1. If 5-10 years of data is used to estimate β_i, the estimate is very noisy.

2. Using longer estimation period won't work because β_i's change over time.

3. Solution is to form portfolios, whose β_i's can be estimated much more precisely.

4. Problem is how to choose the composition of the portfolios:

- If portfolios are formed randomly, portfolio β's will all be close to 1, and test will lack *power*.

- Need a method to get sufficient variation in the portfolio β's.

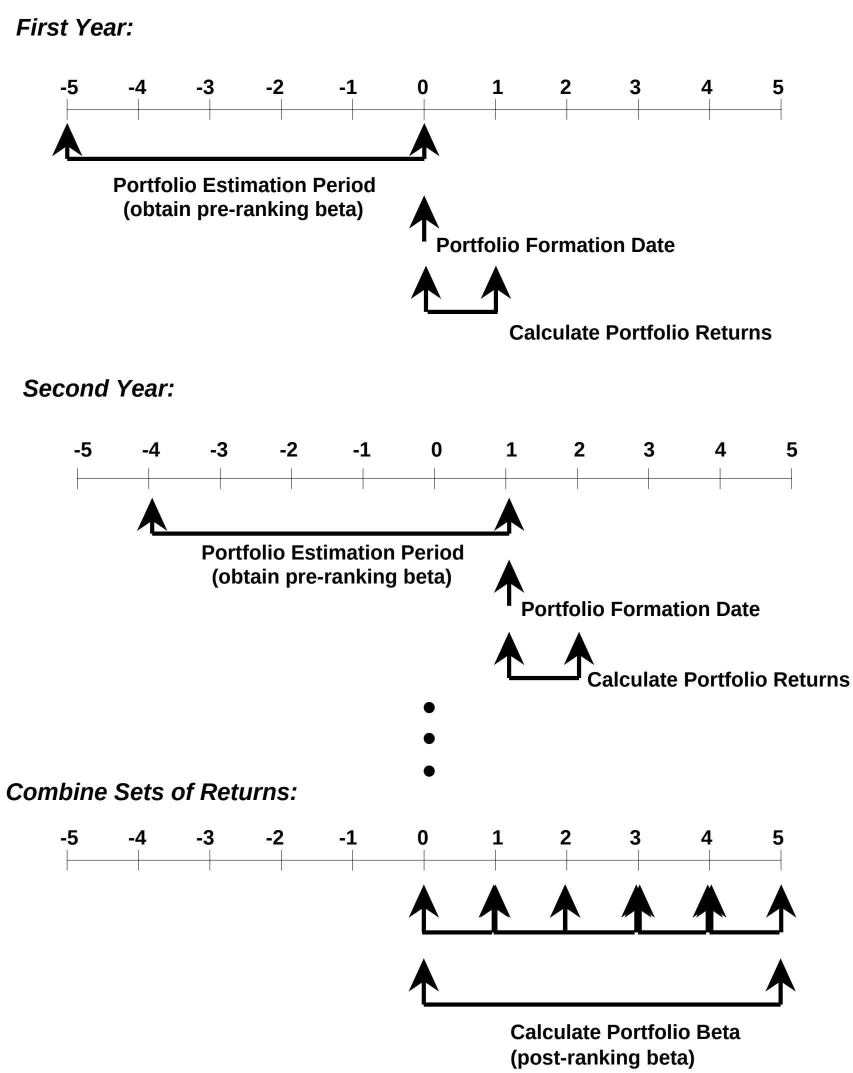
B. BJS Portfolio selection technique (slightly modified):

1. Each year, calculate β's using a regression of each securities return on the market for the past five years.

2. Form 10 portfolios based on estimated β's

- Smallest portfolio has the 10% of the stocks with the lowest estimated β's, etc.

3. Re-estimate the portfolio β's after the formation period.



C. Questions:

1. Why does using (diversified) portfolios help in reducing the measurement error problem?

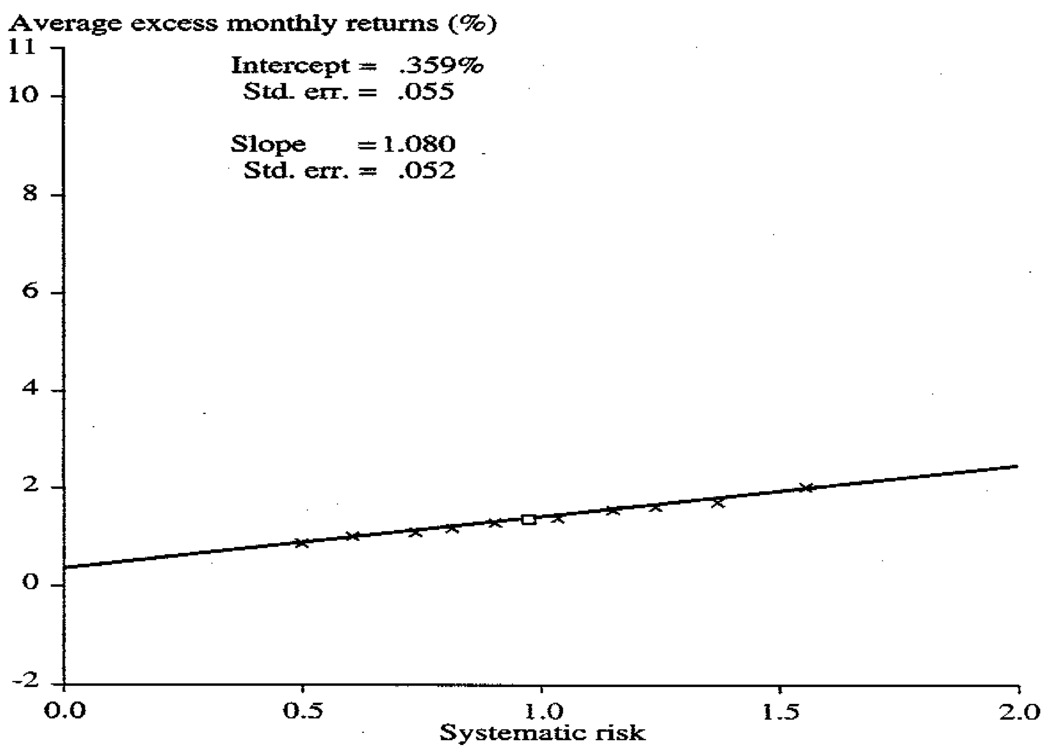
D. BJS obtain the following results (from BKM)

Table 11.1 · Summary of Statistics for Time Series Tests (January 1931–December 1965)

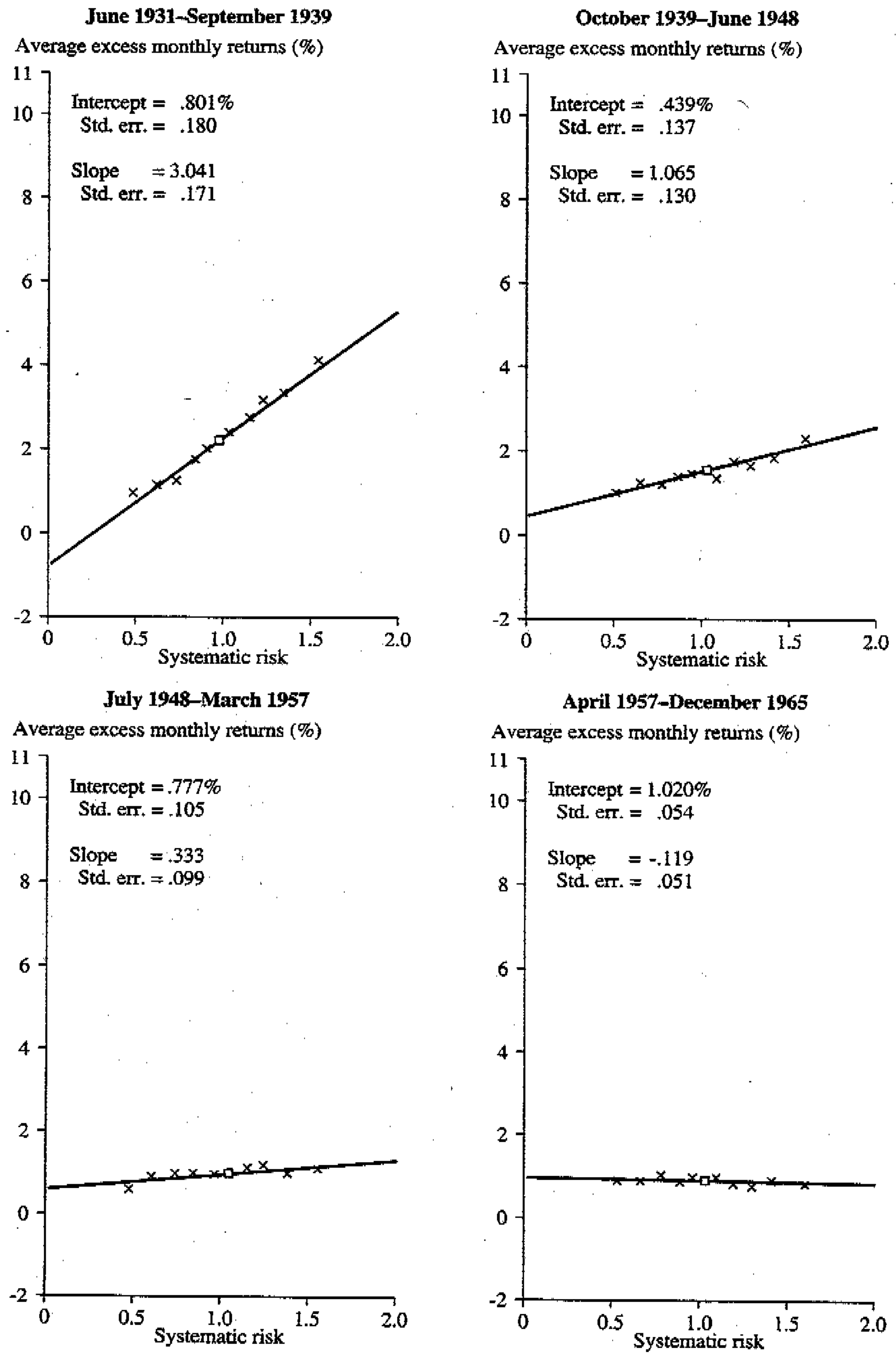
	Portfolio Number*										
Item*	1	2	3	4	5	6	7	8	9	10	$\bar{R}_M$
β	1.5614	1.3838	1.2483	1.1625	1.0572	0.9229	0.8531	0.7534	0.6291	0.4992	1.0000
$\hat{\alpha} \cdot 10^2$	−0.0829	−0.1938	−0.0649	−0.0167	−0.0543	0.0593	0.0462	0.0812	0.1968	0.2012	
$t(\alpha)$	−0.4274	−1.9935	−0.7597	−0.2468	−0.8869	0.7878	0.7050	1.1837	2.3126	1.8684	
$\rho(\bar{R}, \bar{R}_M)$	0.9625	0.9875	0.9882	0.9914	0.9915	0.9833	0.9851	0.9793	0.9560	0.8981	
$\rho(\bar{e}_t, \bar{e}_{t-1})$	0.0549	−0.0638	0.0366	0.0073	−0.0708	−0.1248	0.1294	0.1041	0.0444	0.0992	
$\sigma(\bar{e})$	0.0393	0.0197	0.0173	0.0137	0.0124	0.0152	0.0133	0.0139	0.0172	0.0218	
$\bar{R}$	0.0213	0.0177	0.0171	0.0163	0.0145	0.0137	0.0126	0.0115	0.0109	0.0091	0.0142
σ	0.1445	0.1248	0.1126	0.1045	0.0950	0.0836	0.0772	0.0685	0.0586	0.0495	0.0891

*$\bar{R}$ = Average monthly excess returns, σ = Standard deviation of the monthly excess returns, ρ = Correlation coefficient.
Sample size for each regression, 420.

Or, graphically,



E. Plotting returns over the different subperiods gives (BKM, p. 318):



III. Evidence (apparently) against the CAPM

A. Since these tests, a great deal of *anomalous* evidence has come out suggesting that things other than β are important in determining expected returns:

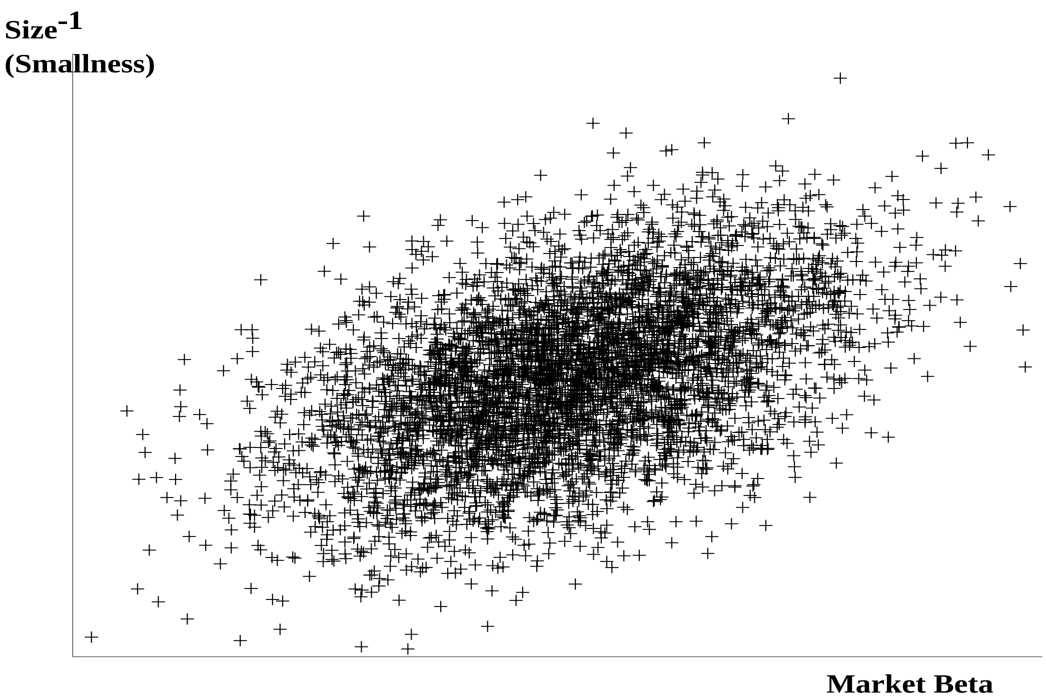
1. The Small Firm Effect (Keim (1981))

2. The P/E Ratio Effect (Ball (1978), Basu (1983))

3. The Leverage Effect (Bhandari (1988))

4. The Book-to-Market Effect (Stattman (1980), Rosenberg, *et. al.*, (1985))

B. However, it could be that these variables are just *proxying* for β, and that they are less noisy measures than time series estimates of β:



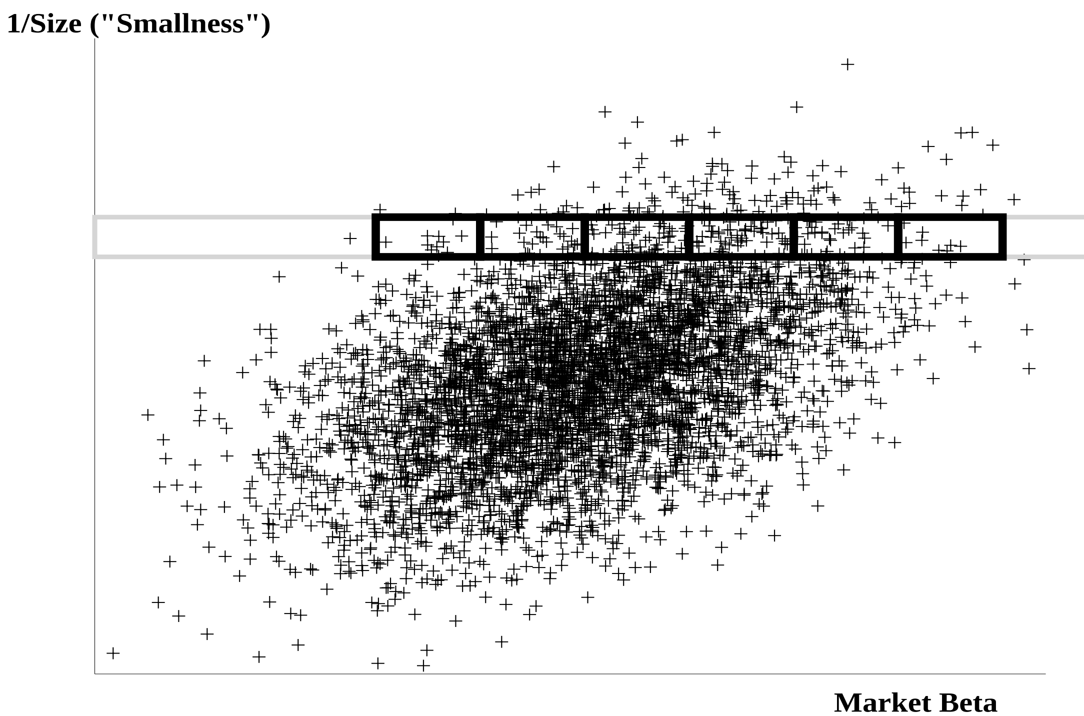
IV. "β is Dead:" *The Fama and French (1992) Study*

A. The FF approach to resolving this was:

1. To resolve this, Fama and French (1992) do a double sort, first on size, then on market β.

2. They find that relation with market β *within size decile* is generally negative

3. They also found that a firm's size and book-to-market ratio (B/M) capture cross-sectional variation in common stock returns.



B. The Fama/French Portfolio Formation Method

1. Split universe of common stocks up into ten size decile portfolios.

2. Each year, sort firms in each size decile portfolio on pre-formation β's.

3. Break each size decile portfolio up into ten sub-portfolios based on pre-β's; hold for one year.

4. To determine β of portfolio, use time series of portfolio returns post-formation (*post*-β).

C. Fama and French (1992) Cross-Sectional Test Results:

Table AII

Properties of Portfolios Formed on Size and Pre-Ranking β: NYSE Stocks Sorted by ME (Down) then Pre-Ranking β (Across): 1941–1990

At the end of year $t-1$, the NYSE stocks on CRSP are assigned to 10 size (ME) portfolios. Each size decile is subdivided into 10 β portfolios using pre-ranking βs of individual stocks, estimated with 24 to 60 monthly returns (as available) ending in December of year $t-1$. The equal-weighted monthly returns on the resulting 100 portfolios are then calculated for year t. The average returns are the time-series averages of the monthly returns, in percent. The post-ranking βs use the full 1941–1990 sample of post-ranking returns for each portfolio. The pre- and post-ranking βs are the sum of the slopes from a regression of monthly returns on the current and prior month's NYSE value-weighted market return. The average size for a portfolio is the time-series average of each month's average value of ln(ME) for stocks in the portfolio. ME is denominated in millions of dollars. There are, on average, about 10 stocks in each size-β portfolio each month. The All column shows parameter values for equal-weighted size-decile (ME) portfolios. The All rows show parameter values for equal-weighted portfolios of the stocks in each β group.

	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
					Panel A: Average Monthly Return (in Percent)						
All		1.22	1.30	1.32	1.35	1.36	1.34	1.29	1.34	1.14	1.10
Small-ME	1.78	1.74	1.76	2.08	1.91	1.92	1.72	1.77	1.91	1.56	1.46
ME-2	1.44	1.41	1.35	1.33	1.61	1.72	1.59	1.40	1.62	1.24	1.11
ME-3	1.36	1.21	1.40	1.22	1.47	1.34	1.51	1.33	1.57	1.33	1.21
ME-4	1.28	1.26	1.29	1.19	1.27	1.51	1.30	1.19	1.56	1.18	1.00
ME-5	1.24	1.22	1.30	1.28	1.33	1.21	1.37	1.41	1.31	0.92	1.06
ME-6	1.23	1.21	1.32	1.37	1.09	1.34	1.10	1.40	1.21	1.22	1.08
ME-7	1.17	1.08	1.23	1.37	1.27	1.19	1.34	1.10	1.11	0.87	1.17
ME-8	1.15	1.06	1.18	1.26	1.25	1.26	1.17	1.16	1.05	1.08	1.04
ME-9	1.13	0.99	1.13	1.00	1.24	1.28	1.31	1.15	1.11	1.09	1.05
Large-ME	0.95	0.99	1.01	1.12	1.01	0.89	0.95	0.95	1.00	0.90	0.68
	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
					Panel B: Post-Ranking β						
All		0.76	0.95	1.05	1.14	1.22	1.26	1.34	1.38	1.49	1.69
Small-ME	1.52	1.17	1.40	1.31	1.50	1.46	1.50	1.69	1.60	1.75	1.92
ME-2	1.37	0.86	1.09	1.12	1.24	1.39	1.42	1.48	1.60	1.69	1.91
ME-3	1.32	0.88	0.96	1.18	1.19	1.33	1.40	1.43	1.56	1.64	1.74
ME-4	1.26	0.69	0.95	1.06	1.15	1.24	1.29	1.46	1.43	1.64	1.83
ME-5	1.23	0.70	0.95	1.04	1.10	1.22	1.32	1.34	1.41	1.56	1.72
ME-6	1.19	0.68	0.86	1.04	1.13	1.20	1.20	1.35	1.36	1.48	1.70
ME-7	1.17	0.67	0.88	0.95	1.14	1.18	1.26	1.27	1.32	1.44	1.68
ME-8	1.12	0.64	0.83	0.99	1.06	1.14	1.14	1.21	1.26	1.39	1.58
ME-9	1.06	0.68	0.81	0.94	0.96	1.06	1.11	1.18	1.22	1.25	1.46
Large-ME	0.97	0.65	0.73	0.90	0.91	0.97	1.01	1.01	1.07	1.12	1.38
					Panel C: Average Size (ln(ME))						
All		4.39	4.39	4.40	4.40	4.39	4.40	4.38	4.37	4.37	4.34
Small-ME	1.93	2.04	1.99	2.00	1.96	1.92	1.92	1.91	1.90	1.87	1.80
ME-2	2.80	2.81	2.79	2.81	2.83	2.80	2.79	2.80	2.80	2.79	2.79
ME-3	3.27	3.28	3.27	3.28	3.27	3.27	3.28	3.29	3.27	3.27	3.26
ME-4	3.67	3.67	3.67	3.67	3.68	3.68	3.67	3.68	3.66	3.67	3.67
ME-5	4.06	4.07	4.06	4.05	4.06	4.07	4.06	4.05	4.05	4.06	4.06
ME-6	4.45	4.45	4.44	4.46	4.45	4.45	4.45	4.45	4.44	4.45	4.45
ME-7	4.87	4.86	4.87	4.86	4.87	4.87	4.88	4.87	4.87	4.85	4.87
ME-8	5.36	5.38	5.38	5.38	5.35	5.36	5.37	5.37	5.36	5.35	5.34
ME-9	5.98	5.96	5.98	5.99	6.00	5.98	5.98	5.97	5.95	5.96	5.96
Large-ME	7.12	7.10	7.12	7.16	7.17	7.20	7.29	7.14	7.09	7.04	6.83

D. The Importance of Book-to-Market and Size:

Table III

Average Slopes (*t*-Statistics) from Month-by-Month Regressions of Stock Returns on β, Size, Book-to-Market Equity, Leverage, and E/P: July 1963 to December 1990

Stocks are assigned the post-ranking β of the size-β portfolio they are in at the end of June of year t (Table I). BE is the book value of common equity plus balance-sheet deferred taxes, A is total book assets, and E is earnings (income before extraordinary items, plus income-statement deferred taxes, minus preferred dividends). BE, A, and E are for each firm's latest fiscal year ending in calendar year $t-1$. The accounting ratios are measured using market equity ME in December of year $t-1$. Firm size ln(ME) is measured in June of year t. In the regressions, these values of the explanatory variables for individual stocks are matched with CRSP returns for the months from July of year t to June of year $t+1$. The gap between the accounting data and the returns ensures that the accounting data are available prior to the returns. If earnings are positive, E(+)/P is the ratio of total earnings to market equity and E/P dummy is 0. If earnings are negative, E(+)/P is 0 and E/P dummy is 1.

The average slope is the time-series average of the monthly regression slopes for July 1963 to December 1990, and the t-statistic is the average slope divided by its time-series standard error.

On average, there are 2267 stocks in the monthly regressions. To avoid giving extreme observations heavy weight in the regressions, the smallest and largest 0.5% of the observations on E(+)/P, BE/ME, A/ME, and A/BE are set equal to the next largest or smallest values of the ratios (the 0.005 and 0.995 fractiles). This has no effect on inferences.

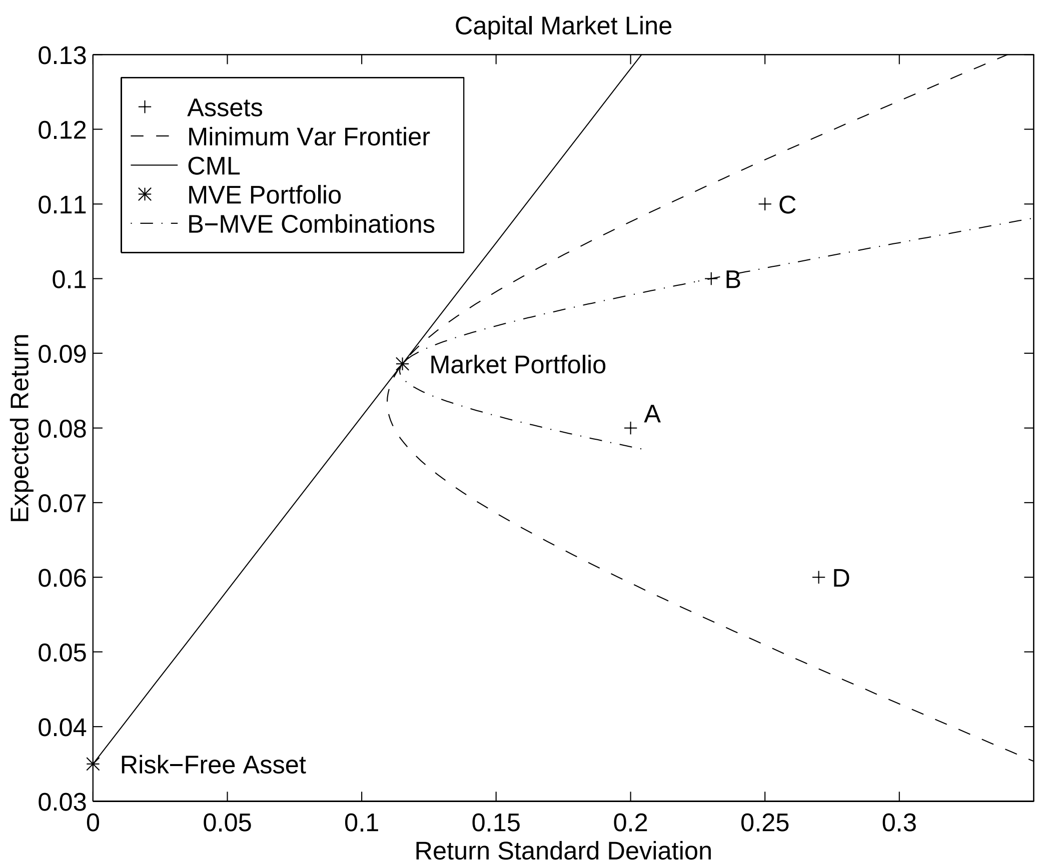
β	ln(ME)	ln(BE/ME)	ln(A/ME)	ln(A/BE)	E/P Dummy	E(+)/P
0.15 (0.46)						
	−0.15 (−2.58)					
−0.37 (−1.21)	−0.17 (−3.41)					
		0.50 (5.71)				
			0.50 (5.69)	−0.57 (−5.34)		
					0.57 (2.28)	4.72 (4.57)
	−0.11 (−1.99)	0.35 (4.44)				
	−0.11 (−2.06)		0.35 (4.32)	−0.50 (−4.56)		
	−0.16 (−3.06)				0.06 (0.38)	2.99 (3.04)
	−0.13 (−2.47)	0.33 (4.46)			−0.14 (−0.90)	0.87 (1.23)
	−0.13 (−2.47)		0.32 (4.28)	−0.46 (−4.45)	−0.08 (−0.56)	1.15 (1.57)

V. The Roll Critique

A. Another important point in this debate is the Roll Critique. Roll makes two points:

1. The only test of the CAPM is whether the market portfolio is mean-variance efficient.

- The CAPM will always hold if the market $proxy$ that is used is MVE.
- If the $proxy$ is not MVE, the relationship between $E(R)$ and β will not hold.



2. Secondly, Roll points out that, since the market portfolio is not identifiable, we cannot really test the CAPM.

- The market proxies that we use don't include
  - Real Estate
  - Human Capital

B. So Roll concludes that the CAPM is useless because it is not testable

- Roll instead advocates the use of the APT, which we shall study next.

C. However, perhaps the usefulness of the Roll critique is in reminding us that if we find that the CAPM tests do not hold, then the so-called "market" (or really market proxy) is not MVE and, unless you have very special reasons for doing so, you should not hold the market proxy!

VI. Implications for Portfolio Risk & Return

A. In another paper we will look at later, Fama and French (1993) construct three portfolios which take advantage of the small firm effect and the book-to-market effect.

- These are called the High-Minus-Low (HML) portfolio and the Small-Minus-Big (SMB) portfolio.

B. The Annualized return, volatility and Sharpe ratio over the 1963:07 to 1992:12 period:[3]

	Mkt	SMB	HML
Mean Return	5.3%	3.3%	4.9%
Std. Deviation	15.7%	10.0%	8.8%
Sharpe Ratio	0.34	0.33	0.56

C. The *ex-post* mean-variance efficient portfolio of the three factor-portfolios has a Sharpe ratio of 0.82.

1. We can correct this to come up with an estimate of the *ex-ante* Sharpe ratio of $\mathbf{0.745}$

2. This means that we can construct a portfolio with the same variance as the market portfolio, but with an expected excess return of 11.9% /year, an extra 5.6% /year, using just the Mkt, HML and SMB portfolios.

3. It is important to note that this portfolio would not track the market portfolio.

[3] Mkt return is in excess of the risk-free rate.

VII. Implications for Rational Asset Pricing Models

A. The Fama and French evidence shows that firms which are *in distress* have had *high average returns*:

1. There is a strong relation between size and B/M and distress:

- Firms which are small usually started out big.
- Distressed firms have high book values (paid a lot for their assets), relative to what they are worth now.
  – *e.g.,* railroads.

B. There are two explanations for the high expected returns, either:

1. Investors become overly pessimistic about stocks which are in distress, making their prices low and the average returns high.

2. Firms in distress (with a high B/M) are risky in a the Fama and French results different way than firms which are not (have a low B/M); these firms have a very low price because of this risk.

C. To discriminate we need to look at how distressed firms respond to economic conditions:

- *For B/M (or size) to represent risk, it must be the case that there are states of the world where high B/M (small size) firms do poorly.*

VIII. Conclusions

A. There are a number of reasons we expect might the CAPM to fail:

1. Imperfect measures of the market portfolio

2. β is an incomplete measure of risk

3. Tax effects

4. Non - normality of returns

5. No riskless asset

6. Divergent borrowing and lending rates

B. So, the CAPM fails! What good is it?

1. Useful benchmark

2. Many alternative asset pricing models have a CAPM like representation. Therefore, understanding the mathematics of the CAPM is very useful.

3. Beginning point to understanding

- Active Portfolio Management
  - what is Passive Portfolio Management?
- Performance Evaluation