How to Write Fast Numerical Code

Spring 2011 Lecture 17

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Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich

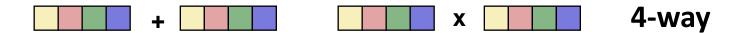
SIMD Extensions and SSE

- Overview
- SSE family, floating point, and x87
- SSE intrinsics
- Compiler vectorization
- This material was developed together with Franz Franchetti, Carnegie Mellon

SIMD (Single Instruction Multiple Data) Vector Extensions

What is it?

 Extension of the ISA. Data types and instructions for the parallel computation on short (length 2-8) vectors of integers or floats



Names: MMX, SSE, SSE2, ...

Why do they exist?

- Useful: Many applications have the necessary fine-grain parallelism
 Then: speedup by a factor close to vector length
- Doable: Chip designers have enough transistors to play with

MMX:

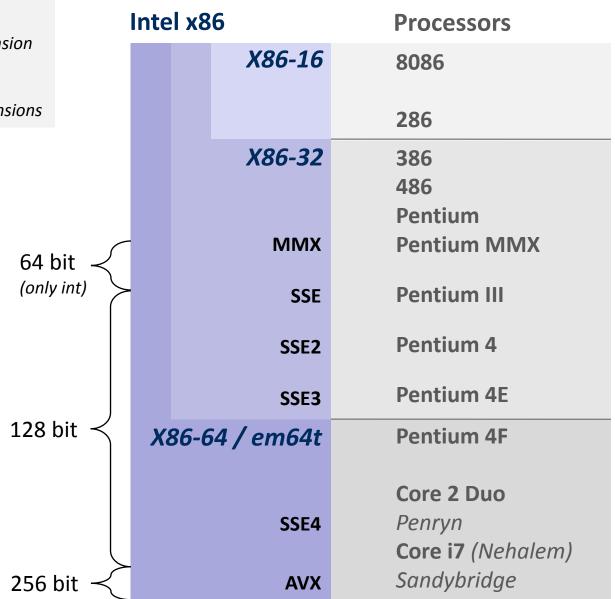
Multimedia extension

SSE:

Streaming SIMD extension

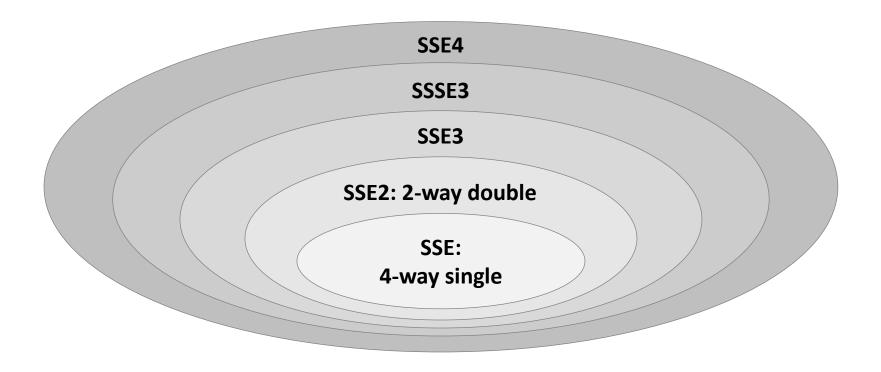
AVX:

Advanced vector extensions



time

SSE Family: Floating Point



- Not drawn to scale
- **■** From SSE3: Only additional instructions
- Every Core 2 has SSE3

Overview Floating-Point Vector ISAs

Vendor	Name		u-way	Precision	Introduced with
Intel	SSE SSE2 SSE3 SSSE3 SSE4 AVX	+	4-way 2-way 8-way 4-way	single double single double	Pentium III Pentium 4 Pentium 4 (Prescott) Core Duo Core2 Extreme (Penryn) Core i7 (Sandybridge)
Intel	IPF		2-way	single	Itanium
Intel	LRB		16-way 8-way	single double	Larrabee
AMD	3DNow! Enhanced 3DNow! 3DNow! Professional AMD64	+++	2-way 4-way 2-way	single single double	K6 K7 Athlon XP Opteron
Motorola	AltiVec		4-way	single	MPC 7400 G4
IBM	VMX SPU	+	4-way 2-way	single double	PowerPC 970 G5 Cell BE
IBM	Double FPU		2-way	double	PowerPC 440 FP2

Within an extension family, newer generations add features to older ones Convergence: 3DNow! Professional = 3DNow! + SSE; VMX = AltiVec;

Core 2

- Has SSE3
- 16 SSE registers

%xmm0	%xmm8
%xmm1	%xmm9
%xmm2	%xmm10
%xmm3	% xmm11
%xmm4	%xmm12
%xmm5	% xmm13
%xmm6	%xmm14
%xmm7	%xmm15

SSE3 Registers





- 16-way byte
- 8-way 2 bytes
- 4-way 4 bytes

			J	128	3 bi	t			LSB

Floating point vectors:

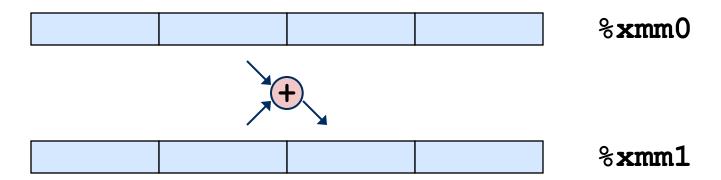
- 4-way single (since SSE)
- 2-way double (since SSE2)

Floating point scalars:

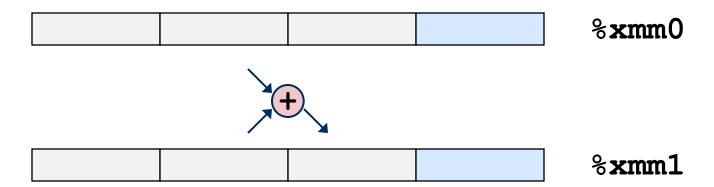
- single (since SSE)
- double (since SSE2)

SSE3 Instructions: Examples

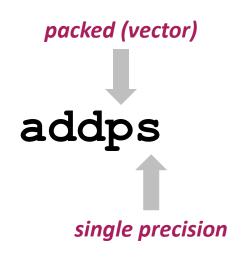
Single precision 4-way vector add: addps %xmm0 %xmm1

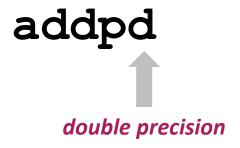


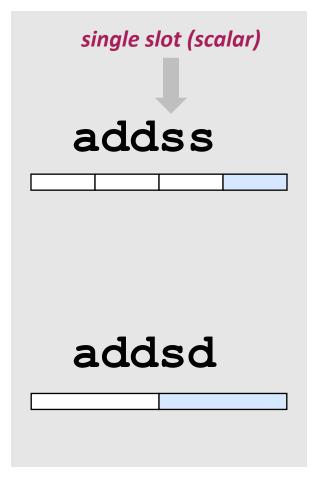
■ Single precision scalar add: addss %xmm0 %xmm1



SSE3 Instruction Names







Compiler will use this for floating point

- on x86-64
- with proper flags if SSE/SSE2 is available

x86-64 FP Code Example

Inner product of two vectors

- Single precision arithmetic
- Compiled: uses SSE instructions

```
ipf:
                               # result = 0.0
  xorps %xmm1, %xmm1
  xorl
          %ecx, %ecx
                               # i = 0
  jmp
          .L8
                                # goto middle
.L10:
                                # loop:
  movslq %ecx,%rax
                                \# icpy = i
  incl
         %ecx
                                # i++
  movss (%rsi,%rax,4), %xmm0
                                # t = y[icpy]
  mulss (%rdi,%rax,4), %xmm0
                                # t *= x[icpy]
                                # result += t
  addss %xmm0, %xmm1
.L8:
                                # middle:
  cmpl %edx, %ecx
                                # i:n
  jl
     .L10
                                # if < goto loop
  movaps %xmm1, %xmm0
                                # return result
  ret
```

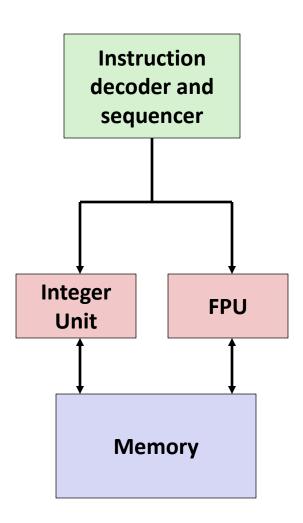
The Other Floating Point (x87)

History

- 8086: first computer to implement IEEE FP (separate 8087 FPU = floating point unit)
- Logically stack based
- 486: merged FPU and Integer Unit onto one chip
- Default on x86-32 (since SSE is not guaranteed)
- Became obsolete with x86-64

Floating Point Formats

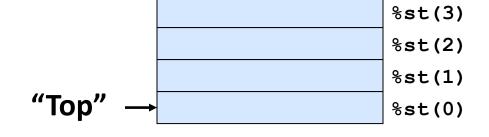
- single precision (C float): 32 bits
- double precision (C double): 64 bits
- extended precision (C long double): 80 bits



x87 FPU Instructions and Register Stack

- Sample instructions:
 - flds (load single precision)
 - fmuls (mult single precision)
 - faddp (add and pop)

- 8 registers %st(0) %st(7)
- Logically form stack
- Top: %st(0)



Bottom disappears (drops out) after too many pushs

FP Code Example (x87)

Inner product of two vectors

Single precision arithmetic

```
pushl %ebp
                         # setup
  mov1 %esp,%ebp
  pushl %ebx
  movl 8(%ebp),%ebx # %ebx=&x
  movl 12(%ebp),%ecx # %ecx=&y
  movl 16(\%ebp), %edx # %edx=n
  fldz
                         # push +0.0
  xorl %eax,%eax
                         # i=0
  cmpl %edx,%eax
                         # if i>=n done
  ige .L3
.L5:
  flds (%ebx,%eax,4) # push x[i]
  fmuls (%ecx, %eax, 4) # st(0)*=y[i]
                         # st(1)+=st(0); pop
  faddp
                         # i++
  incl %eax
                         # if i<n repeat
  cmpl %edx,%eax
  il .L5
.L3:
  movl -4(%ebp),%ebx
                         # finish
  mov1 %ebp, %esp
  popl %ebp
                         # st(0) = result
  ret
```

From Core 2 Manual

	-, -	., .	1
Single-precision (SP) FP MUL	4, 1	4, 1	Issue port 0; Writeback port 0
Double-precision FP MUL	5, 1	5, 1	
FP MUL (X87)	5, 2	5, 2	Issue port 0; Writeback port 0
FP Shuffle	1, 1	1, 1	FP shuffle does not handle QW
DIV/SQRT			shuffle.
			C C

SSE based FP x87 FP

Summary

- On Core 2 there are two different (unvectorized) floating points
 - x87: obsolete, is default on x86-32
 - SSE based: uses only one slot, is default on x86-64
- SIMD vector floating point instructions
 - 4-way single precision: since SSE
 - 2-way double precision: since SSE2
 - Since on Core 2 add and mult are fully pipelined (1 per cycle): possible gain 4x and 2x, respectively

SSE: How to Take Advantage?



- Necessary: fine grain parallelism
- Options:
 - Use vectorized libraries (easy, not always available)
 - Write assembly
 - Use intrinsics (focus of this course)
 - Compiler vectorization (this course)
- We will focus on floating point and single precision (4-way)

SIMD Extensions and SSE

- Overview
- SSE family, floating point, and x87
- SSE intrinsics
- Compiler vectorization

References:

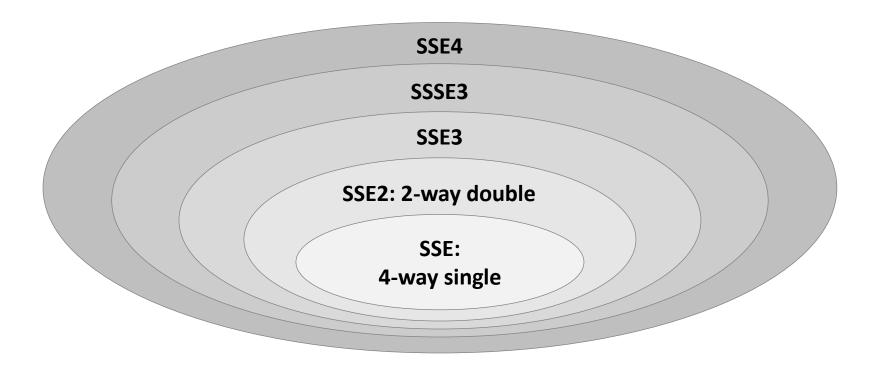
Intel icc manual (currently 12.0) \rightarrow Intrinsics reference

http://software.intel.com/sites/products/documentation/hpc/composerxe/en-us/cpp/lin/index.htm

Visual Studio Manual (also: paste the intrinsic into Google)

http://msdn.microsoft.com/de-de/library/26td21ds.aspx

SSE Family: Floating Point



- Not drawn to scale
- **■** From SSE3: Only additional instructions
- Every Core 2 has SSE3

SSE Family Intrinsics

- Assembly coded C functions
- Expanded inline upon compilation: no overhead
- Like writing assembly inside C
- Floating point:
 - Intrinsics for math functions: log, sin, ...
 - Intrinsics for SSE

- Our introduction is based on icc
 - Most intrinsics work with gcc and Visual Studio (VS)
 - Some language extensions are icc (or even VS) specific

Header files

SSE: xmmintrin.h

SSE2: emmintrin.h

SSE3: pmmintrin.h

SSSE3: tmmintrin.h

SSE4: smmintrin.h and nmmintrin.h

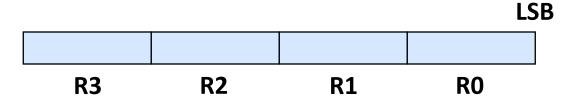
or ia32intrin.h

Visual Conventions We Will Use

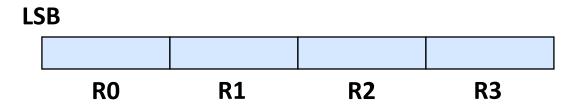
Memory
increasing address
memory

Registers

Before (and common)



Now we will use



SSE Intrinsics (Focus Floating Point)

Data types

```
_m128 f; // = {float f0, f1, f2, f3}
m128d d; // = \{double d0, d1\}
m128i i; // 16 8-bit, 8 16-bit, 4 32-bit, or 2 64-bit ints
                                         ints
                                         ints
                                         ints or floats
                                         ints or doubles
```

SSE Intrinsics (Focus Floating Point)

Instructions

- Naming convention: _mm_<intrin_op>_<suffix>
- Example:

```
// a is 16-byte aligned
float a[4] = {1.0, 2.0, 3.0, 4.0};
__m128 t = _mm_load_ps(a);
```

p: packeds: single

LSB 1.0 2.0 3.0 4.0

Same result as

```
_{m128} t = _{mm_set_ps(4.0, 3.0, 2.0, 1.0)}
```

SSE Intrinsics

Native instructions (one-to-one with assembly)

```
_mm_load_ps()
_mm_add_ps()
_mm_mul_ps()
```

Multi instructions (map to several assembly instructions)

```
_mm_set_ps()
_mm_set1_ps()
```

Macros and helpers

```
_MM_TRANSPOSE4_PS()
_MM_SHUFFLE()
```

•••

What Are the Main Issues?

- Alignment is important (128 bit = 16 byte)
- You need to code explicit loads and stores (what does that remind you of?)
- Overhead through shuffles

SSE Intrinsics

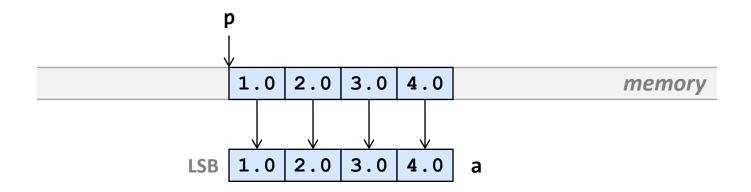
- Load and store
- Constants
- Arithmetic
- Comparison
- Conversion
- Shuffles

Loads and Stores

Intrinsic Name	Operation	Corresponding SSE Instructions
_mm_loadh_pi	Load high	MOVHPS reg, mem
_mm_loadl_pi	Load low	MOVLPS reg, mem
_mm_load_ss	Load the low value and clear the three high values	MOVSS
_mm_load1_ps	Load one value into all four words	MOVSS + Shuffling
_mm_load_ps	Load four values, address aligned	MOVAPS
_mm_loadu_ps	Load four values, address unaligned	MOVUPS
_mm_loadr_ps	Load four values in reverse	MOVAPS + Shuffling

Intrinsic Name	Operation	Corresponding SSE Instruction
_mm_set_ss	Set the low value and clear the three high values	Composite
_mm_set1_ps	Set all four words with the same value	Composite
_mm_set_ps	Set four values, address aligned	Composite
_mm_setr_ps	Set four values, in reverse order	Composite
_mm_setzero_ps	Clear all four values	Composite

Loads and Stores



```
a = _mm_load_ps(p); // p 16-byte aligned
```

avoid (expensive)

How to Align

m128, __m128d, __m128i are 16-byte aligned

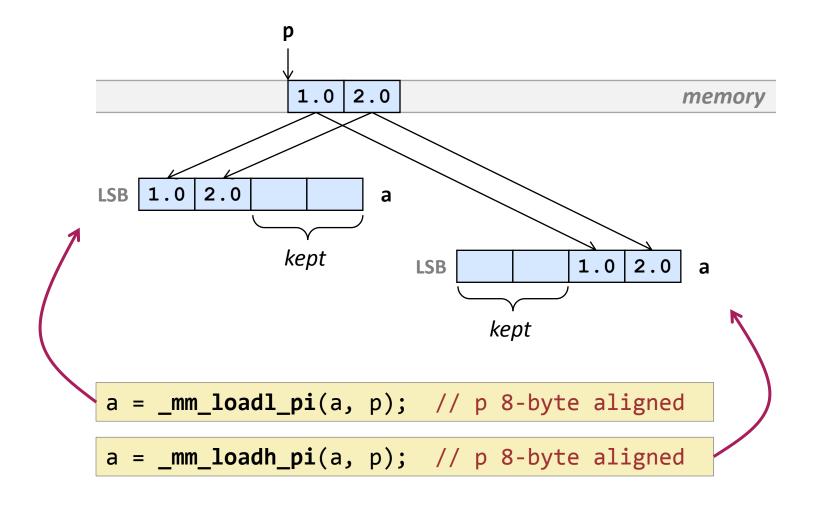
Arrays:

```
__declspec(align(16)) float g[4];
```

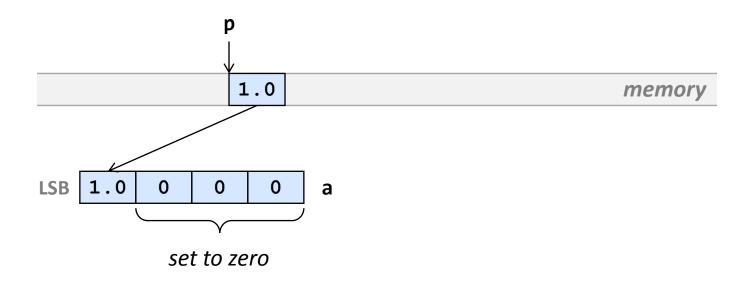
Dynamic allocation

- mm malloc() and mm free()
- Write your own malloc that returns 16-byte aligned addresses
- Some malloc's already guarantee 16-byte alignment

Loads and Stores



Loads and Stores



Stores Analogous to Loads

Intrinsic Name	Operation	Corresponding SSE Instruction
_mm_storeh_pi	Store high	MOVHPS mem, reg
_mm_storel_pi	Store low	MOVLPS mem, reg
_mm_store_ss	Store the low value	MOVSS
_mm_store1_ps	Store the low value across all four words, address aligned	Shuffling + MOVSS
_mm_store_ps	Store four values, address aligned	MOVAPS
_mm_storeu_ps	Store four values, address unaligned	MOVUPS
_mm_storer_ps	Store four values, in reverse order	MOVAPS + Shuffling

Constants

```
d = _mm_setzero_ps();
```

Arithmetic

SSE

Intrinsic Name	Operation	Corresponding SSE Instruction
_mm_add_ss	Addition	ADDSS
_mm_add_ps	Addition	ADDPS
_mm_sub_ss	Subtraction	SUBSS
_mm_sub_ps	Subtraction	SUBPS
_mm_mul_ss	Multiplication	MULSS
_mm_mul_ps	Multiplication	MULPS
_mm_div_ss	Division	DIVSS
_mm_div_ps	Division	DIVPS
_mm_sqrt_ss	Squared Root	SQRTSS
_mm_sqrt_ps	Squared Root	SQRTPS
_mm_rcp_ss	Reciprocal	RCPSS
_mm_rcp_ps	Reciprocal	RCPPS
_mm_rsqrt_ss	Reciprocal Squared Root	RSQRTSS
_mm_rsqrt_ps	Reciprocal Squared Root	RSQRTPS
_mm_min_ss	Computes Minimum	MINSS
_mm_min_ps	Computes Minimum	MINPS
_mm_max_ss	Computes Maximum	MAXSS
_mm_max_ps	Computes Maximum	MAXPS

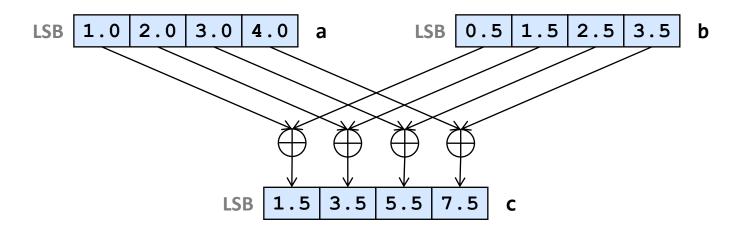
SSE3

Intrinsic Name	Operation	Corresponding SSE3 Instruction	
_mm_addsub_ps	Subtract and add	ADDSUBPS	
_mm_hadd_ps	Add	HADDPS	
_mm_hsub_ps	Subtracts	HSUBPS	

SSE4

Intrinsic	Operation	Corresponding SSE4 Instruction	
_mm_dp_ps	Single precision dot product	DPPS	

Arithmetic



analogous:

Example

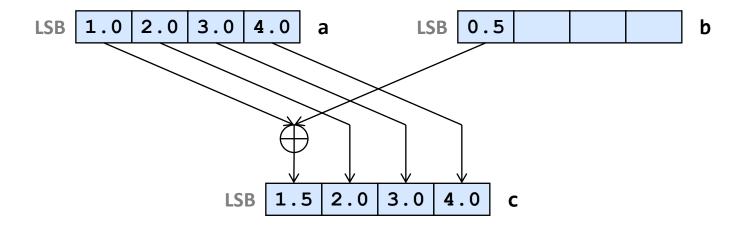
```
void addindex(float *x, int n) {
  for (int i = 0; i < n; i++)
    x[i] = x[i] + i;
}</pre>
```

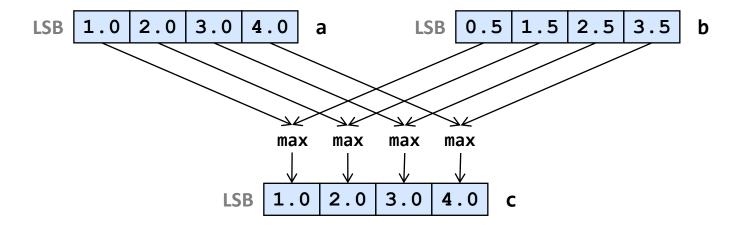
Note how using intrinsics implicitly forces scalar replacement!

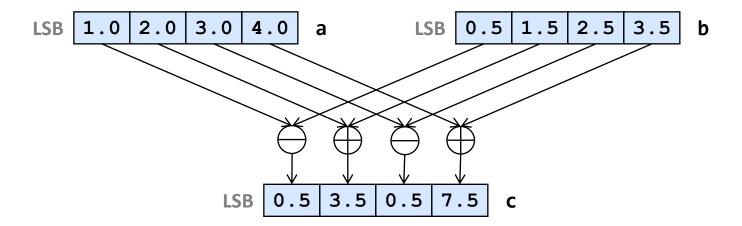
Example: Better Solution

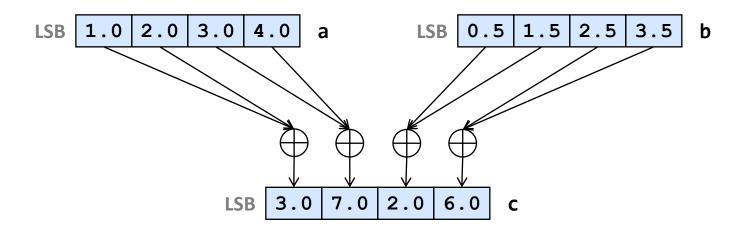
```
void addindex(float *x, int n) {
  for (int i = 0; i < n; i++)
    x[i] = x[i] + i;
}</pre>
```

Note how using intrinsics implicitly forces scalar replacement!









analogous:

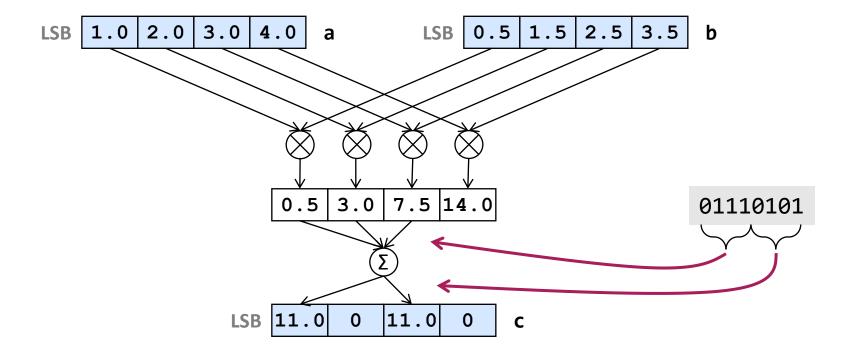
Example

```
// n is even
void lp(float *x, float *y, int n) {
  for (int i = 0; i < n/2; i++)
    y[i] = (x[2*i] + x[2*i+1])/2;
}</pre>
```

```
__m128 _mm_dp_ps(__m128 a, __m128 b, const int mask)
```

(SSE4) Computes the pointwise product of a and b and writes a selected sum of the resulting numbers into selected elements of c; the others are set to zero. The selections are encoded in the mask.

Example: mask = 117 = 01110101

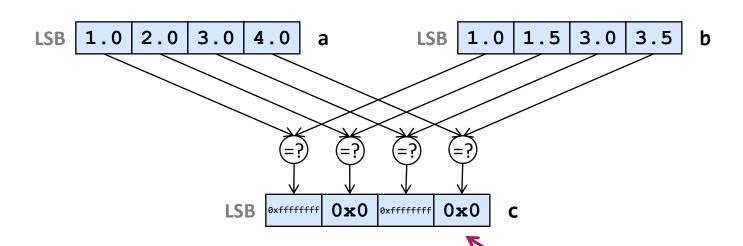


Comparisons

Intrinsic Name	Operation	Corresponding SSE Instruction
_mm_cmpeq_ss	Equal	CMPEQSS
_mm_cmpeq_ps	Equal	CMPEQPS
_mm_cmplt_ss	Less Than	CMPLTSS
_mm_cmplt_ps	Less Than	CMPLTPS
_mm_cmple_ss	Less Than or Equal	CMPLESS
_mm_cmple_ps	Less Than or Equal	CMPLEPS
_mm_cmpgt_ss	Greater Than	CMPLTSS
_mm_cmpgt_ps	Greater Than	CMPLTPS
_mm_cmpge_ss	Greater Than or Equal	CMPLESS
_mm_cmpge_ps	Greater Than or Equal	CMPLEPS
_mm_cmpneq_ss	Not Equal	CMPNEQSS
_mm_cmpneq_ps	Not Equal	CMPNEQPS
_mm_cmpnlt_ss	Not Less Than	CMPNLTSS
_mm_cmpnlt_ps	Not Less Than	CMPNLTPS
_mm_cmpnle_ss	Not Less Than or Equal	CMPNLESS
_mm_cmpnle_ps	Not Less Than or Equal	CMPNLEPS
_mm_cmpngt_ss	Not Greater Than	CMPNLTSS
_mm_cmpngt_ps	Not Greater Than	CMPNLTPS
_mm_cmpnge_ss	Not Greater Than or Equal	CMPNLESS
_mm_cmpnge_ps	Not Greater Than or Equal	CMPNLEPS

Intrinsic Name	Operation	Corresponding SSE Instruction
_mm_cmpord_ss	Ordered	CMPORDSS
_mm_cmpord_ps	Ordered	CMPORDPS
_mm_cmpunord_ss	Unordered	CMPUNORDSS
_mm_cmpunord_ps	Unordered	CMPUNORDPS
_mm_comieq_ss	Equal	COMISS
_mm_comilt_ss	Less Than	COMISS
_mm_comile_ss	Less Than or Equal	COMISS
_mm_comigt_ss	Greater Than	COMISS
_mm_comige_ss	Greater Than or Equal	COMISS
_mm_comineq_ss	Not Equal	COMISS
_mm_ucomieq_ss	Equal	UCOMISS
_mm_ucomilt_ss	Less Than	UCOMISS
_mm_ucomile_ss	Less Than or Equal	UCOMISS
_mm_ucomigt_ss	Greater Than	UCOMISS
_mm_ucomige_ss	Greater Than or Equal	UCOMISS
_mm_ucomineq_ss	Not Equal	UCOMISS

Comparisons



analogous:

etc.

Each field:

0xffffffff if true 0x0 if false

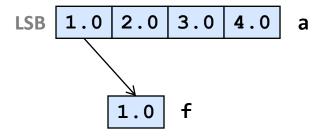
Return type __m128

Conversion

Intrinsic Name	Operation	Corresponding SSE Instruction
_mm_cvtss_si32	Convert to 32-bit integer	CVTSS2SI
_mm_cvtss_si64*	Convert to 64-bit integer	CVTSS2SI
_mm_cvtps_pi32	Convert to two 32-bit integers	CVTPS2PI
_mm_cvttss_si32	Convert to 32-bit integer	CVTTSS2SI
_mm_cvttss_si64*	Convert to 64-bit integer	CVTTSS2SI
_mm_cvttps_pi32	Convert to two 32-bit integers	CVTTPS2PI
_mm_cvtsi32_ss	Convert from 32-bit integer	CVTSI2SS
_mm_cvtsi64_ss*	Convert from 64-bit integer	CVTSI2SS
_mm_cvtpi32_ps	Convert from two 32-bit integers	CVTTPI2PS
_mm_cvtpi16_ps	Convert from four 16-bit integers	composite
_mm_cvtpu16_ps	Convert from four 16-bit integers	composite
_mm_cvtpi8_ps	Convert from four 8-bit integers	composite
_mm_cvtpu8_ps	Convert from four 8-bit integers	composite
_mm_cvtpi32x2_ps	Convert from four 32-bit integers	composite
_mm_cvtps_pi16	Convert to four 16-bit integers	composite
_mm_cvtps_pi8	Convert to four 8-bit integers	composite
_mm_cvtss_f32	Extract	composite

Conversion

```
float _mm_cvtss_f32(__m128 a)
```



```
float f;
f = _mm_cvtss_f32(a);
```

Cast



Reinterprets the four single precision floating point values in a as four 32-bit integers, and vice versa.

No conversion is performed.

Makes integer shuffle instructions usable for floating point.

SSE

Intrinsic Name	Operation	Corresponding SSE Instruction
_mm_shuffle_ps	Shuffle	SHUFPS
_mm_unpackhi_ps	Unpack High	UNPCKHPS
_mm_unpacklo_ps	Unpack Low	UNPCKLPS
_mm_move_ss	Set low word, pass in three high values	MOVSS
_mm_movehl_ps	Move High to Low	MOVHLPS
_mm_movelh_ps	Move Low to High	MOVLHPS
_mm_movemask_ps	Create four-bit mask	MOVMSKPS

SSE3

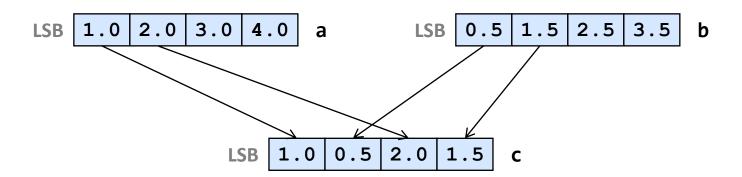
Intrinsic Name	Operation	Corresponding SSE3 Instruction
_mm_movehdup_ps	Duplicates	MOVSHDUP
_mm_moveldup_ps	Duplicates	MOVSLDUP

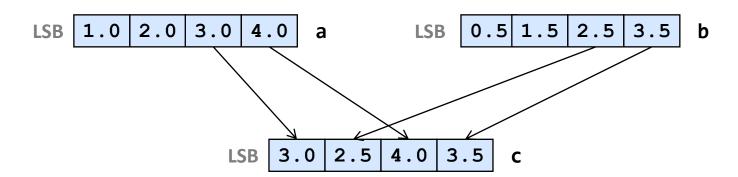
SSSE3

Intrinsic Name	Operation	Corresponding SSSE3 Instruction
_mm_shuffle_epi8	Shuffle	PSHUFB
_mm_alignr_epi8	Shift	PALIGNR

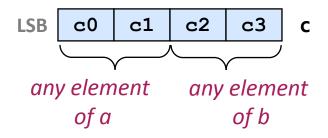
SSE4

Intrinsic Syntax	Operation	Corresponding SSE4 Instruction
m128 _mm_blend_ps(m128 v1,m128 v2, const int mask)	Selects float single precision data from 2 sources using constant mask	BLENDPS
m128 _mm_blendv_ps(m128 v1,m128 v2,m128 v3)	Selects float single precision data from 2 sources using variable mask	BLENDVPS
m128 _mm_insert_ps(m128 dst,m128 src, const int ndx)	Insert single precision float into packed single precision array element selected by index.	INSERTPS
int _mm_extract_ps(m128 src, const int ndx)	Extract single precision float from packed single precision array selected by index.	EXTRACTPS

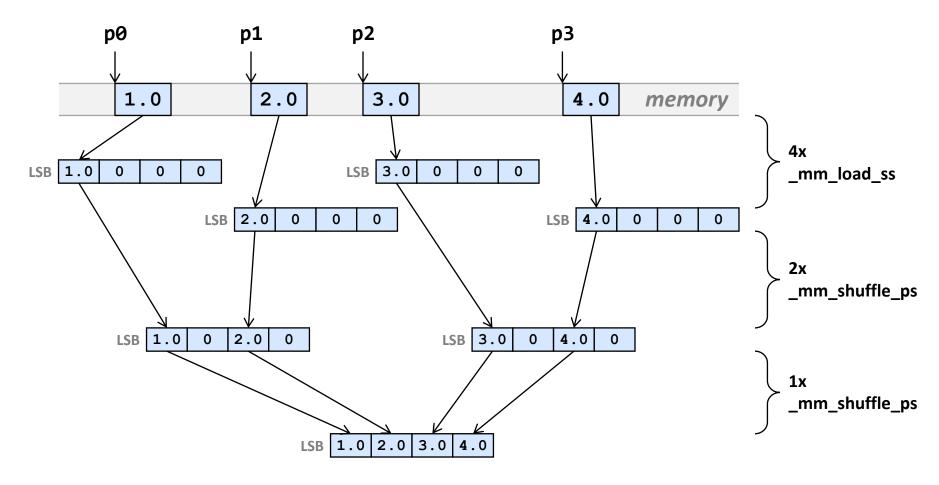




helper macro to create mask



Example: Loading 4 Real Numbers from Arbitrary Memory Locations



7 instructions, this is the right way (before SSE4)

Code For Previous Slide

Example: Loading 4 Real Numbers from Arbitrary Memory Locations (cont'd)

- Whenever possible avoid the previous situation
- Restructure algorithm and use the aligned _mm_load_ps()
- Other possibility (should also yields 7 instructions, trusting the compiler)

```
__m128 vf;
vf = _mm_set_ps(*p3, *p2, *p1, *p0);
```

- SSE4: _mm_insert_epi32 together with _mm_castsi128_ps
 - Not clear whether better

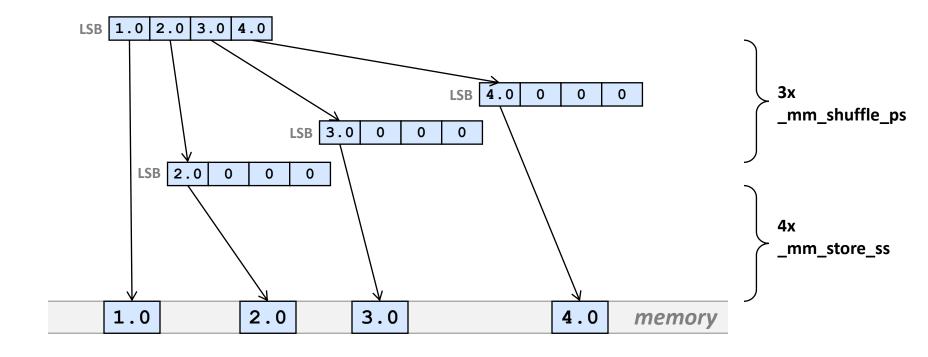
Example: Loading 4 Real Numbers from Arbitrary Memory Locations (cont'd)

Do not do this (why?):

```
__declspec(align(16)) float g[4];
__m128 vf;

g[0] = *p0;
g[1] = *p1;
g[2] = *p2;
g[3] = *p3;
vf = __mm_load_ps(g);
```

Example: Storing 4 Real Numbers to Arbitrary Memory Locations

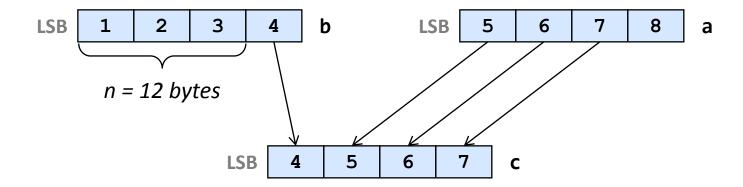


7 instructions, shorter critical path (before SSE4)

```
__m128i _mm_alignr_epi8(__m128i a, __m128i b, const int n)
```

Concatenate a and b and extract byte-aligned result shifted to the right by n bytes

Example: View __m128i as 4 32-bit ints; n = 12



Use with _mm_castsi128_ps to do the same for floating point

Example

```
void shift(float *x, float *y, int n) {
  for (int i = 0; i < n-1; i++)
    y[i] = x[i+1];
  y[n-1] = 0;
}</pre>
```

```
#include <ia32intrin.h>
// n a multiple of 4, x, y are 16-byte aligned
void shift vec(float *x, float *y, int n) {
 m128 f;
 __m128i i1, i2, i3;
 i1 = mm castps si128(mm load ps(x));
                                           // load first 4 floats and cast to int
for (int i = 0; i < n-8; i = i + 4) {
   i2 = _mm_castps_si128(_mm_load_ps(x+4+i)); // load next 4 floats and cast to int
   f = _mm_castsi128_ps(_mm_alignr_epi8(i2,i1,4)); // shift and extract and cast back
   mm store ps(y+i,f);
                                           // store it
   i1 = i2;
                                            // make 2nd element 1st
 // we are at the last 4
 _mm_store_ps(y+n-4,f);
                                            // store it
```

Vectorization

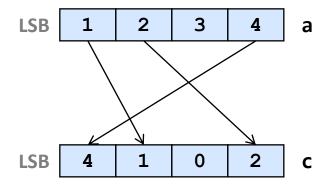


Picture: www.druckundbestell.de

```
__m128i _mm_shuffle_epi8(__m128i a, __m128i mask)
```

Result is filled in each position by any element of a or with 0, as specified by mask

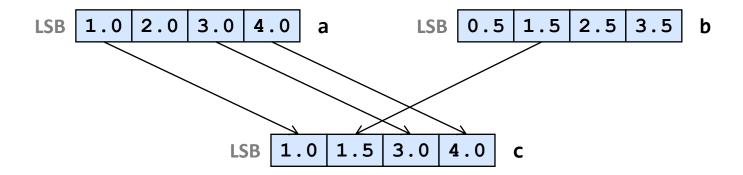
Example: View ___m128i as 4 32-bit ints



Use with _mm_castsi128_ps to do the same for floating point

(SSE4) Result is filled in each position by an element of a or b in the same position as specified by mask

Example: mask = 2 = 0010



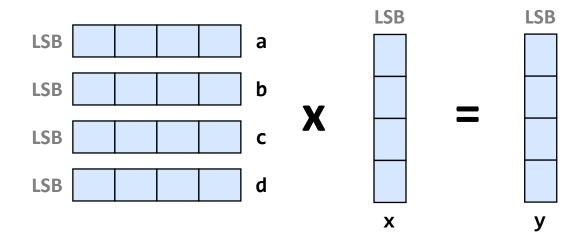
```
_MM_TRANSPOSE4_PS(row0, row1, row2, row3)
```

Macro for 4 x 4 matrix transposition: The arguments row0,..., row3 are __m128 values each containing a row of a 4 x 4 matrix. After execution, row0, .., row 3 contain the columns of that matrix.



In SSE: 8 shuffles (4 mm unpacklo ps, 4 mm unpackhi ps)

Example: 4 x 4 Matrix-Vector Product



Blackboard

Other Intrinsics

- Logical intrinsics (bitwise and, or, ...)
- Cacheability support intrinsics
 - Prefetch:
 void _mm_prefetch(char const *a, int sel)
 - Loads that bypass the cache:
 void _mm_stream_ps(float *p, __m128 a)
- Others

Vectorization With Intrinsics: Key Points

- Use aligned loads and stores
- Minimize overhead (shuffle instructions)= maximize vectorization efficiency
- Definition: Vectorization efficiency

Op count of scalar (unvectorized) code
Op count (including shuffles) of vectorized code

- *Ideally:* Efficiency = v for v-way vector instructions
 (assumes no vector instruction does more than 4 scalar ops)
- Examples (blackboard):
 - Adding two vectors of length 4
 - 4 x 4 matrix-vector multiplication