# **Q1** Team Name

0 Points

ANV

# **Q2** Commands

10 Points

List the commands used in the game to reach the ciphertext.

#### Commands Used:-

- exit1
- exit3
- exit4
- exit4
- exit1
- exit3
- exit4
- exit1
- exit3
- exit2
- $\bullet$  read

After executing the above commands in sequence, we reached the cip

## **Q3** Analysis

60 Points

Give a detailed description of the cryptanalysis used to figure out the password. (Use Latex wherever required. If your solution is not readable, you will lose marks. If necessary the file upload option in this question must be used TO SHARE IMAGES ONLY.)

### ■Analysis:

First, we pressed the command exit1. Then, some hexadecimal numbers were displayed on the screen. We converted those numbers to characters by assuming that they were ascii. Then, we got this following mapping:

a Gold 20 61 20 47 6f 6c 64

You see 59 6f 75 20 73 65 65

-Bug in 2d 42 75 67 20 69 6e

one co 20 6f 6e 65 20 63 6f

rner. I 72 6e 65 72 2e 20 49

t is th 74 20 69 73 20 74 68

e key t 65 20 6b 65 79 20 74

o a tre 6f 20 61 20 74 72 65

asure f 61 73 75 72 65 20 66

ound by 6f 75 6e 64 20 62 79

The message would be like:

You see a Gold-Bug in one corner. It is the key to a treasure found by

We then pressed read. The following appeared:

#### ANV: This door has RSA encryption with exponent 5 and the passwo

We realized that the cryptosystem used here is **RSA with exponent 5**.

To decrypt this, we thought of using **CopperSmith Attack** 

Coppersmith's attack describes a class of cryptographic attacks on the public-key cryptosystem RSA based on the Coppersmith method.

This Coppersmith's attack is generally used when the public exponent e is small.

Let  ${f N}$  be an integer and  $f\in {\Bbb Z}[x]f\in {\Bbb Z}$  [x] be a monic polynomial of degree d over the integers. Set  $X=N^{\frac{1}{d}-\epsilon}$  for  $\frac{1}{d}>\epsilon>0$ . Then, given  $\langle N,f\rangle$ , attacker (Eve) can efficiently find all integers  $x_0< X$  satisfying  $f(x_0)\equiv 0\pmod N$ . The running time is dominated by the time it takes to run the LLL algorithm on a lattice of dimension O(av) with  $av=\min\{\int_0^1 \log_2 N\}$ 

 $O(w) with w = \min \left\{ rac{1}{\epsilon}, \log_2 N 
ight\}.$ 

This theorem states the existence of an algorithm that can efficiently find all roots of  $\mathbf{f} \ \mathbf{modulo} \ \mathbf{N} \ \text{that are smaller than} \ X = N^{\frac{1}{d}}. \ \text{As X gets smaller, the algorithm's runtime} \\ \text{decreases. This theorem's strength is the ability to find all small roots of polynomials} \\ \text{modulo a composite } N.$ 

Coppersmith attack uses  ${f LLL\ reduction}.$ 

In normal RSA, we have a ciphertext c, modulus N and a public exponent e. We find m such that  $m^e = c \mod N$ .

In relaxed model of RSA, we have  $c = (m + x)^e$ , we know a part of the message m, but we do not know x. Coppersmith says that, if you are looking for N^1/e of the message, it is then a small root and you should be able to find it pretty quickly.

let our polynomial be  $f(x) = (m + x)^e - c$  which has a root we want to find modulo N

This m is padding. We did not have any idea of the padding at start. Then, we saw the ciphertext and saw this

**ANV:** This door has **RSA** encryption with exponent 5 and the passwo We assumed that the padding might be similar to this. We manually tried the following paddings:

- 1:- ANV: This door has RSA encryption with exponent 5 and the pass
- 2:- ANV: This door has RSA encryption with exponent 5 and the pass
- 3:- ANV: This door has RSA encryption with exponent 5 and the pass
- 4:- ANV: This door has RSA encryption with exponent 5 and the pass
- 5:- This door has RSA encryption with exponent 5 and the password i

We found out that the correct padding was the following:

ANV: "This door has RSA encryption with exponent 5 and the passw without quotes

We used **Sage Library** for implementing the coppersmith attack. We used some of code for the coppersmith attack implementation from this github link:

#### https://github.com/mimoo/RSA-and-LLL-attacks

We converted each character in the padding to ASCII and converted the total thing into binary padding and converted this to integer.

For finding the length of the unknown part, we tried all the lengths starting from 0 to 500.

For each of this length 'le' from 0 to 500, we left shift the padding by 'le' and add x to it.

That is pol =  $((padding << le) + x)^e - C$ 

The unknown length we got was 88.

We got the solution to be the following:

#### 

The length of this was 86. So, we ignored the first 6 bits and then converted each 8 bits to integer and considered these as ascii values and mapped them to characters to get the following password: C8YP7oLo6Y

We realized that the password looks like the word Cryptology.

The name of the sage script is decode.sage

The final message would be like:

You see a Gold-Bug in one corner. It is the key to a treasure found by

#### Resources used:

1:- https://github.com/mimoo/RSA-and-LLL-attacks

2:- https://www.cryptologie.net/article/222/implementation-of-cop

3:- https://en.wikipedia.org/wiki/Coppersmith%27s\_attack



#### **Q4** Password

10 Points

What was the final command used to clear this level?

The password used to clear this level is :-  ${\bf C8YP7oLo6Y}$ 

### **Q5** Codes

0 Points

It is MANDATORY that you upload the codes used in the cryptanalysis. If you fail to do so, you will be given 0 for the entire assignment.

```
▼ Makefile

1    run:
2    sage decode.sage
3
```

```
2    "74 20 69 73 20 74 68","65 20 6b 65 79 20 74","6f 20 61 20 74 72 65","61 73 75 72 65
    20 66","6f 75 6e 64 20 62 79"]
3    for j in s:
4         r=""
5         for i in j.split():
6             r+=chr(int(i,16))
7         print(r,j)
```

```
♣ Download
▼ decode.sage
     def coppersmith howgrave univariate(pol, modulus, beta, mm, tt, XX):
 1
 2
         dd = pol.degree()
 3
         nn = dd * mm + tt
 4
 5
 6
         if not 0 < beta <= 1:
 7
             raise ValueError("beta should belong in (0, 1]")
 8
         if not pol.is_monic():
 9
             raise ArithmeticError("Polynomial must be monic.")
10
11
         polZ = pol.change ring(ZZ)
12
         x = polZ.parent().gen()
13
14
         gg = []
         for ii in range(mm):
15
             for jj in range(dd):
16
                 gg.append((x * XX)**jj * modulus**(mm - ii) * polZ(x * XX)**ii)
17
         for ii in range(tt):
18
             gg.append((x * XX)**ii * polZ(x * XX)**mm)
19
20
21
         BB = Matrix(ZZ, nn)
22
         for ii in range(nn):
23
             for jj in range(ii+1):
24
25
                 BB[ii, jj] = gg[ii][jj]
26
```

```
# LLL
27
        BB = BB.LLL()
28
29
30
        # transform shortest vector in polynomial
31
        new pol = 0
32
        for ii in range(nn):
33
            new pol += x**ii * BB[0, ii] / XX**ii
34
        # factor polynomial
35
36
        potential roots = new pol.roots()
37
38
        # test roots
39
        roots = []
40
        for root in potential roots:
            if root[0].is_integer():
41
                result = polZ(ZZ(root[0]))
42
43
                if gcd(modulus, result) >= modulus^beta:
                    roots.append(ZZ(root[0]))
44
45
46
47
        return roots
48
    e = 5
49
    N =
    843644437357250348644025545338262791747038934397633433438632603427566786092168950937792
50
    C =
    259250464085150344189639062896456753787636324055158863112997919061591661748817605562071
51
52
    ZmodN = Zmod(N)
53
54
    def decrypt(padding):
        z=""
55
        for i in padding:
56
            i=bin(ord(i))[2:]
57
            z+='0'*(8-len(i))+i
58
59
        padding = int(z,2)
```

```
for le in range(0, 501):
60
61
62
63
            P.<x> = PolynomialRing(ZmodN)
64
            pol = ((padding << le) + x)^e - C
65
            dd = pol.degree()
            beta = 1
66
67
            epsilon = beta / 7
            mm = ceil(beta**2 / (dd * epsilon))
68
69
            tt = floor(dd * mm * ((1/beta) - 1))
70
            XX = ceil(N**((beta**2/dd) - epsilon))
71
72
            roots = coppersmith howgrave univariate(pol, N, beta, mm, tt, XX)
73
            if roots!=[]:
74
                print("length of unknown part is: ",le)
75
                password=bin(roots[0])[2:]
76
                print ("A solution has been found out. It is : "+password)
77
                final password=""
78
                for i in range(len(password)%8,len(password),8):
                    final password+=chr(int(password[i:i+8],2))
79
80
                print("\nDecrypted password is: ",final password)
81
                return
82
83
        print('There is no solution for this padding')
84
    padding="ANV: This door has RSA encryption with exponent 5 and the password is"
    decrypt(padding)
86
```

Assignment 6	• GRADED
GROUP Dibbu Amar Raja Vikas Idamakanti Venkata Nagarjun Reddy  View or edit group	
TOTAL POINTS	
80 / 80 pts	
QUESTION 1	
Team Name	<b>0</b> / 0 pts
QUESTION 2	
Commands	<b>10</b> / 10 pts
QUESTION 3	
Analysis	<b>60</b> / 60 pts
QUESTION 4	
Password	<b>10</b> / 10 pts
QUESTION 5	
Codes	<b>0</b> / 0 pts