# **Q1** Team Name

0 Points

ANV

## **Q2** Commands

10 Points

List the commands used in the game to reach the ciphertext

#### Commands Used: -

- ullet to enter the small chamber :  ${f go}$
- ullet to get into the underground chamber :  ${f enter}$
- to pick Mushrooms : pick
- to return to small Chamber : back
- to put some Mushrooms into small hole after returning to small chamber give
- ullet to go back from the creature : back
- to reach the hidden door in the main Chamber: back
- to enter the hidden door: thrnxxtzy
- to read the cipher text: read

After executing the above commands in sequence, we reached the cir

# **Q3** Analysis

50 Points

Give a detailed analysis of how you figured out the password? (Explain in less than 500 words)

#### Given that: -

Password is an element of the multiplicative group  $Z_p^{st}$  . where p = 455470209427676832372575348833 is a prime.

After looking around, we found three pairs of numbers of the form ( $\mathbf{a}$ ,  $\mathbf{password} * \mathbf{g}^{\mathbf{a}}$ ) where  $\mathbf{g}$  is a element in  $Z_p^*$  and  $\mathbf{a}$  is an integer. The  $\mathbf{g}$  in each pair is the same.

 $a_1, x = (429, 431955503618234519808008749742)$ 

 $a_2, y = (1973, 176325509039323911968355873643)$ 

 $a_3, z = (7596, 98486971404861992487294722613)$ 

After looking for more numbers, we find

5\_\_50\_4\_\_31\_\_94\_9

We thought this was **g** with many digits missing.

We need to figure out the password to complete the level.

## $\blacksquare$ **ANALYSIS**: -

Let us assume

$$egin{aligned} x &= \left(password * g^{a_1}
ight) \mod p & o (1) \ y &= \left(password * g^{a_2}
ight) \mod p & o (2) \ z &= \left(password * g^{a_3}
ight) \mod p & o (3) \end{aligned}$$

ullet Here, we do not know the password. So we try to eliminate the password term. We eliminate the password term by dividing equations (3) and (2),

we get 
$$\Rightarrow z*modinv(y,p) = \left(g^{a_3}*modinv(g^{a_2},p)\right) \mod p$$
  $\Rightarrow z*modinv(y,p) = g^{a_3-a_2} \mod p \qquad o (4)$ 

here, modinv(y, p) means **modulo multiplicative inverse** of "y" under "p".

• We need to eliminate the power  $(a_3 - a_2)$  on the right side of the equation (4) to find "g".

Now, we have to find some value "d" such that  $=> (g^{a_3-a_2} \mod p)^d \mod p = g \qquad ==> \bigstar$ 

{If we recall  $\mathbf{RSAalgorithm}$ , we have a message (M), encryption key (e), decryption key (d) and a number (n) such that n is product of two prime numbers, where M and n are relatively prime and e is coprime to n.

In RSA, we find a number (d) such that  $e*d*mod(\Phi(n))=1$ 

where,

 $\Phi$  function is **Euler's totient function**, such that  $\Phi(n)$  is the count of numbers less than n and relatively prime to n. All of this can be derived from Euler's Theorem. Link to the resources of the proof are provided at the end.

Here, the encrypted message is  $M^e \mod n$  and decrypted message is  $(M^e \mod n)^d \mod n$ .

That means applying the power of d will get rid of the exponent "e" on the message M and we will only be left with M. We can use similar technique in our problem.  $\}$ 

- ullet In our problem, e is analogous to " $a_3-a_2$ ", d is analogous to "d" and M is analogous to g. Instead of n, we are using our given prime number p. g is relatively prime to g.
- ullet So, we need to find a "d" such that

$$=> (a_3-a_2)*d \mod \Phi(p)=1$$

where  $\ (\Phi(p)=p-1,$  as p is a prime number.)

$$\Rightarrow$$
  $(a_3 - a_2) * d \mod (p - 1) = 1$ 

$$\Rightarrow$$
  $\mathbf{d} = modinv(a_3 - a_2, p - 1)$   $\Longrightarrow \star$ 

On applying the power of "d" on both sides of **equation 4**. We get

- $((z*modinv(y,p))^d) \mod p = ((g^{(a_3-a_2)} \mod p)^d) \mod p$
- ullet Now, we can replace  $ig((g^{a_3-a_2} \mod p)^d \mod pig)$   $with \ {f g}$ 
  - $\Rightarrow$   $((z*modinv(y,p))^d) \mod p = \mathbf{g}$
  - =>  $\mathbf{g}=((z*modinv(y,p))^d)\mod p$  ==>

We get the value of  ${f g}$  to be  ${f 52565085417963311027694339}$ 

- ullet Now, substituting  ${f g}$  in  ${f equation}({f 3})$  to find out the value of password.
  - $\Rightarrow z = \left(password * g^{a_3}\right) \mod p$
  - $\Rightarrow password = (z * modinv(g^{a_3}, p)) \mod p$

We get the password to be 134721542097659029845273957

• We can convert the password to **hexadecimal** and convert it to **binary string** to get the password as "**open\_sesame**".

#### Resources:

- 1.) https://leimao.github.io/article/RSA-Algorithm/
- 2.) https://www.educative.io/edpresso/what-is-the-rsa-algorithm

What was the final command used to clear this level?

The final command used to clear this level is 134721542097659029845273957

The number mentioned above is the password that we have derived.

The binary String representation of the password is => "open\_sesame".

### **Q5** Codes

0 Points

Upload any code that you have used to solve this level

```
♣ Download
■ a.py
    import gmpv2 ## We used gmpv2 module since it supports fast multiple precision
     arithmetic.
    import binascii ## We used binascii module to convert between binary and various ACII
     encoded binary representations.
    p=455470209427676832372575348833 ## Value of p given in the cipher text.
3
4
    ## Below are the three pairs of numbers of the form(a,password*g^a)
5
    a1, x=(429, 431955503618234519808008749742)
6
     a2,y=(1973, 176325509039323911968355873643)
7
    a3,z=(7596, 98486971404861992487294722613)
8
9
10
    ## Finding the value of d.
11
    d = gmpy2.invert(a3-a2,p-1) # 51192810307361152064639448775
12
13
    ## Finding the value of g^(a3-a2)
14
    g_pow_a3_minus_a2 = z*gmpy2.invert(y,p) #
15
     22548311363938571801320757867327548215563208824357219231625
```

```
16
17
    ## Finding the value of g
    g = pow(g_pow_a3_minus_a2,d,p) # 52565085417963311027694339
18
19
    ## Finding the value of password
20
    password = (gmpy2.invert(pow(g,a3,p),p)*z)%p #134721542097659029845273957
21
22
    print("The password is ",password)
23
    ## Finding the binary string representation of the password
24
    password string = binascii.unhexlify(hex(password).replace("0x",""))
25
    print("The binary string represented by the password
26
    is",password string.decode("ascii"))
```

# Assignment 3

GRADED

#### **GROUP**

Vikas

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View or edit group

**TOTAL POINTS** 

65 / 70 pts

**QUESTION 1** 

Team Name	<b>o</b> / O pts
QUESTION 2 Commands	<b>10</b> / 10 pts
QUESTION 3 Analysis	R 45 / 50 pts
QUESTION 4 Password	<b>10</b> / 10 pts
QUESTION 5 Codes	<b>o</b> / 0 pts