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Thermal X-ray Emission from Isolated Older Pulsars: A New Heating Mechanism

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Summary. A new pulsar heating mechanism, magnetic dipole radiation of superfluid neutrons, is proposed. It is also a new mechanism for pulsar spindown, $\dot{P} \propto P^2$. The expected surface temperatures are calculated for isolated, older pulsars for different models. These temperatures are compared with measurements from the Einstein Observatory (HEAO 2). According to this mechanism, the internal magnetic field strengths are estimated for older pulsars.

Key words: superfluid vortex – spin down – thermal X-ray – neutron star

I. Introduction

Heating mechanisms that might cause detectable thermal X-ray emission from the surfaces of isolated neutron stars have recently been reviewed (Tsuruta, 1979; Helfand et al., 1980) and the expected surface temperatures have been compared with upper limits set for several pulsars by the Einstein Observatory (HEAO 2) (Helfand et al., 1980). Newer preliminary observations (Helfand, 1980) also suggest detection of *nonpulsed* thermal emission from the surface of at least two pulsars at temperatures at $\sim 3\,10^5$ and $\sim 10^6$ K. Without heat sources, cooling models for neutron stars with magnetic fields $\gtrsim 10^{12}$ G predict (Tsuruta, 1979) temperatures $< 10^4$ K for stars older than 10^5 yr, assuming the stiffest equation of state. Since nearly all of the heating processes, which have been considered, heat only the magnetic polar cap of a pulsar, the resulting thermal emission which would probably be observed as pulsed emission at the earth (Helfand, 1980). The process proposed previously that should lead to global heating and, hence, unpulsed emission is frictional dissipation between the crust and superfluid interior of a neutron star. This process, however, may not be applicable to older pulsars ($> 10^5$ yr for masses $> 0.5\,M_\odot$), since the decelerating torque on the crust may become decoupled from the superfluid interior so that frictional dissipation becomes insignificant (Greenstein, 1975).

The more rapid spindown of such a decoupled crust might be compatible with the higher average values of the spindown rate $\dot{P}$ observed for pulsars with longer periods, P. But we have recently suggested an alternative mechanism for this more rapid spindown of older pulsars. This is neutrino cyclotron radiation from superfluid neutron vortexes in the neutron star interior (Peng et al., 1982). The spindown rate, according to this mechanism, is $\dot{P} \propto P^2$.

In this paper we propose another mechanism contributing to the spindown of older pulsars, named magnetic dipole radiation from superfluid neutron vortexes in the neutron stars (for simplicity we denote it by MDRSN hereafter). The spindown rate caused is also $\dot{P} \propto P^2$.

The interesting thing is that MDRSN may also be an important heating source for isolated, older pulsars, and the thermal X-ray emission could be expected from it. Emphasis of this paper is placed on it because it would add considerably to our understanding of neutron star physics, such as magnetic properties of dense matter, equation of state, pion condensates, and other fundamental problems in high energy physics.

II. Magnetic Dipole Radiation of Superfluid Neutron Vortexes

As is well known, electro-neutral neutrons have an anomalous magnetic moment

$$\mu_z = g_n \mu_0 s_z \tag{1}$$

where the nuclear number $g_n = -3.826$, μ_0 is the nuclear magneton and s_z is the z-component of the spin angular moment. Thus there is an electromagnetic interaction between neutrons and electromagnetic field. This interaction leads to dipole radiation of the neutron magnetic moment for neutrons in circular motion, such as those in superfluid vortexes.

For neutron Cooper pairs coupled by the $^1S_0$ wave interaction, the spins of the two neutrons are anti-parallel, so that the total spin is zero and there is no net magnetic moment. But for neutron pairs coupled by the $^3P_2$ wave interaction the spins are parallel, so that the total spin is equal to 1 and there is a net magnetic moment. If there is also a magnetic field in the interior, the $^3P_2$ superfluid neutrons should lose energy and angular momentum by magnetic dipole radiation, contributing to both the spindown and the heating of the star.

The energy radiated by one neutron is

$$W = \frac{2\omega^4}{3c^3} \sum_f |\langle f|\hat{M}_z|i\rangle|^2 = \frac{2\omega^4}{3c^3} \langle i|\hat{M}_z^+ \hat{M}_z|i\rangle\,, \tag{2}$$

where $|f\rangle$ is the final state, $|i\rangle$ is the initial state, and $\hat{M}_z$ is the operator of the magnetic moment. The frequency of emitted photons is equal to the rotational frequency of the neutrons, ω. Because of the high opacity of the stellar interior this radiation is absorbed and become a heating source.

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If we consider a coherent small volume ΔV, and take

$\langle i|\hat{M}_z^+\hat{M}_z|i\rangle = \langle i||\hat{M}_z|^2|i\rangle \approx M_{\Delta V}^2$,

corresponding to the macroscopic magnetic moment, then

$$W_{\Delta V} = \frac{2\omega^4}{3c^3} M_{\Delta V}^2 . \tag{3}$$

Since for $^3P_2$ superfluid neutrons the magnetic moment is anisotropic, the susceptibility χ is a tensor. In a strong magnetic field, however, the component of susceptibility along the direction of the magnetic field is very large, and very small in other directions, i.e. $\chi_{zz} \approx \chi_n$. Here χ_n is the Pauli paramagnetic susceptibility of normal neutrons

$$\chi_n = \frac{1}{4}\gamma^2\hbar^2\frac{dn}{d\varepsilon}, \tag{4}$$

where $\gamma = \frac{g_n\mu_0}{\hbar}$ is the neutron gyromagnetic ratio and the density of state at the Fermi surface

$$\frac{dn}{d\varepsilon} = \left(\frac{3}{\pi^4}\right)^{1/3}\frac{m_n^{2/3}}{\hbar^2}\varrho_n^{1/3} \tag{5}$$

thus

$$M_{\Delta V} \approx \chi_n B \Delta V , \tag{6}$$

where B is the magnetic field strength.

Taking the same approach considering the coherent effect as we did in the neutrino cyclotron radiation paper (Peng et al., 1982), we obtain the total magnetic dipole radiation power of all the superfluid neutrons in neutron star

$$W_m = \frac{11\pi}{3}\frac{\gamma^4}{c^2\hbar^2} m_n^2 \overline{\Delta^2 B^2 n^3} \overline{n^{-1}} R_p^3 P^{-1} , \tag{7}$$

where the bar denotes an average of some physical quantity for all the vortex lines. Δ is the energy gap, and R_p is the radius of $^3P_2$ region.

Assuming

$$\overline{\Delta^2 B^2 n^3} \approx \overline{\Delta^2 B^2}\,\overline{n^3} \tag{8}$$

then

$$W_m = KQ(n)P^{-1} \tag{9}$$

here

$$K = 3.67\,10^3(\overline{\Delta^2 B^2})R_p^3$$
$$Q(n) = \overline{n^3}/\bar{n} . \tag{10}$$

III. The Effect of MDRSN on the Spindown of Pulsars

As we see that both MDRSN and neutrino cyclotron radiation of superfluid neutron (Peng et al., 1982) lead to pulsar energy loss rate $\propto P^{-1}$. But under certain condition the MDRSN would be the more important of the two processes, that is about

$$\left(\frac{R_p}{R}\right)^3 B_c^2 > 1.7\,10^{54} n^4 \frac{\Delta^8}{\Delta_p^2}\frac{1}{\varrho^{2/3}} . \tag{11}$$

For simplicity, we use maximum quantities instead of average ones, i.e. $\Delta \sim 2.35$ MeV, $\varrho \sim 2\,10^{13}$ g cm^{-3}, and $\Delta_p \sim 0.6$ MeV. Taking $n \sim 10$, we get $B_c \gtrsim 10^{11}$ G for $R_p \gtrsim 0.1\,R$, where R is the pulsar surface radius. This condition on the magnetic field is probably satisfied for most observed pulsars (Manchester and Taylor, 1977) if the ratio of the average interior to surface magnetic field strengths is at least of the order of unity; the condition on the $^3P_2$ radius should be satisfied for pulsars with masses $\gtrsim 0.5\,M_\odot$, assuming even the stiffest equation of state (Pandharipande et al., 1976).

Following the analyses we did in our first paper (Peng et al., 1982), it is easy to see that the MDRSN is also a new mechanism for pulsar spindown, leading to $\dot{P} \propto P^2$, for periods greater than some transition period P_m which is represented by the following formula

$$P_m^3 \simeq 2.5\,10^{-34}\frac{B_s^2}{\overline{\Delta^2 B^2}}\frac{R^6}{R_p^3}\frac{\bar{n}}{\overline{n^3}} . \tag{12}$$

This transition period P_m depends on properties of the neutron star, such as the interior structure and the internal magnetic field which are model-dependent and difficult to estimate. But we can estimate it from the statistical results (Peng et al., 1982) which leads to $0.5\,\mathrm{s} < P_m < 1.25\,\mathrm{s}$.

IV. The MDRSN as a Heating Source

a) Basic Consideration

Rewriting the heating rate W_m

$$W_m = 4\pi^2 f I \dot{P} P^{-3} , \tag{13}$$

where

$$f \equiv \frac{W_m}{W_{\mathrm{rot}}} = [1 + (P_m/P)^3]^{-1} \tag{14}$$

we see that smaller value of P_m leads to larger heating rates. From these two equations we can estimate the expected heating by MDRSN for individual pulsars with measured P and $\dot{P}$ by choosing $P_m \simeq 0.5$ s and a moment of inertia which is model dependent. In order to calculate the expected surface temperature of older pulsars, we should consider two different situation, with or without pion condensates. For older pulsars without pion condensates, where the internal temperature is $< 8\,10^8$ K, we can then calculate the expected surface temperatures from heating rate W_m, assuming that it is equal to the cooling rate by black body radiation from the surface, L_γ, and the ordinary neutrino bremsstrahlung emission in the interior, L_ν^B

$$W_m = L_\gamma + L_\nu^B , \tag{15}$$

where

$$L_\nu^B(C) \approx 3\,10^{39}\left\{\int_{R_i}^{R}\left(\frac{\varrho}{\varrho_N}\right)r^2 dr\right\}\left(\frac{Z^2}{A}\right)T^6 (\mathrm{erg\,s}) , \tag{16}$$

where the radii are in units of 10^6 cm, R is the stellar radius, R_i is the inner boundary radius of the heavy ion crust, $\varrho_N \approx 3\,10^{14}$ g cm^{-3} is the nuclear density, T is the internal temperature in units of 10^9 K, and we set $(Z^2/A) = 1$ following Maxwell (1979). The ratio of the surface to internal temperature is adopted from the calculation of Glen and Sutherland (1980). Finally, from the surface temperature, T_{se}, we calculate the expected temperature, T_∞, seen by an observer infinitely far from

Table 1. Various properties of model A and Model B

Model	$M/M_\odot$	ϱ^s (g cm^{-3})	R (km)	R_s^i (km)	R_π (km)
A	1.32	4.2 10^{14}	15.97	11.20	7.5
B	1.33	3.8 10^{15}	7.77	7.38	7.3
B	0.52	1.0 10^{15}	9.8	7.52	7.2

the surface of the neutron star,

$$T_\infty \simeq T_{s_e}(1+z)^{-1} \tag{17}$$

here $(1+z)^{-1}$ is the redshift correction due to the neutron stars.

If pions are present, the beta processes involving pions are the most important neutrino processes as Bahcall and Wolf (1965) first pointed out, thus instead of L_ν^B in Eq. (15), we consider L_ν^π

Table 2. Estimated pulsar surface temperatures for magnetic dipole radiation heating

PSR	D	f	Model A		Model B		T_{obs}	Model B
	kpc	$(P_m=0.5)$	W_m Erg s^{-1}	T 10^6 K	W_m Erg s^{-1}	T 10^6 K	10^6 K	T 10^6 K
0031−07	.44	.87	3.3E31	.32	1.0E31	.28	< .4	.25
0149−16	.56	.82	1.5E32	.46	4.5E31	.40	< .5	.37
0355+54	1.5	.03	2.7E33	.83	8.3E32	.83	~ .8	.76
0402+61	.87	.63	1.3E33	.70	4.1E32	.69		.64
0450+55	.55	.24	1.1E33	.67	3.5E32	.67		.61
0540+23	2.8	.11	8.7E33	1.1	2.7E33	1.1		1.0
0611+22	3.5	.23	2.9E34	1.5	9.0E33	1.5		1.4
0727−18	2.0	.52	5.8E33	1.0	1.8E33	1.0		.92
0740−28	2.0	.36	1.0E34	1.2	3.2E33	1.2		1.1
0743−53	.39	.07	1.6E33	.73	5.0E32	.73		.67
0809+74	.20	.95	5.5E30	.20	1.7E30	.18		.16
0826−34	.32	.98	1.2E31	.25	3.8E30	.22		.20
0840−48	.42	.68	1.9E33	.76	5.9E32	.76		.70
0905−52	.40	.12	1.0E33	.65	3.2E32	.65		.60
0919+06	1.3	.39	5.3E33	.99	1.6E33	.98		.90
0922−52	.45	.77	5.2E33	.98	1.6E33	.98		.90
0940−55	.52	.70	4.3E33	.94	1.3E33	.93		.86
0941−56	.51	.81	4.8E33	.96	1.5E33	.96		.88
0950+08	.10	.12	1.3E32	.45	4.0E31	.39		.36
1055−52	1.1	.06	3.5E33	.89	1.1E33	.88	~ 1.0	.81
1133+16	.18	.93	1.6E32	.48	5.1E31	.41		.38
1221−63	.99	.08	2.9E33	.85	9.0E32	.84		.78
1449−64	2.6	.04	1.7E33	.74	5.1E32	.73		.68
1451−68	.29	.13	5.3E31	.36	1.6E31	.31		.29
1508+55	.92	.76	7.5E32	.61	2.3E32	.60		.55
1530+27	.63	.92	2.8E31	.31	8.5E30	.26	< .5	.24
1641−45	4.9	.43	7.2E33	1.1	2.2E33	1.1		.98
1642−03	.16	.32	7.7E32	.61	2.4E32	.61	~ .3	.56
1648−17	.16	.88	2.3E32	.52	7.1E31	.45		.41
1706−16	.16	.69	1.2E33	.69	3.9E32	.68	< .3	.63
1727−47	5.9	.82	1.9E34	1.4	5.8E33	1.3		1.2
1742−30	2.9	.28	4.8E33	.96	1.5E33	.96		.88
1747−46	.74	.77	1.9E32	.50	5.9E31	.43		.39
1749−28	1.0	.59	2.1E33	.79	6.6E32	.78		.72
1822−09	.65	.28	7.1E33	1.1	2.2E33	1.1		.97
1844−04	4.9	.63	1.2E34	1.2	3.8E33	1.2		1.1
1916+14	1.0	.93	9.4E33	1.1	2.9E33	1.1		1.0
1930+22	8.1	.02	3.6E34	1.6	1.1E34	1.6		1.5
1952+29	.67	.38	6.2E29	.12	1.9E29	.10	< .3	.09
2151−56	.18	.95	1.2E32	.44	3.8E31	.38		.35
2327−20	.32	.97	8.0E31	.40	2.5E31	.34	< .4	.32

Table 3. Surface temperatures of pulsars with pion cooling

PSR	T	$T_{\rm obs}$
0355+54	0.57	~0.8
1055−52	0.58	~1.0
1642−03	0.52	~0.3

expressed by the following equation (Tsuruta, 1979)

$$L_\nu^\pi = 10^{46} a R_\pi^3 T^6 , \tag{18}$$

where

$a=1$ for $\varrho > \varrho_\mu$

$a=0.5$ for $\varrho < \varrho_\mu$.

R_π is radius (in units of 10^6 cm) of the core within which pions are present, ϱ_μ is the density where muons appear $\sim 8\,10^{14}\,\mathrm{g\,cm^{-3}}$.

Obviously, all of these calculations, L_ν^π, L_ν^B, and L_γ are model dependent because they generally depend on density profiles. We should then expect that different surface temperatures which might be detected could be caused by different equations of state or different stellar masses for each pulsar. In order to see the effects of equations of state, we choose two extreme cases: the stiffest equation of state by Pandharipande et al. (1976; hereafter referred to as Model A); and the very soft equation of state by Baym et al. (1971; hereafter referred to as Model B). For the sake of comparison we set the stellar mass at $M=1.3\,M_\odot$.

The various properties of neutron star model A and B are listed in Table 1, where $M/M_\odot$ is the mass in solar mass units, ϱ^c is the central density in cgs units, R is the stellar radius in km. R_c^i and R_π are, respectively, the distance from the center of the star to the point where the heavy ion crust, the pion core start, expressed in km. The critical densities at these boundaries are, respectively, $\varrho_c^i = 2\,10^{14}\,\mathrm{g\,cm^{-3}}$, $\varrho_\pi = 3\,10^{14}\,\mathrm{g\,cm^{-3}}$.

On the other hand, we choose another stellar mass for Model B to estimate the effects of mass, various properties are also listed in Table 1.

b) Results

The effects of equations of state on the thermal X-ray radiations could be seen in Table 2. There we list the fraction, f, of the rotational energy loss dissipated by MDRSN as heat in column 3, the heating rate W_m and the resultant observable surface temperatures, T_∞, for Model A and B in columns 4, 5, 6, 7 for each of the pulsars, some of which are in the Einstein Observatory Program (Helfand et al., 1980; Einstein X-ray Observatory Consortium Observing Program). Also listed in column 8 are the preliminary temperatures reported (Helfand, 1980) for the three detected pulsars and upper limits on six others set by the Einstein Observations (Helfand et al., 1980), assuming an interstellar hydrogen density of $0.3\,\mathrm{cm^{-3}}$. We also list the estimated distance of each pulsar in column 2 (Taylor and Manchester, 1980; Newton et al., 1980).

From the resultant surface temperatures, we see that the two models make no difference indeed. For pulsars with lower surface temperatures, say $<5\,10^5$ K, the differences are rather large, $\Delta T \sim 0.1\,T$. Within the uncertainties in the distance estimate, however, the temperatures of, say, $4\,10^5$ K and $4.6\,10^5$ K could be considered to be quite consistent.

The interesting thing is that the quite consistent surface temperatures between the two models are just caused by the very differences of these two models. As we know, a harder equation of state generally gives higher masses at lower densities, this model (Model A) is larger and lighter. If the MDRSN mechanism works in Model A, the heating rate W_m would be larger than that in Model B because of larger moment of inertia. In our case

$$W_m|_{\rm Model\,A} \approx 3\, W_m|_{\rm Model\,B} .$$

But Model A has a larger heavy ion crust region, then the cooling processes through the neutrino bremsstrahlung involving heavy ions is an important cooling factor in Model A. While in Model B, the heavy ion crust region is very thin, the cooling rate through neutrino bremsstrahlung is insignificant indeed. Although the heating rate W_m in Model B is rather small, the only cooling process we have to consider in Model B is the back body radiation from the surface of neutron stars. Besides, the redshift correction are also different. The correction in Model A is rather small. That is why the two models make no difference to the thermal X-ray radiation. But when the heating rate is rather low, the neutrino cooling process will be negligible in Model A compared with the L_γ because of the low internal temperature. So the difference between the two models appears, though it could not be detected.

In the last column of Table 2 we list the resultant surface temperatures for Model B with a mass of $0.5\,M_\odot$, from which we can see the effect of stellar masses on the thermal X-ray radiation. Obviously, the temperatures decrease with decreasing masses. Although the mass of $1.33\,M_\odot$ seems to be more probable from the comparison with the temperatures reported for the three detected pulsars, the mass of $0.5\,M_\odot$ still could not be rejected according to MDRSN mechanism. The main reason is that the differences are not large enough to draw a definite conclusion.

In Table 3 we list the resultant surface temperatures obtained if we consider the presence of pion condensates for Model B with mass of $1.33\,M_\odot$, together with the temperatures reported for these three detected pulsars: PSR 0355+54, 1055−52, and 1642−03. Still no decisive conclusion could be drawn from the comparison. The temperatures predicted from MDRSN for PSR 0355+54 and 1055−52 are lower than that from observation quite large. PSR 1642−03, however, is a puzzeling one again. Its temperature could not be decreased through decreasing the stellar mass in this case because small pion condensates region caused by small stellar mass will yield a higher surface temperature. But if we change the transition period P_m of 0.5 s into 1.25 s, the heating rate will be reduced to 0.1 times of that derived from $P_m = 0.5$ s. In that case the pion cooling will be negligible, and the resultant surface temperature of PSR 1642−03 could be decreased to $3.3\,10^5$ K, pretty close to the detected value. In conclusion, there is no need to consider the pion cooling for PSR 1642−03 if MDRSN mechanism works. As for PSR 0355+54 and 1055−52, we would leave the pion cooling out of consideration.

c) Internal Magnetic Field

Now it is quite certain that the intense magnetic fields of neutron stars must be a fossil of the field it achieved at birth. Assuming that the increasing pulsar periods are caused by the loss of rotational energy from an oblique, magnetic-dipole radiator, one can derive the surface magnetic field from the spindown rate of each pulsar

$$B_s^2 \simeq P\dot{P}(3Ic^3/8\pi^2 R^6) . \tag{19}$$

It gives B_s in the range 10^{11}–10^{12} G. In one case, however, we can get the magnetic field strength directly from observation, that is

Table 4. Estimation of internal magnetic fields for different models

PSR	Model A		Model B	
	B_p (G)	B_p/B_s	B_p (G)	B_p/B_s
0031-07	1.8E11	.82	5.5E11	0.55
0149-16	3.5E11	.96	1.0E12	0.60
0355+54	6.5E11	2.2	2.0E12	1.48
0402+61	8.9E11	1.4	2.8E12	0.93
0450+55	6.3E11	2.0	2.0E12	1.32
0540+23	1.5E12	2.2	4.6E12	1.43
0611+22	3.1E12	2.0	9.0E12	1.32
0727-18	1.7E12	1.6	5.5E12	1.04
0740-28	1.3E12	2.2	4.0E12	1.48
0743-53	5.9E11	2.2	1.8E12	1.43
0809+74	8.5E10	.53	2.7E11	0.33
0826-34	1.5E11	.32	4.8E11	0.22
0840-48	1.1E12	1.3	3.6E12	0.82
0905-52	5.1E11	2.1	1.7E12	1.43
0919+06	1.5E12	1.8	4.8E12	1.15
0922-52	2.0E12	1.1	6.6E12	0.71
0940-55	1.7E12	1.2	5.4E12	0.82
0941-56	2.0E12	.99	6.6E12	0.66
0950+08	1.8E11	2.1	6.0E11	1.43
1055-52	8.3E11	2.2	2.6E12	1.43
1133+16	4.4E11	.60	1.4E12	0.38
1221-63	8.0E11	2.2	2.5E12	1.43
1449-64	5.5E11	2.2	1.7E12	1.48
1451-68	1.2E11	2.1	3.8E11	1.37
1508+55	7.5E11	1.1	2.4E12	0.71
1530+27	1.8E11	.64	5.5E11	0.44
1641-45	1.8E12	1.7	6.0E12	1.15
1642-03	5.5E11	1.9	1.7E12	1.21
1648-17	4.7E11	.78	1.5E12	0.49
1706-16	9.1E11	1.3	2.9E12	0.82
1727-47	4.0E12	.96	1.3E13	0.66
1742-30	1.3E12	1.9	4.3E12	1.26
1747-46	3.8E11	1.1	1.2E12	0.71
1749-28	1.1E12	1.5	3.5E12	0.93
1822-09	2.4E12	1.1	7.7E12	0.71
1844-04	2.7E12	1.4	8.8E12	0.93
1916+14	3.4E12	.60	1.0E13	0.38
1930+22	2.3E12	2.2	7.1E12	1.48
1952+29	1.6E10	1.8	5.1E10	1.15
2151-56	4.1E11	.49	1.3E12	0.33
2327-20	3.7E11	.38	1.2E11	0.27

observations of a line in the Her X-1 X-ray spectrum near 53 KeV which is most likely an electron cyclotron emission. This observation gives the surface magnetic field of about $5\,10^{12}$ G at polar cap of Her X-1.

As pointed out by Ruderman (1980), the surface field of a neutron star does not necessarily reflect the interior field. A large toroidal magnetic field could exist in the interior of neutron star analogous to the sun, in which toroidal fields 10^2–10^3 times the average surface 1 G dipole field almost certainly exist beneath the surface of the Sun. That means 10^{14}–10^{15} G toroidal field in a neutron star.

Here we propose another way to estimate the internal magnetic field strength using MDRSN mechanism if it works. From Eqs. (11) and (19), we obtained the ratio of internal to surface magnetic field

$$\left(\frac{B_p}{B_s}\right)^2 = \frac{32\pi^3}{11c} \frac{\hbar^2}{\gamma^4 m_n^2 \Delta_p^2 n^2} \frac{R^6}{R_p^3} (P^3 + P_m^3)^{-1} . \tag{20}$$

For Model B we use $(R_p - R_I)(R_p^2 - R_I^2)$ instead of R_p^3, here R_I is the distance from the center of the star to the point where the density is equal to $10^{15}\,\mathrm{g\,cm^{-3}}$.

Taking $n=1$ and $P_m=0.5$ s, we can estimate the upper limit of internal magnetic field and the ratio for each pulsar in different models. The results are listed in Table 4. $10^{11} \rightarrow 10^{12}$ G is the order of magnitude in which the internal fields could be, no matter what model we take. Of course, a major premise is that MDRSN mechanism works.

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