I. Objective

The aim of this project is to demonstrate your understanding of volatility and multivariate analysis by investigating and analyzing a particular stock return and foreign exchange trading.

Data Source: Apple stock historical prices from Yahoo Finance

Price considered in the analysis: Close price adjusted for dividends and splits

3.2.1 Volatility Analysis

1. Select GARCH model (ARCH, GARCH-M, IGARCH, EGARCH, TARCH, multivariate GARCH etc). Explain your choice.
2. Forecast next period daily return (t+1) using the chosen model. Select the timeframe in the analysis. Provide charts and comment.

3.2.2 Multivariate Analysis

1. Indicate economic theories and models for calculating equilibrium FX.
2. Indicate macroeconomic variables used for calculating equilibrium FX.
3. Explain the connection between linear regression and Vector Error Correction (VEC).
4. Calculate equilibrium FX using VEC. You can use the Behavioral Equilibrium Exchange Rate (BEER) approach. Comment results.

II. Introduction

Volatility Analysis

Before going deep into Volatility Analysis, Let's first get grasp the importance of volatility in stock price prediction. We have already discussed definition of it in previous submission, now in this submission we will focus on practical implementation of it.

Volatility refers to the amount of uncertainty or risk related to the size of changes in a security's value. A higher volatility means that a security's value can potentially be spread out over a larger range of values. This means that the price of the security can change dramatically over a short time period in either direction. A lower volatility means that a security's value does not fluctuate dramatically, and tends to be steadier. Volatility is often calculated using variance and standard deviation. The standard deviation is the square root of the variance.

It is useful to have means to compute this volatility at any instant, to analyze how it varies over time and possibly, to forecast its future values. We have already discussed ARIMA models in previous submission, in this submission we'll give attention on GARCH Models.

The generalized autoregressive conditional heteroskedasticity (GARCH) process is an econometric term developed in 1982 by Robert F. Engle, an economist and 2003 winner of the Nobel Memorial Prize for Economics. Heteroskedasticity describes the irregular pattern of variation of an error term or variable, in a statistical model. Essentially, where there is heteroskedasticity, observations do not conform to a linear pattern. Instead, they tend to cluster. GARCH is a statistical model that can be used to analyze a number of different types of financial data.

Just like ARCH(p) is AR(p) applied to the variance of a time series, GARCH(p, q) is an ARMA(p,q) model applied to the variance of a time series. The AR(p) models the variance of the residuals (squared errors) or simply our time series squared. The MA(q) portion models the variance of the process.

Following are majorly types of GARCH used in industry:

- GARCH-M model – It addresses the limitations of standard ARCH / GARCH models which do not allow the realization of the conditional variance process to affect the conditional expectation. An asset that is systematically riskier than another must offer a higher return to be invested in.

- TGARCH – It is volatility model commonly used to handle leverage effects, like considering the fact that negative shocks have larger (positive) impacts on volatility than positive shocks.

- EGARCH - It employs a logarithmic transformation to ensure positive variances (in all the other models, all the estimated coefficients need to be positive, which is not the case here) as well as working in the levels of the residuals (not their squares) and additionally, standardizing by their standard deviation.

- IGARCH – Integrated GARCH is a model where a shock to the conditional variance (and/or the conditional expectation) never fades out. We will obtain the same by augment and integrate, i.e. ARIMA (p,,) with d>0.

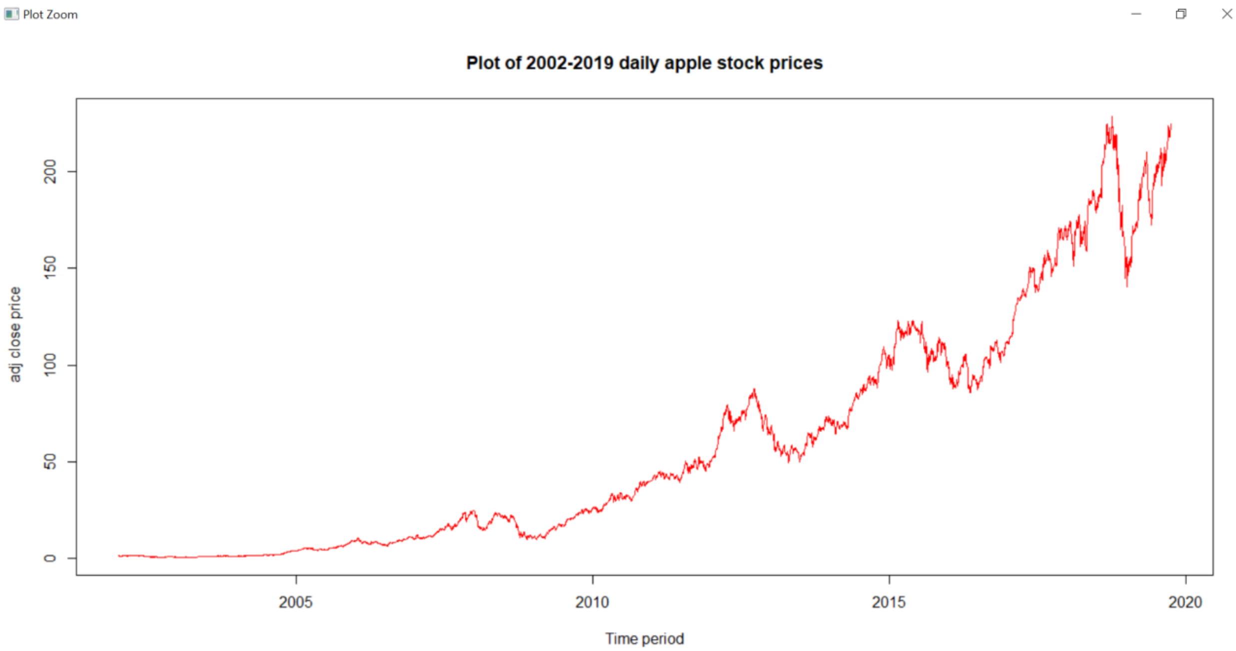
- NGARCH - This models are typically not risk-neutral, although local risk-neutrality may exist. It is more complex than the preceding GARCH models.

III. Results

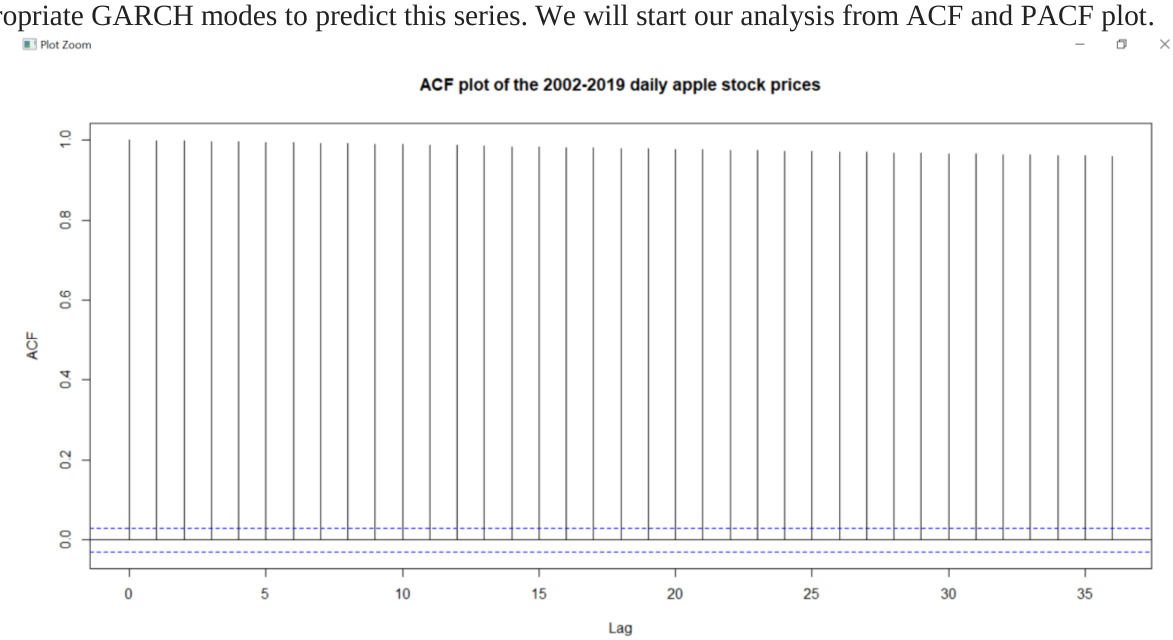
3.2.1 Volatility Analysis

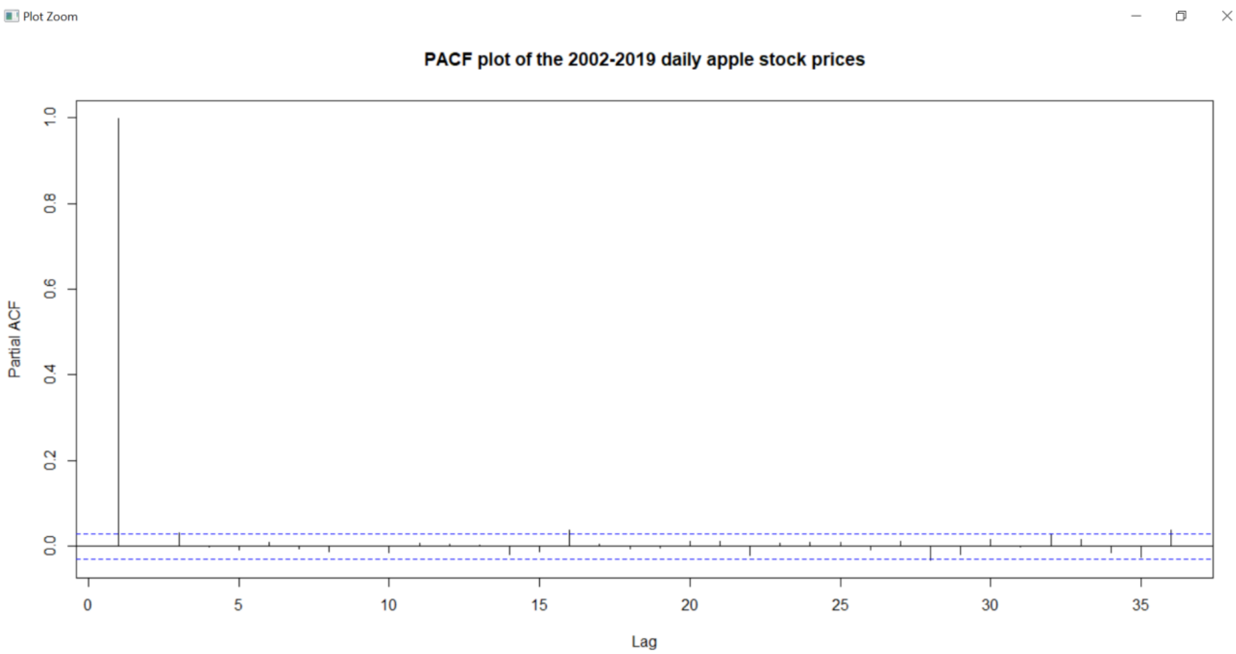
Historical data of apple stock is downloaded from yahoo finance in CSV format. We have now restricted our analysis to time period 1 Jan 2002 to 1 October 2019.

Following is time series chart of Apple data from given time period.



As it is clear from above chart that stock does not has constant volatility over given time period, hence we will appropriate GARCH modes to predict this series. We will start our analysis from ACF and PACF plot.



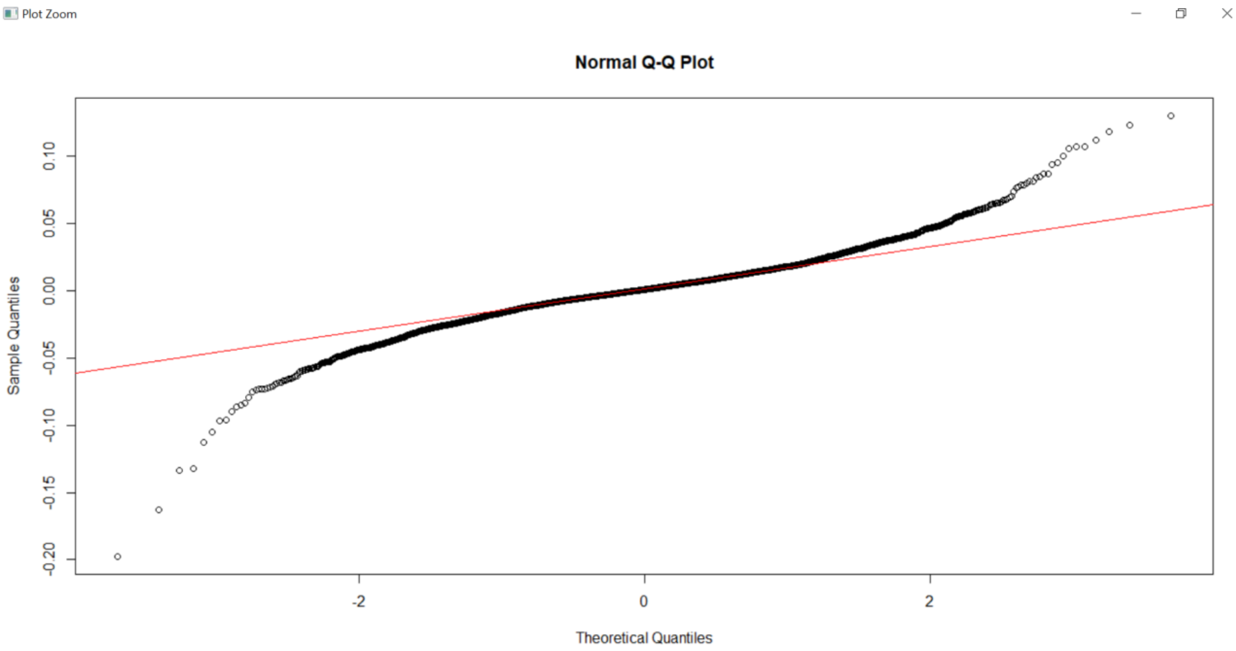


From the ACF plot, we observe that the plot decays to zero slowly, meaning the shock affects the process permanently. We can conclude that we need to perform time series analysis on the daily return (log return) of the stock prices.

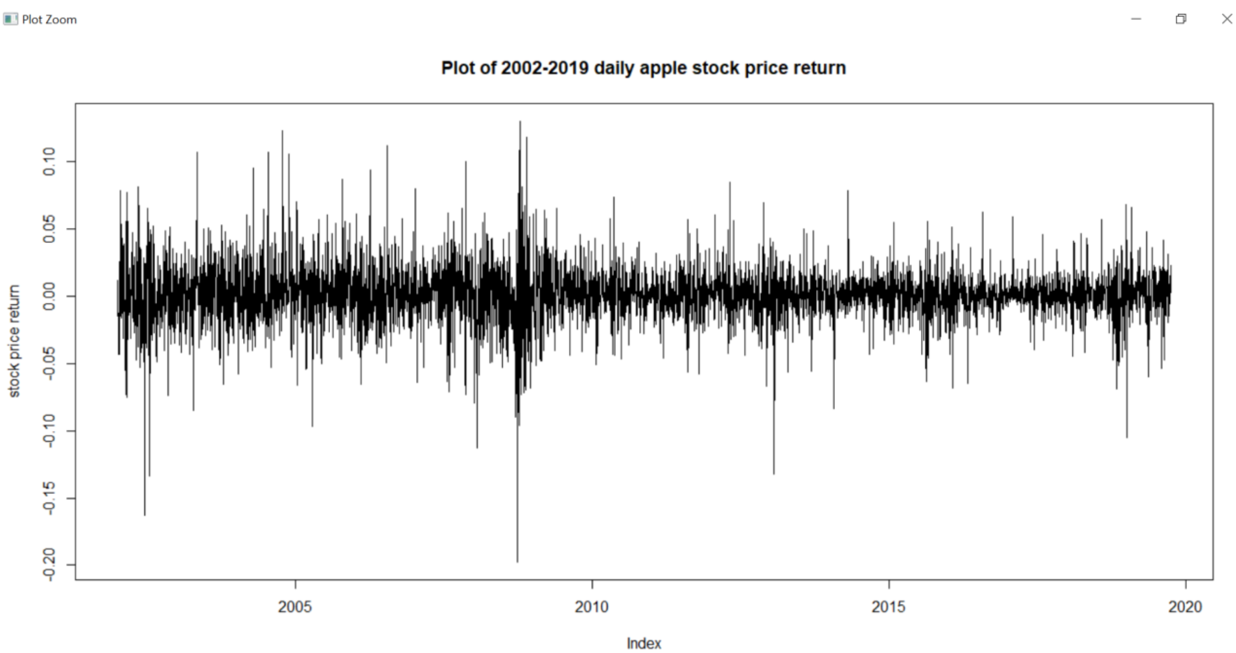
Computing the basic statistics on applying log return of the stock prices:

```
Console   Terminal ×
C:/Users/RAWAT/Downloads/ECO_GWP_2/
> basicStats(apple_rets)
                      x
nobs        4467.000000
NAs            0.000000
Minimum       -0.197470
Maximum        0.130194
1. Quartile   -0.009183
3. Quartile    0.012053
Mean           0.001129
Median         0.000952
Sum            5.041029
SE Mean        0.000323
LCL Mean       0.000494
UCL Mean       0.001763
Variance       0.000467
Stdev          0.021619
Skewness      -0.199284
Kurtosis       5.663092
> 
```

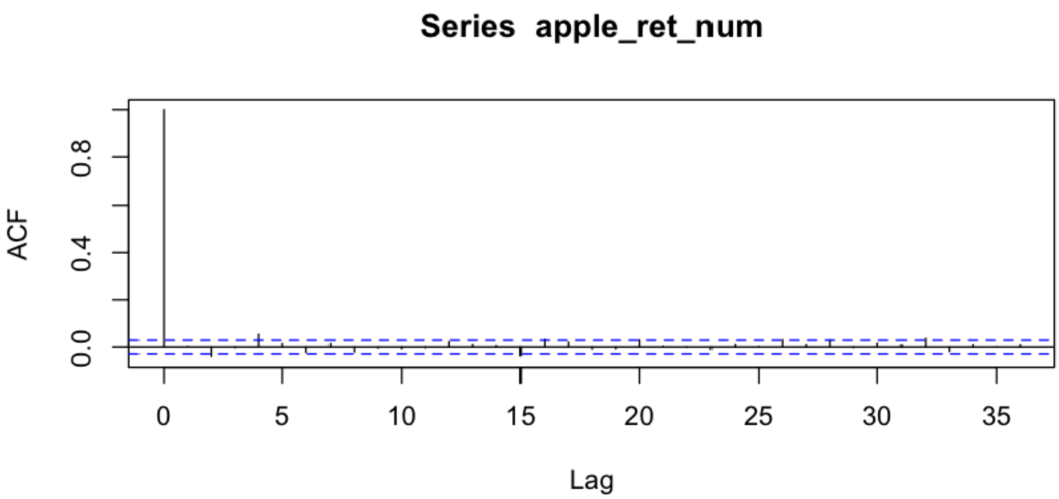
From the basic statistics of the log return of the stock prices, we observe that the mean is 0 and the distribution of log returns has large kurtosis(fat tails). We observe this further using histogram and Q-Q plot.

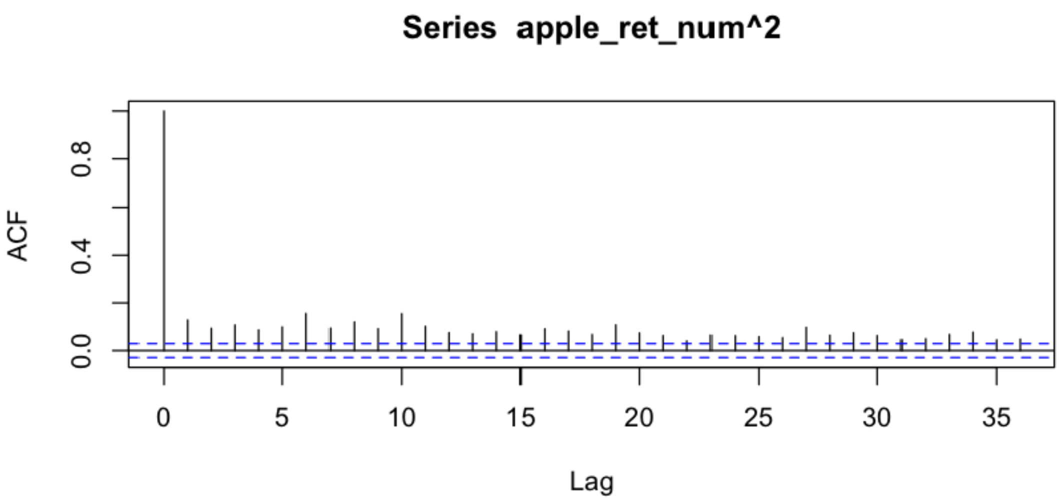


Now, we will look into analysis of log returns of stock.

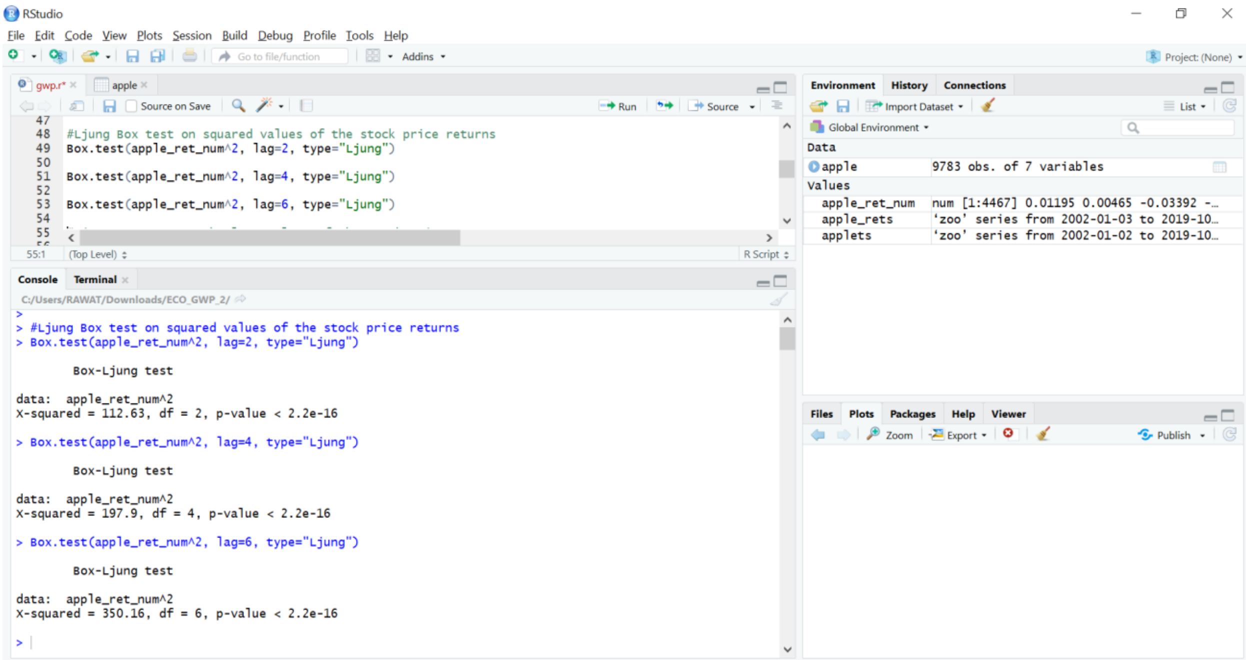


From the time plot we observe that the returns vary along the zero line with the largest log return of stock prices observed around 2002 having a value of -0.16, around 2009 having a value of -0.19 and around 2013 having a value of 0.13. The period after shows signs of volatility. During the years 2002, 2008-2009 and 2013, there is spike in volility indicating non-constant conditional volatility. The high volatility doesn't decrease as fast because the negative shocks have an effect on the process.

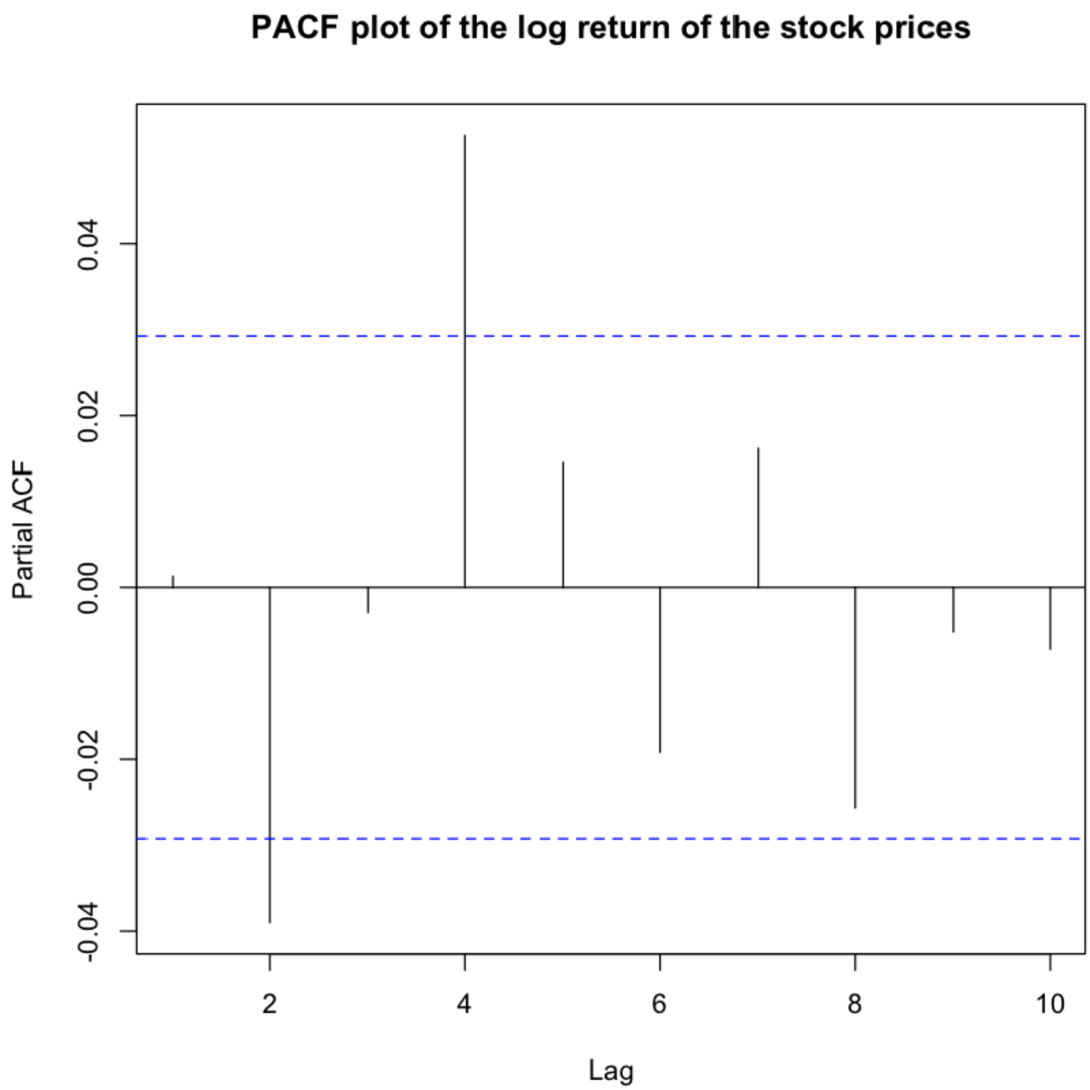


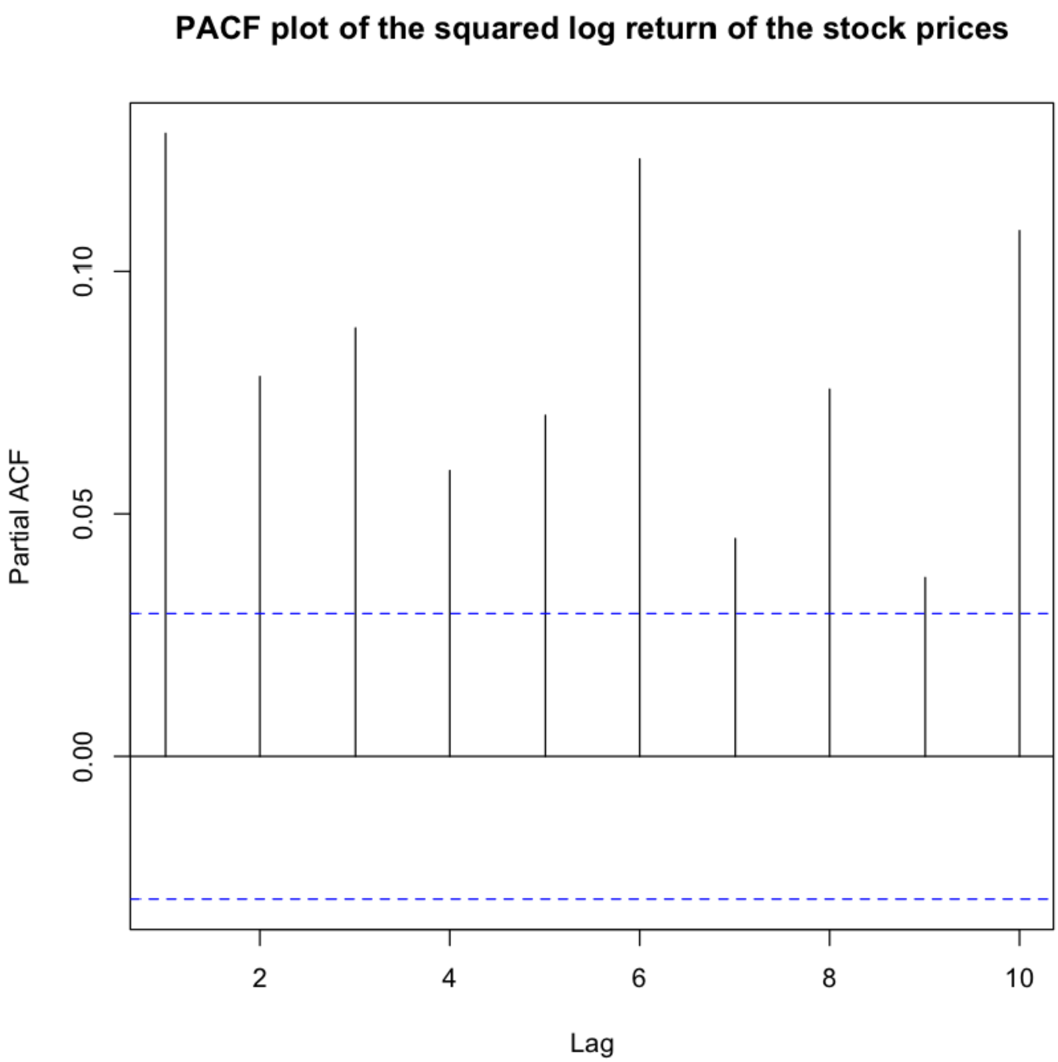


The statistics showed that the mean was constant and nearly 0. This is further confirmed by the time series plot. The ACF plot further shows that since, the log stock price returns are not correlated, the mean is constant for the time series. However, the squared stock price return values have high correlation. Thus, we may conclude that the log returns process has a strong non-linear dependence.



The null hypothesis is there exists no autocorrelation. We perform the Ljung Box's test to test the independence of the stock return prices. From all the above Ljung Box Tests, we observe that the log returns are not correlated as the p-values<<0.05 and hence we can't reject the null hypothesis of no autocorrelation. However, it shows signs of ARCH effect on the log returns of the stock prices since the Ljung Box test on the squared values of the stock price returns are significant.


PACF plot of the log return of the stock prices
Partial ACF
0.04
0.02
0.00
-0.02
-0.04
2
4
6
8
10
Lag



Now, We will start our model assumption from **Model 1**: ARMA (0,0)-GARCH (1,1) with normally distributed errors. Following is output on running mentioned model on data:

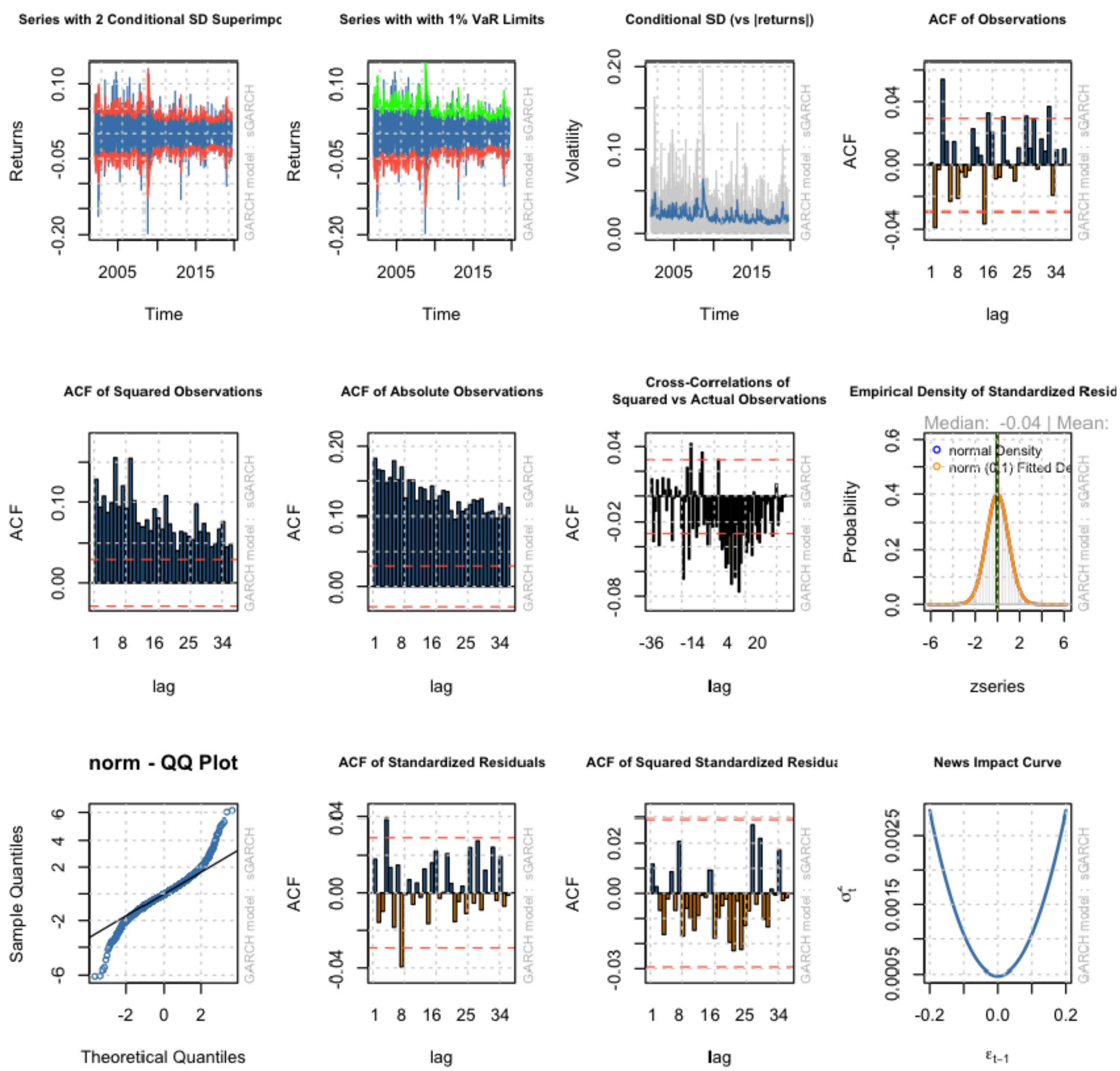
```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model    : sGARCH(1,1)
Mean Model     : ARFIMA(0,0,0)
Distribution   : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001722    0.000270   6.3731   0.0000
omega   0.000006    0.000004   1.5189   0.1288
alpha1  0.059410    0.006710   8.8536   0.0000
beta1   0.928518    0.012393  74.9247   0.0000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
```

```
mu      0.001722    0.000317  5.43085   0.0000
omega   0.000006    0.000029  0.20637   0.8365
alpha1  0.059410    0.011435  5.19556   0.0000
beta1   0.928518    0.074158 12.52082   0.0000

LogLikelihood : 11194.3

Information Criteria
------------------------------------

Akaike       -5.0102
Bayes        -5.0045
Shibata      -5.0102
Hannan-Quinn -5.0082

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      1.436  0.2307
Lag[2*(p+q)+(p+q)-1][2]     1.989  0.2646
Lag[4*(p+q)+(p+q)-1][5]     5.454  0.1206
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.6067  0.4360
Lag[2*(p+q)+(p+q)-1][5]    1.2407  0.8032
Lag[4*(p+q)+(p+q)-1][9]    2.3968  0.8527
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.2047 0.500 2.000  0.6509
ARCH Lag[5]    1.1797 1.440 1.667  0.6804
ARCH Lag[7]    1.5283 2.315 1.543  0.8157

Nyblom stability test
------------------------------------
Joint Statistic:  2.2842
Individual Statistics:
mu     0.2921
omega  0.6027
alpha1 1.1171
beta1  1.3338

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.07 1.24 1.6
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value    prob sig
Sign Bias           0.8331 0.40483
Negative Sign Bias  1.6526 0.09849   *
Positive Sign Bias  0.1742 0.86173
Joint Effect        8.5866 0.03532  **


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
```

```
1    20     177.4    9.880e-28
2    30     190.3    1.206e-25
3    40     201.4    9.282e-24
4    50     219.3    2.076e-23

Elapsed time : 7.040918
```

Fitted model: r_t = 0.0017 + a_t; a_t= sigma_te_t; sigma2t=0.00+0.059a2(t-1)+0.9285sigma^2_(t-1) AIC value = -4.9238, BIC value = -4.9172

Residual diagnostics: Ljung Box test for white noise behaviour in residuals. Since the residuals have pvalues > 0.05 and we fail to reject the null hypothesis, there is no evidence of autocorrelation in the residuals. Hence, we may conclude that the residuals behave as white noise. Test for ARCH beaviour in residuals: Looking at the standaridized squared residuals and ARCH LM Tests, the p-values > 0.05 and we fail to reject the null hypothesis hence there is no evidence of serial correlation in squared residuals. This confirms that the residuals behave as a white noise process. Looking at the output for the goodness of fit test, since the p-values>0.05, the normal distribution assumption is strongly rejected.



Now trying with new **Model 2**: ARMA(0,0)-GARCH(1,1) model with t-distribution. Following are output from model:

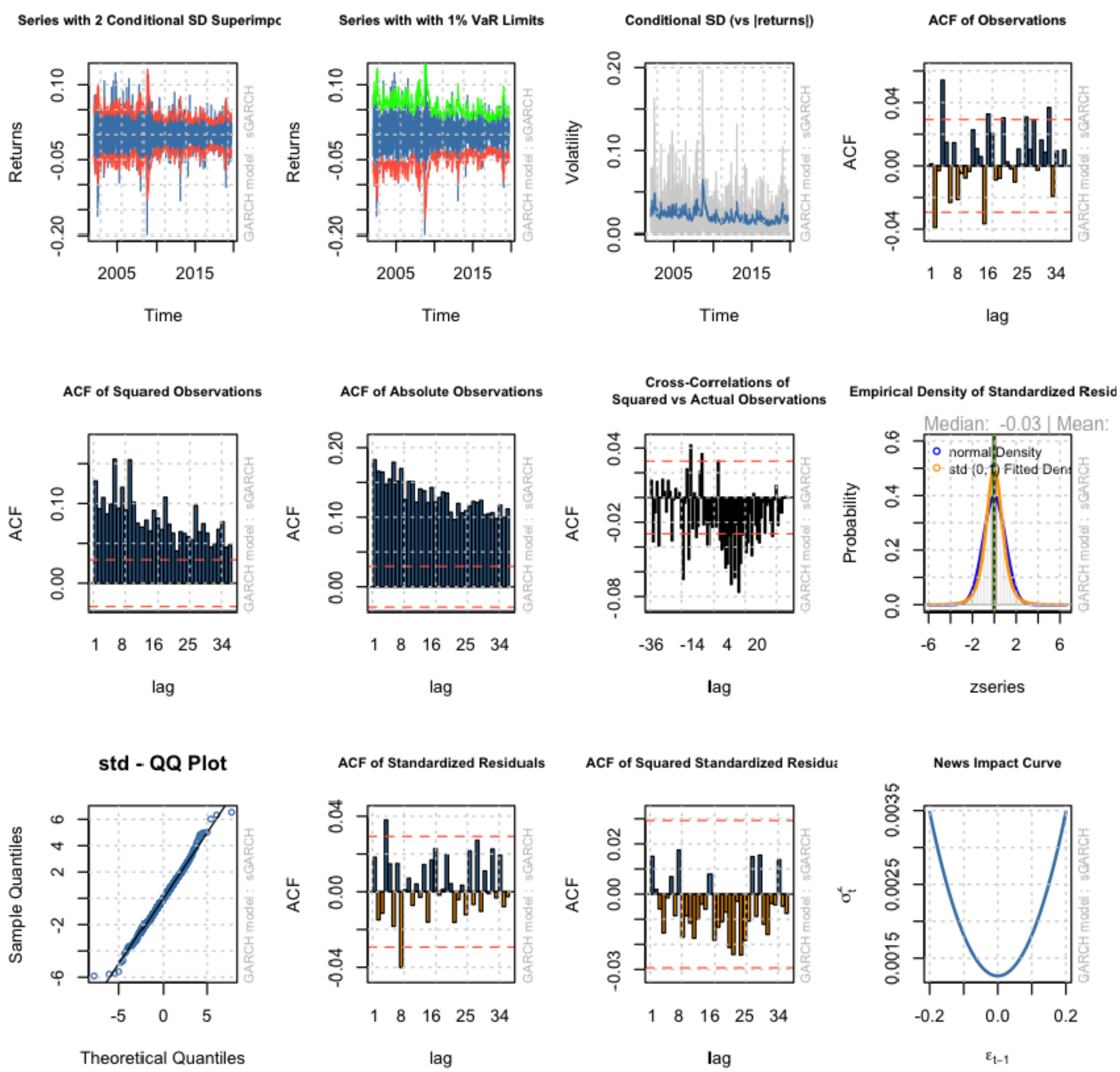
```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001412    0.000233   6.0562 0.000000
omega   0.000003    0.000001   2.4285 0.015163
alpha1  0.055449    0.006688   8.2913 0.000000
```

```
beta1   0.942675    0.007120 132.4029 0.000000
shape   4.627227    0.312897  14.7884 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001412    0.000240   5.8899 0.000000
omega   0.000003    0.000002   1.0486 0.294376
alpha1  0.055449    0.020587   2.6934 0.007072
beta1   0.942675    0.019998  47.1382 0.000000
shape   4.627227    0.385943  11.9894 0.000000

LogLikelihood : 11422.53

Information Criteria
------------------------------------

Akaike       -5.1119
Bayes        -5.1048
Shibata      -5.1119
Hannan-Quinn -5.1094

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      1.493  0.2218
Lag[2*(p+q)+(p+q)-1][2]     1.989  0.2646
Lag[4*(p+q)+(p+q)-1][5]     5.408  0.1236
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                      1.012  0.3145
Lag[2*(p+q)+(p+q)-1][5]     1.548  0.7282
Lag[4*(p+q)+(p+q)-1][9]     2.525  0.8341
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.1553 0.500 2.000  0.6935
ARCH Lag[5]    1.0159 1.440 1.667  0.7282
ARCH Lag[7]    1.3428 2.315 1.543  0.8523

Nyblom stability test
------------------------------------
Joint Statistic:  27.0266
Individual Statistics:
mu     0.1401
omega  2.8216
alpha1 1.3545
beta1  1.6617
shape  1.8909

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.28 1.47 1.88
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value    prob sig
Sign Bias           0.9348 0.34995
Negative Sign Bias  1.4189 0.15601
```

```
Positive Sign Bias  0.3970 0.69137
Joint Effect        8.7198 0.03326  **


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     27.18       0.1005
2    30     30.36       0.3962
3    40     50.40       0.1044
4    50     42.75       0.7232


Elapsed time : 1.125968
```

Fitted model: 0.0014 + at, at = stet s2t = 0.00 55+ 0.0477a2t-1 + 0.949 s2t-1 with the t distribution of 5 degrees of freedom (4.97).

The shape parameter has p-value = 0<0.05 and hence, is significant. Thus, this model could be significant and a good choice. AIC value = -5.1119 and BIC value = -5.1048 Taking a look at the R output, in particular the weighted Ljung-Box test on squared residuals, there is no evidence of serial correlation as the p-values>0.05 and hence the null hypothesis of serial correlation can be rejected and we may conclude that the residuals behave as a white noise process. Looking at the Goodness-of-fit test, we observe that for group 20 and group 40, the p-value<0.05 and hence we can reject the null hypothesis that this model is adequate for this process.

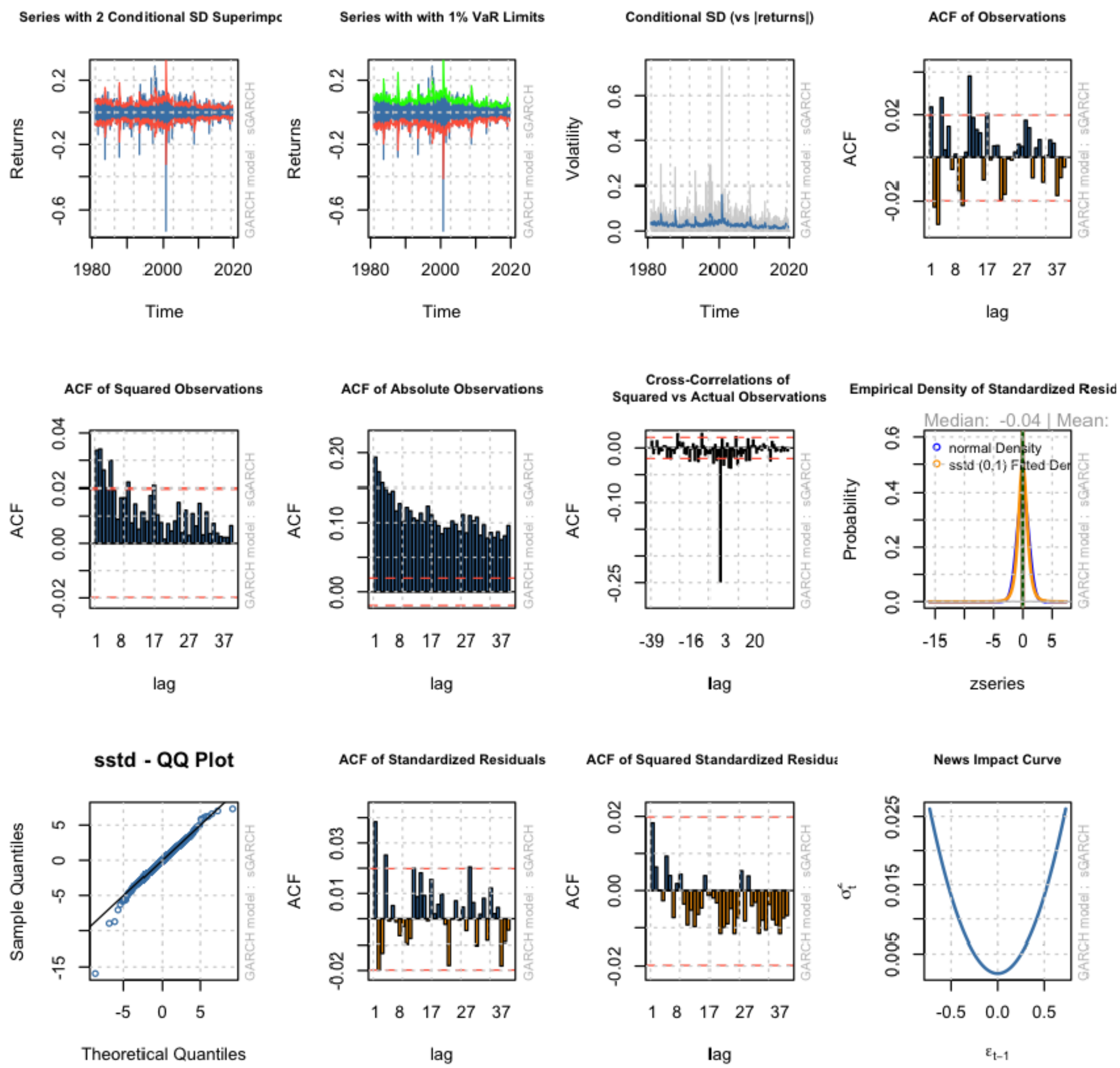


Now trying with **Model 3**: ARMA(0,0)-GARCH(1,1) model with skewed t-distribution. Following is output from model:

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001500    0.000258   5.8230  0.00000
omega   0.000003    0.000001   2.4316  0.01503
alpha1  0.055842    0.006735   8.2913  0.00000
```

```
beta1   0.942367    0.007160 131.6154  0.00000
skew    1.016172    0.020654  49.2007  0.00000
shape   4.616734    0.312116  14.7917  0.00000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001500    0.000273   5.5028 0.000000
omega   0.000003    0.000002   1.0519 0.292855
alpha1  0.055842    0.020748   2.6915 0.007114
beta1   0.942367    0.020078  46.9362 0.000000
skew    1.016172    0.020989  48.4144 0.000000
shape   4.616734    0.386390  11.9484 0.000000

LogLikelihood : 11422.84

Information Criteria
------------------------------------

Akaike       -5.1116
Bayes        -5.1030
Shibata      -5.1116
Hannan-Quinn -5.1086

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      1.502  0.2203
Lag[2*(p+q)+(p+q)-1][2]     1.997  0.2632
Lag[4*(p+q)+(p+q)-1][5]     5.404  0.1238
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.9999  0.3173
Lag[2*(p+q)+(p+q)-1][5]    1.5378  0.7306
Lag[4*(p+q)+(p+q)-1][9]    2.5095  0.8363
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.1608 0.500 2.000  0.6884
ARCH Lag[5]    1.0186 1.440 1.667  0.7274
ARCH Lag[7]    1.3411 2.315 1.543  0.8526

Nyblom stability test
------------------------------------
Joint Statistic:  26.8193
Individual Statistics:
mu     0.1414
omega  2.7525
alpha1 1.3366
beta1  1.6439
skew   0.2026
shape  1.8977

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.49 1.68 2.12
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
------------------------------------
```

```
                       t-value    prob sig
Sign Bias               0.9577 0.33827    
Negative Sign Bias      1.3861 0.16579    
Positive Sign Bias      0.3945 0.69323    
Joint Effect            8.6770 0.03391  **


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     27.17       0.1007
2    30     29.53       0.4378
3    40     45.87       0.2086
4    50     47.29       0.5425


Elapsed time : 1.03590
```



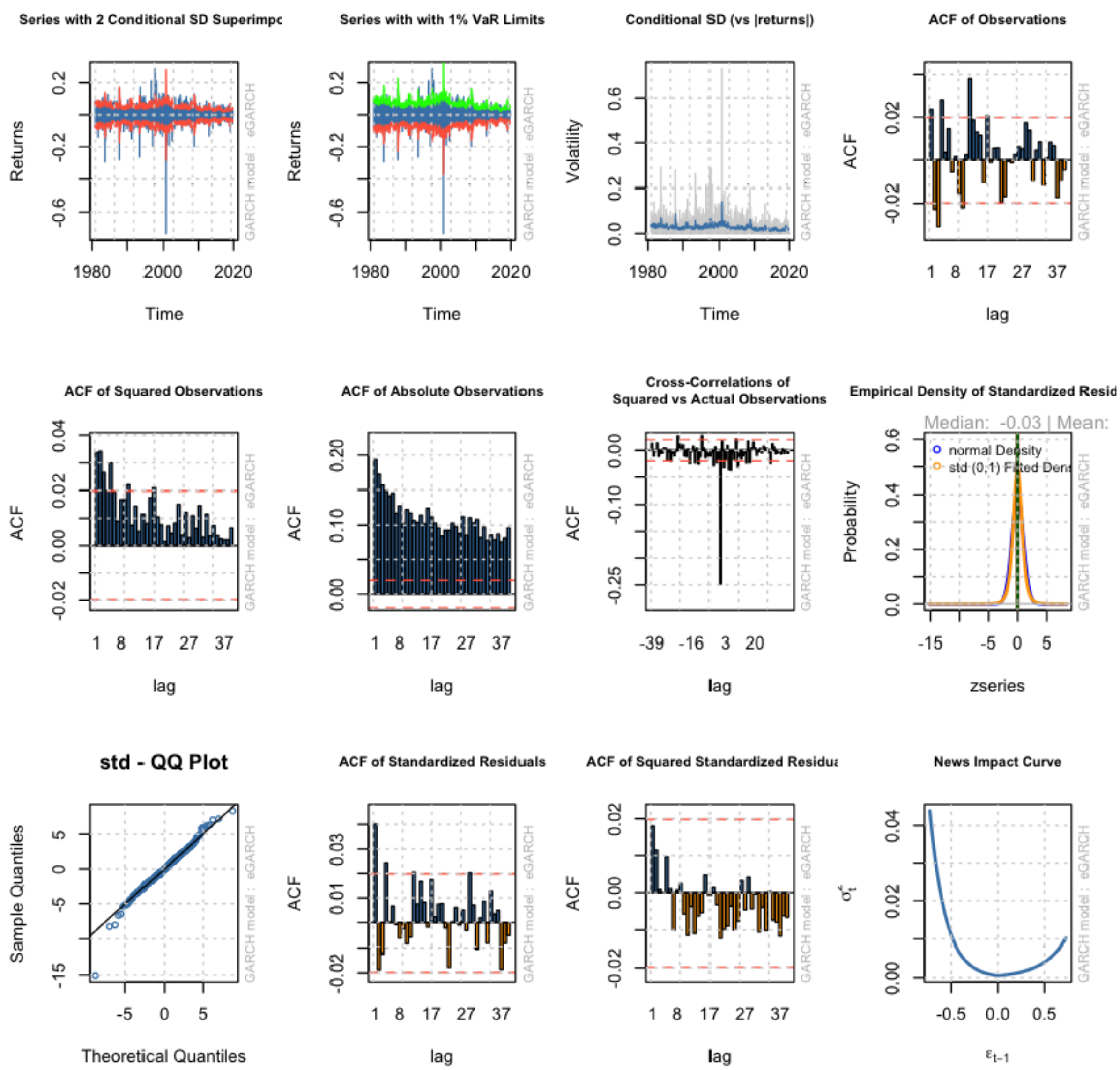
Fitted model: rt=0.0012 + at, at=stet s2t = 0.000002 + 0.0444a2t-1 + 0.95s2t-1, with the t-distribution of 5 degrees of freedom (4.92).

Looking at the output, we observe that the skewness value has p-value = 0<0.05 and hence, is significant. Since, the skew value>1(1.02), it indicates that the t-distribution is skewed to the right. The shape value has p-value=0<.05 and is significant. We might be interested in this model for the process looking further into the output. AIC value = -4.6010 and BIC value = -4.5966 Residual diagnostics: Ljung Box test for white noise behaviour in residuals. Since the residuals have p-values>0.05 and we fail to reject the null hypothesis, there is no evidence of autocorrelation in the residuals.Hence, we may conclude that the residuals behave as hite noise. Test for ARCH beaviour in residuals: Looking at the standaridized squared residuals and ARCH LM Tests, the p-values>0.05 and we fail to reject the null hypothesis hence there is no evidence of serial correlation in squared residuals. This confirms that the residuals behave as a white noise process. Looking at the output for the goodness of fit test, since the p-values>0.05, the null hypothesis can't be rejected and hence this model is a good fit.

Now trying with **Model 4**: Fit ARMA(0,0)-eGARCH(1,1) model with t-distribution. Following is output from the model:

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : eGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001248    0.000212   5.8955        0
omega  -0.131642    0.009992 -13.1744        0
alpha1 -0.061422    0.009910  -6.1983        0
beta1   0.983296    0.001264 778.0512        0
gamma1  0.161226    0.017123   9.4157        0
shape   4.851953    0.353652  13.7196        0

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.001248    0.000198    6.2982        0
omega  -0.131642    0.005047  -26.0824        0
alpha1 -0.061422    0.010327   -5.9480        0
beta1   0.983296    0.000674 1457.8514        0
gamma1  0.161226    0.019509    8.2643        0
shape   4.851953    0.370205   13.1061        0

LogLikelihood : 11459.9

Information Criteria
------------------------------------

Akaike       -5.1282
Bayes        -5.1196
Shibata      -5.1282
Hannan-Quinn -5.1252

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      3.346 0.06737
```

```
Lag[2*(p+q)+(p+q)-1][2]     3.661 0.09301
Lag[4*(p+q)+(p+q)-1][5]     6.638 0.06341
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.2736  0.6009
Lag[2*(p+q)+(p+q)-1][5]    1.2341  0.8048
Lag[4*(p+q)+(p+q)-1][9]    1.7978  0.9277
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.9811 0.500 2.000  0.3219
ARCH Lag[5]    1.0719 1.440 1.667  0.7117
ARCH Lag[7]    1.3222 2.315 1.543  0.8562

Nyblom stability test
------------------------------------
Joint Statistic:  4.1697
Individual Statistics:
mu     0.67978
omega  2.94294
alpha1 0.66173
beta1  2.89550
gamma1 0.09944
shape  1.09817

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.49 1.68 2.12
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias          1.28694 0.1982
Negative Sign Bias 0.58348 0.5596
Positive Sign Bias 0.02955 0.9764
Joint Effect       2.40405 0.4929


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     22.70       0.2507
2    30     25.98       0.6265
3    40     48.40       0.1439
4    50     48.17       0.5068


Elapsed time : 1.046004
```



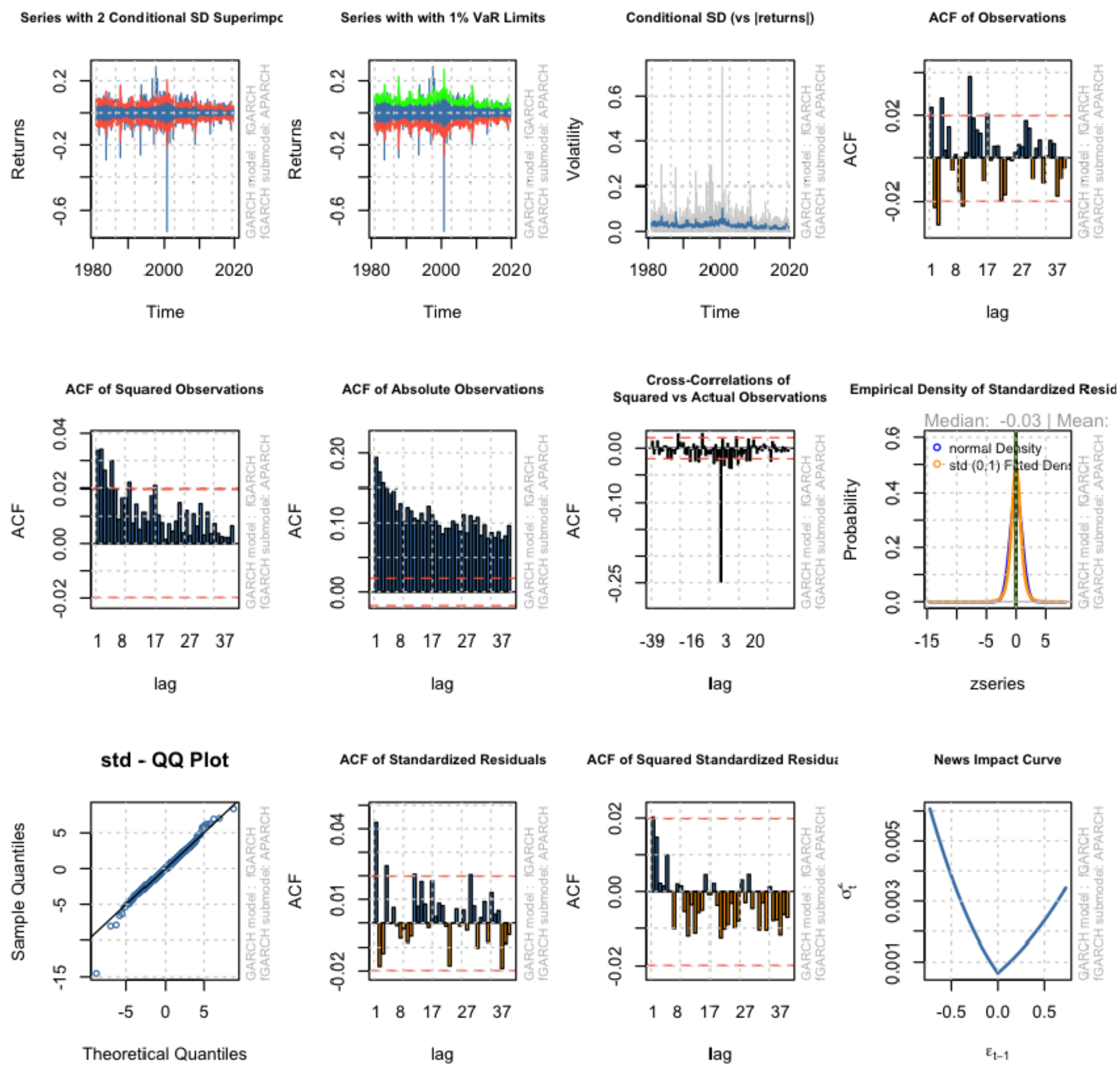
The shape parameter is significant as the p-value < 0.05, indicating that the t-distibution is a good choice. AIC value = -5.1282 and BIC value = -5.1196 Residual diagnostics: All the p-values for the Ljung Box Test of residuals are > 0.05, thus indicating that there is no evidence of serial correlation in the squared residuals and hence, they behave as white noise process. Looking at the test for goodness-of-fit, since all the p-values > 0.05, we cant reject the null hypothesis, and hence we may conclude that the Egarch model with the t-distribution is a good choice.

Again trying with new **Model 5**: Fit ARMA(0,0)-fGARCH(1,1) model with t-distribution. Following are output from model:

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : fGARCH(1,1)
fGARCH Sub-Model        : APARCH
```

```
Mean Model      : ARFIMA(0,0,0)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001237    0.000214   5.7718  0.00000
omega   0.000396    0.000248   1.5984  0.10996
alpha1  0.091987    0.013692   6.7185  0.00000
beta1   0.915802    0.013934  65.7266  0.00000
eta11   0.399875    0.066989   5.9693  0.00000
lambda  0.972900    0.142645   6.8204  0.00000
shape   4.848757    0.348962  13.8948  0.00000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001237    0.000204   6.0749 0.000000
omega   0.000396    0.000235   1.6852 0.091957
alpha1  0.091987    0.022540   4.0812 0.000045
beta1   0.915802    0.023968  38.2088 0.000000
eta11   0.399875    0.066885   5.9785 0.000000
lambda  0.972900    0.147839   6.5808 0.000000
shape   4.848757    0.360459  13.4516 0.000000

LogLikelihood : 11459.53

Information Criteria
------------------------------------

Akaike       -5.1276
Bayes        -5.1176
Shibata      -5.1276
Hannan-Quinn -5.1241

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      3.810 0.05095
Lag[2*(p+q)+(p+q)-1][2]     4.059 0.07271
Lag[4*(p+q)+(p+q)-1][5]     6.912 0.05446
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.3884  0.5332
Lag[2*(p+q)+(p+q)-1][5]    1.6918  0.6928
Lag[4*(p+q)+(p+q)-1][9]    2.2640  0.8713
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]     1.015 0.500 2.000  0.3138
ARCH Lag[5]     1.067 1.440 1.667  0.7131
ARCH Lag[7]     1.291 2.315 1.543  0.8621

Nyblom stability test
------------------------------------
Joint Statistic:  4.9206
Individual Statistics:
mu     0.7577
omega  3.0860
```

```
alpha1 2.6832
beta1  3.1435
eta11  1.2729
lambda 3.2036
shape  2.2319

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.69 1.9 2.35
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           1.1094 0.2673
Negative Sign Bias  0.7491 0.4539
Positive Sign Bias  0.1973 0.8436
Joint Effect        2.0295 0.5663


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     19.98       0.3958
2    30     27.77       0.5303
3    40     39.07       0.4669
4    50     45.55       0.6139


Elapsed time : 13.7758
```



The shape parameter is significant as the p-value < 0.05, indicating that the t-distibution is a good choice. AIC value = -5.1276 and BIC value = -5.1176.

Residual diagnostics: All the p-values for the Ljung Box Test of residuals are > 0.05, thus indicating that there is no evidence of serial correlation in the squared residuals and hence, they behave as white noise process.

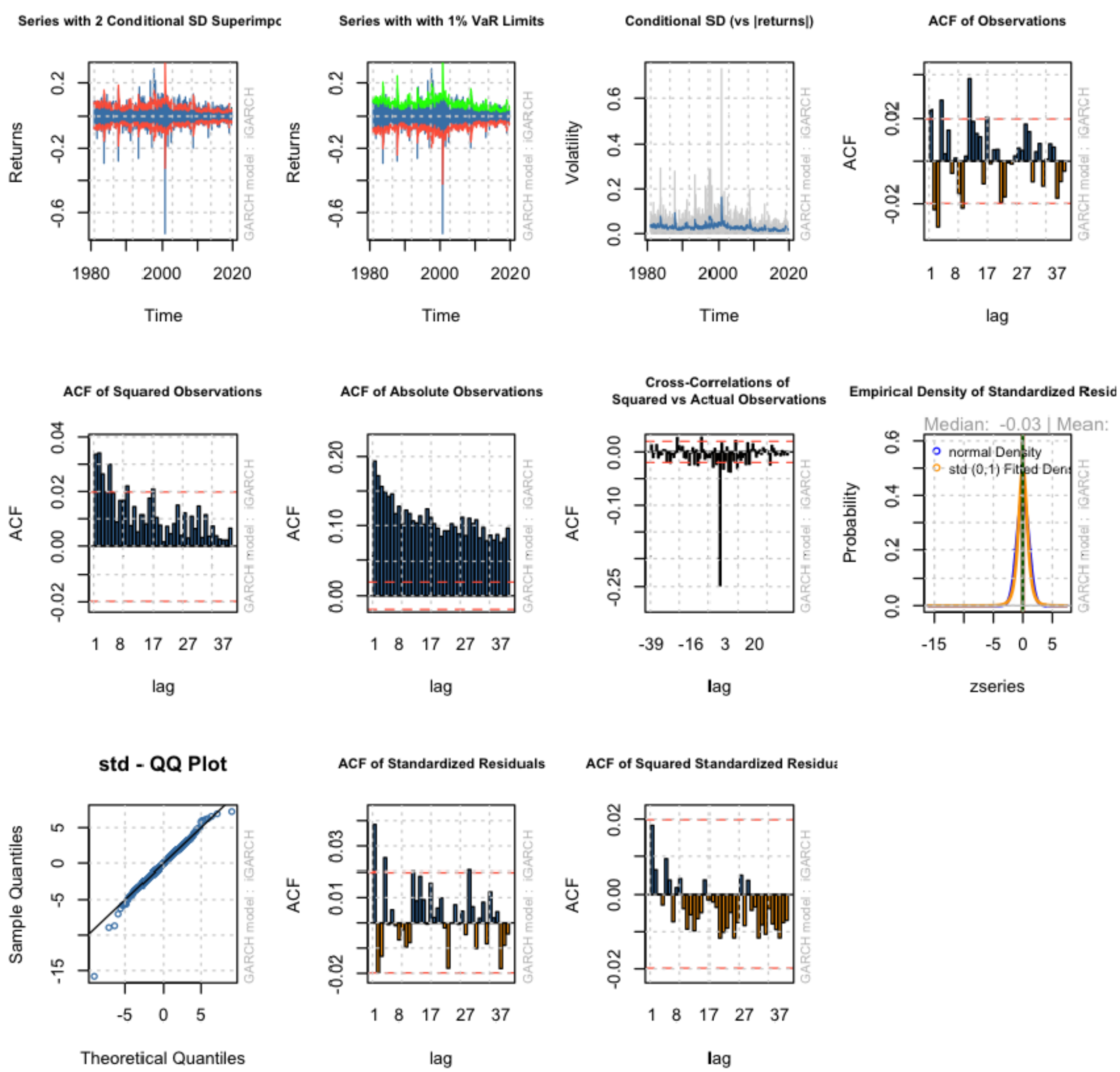
Looking at the test for goodness-of-fit, since all the p-values > 0.05, we cant reject the null hypothesis, and hence we may conclude that the fgarch model with the t-distribution is a good choice.

Now coming to our last trial **Model: 5**, Igarch model. Following is output from it:

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : iGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : std
```

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001407    0.000233   6.0522  0.00000
omega   0.000002    0.000001   1.5471  0.12184
alpha1  0.056209    0.008288   6.7822  0.00000
beta1   0.943791          NA       NA       NA
shape   4.536285    0.240899  18.8307  0.00000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001407    0.000234  6.00391 0.000000
omega   0.000002    0.000003  0.65635 0.511600
alpha1  0.056209    0.024455  2.29842 0.021538
beta1   0.943791          NA       NA       NA
shape   4.536285    0.287919 15.75541 0.000000

LogLikelihood : 11422.39

Information Criteria
------------------------------------

Akaike       -5.1123
Bayes        -5.1066
Shibata      -5.1123
Hannan-Quinn -5.1103

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      1.508  0.2194
Lag[2*(p+q)+(p+q)-1][2]     1.997  0.2633
Lag[4*(p+q)+(p+q)-1][5]     5.400  0.1241
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                      1.026  0.3110
Lag[2*(p+q)+(p+q)-1][5]     1.549  0.7278
Lag[4*(p+q)+(p+q)-1][9]     2.490  0.8392
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.1572 0.500 2.000  0.6918
ARCH Lag[5]    0.9942 1.440 1.667  0.7347
ARCH Lag[7]    1.3140 2.315 1.543  0.8578

Nyblom stability test
------------------------------------
Joint Statistic:  18.2753
Individual Statistics:
mu     0.1409
omega  3.6723
alpha1 1.1641
shape  1.7534

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.07 1.24 1.6
Individual Statistic:   0.35 0.47 0.75
```

```
Sign Bias Test
------------------------------------
                     t-value    prob sig
Sign Bias             0.9529 0.34071
Negative Sign Bias    1.3494 0.17726
Positive Sign Bias    0.4452 0.65618
Joint Effect          8.6379 0.03451  **


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     27.05       0.1036
2    30     33.02       0.2769
3    40     49.02       0.1305
4    50     47.97       0.5151


Elapsed time : 0.4906211
```

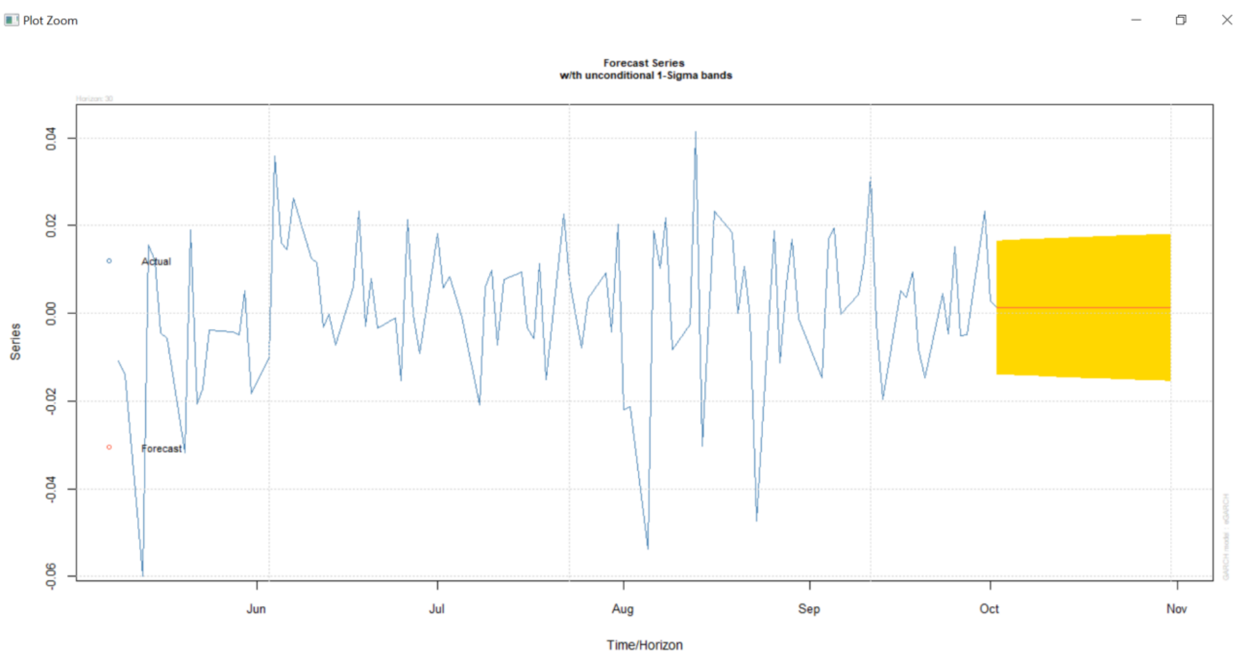


Comparing results with all the possible models that are a good choice for our model, our final model is selected based on the lowest BIC value.

Model name BIC value Model 1: -5.0045; Model 2: -5.1048; Model 3: -4.5966; Model 4: -5.1196; Model 5: -5.1176; Model 6: -4.5980;

From the above table, we observe that the *best model is Model 4 using Egarch with t-distribution as the model has the lowest BIC value of -5.1196.*

Now let's forecast the t+1 return (for next 30 days) usign the Egarch model with t-distribution. We have used "ugarchforecast" function for prediction.



3.2.2 Multivariate Analysis

Calculating Equilibrium FX

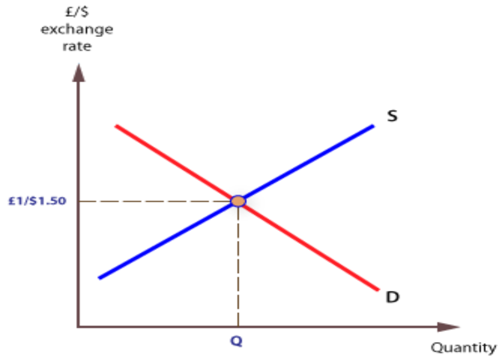
Currencies are bought and sold, just like other commodities, in markets called foreign exchange markets. A ***(foreign) exchange rate*** is the rate at which one currency is exchanged for another. Thus, an exchange rate can be regarded as the price of one currency in terms of another. An exchange rate is a ratio between two currencies.

Eg. If 5 UK pounds or 5 US dollars buy Indian goods worth Rs. 435 and Rs. 355 then pound-rupee or dollar-rupee, then exchange rate becomes Rs. 87 = £1 or Rs. 71 = $1, respectively.

Equilibrium exchange rate (Equilibrium FX)- The exchange rate at which the supply for a currency meets the demand of the same currency. It is the rate which equates demand and supply for a particular currency against another currency. Hence, equilibrium is achieved when a currency's demand is equal to its supply.

The demand for currency
The demand for currencies is derived from the demand for a country's exports, and from spectaculars looking to make a profit on changes in currency values.

The supply of currency
The supply of a currency is determined by the domestic demand for imports from abroad.



$$\mathbf{e_t = \beta' Z_t + \theta' T_t + \varepsilon_t}$$

where e_t is the exchange rate in time t, Z is a vector of economic fundamentals that are expected to influence the exchange rate in the medium to long term, T is a vector of transitory factors (including current and lagged variables as well as dynamic effects from the fundamentals, Z) which have an impact on exchange rate in the short term, ε_t is a random disturbance and β and θ are vectors of coefficients

There are ***three*** different types of equilibrium exchange rate concepts which differ according to the time horizon to which they apply:

- **Long-Run Equilibrium**
- **Medium-Run Equilibrium**
- **Short-Run Equilibrium**

1. Indicate economic theories and models for calculating equilibrium FX
   The difference in different models lie in the treatment of dynamics and the time frame they concentrate on- Model based, estimation based, monetary -based models.

PPP	Balassa Samuelson	Monetary Models	CHEERs	ITMEERs	BEERs	FEERs	DEERs	APEERs	PEERs	NATREX	SVARs	DSGE
Purchasing Power Parity	BalassaSamuelson	Monetary and Portfolio balance models	Capital Enhanced Equilibrium Exchange Rates	Intermediate Term Model Based Equilibrium Exchange Rates	Behavioural Equilibrium Exchange Rates	Fundamental Equilibrium Exchange Rates	Desired Equilibrium Exchange Rates	Atheoretical Permanent Equilibrium Exchange Rates	Permanent Equilibrium Exchange Rates	Natural Real Exchange Rates	Structural Vector Auto Regression	Dynamic Stochastic General Equilibrium models
Constant Equilibrium Exchange Rate	PPP for tradable goods. Productivity differentials between traded and non-traded goods	PPP in long run (or short run) plus demand for money	PPP plus nominal UIP without risk premia	Nominal UIP including a risk premia plus expected future movements in real exchange rates determined by fundamentals	Real UIP with a risk premia and/or expected future movements in real exchange rates determined by fundamentals	Real exchange rate compatible with both internal and external balance. Flow not full stock equilibrium	As with FEERs, but the definition of external balance based on optimal policy	None	As BEERs	As with FEERs, but with the assumption of portfolio balance (so domestic real interest rate is equal to the world rate)	Real exchange rate affected by supply and demand (but not nominal) shocks in the long run	Models designed to explore movements in real and/or nominal exchange rates in response to shocks
Long Run	Long Run	Short Run	Short run (forecast)	Short run (forecast)	Short run (forecast)	Medium run	Medium run	Medium / Long run	Medium / Long run	Long run	Short (and long) run	Short (and long) run
Stationary	Non-stationary	Non- stationary	Stationary, with emphasis on speed of convergence	None	Non-stationary	Non-stationary	Non-stationary	Non- stationary (extract permanent component)	Non- stationary (extract permanent component)	Non- stationary	As with theoretical	As with theoretical
Real or nominal	Real	Nominal	Nominal	Future change in the Nominal	Real	Real Effective	Real Effective	Real	Real	Real	Change in the Real	Change relative to long-run steady state
Test for stationarity	Direct	Direct	Direct	Direct	Direct	Underlying Balance	Underlying Balance	Direct	Direct	Direct	Direct	Simulation

<u>Balance of Payments (BOP) Approach Model-</u> Under the BOP approach, the domestic price of a foreign currency is determined just like the price of any commodity, i.e., by the intersection of the market demand and supply curves for that foreign currency. The BOP approach models the demand and supply for foreign exchange as determined by the flows of currency created by international transactions. According to the BOP theory of exchange rates, the supply and demand for a currency arise from the flows related to the BOP, that is, trade in goods and services, portfolio investment, and direct investment. Equilibrium exchange rates are determined when the BOP is in equilibrium

1.A. The BOP

The BOP tracks all financial flows crossing the borders of a country during a given period. For example, an import creates a negative financial inflow (positive financial outflow) for the country, whereas an export creates a positive financial inflow (negative financial outflow). The convention is to treat all financial inflows as a credit to the balance of payments. A BOP is not an income statement or a balance sheet but rather a cash balance of the country relative to the rest of the world. As long as the country is not bankrupt, the balance of all financial flows must be equilibrated, like any cash balance. In other words, the final balance must be zero.

1.B The Absorption Approach to the Balance of Trade

The absorption approach to the balance of trade studies how domestic spending on domestic goods changes relative to domestic output. That is, the CA component is viewed as the difference between what the economy produces and what it consumes, or absorbs, for domestic purposes.

1.D The Monetary Approach to the BOP

The BOP analysis becomes more complex when the capital account is taken into consideration. The monetary approach to the BOP incorporates international financial flows to the model. The monetary approach views any BOP disequilibrium as a monetary disequilibrium, which is manifested through the capital account.

II. Asset Approach

Under this view, exchange rates are asset prices that adjust to equilibrate international trade in financial assets. Exchange rates are relative prices between two currencies and these relative prices are determined by the desire of residents to hold domestic and foreign financial assets. Like other asset prices, exchange rates are determined by expectations about the future. Therefore, past or present trade flows cannot influence exchange rates to the extent that they have already been expected. This approach, which treats currencies as assets, is appropriately called the asset approach.

2.A Monetary Approach

The monetary asset assumes a high degree of capital mobility between assets denominated in different currencies. The difficult part of this approach is to specify the domestic and foreign assets to be included in the portfolio of a domestic resident. Since exchange rates are relative prices between two currencies, a simple model is to consider domestic money and foreign money. This simple asset model is called the monetary approach model. Eg. PPP Model

2.B Portfolio-Balance Approach

The monetary approach emphasizes the monetary phenomenon; other financial assets are excluded from consideration. The portfolio-balance approach considers other assets. Under this approach, investors compose their portfolios with money and bonds (domestic and foreign), but the assets may not be perfect substitutes. Portfolio decisions are affected by the relative expected rates of returns, adjusted by risk, of the assets. Then, the supply and demand of bonds, interest rates, income and wealth play a role in the determination of Exchange Rate.

III. Structural Models: Evidence

Many economists suggested that structural models were mis specified, because of the so-called structural change problem. When economic models are used, in general, they assume that the parameters are constant. Changes in the structure of the economy, however, can cause the parameters to also change. even though regression-based structural models tend to perform very poorly, the variables used in structural models have power to explain the movement of exchange rates.

IV. The Martingale-Random Walk Model

The random walk model assumes that price changes are independently and identically distributed. The martingale model only requires that price changes are uncorrelated. Technically, when a random walk is specified, we need to consider not only the equilibrium expected value, but also the entire distribution. This model has a testable implication: changes in exchange rates are serially uncorrelated, and, then, they appear random. That is, firms should not spend any resources forecasting exchange rates based on past history

2. Indicate macroeconomic variables used for calculating equilibrium FX.

   Exchange rate(XR) fluctuation is defined as the risk associated with unpredicted movements in exchange rate. Macroeconomic variables such as interest rate, inflation rate, the balance of payments, tax rate etc influence the XR randomly. These macroeconomic variables are unstable and volatile depending on the state of the economy prevailing in their countries. Interest rates, inflation, and exchange rates are all highly correlated.

   -Balance of Payments: The balance of payments summarizes the flow of economic transactions between residents of a given country and the residents of other countries during a certain period of time. Balance of payments represents the demand and supply of foreign exchange which ultimately determine the value of the currency. When the balance of payments of a country is continuously deficit, it implies that the demand for the currency of the country is lesser than its supply. Therefore, its value in the market declines. If the balance of payments is surplus continuously, it shows that the demand for the currency in the exchange market is higher than its supply and therefore the currency gains value.
   -Relative Inflation Rates: A nation running a relatively high rate of inflation will find its currency declining in value relative to the currencies of countries with lower inflation rates. This is also usually accompanied by higher interest rates. Alternatively, a country with a consistently lower inflation rate exhibits a rising currency value, as its purchasing power increases relative to other currencies.
   Relative Interest rates: Higher interest rates attract foreign capital and cause the exchange rate to rise. The

impact of higher interest rates is mitigated, however, if inflation in the country is much higher than in others, or if additional factors serve to drive the currency down. The opposite relationship exists for decreasing interest rates – that is, lower interest rates tend to decrease exchange rates. interest rates mentioned here are real interest rates. The real interest rate equals the nominal or actual interest rate minus the rate of inflation.

-Current Account Deficits: The current account is the balance of trade between a country and its trading partners, reflecting all payments between countries for goods, services, interest, and dividends. A deficit in the current account shows the country is spending more on foreign trade than it is earning, and that it is borrowing capital from foreign sources to make up the deficit. In other words, the country requires more foreign currency than it receives through sales of exports, and it supplies more of its own currency than foreigners demand for its products. The excess demand for foreign currency lowers the country's exchange rate until domestic goods and services are cheap enough for foreigners, and foreign assets are too expensive to generate sales for domestic interests.

-Public Debt: Countries will engage in large-scale deficit financing to pay for public sector projects and governmental funding. While such activity stimulates the domestic economy, nations with large public deficits and debts are less attractive to foreign investors. The reason? A large debt encourages inflation, and if inflation is high, the debt will be serviced and ultimately paid off with cheaper real dollars in the future.
In the worstcase scenario, a government may print money to pay part of a large debt, but increasing the money supply inevitably causes inflation. Moreover, if a government is not able to service its deficit through domestic means (selling domestic bonds, increasing the money supply), then it must increase the supply of securities for sale to foreigners, thereby lowering their prices.

- Terms of Trade: A ratio comparing export prices to import prices, the terms of trade is related to current accounts and the balance of payments. If the price of a country's exports rises by a greater rate than that of its imports, its terms of trade have favorably improved. Increasing terms of trade shows' greater demand for the country's exports. This, in turn, results in rising revenues from exports, which provides increased demand for the country's currency (and an increase in the currency's value). If the price of exports rises by a smaller rate than that of its imports, the currency's value will decrease in relation to its trading partners.

-Strong Economic Performance: Country with such positive attributes will draw investment funds away from other countries perceived to have more political and economic risk. Political turmoil, for example, can cause a loss of confidence in a currency and a movement of capital to the currencies of more stable countries.

-Cross border currency flows due to foreign direct investment and service like banking, insurance, education, tourism cause the exchange rate fluctuate randomly

-An ultimately **culminate in current account balance and foreign currency reserves held by the central banks.**Country with an appreciating home currency will experience its goods become more expensive in international market which may affect the exports and at the same time imports become inexpensive

-To bridge the gap between **tax payers**, the governments increase the tax rates and also bring in new taxes such as service tax and surcharges. These measures bring in some disparity and imbalance in economic alignment which affect the exchange rates ultimately

-The national governments should spend within the national income by collecting tax. If expenditure exceeds the revenue the governments will finance the gap by borrowing or by printing currency notes. These actions erode the confidence of the external parties who have financial dealings with the home country. If the governments fail to raise finance by taxes they go in for foreign debt. **Foreign debts and budget deficit** create financial imbalance which leads to exchange rate fluctuation

-**Multinational companies** establish subsidiary companies in. other countries to reduce their cost of production as the input costs are cheaper in the host countries. These produce goods in large volume and export. This results in massive cash flows which affect XRs

- There is a positive relationship between **corruption and exchange rate** which leads to depreciation of home currency. Corruption also results in insecurity and fixed costs for the international trade in the form of extortion and bribes which ultimately affects the exchange rates.

3. Explain the connection between linear regression and Vector Error Correction (VEC).

Given a data set $\{y_i,\, x_{i1}, \ldots, x_{ip}\}_{i=1}^{n}$ of n statistical units, a ***linear regression*** model assumes that the relationship between the dependent variable y and the p-vector of regressors x is linear. This relationship is modeled through a disturbance term or error variable ε — an unobserved random variable that adds "noise" to the linear relationship between the dependent variable and regressors. Thus the model takes the form

$$y_i = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_p x_{ip} + \varepsilon_i = \mathbf{x}_i^{\mathsf{T}} \boldsymbol\beta + \varepsilon_i, \qquad i = 1, \ldots, n$$

where T denotes the transpose, so that $\mathbf{x}_i^{\mathsf{T}}\boldsymbol\beta$ is the inner product between vectors $\mathbf{x}_i$ and β.

Often these n equations are stacked together and written in matrix notation as

$$\mathbf{y} = X\boldsymbol\beta + \boldsymbol\varepsilon$$

A vector error correction (VEC) model is a restricted VAR designed for use with nonstationary series that are known to be cointegrated. You may test for cointegration using an estimated VAR object, Equation object estimated using nonstationary regression methods, or using a Group object (see “Cointegration Testing”).

The VEC has cointegration relations built into the specification so that it restricts the long-run behavior of the endogenous variables to converge to their cointegrating relationships while allowing for short-run adjustment dynamics. The cointegration term is known as the error correction term since the deviation from long-run equilibrium is corrected gradually through a series of partial short-run adjustments.

To take the simplest possible example, consider a two variable system with one cointegrating equation and no lagged difference terms. The cointegrating equation is:

$$y_{2,t} = \beta y_{1,t}$$

The corresponding VEC model is:

$$\Delta y_{1,t} = \alpha_1 (y_{2,t-1} - \beta y_{1,t-1}) + \epsilon_{1,t}$$
$$\Delta y_{2,t} = \alpha_2 (y_{2,t-1} - \beta y_{1,t-1}) + \epsilon_{2,t}$$

It is important to discriminate between two situations:
1. Spurious regressions. Apparently significant relationship between unrelated series.
2. Genuine relationships which arise when the time series are cointegrated.

In time series linear regression analysis, we have to be careful from spurious results i.e. When non-stationary time series are used in a regression model, one may obtain apparently significant relationships from unrelated variables. For that, in these cases, we have to pass by unit root test, cointegration and Granger Engels causality through Vector Error Correction Model (VECM)

We can run an OLS regression on the system generated VECM Equation to check the p values associated with the coefficients and to run a Wald Test to check the restrictions on the coefficients of VECM using OLS regression with the help statistical software like SPSS and Eviews.

>***First Step:*** If cointegration holds, the OLS estimator of Estimate the long-run (equilibrium) equation, is said to be super-consistent. Implications: as $T \rightarrow \infty$ (i) there is no need to include I(0) variables in the cointegrating equation

> ***Second step:*** estimate the Error Correction Model by OLS as the equation has only I(0) variables, standard hypothesis testing using t-ratios and diagnostic testing of the error term is appropriate. The adjustment coefficient α must be negative.

4 Calculate equilibrium FX using VEC. You can use the Behavioural Equilibrium

Currency Pair – USD / GBP

For Calculation of Equilibrium FX following data set is being used –

- USD/GBP Exchange Rate (2014-2019)
- CPI for US
- CPI for UK

Period Dec 2014 to 1 October 2019.

Following is time series chart of data from given time period –

- USD/GBP exchange rate


Plot Zoom
USD/GBP exchange Rate
1.5
1.4
1.3
2015
2016
2017
2018
2019
Type here to search
ENG
19:31
08-10-2019

- CPI US


Plot Zoom
CPI US
3
2
1
0
2015
2016
2017
2018
2019
Type here to search
ENG
19:33
08-10-2019

- CPI UK

By analyzing the time series all of them are non-stationary. So we need to difference the data to order 1.

Computing of correlation of FX_RATES, UK CPI & US CPI.

```r
# Differencing the data to level1
D_US_UK_Intreset_rate <- diff(fx_rate$DEXUSUK,trim=TRUE)
D_US_CPI <- diff(fx_rate$CPIUS,trim=TRUE)
D_UK_CPI <- diff(fx_rate$CPIUK,trim=TRUE)

D_Equilibrum_fx1 = cbind(D_US_UK_Intreset_rate, D_US_CPI, D_UK_CPI)
complete.cases(Equilibrum_fx1)
D_Equilibrum_fx <- na.omit(D_Equilibrum_fx1)
#computing correlation
cor(D_Equilibrum_fx)

VAR_model <- VAR(D_Equilibrum_fx, lag.max=2, type = "none", ic = "AIC")
```

25:1 (Top Level) ÷

Console Terminal × Jobs ×

~/

```
> cor(D_Equilibrum_fx)
                      D_US_UK_Intreset_rate  D_US_CPI  D_UK_CPI
D_US_UK_Intreset_rate             1.0000000 0.0826675 0.1782247
D_US_CPI                          0.0826675 1.0000000 0.4940307
D_UK_CPI                          0.1782247 0.4940307 1.0000000
```

It has CPI low level correlation with Exchange Rate. CPI among themselves has strong correlation.

Since we do not know what the appropriate lag length of the VAR should be, several tests have been developed for choosing the lag value. Lag value was selected using the Akaike Information Criterion.

LAG = 2 was selected.

```
#Estimate a Vector Auto Regression

VAR_model <- VAR(D_Equilibrum_fx, lag.max=2, type = "none", ic = "AIC")
summary(VAR_model)
```

```
> summary(VAR_model)

VAR Estimation Results:
=========================
Endogenous variables: D_US_UK_Intreset_rate, D_US_CPI, D_UK_CPI
Deterministic variables: none
Sample size: 38
Log Likelihood: 101.554
Roots of the characteristic polynomial:
0.7358 0.6245 0.6245 0.5858 0.5858 0.3116
Call:
VAR(y = D_Equilibrum_fx, type = "none", lag.max = 2, ic = "AIC")
```

Johnson cointegration test is applied to identify number of statistically non-zero eigenvalues

```
US_UK_Intreset_rate <- fx_rate$DEXUSUK
US_CPI <- fx_rate$CPIUS
UK_CPI <- fx_rate$CPIUK

Equilibrum_fx = cbind(US_UK_Intreset_rate, US_CPI, UK_CPI)
jotest1=ca.jo(Equilibrum_fx, type="eigen", K=2, ecdet="none", spec="longrun
summary(jotest1)
jotest2=ca.jo(Equilibrum_fx, type="trace", K=2, ecdet="none", spec="longrun
summary(jotest2)
```

```
> summary(jotest1)

######################
# Johansen-Procedure #
######################

Test type: maximal eigenvalue statistic (lambda max) , with linear trend

Eigenvalues (lambda):
[1] 0.4571709 0.3428825 0.1465089

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 2 |  6.18  6.50  8.18 11.65
r <= 1 | 16.38 12.91 14.90 19.19
r = 0  | 23.83 18.90 21.07 25.75

Eigenvectors, normalised to first column:
(These are the cointegration relations)

                       US_UK_Intreset_rate.l2  US_CPI.l2 UK_CPI.l2
US_UK_Intreset_rate.l2            1.000000000  1.0000000 1.0000000
US_CPI.l2                         0.082880426 -0.6469243 0.1352980
UK_CPI.l2                         0.007029536  0.6996988 0.6600746
```

```
> summary(jotest2)

######################
# Johansen-Procedure #
######################

Test type: trace statistic , with linear trend

Eigenvalues (lambda):
[1] 0.4571709 0.3428825 0.1465089

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 2 |  6.18  6.50  8.18 11.65
r <= 1 | 22.55 15.66 17.95 23.52
r = 0  | 46.38 28.71 31.52 37.22

Eigenvectors, normalised to first column:
(These are the cointegration relations)

                       US_UK_Intreset_rate.l2  US_CPI.l2 UK_CPI.l2
US_UK_Intreset_rate.l2            1.000000000  1.0000000 1.0000000
US_CPI.l2                         0.082880426 -0.6469243 0.1352980
UK_CPI.l2                         0.007029536  0.6996988 0.6600746
```

These tests show strong evidence of cointegration at significance level 10 & 5, satisfying the requirements we place on the **Π.**

To keep the output simple, we estimate with only 2 lagged differences –

```r
# Fit cointegrated VECM
VECM_fit = VECM(Equilibrum_fx,1,r = 2, include = "const",estim = "ML", LRin
summary(VECM_fit)
```

```
#############
Full sample size: 41    End sample size: 39
Number of variables: 3  Number of estimated slope parameters 18
AIC -497.8756   BIC -464.6044   SSR 3.218046
Cointegrating vector (estimated by ML):
   US_UK_Intreset_rate US_CPI      UK_CPI
r1                   1      0  0.08569267
r2                   0      1 -0.94911594


                              ECT1                ECT2
Equation US_UK_Intreset_rate -0.2844(0.1161)*     -0.0506(0.0183)**
Equation US_CPI              -1.0803(0.8719)      -0.4141(0.1375)**
Equation UK_CPI              -2.0923(0.4825)***   -0.0692(0.0761)
                              Intercept          US_UK_Intreset_rate -1
Equation US_UK_Intreset_rate 0.4220(0.1760)*     -0.0877(0.1747)
Equation US_CPI              1.6667(1.3226)      -1.2729(1.3127)
Equation UK_CPI              3.1984(0.7319)***   0.1178(0.7264)
                              US_CPI -1          UK_CPI -1
Equation US_UK_Intreset_rate 0.0453(0.0227).    -0.0814(0.0370)*
Equation US_CPI              0.4975(0.1702)**   -0.1788(0.2780)
Equation UK_CPI              0.2939(0.0942)**   -0.3464(0.1539)*
```

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