— *course overview* —

18.06: Linear Algebra

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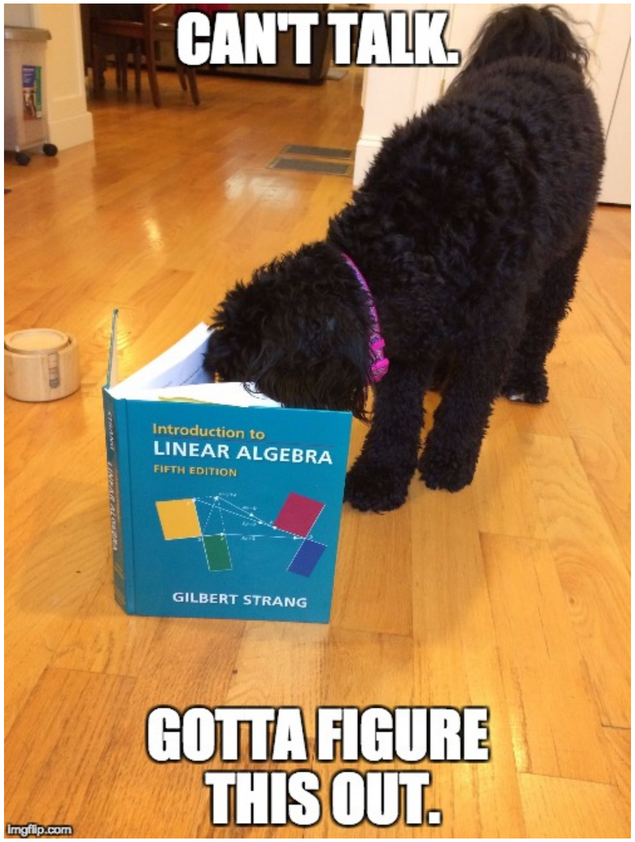
Spring 2018

http://web.mit.edu/18.06

Textbook: Strang, *Introduction to Linear Algebra*, 5^{th} edition
+ supplementary notes

Help wanted:

arrive 10 minutes early
and get paid $10 to erase the boards



The text

For an autographed copy of the textbook, you could see Professor Strang in 2-245 (teaching MWF 1-2). In his office payment is by cash or check.

Administrative Details

Lectures MWF11, 10-250
Tuesday recitations — use Stellar to switch sections

Weekly psets, due Wednesday 10:55am. Electronic submission through Stellar
— no extensions or makeup, but lowest pset score will be dropped
— pset 1 to be posted shortly

Grading: homework 15%, 3 exams 45% (3/2, 4/6, & 5/4 in 50-340),
& final exam 40%

Collaboration policy: talk to anyone you want, read anything you want, but:

- Make an effort on a problem before collaborating.
- Write up your solutions independently (from "blank sheet of paper").
- List your collaborators and external sources (not course materials).

Syllabus and Calendar

- Significant overlap with Strang's OCW video lectures: these are a useful supplement but not a replacement for attending lecture. Likely topics:
- Exam 1: Friday 3/2. Elimination, LU factorization, nullspaces and other subspaces, bases and dimensions, vector spaces, complexity. (Book: 1–3.5, 11.1)
- Exam 2: Friday 4/6. Orthogonality, projections, least-squares, QR, Gram–Schmidt, orthogonal functions (Book: Chapters 1 to 6.2, and 8.2, 8.5).
- Exam 3: Friday 5/4. Eigenvectors, determinants, similar matices, Markov matrices, ODEs, symmetric matrices, definite matrices, matrices from graphs and engineering. (Book: 1–7, 10.1–3.)
- Other topics: defective matrices, SVD and principal-components analysis, sparse matrices and iterative methods, complex matrices, symmetric linear operators on functions.
- Final exam: all of the above.

What is 18.06 about?

High school:

3 "linear" equations
(only ± and × constants)
in 3 unknowns

$$2x_1 + 4x_2 - 2x_3 = 2$$
$$\boxed{4x_1 + 9x_2 - 3x_3 = 8}$$
$$-2x_1 - 3x_2 + 7x_3 = 10$$

Method: eliminate unknowns one at a time.

Equivalent matrix problem

take "dot products" of rows × columns

$$\begin{pmatrix} 2 & 4 & -2 \\ \boxed{4 \quad 9 \quad -3} \\ -2 & -3 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ \boxed{8} \\ 10 \end{pmatrix}$$

Ax = b

A matrix of coefficients

x vector of unknowns

b vector of right-hand sides

Ax is a "linear operation:"

A(x+y) = Ax + Ay

A(3x) = 3Ax

What is 18.06 about?

Linear system of equations,
in matrix form

$Ax = b$

$$\underbrace{\begin{pmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} 2 \\ 8 \\ 10 \end{pmatrix}}_{b}$$

Will we learn faster methods to solve this? No. (Except if A is special.) The standard "Gaussian elimination" (and "LU factorization") matrix methods are just a slightly more organized version of the high-school algebra elimination technique.

Will we get better at doing these calculations by hand? Maybe, but who cares? Nowadays, all important matrix calculations are done by computers.

Will we learn more about the computer algorithms? A little. But mostly the techniques for "serious" numerical linear-algebra are topics for another course (e.g. 18.335).

How do we *think* about linear systems?
(imagine someone gives you a $10^6 \times 10^6$ matrix)

- All the formulas for 2×2 and 3×3 matrices would fit on one piece of paper. They aren't the reason why linear algebra is important (as a class or a field of study).
- Large problems are solved by computers, but must be understood by human beings. (And we need to give computers the right tasks!)
- Understand non-square problems: #equations > #unknowns or vice versa
- Break up matrices into simpler pieces
  - Factorize matrices into products of simpler matrices: A=LU (triangular: Gauss), A=QR (orthogonal/triangular), $A=X\Lambda X^{-1}$ (diagonal: eigenvecs/vals), $A=U\Sigma V^*$ (orthogonal/diagonal: SVD)
  - Submatrices (matrices of matrices).
- Break up vectors into simpler pieces: subspaces and basis choices.
- Algebraic manipulations to turn harder/unfamiliar problems (e.g. minimization or differential equations) into simpler/familiar ones: algebra on whole matrices at once

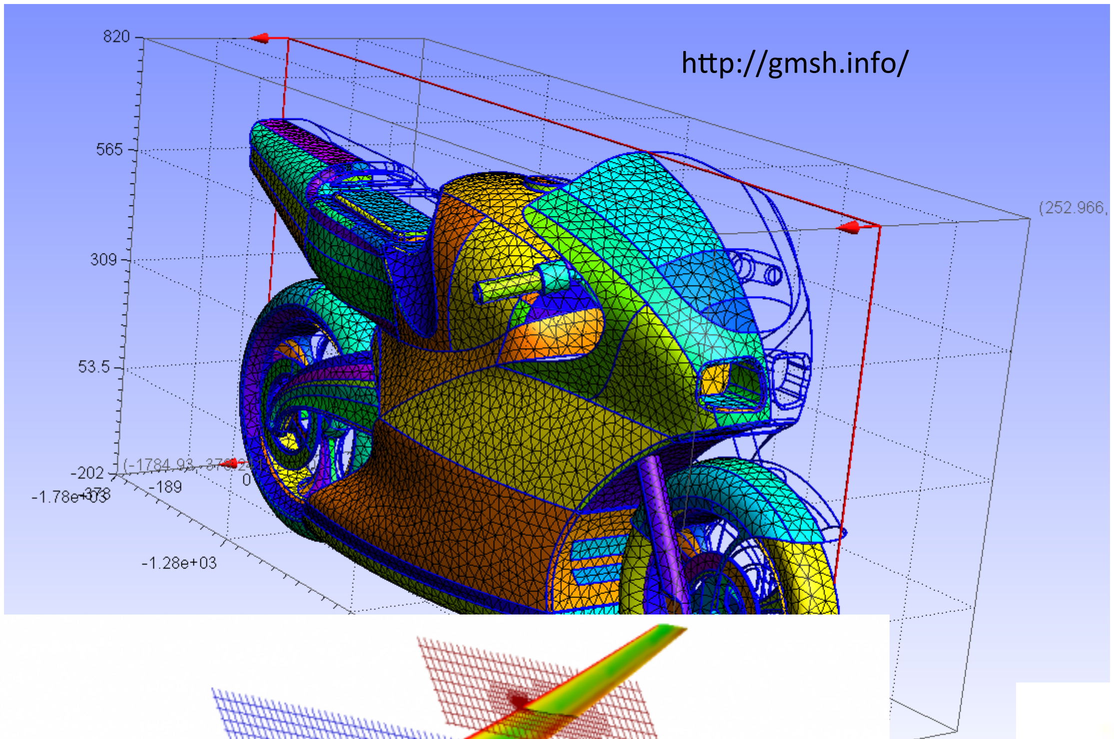
Don’t expect a lot of “turn the crank” problems
on psets or exams of the form
“solve this system of equations.”

… we will turn it upside-down, give you the answer and ask the question, ask about properties of the solution from partial information, … the general goal is to require you to understand the crank rather than just turn it.

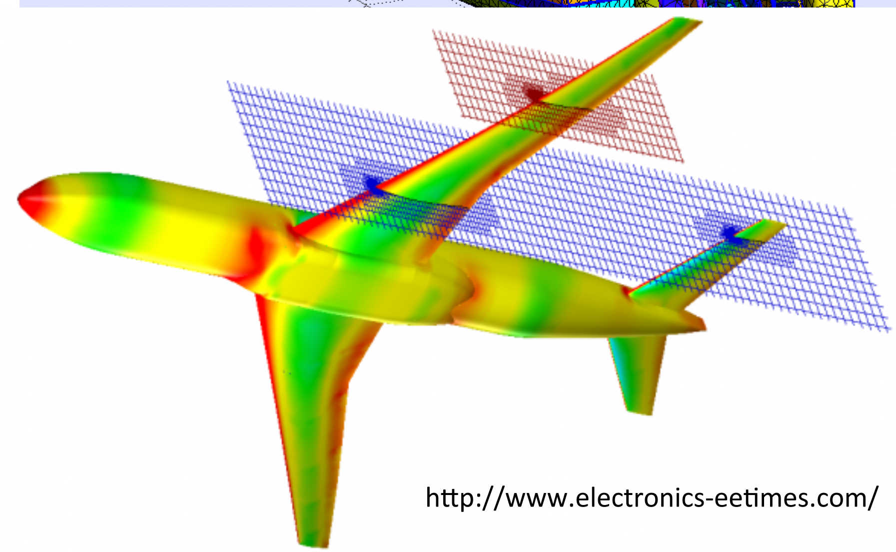
Where do big matrices come from?

Lots of examples in many fields, but here are a couple that are relatively easy to understand…

Engineering & Scientific Modeling
[18.303, 18.330, 6.336, 6.339, …]



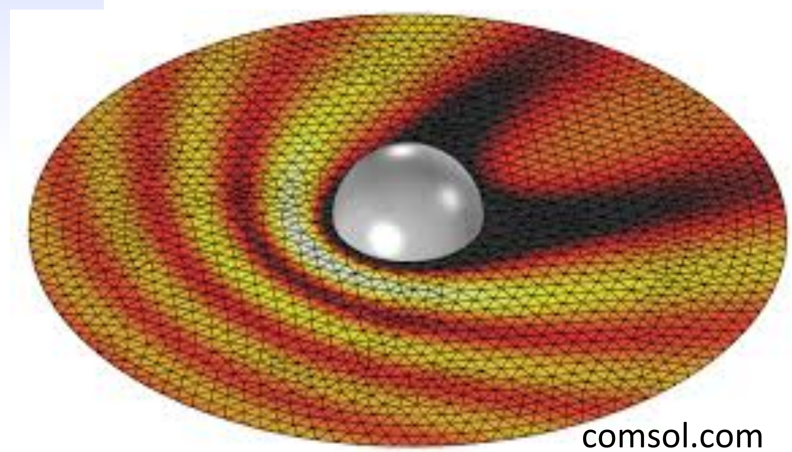
http://gmsh.info/



http://www.electronics-eetimes.com/

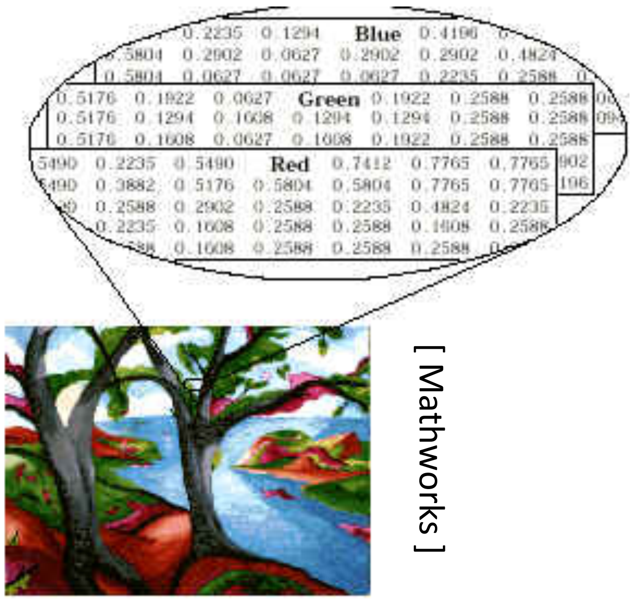
Unknown functions
(fluid flow, mechanical stress, electromagnetic fields, …)
approximated by values on a discrete mesh/grid

e.g. 100x100x100 grid
= 10^6 unknowns!



comsol.com

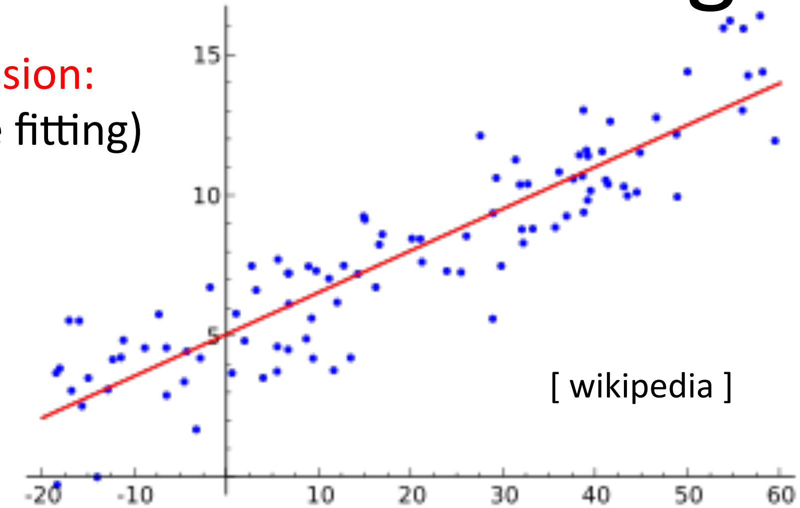
Data analysis
and Machine Learning



[Mathworks]

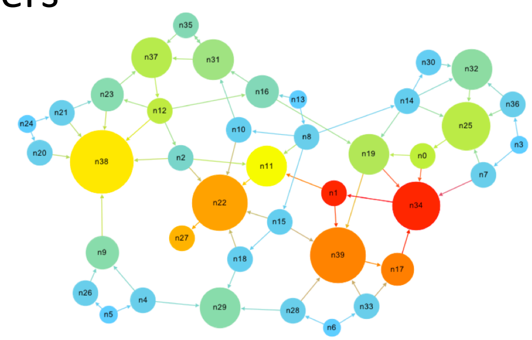
Image processing:
images are just matrices of numbers
(red/green/blue intensity)

Regression:
(curve fitting)



[wikipedia]

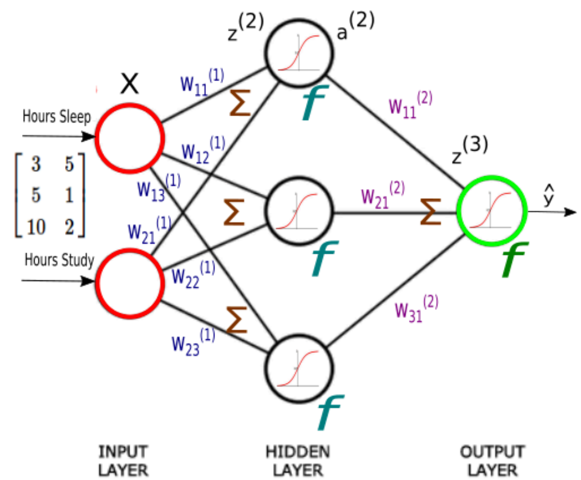
Google "page rank" problem
(also for gene networks etc.)



[computationalculture.net]

Determine the "most important"
web pages just from how they link.

matrix = (# web pages) × (# web pages)
(*entry = 1 if they link, 0 otherwise*)



Machine Learning

Not just matrices of numbers

- There are lots of surprising and important generalizations of the ideas in linear algebra.
- Instead of vectors with a finite number of unknowns, similar ideas apply to functions with an infinite number of unknowns.
- Instead of matrices multiplying vectors, we can think about linear operators on functions

Poisson's equation
(e.g. 18.303)

$$\nabla^2 u = f$$

"A"
linear
operator
∇^2

"x"
unknown
function
$u(x,y,z)$

"b"
right-hand side
$f(x,y,z)$

18.06 vs. 18.700

"applied" vs. "pure" math

few proofs vs. formal proofs expected

(deduce patterns from examples, informal arguments)

(definitions to lemmas to theorems … training in proof writing)

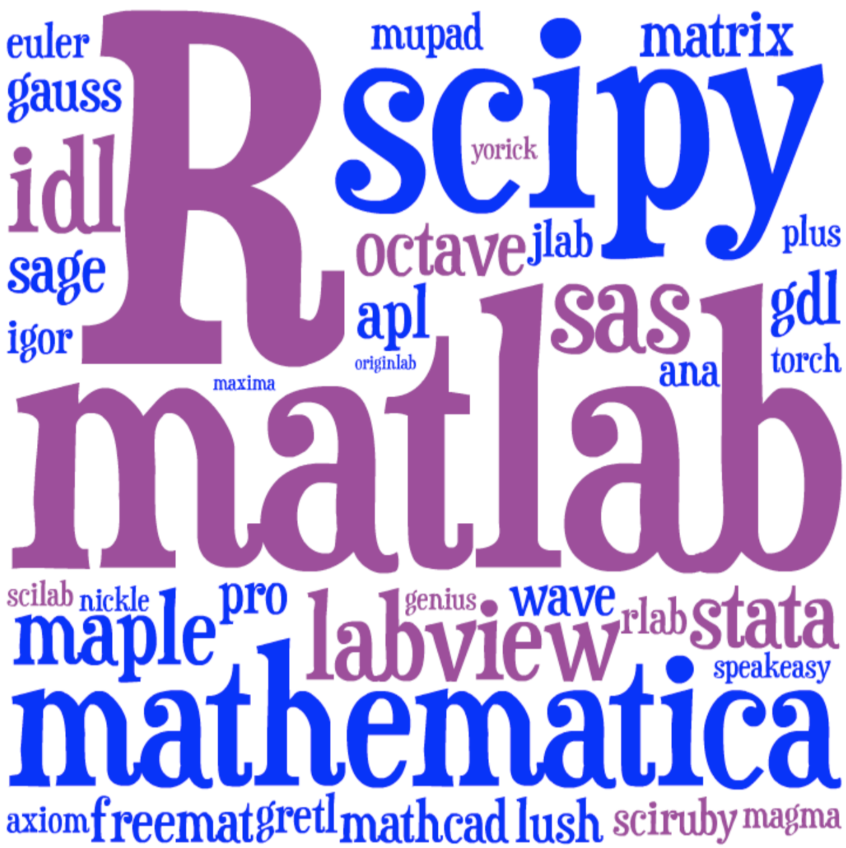
more applications vs. more theorems

more concrete vs. more abstract

some computers vs. only pencil-and-paper

Computer software

Lots of choices:



[image: Viral Shah]

This semester: a relatively new language that scales better to real problems.

julia

julialang.org

No programming required for 18.06,
just a "glorified calculator" to turn the crank.

Use it online: log in at juliabox.com
see "Julia" link on Stellar

Optional tutorial: Friday 5pm 32-141