Markov chains

Quiz, 5 questions

5/5 points (100%)

Congratulations! You passed! Next Item

1 / 1 points

1. All but one of the following scenarios describes a valid Markov chain. Which one is not a Markov chain?

- At any given hour, the number of customers entering a grocery store follows a Poisson distribution. The number of customers in the store who leave during that hour also follows a Poisson distribution (only up to as many people are in the store). A clerk reports the total number of customers in the store X_t at the end of hour t.
- Three friends take turns playing chess with the following rules: the player who sits out the current round plays the winner in the next round. Player A, who has 0.7 probability of winning any game regardless of opponent, keeps track of whether he plays in game t with an indicator variable X_t.
- **(selected)** Suppose you have a special savings account which accrues interest according to the following rules: the total amount deposited in a given month will earn $10(1/2)^{(r-1)}$% interest in the rth month after the deposit. For example, if the deposits in January total $100, then you will earn $10 interest in January, $5 interest at the end of February, $2.50 in March, etc. In addition to the interest from January, if you deposit $80 in February, you will earn an additional $8 at the end of February, $4 at the end of March, and so forth. The total amount of money deposited in a given month follows a gamma distribution. Let X_t be the total dollars in your account, including all deposits and interest up to the end of month t.

Correct
Because of these particular rules, the total in the account at the end of the previous month does not provide enough information to create a probability distribution for the current month. You need to know the amount deposited in each month to determine how much interest it qualifies for this month. To be a Markov chain, the probability distribution for X_{t+1} must depend only on X_t and not the older history $X_{t-1}, X_{t-2}, \ldots$.

- While driving through a city with square blocks, you roll a six-sided die each time you come to an intersection. If the die shows 1, 2, 3, or 4, then you turn left. If the die shows 5 or 6, you turn right. Each time you reach an intersection, you report your coordinates X_t.

1 / 1 points

2. Which of the following gives the transition probability matrix for the chess example in the previous question? The first row and column correspond to $X = 0$ (player A not playing) while the second row and column correspond to $X = 1$ (player A playing).

- $\begin{pmatrix} 0.7 & 0 \\ 0.3 & 1 \end{pmatrix}$
- $\begin{pmatrix} 0.3 & 0 \\ 0.7 & 1 \end{pmatrix}$
- **(selected)** $\begin{pmatrix} 0 & 1 \\ 0.3 & 0.7 \end{pmatrix}$

Correct

- $\begin{pmatrix} 0 & 0.3 \\ 1 & 0.7 \end{pmatrix}$

1 / 1 points

3. Continuing the chess example, suppose that the first game is between Players B and C. What is the probability that Player A will play in Game 4? Round your answer to two decimal places.

0.79

Correct Response
The distribution for X in Game 4 in this case is $(1,0)Q^3$ (using the initial distribution and three transitions).

1 / 1 points

4. Which of the following is the stationary distribution for X in the chess example?

- (0.0, 1.0)
- **(selected)** (.231, .769)

Correct

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(.769, .231)

(.750, .250)

(.250, .750)

1 / 1 points

5. If the players draw from the stationary distribution in Question 4 to decide whether Player A participates in Game 1, what is the probability that Player A will participate in Game 4? Round your answer to two decimal places.

0.77

Correct Response

This is just the stationary probability of Player A playing. If the chain starts in the stationary distribution, the probability of Player A playing in the next game, the game after that, and so forth, is always this stationary probability.