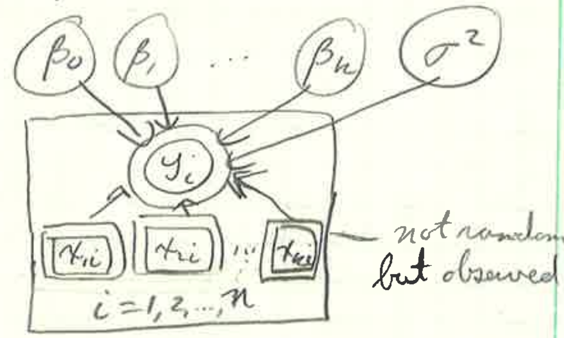
$$y_i = \underbrace{\beta_0 + \beta_1 x_{1i} + \dots + \beta_k x_{ki}}_{\text{mean}} + \underbrace{\varepsilon_i}_{\text{error}} \quad , \quad \varepsilon_i \overset{iid}{\sim} N(0, \sigma^2), \; i = 1, \dots, n$$

$$y_i \mid \underset{\sim}{x}_i, \underset{\sim}{\beta}, \sigma^2 \overset{ind}{\sim} N(\beta_0 + \beta_1 x_{1i} + \dots + \beta_k x_{ki}, \sigma^2)$$

$\beta_0 \sim p(\beta_0)$ , $\beta_1 \sim p(\beta_1)$, ...

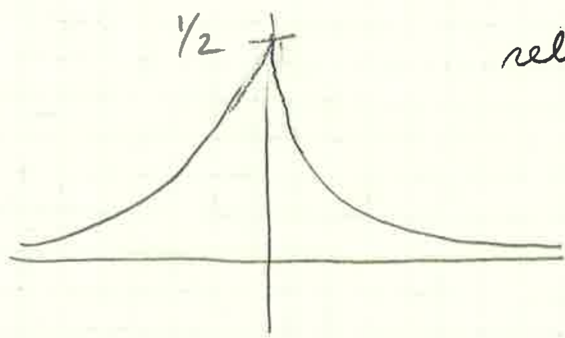
$\sigma^2 \sim p(\sigma^2)$ (maybe IG)

usually normal or multivariate normal




Laplace prior

$$p(\beta) = \frac{1}{2} e^{-|\beta|}$$




related to the LASSO

| Factors: | sound | font size |
|---|---|---|
| Levels | music | small |
| | no music | medium |
| | | large |

$g_i \sim$ group of subject $i$

$\underset{\sim}{\mu} \sim$ vector of $G$ group means

$$y_i \mid g_i, \underset{\sim}{\mu}, \sigma^2 \overset{ind}{\sim} N(\mu_{g_i}, \sigma^2)$$

$$g_i \in \{1, \dots, G\} \quad , i = 1, \dots, n$$

Alternative

$$E(y_i) = \beta_0 + \beta_1 x_{1i} + \cdots + \beta_{G-1} x_{G-1,i}$$

$$x_{1i} = I(g_i = 1) \qquad x_{G-1,i} = I(g_i = G-1)$$

$x$'s are indicators or dummy variables

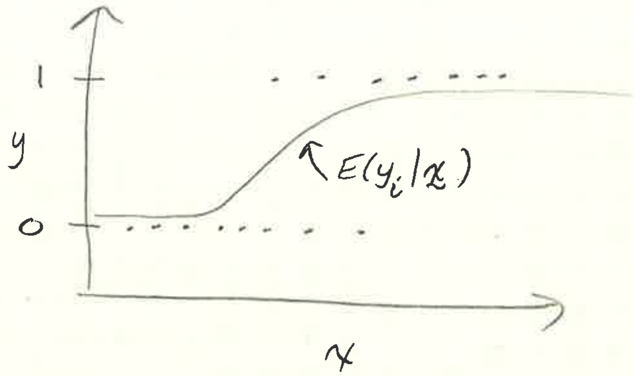
$$\begin{array}{cc|ccc} & & & B & \\ & & 1 & 2 & 3 \\ \hline A & 1 & \mu_{1,1} & \mu_{1,2} & \mu_{1,3} \\ & 2 & \mu_{2,1} & \mu_{2,2} & \mu_{2,3} \end{array}$$

$A, B \sim$ Factors

6 treatment groups

Additive model: (good for no interactions between factors)

$$E(y_i) = \mu + \alpha_2 I(a_i = 2) + \beta_2 I(b_i = 2) + \beta_3 I(b_i = 3)$$




$y_i | \phi_i \overset{ind}{\sim} \text{Bernoulli}(\phi_i) \quad , i = 1, \dots, n$ $\qquad$ $\phi$ is probability of success

In linear regression : $E(y_i) = \beta_0 + \beta_1 x_{1i}$

$\dfrac{\phi}{1-\phi} \Leftrightarrow$ odds

$\log\left(\dfrac{\phi}{1-\phi}\right) \Leftrightarrow$ logit link

$$\text{logit}(\phi_i) = \log\left(\frac{\phi_i}{1-\phi_i}\right) = \beta_0 + \beta_1 x_{1i}$$

$$E(y_i) = \phi_i = \frac{\exp(\beta_0 + \beta_1 x_{1i})}{1 + \exp(\beta_0 + \beta_1 x_{1i})} = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_{1i})}}$$

$$y_i | x_i, \underset{\sim}{\beta} \overset{ind}{\sim} \text{Bernoulli}\left(\frac{1}{1 + \exp[-\beta_0 + \beta_1 x_{1i} + \dots + \beta_k x_{ki}]}\right)$$

$y_i \mid \lambda_i \overset{ind}{\sim} Pois(\lambda_i)$ , $i = 1, \ldots, n$

linear regression: $E(y_i) = \beta_0 + \beta_1 x_{1i}$ is a problem

log link: $\log(\lambda_i) = \beta_0 + \beta_1 x_{1i}$

$\Rightarrow E(y_i) = \lambda_i = e^{\beta_0 + \beta_1 x_{1i}}$

$\lambda_i$ is the mean and variance of the poisson distribution

$E(\log(y)) \neq \log(E(y))$

$y_i \mid x_i, \underset{\sim}{\beta} \overset{ind}{\sim} Pois\left(\exp[\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_k x_{ki}]\right)$