MCMC

Quiz, 5 questions

5/5 points (100%)

Congratulations! You passed!

Next Item

1 / 1 points

1. For Questions 1 through 3, consider the following model for data that take on values between 0 and 1:

$$x_i \mid \alpha, \beta \overset{\text{iid}}{\sim} \text{Beta}(\alpha, \beta)\,, \quad i = 1, \ldots, n\,,$$
$$\alpha \sim \text{Gamma}(a, b)\,,$$
$$\beta \sim \text{Gamma}(r, s)\,,$$

where α and β are independent a priori. Which of the following gives the full conditional density for α up to proportionality?

- $p(\alpha \mid \beta, x) \propto \left[\prod_{i=1}^n x_i\right]^{\alpha-1} \alpha^{a-1} e^{-b\alpha} I_{(\alpha>0)}$
- $p(\alpha \mid \beta, x) \propto \frac{\Gamma(\alpha+\beta)^n}{\Gamma(\alpha)^n \Gamma(\beta)^n} \left[\prod_{i=1}^n x_i\right]^{\alpha-1} \left[\prod_{i=1}^n (1-x_i)\right]^{\beta-1} \alpha^{a-1} e^{-b\alpha} \beta^{r-1} e^{-s\beta} I_{(0<\alpha<1)} I_{(0<\beta<1)}$
- $p(\alpha \mid \beta, x) \propto \frac{\Gamma(\alpha+\beta)^n}{\Gamma(\alpha)^n} \left[\prod_{i=1}^n x_i\right]^{\alpha-1} \alpha^{a-1} e^{-b\alpha} I_{(0<\alpha<1)}$
- (selected) $p(\alpha \mid \beta, x) \propto \frac{\Gamma(\alpha+\beta)^n}{\Gamma(\alpha)^n} \left[\prod_{i=1}^n x_i\right]^{\alpha-1} \alpha^{a-1} e^{-b\alpha} I_{(\alpha>0)}$

Correct

When we treat the data and β as known constants, the full joint distribution of all quantities x, α, and β is proportional to this expression when viewed as a function of α.

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1 / 1 points

2. Suppose we want posterior samples for α from the model in Question 1. What is our best option?

- The joint posterior for α and β is a common probability distribution which we can sample directly. Thus we can draw Monte Carlo samples for both parameters and keep the samples for α.
- (selected) The full conditional for α is not proportional to any common probability distribution, and the marginal posterior for β is not any easier, so we will have to resort to a Metropolis-Hastings sampler.

Correct

Another option is to approximate the posterior distribution for α by considering a set of discrete values such as $0.1, 0.2, \ldots, 0.9$ etc. You could use a discrete uniform prior, or discrete prior probabilities proportional to the beta prior evaluated at these specific values. Either way, the full conditional distribution for α looks like the discrete version of Bayes' theorem, which is easy to compute.

- The full conditional for α is proportional to a common distribution which we can sample directly, so we can draw from that.
- The full conditional for α is not a proper distribution (it doesn't integrate to 1), so we cannot sample from it.

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1 / 1 points

3. If we elect to use a Metropolis-Hastings algorithm to draw posterior samples for α, the Metropolis-Hastings candidate acceptance ratio is computed using the full conditional for α as

$$\frac{\Gamma(\alpha)^n \Gamma(\alpha^*+\beta)^n \left[\prod_{i=1}^n x_i\right]^{\alpha^*} \alpha^{*a-1} e^{-b\alpha^*} q(\alpha^*|\alpha) I_{\alpha^*>0}}{\Gamma(\alpha^*)^n \Gamma(\alpha+\beta)^n \left[\prod_{i=1}^n x_i\right]^{\alpha} \alpha^{a-1} e^{-b\alpha} q(\alpha|\alpha^*) I_{\alpha>0}}$$

where α^* is a candidate value drawn from proposal distribution $q(\alpha^*|\alpha)$. Suppose that instead of the full conditional for α, we use the full joint posterior distribution of α and β and simply plug in the current (or known) value of β. What is the Metropolis-Hastings ratio in this case?

- $\frac{\alpha^{*a-1} e^{-b\alpha^*} q(\alpha^*|\alpha) I_{\alpha^*>0}}{\alpha^{a-1} e^{-b\alpha} q(\alpha|\alpha^*) I_{\alpha>0}}$
- (selected) $\frac{\Gamma(\alpha)^n \Gamma(\alpha^*+\beta)^n \left[\prod_{i=1}^n x_i\right]^{\alpha^*} \alpha^{*a-1} e^{-b\alpha^*} q(\alpha^*|\alpha) I_{\alpha^*>0}}{\Gamma(\alpha^*)^n \Gamma(\alpha+\beta)^n \left[\prod_{i=1}^n x_i\right]^{\alpha} \alpha^{a-1} e^{-b\alpha} q(\alpha|\alpha^*) I_{\alpha>0}}$

Correct

All of the terms involving only β are identical in the numerator and denominator, and thus cancel out. The acceptance ratio is the same whether we use the full joint posterior or the full conditional in a Gibbs sampler.

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$$\frac{\Gamma(\alpha^*+\beta)^n \left[\prod_{i=1}^n x_i\right]^{\alpha^*-1} \left[\prod_{i=1}^n (1-x_i)\right]^{\beta-1} \alpha^{*a-1} e^{-b\alpha^*} \beta^{r-1} e^{-s\beta} q(\alpha|\alpha^*) I_{(0<\alpha^*)} I_{(0<\beta)}}{\Gamma(\alpha^*)^n \Gamma(\beta)^n q(\alpha^*|\alpha)}$$

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- ( ) $$\frac{\Gamma(\alpha)^n \Gamma(\alpha^*+\beta)^n \left[\prod_{i=1}^n x_i\right]^{\alpha^*} q(\alpha^*|\alpha) I_{\alpha^*>0}}{\Gamma(\alpha^*)^n \Gamma(\alpha+\beta)^n \left[\prod_{i=1}^n x_i\right]^{\alpha} q(\alpha|\alpha^*) I_{\alpha>0}}$$

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4. For Questions 4 and 5, re-run the Metropolis-Hastings algorithm from Lesson 4 to draw posterior samples from the model for mean company personnel growth for six new companies: (-0.2, -1.5, -5.3, 0.3, -0.8, -2.2). Use the same prior as in the lesson.

1 / 1 points

Below are four possible values for the standard deviation of the normal proposal distribution in the algorithm. Which one yields the best sampling results?

- ( ) 0.5
- (x) 1.5

Correct
The candidate acceptance rate for this proposal distribution is about 0.3 which yields good results.

- ( ) 3.0
- ( ) 4.0

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5. Report the posterior mean point estimate for μ, the mean growth, using these six data points. Round your answer to two decimal places.

1 / 1 points

-1.43

Correct Response
The sample mean of the six points is -1.62. Clearly the prior has some influence on this estimate.
