Lesson 4

8/8 points (100%)

Quiz, 8 questions

✔ **Congratulations! You passed!**

<div style="border:1px solid">Next Item</div>

✔ 1 / 1
points

1.

In which situation would we choose to use a Metropolis-Hastings (or any MCMC) sampler rather than straightforward Monte Carlo sampling?

○ Monte Carlo estimation is easier than calculating the integral required to obtain the mean of the target distribution.

○ The target distribution follows a Markov chain.

○ The data (likelihood) come from a Markov chain.

○ There is no easy way to simulate independent draws from the target distribution.

Correct
If we could, straightforward Monte Carlo sampling would be preferable.

✔ 1 / 1
points

2.

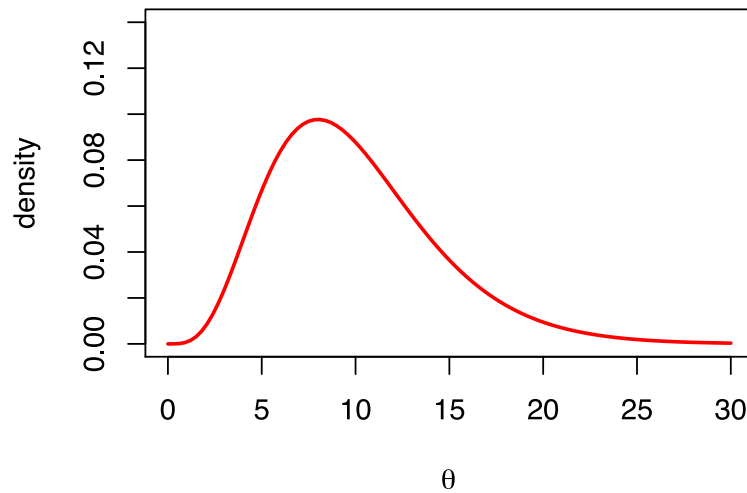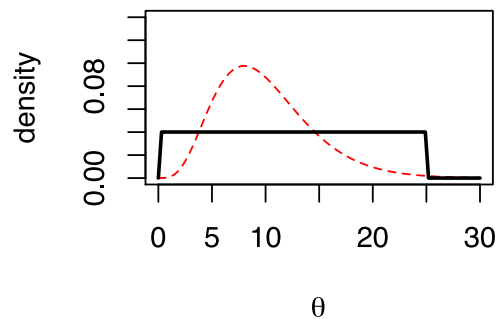## Lesson 4

Which of the following candidate-generating distributions would be best for an independent Metropolis-Hastings algorithm to sample the target distribution whose PDF is shown below?
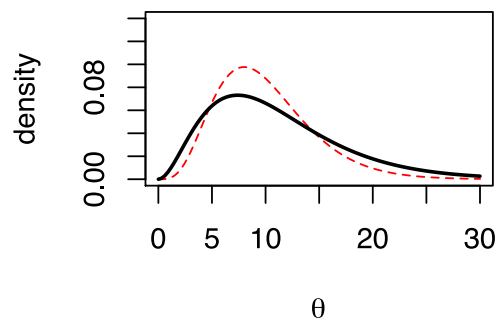
8/8 points (100%)

Note: In independent Metropolis-Hastings, the candidate-generating distribution q does not depend on the previous iteration of the chain.

○ $q = \mathrm{Unif}(0.05, 25.0)$
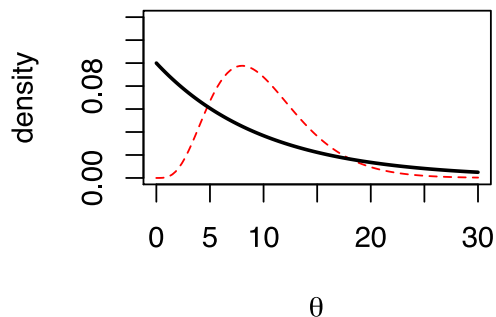

◉ $q = \mathrm{Gamma}(3.0, 0.27)$
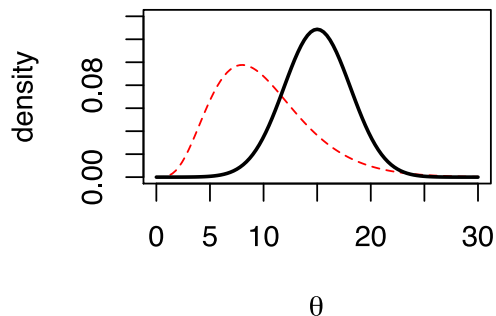
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Correct

This candidate-generating distribution approximates the target distribution well, and even has slightly larger variance.

8/8 points (100%)

○ $q = \text{Exp}(0.1)$

○ $q = \text{N}(15, 3.1^2)$

✔ 1 / 1
points

3.

If we employed an independent Metropolis-Hastings algorithm (in which the candidate-generating distribution q does not depend on the previous iteration of the chain), what would happen if we skipped the acceptance ratio step and always accepted candidate draws?

○   The chain would explore the posterior distribution very slowly, requiring more samples.

○   Each draw could be considered as a sample from the target distribution.

○   The sampler would become more efficient because we are no longer discarding draws.

○ The resulting sample would be a Monte Carlo simulation from q instead of
     from the target distribution.

▲

Correct
Accepting all candidates just means we are simulating from the candidate-
generating distribution. The acceptance step in the algorithm acts as a
correction, so that the samples reflect the target distribution more than the
candidate-generating distribution.

✔   1 / 1
         points

4.
If the target distribution $p(\theta) \propto g(\theta)$ is for a positive-valued random variable so that
$p(\theta)$ contains the indicator function $I_{\theta>0}(\theta)$, what would happen if a random walk
Metropolis sampler proposed the candidate $\theta^* = -0.3$?

○ The candidate would be rejected with probability 1 because $g(\theta^*) = 0$,
     yielding an acceptance ratio $\alpha = 0$.

▲

Correct
This strategy usually works, but sometimes runs into problems. Another solution
is to draw candidates for the logarithm of θ (which of course has a different
target distribution that you must derive) using normal proposals.

○ The candidate would be accepted with probability 1 because $g(\theta^*) = 0$,
     yielding an acceptance ratio $\alpha = 1$.

○ The candidate would be accepted with probability 1 because $g(\theta^*) = 0$,
     yielding an acceptance ratio $\alpha = \infty$.

○ The candidate would be accepted with probability 0.3 because $g(\theta^*) = |\theta^*|$,
   yielding an acceptance ratio $\alpha = 0.3$.

✔   1 / 1
         points

5.

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Quiz, 8 questions

Suppose we use a random walk Metropolis sampler with normal proposals (centered on the current value of the chain) to sample from the target distribution whose PDF is shown below. The chain is currently at $\theta_i = 15.0$. Which of the other points, if used as candidate θ^* for the next step, would yield the largest acceptance ratio α?

8/8 points (100%)
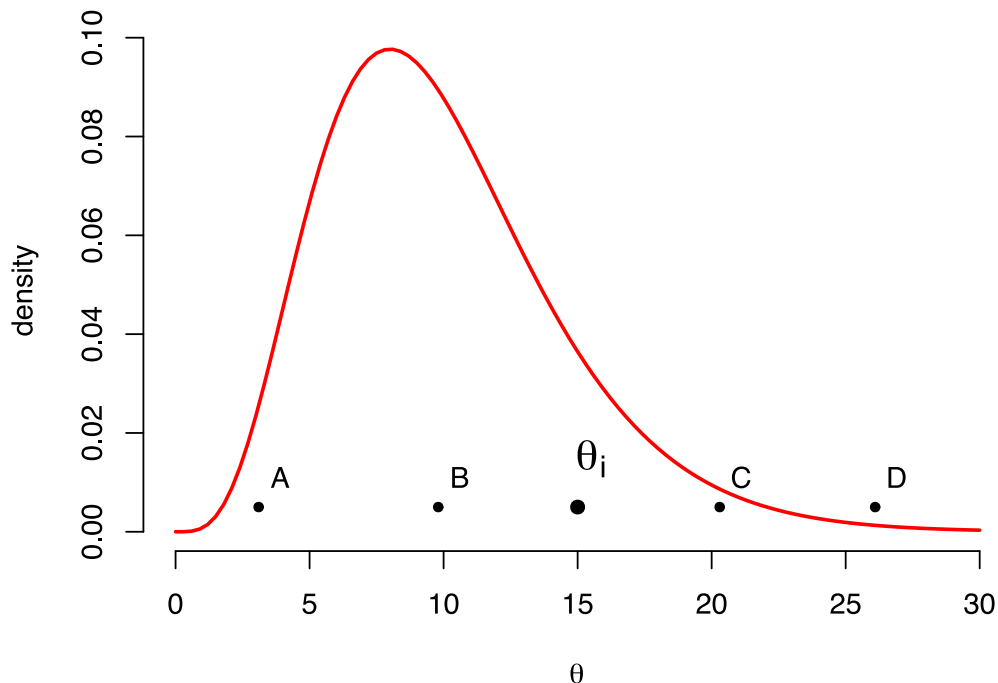
A) $\theta^* = 3.1$

B) $\theta^* = 9.8$

Correct

B is the only point with a target density value (close to 0.09) higher than that of θ_i (close to 0.04).

Since this is a random walk Metropolis sampler with symmetric proposal distribution, the expression for calculating the acceptance ratio for iteration $i + 1$ is $\alpha = g(\theta^*)/g(\theta_i)$. In this case α would be close to 2, whereas for A, C, and D, we have $\alpha < 1$. If point B were proposed, it would be accepted in this case.

C) $\theta^* = 20.3$

D) $\theta^* = 26.1$

1 / 1
points

6.

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Suppose you are using a random walk Metropolis sampler with normal proposals. After sampling the chain for 1000 iterations, you notice that the acceptance rate for the candidate draws is only 0.02. Which corrective action is most likely to help you approach a better acceptance rate (between 0.23 and 0.50)?

8/8 points (100%)

○   Increase the variance of the normal proposal distribution q.

◉   Decrease the variance of the normal proposal distribution q.

Correct

A low acceptance rate in a random walk Metropolis sampler usually indicates that the candidate-generating distribution is too wide and is proposing draws too far away from most of the target mass.

○   Replace the normal proposal distribution with a uniform proposal distribution centered on the previous value and variance equal to that of the old normal proposal distribution.

○   Fix the mean of the normal proposal distribution at the last accepted candidate's value. Use the new mean for all future proposals.

✔   1 / 1 points

7.

Suppose we use a random walk Metropolis sampler to sample from the target distribution $p(\theta) \propto g(\theta)$ and propose candidates θ^* using the $\text{Unif}(\theta_{i-1} - \epsilon,\, \theta_{i-1} + \epsilon)$ distribution where ϵ is some positive number and θ_{i-1} is the previous iteration's value of the chain. What is the correct expression for calculating the acceptance ratio α in this scenario?

Hint: Notice that the $\text{Unif}(\theta_{i-1} - \epsilon,\, \theta_{i-1} + \epsilon)$ distribution is centered on the previous value and is symmetric (since the PDF is flat and extends the same distance ϵ on either side).

○ $\alpha = \frac{\text{Unif}(\theta^*|\theta_{i-1}-\epsilon,\, \theta_{i-1}+\epsilon)}{\text{Unif}(\theta_{i-1}|\theta^*-\epsilon,\, \theta^*+\epsilon)}$ where $\text{Unif}(\theta \mid a, b)$ represents the PDF of a $\text{Unif}(a, b)$ evaluated at θ.

○ $\alpha = \frac{g(\theta_{i-1})}{g(\theta^*)}$

○ $\alpha = \frac{\text{Unif}(\theta_{i-1}|\theta^*-\epsilon,\, \theta^*+\epsilon)}{\text{Unif}(\theta^*|\theta_{i-1}-\epsilon,\, \theta_{i-1}+\epsilon)}$ where $\text{Unif}(\theta \mid a, b)$ represents the PDF of a $\text{Unif}(a, b)$ evaluated at θ.

◉ $\alpha = \frac{g(\theta^*)}{g(\theta_{i-1})}$

Correct

Since the proposal distribution is centered on the previous value and is symmetric, evaluations of q drop from the calculation of α.

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✔   1 / 1
       points

8.

The following code completes one iteration of an algorithm to simulate a chain whose stationary distribution is $p(\theta) \propto g(\theta)$. Which algorithm is employed here?

```
 1   # draw candidate
 2     theta_cand = rnorm(n=1, mean=0.0, sd=10.0)
 3
 4   # evaluate log of g with the candidate
 5     lg_cand = lg(theta=theta_cand)
 6
 7   # evaluate log of g at the current value
 8     lg_now = lg(theta=theta_now)
 9
10   # evaluate log of q at candidate
11     lq_cand = dnorm(theta_cand, mean=0.0, sd=10.0, log=TRUE)
12
13   # evaluate log of q at the current value
14     lq_now = dnorm(theta_now, mean=0.0, sd=10.0, log=TRUE)
15
16   # calculate the acceptance ratio
17     lalpha = lg_cand + lq_now - lg_now - lq_cand
18     alpha = exp(lalpha)
19
20   # draw a uniform variable which will be less than alpha with probability min
         (1, alpha)
21     u = runif(1)
22
23     if (u < alpha) { # then accept the candidate
24       theta_now = theta_cand
25       accpt = accpt + 1 # to keep track of acceptance
26     }
```

⊙   Independent Metropolis-Hastings (q does not condition on the previous value
    of the chain) with normal proposal

Correct

Candidates are always drawn from the same $\mathrm{N}(0, 10^2)$ distribution.

○   Random walk Metropolis with normal proposal

○   Independent Metropolis-Hastings (q does not condition on the previous value
    of the chain) with uniform proposal

○   Random walk Metropolis with uniform proposal