MAP Message Passing

返回第 3 周课程

2/2 得分（100%）

测验通过！

1 / 1 分

1.
Real-World Applications of MAP Estimation. Suppose that you are in charge of setting up a soccer league for a bunch of kindergarten kids, and your job is to split the N children into K teams. The parents are very controlling and also uptight about which friends their kids associate with. So some of them bribe you to set up the teams in certain ways.

The parents' bribe can take two forms: For some children i, the parent says "I will pay you A_{ij} dollars if you put my kid i on the same team as kid j"; in other cases, the parent of child i says "I will pay you B_i dollars if you put my kid on team k." In our notation, this translates to factor $f_{i,j}(x_i, x_j) = A_{ij} \cdot \mathbf{1}\{x_i = x_j\}$ or $g_i(x_i) = B_i \cdot \mathbf{1}\{x_i = k\}$, respectively, where x_i is the assigned team of child i and $\mathbf{1}\{\}$ is the indicator function. More formally, if we define x_i to be the assigned team of child i, the amount of money you get for the first type of bribe will be $f_{i,j}(x_i, x_j)$.

Being greedy and devoid of morality, you want to make as much money as possible from these bribes. What are you trying to find?

- $\text{argmax}_{\bar{x}} \sum_i g_i(x_i)$
- $\text{argmax}_{\bar{x}} \prod_i g_i(x_i) \cdot \prod_{i,j} f_{i,j}(x_i, x_j)$
- (selected) $\text{argmax}_{\bar{x}} \sum_i g_i(x_i) + \sum_{i,j} f_{i,j}(x_i, x_j)$

正确
Correct. The total amount of money is the sum of the indicator functions, so you want to find the assignment that maximizes the sum.

- $\text{argmax}_{\bar{x}} \prod_i g_i(x_i)$

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2.

***Decoding MAP Assignments.** You want to find the optimal solution to the above problem using a clique tree over a set of factors ϕ . How could you accomplish this such that you are guaranteed to find the optimal solution? (Ignore issues of tractability, and assume that if you specify a set of factors ϕ , you will be given a valid clique tree of minimum tree width.)

- ○ The optimal solution is not guaranteed to be found in this manner using clique trees.
- ○ Set $\phi_{i,j} = \exp(f_{i,j})$, $\phi_i = \exp(g_i)$, get the clique tree, run sum-product message passing, and decode the marginals.
- ○ Set $\phi_{i,j} = f_{i,j}$, $\phi_i = g_i$, get the clique tree over this set of factors, run max-sum message passing on this clique tree, and decode the marginals.
- ○ Set $\phi_{i,j} = f_{i,j}$, $\phi_i = g_i$, get the clique tree, run sum product message passing, and decode the marginals.
- ◉ Set $\phi_{i,j} = \exp(f_{i,j})$, $\phi_i = \exp(g_i)$, get the clique tree over this set of factors, run max-sum message passing on this clique tree, and decode the marginals.

正确

We want to compute
$\text{argmax}_{\bar{x}} \sum_i g_i(x_i) + \sum_{i,j} f_{i,j}(x_i, x_j) = \text{argmax}_{\bar{x}} \log\left[\prod_i \exp(g_i(x_i)) \cdot \prod_{i,j} \exp(f_{i,j}(x_i, x_j))\right]$
. Since maximizing $\log(z)$ over z is the same as maximizing z over z , we can simply compute $\text{argmax}_{\bar{x}} \prod_i \exp(g_i(x_i)) \cdot \prod_{i,j} \exp(f_{i,j}(x_i, x_j))$, which is what max-sum message passing returns. So setting the potentials appropriately and running clique tree inference (which is exact) is guaranteed to get the optimal solution.

(Remember that max-sum message passing involves taking a log-transform of the factors first, and summing up log-transformed factors is equivalent to multiplying them together; don't be tricked by the "sum"!)