Bayesian Network Independencies
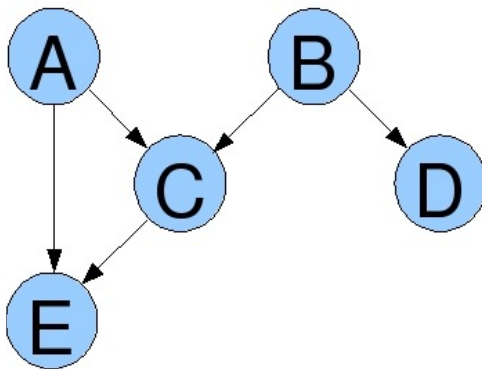
✓ **5/5** 得分（100%）

测验通过！

✔ 1 / 1 分

1.

Independencies in a graph.

Which pairs of variables are independent in the graphical model below, given that none of them have been observed? You may select 1 or more options.

☑ A, B

正确回答
There are no active trails between A and B, so they are independent.

☐ A, E

正确回答
There is a directed edge from A to E.

☐ D, E

正确回答
There is an active trail connecting D and E that goes through B and C.

☐ A, C

正确回答
There is a directed edge from A to C.

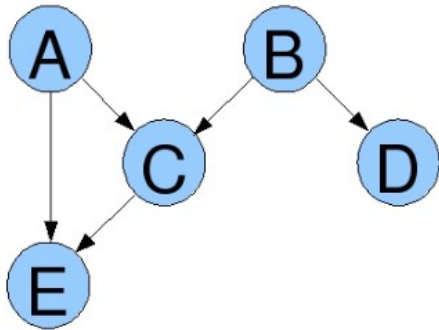☐ None - there are no pairs of independent variables.

✔ 1 / 1 分

2.
*Independencies in a graph. (An asterisk marks a question that is more challenging. Congratulations if you get it right!)

Now assume that the value of E is known. (E is observed. A, B, C, and D are not observed.) Which pairs of variables (not including E) are independent in the same graphical model, given E? You may select 1 or more options.

☑ None - given E, there are no pairs of variables that are independent.

Observing E activates the V-structures around C and E, giving rise to active trails between every pair of variables in the network.

☐ A, B

Observing E activates the V-structures around C and E. Hence, influence can flow from A to B through C; knowing B will now affect the probabilities of A taking on each of its values.

☐ A, C

There is a directed edge from A to C.

☐ A, D

Observing E activates the V-structures around C and E. Hence, influence can flow from A to B through C, and therefore from A to D through C and B.

☐ B, D

There is a directed edge from B to D.

☐ D, C

Influence can flow along the active trail $D \leftarrow B \rightarrow C$.

☐ B, C

There is a directed edge from B to C.

✔ 1 / 1 分

3.

I-maps. I-maps can also be defined directly on graphs as follows. Let $I(G)$ be the set of independencies encoded by a graph G . Then G_1 is an I-map for G_2 if $I(G_1) \subseteq I(G_2)$.

Which of the following statements about I-maps are true? You may select 1 or more options.

☑ A graph K is an I-map for a graph G if and only if all of the independencies encoded by K are also encoded by G.

K is an I-map for G if K does not make independence assumptions that are not true in G. An easy way to remember this is that the complete graph, which has no independencies, is an I-map of all distributions.

☐ A graph K is an I-map for a graph G if and only if K and G are identical, i.e., they have exactly the same nodes and edges.

K is an I-map for G if K and G are identical, but there might be other I-maps for G.

☐ An I-map is a function f that maps a graph G to itself, i.e., $f(G) = G$.

This is an identity function, not an I-map.

☐ The graph K that is the same as the graph G, except that all of the edges are oriented in the opposite direction as the corresponding edges in G, is always an I-map for G, regardless of the structure of G.

This is not always true; consider the V-structure $A \rightarrow B \leftarrow C$.

☐ I-maps are Apple's answer to Google Maps.
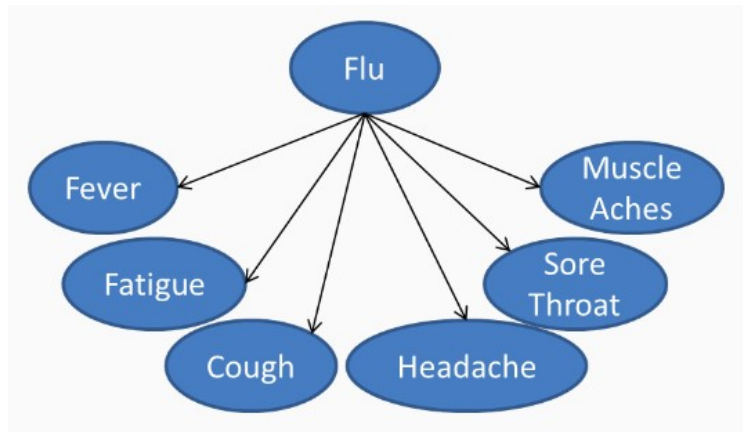
This is not true -- yet!

✔ 1 / 1 分

4.

Naive Bayes.

Consider the following Naive Bayes model for flu diagnosis:

Assume a population size of 10,000. Which of the following statements are true in this model? You may select 1 or more options.

☑ Say we observe that 1000 people have the flu, out of which 500 people have a headache (and possibly other symptoms) and 500 have a fever (and possibly other symptoms).

We would expect that approximately 250 people with the flu also have both a headache and fever.

▲

正确回答

Given that someone has the flu, whether he has a headache is independent of whether he has a fever. We can thus calculate:

$$P(Headache = 1, Fever = 1 | Flu = 1) = P(Headache = 1 | Flu = 1) \times P(Fever = 1 | Flu = 1)$$
$$\approx 0.5 * 0.5$$
$$= 0.25.$$

Since 1000 people have the flu, we can estimate that 250 of these people will have both a headache and fever.

Note that this is only an estimate: we can assert with high confidence that $P(Headache = 1, Fever = 1 | Flu = 1)$ is near to 0.25, but in general it will not be exactly 0.25. Moreover, even if it is exactly 0.25, the number of people with the flu, a headache and a fever need not be exactly 250 all the time. Think of this as analogous to flipping a fair coin: even though the probability of seeing a heads is exactly 0.5, in any given sequence of coin flips we need not see exactly half of the coins turning up heads.

☐ Say we observe that 1000 people have the flu, out of which 500 people have a headache (and possibly other symptoms) and 500 people have a fever (and possibly other symptoms).

Without more information, we cannot estimate how many people with the flu also have both a headache and fever.

▲

正确回答

Given that someone has the flu, whether he has a headache is independent of whether he has a fever. We can thus calculate:

$$P(Headache = 1, Fever = 1 | Flu = 1) = P(Headache = 1 | Flu = 1) \times P(Fever = 1 | Flu = 1)$$

We can use the data given in the option to estimate this probability, and hence the number of people with the flu who also have both a headache and fever.

☑ Say we observe that 1000 people have a headache (and possibly other symptoms), out of which 500 people have the flu (and possibly other symptoms), and 500 people have a fever (and possibly other symptoms).

Without more information, we cannot estimate how many people with a headache also have both the flu and a fever.

正确回答

Even after observing the Headache variable, there is still an active trail from Flu to Fever. Thus, the probability of someone with a headache also having a flu is dependent on the probability of his having a fever as well. For example, if someone has a flu, he could be more likely to have a fever, irrespective of whether he has a headache or not.

We therefore cannot estimate $P(Flu = 1, Fever = 1|Headache = 1)$ from the conditional marginal probabilities $P(Flu = 1|Headache = 1)$ and $P(Fever = 1|Headache = 1)$.

☑ Say we observe that 500 people have a headache (and possibly other symptoms) and 500 people have a fever (and possibly other symptoms).

Without more information, we cannot estimate how many people have both a headache and fever.

正确回答

Without having observed the Flu variable, there is an active trail from Headache and Fever. Thus, the probability of someone having a headache (without observing flu status) is not independent of the probability of the same person having a fever. For example, if someone has a headache, he might be more likely to have the flu, which would correspondingly increase the probability that he has a fever as well.

We therefore cannot estimate $P(Headache = 1, Fever = 1)$ from the marginal probabilities $P(Headache = 1)$ and $P(Fever = 1)$.

✔ 1 / 1 分

5.
I-maps.

Suppose $(A \perp B) \in \mathcal{I}(P)$, and G is an I-map of P, where G is a Bayesian network and P is a probability distribution. Is it necessarily true that $(A \perp B) \in \mathcal{I}(G)$?

○ Yes

◉ No

正确回答

Since G is an I-map of P, all independencies in G are also in P. However, this doesn't mean that all independencies in P are also in G. An easy way to remember this is that the complete graph, which has no independencies, is an I-map of all distributions.