

# Kolmogorov Backward Equations

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```
using Flux, StochasticDiffEq
using NeuralNetDiffEq
using Plots
using CuArrays
using CUDAnative
```

## 0.1 Introduction on Backward Kolmogorov Equations

The backward Kolmogorov Equation deals with a terminal condition. The one dimensional backward kolmogorov equation that we are going to deal with is of the form :

$$\frac{\partial p}{\partial t} = -\mu(x) \frac{\partial p}{\partial x} - \frac{1}{2} \sigma^2(x) \frac{\partial^2 p}{\partial x^2}, \quad p(T, x) = \varphi(x)$$

for all  $t \in [0, T]$  and for all  $x \in \mathbb{R}^d$

**The Black Scholes Model** The Black-Scholes Model governs the price evolution of the European put or call option. In the below equation  $V$  is the price of some derivative,  $S$  is the Stock Price,  $r$  is the risk free interest rate and  $\sigma$  the volatility of the stock returns. The payoff at a time  $T$  is known to us. And this makes it a terminal PDE. In case of an European put option the PDE is:

$$\frac{\partial V}{\partial t} + rS \frac{\partial V}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rV = 0, \quad V(T, S) = \max\{K - S, 0\}$$

for all  $t \in [0, T]$  and for all  $S \in \mathbb{R}^d$

In order to make the above equation in the form of the Backward - Kolmogorov PDE we should substitute

$$V(S, t) = e^{r(t-T)} p(S, t)$$

and thus we get

$$e^{r(t-T)} \frac{\partial p}{\partial t} + r e^{r(t-T)} p(S, t) = -\mu(x) \frac{\partial p}{\partial x} e^{r(t-T)} - \frac{1}{2} \sigma^2(x) \frac{\partial^2 p}{\partial x^2} e^{r(t-T)} + r e^{r(t-T)} p(S, t)$$

And the terminal condition

$$p(S, T) = \max\{\mathcal{K} - x, 0\}$$

We will train our model and the model itself will be the solution of the equation

## 0.2 Defining the problem and the solver

We should start defining the terminal condition for our equation:

```
function phi(xi)
    y = Float64[]
    K = 100
    for x in eachcol(xi)
        val = max(K - maximum(x) , 0.00)
        y = push!(y , val)
    end
    y = reshape(y , 1 , size(y)[1] )
    return y
end
```

phi (generic function with 1 method)

Now we shall define the problem : We will define the  $\sigma$  and  $\mu$  by comparing it to the original equation. The xspan is the span of initial stock prices.

```
d = 1
r = 0.04
sigma = 0.2
xspan = (80.00 , 115.0)
tspan = (0.0 , 1.0)
σ(du , u , p , t) = du .= sigma.*u
μ(du , u , p , t) = du .= r.*u
prob = KolmogorovPDEProblem(μ , σ , phi , xspan , tspan, d)
```

```
KolmogorovPDEProblem
timespan: (0.0, 1.0)xspan: (80.0, 115.0)μ
Main.##WeaveSandBox#563.μSigma
Main.##WeaveSandBox#563.σ
```

Now once we have defined our problem it is necessary to define the parameters for the solver.

```
sdealg = EM()
ensemblealg = EnsembleThreads()
dt = 0.01
dx = 0.01
trajectories = 100000

100000
```

Now lets define our model m and the optimiser

```
m = Chain(Dense(d, 64, elu),Dense(64, 128, elu),Dense(128 , 16 , elu) , Dense(16 , 1))
use_gpu = false
if CUDAactive.functional() == true
    m = fmap(CuArrays.cu , m)
    use_gpu = true
end
opt = Flux.ADAM(0.0005)
```

```
Flux.Optimise.ADAM(0.0005, (0.9, 0.999), IdDict{Any,Any}())
```

And then finally call the solver

```
@time sol = solve(prob, NeuralNetDiffEq.NNKolmogorov(m, opt, sdealg, ensemblealg),  
verbose = true, dt = dt,  
          dx = dx , trajectories = trajectories , abstol=1e-6, maxiters = 1000 ,  
use_gpu = use_gpu)
```

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(Float32[98.92 110.1 ... 97.88 91.6], Float32[6.7236214 3.1487408 ... 7.1996875
10.386402])

```

### 0.3 Analyzing the solution

Now let us find a Monte-Carlo Solution and plot the both:

```

monte_carlo_sol = []
x_out = collect(85:2.00:110.00)
for x in x_out
    u_0= [x]
    g_val(du , u , p , t) = du . = 0.2.*u
    f_val(du , u , p , t) = du . = 0.04.*u
    dt = 0.01
    tspan = (0.0,1.0)
    prob = SDEProblem(f_val,g_val,u_0,tspan)
    output_func(sol,i) = (sol[end],false)
    ensembleprob_val = EnsembleProblem(prob , output_func = output_func )
    sim_val = solve(ensembleprob_val, EM(), EnsembleThreads() , dt=0.01,
trajectories=100000,adaptive=false)
    s = reduce(hcat , sim_val.u)
    mean_phi = sum(phi(s))/length(phi(s))
    global monte_carlo_sol = push!(monte_carlo_sol , mean_phi)
end

```

##Plotting the Solutions We should reshape the inputs and outputs to make it compatible with our model. This is the most important part. The algorithm gives a distributed function over all initial prices in the xspan.

```

x_model = reshape(x_out, 1 , size(x_out)[1])
if use_gpu == true
    m = fmap(cpu , m)
end
y_out = m(x_model)
y_out = reshape(y_out , 13 , 1)

```

13×1 Array{Float32,2}:

```

14.734822
13.272918
11.932178
10.727601
9.620036
8.586618
7.612422
6.687542
5.841154
5.129661
4.5073957
3.9442136
3.4223313

```

And now finally we can plot the solutions

```

plot(x_out , y_out , lw = 3 ,  xaxis="Initial Stock Price", yaxis="Payoff" , label =
"NNKolmogorov")
plot!(x_out , monte_carlo_sol , lw = 3 ,  xaxis="Initial Stock Price", yaxis="Payoff"
,label = "Monte Carlo Solutions")

```

