

The Outer Solar System

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0.1 Data

The chosen units are: masses relative to the sun, so that the sun has mass 1. We have taken $m_0 = 1.00000597682$ to take account of the inner planets. Distances are in astronomical units, times in earth days, and the gravitational constant is thus $G = 2.95912208286 \cdot 10^{-4}$.

planet	mass	
Jupiter	$m_1 = 0.000954786104043$	$\langle \text{ul} \rangle \langle \text{li} \rangle -3.5023653 \langle / \text{li} \rangle \langle \text{li} \rangle -3.8169847 \langle / \text{li} \rangle \langle \text{li} \rangle -1.5$
Saturn	$m_2 = 0.000285583733151$	$\langle \text{ul} \rangle \langle \text{li} \rangle 9.0755314 \langle / \text{li} \rangle \langle \text{li} \rangle -3.0458353 \langle / \text{li} \rangle \langle \text{li} \rangle -1.6$
Uranus	$m_3 = 0.0000437273164546$	$\langle \text{ul} \rangle \langle \text{li} \rangle 8.3101420 \langle / \text{li} \rangle \langle \text{li} \rangle -16.2901086 \langle / \text{li} \rangle \langle \text{li} \rangle -7.2$
Neptune	$m_4 = 0.0000517759138449$	$\langle \text{ul} \rangle \langle \text{li} \rangle 11.4707666 \langle / \text{li} \rangle \langle \text{li} \rangle -25.7294829 \langle / \text{li} \rangle \langle \text{li} \rangle -10.8$
Pluto	$m_5 = 1/(1.3 \cdot 10^8)$	$\langle \text{ul} \rangle \langle \text{li} \rangle -15.5387357 \langle / \text{li} \rangle \langle \text{li} \rangle -25.2225594 \langle / \text{li} \rangle \langle \text{li} \rangle -3.1$

The data is taken from the book "Geometric Numerical Integration" by E. Hairer, C. Lubich and G. Wanner.

```
using Plots, OrdinaryDiffEq, DiffEqPhysics, RecursiveArrayTools
gr()

G = 2.95912208286e-4
M = [1.00000597682, 0.000954786104043, 0.000285583733151, 0.0000437273164546,
0.0000517759138449, 1/1.3e8]
planets = ["Sun", "Jupiter", "Saturn", "Uranus", "Neptune", "Pluto"]

pos_x = [0.0, -3.5023653, 9.0755314, 8.3101420, 11.4707666, -15.5387357]
pos_y = [0.0, -3.8169847, -3.0458353, -16.2901086, -25.7294829, -25.2225594]
pos_z = [0.0, -1.5507963, -1.6483708, -7.2521278, -10.8169456, -3.1902382]
pos = ArrayPartition(pos_x, pos_y, pos_z)

vel_x = [0.0, 0.00565429, 0.00168318, 0.00354178, 0.00288930, 0.00276725]
vel_y = [0.0, -0.00412490, 0.00483525, 0.00137102, 0.00114527, -0.00170702]
vel_z = [0.0, -0.00190589, 0.00192462, 0.00055029, 0.00039677, -0.00136504]
vel = ArrayPartition(vel_x, vel_y, vel_z)

tspan = (0., 200_000)

(0.0, 200000)
```

The N-body problem's Hamiltonian is

$$H(p, q) = \frac{1}{2} \sum_{i=0}^N \frac{p_i^T p_i}{m_i} - G \sum_{i=1}^N \sum_{j=0}^{i-1} \frac{m_i m_j}{\|q_i - q_j\|}$$

Here, we want to solve for the motion of the five outer planets relative to the sun, namely, Jupiter, Saturn, Uranus, Neptune and Pluto.

```
const sum = sum
const N = 6
potential(p, t, x, y, z, M) = -G*sum(i->sum(j->(M[i]*M[j])/sqrt((x[i]-x[j])^2 +
(y[i]-y[j])^2 + (z[i]-z[j])^2), 1:i-1), 2:N)

potential (generic function with 1 method)
```

0.2 Hamiltonian System

NBodyProblem constructs a second order ODE problem under the hood. We know that a Hamiltonian system has the form of

$$\dot{p} = -H_q(p, q) \quad \dot{q} = H_p(p, q)$$

For an N-body system, we can symplify this as:

$$\dot{p} = -\nabla V(q) \quad \dot{q} = M^{-1}p.$$

Thus \dot{q} is defined by the masses. We only need to define \dot{p} , and this is done internally by taking the gradient of V . Therefore, we only need to pass the potential function and the rest is taken care of.

```
nprob = NBodyProblem(potential, M, pos, vel, tspan)
sol = solve(nprob, Yoshida6(), dt=100);
```

```
retcode: Success
```

```
Interpolation: 3rd order Hermite
```

```
t: 2001-element Array{Float64,1}:
```

```
  0.0
 100.0
 200.0
 300.0
 400.0
 500.0
 600.0
 700.0
 800.0
 900.0
```

```
⋮
```

```
199200.0
199300.0
199400.0
199500.0
199600.0
199700.0
199800.0
199900.0
200000.0
```

```
u: 2001-element Array{RecursiveArrayTools.ArrayPartition{Float64,Tuple{RecursiveArrayTools.ArrayPartition{Float64,Tuple{Array{Float64,1},Array{Float64,1},Array{Float64,1}}},RecursiveArrayTools.ArrayPartition{Float64,Tuple{Array{Float64,1},Array{Float64,1},Array{Float64,1}}}},1}:
 [0.0, -3.5023653, 9.0755314, 8.310142, 11.4707666, -15.5387357] [0.0, -3.81
```

69847, -3.0458353, -16.2901086, -25.7294829, -25.2225594] [0.0, -1.5507963, -1.6483708, -7.2521278, -10.8169456, -3.1902382] [0.0, 0.00565429, 0.00168318, 0.00354178, 0.0028893, 0.00276725] [0.0, -0.0041249, 0.00483525, 0.00137102, 0.00114527, -0.00170702] [0.0, -0.00190589, 0.00192462, 0.00055029, 0.00039677, -0.00136504]

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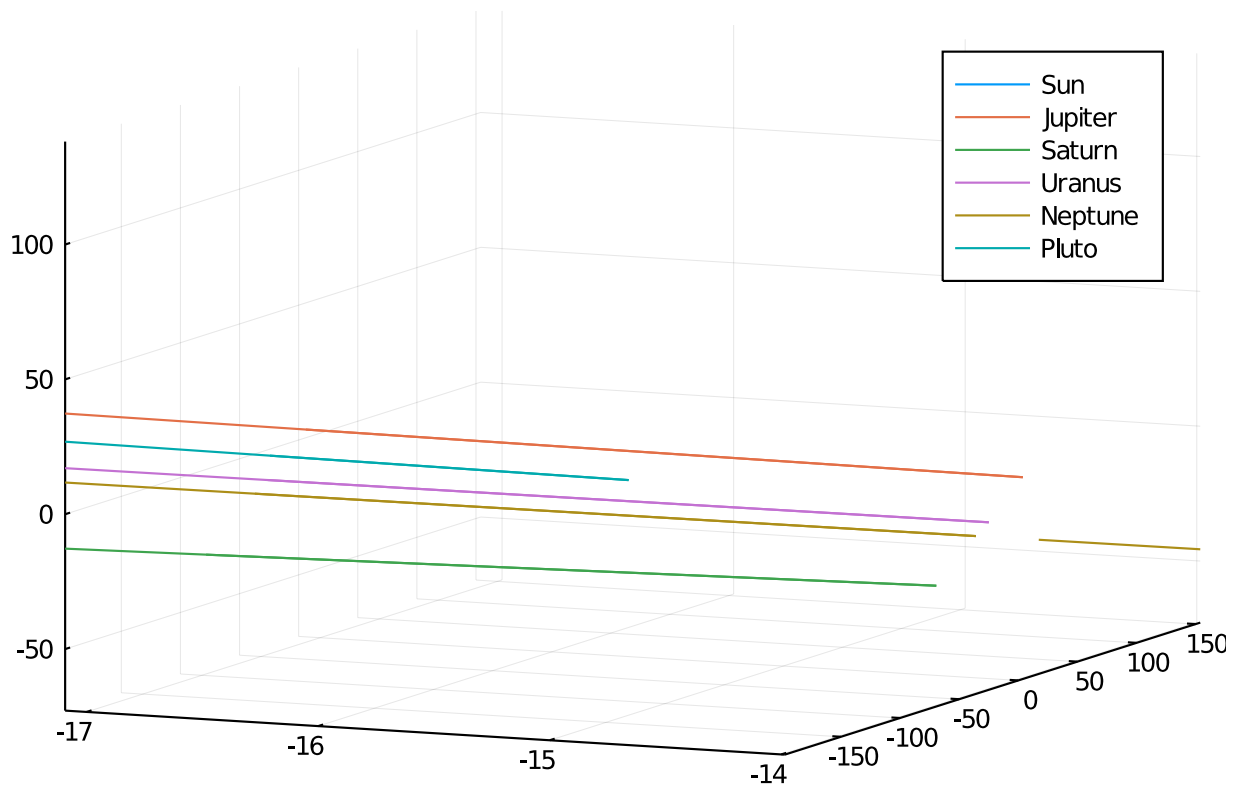
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-638047.639331222]

```

```
orbitplot(sol,body_names=planets)
```



0.3 Appendix

This tutorial is part of the DiffEqTutorials.jl repository, found at: <https://github.com/JuliaDiffEq/DiffEqTutorials>

To locally run this tutorial, do the following commands:

```
using DiffEqTutorials
```

```
DiffEqTutorials.weave_file("models","07-outer_solar_system.jmd")
```

Computer Information:

Julia Version 1.4.2

Commit 44fa15b150* (2020-05-23 18:35 UTC)

Platform Info:

OS: Linux (x86_64-pc-linux-gnu)

CPU: Intel(R) Core(TM) i7-9700K CPU @ 3.60GHz

WORD_SIZE: 64

LIBM: libopenlibm

LLVM: libLLVM-8.0.1 (ORCJIT, skylake)

Environment:

JULIA_DEPOT_PATH = /builds/JuliaGPU/DiffEqTutorials.jl/.julia

JULIA_CUDA_MEMORY_LIMIT = 536870912

JULIA_PROJECT = @.

JULIA_NUM_THREADS = 4

Package Information:

Status `~/builds/JuliaGPU/DiffEqTutorials.jl/tutorials/models/Project.toml`

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```