Lesson 5.1–5.2

9/9 points (100%)

Quiz, 9 questions

✔ **Congratulations! You passed!**   [Next Item]
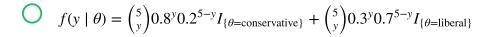
✔   1 / 1
      points

1.
For Questions 1-5, consider the following scenario:

You are trying to ascertain your American colleague's political preferences. To do so, you design a questionnaire with five yes/no questions relating to current issues. The questions are all worded so that a "yes" response indicates a conservative viewpoint.

Let θ be the unknown political viewpoint of your colleague, which we will assume can only take values $\theta = \text{conservative}$ or $\theta = \text{liberal}$. You have no reason to believe that your colleague leans one way or the other, so you assign the prior $P(\theta = \text{conservative}) = 0.5$.

Assume the five questions are independent and let Y count the number of "yes" responses. If your colleague is conservative, then the probability of a "yes" response on any given question is 0.8. If your colleague is liberal, the probability of a "no" response on any given question is 0.7.

- What is an appropriate likelihood for this scenario?

○ $f(y \mid \theta) = \binom{5}{y} 0.8^y 0.2^{5-y} I_{\{\theta=\text{conservative}\}} + \binom{5}{y} 0.3^y 0.7^{5-y} I_{\{\theta=\text{liberal}\}}$

▲

Correct
If your colleague is conservative, the number of "yes" responses will follow a Binomial(5, 0.8). If liberal, the number of "yes" responses will follow a Binomial(5, 0.3).

○ $f(y \mid \theta) = \binom{5}{y} 0.8^y 0.2^{5-y}$

○ $f(y \mid \theta) = \binom{5}{y} 0.3^y 0.7^{5-y} I_{\{\theta=\text{conservative}\}} + \binom{5}{y} 0.8^y 0.2^{5-y} I_{\{\theta=\text{liberal}\}}$

○ $f(y \mid \theta) = \theta^y e^{-\theta}/y!$

○ $f(y \mid \theta) = \binom{5}{y} 0.2^y 0.8^{5-y}$

Lesson 5.1–5.2

9/9 points (100%)

Quiz, 9 questions   ✔   1 / 1 points

2.
Political preferences:

- Suppose you ask your colleague the five questions and he answers "no" to all of them. What is the MLE for θ?

  ○ $\hat\theta = \text{conservative}$

  ◉ $\hat\theta = \text{liberal}$

  ▲

  Correct

  This is
  $$\binom{5}{0}.8^0.2^5 I_{\{\theta=\text{conservative}\}} + \binom{5}{0}.3^0.7^5 I_{\{\theta=\text{liberal}\}} = .2^5 I_{\{\theta=\text{conservative}\}} + .7^5 I_{\{\theta=\text{liberal}\}}$$
  which evaluates to $.2^5$ if $\theta = \text{conservative}$ and $.7^5$ if $\theta = \text{liberal}$. Hence,
  $\hat\theta = \text{liberal}$ returns a higher likelihood value.

  This result is intuitive because if your colleague answered "no" to all conservative-leaning questions, he is likely not conservative.

  ○   None of the above. The MLE is a number.

✔   1 / 1 points

3.
Political preferences:

- Recall that Bayes' theorem gives $f(\theta \mid y) = \frac{f(y|\theta)f(\theta)}{\sum_\theta f(y|\theta)f(\theta)}$. What is the corresponding expression for this problem?

  ○ $f(\theta \mid y) = \dfrac{\theta^y e^{-\theta}(0.5)/y!}{0.8^y e^{-.8}(0.5)/y!+0.3^y e^{-.3}(0.5)/y!}$

  ○ $f(\theta \mid y) = \dfrac{\binom{5}{y}0.8^y 0.2^{5-y}(0.5)+\binom{5}{y}0.3^y 0.7^{5-y}(0.5)}{\binom{5}{y}0.8^y 0.2^{5-y}(0.5)+\binom{5}{y}0.3^y 0.7^{5-y}(0.5)}$

  ○ $f(\theta \mid y) = \dfrac{\binom{5}{y}0.8^y 0.2^{5-y}(0.5)^2}{\binom{5}{y}0.8^y 0.2^{5-y}(0.5)+\binom{5}{y}0.3^y 0.7^{5-y}(0.5)}$

  ○

$$f(\theta \mid y) = \frac{\binom{5}{y}0.8^{y}0.2^{5-y}(0.5)I_{\{\theta=\text{conservative}\}} + \binom{5}{y}0.3^{y}0.7^{5-y}(0.5)I_{\{\theta=\text{liberal}\}}}{\binom{5}{y}0.8^{y}0.2^{5-y}(0.5) + \binom{5}{y}0.3^{y}0.7^{5-y}(0.5)}$$

Lesson 5.1–5.2

Quiz, 9 questions

9/9 points (100%)

Correct

The prior probability was 0.5 for both values of θ.

Notice we have added over all possibilities of θ to get the denominator. Hence, the denominator contains no θ and evaluates to a number when we plug in y. This is the marginal probability of observing y, which gives us the normalizing constant.

○ $$f(\theta \mid y) = \frac{\binom{5}{y}0.8^{y}0.2^{5-y}(0.2)I_{\{\theta=\text{conservative}\}} + \binom{5}{y}0.3^{y}0.7^{5-y}(0.7)I_{\{\theta=\text{liberal}\}}}{\binom{5}{y}0.8^{y}0.2^{5-y}(0.2) + \binom{5}{y}0.3^{y}0.7^{5-y}(0.7)}$$

✔ **1 / 1 points**

4.
Political preferences:

- Evaluate the expression in Question 3 for $y = 0$ and report the posterior probability that your colleague is conservative, given that he responded "no" to all of the questions. Round your answer to three decimal places.

0.002

Correct Response

This is $P(\theta = \text{conservative} \mid Y = 0) = \frac{.2^{5}(.5)}{.2^{5}(.5)+.7^{5}(.5)} = \frac{.2^{5}}{.2^{5}+.7^{5}}$.

✔ **1 / 1 points**

5.
Political preferences:

- Evaluate the expression in Question 3 for $y = 0$ and report the posterior probability that your colleague is liberal, given that he responded "no" to all of the questions. Round your answer to three decimal places.

0.998

Correct Response

This is $P(\theta = \text{liberal} \mid Y = 0) = \dfrac{.7^5(.5)}{.2^5(.5)+.7^5(.5)} = \dfrac{.7^5}{.2^5+.7^5}$.

Lesson 5.1–5.2

Quiz, 9 questions

9/9 points (100%)

Note that once we have calculated the posterior probability for this hypothesis, we can easily obtain the posterior probability for the opposite (complementary) hypothesis with
$P(\theta = \text{conservative} \mid Y = 0) = 1 - P(\theta = \text{liberal} \mid Y = 0)$.

✔ 1 / 1 points

6.

For Questions 6-9, consider again the loaded coin example from the lesson.

Recall that your brother has a fair coin which comes up heads 50% of the time and a loaded coin which comes up heads 70% of the time.

Suppose now that he has a third coin which comes up tails 70% of the time. Again, you don't know which coin your brother has brought you, so you are going to test it by flipping it 4 times, where counts the number of heads. Let identify the coin so that there are three possibilities , , and .

Suppose the prior is now , , and . Our prior probability that the coin is loaded is still 0.6, but we do not know which loaded coin it is, so we split the probability evenly between the two options.

- What is the form of the likelihood now that we have three options?

○

⯅

Correct

○

○

○

✔ 1 / 1 points

7.

Loaded coins:

Lesson 5.1–5.2 Suppose you flip the coin four times and it comes up heads twice. What is the MLE for **9/9 points (100%)** $$\theta$$?

Quiz, 9 questions

     ⊙ $$\hat\theta = \text{fair}$$

Correct
$$ {4 \choose 2} 0.5^4 $$ is the highest value among the three when $$X=2$$.

     ○ $$\hat\theta = \text{loaded heads}$$

     ○ $$\hat\theta = \text{loaded tails}$$

     ○   None of the above. The MLE is a number.

✓   1 / 1
points

8.
Loaded coins:

- Suppose you flip the coin four times and it comes up heads twice. What is the posterior probability that this is the fair coin? Round your answer to two decimal places.

> 0.49

Correct Response
This is $$ P(\theta = \text{fair} \mid X=2) = \frac{ {4 \choose 2} 0.5^2 0.5^2 (0.4) } { {4 \choose 2} \left[0.5^4 (0.4) + 0.7^2 0.3^{2} (0.3) + 0.3^2 0.7^{2} (0.3) \right] }$$.

✓   1 / 1
points

9.
Loaded coins:

Lesson 5.1–5.2

9/9 points (100%)

Quiz, 9 questions

- Suppose you flip the coin four times and it comes up heads twice. What is the posterior probability that this is a loaded coin (favoring either heads or tails)? Round your answer to two decimal places.

Hint: $$ P(\theta=\text{fair} \mid X=2) = 1 - P(\theta=\text{loaded} \mid X=2)$$, so you can use your answer from the previous question rather than repeat the calculation from Bayes' theorem (both approaches yield the same answer).

0.51

Correct Response
This is 1 - .486 (from the previous question).