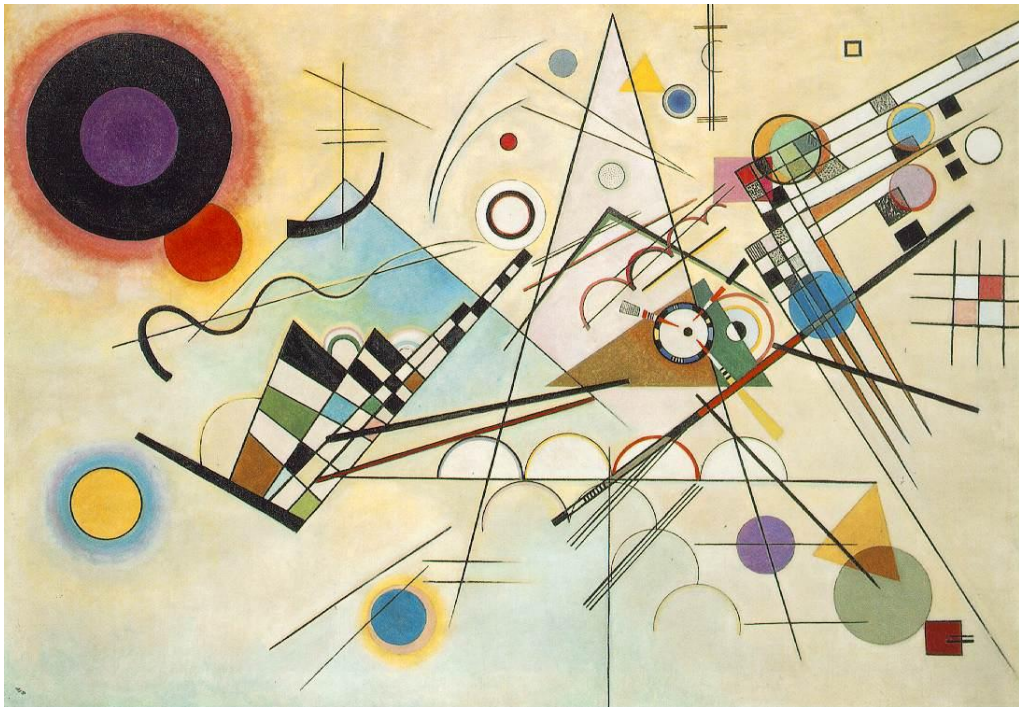


# INTENSIONAL SEMANTICS



KAI VON FINTEL      IRENE HEIM

1997 –  $\infty$

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## About these lecture notes

These lecture notes have been evolving for years now, starting with some old notes from the early 1990s by Angelika Kratzer, Irene Heim, and Kai von Fintel, which have since been modified and expanded many times by Irene and/or Kai.

We encourage the use of these notes in courses at other institutions. Of course, you need to give full credit to the authors and you may not use the notes for any commercial purposes. If you use the notes, we would like to be notified and we would very much appreciate any comments, criticism, and advice on these materials.

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Link to the latest full version (not necessarily up-to-date):

<http://kvf.me/intensional>

GitHub repository with the current & up-to-date development version:

<https://github.com/fintelkai/fintel-heim-intensional-notes>

Most recent homepage of the class these notes are designed for:

<http://kvf.me/advsem>

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## Some advice

1. These notes presuppose familiarity with the material, concepts, and notation of the Heim & Kratzer textbook.
2. There are numerous exercises throughout the notes. It is highly recommended to do all of them and it is certainly necessary to do so if you at all anticipate doing semantics-related work in the future.
3. The notes are designed to go along with explanatory lectures. You should ask questions and make comments as you work through the notes.
4. Students with semantic ambitions should also at an early point start reading supplementary material (as for example listed at the end of each chapter of these notes).
5. Prospective semanticists may start thinking about how *they* would teach this material.
6. For more advice along these lines, see <http://kaivonfintel.org/prerequisites/>.

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# CONTENTS

## I Intensionality: Conditionals, Attitudes, Modals I

- I Beginnings 3
  - 1.1 Displacement 3
  - 1.2 An Intensional Semantics in 10 Easy Steps 6
  - 1.3 Comments and Complications 14
  - 1.4 \*Issues with an informal meta-language 18
  - 1.5 Supplemental Readings 19
- 2 Conditionals 23
  - 2.1 *If* ... the biggest little word 23
  - 2.2 The Restricted Strict Implication Analysis 24
  - 2.3 A red herring: Material implication 25
  - 2.4 Supplemental Readings 28
- 3 Propositional Attitudes 29
  - 3.1 Hintikka's Idea 29
  - 3.2 Accessibility Relations 32
  - 3.3 Supplemental Readings 36

## II DPs in Intensional Contexts 39

### Bibliography 41

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## PART I

INTENSIONALITY: CONDITIONALS, ATTITUDES, MODALS



# CHAPTER ONE

## BEGINNINGS

Language is the main  
instrument of man's refusal  
to accept the world as it is.

---

George Steiner, *After Babel*,  
p. 228

*We introduce the idea of extension vs. intension and its main use: taking us from the actual here and now to past, future, possible, counterfactual situations. We develop a compositional framework for intensional semantics.*

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I.1	Displacement	3
I.2	An Intensional Semantics in 10 Easy Steps	6
I.2.1	Laying the Foundations	6
I.2.2	Intensional Operators	9
I.3	Comments and Complications	14
I.3.1	Intensions All the Way?	14
I.3.2	Why Talk about Other Worlds?	15
I.3.3	The Worlds of Sherlock Holmes	16
I.3.4	What's next	17
I.4	*Issues with an informal meta-language	18
I.5	Supplemental Readings	19

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### I.1 Displacement

Hockett (1960) in a famous article (and a follow-up, Hockett & Altmann (1968)) presented a list of DESIGN FEATURES OF HUMAN LANGUAGE. This list continues to

Charles F. Hockett. 1960.  
The origin of speech. *Scientific American* 203. 89–96

play a role in current discussions of animal communication. One of the design features is **DISPLACEMENT**. Human language is not restricted to discourse about the *actual here and now*.<sup>1</sup>

How does natural language untie us from the actual here and now? One degree of freedom is given by the ability to name entities and refer to them even if they are not where we are when we speak:

- (1) Thomas is in Hamburg.

This kind of displacement is not something we will explore here. We'll take it for granted.

Consider a sentence with no names of absent entities in it:

- (2) It is snowing (in Cambridge).

On its own, (2) makes a claim about what is happening right now here in Cambridge. But there are devices at our disposal that can be added to (2), resulting in claims about snow in displaced situations. Displacement can occur in the **TEMPORAL** dimension and/or in what might be called the **MODAL** dimension. Here's an example of temporal displacement:

- (3) At noon yesterday, it was snowing in Cambridge.

This sentence makes a claim not about snow now but about snow at noon yesterday, a different time from now.

Here's an example of modal displacement:

- (4) If the storm system hadn't been deflected by the jet stream, it would have been snowing in Cambridge.

This sentence makes a claim not about snow in the actual world but about snow in the world as it would have been if the storm system hadn't been deflected by the jet stream, a world distinct from the actual one (where the system did not hit us), a merely **POSSIBLE WORLD**.

Natural language abounds in modal constructions. (4) is a so-called **COUNTERFACTUAL CONDITIONAL**. Here are some other examples:

- (5) **MODAL AUXILIARIES**  
It may be snowing in Cambridge.
- (6) **MODAL ADVERBS**  
Possibly, it will snow in Cambridge tomorrow.

The terms **MODAL** and **MODALITY** descend from the Latin *modus*, "way", and are ancient terms pertaining to the way a proposition holds, necessarily, contingently, etc.

See Kratzer (1981, 1991) for more examples of modal constructions.

<sup>1</sup> Steiner (1998: 226) writes: "Hypotheticals, 'imaginaries', conditionals, the syntax of counterfactuality and contingency may well be the generative centres of human speech".

- (7) PROPOSITIONAL ATTITUDES  
Jens believes that it is snowing in Cambridge.
- (8) HABITUALS  
Jane smokes.
- (9) GENERICS  
Bears like honey.

In this chapter, we will put in place the basic framework of INTENSIONAL SEMANTICS, the kind of semantics that models displacement of the point of evaluation in temporal and modal dimensions. To do this, we will start with one rather special example of modal displacement:

- (10) In the world of Sherlock Holmes, a detective lives at 221B Baker Street.

(10) doesn't claim that a detective lives at 221B Baker Street in the actual world (presumably a false claim), but that in the world as it is described in the Sherlock Holmes stories of Sir Arthur Conan Doyle, a detective lives at 221B Baker Street (a true claim, of course). We choose this example rather than one of the more run-of-the-mill displacement constructions because we want to focus on conceptual and technical matters before we do serious empirical work.

The questions we want to answer are: How does natural language achieve this feat of modal displacement? How do we manage to make claims about other possible worlds? And why would we want to?

To make displacement possible and compositionally tractable, we need meanings of natural language expressions, and of sentences in particular, to be displaceable in the first place. They need to be “portable”, so to speak, able to make claims about more than just the actual here and now. And need we need other natural language expressions that take that portable meaning and apply it to some situation other than the actual here and now. That is what intensionality is all about.

The basic idea of the account we'll develop is this:

- expressions are assigned their semantic values relative to a possible world;
- in particular, sentences have truth-values in possible worlds;
- in the absence of modal displacement, we evaluate sentences with respect to the “actual” world, the world in which we are speaking;
- modal displacement changes the world of evaluation;
- displacement is effected by special operators, whose semantics is our primary concern here.

A terminological note: we will call the sister of the intensional operator its PREJACENT, a useful term introduced by our medieval colleagues.



[https://en.wikipedia.org/wiki/Sherlock\\_Holmes](https://en.wikipedia.org/wiki/Sherlock_Holmes)

## 1.2 An Intensional Semantics in 10 Easy Steps

### 1.2.1 Laying the Foundations

STEP I: POSSIBLE WORLDS. Our first step is to introduce possible worlds. This is not the place to discuss the metaphysics of possible worlds in any depth. Instead, we will just start working with them and see what they can do for us. Basically, a possible world is a way that things might have been. In the actual world, there are two coffee mugs on my desk, but there could have been more or less. So, there is a possible world — albeit a rather bizarre one — where there are 17 coffee mugs on my desk. We join Heim & Kratzer in adducing this quote from [D. Lewis \(1986: 1f.\)](#):



David Lewis

The world we live in is a very inclusive thing. Every stick and every stone you have ever seen is part of it. And so are you and I. And so are the planet Earth, the solar system, the entire Milky Way, the remote galaxies we see through telescopes, and (if there are such things) all the bits of empty space between the stars and galaxies. There is nothing so far away from us as not to be part of our world. Anything at any distance at all is to be included. Likewise the world is inclusive in time. No long-gone ancient Romans, no long-gone pterodactyls, no long-gone primordial clouds of plasma are too far in the past, nor are the dead dark stars too far in the future, to be part of the same world. . . .

The way things are, at its most inclusive, means the way the entire world is. But things might have been different, in ever so many ways. This book of mine might have been finished on schedule. Or, had I not been such a commonsensical chap, I might be defending not only a plurality of possible worlds, but also a plurality of impossible worlds, whereof you speak truly by contradicting yourself. Or I might not have existed at all — neither myself, nor any counterparts of me. Or there might never have been any people. Or the physical constants might have had somewhat different values, incompatible with the emergence of life. Or there might have been altogether different laws of nature; and instead of electrons and quarks, there might have been alien particles, without charge or mass or spin but with alien physical properties that nothing in this world shares. There are ever so many ways that a world might be: and one of these many ways is the way that this world is.

Previously, our “metaphysical inventory” included a domain of entities and a set of two truth-values and increasingly complex functions between entities, truth-values, and functions thereof. Now, we will add possible worlds to the inventory. Let’s assume we are given a set  $W$ , the set of all possible worlds, which

is a vast space since there are so many ways that things might have been different from the way they are. Each world has as among its parts entities like you and me and these coffee mugs. Some of them may not exist in other possible worlds. So, strictly speaking each possible worlds has its own, possibly distinctive, domain of entities. What we will use in our system, however, will be the grand union of all these world-specific domains of entities. We will use  $D$  to stand for the set of all possible individuals.

Among the many possible worlds that there are — according to Lewis, there is a veritable plenitude of them — is the world as it is described in the Sherlock Holmes stories by Sir Arthur Conan Doyle. In that world, there is a famous detective Sherlock Holmes, who lives at 221B Baker Street in London and has a trusted sidekick named Dr. Watson. Our sentence *In the world of Sherlock Holmes, a detective lives at 221B Baker Street* displaces the claim that a famous detective lives at 221B Baker Street from the actual world to the world as described in the Sherlock Holmes stories. In other words, the following holds:<sup>2</sup>

- (11) The sentence *In the world of Sherlock Holmes, a detective lives at 221B Baker Street* is true in a world  $w$  iff the sentence *a detective lives at 221B Baker Street* is true in the world as it is described in the Sherlock Holmes stories.

What this suggests is that we need to make space in our system for having devices that control in what world a claim is evaluated. This is what we will do now.

STEP 2: THE EVALUATION WORLD PARAMETER. Recall from H&K that we were working with a semantic interpretation function that was relativized to an assignment function  $g$ , which was needed to take care of pronouns, traces, variables, etc. From now on, we will relativize the semantic values in our system to possible worlds as well. What this means is that from now on, our interpretation function will have two superscripts: a world  $w$  and an assignment  $g$ :  $\llbracket \cdot \rrbracket^{w,g}$ .

So, the prejacent embedded in (10) will have its truth-conditions described as follows:<sup>3</sup>

- (12)  $\llbracket \text{a famous detective lives at 221B Baker Street} \rrbracket^{w,g} = 1$   
iff a famous detective lives at 221B Baker Street in world  $w$ .

It is customary to refer to the world for which we are calculating the extension of a given expression as the EVALUATION WORLD. In the absence of any shifting devices, we would normally evaluate a sentence in the actual world. But then

<sup>2</sup> We will see in Section (1.3.2) that this is not quite right. It'll do for now.

<sup>3</sup> Recall from H&K, pp.22f, that what's inside the interpretation brackets is a mention of an object language expression. They make this clear by bold-facing all object language expressions inside interpretation brackets. In these notes, we will follow common practice in the field and not use a special typographic distinction, but let it be understood that what is interpreted are object language expressions.

there are shifting devices such as our *in the world of Sherlock Holmes*. We will soon see how they work. But first some more pedestrian steps: adding lexical entries and composition principles that are formulated relative to a possible world. This will allow us to derive the truth-conditions as stated in (12) in a compositional manner.

STEP 3: LEXICAL ENTRIES. Among our lexical items, we can distinguish between items which have a WORLD-DEPENDENT semantic value and those that are world-independent. Predicates are typically world-dependent. Here are some sample entries.

(13) For any  $w \in W$  and any assignment function  $g$ :

- a.  $\llbracket \text{famous} \rrbracket^{w,g} = \lambda x \in D. x \text{ is famous in } w.$ <sup>4,5</sup>
- b.  $\llbracket \text{detective} \rrbracket^{w,g} = \lambda x \in D. x \text{ is a detective in } w.$
- c.  $\llbracket \text{lives-at} \rrbracket^{w,g} = \lambda x \in D. \lambda y \in D. y \text{ lives-at } x \text{ in } w.$

The set of detectives will obviously differ from world to world, and so will the set of famous individuals and the set of pairs where the first element lives at the second element.

Other items have semantic values which do not differ from world to world. The most important such items are certain “logical” expressions, such as truth-functional connectives and determiners:

- (14) a.  $\llbracket \text{and} \rrbracket^{w,g} = \lambda u \in D_t. \lambda v \in D_t. u = v = \mathbf{I}.$   
 b.  $\llbracket \text{the} \rrbracket^{w,g} = \lambda f \in D_{\langle e,t \rangle} : \exists! x [f(x) = \mathbf{I}]. \text{ the } y \text{ such that } f(y) = \mathbf{I}.$   
 c.  $\llbracket \text{every} \rrbracket^{w,g} = \lambda f_{\langle e,t \rangle}. \lambda h_{\langle e,t \rangle}. \forall x_e : f(x) = \mathbf{I} \rightarrow h(x) = \mathbf{I}.$   
 d.  $\llbracket \text{a/some} \rrbracket^{w,g} = \lambda f_{\langle e,t \rangle}. \lambda h_{\langle e,t \rangle}. \exists x_e : f(x) = \mathbf{I} \ \& \ h(x) = \mathbf{I}.$

Note that there is no occurrence of  $w$  on the right-hand side of the entries in (14). That’s the tell-tale sign of the world-independence of the semantics of these items.

We will also assume that proper names have world-independent semantic values, that is, they refer to the same individual in any possible world.

Note the ruthless condensation of the notation in (c) and (d): variables are subscripted with the type of the domain that their values are constrained to come from.

4 Of course, “ $\lambda x \in D. \dots$ ” is short for “ $\lambda x : x \in D. \dots$ ”. Get used to semanticists condensing their notation whenever convenient! A further step of condensation is taken below: “ $\lambda x : x \in D_e. \dots$ ” becomes “ $\lambda x_e. \dots$ ”.

5 Always make sure that you actually understand what the notation means. Here, for example, we are saying that the semantic value of the word *famous* with respect to a given possible world  $w$  and a variable assignment  $g$  is that function that is defined for an argument  $x$  only if  $x$  is a member of the domain of individuals and that, if it is defined, yields the truth-value  $\mathbf{I}$  if and only if  $x$  is famous in  $w$ . (12)[a] does *not* mean that the function maps  $x$  to “ $x$  is famous in  $w$ ”, which would be very weird: mapping an individual to a meta-language statement!



- (15) a.  $\llbracket \text{Noam Chomsky} \rrbracket^{w,g} = \text{Noam Chomsky}.$   
 b.  $\llbracket \text{Sherlock Holmes} \rrbracket^{w,g} = \text{Sherlock Holmes}.$   
 c.  $\llbracket \text{221B Baker Street} \rrbracket^{w,g} = \text{221B Baker Street}.$

STEP 4: COMPOSITION PRINCIPLES. The old rules of Functional Application, Predicate Modification, and  $\lambda$ -Abstraction can be retained almost intact. We just need to modify them by adding world-superscripts to the interpretation function. For example:

- (16) FUNCTIONAL APPLICATION (FA)  
 If  $\alpha$  is a branching node and  $\{\beta, \gamma\}$  the set of its daughters, then, for any world  $w$  and assignment  $g$ : if  $\llbracket \beta \rrbracket^{w,g}$  is a function whose domain contains  $\llbracket \gamma \rrbracket^{w,g}$ , then  $\llbracket \alpha \rrbracket^{w,g} = \llbracket \beta \rrbracket^{w,g}(\llbracket \gamma \rrbracket^{w,g})$ .

The rule simply passes the world parameter down.

STEP 5: TRUTH. Lastly, we will want to connect our semantic system to the notion of the TRUTH OF AN UTTERANCE. We first adopt the “Appropriateness Condition” from Heim & Kratzer (p.243):

- (17) APPROPRIATENESS CONDITION  
 A context  $c$  is *appropriate* for an LF  $\phi$  only if  $c$  determines a variable assignment  $g_c$  whose domain includes every index which has a free occurrence in  $\phi$ .

We then intensionalize Heim & Kratzer’s definition of truth and falsity of utterances:

- (18) TRUTH AND FALSITY CONDITIONS FOR UTTERANCES  
 An utterance of a sentence  $\phi$  in a context  $c$  in a possible world  $w$  is *true* iff  $\llbracket \phi \rrbracket^{w,g_c} = \mathbf{1}$  and *false* if  $\llbracket \phi \rrbracket^{w,g_c} = \mathbf{0}$ .

EXERCISE 1.1: Compute under what conditions an utterance in possible world  $w_7$  (which may or may not be the one we are all living in) of the sentence *a famous detective lives at 221B Baker Street* is true. [Since this is the first exercise in this framework, please do this in excruciating detail, not skipping any steps.]  $\square$

### 1.2.2 Intensional Operators

So far we have merely “redecorated” the system inherited from Heim & Kratzer. We have introduced possible worlds into our inventory, our lexical entries and our old composition principles. But with the tools we have now, all we can do so far is to keep track of the world in which we evaluate the semantic value of an expression, complex or lexical. We will get real mileage once we introduce INTENSIONAL OPERATORS which are capable of shifting the world parameter.

We mentioned that there are a number of devices for modal displacement. As advertised, for now, we will just focus on a very particular one: the expression *in the world of Sherlock Holmes*. We will assume, as seems reasonable, that this expression is a sentence-modifier both syntactically and semantically.

STEP 6: A SYNCATEGOREMATIC ENTRY. We begin with a heuristic step. We want to derive something like the following truth-conditions for our sentence:

- (19)  $\llbracket \text{in the world of Sherlock Holmes, a famous detective lives at 221B Baker Street} \rrbracket^{w,g} = 1$   
 iff the world  $w'$  as it is described in the Sherlock Holmes stories is such that there exists a famous detective in  $w'$  who lives at 221B Baker Street in  $w'$ .

We would get this if in general we had this rule for *in the world of Sherlock Holmes*:

- (20) For any sentence  $\phi$ , any world  $w$ , and any assignment  $g$ :  
 $\llbracket \text{in the world of Sherlock Holmes } \phi \rrbracket^{w,g} = 1$   
 iff the world  $w'$  as it is described in the Sherlock Holmes stories is such that  $\llbracket \phi \rrbracket^{w',g} = 1$ .

This is a so-called SYNCATEGOREMATIC treatment of the meaning of this expression. Instead of giving an explicit semantic value to the expression, we specify what effect it has on the meaning of a complex expression that contains it. In (20), we do not compute the meaning for *in the world of Sherlock Holmes*,  $\phi$  from the combination of the meanings of its parts, since *in the world of Sherlock Holmes* is not given a separate meaning, but in effect triggers a special composition principle. This format is very common in modal logic systems, which usually give a syncategorematic semantics for the two modal operators (the necessity operator  $\Box$  and the possibility operator  $\Diamond$ ). When one only has a few closed class expressions to deal with that may shift the world parameter, employing syncategorematic entries is a reasonable strategy. But we are facing a multitude of displacement devices. So, we will need to make our system more modular.

So, we want to give *in the world of Sherlock Holmes* its own meaning and combine that meaning with that of its prejacent by a general composition principle. The Fregean slogan we adopted says that all composition is function application (modulo the need for  $\lambda$ -abstraction and the possible need for predicate modification).<sup>6</sup> So, what we will want to do is to make 19 be the result of functional application. But we can immediately see that it cannot be the result of our usual rule of functional application, since that would feed to *in the world of Sherlock*

The diamond  $\Diamond$  symbol for possibility is due to C.I. Lewis, first introduced in C. I. Lewis & Langford (1932), but he made no use of a symbol for the dual combination  $\neg\Diamond\neg$ . The dual symbol  $\Box$  was later devised by F.B. Fitch and first appeared in print in 1946 in a paper by his doctoral student Barcan (1946). See footnote 425 of Hughes & Cresswell (1968). Another notation one finds is L for necessity and M for possibility, the latter from the German *möglich* 'possible'.

6 See Heim & Kratzer, Section 4.3, pp. 63–72 for a reminder about the status of predicate modification.

*Holmes* the semantic value of *a famous detective lives in 221B Baker Street* in  $w$ , which would be a particular truth-value, 1 if a famous detective lives at 221B Baker Street in  $w$  and 0 if there doesn't. And whatever the semantics of *in the world of Sherlock Holmes* is, it is certainly *not* a truth-functional operator.

So, we need to feed something else to *in the world of Sherlock Holmes*. At the same time, we want the operator to be able to shift the evaluation world of its prejacent. Can we do this?

EXERCISE 1.2: How would you show that *in the world of Sherlock Holmes* is not a truth-functional operator?  $\square$

STEP 7: INTENSIONS. We will define a richer notion of semantic value, the INTENSION of an expression. This will be a function from possible worlds to the extension of the expression in that world. The intension of a sentence can be applied to any world and give the truth-value of the sentence in that world. Intensional operators take the intension of their prejacent as their argument, that is we will feed the intension of the embedded sentence to the shifting operator. The operator will use that intension and apply it to the world it wants the evaluation to happen in. Voilà.

Now let's spell that account out. Our system actually provides us with two kinds of meanings. For any expression  $\alpha$ , we have  $\llbracket \alpha \rrbracket^{w,g}$ , the semantic value of  $\alpha$  in  $w$ , also known as the EXTENSION of  $\alpha$  in  $w$ . But we can also calculate  $\lambda w. \llbracket \alpha \rrbracket^{w,g}$ , the function that assigns to any world  $w$  the extension of  $\alpha$  in that world. This is usually called the INTENSION of  $\alpha$ . We will sometimes use an abbreviatory notation<sup>7</sup> for the intension of  $\alpha$ :

$$(21) \quad \llbracket \alpha \rrbracket_{\mathfrak{c}}^g := \lambda w. \llbracket \alpha \rrbracket^{w,g}.$$

It should be immediately obvious that since the definition of intension abstracts over the evaluation world, intensions are not world-dependent.<sup>8,9</sup>

Note that strictly speaking, it now makes no sense anymore to speak of “*the* semantic value” of an expression  $\alpha$ . What we have is a semantic system that allows us to calculate extensions (for a given possible world  $w$ ) as well as intensions for all (interpretable) expressions. We will see that when  $\alpha$  occurs in a

As before in H&K, we make no claim that the semantic values that are attributed to expressions in our framework fully capture what is informally meant by “meaning”. But certainly, intensions come closer to “meaning” than extensions.

<sup>7</sup> The notation with the subscripted cent-sign comes from Montague Grammar. See e.g. Dowty, Wall & Peters (1991: 147).

<sup>8</sup> Since intensions are by definition not dependent on the choice of a particular world, it makes no sense to put a world-superscript on the intension-brackets. So don't ever write “ $\llbracket \dots \rrbracket_{\mathfrak{c}}^{w,g}$ ”; we'll treat that as undefined nonsense.

<sup>9</sup> The definition here is simplified, in that it glosses over the fact that some expressions, in particular those that contain PRESUPPOSITION TRIGGERS, may fail to have an extension in certain worlds. In such a case, the intension has no extension to map such a world to. Therefore, the intension will have to be a partial function. So, the official, more “pedantic”, definition will have to be as follows:  $\llbracket \alpha \rrbracket_{\mathfrak{c}}^g := \lambda w: \alpha \in \text{dom}(\llbracket \dots \rrbracket^{w,g}). \llbracket \alpha \rrbracket^{w,g}$ .

particular bigger tree, it will always be determinate which of the two “semantic values” of  $\alpha$  is the one that enters into the compositional semantics. So, that one — whichever one it is, the extension or the intension of  $\alpha$  — might then be called “*the* semantic value of  $\alpha$  in the tree  $\beta$ ”.

It should be noted that the terminology of EXTENSION vs. INTENSION is time-honored but that the possible worlds interpretation thereof is more recent. The technical notion we are using is certainly less rich a notion of meaning than traditionally assumed.<sup>10</sup>

STEP 8: SEMANTIC TYPES AND SEMANTIC DOMAINS. If we want to be able to feed the intensions to lexical items like *in the world of Sherlock Holmes*, we need to have the appropriate types in our system.

Recall that  $W$  is the set of all possible worlds. And recall that  $D$  is the set of all POSSIBLE INDIVIDUALS and thus contains all individuals existing in the actual world *plus* all individuals existing in any of the merely possible worlds.

We now expand the set of semantic types, to add intensions. Intensions are functions from possible worlds to all kinds of extensions. So, basically we want to add for any kind of extension we have in our system, a corresponding kind of intension, a function from possible worlds to that kind of extension.

We add a new clause, (22c), to the definition of semantic types:

- (22) SEMANTIC TYPES
- a.  $e$  and  $t$  are semantic types.
  - b. If  $\sigma$  and  $\tau$  are semantic types, then  $\langle \sigma, \tau \rangle$  is a semantic type.
  - c. If  $\sigma$  is a semantic type, then  $\langle s, \sigma \rangle$  is a semantic type.
  - d. Nothing else is a semantic type.

We also add a fourth clause to the previous definition of semantic domains:

- (23) SEMANTIC DOMAINS
- a.  $D_e = D$ , the set of all possible individuals
  - b.  $D_t = \{0, 1\}$ , the set of truth-values
  - c. If  $\sigma$  and  $\tau$  are semantic types, then  $D_{\langle \sigma, \tau \rangle}$  is the set of all functions from  $D_\sigma$  to  $D_\tau$ .
  - d. INTENSIONS: If  $\sigma$  is a type, then  $D_{\langle s, \sigma \rangle}$  is the set of all functions from  $W$  to  $D_\sigma$ .

Clause (d) is the addition to our previous system of types. The functions of the schematic type  $\langle s, \dots \rangle$  are intensions.<sup>11</sup> Here are some examples of intensions:

<sup>10</sup> For example, Frege’s “modes of presentation” are not obviously captured by this possible worlds implementation of extension/intension.

<sup>11</sup> Note a curious feature of this set-up: there is no type  $s$  and no associated domain. This corresponds to the assumption that there are no expressions of English that take as their extension

The Port-Royal logicians distinguished EXTENSION from COMPREHENSION. Leibniz preferred the term INTENSION rather than COMPREHENSION. The notion probably goes back even further. See [Spencer \(1971\)](#) for some notes on this. The possible worlds interpretation is due to [Carnap \(1947\)](#).

- The intensions of sentences are of type  $\langle s, t \rangle$ , functions from possible worlds to truth values. These are usually called PROPOSITIONS. Note that if the function is total, then we can see the sentence as picking out a set of possible worlds, those in which the sentence is true. More often than not, however, propositions will be PARTIAL functions from worlds to truth-values, that is functions that fail to map certain possible worlds into either truth-value. This will be the case when the sentence contains a presupposition trigger, such as *the*. The famous sentence *The King of France is bald* has an intension that (at least in the analysis sketched in Heim & Kratzer) is undefined for any world where there fails to be a unique King of France.
- The intensions of one-place predicates are of type  $\langle s, \langle e, t \rangle \rangle$ , functions from worlds to set of individuals. These are usually called PROPERTIES.
- The intensions of expressions of type  $e$  are of type  $\langle s, e \rangle$ , functions from worlds to individuals. These are usually called INDIVIDUAL CONCEPTS.

STEP 9: A LEXICAL ENTRY FOR A SHIFTER. We are ready to formulate the lexical entry for *in the world of Sherlock Holmes*.<sup>12</sup>

- (24)  $\llbracket \text{in the world of Sherlock Holmes} \rrbracket^{w,g} =$   
 $\lambda p_{\langle s, t \rangle}. \text{the world } w' \text{ as it is described in the Sherlock Holmes stories}$   
 $\text{is such that } p(w') = 1.$

That is, *in the world of Sherlock Holmes* expects as its argument a function of type  $\langle s, t \rangle$ , a proposition. It yields the truth-value 1 iff the proposition is true in the world as it is described in the Sherlock Holmes stories.

All that's left to do now is to provide *in the world of Sherlock Holmes* with a proposition as its argument. This is the job of a new composition principle.

STEP 10: INTENSIONAL FUNCTIONAL APPLICATION. We add the new rule of Intensional Functional Application.

- (25) INTENSIONAL FUNCTIONAL APPLICATION (IFA)  
 If  $\alpha$  is a branching node and  $\{\beta, \gamma\}$  the set of its daughters, then, for any world  $w$  and assignment  $g$ : if  $\llbracket \beta \rrbracket^{w,g}$  is a function whose domain contains  $\llbracket \gamma \rrbracket_c^g$ , then  $\llbracket \alpha \rrbracket^{w,g} = \llbracket \beta \rrbracket^{w,g}(\llbracket \gamma \rrbracket_c^g).$

---

a possible world, that is, there are no pronouns or names referring to possible worlds. We will actually question this assumption in a later chapter. For now, we will stay with this more conventional set-up.

12 This is not yet the final semantics, see Section (1.3) for complications. One complication we will not even start to discuss is that obviously it is not a necessity that there are Sherlock Holmes stories in the first place and that the use of this operator *presupposes* that they exist; so a more fully explicit semantics would need to build in that presuppositional component. Also, note again the condensed notation: “ $\lambda p_{\langle s, t \rangle}. \dots$ ” stands for the fully official “ $\lambda p: p \in D_{\langle s, t \rangle}. \dots$ ”.

This is the crucial move. It makes space for expressions that want to take the intension of their sister as their argument and do stuff to it. Now, everything is in place. Given 24, the semantic argument of *in the world of Sherlock Holmes* will not be a truth-value but a proposition. And thus, *in the world of Sherlock Holmes* will be able to check the truth-value of its prejacent in various possible worlds. To see in practice that we have all we need, please do the following exercise.

EXERCISE 1.3: Calculate the conditions under which an utterance in a given possible world  $w_7$  of the sentence *in the world of the Sherlock Holmes stories, a famous detective lives at 221B Baker Street* is true.  $\square$

EXERCISE 1.4: What in our system prevents us from computing the extension of *Watson is slow*, for example, by applying the intension of *slow* to the extension of *Watson*? What in our system prevents us from computing the extension of *Watson is slow* by applying the intension of *slow* to the intension of *Watson*?  $\square$

Please think about this exercise before looking at Section (1.4), which explores this issue.

EXERCISE 1.5: What is wrong with the following equation:

$$(26) \quad (\lambda x. x \text{ is slow in } w) (\text{Watson}) = \text{Watson is slow in } w.$$

[ Hint: there is nothing wrong with the following:

$$(27) \quad . (\lambda x. x \text{ is slow in } w) (\text{Watson}) = 1 \text{ iff Watson is slow in } w. ]$$

$\square$

## 1.3 Comments and Complications

### 1.3.1 Intensions All the Way?

We have seen that to adequately deal with expressions like *in the world of Sherlock Holmes*, we need an intensional semantics, one that gives us access to the extensions of expressions across the multitude of possible worlds. At the same time, we have kept the semantics for items like *and*, *every*, and *a* unchanged and extensional. This is not the only way one can set up an intensional semantics. The following exercise demonstrates this.

EXERCISE 1.6: Consider the following “intensional” meaning for *and*:

$$(28) \quad \llbracket \text{and} \rrbracket^{w,g} = \lambda p_{\langle s,t \rangle}. \lambda q_{\langle s,t \rangle}. p(w) = q(w) = 1.$$

With this semantics, *and* would operate on the intensions of the two conjoined sentences. In any possible world  $w$ , the complex sentence will be true iff the component propositions are both true of that world.

Compute the truth-conditions of the sentence *In the world of Sherlock Holmes, Holmes is quick and Watson is slow* both with the extensional meaning for *and*

given earlier and the intensional meaning given here. Is there any difference in the results? □

There are then at least two ways one could develop an intensional system.

- (i) We could “generalize to the worst case” and make the semantics deliver intensions as *the* semantic value of an expression. Such systems are common in the literature (see [D. Lewis 1970](#), [Cresswell 1973](#)).
- (ii) We could maintain much of the extensional semantics we have developed so far and extend it conservatively so as to account for non-extensional contexts.

We have chosen to pursue (ii) over (i), because it allows us to keep the semantics of extensional expressions simpler. The philosophy we follow is that we will only move to the intensional sub-machinery when triggered by an expression that creates a non-extensional context. As the exercise just showed, this is more a matter of taste than a deep scientific decision.

### 1.3.2 Why Talk about Other Worlds?

Why would natural language bother having such elaborate mechanisms to talk about other possible worlds? While having devices for spatial and temporal displacement (talking about Hamburg or what happened yesterday) seems eminently reasonable, talking about worlds other than the actual world seems only suitable for poets and the like. So, why?

The solution to this puzzle lies in a fact that our current semantics of the shifter *in the world of Sherlock Holmes* does not yet accurately capture: modal sentences have empirical content, they make CONTINGENT claims, claims that are true or false depending on the circumstances in the actual world.

Our example sentence *In the world of Sherlock Holmes, a famous detective lives at 221B Baker Street* is true in this world but it could easily have been false. There is no reason why Sir Arthur Conan Doyle could not have decided to locate Holmes’ abode on Abbey Road.

To see that our semantics does not yet capture this fact, notice that in the semantics we gave for *in the world of Sherlock Holmes*:

- (29)  $\llbracket \text{in the world of Sherlock Holmes} \rrbracket^{w,g} =$   
 $\lambda p_{\langle s,t \rangle}. \text{the world } w' \text{ as it is described in the Sherlock Holmes stories}$   
 $\text{is such that } p(w') = 1.$

there is no occurrence of  $w$  on the right hand side. This means that the truth-conditions for sentences with this shifter are world-independent. In other words, they are predicted to make non-contingent claims that are either true no-matter-what or false no-matter-what. This needs to be fixed.



The fix is obvious: what matters to the truth of our sentence is the content of the Sherlock Holmes stories as they are in the evaluation world. So, we need the following semantics for our shifter:

- (30)  $\llbracket \text{in the world of Sherlock Holmes} \rrbracket^{w,g} =$   
 $\lambda p_{\langle s,t \rangle}. \text{the world } w' \text{ as it is described in the Sherlock Holmes stories}$   
*in*  $w$  is such that  $p(w') = 1$ .

We see now that sentences with this shifter do make a claim about the evaluation world: namely, that the Sherlock Holmes stories as they are in the evaluation world describe a world in which such-and-such is true. So, what is happening is that although it appears at first as if modal statements concern other possible worlds and thus couldn't really be very informative, they actually only talk about certain possible worlds, those that stand in some relation to what is going on at the ground level in the actual world. As a crude analogy, consider:

- (31) My grandmother is sick.

At one level this is a claim about my grandmother. But it is also a claim about me: namely that I have a grandmother who is sick. Thus it is with modal statements. They talk about possible worlds that stand in a certain relation to the actual world and thus they make claims about the actual world, albeit slightly indirectly.

### 1.3.3 The Worlds of Sherlock Holmes

So far, we have played along with colloquial usage in talking of *the* world of Sherlock Holmes. But it is important to realize that this is sloppy talk. D. Lewis (1978) writes:

[I]t will not do to follow ordinary language to the extent of supposing that we can somehow single out a single one of the worlds [as the one described by the stories]. Is the world of Sherlock Holmes a world where Holmes has an even or an odd number of hairs on his head at the moment when he first meets Watson? What is Inspector Lestrade's blood type? It is absurd to suppose that these questions about the world of Sherlock Holmes have answers. The best explanation of that is that the worlds of Sherlock Holmes are plural, and the questions have different answers at different ones.

The usual move at this point is to talk about the set of worlds "COMPATIBLE WITH the (content of) Sherlock Holmes stories in  $w$ ". We imagine that we ask of each possible world whether what is going on in it is compatible with the stories as they were written in our world. Worlds where Holmes lives on Abbey Road are not compatible. Some worlds where he lives at 221B Baker Street are compatible (again not all, because in some such worlds he is not a famous detective but



an obscure violinist). Among the worlds compatible with the stories are ones where he has an even number of hairs on his head at the moment when he first meets Watson and there are others where he has an odd number of hairs at that moment.

What the operator *in the world of Sherlock Holmes* expresses is that its complement is true throughout the worlds compatible with the stories. In other words, the operator *universally quantifies* over the compatible worlds. Our next iteration of the semantics for the operator is therefore this:

- (32)  $\llbracket \text{in the world of Sherlock Holmes} \rrbracket^{w,g} =$   
 $\lambda p_{\langle s,t \rangle}. \forall w' \text{ compatible with the Sherlock Holmes stories in } w:$   
 $p(w') = 1.$

At a very abstract level, the way we parse sentences of the form *in the world of Sherlock Holmes*,  $\phi$  is that both components, the *in*-phrase and the prejacent, determine sets of possible worlds and that the set of possible worlds representing the content of the fiction mentioned in the *in*-phrase is a subset of the set of possible worlds determined by the prejacent. We will see the same rough structure of relating sets of possible worlds in other intensional constructions.

This is where we will leave things. There is more to be said about fiction operators like *in the world of Sherlock Holmes*, but we will just refer to you to the relevant literature. In particular, one might want to make sense of Lewis' idea that a special treatment is needed for cases where the sentence makes a claim about things that are left open by the fiction (no truth-value, perhaps?). One also needs to figure out how to deal with cases where the fiction is internally inconsistent. In any case, for our purposes we're done with this kind of operator.

#### 1.3.4 What's next

With the basic framework of intensional semantics in place, we can now look at a succession of intensional operators. In particular, we will explore the semantics of propositional attitude predicates such as *believe* or *want*, modal auxiliaries such as *must* or *might*, and conditional sentences. In each case, we will see that they shift the evaluation of their prejacent to a certain set of possible worlds and that which worlds we are taken to depends on certain facts about the original evaluation world. We will look at the non-trivial issues that arise when several intensional operators interact (modals under attitudes, modals in the consequent of a conditional, etc.). We will also see that constituents of the prejacent can sometimes be evaluated with respect to a world that is not the world that the intensional operator is taking us to (so-called *de re* readings). Further, we will move from worlds to times and explore the semantics of tense and aspect. And, for the intrepid, this can all come together by exploring how tense and aspect interact with attitudes, modality, and conditionals.

## 1.4 \*Issues with an informal meta-language

Starred sections are optional on a first pass.

Exercise (1.5) asks what is wrong with writing something like<sup>13</sup>

$$(33) \quad (\lambda x. x \text{ is slow in } w) (\text{Watson}) = \text{Watson is slow in } w.$$

Think about it. On the left hand side of the “=” sign is a meta-language expression consisting of a  $\lambda$ -expression (so some kind of function) applied to an individual (contributed by the meta-language name “Watson”). The function is a function from individuals to truth-values that will deliver the truth-value 1 if the individual is slow in world  $w$ . So, what we have on the left hand side is the result of a function from individuals to truth-values applied to an individual. In other words, on the left hand side we have a truth-value, namely the truth-value 1 if Watson is slow in  $w$  and the truth-value 0 if Watson is not slow in  $w$ .

Now, what do we have on the right hand side of the “=”? We have the meta-language sentence “Watson is slow in  $w$ ”. That is not nor does it contribute a truth-value. It is a statement of fact. Truth-values are not the same as statements of fact.

The proper thing to do is to write

$$(34) \quad (\lambda x. x \text{ is slow in } w) (\text{Watson}) = 1 \text{ iff Watson is slow in } w.$$

There are actually two ways to parse the statement in (34), both legitimate it appears.

On one parse, the major connective is the meta-language expression “iff”. On its left hand side is a meta-language statement (that applying the function to the individual Watson gives the truth-value 1) and on the right hand side of the “iff” we have another meta-language statement (that Watson is slow in  $w$ ). So, the whole thing says that these two statements are equivalent: (i) that function applied to that individual gives us the truth-value 1, and (ii) that Watson is slow in  $w$ .

The other parse is perhaps more conspicuously represented as follows:

$$(35) \quad (\lambda x. x \text{ is slow in } w) (\text{Watson}) = \begin{cases} 1 & \text{if Watson is slow in } w \\ 0 & \text{if Watson is not slow in } w \end{cases}$$

Here, the “=” sign is the major connective. The left hand side is a meta-language expression that resolves to a truth-value and the right hand side as well contributes a truth-value: 1 if such and such and 0 if such and such.

H&K, of course, introduced a convention that allowed meta-language statements to be used in a place where a truth-value was expected (p.37, (9)):

Read “[ $\lambda\alpha: \phi. \gamma$ ]” as either (i) or (ii), whichever makes sense.

(i) “the function which maps every  $\alpha$  such that  $\phi$  to  $\gamma$ ”

Is this weird? It turns out that natural language, not just our semi-formal meta-language, has conditionals that seem very similar: *I fear [the consequences if we fail]*. See Lasersohn 1996 for some discussion.

<sup>13</sup> Thanks to Magda Kaufmann, Angelika Kratzer, and Ede Zimmermann for discussion on the issues explored in this section.

- (ii) “the function which maps every  $\alpha$  such that  $\gamma$  to 1, if  $\gamma$ , and to 0 otherwise”

Since it never makes sense to map anything to a meta-language statement, no ambiguity will ever arise.

So, one might want to extend this leeway and use it in the case of (33) as well. We could say that in general, meta-language statements supply truth-values wherever that makes sense. In that case, (33) is just shorthand for (34).

This is the approach of von Stechow 1991.

Alternatively, one can introduce a new notation that indicates that a meta-language statement is being used to contribute a truth-value:

This is the approach Ede Zimmermann (pc) advocates and has been using in his classes.

$$(36) \quad \vdash \alpha \dashv = \begin{cases} 1 & \text{if } \alpha \\ 0 & \text{if otherwise} \end{cases}$$

Lastly, one could abandon the H & K informal meta-language approach altogether and introduce a rigidly formalized meta-language.

These lecture notes will proceed to follow H & K's approach and will not introduce any further innovations. So, (33) is illicit and only (34) is acceptable.

## 1.5 Supplemental Readings

There is considerable overlap between this chapter and Chapter 12 of Heim & Kratzer's textbook:

Irene Heim & Angelika Kratzer. 1998. *Semantics in generative grammar*. Oxford: Blackwell.

Here, we approach intensional semantics from a different angle. It would probably be beneficial if you read H & K's Chapter 12 in addition to this chapter and if you did the exercises in there.

Come to think of it, some other ancillary reading is also recommended. You may want to look at relevant chapters in other textbooks:

David R. Dowty, Robert Wall & Stanley Peters. 1981. *Introduction to Montague semantics*. Kluwer. [Chapters 5 & 6].

L. T. F. Gamut. 1991. *Logic, language, and meaning*. Chicago University Press. [Volume II: Intensional Logic and Logical Grammar].

Gennaro Chierchia & Sally McConnell-Ginet. 2000. *Meaning and grammar: An introduction to semantics (2<sup>nd</sup> edition)*. MIT Press. [Chapter 5: Intensionality].

Thomas Ede Zimmermann & Wolfgang Sternefeld. 2013. *Introduction to semantics: An essential guide to the composition of meaning*. de Gruyter Mouton.

The *Stanford encyclopedia of philosophy* is always a good resource. Here's the entry on possible worlds:

Christopher Menzel. 2016. Possible worlds. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Spring 2016. <http://plato.stanford.edu/entries/possible-worlds/>.

A couple of influential philosophical works on the metaphysics and uses of possible worlds:

Saul Kripke. 1980. *Naming and necessity*. Oxford: Blackwell.

David Lewis. 1986. *On the plurality of worlds*. Oxford: Blackwell.

An interesting paper on the origins of the modern possible worlds semantics for modal logic:

B. Jack Copeland. 2002. The genesis of possible worlds semantics. *Journal of Philosophical Logic* 31(2). 99–137. <http://dx.doi.org/10.1023/A:1015273407895>.

A personal history of formal semantics:

Barbara H. Partee. 2005. Reflections of a formal semanticist as of Feb 2005. ms. (longer version of introductory essay in 2004 book). [http://people.umass.edu/partee/docs/BHP\\_Essay\\_Feb05.pdf](http://people.umass.edu/partee/docs/BHP_Essay_Feb05.pdf).

A must read for students who plan to go on to becoming specialists in semantics, together with a handbook article putting it in perspective:

Richard Montague. 1973. The proper treatment of quantification in ordinary English. In Jaako Hintikka, Julius Moravcsik & Patrick Suppes (eds.), *Approaches to natural language*, 221–242. Reprinted in Portner & Partee (2002), pp. 17–34. Dordrecht: Reidel. [http://www.blackwellpublishing.com/content/BPL/Images/Content\\_store/Sample\\_chapter/9780631215417/Portner.pdf](http://www.blackwellpublishing.com/content/BPL/Images/Content_store/Sample_chapter/9780631215417/Portner.pdf).

Barbara H. Partee & Herman L.W. Hendriks. 1997. Montague grammar. In Johan van Benthem & Alice ter Meulen (eds.), *Handbook of logic and language*, 5–91. Elsevier.

To learn more about discourse about fiction, read Lewis:

David Lewis. 1978. Truth in fiction. *American Philosophical Quarterly* 15(1). Reprinted with postscripts in D. Lewis (1983), pp. 261–280, 37–46. <http://www.jstor.org/stable/20009693>.

Recent reconsiderations:

Andrea Bonomi & Sandro Zucchi. 2003. A pragmatic framework for truth in fiction. *Dialectica* 57(2). Preprint <http://filosofia.dipafilo.unimi.it/~bonomi/Pragmatic.pdf>, 103–120. <http://dx.doi.org/10.1111/j.1746-8361.2003.tb00259.x>.

Richard Hanley. 2004. As good as it gets: Lewis on truth in fiction. *Australasian Journal of Philosophy* 82(1). 112–128. <http://dx.doi.org/10.1080/713659790>.

Diane Proudfoot. 2006. Possible worlds semantics and fiction. *Journal of Philosophical Logic* 35(1). 9–40. <http://dx.doi.org/10.1007/s10992-005-9005-8>.

Inconsistencies in fictions and elsewhere are discussed in:

Achille Varzi. 1997. Inconsistency without contradiction. *Notre Dame Journal of Formal Logic* 38(4). 621–638. <http://dx.doi.org/10.1305/ndjfl/1039540773>.

David Lewis. 1982. Logic for equivocators. *Noûs* 16(3). Reprinted in D. Lewis (1998: pp. 97–110), 431–441. <http://dx.doi.org/10.2307/2216219>.

Some other interesting work on stories and pictures and their content:

Jeff Ross. 1997. *The semantics of media* (Studies in Linguistics and Philosophy (SLAP) 64). Dordrecht: Kluwer.

Sandro Zucchi. 2001. Tense in fiction. In Carlo Cecchetto, Gennaro Chierchia & Maria Teresa Guasti (eds.), *Semantic interfaces: Reference, anaphora and aspect*, 320–355. CSLI Publications. <http://tinyurl.com/5ulwxwg>.

Ben Blumson. 2009. Pictures, perspective and possibility. *Philosophical Studies*. <http://dx.doi.org/10.1007/s11098-009-9337-2>.

Astonishingly, Lewis’ doctrine of the reality of the plurality of possible worlds is being paralleled (pun absolutely intended) by theoretical physicists in a number of ways. There is a controversial “many worlds” interpretation of quantum mechanics, for example. Other terms found are the “multiverse” and “parallel universes”. See for starters, Kai’s blog entry on a popular book on the issue, <http://kaivonfintel.org/many-worlds/>, MIT physics professor Max Tegmark’s page on the topic, <http://space.mit.edu/home/tegmark/crazy.html>, and a Fresh Air interview with physicist Brian Greene, who just wrote a book called *The Hidden Reality: Parallel Universes and the Deep Laws of the Cosmos*: <http://www.npr.org/2011/01/24/132932268/a-physicist-explains-why-parallel-universes-may-exist>.

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# CHAPTER TWO

## CONDITIONALS

The word “if”, just two tiny letters  
Says so much for something so small  
The biggest little word in existence;  
Never answers, just questions us all

If regrets were gold, I’d be rich as a queen  
If teardrops were diamonds, how my face would gleam  
If I’d loved you better, I wouldn’t be lonely  
If only, if only, if only

---

Dolly Parton, *If Only*

*We develop a possible worlds semantics for conditionals that treats the if-clause as an intensional operator, with a bit of context-dependency thrown in.*

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2.1	<i>If</i> ... the biggest little word	23
2.2	The Restricted Strict Implication Analysis	24
2.3	A red herring: Material implication	25
2.4	Supplemental Readings	28

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### 2.1 *If* ... the biggest little word

In many ways, conditionals are the archetypal construction of displacement: the consequent is evaluated not against the actual here and now but against the scenario conjured up by the antecedent. Consider a few conditional sentences:

- (1) a. If Kim left before 6am, she got here in time.  
b. If there’s an earthquake tomorrow, this house will collapse.  
c. If there had been a massive snowstorm last night, Kai would have stayed home.

These represent the three main subtypes of conditionals (there are more): (1a) is an “indicative” conditional about the past, (1b) is an indicative conditional about the future, and (1c) is a “subjunctive” conditional. For the moment, the differences will be left aside.

The basic idea of how conditionals work is this: the *if*-clause takes us to a particular possible world (or maybe a set thereof) and the consequent clause is asserted to be true of that world (or those worlds). But what world(s) are we being taken to? The most obvious requirement is that the antecedent of the conditional needs to be true of the world(s). But there’s more.

Given our discussion of how the semantics of fiction operators anchors them in facts about the actual world (the content of the relevant body of fiction), it shouldn’t come as a surprise that conditionals are similarly anchored. So, look at the examples in (1): what in the actual world are they about?

Here’s a sketch: (1a) is about the local transportation system, the weather, the traffic, and so on. (1b) is about the sturdiness of this house, facts of geology, laws of physics, and so on. (1c) is about Kai’s proclivities (such as avoiding traffic snarls), the local climate, and so on. Since the conditionals are anchored in real world facts, they are no mere flights of fancy and whether they are true depends on those facts. If today’s traffic was particularly bad, it may be false that Kim’s leaving before 6am would have got her here in time. If the architects went to great lengths to make the house earthquake-safe, (1b) may well be false. And if there was an attendance-mandatory faculty meeting, Kai may well have come in in spite of a massive snowstorm.

So, the outlines of the semantics of conditionals are clear: *if* takes us to worlds where the antecedent is true but that match the actual world in certain relevant features. And the consequent then is evaluated in those worlds. There are many details to work out and we’ll return to that task in Part III of these notes. But for now, we put forward a placeholder analysis.

## 2.2 The Restricted Strict Implication Analysis

We will treat *if* as a higher-order operator that together with the antecedent creates an intensional operator with a semantics very similar to the final analysis we gave to *in the world of Sherlock Holmes* in the previous chapter. But where the fiction operator directly encoded what features of the actual world it’s sensitive to (the Sherlock Holmes fiction), conditionals rely on context for this job. Here’s the proposal:

$$(2) \quad \llbracket \text{if} \rrbracket^{w,g} = \lambda p \in D_{\langle s,t \rangle}. \lambda q \in D_{\langle s,t \rangle}. \\ \forall w': p(w') = 1 \ \& \ w' \text{ is relevantly like } w \rightarrow q(w') = 1.$$

The contextual anchoring to features of the evaluation world  $w$  is here effected by the placeholder “relevantly like  $w$ ”. This is crucial because otherwise the

It is perhaps unfortunate that David Lewis used a rather whimsical example to start off his seminal book on counterfactuals (“If kangaroos had not tails, they would topple over”). Or consider this scene from the TV show “Big Bang Theory”: <https://www.youtube.com/watch?v=olpYoKt4bn8>. As the examples in the text make clear, conditionals can be very down-to-earth.



conditional would talk about any world whatsoever where the antecedent is true. This would make the truth-conditions not just not contingent on the actual world but also far too strong to allow most sensible conditionals to be true ever.

Think about the earthquake conditional (1b): we would derive the absurdly strong truth-conditions that the conditional is true iff *all* of the worlds where there is a major earthquake in Cambridge tomorrow are worlds where my house collapses.

There are some obvious and immediate problems with this analysis. For one, while it's easy to imagine circumstances where the conditional (1b) is judged to be true, there surely are possible worlds where there's an earthquake but my house does not collapse: perhaps, the builders in that world used all the recommended best practices to make the building earthquake-safe, perhaps it's a world where I'm simply unreasonably lucky, or the house is immediately adjacent to much sturdier neighboring buildings which keep it propped up, or Harry Potter flies by and protects the house at the last minute (he owes me a favor, after all). This problem (that the house doesn't in fact collapse in *all* possible worlds where there's an earthquake but that the conditional can still be judged true in some worlds) is accompanied with another problem: whether the conditional is true depends on what the world is like. Was the house built to exacting standards? Is it propped up by its neighbors? Does Harry Potter owe me a favor? That is the problem solved by restricting the quantifier over worlds to world “relevantly like *w*”.

Obviously, this is a semantics with a “placeholder”, because what does “relevantly like” mean precisely? Now, just because the semantics is therefore rather vague and context-dependent doesn't mean it is wrong. As D. Lewis (1973) writes:

Counterfactuals are notoriously vague. That does not mean that we cannot give a clear account of their truth conditions. It does mean that such an account must either be stated in vague terms — which does not mean ill-understood terms — or be made relative to some parameter that is fixed only within rough limits on any given occasion of language use.

In a later chapter, we will revisit the formulation in (2) and fill it out more explicitly. Before we go on to consider other intensional constructions, we'll briefly talk about the astonishingly silly idea that conditionals are not intensional constructions.

## 2.3 A red herring: Material implication

Consider the following example:

- (3) If I am healthy, I will come to class.

The simplest analysis of such conditional constructions is the so-called MATERIAL IMPLICATION analysis,<sup>1</sup> which treats *if* as contributing a truth-function operating on the truth-values of the two component sentences (which are called the ANTECEDENT and CONSEQUENT — from Latin — or PROTASIS and APODOSIS — from Greek). The lexical entry for *if* would look as follows:

$$(4) \quad \llbracket \text{if} \rrbracket = \lambda u \in D_t. \lambda v \in D_t. u = 0 \text{ or } v = 1.$$

Note that as a truth-functional connective, this *if* does not vary its denotation depending on the evaluation world. It's its arguments that vary with the evaluation world.

Applied to example in (3), this semantics would predict that the example is false just in case the antecedent is true, I am healthy, but the consequent false, I do not come to class. Otherwise, the sentence is true. We will see that there is much to complain about here. But one should realize that under the assumption that *if* denotes a truth-function, *this one* is the most plausible candidate.

Suber (1997) does a good job of persuading (or at least trying to persuade) recalcitrant logic students:

After saying all this, it is important to note that material implication does conform to some of our ordinary intuitions about implication. For example, take the conditional statement, *If I am healthy, I will come to class*. We can symbolize it:  $H \supset C$ .<sup>2</sup>

The question is: when is this statement false? When will I have broken my promise? There are only four possibilities:

H	C	$H \supset C$
T	T	?
T	F	?
F	T	?
F	F	?

- In case #1, I am healthy and I come to class. I have clearly kept my promise; the conditional is true.
- In case #2, I am healthy, but I have decided to stay home and read magazines. I have broken my promise; the conditional is false.
- In case #3, I am not healthy, but I have come to class anyway. I am sneezing all over you, and you're not happy about it, but I did not violate my promise; the conditional is true.

<sup>1</sup> Quoth the Stoic philosopher Philo of Megara: “a true conditional is one which does not have a true antecedent and a false consequent” (according to Empiricus (c. 200: II, 110–112)).

<sup>2</sup> The symbol  $\supset$  which Suber uses here is called the “horseshoe”. We have been using the right arrow  $\rightarrow$  as the symbol for implication. We think that this is much preferable to the confusing horseshoe symbol. There is an intimate connection between universal quantification, material implication, and the subset relation, usually symbolized as  $\subset$ , which is the other way round from the horseshoe. The horseshoe can be traced back to the notation introduced by Peano (1889), a capital C standing for ‘consequenza’ facing backwards. The C facing in the other (more “logical”) direction was actually introduced first by Gergonne (1817), but didn't catch on.

- In case #4, I am not healthy, and I did not come to class. I did not violate my promise; the conditional is true.

But this is exactly the outcome required by the material implication. The compound is only false when the antecedent is true and the consequence is false (case #2); it is true every other time.

Despite the initial plausibility of the analysis, it cannot be maintained. Consider again our earthquake example:

- (1b) If there is a major earthquake in Cambridge tomorrow, my house will collapse.

If we adopt the material implication analysis, we predict that (1b) will be false just in case there is indeed a major earthquake in Cambridge tomorrow but my house fails to collapse. This makes a direct prediction about when the negation of (1b) should be true. A false prediction, if ever there was one:

- (5) a. It's not true that if there is a major earthquake in Cambridge tomorrow, my house will collapse.  
 b.  $\neq$  There will be a major earthquake in Cambridge tomorrow, and my house will fail to collapse.

Clearly, one might think that (5a) is true without at all being committed to what the material implication analysis predicts to be the equivalent statement in (5b). This is one of the inadequacies of the material implication analysis.

These inadequacies are sometimes referred to as the “paradoxes of material implication”. But that is misleading. As far as logic is concerned, there is nothing wrong with the truth-function of material implication. It is well-behaved and quite useful in logical systems. What is arguable is that it is not to be used as a reconstruction of what conditionals mean in natural language.

EXERCISE 2.1: What prediction does the restricted strict implication analysis in (2) make about the negated conditional in (5a)? □

EXERCISE 2.2: Under the assumption that *if* has the meaning in (4), calculate the truth-conditions predicted for (6):

- (6) a. No student will succeed if he goofs off.  
 b. No student  $\lambda x$  (if  $x$  goofs off,  $x$  will succeed)

State the predicted truth-conditions in words and evaluate whether they correspond to the actual meaning of (6). □

## 2.4 Supplemental Readings

We'll come back to conditionals soon enough, but here are two introductory readings:

Kai von Fintel. 2011. Conditionals. In Claudia Maienborn, Klaus von Heusinger & Paul Portner (eds.), *Semantics: An international handbook of meaning*, vol. 2, 1515–1538. de Gruyter. <http://dx.doi.org/10.1515/9783110255072.1515>. <http://mit.edu/fintel/fintel-2011-hsk-conditionals.pdf>.

Kai von Fintel. 2012. Subjunctive conditionals. In Gillian Russell & Delia Graff Fara (eds.), *The Routledge companion to philosophy of language*, 466–477. New York: Routledge. <http://dx.doi.org/1721.1/95784>. <http://mit.edu/fintel/fintel-2012-subjunctives.pdf>.

Here is a not so introductory reading:

Anthony S. Gillies. 2016. Conditionals. To appear in: Bob Hale, Alex Miller, and Crispin Wright, eds. *Blackwell Companion to the Philosophy of Language*, Blackwell. Pre-final draft. <http://semanticsarchive.net/Archive/TAYOTArY/gillies-conditionals-blackwell.pdf>.

A couple of defenses of the material conditional:

Barbara Abbott. 2010. Conditionals in English and FOPL. In Dingfang Shu & Ken Turner (eds.), *Contrasting meanings in languages of the East and West*. Peter Lang. <http://semanticsarchive.net/Archive/jUoM2ZmZ>.

Adam Rieger. 2015. Defending a simple theory of conditionals. *American Philosophical Quarterly* 52(3). 253–260. <http://apq.press.illinois.edu/52/3/rieger.html>.

# CHAPTER THREE

## PROPOSITIONAL ATTITUDES

*We start with the basic possible worlds semantics for propositional attitude ascriptions. We talk briefly about the formal properties of accessibility relations.*

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3.1	Hintikka's Idea	29
3.2	Accessibility Relations	32
3.2.1	Reflexivity	32
3.2.2	*Transitivity	33
3.2.3	*Symmetry	35
3.3	Supplemental Readings	36

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### 3.1 Hintikka's Idea

Expressions like *believe*, *know*, *doubt*, *expect*, *regret*, and so on are usually said to describe PROPOSITIONAL ATTITUDES, expressing relations between individuals (the attitude holder) and propositions (intensions of sentences).

The simple idea is that *George believes that Henry is a spy* claims that George believes of the proposition that Henry is a spy that it is true. Note that for the attitude ascription to be true it does not have to hold that Henry is actually a spy. But where—in which world(s)—does Henry have to be a spy for it be true that George believes that Henry is a spy? We might want to be inspired by the colloquial phrase “in the world according to George” and say that *George believes that Henry is a spy* is true iff in the world according to George's beliefs, Henry is a spy. We immediately recall from Chapter 1 that we need to fix this idea up by making space for multiple worlds compatible with George's beliefs and by tying the truth-conditions to contingent facts about the evaluation world. That is, what George believes is different in different possible worlds.

According to [Hintikka \(1969\)](#), the term PROPOSITIONAL ATTITUDE goes back to [Russell \(1940\)](#).

Of course, the possible worlds semantics for propositional attitudes was in place long before the extension to fiction contexts was proposed. Our discussion here has inverted the historical sequence for pedagogical purposes.

The following lexical entry thus offers itself:

$$(I) \quad \llbracket \text{believe} \rrbracket^{w,g} = \lambda p_{\langle s,t \rangle} . \lambda x . \forall w' \text{ compatible with } x \text{'s beliefs in } w : p(w') = 1.$$

It is important to realize the modesty of this semantics: we are not trying to figure out what belief systems are and particularly not what their internal workings are. That is the job of psychologists (and philosophers of mind, perhaps). For our semantics, we treat the belief system as a black box that determines for each possible world whether it considers it possible that it is the world it is located in.

What is going on in this semantics? We conceive of George's beliefs as a state of his mind about whose internal structure we will remain agnostic, a matter left to other cognitive scientists. What we require of it is that it embody opinions about what the world he is located in looks like. In other words, if his beliefs are confronted with a particular possible world  $w'$ , they will determine whether that world may or may not be the world as they think it is. What we are asking of George's mental state is whether any state of affairs, any event, anything in  $w'$  is in contradiction with anything that George believes. If not, then  $w'$  is compatible with George's beliefs. For all George believes,  $w'$  may well be the world where he lives. Many worlds will pass this criterion, just consider as one factor that George is unlikely to have any precise opinions about the number of leaves on the tree in front of my house. George's belief system determines a set of worlds compatible with his beliefs: those worlds that are viable candidates for being the actual world, as far as his belief system is concerned.

Now, George believes a proposition iff that proposition is true in all of the worlds compatible with his beliefs. If there is just one world compatible with his beliefs where the proposition is not true, that means that he considers it possible that the proposition is not true. In such a case, we can't say that he believes the proposition. Here is the same story in the words of [Hintikka \(1969\)](#), the source for this semantics for propositional attitudes:



Jaakko Hintikka

My basic assumption (slightly simplified) is that an attribution of any propositional attitude to the person in question involves a division of all the possible worlds (...) into two classes: into those possible worlds which are in accordance with the attitude in question and into those which are incompatible with it. The meaning of the division in the case of such attitudes as knowledge, belief, memory, perception, hope, wish, striving, desire, etc. is clear enough. For instance, if what we are speaking of are (say)  $a$ 's memories, then these possible worlds are all the possible worlds compatible with everything he remembers. [...]

How are these informal observations to be incorporated into a more explicit semantical theory? According to what I have said, understanding attributions of the propositional attitude in question (...) means being able to make a distinction between two kinds of possible worlds, according to whether they are compatible with the relevant attitudes of the person in question. The semantical counterpart to this is of course a function which to a given individual person assigns a set of possible worlds.

However, a minor complication is in order here. Of course, the person in question may himself have different attitudes in the different worlds we are considering. Hence this function in effect becomes a relation which to a given individual and to a given possible world  $\mu$  associates a number of possible worlds which we shall call the *ALTERNATIVES* to  $\mu$ . The relation will be called the *alternativeness* relation. (For different propositional attitudes, we have to consider different alternativeness relations.)

EXERCISE 3.1: Let's adopt Hintikka's idea that we can use a function that maps  $x$  and  $w$  into the set of worlds  $w'$  compatible with what  $x$  believes in  $w$ . Call this function  $\mathcal{B}$ . That is,

$$(2) \quad \mathcal{B} = \lambda x. \lambda w. \{w' : w' \text{ is compatible with what } x \text{ believes in } w\}.$$

Using this notation, our lexical entry for *believe* would look as follows:

$$(3) \quad \llbracket \text{believe} \rrbracket^{w,g} = \lambda p_{\langle s,t \rangle}. \lambda x. \mathcal{B}(x)(w) \subseteq p.$$

We are here indulging in the usual sloppiness in treating  $p$  both as a function from worlds to truth-values and as the set characterized by that function.

Here now are two “alternatives” for the semantics of *believe*:

$$(4) \quad \text{ATTEMPT 1 (VERY WRONG)}$$

$$\llbracket \text{believe} \rrbracket^{w,g} = \lambda p \in D_{\langle s,t \rangle}. [\lambda x \in D. p = \mathcal{B}(x)(w)].$$

$$(5) \quad \text{ATTEMPT 2 (ALSO VERY WRONG)}$$

$$\llbracket \text{believe} \rrbracket^{w,g} = \lambda p \in D_{\langle s,t \rangle}. [\lambda x \in D. p \cap \mathcal{B}(x)(w) \neq \emptyset].$$

Explain why these do not adequately capture the meaning of *believe*.  $\square$

EXERCISE 3.2: Follow-up: The semantics in (5) would have made *believe* into an existential quantifier of sorts: it would say that *some* of the worlds compatible with what the subject believes are such-and-such. You have argued (successfully, of course) that such an analysis is wrong for *believe*. But *are* there attitude predicates with such an “existential” meaning? Discuss some candidates. If you can't find any candidates that survive scrutiny, can you speculate why there might be no existential attitude predicates? [Warning: this is underexplored territory!]

$\square$

$\mathcal{BS}$  is meant to stand for ‘belief state’, not for what you might have thought!

We can also think of belief states as being represented by a function  $\mathcal{BS}$ , which maps an individual and a world into a set of propositions: those that the individual believes. From there, we could calculate the set of worlds compatible with an individual  $x$ ’s beliefs in world  $w$  by retrieving the set of those possible worlds in which all of the propositions in  $\mathcal{BS}(x)(w)$  are true:  $\{w' : \forall p \in \mathcal{BS}(x)(w) : p(w') = 1\}$ , which in set talk is simply the big intersection of all the propositions in the set:  $\cap \mathcal{BS}(x)(w)$ . Our lexical entry then would be:

$$(6) \quad \llbracket \text{believe} \rrbracket^{w,g} = \lambda p_{\langle s,t \rangle} . \lambda x . \cap \mathcal{BS}(x)(w) \subseteq p.$$

EXERCISE 3.3: Imagine that our individual  $x$  forms a new opinion. Imagine that we model this by adding a new proposition  $p$  to the pool of opinions. So,  $\mathcal{BS}(x)(w)$  now contains one further element. There are now more opinions. What happens to the set of worlds compatible with  $x$ ’s beliefs? Does it get bigger or smaller? Is the new set a subset or superset of the previous set of compatible worlds?  $\square$

## 3.2 Accessibility Relations

Another way of reformulating Hintikka’s semantics for propositional attitudes is via the notion of an ACCESSIBILITY RELATION. We talk of a world  $w'$  being accessible from  $w$ . Each attitude can be associated with such an accessibility relation. For example, we can introduce the relation  $w\mathcal{R}_\alpha^B w'$  which holds iff  $w'$  is compatible with  $\alpha$ ’s belief state in  $w$ . We have then yet another equivalent way of specifying the lexical entry for *believe*:

$$(7) \quad \llbracket \text{believe} \rrbracket^{w,g} = \lambda p_{\langle s,t \rangle} . \lambda x . \forall w' : w\mathcal{R}_x^B w' \rightarrow p(w') = 1.$$

It is profitable to think of different attitudes (belief, knowledge, hope, regret, memory, ...) as corresponding to different accessibility relations. Recall now that the linguistic study of determiners benefitted quite a bit from an investigation of the formal properties of the relations between sets of individuals that determiners express. We can do the same thing here and ask about the formal properties of the accessibility relation associated with belief versus the one associated with knowledge, etc. The obvious properties to think about are reflexivity, transitivity, and symmetry.

### 3.2.1 Reflexivity

A relation is reflexive iff for any object in the domain of the relation we know that the relation holds between that object and itself. Which accessibility relations are reflexive? Take knowledge:

Kirill Shkolovsky (in class) asked why we call reflexivity, transitivity, and symmetry “formal” properties of relations. The idea is that certain properties are “formal” or “logical”, while others are more substantial. So, the fact that the relation “have the same birthday as” is symmetric seems a more formal fact about it than the fact that the relation holds between my daughter and my brother-in-law. Nevertheless, one of the most common ways of characterizing formal/logical notions (permutation-invariance, if you’re curious) does not in fact make symmetry etc. a formal/logical notion. So, while intuitively these do seem to be formal/logical properties, we do not know how to substantiate that intuition. See MacFarlane (2005) for discussion.



(8)  $w\mathcal{R}_x^{\mathcal{K}}w'$  iff  $w'$  is compatible with what  $x$  knows in  $w$ .

We are asking whether for any given possible world  $w$ , we know that  $\mathcal{R}_x^{\mathcal{K}}$  holds between  $w$  and  $w$  itself. It will hold if  $w$  is a world that is compatible with what we know in  $w$ . And clearly that must be so. Take our body of knowledge in  $w$ . The concept of knowledge crucially contains the concept of truth: what we know must be true. So if in  $w$  we know that something is the case then it must be the case in  $w$ . So,  $w$  must be compatible with all we know in  $w$ .  $\mathcal{R}_x^{\mathcal{K}}$  is reflexive.

We talk here about knowledge entailing (or even presupposing) truth but we do not mean to say that knowledge simply equals true belief. Professors Socrates and Gettier and their exegetes have further considerations.

Now, if an attitude  $X$  corresponds to a reflexive accessibility relation, then we can conclude from a  $X$ s *that*  $p$  being true in  $w$  that  $p$  is true in  $w$ . This property of an attitude predicate is often called VERIDICALITY. It is to be distinguished from FACTIVITY, which is a property of attitudes which *presuppose* – rather than (merely) entail – the truth of their complement.

In modal logic notation:  $\Box p \rightarrow p$ . This pattern is sometimes called T or M, as is the corresponding system of modal logic.

If we consider the relation  $\mathcal{R}_x^{\mathcal{B}}$  pairing with a world  $w$  those worlds  $w'$  which are compatible with what  $x$  *believes* in  $w$ , we no longer have reflexivity: belief is not a veridical attitude. It is easy to have false beliefs, which means that the actual world is not in fact compatible with one's beliefs, which contradicts reflexivity. And many other attitudes as well do not involve veridicality/reflexivity: what we hope may not come true, what we remember may not be what actually happened, etc.

The difference between *believe* and *know* in natural discourse is quite delicate, especially when one considers first person uses (*I believe the earth is flat* vs. *I know the earth is flat*).

In modal logic, the correspondence between formal properties of the accessibility relation and the validity of inference patterns is well-studied. What we have just seen is that reflexivity of the accessibility relation corresponds to the validity of  $\Box p \rightarrow p$ . Other properties correspond to other characteristic patterns. Let's see this for transitivity and symmetry.

### 3.2.2 \*Transitivity

Transitivity of the accessibility relation corresponds to the inference  $\Box p \rightarrow \Box \Box p$ . The pattern seems not obviously wrong for knowledge: if one knows that  $p$ , doesn't one thereby know that one knows that  $p$ ? But before we comment on that, let's establish the formal correspondence between transitivity and that inference pattern. This needs to go in both directions.

What does it take for the pattern to be valid? Assume that  $\Box p$  holds for an arbitrary world  $w$ , i.e. that  $p$  is true in all worlds  $w'$  accessible from  $w$ . Now, the inference is to the fact that  $p$  again holds in any world  $w''$  accessible from any of those worlds  $w'$  accessible from  $w$ . But what would prevent  $p$  from being false in some  $w''$  accessible from some  $w'$  accessible from  $w$ ? That could only be prevented from happening if we knew that  $w''$  itself is accessible from  $w$  as well, because then we would know from the premiss that  $p$  is true in it (since  $p$  is true in *all* worlds accessible from  $w$ ). Ah, but  $w''$  (some world accessible from a world  $w'$  accessible from  $w$ ) is only guaranteed to be accessible from

In the literature on epistemic modal logic, the pattern is known as the KK THESIS or POSITIVE INTROSPECTION. In general modal logic, it is the characteristic axiom 4 of the modal logic system S4, which is a system that adds 4 to the previous axiom M/T. Thus, S4 is the logic of accessibility relations that are both reflexive and transitive.

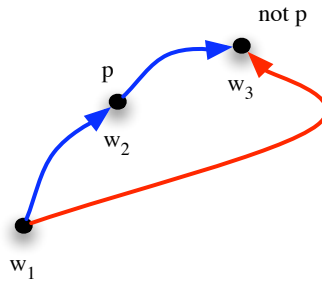


Figure 3.1: Transitivity

$w$  if the accessibility relation is transitive (if  $w'$  is accessible from  $w$  and  $w''$  is accessible from  $w'$ , then transitivity ensures that  $w''$  is accessible from  $w$ ). This reasoning has shown that validity of the pattern requires transitivity. The other half of proving the correspondence is to show that transitivity entails that the pattern is valid.

The proof proceeds by reductio. Assume that the accessibility relation is transitive. Assume that (i)  $\Box p$  holds for some world  $w$  but that (ii)  $\Box\Box p$  doesn't hold in  $w$ . We will show that this situation cannot obtain. By (i),  $p$  is true in all worlds  $w'$  accessible from  $w$ . By (ii), there is some non- $p$  world  $w''$  accessible from some world  $w'$  accessible from  $w$ . But by transitivity of the accessibility relation, that non- $p$  world  $w''$  must be accessible from  $w$ . And since *all* worlds accessible from  $w$  are  $p$  worlds,  $w''$  must be a  $p$  world, in contradiction to (ii). So, as soon as we assume transitivity, there is no way for the inference not to go through.

Now, do any of the attitudes have the transitivity property? It seems rather obvious that as soon as you believe something, you thereby believe that you believe it (and so it seems that belief involves a transitive accessibility relation). And in fact, as soon as you believe something, you believe that you *know* it. But one might shy away from saying that knowing something automatically amounts to knowing that you know it. For example, many are attracted to the idea that to know something requires that (i) that it is true, (ii) that you believe it, and (iii) that you are justified in believing it: the justified true belief analysis of knowledge. So, now couldn't it be that you know something, and thus (?) that you believe you know it, and thus that you believe that you are justified in believing it, but that you are not justified in believing that you are *justified* in believing it? After all, one's source of knowledge, one's reliable means of acquiring knowledge, might be a mechanism that one has no insight into. So, while one can implicitly trust (believe) in its reliability, and while it is in fact reliable, one might not have any means to have trustworthy beliefs about it. [Further worries about the KK Thesis are discussed by [Williamson \(2000\)](#).]

### 3.2.3 \*Symmetry

What would the consequences be if the accessibility relation were symmetric? Symmetry of the accessibility relation  $\mathcal{R}$  corresponds to the validity of the following principle:

(9) Brouwer's Axiom :

$$\forall p \forall w : w \in p \rightarrow \left[ \forall w' [w \mathcal{R} w' \rightarrow \exists w'' [w' \mathcal{R} w'' \& w'' \in p]] \right]$$

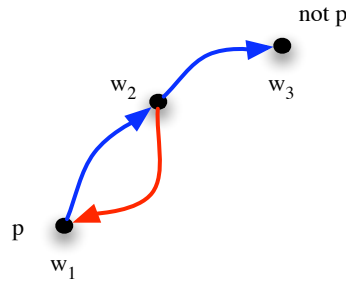


Figure 3.2: Symmetry

In modal logic notation:  $p \rightarrow \Box \Diamond p$ , known simply as B in modal logic. The system that combines T/M with B is often called Brouwer's System (B), after the mathematician L.E.J. Brouwer, not because he proposed it but because it was thought that it had some connections to his doctrines.



L.E.J. Brouwer

Here's the reasoning: Assume that  $\mathcal{R}$  is in fact symmetric. Pick a world  $w$  in which  $p$  is true. Now, could it be that the right hand side of the inference fails to hold in  $w$ ? Assume that it does fail. Then, there must be some world  $w'$  accessible from  $w$  in which  $\Diamond p$  is false. In other words, from that world  $w'$  there is no accessible world  $w''$  in which  $p$  is true. But since  $\mathcal{R}$  is assumed to be symmetric, one of the worlds accessible from  $w'$  is  $w$  and in  $w$ ,  $p$  is true, which contradicts the assumption that the inference doesn't go through. So, symmetry ensures the validity of the inference.

The other way (validity of the inference requires symmetry): the inference says that from any  $p$  world we only have worlds accessible from which there is at least one accessible  $p$  world. But imagine that  $p$  is true in  $w$  but not true in any other world. So, the only way for the conclusion of the inference to hold automatically is to have a guarantee that  $w$  (the only  $p$  world) is accessible from any world accessible from it. That is, we need to have symmetry. QED.

To see whether a particular kind of attitude is based on a symmetric accessibility relation, we can ask whether Brouwer's Axiom is intuitively valid with respect to this attitude. If it is not valid, this shows that the accessibility relation can't be symmetric. In the case of a knowledge-based accessibility relation (epistemic accessibility), one can argue that *symmetry does not hold*:<sup>1</sup>

The symmetry condition would imply that if something happens to be true in the actual world, then you know that it is compatible

<sup>1</sup> Thanks to Bob Stalnaker (pc to Kai von Fintel) for help with the following reasoning.

with your knowledge (Brouwer's Axiom). This will be violated by any case in which your beliefs are consistent, but mistaken. Suppose that while  $p$  is in fact true, you feel certain that it is false, and so think that you know that it is false. Since you think you know this, it is compatible with your knowledge that you know it. (Since we are assuming you are consistent, you can't both believe that you know it, and know that you do not). So it is compatible with your knowledge that you know that *not*  $p$ . Equivalently<sup>2</sup>: you don't know that you don't know that *not*  $p$ . Equivalently: you don't know that it's compatible with your knowledge that  $p$ . But by Brouwer's Axiom, since  $p$  is true, you would have to know that it's compatible with your knowledge that  $p$ . So if Brouwer's Axiom held, there would be a contradiction. So Brouwer's Axiom doesn't hold here, which shows that epistemic accessibility is not symmetric.

Game theorists and theoretical computer scientists who traffic in logics of knowledge often assume that the accessibility relation for knowledge is an equivalence relation (reflexive, symmetric, and transitive). But this is appropriate only if one abstracts away from any error, in effect assuming that belief and knowledge coincide. One striking consequence of working with an equivalence relation as the accessibility relation for knowledge is that one predicts the principle of NEGATIVE INTROSPECTION to hold:

- (IO) NEGATIVE INTROSPECTION (NI)  
 If one doesn't know that  $p$ , then one knows that one doesn't know that  $p$ .  
 $(\neg \Box p \rightarrow \Box \neg \Box p)$ .

This surely seems rather dubious: imagine that one strongly believes that  $p$  but that nevertheless  $p$  is false, then one doesn't know that  $p$ , but one doesn't seem to believe that one doesn't know that  $p$ , in fact one believes that one does know that  $p$ .

### 3.3 Supplemental Readings

A recent survey on attitudes:

Eric Swanson. 2011. Propositional attitudes. In Klaus von Heusinger, Claudia Maienborn & Paul Portner (eds.), *Semantics: An international handbook of meaning*, vol. 2 (Handbücher zur Sprach- und Kommunikationswissenschaft 33.2), chap. 60, 1538–1561. Berlin/Boston: de Gruyter Mouton. <http://dx.doi.org/10.1515/9783110255072.1538>. <http://tinyurl.com/swanson-hsk>.

<sup>2</sup> This and the following step rely on the duality of necessity and possibility:  $q$  is compatible with your knowledge iff you don't know that *not*  $q$ .

All one really needs to make NI valid is to have a EUCLIDEAN accessibility relation: any two worlds accessible from the same world are accessible from each other. It is a nice little exercise to prove this, if you have become interested in this sort of thing. Note that all reflexive and Euclidean accessibility relations are transitive and symmetric as well — another nice little thing to prove.

Further connections between mathematical properties of accessibility relations and logical properties of various notions of necessity and possibility are studied extensively in modal logic:

G.E. Hughes & Max Cresswell. 1996. *A new introduction to modal logic*. London: Routledge.

James Garson. 2008. Modal logic. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*. <http://plato.stanford.edu/entries/logic-modal/>, especially section 7 and 8, “Modal Axioms and Conditions on Frames”, “Map of the Relationships between Modal Logics”.

A thorough discussion of the possible worlds theory of attitudes, and some of its potential shortcomings, can be found in Bob Stalnaker’s work:

Robert Stalnaker. 1984. *Inquiry*. MIT Press.

Robert Stalnaker. 1999. *Context and content*. Oxford: Oxford University Press.

A quick and informative surveys about the notion of knowledge:

Matthias Steup. 2008. The analysis of knowledge. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Fall 2008. <http://plato.stanford.edu/archives/fall2008/entries/knowledge-analysis/>.

Linguistic work on attitudes has often been concerned with various co-occurrence patterns, particularly which moods (indicative or subjunctive or infinitive) occur in the complement and whether negative polarity items are licensed in the complement.

Mood licensing:

Paul Portner. 1997. The semantics of mood, complementation, and conversational force. *Natural Language Semantics* 5(2). 167–212. <http://dx.doi.org/10.1023/A:1008280630142>.

NPI-Licensing:

Nirit Kadmon & Fred Landman. 1993. *Any*. *Linguistics and Philosophy* 16(4). 353–422. <http://dx.doi.org/10.1007/BF00985272>.

Kai von Stechow. 1999. NPI licensing, Strawson entailment, and context dependency. *Journal of Semantics* 16(2). 97–148. <http://dx.doi.org/10.1093/jos/16.2.97>.

Anastasia Giannakidou. 1999. Affective dependencies. *Linguistics and Philosophy* 22(4). 367–421. <http://dx.doi.org/10.1023/A:1005492130684>.

There is some interesting work out of Amherst rethinking the way attitude predicates take their complements:

Angelika Kratzer. 2006. Decomposing attitude verbs. Handout from a talk honoring Anita Mittwoch on her 80th birthday at the Hebrew University of Jerusalem July 4, 2006. <http://semanticsarchive.net/Archive/DcwY2JkM/attitude-verbs2006.pdf>.

Keir Moulton. 2008. Clausal complementation and the *Wager*-class. *North East Linguistics Society (NELS)* 38. <http://sites.google.com/site/keirmoulton/Moultonnels2008wager.pdf>. <http://people.umass.edu/keir/Wager.pdf>.

Keir Moulton. 2009. *Natural selection and the syntax of clausal complementation*. University of Massachusetts at Amherst dissertation. [http://scholarworks.umass.edu/open\\_access\\_dissertations/99/](http://scholarworks.umass.edu/open_access_dissertations/99/).

Tamina Stephenson in her MIT dissertation and related work explores the way attitude predicates interact with epistemic modals and taste predicates in their complements:

Tamina Stephenson. 2007a. Judge dependence, epistemic modals, and predicates of personal taste. *Linguistics and Philosophy* 30(4). 487–525. <http://dx.doi.org/10.1007/s10988-008-9023-4>.

Tamina Stephenson. 2007b. *Towards a theory of subjective meaning*. Massachusetts Institute of Technology dissertation. <http://semanticsarchive.net/Archive/2QxMjkoO/Stephenson-2007-thesis.pdf>.

Jon Gajewski in his MIT dissertation and subsequent work explores the distribution of the NEG-RAISING property among attitude predicates and traces it back to presuppositional components of the meaning of the predicates:

Jon Gajewski. 2005. *Neg-raising: Polarity and presupposition*. Massachusetts Institute of Technology dissertation. <http://dx.doi.org/1721.1/33696>.

Jon Gajewski. 2007. Neg-raising and polarity. *Linguistics and Philosophy*. <http://dx.doi.org/10.1007/s10988-007-9020-z>.

Interesting work has also been done on presupposition projection in attitude contexts:

Nicholas Asher. 1987. A typology for attitude verbs and their anaphoric properties. *Linguistics and Philosophy* 10(2). 125–197. <http://dx.doi.org/10.1007/BF00584317>.

Irene Heim. 1992. Presupposition projection and the semantics of attitude verbs. *Journal of Semantics* 9(3). 183–221. <http://dx.doi.org/10.1093/jos/9.3.183>.

Bart Geurts. 1998. Presuppositions and anaphors in attitude contexts. *Linguistics and Philosophy* 21(6). 545–601. <http://dx.doi.org/10.1023/A:1005481821597>.

## PART II

### DPS IN INTENSIONAL CONTEXTS





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