

# CHAPTER ONE

## MODALITY

*We turn to modal auxiliaries and related constructions. The main difference from attitude constructions is that their semantics is more context-dependent. Otherwise, we are still quantifying over possible worlds.*

### 1.1 The Quantificational Theory of Modality

We will now be looking at modal auxiliaries like *may*, *must*, *can*, *have to*, etc. Most of what we say here should carry over straightforwardly to modal adverbs like *maybe*, *possibly*, *certainly*, etc. We will make certain syntactic assumptions, which make our work easier but which leave aside many questions that at some point deserve to be addressed.

#### 1.1.1 Syntactic Assumptions

We will assume, at least for the time being, that a modal like *may* is a RAISING predicate (rather than a CONTROL predicate), i.e., its subject is not its own argument, but has been moved from the subject-position of its infinitival complement. So, we are dealing with the following kind of structure:

- (1)    a.    Ann may be smart.  
      b.    [ Ann [  $\lambda_t$  [ may [  $t_t$  be smart ] ] ] ]

Actually, we will be working here with the even simpler structure below, in which the subject has been reconstructed to its lowest trace position. (E.g., these could be generated by deleting all but the lowest copy in the movement chain.) We will be able to prove that movement of a name or pronoun never affects truth-conditions, so at any rate the interpretation of the structure in (1b) would be the same as of (2). As a matter of convenience, then, we will take the reconstructed structures, which allow us to abstract away from the (here irrelevant) mechanics of variable binding.

The issue of raising vs. control will probably be taken up later. If you are eager to get started on it and other questions of the morphosyntax of modals, read the handout from an LSA class Sabine and Kai taught a few years ago: <http://web.mit.edu/fintel/lsa220-class-2-handout.pdf>.

We will talk about reconstruction in more detail later.

- (2) may [ Ann be smart ]

So, for now at least, we are assuming that modals are expressions that take a full sentence as their semantic argument.<sup>1</sup> Now then, what do modals mean?

### 1.1.2 Quantification over Possible Worlds

This idea goes back a long time. It was famously held by Leibniz, but there are precedents in the medieval literature, see Knuuttila (2003). See Copeland (2002) for the modern history of the possible worlds analysis of modal expressions.

The basic idea of the possible worlds semantics for modal expressions is that they are quantifiers over possible worlds. Toy lexical entries for *must* and *may*, for example, would look like this:

- (3)  $\llbracket \text{must} \rrbracket^{w,g} = \lambda p_{\langle s,t \rangle}. \forall w': p(w') = 1.$   
 (4)  $\llbracket \text{may} \rrbracket^{w,g} = \lambda p_{\langle s,t \rangle}. \exists w': p(w') = 1.$

This analysis is too crude (in particular, notice that it would make modal sentences non-contingent — there is no occurrence of the evaluation world on the right hand side!). But it does already have some desirable consequences that we will seek to preserve through all subsequent refinements. It correctly predicts a number of intuitive judgments about the logical relations between *must* and *may* and among various combinations of these items and negations. To start with some elementary facts, we feel that *must*  $\phi$  entails *may*  $\phi$ , but not vice versa:

- (5) You must stay.  
 Therefore, you may stay. VALID
- (6) You may stay.  
 Therefore, you must stay. INVALID
- (7) a. You may stay, but it is not the case that you must stay.<sup>2</sup>  
 b. You may stay, but you don't have to stay. CONSISTENT

We judge *must*  $\phi$  incompatible with its “inner negation” *must* [ *not*  $\phi$  ], but find *may*  $\phi$  and *may* [ *not*  $\phi$  ] entirely compatible:

<sup>1</sup> We will assume that even though *Ann be smart* is a non-finite sentence, this will not have any effect on its semantic type, which is that of a sentence, which in turn means that its semantic value is a truth-value. This is hopefully independent of the (interesting) fact that *Ann be smart* on its own cannot be used to make a truth-evaluable assertion.

<sup>2</sup> The somewhat stilted *it is not the case*-construction is used in to make certain that negation takes scope over *must*. When modal auxiliaries and negation are together in the auxiliary complex of the same clause, their relative scope seems not to be transparently encoded in the surface order; specifically, the scope order is not reliably negation  $\succ$  modal. (Think about examples with *mustn't*, *can't*, *shouldn't*, *may not* etc. What's going on here? This is an interesting topic which we must set aside for now. See the references at the end of the chapter for relevant work.) With modal *main* verbs (such as *have to*), this complication doesn't arise; they are consistently inside the scope of clause-mate auxiliary negation. Therefore we can use (b) to (unambiguously) express the same scope order as (a), without having to resort to a biclausal structure.

- (8) You must stay, and/but also, you must leave. (leave = not stay).

CONTRADICTION

- (9) You may stay, but also, you may leave.

CONSISTENT

We also judge that in each pair below, the (a)-sentence and the (b)-sentences say the same thing.

- (10) a. You must stay.  
 b. It is not the case that you may leave.  
 You aren't allowed to leave.  
 (You may not leave.)<sup>3</sup>  
 (You can't leave.)

- (11) a. You may stay.  
 b. It is not the case that you must leave.  
 You don't have to leave.  
 You don't need to leave.  
 (You needn't leave.)

Given that *stay* and *leave* are each other's negations (i.e.  $\llbracket \text{leave} \rrbracket^{w,g} = \llbracket \text{not stay} \rrbracket^{w,g}$ , and  $\llbracket \text{stay} \rrbracket^{w,g} = \llbracket \text{not leave} \rrbracket^{w,g}$ ), the LF-structures of these equivalent pairs of sentences can be seen to instantiate the following schemata:<sup>4</sup>

- (12) a.  $\text{must } \phi \equiv \text{not } [\text{may } [\text{not } \phi]]$   
 b.  $\text{must } [\text{not } \psi] \equiv \text{not } [\text{may } \psi]$

- (13) a.  $\text{may } \phi \equiv \text{not } [\text{must } [\text{not } \phi]]$   
 b.  $\text{may } [\text{not } \psi] \equiv \text{not } [\text{must } \psi]$

Our present analysis of *must*, *have-to*, ... as universal quantifiers and of *may*, *can*, ... as existential quantifiers straightforwardly predicts all of the above judgments, as you can easily prove.

- (14) a.  $\forall x \phi \equiv \neg \exists x \neg \phi$   
 b.  $\forall x \neg \phi \equiv \neg \exists x \phi$

- (15) a.  $\exists x \phi \equiv \neg \forall x \neg \phi$   
 b.  $\exists x \neg \phi \equiv \neg \forall x \phi$

More linguistic data regarding the “parallel logic” of modals and quantifiers can be found in Larry Horn’s dissertation (Horn 1972).

<sup>3</sup> The parenthesized variants of the (b)-sentences are pertinent here only to the extent that we can be certain that negation scopes over the modal. In these examples, apparently it does, but as we remarked above, this cannot be taken for granted in all structures of this form.

<sup>4</sup> In logicians’ jargon, *must* and *may* behave as DUALS of each other. For definitions of “dual”, see Barwise & Cooper (1981, p. 197) or Gamut (1991, vol.2,238).

## 1.2 Flavors of Modality

### 1.2.1 Contingency

We already said that the semantics we started with is too simple-minded. In particular, we have no dependency on the evaluation world, which would make modal statements non-contingent. This is not correct.

If one says *It may be snowing in Cambridge*, that may well be part of useful, practical advice about what to wear on your upcoming trip to Cambridge. It may be true or it may be false. The sentence seems true if said in the dead of winter when we have already heard about a Nor'Easter that is sweeping across New England. The sentence seems false if said by a clueless Australian acquaintance of ours in July.

The contingency of modal claims is not captured by our current semantics. All the *may*-sentence would claim under that semantics is that there is some possible world where it is snowing in Cambridge. And surely, once you have read Lewis' quote in Chapter I, where he asserts the existence of possible worlds with different physical constants than we enjoy here, you must admit that there have to be such worlds even if it is July. The problem is that in our semantics, repeated here

$$(16) \quad \llbracket \text{may} \rrbracket^{w,g} = \lambda p_{\langle s,t \rangle}. \exists w': p(w') = 1.$$

there is no occurrence of  $w$  on the right hand side. This means that the truth-conditions for *may*-sentences are world-independent. In other words, they make non-contingent claims that are either true whatever or false whatever, and because of the plenitude of possible worlds they are more likely to be true than false. This needs to be fixed. But how?

Well, what makes *it may be snowing in Cambridge* seem true when we know about a Nor'Easter over New England? What makes it seem false when we know that it is summer in New England? The idea is that we only consider possible worlds COMPATIBLE WITH THE EVIDENCE AVAILABLE TO US. And since what evidence is available to us differs from world to world, so will the truth of a *may*-statement.

$$(17) \quad \llbracket \text{may} \rrbracket^{w,g} = \lambda p. \exists w' \text{ compatible with the evidence in } w: p(w') = 1.^5$$

$$(18) \quad \llbracket \text{must} \rrbracket^{w,g} = \lambda p. \forall w' \text{ compatible with the evidence in } w: p(w') = 1.$$

Let us consider a different example:

$$(19) \quad \text{You have to be quiet.}$$

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5 From now on, we will leave off type-specifications such as that  $p$  has to be of type  $\langle s, t \rangle$ , whenever it is obvious what they should be and when saving space is aesthetically called for.

Imagine this sentence being said based on the house rules of the particular dormitory you live in. Again, this is a sentence that could be true or could be false. Why do we feel that this is a contingent assertion? Well, the house rules can be different from one world to the next, and so we might be unsure or mistaken about what they are. In one possible world, they say that all noise must stop at 11pm, in another world they say that all noise must stop at 10pm. Suppose we know that it is 10:30 now, and that the dorm we are in has either one or the other of these two rules, but we have forgotten which. Then, for all we know, *you have to be quiet* may be true or it may be false. This suggests a lexical entry along these lines:

$$(20) \quad \llbracket \text{have-to} \rrbracket^{w,g} = \lambda p. \forall w' \text{ compatible with the rules in } w: p(w') = 1.$$

Again, we are tying the modal statement about other worlds down to certain worlds that stand in a certain relation to actual world: those worlds where the rules as they are here are obeyed.

A note of caution: it is very important to realize that the worlds compatible with the rules as they are in  $w$  are those worlds where nothing happens that violates any of the  $w$ -rules. This is not at all the same as saying that the worlds compatible with the rules in  $w$  are those worlds where the same rules are in force. Usually, the rules do not care what the rules are, unless the rules contain some kind of meta-statement to the effect that the rules have to be the way they are, i.e. that the rules cannot be changed. So, in fact, a world  $w'$  in which nothing happens that violates the rules as they are in  $w$  but where the rules are quite different and in fact what happens violates the rules as they are in  $w'$  is nevertheless a world compatible with the rules in  $w$ . For example, imagine that the only relevant rule in  $w$  is that students go to bed before midnight. Take a world  $w'$  where a particular student goes to bed at 11:30 pm but where the rules are different and say that students have to go to bed before 11 pm. Such a world  $w'$  is compatible with the rules in  $w$  (but of course not with the rules in  $w'$ ).

Apparently, there are different flavors of modality, varying in what kind of facts in the evaluation world they are sensitive to. The semantics we gave for *must* and *may* above makes them talk about evidence, while the semantics we gave for *have-to* made it talk about rules. But that was just because the examples were hand-picked. In fact, in the dorm scenario we could just as well have said *You must be quiet*. And, vice versa, there is nothing wrong with using *it has to be snowing in Cambridge* based on the evidence we have. In fact, many modal expressions seem to be multiply ambiguous.

Traditional descriptions of modals often distinguish a number of “readings”: EPISTEMIC, DEONTIC, ABILITY, CIRCUMSTANTIAL, DYNAMIC, . . . . (Beyond “epistemic” and “deontic,” there is a great deal of terminological variety. Sometimes all non-epistemic readings are grouped together under the term ROOT MODALITY.) Here are some initial illustrations.

## (21) EPISTEMIC MODALITY

A: Where is John?

B: I don't know. He *may* be at home.

## (22) DEONTIC MODALITY

A: Am I allowed to stay over at Janet's house?

B: No, but you *may* bring her here for dinner.

## (23) CIRCUMSTANTIAL/DYNAMIC MODALITY

A: I will plant the rhododendron here.

B: That's not a good idea. It *can* grow very tall.

How are *may* and *can* interpreted in each of these examples? What do the interpretations have in common, and where do they differ?

In all three examples, the modal makes an existentially quantified claim about possible worlds. This is usually called the MODAL FORCE of the claim. What differs is what worlds are quantified over. In EPISTEMIC modal sentences, we quantify over worlds compatible with the available evidence. In DEONTIC modal sentences, we quantify over worlds compatible with the rules and/or regulations. And in the CIRCUMSTANTIAL modal sentence, we quantify over the set of worlds which conform to the laws of nature (in particular, plant biology). What speaker B in (23) is saying, then, is that there are some worlds conforming to the laws of nature in which this rhododendron grows very tall. (Or is this another instance of an epistemic reading? See below for discussion of the distinction between circumstantial readings and epistemic ones.)



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How can we account for this variety of readings? One way would be to write a host of lexical entries, basically treating this as a kind of (more or less principled) ambiguity. Another way, which is preferred by many people, is to treat this as a case of context-dependency, as argued in seminal work by Kratzer (1977, 1978, 1981, 1991).

According to Kratzer, what a modal brings with it intrinsically is just a modal force, that is, whether it is an existential (possibility) modal or a universal (necessity) modal. What worlds it quantifies over is determined by context. In essence, the context has to supply a restriction to the quantifier. How can we implement this idea?

We encountered context-dependency before when we talked about pronouns and their referential (and E-Type) readings (H&K, chapters 9–11). We treated referential pronouns as free variables, appealing to a general principle that free variables in an LF need to be supplied with values from the utterance context. If we want to describe the context-dependency of modals in a technically analogous fashion, we can think of their LF-representations as incorporating or subcategorizing for a kind of invisible pronoun, a free variable that stands for a set of possible worlds. So we posit LF-structures like this:

It is well-known that natural language quantification is in general subject to contextual restriction. See Stanley & Szabó (2000) for a recent discussion.

(24) [<sub>I'</sub> [<sub>I</sub> must  $p_{\langle n, \langle s, t \rangle \rangle}$ ] [<sub>VP</sub> you quiet]]

$p_{\langle n, \langle s, t \rangle \rangle}$  here is a variable over (characteristic functions of) sets of worlds, which — like all free variables — needs to receive a value from the utterance context. Possible values include: the set of worlds compatible with the speaker's current knowledge; the set of worlds in which everyone obeys all the house rules of a certain dormitory; and many others. The denotation of the modal itself now has to be of type  $\langle st, \langle st, t \rangle \rangle$  rather than  $\langle st, t \rangle$ , thus it will be more like a quantificational determiner rather than a complete generalized quantifier. Only after the modal has been combined with its covert restrictor do we obtain a value of type  $\langle st, t \rangle$ .

We are using the notation for variables of types other than  $e$ , introduced by Heim & Kratzer.

See p. 213. An index on a variable now is an ordered pair of a natural number and a type.

Q: Can you think of overt anaphoric expressions that are arguably of the type  $\langle s, t \rangle$ , a proposition?

- (25) a.  $\llbracket \text{must} \rrbracket^{w,g} = \llbracket \text{have-to} \rrbracket^{w,g} = \llbracket \text{need-to} \rrbracket^{w,g} = \dots =$   
 $\lambda p \in D_{\langle s, t \rangle}. \lambda q \in D_{\langle s, t \rangle}. \forall w \in W [p(w) = 1 \rightarrow q(w) = 1]$   
 b.  $\llbracket \text{may} \rrbracket^{w,g} = \llbracket \text{can} \rrbracket^{w,g} = \llbracket \text{be-allowed-to} \rrbracket^{w,g} = \dots =$   
 $\lambda p \in D_{\langle s, t \rangle}. \lambda q \in D_{\langle s, t \rangle}. \exists w \in W [p(w) = 1 \ \& \ q(w) = 1]$

in set talk:  $p \subseteq q$

in set talk:  $p \cap q \neq \emptyset$

On this approach, the epistemic, deontic, etc. “readings” of individual occurrences of modal verbs come about by a combination of two separate things. The lexical semantics of the modal itself encodes just a quantificational force, a *relation* between sets of worlds. This is either the subset-relation (universal quantification; necessity) or the relation of non-disjointness (existential quantification; possibility). The covert variable next to the modal picks up a contextually salient set of worlds, and this functions as the quantifier's restrictor. The labels “epistemic”, “deontic”, “circumstantial” etc. group together certain conceptually natural classes of possible values for this covert restrictor.

Notice that, strictly speaking, there is not just one deontic reading (for example), but many. A speaker who utters

(26) You have to be quiet.

might mean: ‘I want you to be quiet,’ (i.e., you are quiet in all those worlds that conform to my preferences). Or she might mean: ‘unless you are quiet, you won't succeed in what you are trying to do,’ (i.e., you are quiet in all those worlds in which you succeed at your current task). Or she might mean: ‘the house rules of this dormitory here demand that you be quiet,’ (i.e., you are quiet in all those worlds in which the house rules aren't violated). And so on. So the label “deontic” appears to cover a whole open-ended set of imaginable “readings”, and which one is intended and understood on a particular utterance occasion may depend on all sorts of things in the interlocutors' previous conversation and tacit shared assumptions. (And the same goes for the other traditional labels.)

### 1.2.2 Epistemic vs. Circumstantial Modality

Is it all context-dependency? Or do flavors of modality correspond to some sorts of signals in the structure of sentences? Read the following famous passage from Kratzer and think about how the two sentences with their very different modal meanings differ in structure:

Quoted from Kratzer (1991). In Kratzer (1981), the hydrangeas were *Zwetschenbäume* 'plum trees'. The German word *Zwetschge*, by the way, is etymologically derived from the name of the city Damascus (Syria), the center of the ancient plum trade.

Consider sentences (27) and (28):

- (27) Hydrangeas can grow here.
- (28) There might be hydrangeas growing here.

The two sentences differ in meaning in a way which is illustrated by the following scenario.

“Hydrangeas”

Suppose I acquire a piece of land in a far away country and discover that soil and climate are very much like at home, where hydrangeas prosper everywhere. Since hydrangeas are my favorite plants, I wonder whether they would grow in this place and inquire about it. The answer is (27). In such a situation, the proposition expressed by (27) is true. It is true regardless of whether it is or isn't likely that there are already hydrangeas in the country we are considering. All that matters is climate, soil, the special properties of hydrangeas, and the like. Suppose now that the country we are in has never had any contacts whatsoever with Asia or America, and the vegetation is altogether different from ours. Given this evidence, my utterance of (28) would express a false proposition. What counts here is the complete evidence available. And this evidence is not compatible with the existence of hydrangeas.

(27) together with our scenario illustrates the pure CIRCUMSTANTIAL reading of the modal *can*. [...]. (28) together with our scenario illustrates the epistemic reading of modals. [...] circumstantial and epistemic conversational backgrounds involve different kinds of facts. In using an epistemic modal, we are interested in what else may or must be the case in our world given all the evidence available. Using a circumstantial modal, we are interested in the necessities implied by or the possibilities opened up by certain sorts of facts. Epistemic modality is the modality of curious people like historians, detectives, and futurologists. Circumstantial modality is the modality of rational agents like gardeners, architects, and engineers. A historian asks what might have been the case, given all the available facts. An engineer asks what can be done given certain relevant facts.



Consider also the very different prominent meanings of the following two sentences, taken from Kratzer as well:

- (29) a. Cathy can make a pound of cheese out of this can of milk.  
b. Cathy might make a pound of cheese out of this can of milk.

EXERCISE 1.1: Come up with examples of epistemic, deontic, and circumstantial uses of the necessity verb *have to*. Describe the set of worlds that constitutes the understood restrictor in each of your examples. □

### 1.2.3 Contingency Again

We messed up. If you inspect the context-dependent meanings we have on the table now for our modals, you will see that the right hand sides again do not mention the evaluation world  $w$ . Therefore, we will again have the problem of not making contingent claims, indirectly about the actual world. This needs to be fixed. We need a semantics that is both context-dependent and contingent.

The problem, it turns out, is with the idea that the utterance context supplies a *determinate set of worlds* as the restrictor. When I understand that you meant your use of *must*, in *you must be quiet*, to quantify over the set of worlds in which the house rules of our dorm are obeyed, this does not imply that you and I have to know or agree on which set exactly this is. That depends on what the house rules in our world actually happen to say, and this may be an open question at the current stage of our conversation. What we do agree on, if I have understood your use of *must* in the way that you intended it, is just that it quantifies over *whatever set of worlds it may be* that the house rules pick out.

The technical implementation of this insight requires that we think of the context's contribution not as a set of worlds, but rather as a function which for each world it applies to picks out such a set. For example, it may be the function which, for any world  $w$ , yields the set  $\{w' : \text{the house rules that are in force in } w \text{ are obeyed in } w'\}$ . If we apply this function to a world  $w_1$ , in which the house rules read “no noise after 10 pm”, it will yield a set of worlds in which nobody makes noise after 10 pm. If we apply the same function to a world  $w_2$ , in which the house rules read “no noise after 11 pm”, it will yield a set of worlds in which nobody makes noise after 11 pm.

Suppose, then, that the covert restrictor of a modal predicate denotes such a function, i.e., its value is of type  $\langle s, st \rangle$ .

- (30)  $[\Gamma \ [I \text{ must } R_{\langle n, \langle s, st \rangle \rangle}] \ [VP \text{ you quiet}]]$

And the new lexical entries for *must* and *may* that will fit this new structure are these:

- (31) a.  $\llbracket \text{must} \rrbracket^{w,g} = \llbracket \text{have-to} \rrbracket^{w,g} = \llbracket \text{need-to} \rrbracket^{w,g} = \dots =$   
 $\lambda R \in D_{\langle s, st \rangle}. \lambda q \in D_{\langle s, t \rangle}. \forall w' \in W [R(w)(w') = 1 \rightarrow q(w') = 1]$

in set talk:  $(R(w) \subseteq q$

You will of course recognize that functions of type  $\langle s, st \rangle$  are simply a schönfinkled version of the ACCESSIBILITY RELATIONS we introduced in the previous chapter.

in set talk:  $(R(w) \cap q \neq \emptyset$

$$\text{b. } \llbracket \text{may} \rrbracket^{w,g} = \llbracket \text{can} \rrbracket^{w,g} = \llbracket \text{be-allowed-to} \rrbracket^{w,g} = \dots = \\ \lambda R \in D_{\langle s, st \rangle}. \lambda q \in D_{\langle s, t \rangle}. \exists w' \in W [R(w)(w') = 1 \ \& \ q(w') = 1]$$

Let us see now how this solves the contingency problem.

- (32) Let  $w$  be a world, and assume that the context supplies an assignment  $g$  such that  $g(R_{\langle 17, \langle s, st \rangle \rangle}) = \lambda w. \lambda w'. \text{ the house rules in force in } w \text{ are obeyed in } w'$

$$\llbracket \text{must } R_{\langle 17, \langle s, st \rangle \rangle} \text{ you quiet} \rrbracket^{w,g} = \quad (\text{IFA})$$

$$\llbracket \text{must } R_{\langle 17, \langle s, st \rangle \rangle} \rrbracket^{w,g} (\lambda w' \llbracket \text{you quiet} \rrbracket^{w'}) = \quad (\text{FA})$$

$$\llbracket \text{must} \rrbracket^{w,g} (\llbracket R_{\langle 17, \langle s, st \rangle \rangle} \rrbracket^{w,g} (\lambda w' \llbracket \text{you quiet} \rrbracket^{w'})) = \quad (\text{lex. entries } \textit{you}, \textit{quiet})$$

$$\llbracket \text{must} \rrbracket^{w,g} (\llbracket R_{\langle 17, \langle s, st \rangle \rangle} \rrbracket^{w,g} (\lambda w'. \text{ you are quiet in } w')) = \quad (\text{lex. entry } \textit{must})$$

$$\forall w' \in W : \llbracket R_{\langle 17, \langle s, st \rangle \rangle} \rrbracket^{w,g}(w)(w') = 1 \rightarrow \text{you are quiet in } w' = \quad (\text{pronoun rule})$$

$$\forall w' \in W : g(R_{\langle 17, \langle s, st \rangle \rangle})(w)(w') = 1 \rightarrow \text{you are quiet in } w' = \quad (\text{def. of } g)$$

$$\forall w' \in W [\text{the house rules in force in } w \text{ are obeyed in } w' \\ \rightarrow \text{you are quiet in } w']$$

As we see in the last line of (32), the truth-value of (30) depends on the evaluation world  $w$ .

EXERCISE 1.2: Describe two worlds  $w_1$  and  $w_2$  so that

$$\llbracket \text{must } R_{\langle 17, \langle s, st \rangle \rangle} \text{ you quiet} \rrbracket^{w_1, g} = 1 \text{ and } \llbracket \text{must } R_{\langle 17, \langle s, st \rangle \rangle} \text{ you quiet} \rrbracket^{w_2, g} = 0. \quad \square$$

EXERCISE 1.3: In analogy to the deontic relation  $g(R_{\langle 17, \langle s, st \rangle \rangle})$  defined in (32), define an appropriate relation that yields an epistemic reading for a sentence like *You may be quiet*.  $\square$

### 1.2.4 Iteration

Consider the following example:

- (33) You might have to leave.

What does this mean? Under one natural interpretation, we learn that the speaker considers it possible that the addressee is under the obligation to leave. This seems to involve one modal embedded under a higher modal. It appears that this sentence should be true in a world  $w$  iff some world  $w'$  compatible with what the speaker knows in  $w$  is such that every world  $w''$  in which the rules as they are in  $w'$  are followed is such that you leave in  $w''$ .

Assume the following LF:

$$(34) \quad [I' [\text{might } R_{\langle 1, \langle s, st \rangle \rangle}] [\text{VP} [\text{have-to } R_{\langle 2, \langle s, st \rangle \rangle}] [\text{IP you leave}]]]$$

There is more to be said about which modals can embed under which other modals. See for some discussion the handout mentioned earlier: <http://web.mit.edu/fintel/lsa220-class-2-handout.pdf>.

Suppose  $w$  is the world for which we calculate the truth-value of the whole sentence, and the context maps  $R_1$  to the function which maps  $w$  to the set of all those worlds compatible with what is known in  $w$ . *might* says that some of those worlds are worlds  $w'$  that make the tree below *might* true. Now assume further that the context maps  $R_2$  to the function which assigns to any such world  $w'$  the set of all those worlds in which the rules as they are in  $w'$  are followed. *have to* says that all of those worlds are worlds  $w''$  in which you leave.

From now on, we will omit the type-designation of variables whenever we feel confident that their type is easy to figure out from the context.

In other words, while it is not known to be the case that you have to leave, for all the speaker knows it might be the case.

EXERCISE 1.4: Describe values for the covert  $\langle s, st \rangle$ -variable that are intuitively suitable for the interpretation of the modals in the following sentences:

- (35) As far as John's preferences are concerned, you *may* stay with us.
- (36) According to the guidelines of the graduate school, every PhD candidate *must* take 9 credit hours outside his/her department.
- (37) John *can* run a mile in 5 minutes.
- (38) This *has* to be the White House.
- (39) This elevator *can* carry up to 3000 pounds.

For some of the sentences, different interpretations are conceivable depending on the circumstances in which they are uttered. You may therefore have to sketch the utterance context you have in mind before describing the accessibility relation.  $\square$

EXERCISE 1.5: Collect two naturally occurring examples of modalized sentences (e.g., sentences that you overhear in conversation, or read in a newspaper or novel – not ones that are being used as examples in a linguistics or philosophy paper!), and give definitions of values for the covert  $\langle s, st \rangle$ -variable which account for the way in which you actually understood these sentences when you encountered them. (If the appropriate interpretation is not salient for the sentence out of context, include information about the relevant preceding text or non-linguistic background.)  $\square$

### 1.2.5 A technical variant of the analysis

In our account of the contingency of modalized sentences, we adopted lexical entries for the modals that gave them world-dependent extensions of type  $\langle \langle s, st \rangle, \langle st, t \rangle \rangle$ :

- (40) (repeated from earlier):  
 For any  $w \in W$ :  $\llbracket \text{must} \rrbracket^{w,9}$   
 $\lambda R \in D_{\langle s, st \rangle}. \lambda q \in D_{\langle s, t \rangle}. \forall w' \in W [R(w)(w') = 1 \rightarrow q(w') = 1]$   
 (in set talk:  $\lambda R_{\langle s, st \rangle}. \lambda q_{\langle s, t \rangle}. (R(w) \subseteq q)$ ).

Unfortunately, this treatment somewhat obscures the parallel between the modals and the quantificational determiners, which have world-independent extensions of type  $\langle et, \langle et, t \rangle \rangle$ .

Let's explore an alternative solution to the contingency problem, which will allow us to stick with the world-independent type- $\langle st, \langle st, t \rangle \rangle$ -extensions that we assumed for the modals at first:

- (41) (repeated from even earlier):  

$$\llbracket \text{must} \rrbracket^{w,g} = \lambda p \in D_{\langle s, t \rangle}. \lambda q \in D_{\langle s, t \rangle}. \forall w \in W [p(w) = 1 \rightarrow q(w) = 1]$$
 (in set talk:  $\lambda p \in D_{\langle s, t \rangle}. \lambda q \in D_{\langle s, t \rangle}. p \subseteq q$ ).

We posit the following LF-representation:

- (42)  $[I' [I \text{ must } [R_{\langle 4, \langle s, st \rangle \rangle} w^*]] [_{VP} \text{ you quiet}]]$

What is new here is that the covert restrictor is complex. The first part,  $R_{\langle 4, \langle s, st \rangle \rangle}$ , is (as before) a free variable of type  $\langle s, st \rangle$ , which gets assigned an accessibility relation by the context of utterance. The second part is a special terminal symbol which is interpreted as picking out the evaluation world:

- (43) For any  $w \in W$ :  $\llbracket w^* \rrbracket^{w,g} = w$ .<sup>6</sup>

When  $R_{\langle 4, \langle s, st \rangle \rangle}$  and  $w^*$  combine (by Functional Application), we obtain a constituent whose extension is of type  $\langle s, t \rangle$  (a proposition or set of worlds). This is the same type as the extension of the free variable  $p$  in the previous proposal, hence suitable to combine with the old entry for *must* (by FA). However, while the extension of  $p$  was completely fixed by the variable assignment, and did not vary with the evaluation world, the new complex constituent's extension depends on both the assignment and the world:

- (44) For any  $w \in W$  and any assignment  $g$ :  

$$\llbracket R_{\langle 4, \langle s, st \rangle \rangle} (w^*) \rrbracket^{w,g} = g(\langle 4, \langle s, st \rangle \rangle)(w).$$

As a consequence of this, the extensions of the higher nodes  $I$  and  $I'$  will also vary with the evaluation world, and this is how we capture the fact that (42) is contingent.

Maybe this variant is more appealing. But for the rest of this chapter, we continue to assume the original analysis as presented earlier. In the next chapter on conditionals, we will however make crucial use of this way of formulating the semantics for modals. So, make sure you understand what we just proposed.

<sup>6</sup> Dowty (1982) introduced an analogous symbol to pick out the evaluation *time*. We have chosen the star-notation to allude to this precedent.

Note that as soon as we're introducing an object language expression whose extension is a possible world, we will now have expressions of type  $s$  and should also introduce the domain of things of type  $s$ :  $D_s = W$ .

### 1.3 \*Kratzer's Conversational Backgrounds

Angelika Kratzer has some interesting ideas on how accessibility relations are supplied by the context. She argues that what is really floating around in a discourse is a CONVERSATIONAL BACKGROUND. Accessibility relations can be computed from conversational backgrounds (as we shall do here), or one can state the semantics of modals directly in terms of conversational backgrounds (as Kratzer does).

A conversational background is the sort of thing that is identified by phrases like *what the law provides*, *what we know*, etc. Take the phrase *what the law provides*. What the law provides is different from one possible world to another. And what the law provides in a particular world is a *set of propositions*. Likewise, what we know differs from world to world. And what we know in a particular world is a set of propositions. The intension of *what the law provides* is then that function which assigns to every possible world the set of propositions  $p$  such that the law provides in that world that  $p$ . Of course, that doesn't mean that  $p$  holds in that world itself: the law can be broken. And the intension of *what we know* will be that function which assigns to every possible world the set of propositions we know in that world. Quite generally, conversational backgrounds are functions of type  $\langle s, \langle st, t \rangle \rangle$ , functions from worlds to (characteristic functions of) sets of propositions.

Now, consider:

- (45) (In view of what we know,) Brown must have murdered Smith.

The *in view of*-phrase may explicitly signal the intended conversational background. Or, if the phrase is omitted, we can just infer from other clues in the discourse that such an epistemic conversational background is intended. We will focus on the case of pure context-dependency.

How do we get from a conversational background to an accessibility relation? Take the conversational background at work in (45). It will be the following:

- (46)  $\lambda w. \lambda p. p$  is one of the propositions that we know in  $w$ .

This conversational background will assign to any world  $w$  the set of propositions  $p$  that in  $w$  are known by us. So we have a set of propositions. From that we can get the set of worlds in which all of the propositions in this set are true. These are the worlds that are compatible with everything we know. So, this is how we get an accessibility relation:

- (47) For any conversational background  $f$  of type  $\langle s, \langle st, t \rangle \rangle$ , we define the corresponding accessibility relation  $R_f$  of type  $\langle s, st \rangle$  as follows:  
 $R_f := \lambda w. \lambda w'. \forall p [f(w)(p) = 1 \rightarrow p(w') = 1]$ .

In words,  $w'$  is  $f$ -accessible from  $w$  iff all propositions  $p$  that are assigned by  $f$  to  $w$  are true in  $w'$ .

Kratzer uses the term **MODAL BASE** for the conversational background that determines the set of accessible worlds. We can be sloppy and use this term for a number of interrelated concepts:

- (i) the conversational background (type  $\langle s, \langle st, t \rangle \rangle$ ),
- (ii) the set of propositions assigned by the conversational background to a particular world (type  $\langle st, t \rangle$ ),
- (iii) the accessibility relation (type  $\langle s, st \rangle$ ) determined by (i),
- (iv) the set of worlds accessible from a particular world (type  $\langle s, t \rangle$ ).

Kratzer calls a conversational background (modal base) **REALISTIC** iff it assigns to *any* world a set of propositions that are all true in that world. The modal base *what we know* is realistic, the modal bases *what we believe* and *what we want* are not.

What follows are some (increasingly technical exercises) on conversational backgrounds.

**EXERCISE 1.6:** Show that a conversational background  $f$  is realistic iff the corresponding accessibility relation  $R_f$  (defined as in (47)) is reflexive.  $\square$

**EXERCISE 1.7:** Let us call an accessibility relation **TRIVIAL** if it makes every world accessible from every world.  $R$  is **TRIVIAL** iff  $\forall w \forall w' : w' \in R(w)$ . What would the conversational background  $f$  have to be like for the accessibility relation  $R_f$  to be trivial in this sense?  $\square$

**EXERCISE 1.8:** The definition in (47) specifies, in effect, a function from  $D_{\langle s, \langle st, t \rangle \rangle}$  to  $D_{\langle s, st \rangle}$ . It maps each function  $f$  of type  $\langle s, \langle st, t \rangle \rangle$  to a unique function  $R_f$  of type  $\langle s, st \rangle$ . This mapping is not one-to-one, however. Different elements of  $D_{\langle s, \langle st, t \rangle \rangle}$  may be mapped to the same value in  $D_{\langle s, st \rangle}$ .<sup>7</sup>

<sup>7</sup> In this exercise, we systematically substitute sets for their characteristic functions. I.e., we pretend that  $D_{\langle s, t \rangle}$  is the power set of  $W$  (i.e., elements of  $D_{\langle s, t \rangle}$  are sets of worlds), and  $D_{\langle st, t \rangle}$  is the power set of  $D_{\langle s, t \rangle}$  (i.e., elements of  $D_{\langle st, t \rangle}$  are sets of sets of worlds). On these assumptions, the definition in (47) can take the following form:

- (i) For any conversational background  $f$  of type  $\langle s, \langle st, t \rangle \rangle$ ,  
we define the corresponding accessibility relation  $R_f$  of type  $\langle s, st \rangle$  as follows:  
 $R_f := \lambda w. \{w' : \forall p [p \in f(w) \rightarrow w' \in p]\}.$

The last line of this can be further abbreviated to:

- (ii)  $R_f := \lambda w. \cap f(w)$

This formulation exploits a set-theoretic notation which we have also used in condition (48) of the second part of the exercise. It is defined as follows:

- (iii) If  $S$  is a set of sets, then  $\cap S := \{x : \forall Y [Y \in S \rightarrow x \in Y]\}.$

- Prove this claim. I.e., give an example of two functions  $f$  and  $f'$  in  $D_{\langle s, \langle st, t \rangle \rangle}$  for which (47) determines  $R_f = R_{f'}$ .
- As you have just proved, if every function of type  $\langle s, \langle st, t \rangle \rangle$  qualifies as a ‘conversational background’, then two different conversational backgrounds can collapse into the same accessibility relation. Conceivably, however, if we imposed further restrictions on conversational backgrounds (i.e., conditions by which only a proper subset of the functions in  $D_{\langle s, \langle st, t \rangle \rangle}$  would qualify as conversational backgrounds), then the mapping between conversational backgrounds and accessibility relations might become one-to-one after all. In this light, consider the following potential restriction:

(48) Every conversational background  $f$  must be “closed under entailment”; i.e., it must meet this condition:  
 $\forall w. \forall p [\cap f(w) \subseteq p \rightarrow p \in f(w)]$ .

(In words: if the propositions in  $f(w)$  taken together entail  $p$ , then  $p$  must itself be in  $f(w)$ .) Show that this restriction would ensure that the mapping defined in (47) will be one-to-one.  $\square$

## 1.4 Supplementary Readings

The most important background readings for this chapter are the following two papers by Kratzer:

Angelika Kratzer. 1981. The notional category of modality. In Hans-Jürgen Eikmeyer & Hannes Rieser (eds.), *Words, worlds, and contexts: New approaches in word semantics* (Research in Text Theory 6), 38–74. Berlin: de Gruyter.

Angelika Kratzer. 1991. Modality. In Arnim von Stechow & Dieter Wunderlich (eds.), *Semantics: An international handbook of contemporary research*, 639–650. Berlin: de Gruyter. <https://udrive.oit.umass.edu/kratzer/kratzer-modality.pdf>.

Kratzer has been updating her classic papers for a volume of her collected work on modality and conditionals. These are very much worth studying: <http://semanticsarchive.net/Archive/Tc2NjA1M/>.

A major new resource on modality is Paul Portner’s book:

Paul Portner. 2009. *Modality*. Oxford University Press.

You might also profit from other survey-ish type papers:

Kai von Fintel. 2005. Modality and language. In Donald M. Borchert (ed.), *Encyclopedia of philosophy – second edition*. MacMillan. <http://mit.edu/fintel/fintel-2005-modality.pdf>.

Kai von Fintel & Anthony S. Gillies. 2007. An opinionated guide to epistemic modality. In Tamar Szabó Gendler & John Hawthorne (eds.), *Oxford studies in epistemology: Volume 2*, 32–62. Oxford University Press. <http://mit.edu/fintel/fintel-gillies-2007-ose2.pdf>.

Eric Swanson. 2008. Modality in language. *Philosophy Compass* 3(6). 1193–1207. <http://dx.doi.org/10.1111/j.1747-9991.2008.00177.x>.

Valentine Hacquard. 2009. Modality. ms, prepared for *Semantics: An international handbook of meaning*, edited by Klaus von Heusinger, Claudia Maienbon, and Paul Portner. [http://ling.umd.edu/~hacquard/papers/HoS\\_Modality\\_Hacquard.pdf](http://ling.umd.edu/~hacquard/papers/HoS_Modality_Hacquard.pdf).

On the syntax of modals, there are only a few papers of uneven quality. Some of the more recent work is listed here. Follow up on older references from the bibliographies in these papers.

Rajesh Bhatt. 1997. Obligation and possession. In Heidi Harley (ed.), *Papers from the upenn/mit roundtable on argument structure and aspect*, vol. 32 (MIT Working Papers in Linguistics), 21–40. <http://people.umass.edu/bhatt/papers/bhatt-haveto.pdf>.

Susi Wurmbrand. 1999. Modal verbs must be raising verbs. *West Coast Conference on Formal Linguistics (WCCFL)* 18. 599–612. <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.35.7442&rep=rep1&type=pdf>.

Annabel Cormack & Neil Smith. 2002. Modals and negation in English. In Sjef Barbiers, Frits Beukema & Wim van der Wurff (eds.), *Modality and its interaction with the verbal system*, 133–163. Benjamins.

Jonny Butler. 2003. A minimalist treatment of modality. *Lingua* 113(10). 967–996. [http://dx.doi.org/10.1016/S0024-3841\(02\)00146-8](http://dx.doi.org/10.1016/S0024-3841(02)00146-8).

The following paper explores some issues in the LF-syntax of epistemic modals:

Kai von Fintel & Sabine Iatridou. 2003. Epistemic containment. *Linguistic Inquiry* 34(2). 173–198. <http://dx.doi.org/10.1162/002438903321663370>.

Valentine Hacquard's MIT dissertation is a rich source of cross-linguistic issues in modality, as is Fabrice Nauze's Amsterdam dissertation:

Valentine Hacquard. 2006. *Aspects of modality*. Massachusetts Institute of Technology PhD thesis. [http://people.umass.edu/hacquard/hacquard\\_thesis.pdf](http://people.umass.edu/hacquard/hacquard_thesis.pdf).

Fabrice Nauze. 2008. *Modality in typological perspective*. Universiteit van Amsterdam PhD thesis. <http://www.illc.uva.nl/Publications/Dissertations/DS-2008-08.text.pdf>.

The semantics of epistemic modals has become a hot topic recently. Here are some of the main references:



- Ian Hacking. 1967. Possibility. *The Philosophical Review* 76(2). 143–168. <http://dx.doi.org/10.2307/2183640>. <http://www.jstor.org/stable/2183640>.
- Paul Teller. 1972. Epistemic possibility. *Philosophia* 2(4). 302–320. <http://dx.doi.org/10.1007/BF02381591>.
- Keith DeRose. 1991. Epistemic possibilities. *The Philosophical Review* 100(4). 581–605. <http://dx.doi.org/10.2307/2185175>.
- Andy Egan, John Hawthorne & Brian Weatherson. 2005. Epistemic modals in context. In Gerhard Preyer & Georg Peter (eds.), *Contextualism in philosophy: Knowledge, meaning, and truth*, 131–170. Oxford: Oxford University Press.
- Andy Egan. 2007. Epistemic modals, relativism, and assertion. *Philosophical Studies* 133(1). 1–22. <http://dx.doi.org/10.1007/s11098-006-9003-x>.
- John MacFarlane. 2006. Epistemic modals are assessment-sensitive. ms, University of California, Berkeley, forthcoming in an OUP volume on epistemic modals, edited by Brian Weatherson and Andy Egan. <http://sophos.berkeley.edu/macfarlane/epistmod.pdf>.
- Tamina Stephenson. 2007. Judge dependence, epistemic modals, and predicates of personal taste. *Linguistics and Philosophy* 30(4). 487–525. <http://dx.doi.org/10.1007/s10988-008-9023-4>.
- John Hawthorne. 2007. Eavesdroppers and epistemic modals. ms, Rutgers University, to appear in the proceedings of the 2007 Sofia Conference in Mexico, in a supplement to *Noûs*.
- Kai von Fintel & Anthony S. Gillies. 2008a. CIA leaks. *The Philosophical Review* 117(1). 77–98. <http://dx.doi.org/10.1215/00318108-2007-025>.
- Kai von Fintel & Anthony S. Gillies. 2008b. *Might* made right. To appear in a volume on epistemic modality, edited by Andy Egan and Brian Weatherson, Oxford University Press. <http://mit.edu/fintel/fintel-gillies-2008-mm.pdf>.
- A recent SALT paper by Pranav Anand and Valentine Hacquard tackles what happens to epistemic modals under attitude predicates:
- Pranav Anand & Valentine Hacquard. 2008. Epistemics with attitude. *Semantics and Linguistic Theory (SALT)* 18. <http://dx.doi.org/1813/13025>.
- Evidentiality is a topic closely related to epistemic modality. Some references:
- Thomas Willett. 1988. A cross-linguistic survey of the grammaticalization of evidentiality. *Studies in Language* 12(1). 51–97.
- Alexandra Y. Aikhenvald. 2004. *Evidentiality*. Oxford: Oxford University Press.

- Hans Bernhard Drubig. 2001. On the syntactic form of epistemic modality. ms, Universität Tübingen. <http://www.sfb441.uni-tuebingen.de/b2/papers/DrubigModality.pdf>.
- Eleanor M. Blain & Rose-Marie Déchaine. 2007. Evidential types: Evidence from Cree dialects. *International Journal of American Linguistics* 73(3). 257–291. <http://dx.doi.org/10.1086/521728>.
- Eric McCready & Norry Ogata. 2007. Evidentiality, modality and probability. *Linguistics and Philosophy* 30(2). 147–206. <http://dx.doi.org/10.1007/s10988-007-9017-7>.
- Peggy Speas. 2008. On the syntax and semantics of evidentials. *Language and Linguistics Compass* 2(5). 940–965. <http://dx.doi.org/10.1111/j.1749-818X.2008.00069.x>.
- Kai von Fintel & Anthony S. Gillies. 2010. *Must ... stay ... strong!* *Natural Language Semantics* 18(4). 351–383. <http://dx.doi.org/10.1007/s11050-010-9058-2>.
- Modals interact with disjunction and indefinites to generate so-called FREE CHOICE-readings, which are a perennial puzzle. Here is just a very small set of initial references:
- Hans Kamp. 1973. Free choice permission. *Proceedings of the Aristotelian Society, New Series* 74. 57–74. <http://www.jstor.org/stable/4544849>.
- Thomas Ede Zimmermann. 2000. Free choice disjunction and epistemic possibility. *Natural Language Semantics* 8(4). 255–290. <http://dx.doi.org/10.1023/A:1011255819284>.
- Katrin Schulz. 2005. A pragmatic solution for the paradox of free choice permission. *Synthese* 147(2). 343–377. <http://dx.doi.org/10.1007/s11229-005-1353-y>.
- Maria Aloni. 2007. Free choice, modals, and imperatives. *Natural Language Semantics* 15(1). 65–94. <http://dx.doi.org/10.1007/s11050-007-9010-2>.
- Luis Alonso-Ovalle. 2006. *Disjunction in alternative semantics*. University of Massachusetts at Amherst PhD thesis. [http://alonso-ovalle.net/index.php?page\\_id=28](http://alonso-ovalle.net/index.php?page_id=28).
- Danny Fox. 2007. Free choice and the theory of scalar implicatures. In Uli Sauerland & Penka Stateva (eds.), *Presupposition and implicature in compositional semantics*, 537–586. New York: Palgrave Macmillan. [http://web.mit.edu/linguistics/people/faculty/fox/free\\_choice.pdf](http://web.mit.edu/linguistics/people/faculty/fox/free_choice.pdf).
- Robert van Rooij. 2006. Free choice counterfactual donkeys. *Journal of Semantics* 23(4). 383–402. <http://dx.doi.org/10.1093/jos/ffl004>.