

INTENSIONAL SEMANTICS

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About these lecture notes

These lecture notes have been evolving for years now, starting with some old notes from the early 1990s by Angelika Kratzer, Irene Heim, and myself, which have since been modified and expanded many times by Irene or myself. Because this version of the notes has not been seen by my co-author, I alone am responsible for any defects.

We encourage the use of these notes in courses at other institutions. Of course, you need to give full credit to the authors and you may not use the notes for any commercial purposes. If you use the notes, we would like to be notified and we would very much appreciate any comments, criticism, and advice on these materials.

Link to the latest full version (currently the 2011 edition):

<http://kvf.me/intensional>

GitHub repository with the current development version:

<https://github.com/fintelkai/fintel-heim-intensional-notes>

Homepage of the class these notes are designed for:

<http://stellar.mit.edu/S/course/24/sp15/24.973>

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Some advice

1. These notes presuppose familiarity with the material, concepts, and notation of the Heim & Kratzer textbook.
2. There are numerous exercises throughout the notes. It is highly recommended to do all of them and it is certainly necessary to do so if you at all anticipate doing semantics-related work in the future.
3. The notes are designed to go along with explanatory lectures. You should ask questions and make comments as you work through the notes.
4. Students with semantic ambitions should also at an early point start reading supplementary material (as for example listed at the end of each chapter of these notes).
5. Lastly, prospective semanticists may start thinking about how *they* would teach this material.

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CHAPTER ONE

BEGINNINGS

Language is the main instrument
of man's refusal to accept the world
as it is.

George Steiner, *After Babel*, p. 228

We introduce the idea of extension vs. intension and its main use: taking us from the actual here and now to past, future, possible, counterfactual situations. We develop a compositional framework for intensional semantics.

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1.1 Displacement

Hockett (1960) in a famous article (and a follow-up, Hockett & Altmann (1968)) presented a list of DESIGN FEATURES OF HUMAN LANGUAGE. This list continues to play a role in current discussions of animal communication. One of the design

Charles F. Hockett. 1960.
The origin of speech. *Scientific American* 203. 89–96

features is **DISPLACEMENT**. Human language is not restricted to discourse about the *actual here and now*.¹

How does natural language untie us from the actual here and now? One degree of freedom is given by the ability to name entities and refer to them even if they are not where we are when we speak:

- (1) Thomas is in Hamburg.

This kind of displacement is not something we will explore here. We'll take it for granted.

Consider a sentence with no names of absent entities in it:

- (2) It is snowing (in Cambridge).

On its own, (2) makes a claim about what is happening right now here in Cambridge. But there are devices at our disposal that can be added to (2), resulting in claims about snow in displaced situations. Displacement can occur in the **TEMPORAL** dimension and/or in what might be called the **MODAL** dimension. Here's an example of temporal displacement:

- (3) At noon yesterday, it was snowing in Cambridge.

This sentence makes a claim not about snow now but about snow at noon yesterday, a different time from now.

Here's an example of modal displacement:

- (4) If the storm system hadn't been deflected by the jet stream, it would have been snowing in Cambridge.

This sentence makes a claim not about snow in the actual world but about snow in the world as it would have been if the storm system hadn't been deflected by the jet stream, a world distinct from the actual one (where the system did not hit us), a merely **POSSIBLE WORLD**.

Natural language abounds in modal constructions. (4) is a so-called **COUNTERFACTUAL CONDITIONAL**. Here are some other examples:

- (5) **MODAL AUXILIARIES**
It may be snowing in Cambridge.
- (6) **MODAL ADVERBS**
Possibly, it will snow in Cambridge tomorrow.
- (7) **PROPOSITIONAL ATTITUDES**
Jens believes that it is snowing in Cambridge.

The terms **MODAL** and **MODALITY** descend from the Latin *modus*, "way", and are ancient terms pertaining to the way a proposition holds, necessarily, contingently, etc.

See Kratzer (1981, 1991) for more examples of modal constructions.

¹ Steiner (1998, p. 226) writes: "Hypotheticals, 'imaginaries', conditionals, the syntax of counterfactuality and contingency may well be the generative centres of human speech".

- (8) HABITUALS
Jane smokes.
- (9) GENERICS
Bears like honey.

In this chapter, we will put in place the basic framework of INTENSIONAL SEMANTICS, the kind of semantics that models displacement of the point of evaluation in temporal and modal dimensions. To do this, we will start with one rather special example of modal displacement:

(10) In the world of Sherlock Holmes, a detective lives at 221B Baker Street.

(10) doesn't claim that a detective lives at 221B Baker Street in the actual world (presumably a false claim), but that in the world as it is described in the Sherlock Holmes stories of Sir Arthur Conan Doyle, a detective lives at 221B Baker Street (a true claim, of course). We choose this example rather than one of the more run-of-the-mill displacement constructions because we want to focus on conceptual and technical matters before we do serious empirical work.

The questions we want to answer are: How does natural language achieve this feat of modal displacement? How do we manage to make claims about other possible worlds? And why would we want to?

To make displacement possible and compositionally tractable, we need meanings of natural language expressions, and of sentences in particular, to be displaceable in the first place. They need to be “portable”, so to speak, able to make claims about more than just the actual here and now. And need we need other natural language expressions that take that portable meaning and apply it to some situation other than the actual here and now. That is what intensionality is all about.

The basic idea of the account we'll develop is this:

- expressions are assigned their semantic values relative to a possible world;
- in particular, sentences have truth-values in possible worlds;
- in the absence of modal displacement, we evaluate sentences with respect to the “actual” world, the world in which we are speaking;
- modal displacement changes the world of evaluation;
- displacement is effected by special operators, whose semantics is our primary concern here.

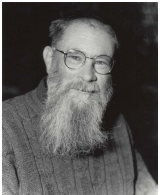
A terminological note: we will call the sister of the intensional operator its PREJACENT, a useful term introduced by our medieval colleagues.



https://en.wikipedia.org/wiki/Sherlock_Holmes

1.2 An Intensional Semantics in 10 Easy Steps

1.2.1 Laying the Foundations



David Lewis

STEP I: POSSIBLE WORLDS. Our first step is to introduce possible worlds. This is not the place to discuss the metaphysics of possible worlds in any depth. Instead, we will just start working with them and see what they can do for us. Basically, a possible world is a way that things might have been. In the actual world, there are two coffee mugs on my desk, but there could have been more or less. So, there is a possible world — albeit a rather bizarre one — where there are 17 coffee mugs on my desk. We join Heim & Kratzer in adducing this quote from [D. Lewis \(1986, 1f.\)](#):

The world we live in is a very inclusive thing. Every stick and every stone you have ever seen is part of it. And so are you and I. And so are the planet Earth, the solar system, the entire Milky Way, the remote galaxies we see through telescopes, and (if there are such things) all the bits of empty space between the stars and galaxies. There is nothing so far away from us as not to be part of our world. Anything at any distance at all is to be included. Likewise the world is inclusive in time. No long-gone ancient Romans, no long-gone pterodactyls, no long-gone primordial clouds of plasma are too far in the past, nor are the dead dark stars too far in the future, to be part of the same world. . . .

The way things are, at its most inclusive, means the way the entire world is. But things might have been different, in ever so many ways. This book of mine might have been finished on schedule. Or, had I not been such a commonsensical chap, I might be defending not only a plurality of possible worlds, but also a plurality of impossible worlds, whereof you speak truly by contradicting yourself. Or I might not have existed at all — neither myself, nor any counterparts of me. Or there might never have been any people. Or the physical constants might have had somewhat different values, incompatible with the emergence of life. Or there might have been altogether different laws of nature; and instead of electrons and quarks, there might have been alien particles, without charge or mass or spin but with alien physical properties that nothing in this world shares. There are ever so many ways that a world might be: and one of these many ways is the way that this world is.

Previously, our “metaphysical inventory” included a domain of entities and a set of two truth-values and increasingly complex functions between entities, truth-values, and functions thereof. Now, we will add possible worlds to the inventory. Let’s assume we are given a set W , the set of all possible worlds, which

is a vast space since there are so many ways that things might have been different from the way they are. Each world has as among its parts entities like you and me and these coffee mugs. Some of them may not exist in other possible worlds. So, strictly speaking each possible worlds has its own, possibly distinctive, domain of entities. What we will use in our system, however, will be the grand union of all these world-specific domains of entities. We will use D to stand for the set of all possible individuals.

Among the many possible worlds that there are — according to Lewis, there is a veritable plenitude of them — is the world as it is described in the Sherlock Holmes stories by Sir Arthur Conan Doyle. In that world, there is a famous detective Sherlock Holmes, who lives at 221B Baker Street in London and has a trusted sidekick named Dr. Watson. Our sentence *In the world of Sherlock Holmes, a detective lives at 221B Baker Street* displaces the claim that a famous detective lives at 221B Baker Street from the actual world to the world as described in the Sherlock Holmes stories. In other words, the following holds:²

- (11) The sentence *In the world of Sherlock Holmes, a detective lives at 221B Baker Street* is true in a world w iff the sentence *a detective lives at 221B Baker Street* is true in the world as it is described in the Sherlock Holmes stories.

What this suggests is that we need to make space in our system for having devices that control in what world a claim is evaluated. This is what we will do now.

STEP 2: THE EVALUATION WORLD PARAMETER. Recall from H&K that we were working with a semantic interpretation function that was relativized to an assignment function g , which was needed to take care of pronouns, traces, variables, etc. From now on, we will relativize the semantic values in our system to possible worlds as well. What this means is that from now on, our interpretation function will have two superscripts: a world w and an assignment g : $\llbracket \cdot \rrbracket^{w,g}$.

So, the prejacent embedded in (10) will have its truth-conditions described as follows:³

- (12) $\llbracket \text{a famous detective lives at 221B Baker Street} \rrbracket^{w,g} = 1$
iff a famous detective lives at 221B Baker Street in world w .

It is customary to refer to the world for which we are calculating the extension of a given expression as the EVALUATION WORLD. In the absence of any shifting

² We will see in Section 1.3.2 that this is not quite right. It'll do for now.

³ Recall from H&K, pp.22f, that what's inside the interpretation brackets is a mention of an object language expression. They make this clear by bold-facing all object language expressions inside interpretation brackets. In these notes, we will follow common practice in the field and not use a special typographic distinction, but let it be understood that what is interpreted are object language expressions.

devices, we would normally evaluate a sentence in the actual world. But then there are shifting devices such as our *in the world of Sherlock Holmes*. We will soon see how they work. But first some more pedestrian steps: adding lexical entries and composition principles that are formulated relative to a possible world. This will allow us to derive the truth-conditions as stated in (I2) in a compositional manner.

STEP 3: LEXICAL ENTRIES. Among our lexical items, we can distinguish between items which have a WORLD-DEPENDENT semantic value and those that are world-independent. Predicates are typically world-dependent. Here are some sample entries.

- (I3) For any $w \in W$ and any assignment function g :
- a. $\llbracket \text{famous} \rrbracket^{w,g} = \lambda x \in D. x \text{ is famous in } w.$ ^{4,5}
 - b. $\llbracket \text{detective} \rrbracket^{w,g} = \lambda x \in D. x \text{ is a detective in } w.$
 - c. $\llbracket \text{lives-at} \rrbracket^{w,g} = \lambda x \in D. \lambda y \in D. y \text{ lives-at } x \text{ in } w.$

The set of detectives will obviously differ from world to world, and so will the set of famous individuals and the set of pairs where the first element lives at the second element.

Other items have semantic values which do not differ from world to world. The most important such items are certain “logical” expressions, such as truth-functional connectives and determiners:

- (I4) a. $\llbracket \text{and} \rrbracket^{w,g} = \lambda u \in D_t. \lambda v \in D_t. u = v = \mathbf{I}.$
 b. $\llbracket \text{the} \rrbracket^{w,g} = \lambda f \in D_{\langle e,t \rangle} : \exists! x [f(x) = \mathbf{I}]. \text{ the } y \text{ such that } f(y) = \mathbf{I}.$
 c. $\llbracket \text{every} \rrbracket^{w,g} = \lambda f_{\langle e,t \rangle}. \lambda h_{\langle e,t \rangle}. \forall x_e : f(x) = \mathbf{I} \rightarrow h(x) = \mathbf{I}.$
 d. $\llbracket \text{a/some} \rrbracket^{w,g} = \lambda f_{\langle e,t \rangle}. \lambda h_{\langle e,t \rangle}. \exists x_e : f(x) = \mathbf{I} \ \& \ h(x) = \mathbf{I}.$

Note the ruthless condensation of the notation in (c) and (d): variables are subscripted with the type of the domain that their values are constrained to come from.

Note that there is no occurrence of w on the right-hand side of the entries in (I4). That’s the tell-tale sign of the world-independence of the semantics of these items.

We will also assume that proper names have world-independent semantic values, that is, they refer to the same individual in any possible world.

- (I5) a. $\llbracket \text{Noam Chomsky} \rrbracket^{w,g} = \text{Noam Chomsky}.$

4 Of course, “ $\lambda x \in D. \dots$ ” is short for “ $\lambda x : x \in D. \dots$ ”. Get used to semanticists condensing their notation whenever convenient! A further step of condensation is taken below: “ $\lambda x : x \in D_e. \dots$ ” becomes “ $\lambda x_e. \dots$ ”.

5 Always make sure that you actually understand what the notation means. Here, for example, we are saying that the semantic value of the word *famous* with respect to a given possible world w and a variable assignment g is that function that is defined for an argument x only if x is a member of the domain of individuals and that, if it is defined, yields the truth-value \mathbf{I} if and only if x is famous in w .

- b. $\llbracket \text{Sherlock Holmes} \rrbracket^{w,g} = \text{Sherlock Holmes.}$
- c. $\llbracket 221B \text{ Baker Street} \rrbracket^{w,g} = 221B \text{ Baker Street.}$

STEP 4: COMPOSITION PRINCIPLES. The old rules of Functional Application, Predicate Modification, and λ -Abstraction can be retained almost intact. We just need to modify them by adding world-superscripts to the interpretation function. For example:

- (I6) FUNCTIONAL APPLICATION (FA)
 If α is a branching node and $\{\beta, \gamma\}$ the set of its daughters, then, for any world w and assignment g : if $\llbracket \beta \rrbracket^{w,g}$ is a function whose domain contains $\llbracket \gamma \rrbracket^{w,g}$, then $\llbracket \alpha \rrbracket^{w,g} = \llbracket \beta \rrbracket^{w,g}(\llbracket \gamma \rrbracket^{w,g})$.

The rule simply passes the world parameter down.

STEP 5: TRUTH. Lastly, we will want to connect our semantic system to the notion of the TRUTH OF AN UTTERANCE. We first adopt the “Appropriateness Condition” from Heim & Kratzer (p.243):

- (I7) APPROPRIATENESS CONDITION
 A context c is *appropriate* for an LF ϕ only if c determines a variable assignment g_c whose domain includes every index which has a free occurrence in ϕ .

We then intensionalize Heim & Kratzer’s definition of truth and falsity of utterances:

- (I8) TRUTH AND FALSITY CONDITIONS FOR UTTERANCES
 An utterance of a sentence ϕ in a context c in a possible world w is *true* iff $\llbracket \phi \rrbracket^{w,g_c} = 1$ and *false* if $\llbracket \phi \rrbracket^{w,g_c} = 0$.

EXERCISE 1.1: Compute under what conditions an utterance in possible world w_7 (which may or may not be the one we are all living in) of the sentence *a famous detective lives at 221B Baker Street* is true. [Since this is the first exercise of the semester, please do this in excruciating detail, not skipping any steps.] \square

1.2.2 Intensional Operators

So far we have merely “redecorated” our old system inherited from last semester. We have introduced possible worlds into our inventory, our lexical entries and our old composition principles. But with the tools we have now, all we can do so far is to keep track of the world in which we evaluate the semantic value of an expression, complex or lexical. We will get real mileage once we introduce INTENSIONAL OPERATORS which are capable of shifting the world parameter. We mentioned that there are a number of devices for modal displacement. As

advertised, for now, we will just focus on a very particular one: the expression *in the world of Sherlock Holmes*. We will assume, as seems reasonable, that this expression is a sentence-modifier both syntactically and semantically.

STEP 6: A SYNCATEGOREMATIC ENTRY. We begin with a heuristic step. We want to derive something like the following truth-conditions for our sentence:

- (19) $\llbracket \text{in the world of Sherlock Holmes,}$
 a famous detective lives at 221B Baker Street $\rrbracket^{w,g} = 1$
 iff the world w' as it is described in the Sherlock Holmes stories is such
 that there exists a famous detective in w' who lives at 221B Baker Street
 in w' .

We would get this if in general we had this rule for *in the world of Sherlock Holmes*:

- (20) For any sentence ϕ , any world w , and any assignment g :
 $\llbracket \text{in the world of Sherlock Holmes } \phi \rrbracket^{w,g} = 1$
 iff the world w' as it is described in the Sherlock Holmes stories is such
 that $\llbracket \phi \rrbracket^{w',g} = 1$.

This is a so-called SYNCATEGOREMATIC treatment of the meaning of this expression. Instead of giving an explicit semantic value to the expression, we specify what effect it has on the meaning of a complex expression that contains it. In (20), we do not compute the meaning for *in the world of Sherlock Holmes*, ϕ from the combination of the meanings of its parts, since *in the world of Sherlock Holmes* is not given a separate meaning, but in effect triggers a special composition principle. This format is very common in modal logic systems, which usually give a syncategorematic semantics for the two modal operators (the necessity operator \Box and the possibility operator \Diamond). When one only has a few closed class expressions to deal with that may shift the world parameter, employing syncategorematic entries is a reasonable strategy. But we are facing a multitude of displacement devices. So, we will need to make our system more modular.

So, we want to give *in the world of Sherlock Holmes* its own meaning and combine that meaning with that of its prejacent by a general composition principle. The Fregean slogan we adopted says that all composition is function application (modulo the need for λ -abstraction and the possible need for predicate modification).⁶ So, what we will want to do is to make (19) be the result of functional application. But we can immediately see that it cannot be the result of our usual rule of functional application, since that would feed to *in the world of Sherlock Holmes* the semantic value of *a famous detective lives in 221B Baker Street* in w , which would be a particular truth-value, 1 if a famous detective lives at 221B

The diamond \Diamond symbol for possibility is due to C.I. Lewis, first introduced in C. I. Lewis & Langford (1932), but he made no use of a symbol for the dual combination $\neg\Diamond\neg$. The dual symbol \Box was later devised by F.B. Fitch and first appeared in print in 1946 in a paper by his doctoral student Barcan (1946). See footnote 425 of Hughes & M.J. Cresswell (1968). Another notation one finds is L for necessity and M for possibility, the latter from the German *möglich* 'possible'.

6 See Heim & Kratzer, Section 4.3, pp. 63–72 for a reminder about the status of predicate modification.

Baker Street in w and o if there doesn't. And whatever the semantics of *in the world of Sherlock Holmes* is, it is certainly *not* a truth-functional operator.

So, we need to feed something else to *in the world of Sherlock Holmes*. At the same time, we want the operator to be able to shift the evaluation world of its prejacent. Can we do this?

EXERCISE 1.2: How would you show that *in the world of Sherlock Holmes* is not a truth-functional operator? \square

STEP 7: INTENSIONS. We will define a richer notion of semantic value, the INTENSION of an expression. This will be a function from possible worlds to the extension of the expression in that world. The intension of a sentence can be applied to any world and give the truth-value of the sentence in that world. Intensional operators take the intension of their prejacent as their argument, that is we will feed the intension of the embedded sentence to the shifting operator. The operator will use that intension and apply it to the world it wants the evaluation to happen in. Voilà.

Now let's spell that account out. Our system actually provides us with two kinds of meanings. For any expression α , we have $\llbracket \alpha \rrbracket^{w,g}$, the semantic value of α in w , also known as the EXTENSION of α in w . But we can also calculate $\lambda w. \llbracket \alpha \rrbracket^{w,g}$, the function that assigns to any world w the extension of α in that world. This is usually called the INTENSION of α . We will sometimes use an abbreviatory notation⁷ for the intension of α :

As before in H&K, we make no claim that the semantic values that are attributed to expressions in our framework fully capture what is informally meant by “meaning”. But certainly, intensions come close to “meaning” than extensions.

$$(21) \quad \llbracket \alpha \rrbracket_{\text{c}}^g := \lambda w. \llbracket \alpha \rrbracket^{w,g}.$$

It should be immediately obvious that since the definition of intension abstracts over the evaluation world, intensions are not world-dependent.^{8,9}

Note that strictly speaking, it now makes no sense anymore to speak of “*the* semantic value” of an expression α . What we have is a semantic system that allows us to calculate extensions (for a given possible world w) as well as intensions for all (interpretable) expressions. We will see that when α occurs in a particular bigger tree, it will always be determinate which of the two “semantic values” of α is the one that enters into the compositional semantics. So, that

7 The notation with the subscripted cent-sign comes from Montague Grammar. See e.g. D. Dowty, Wall & Peters (1981, p. 147).

8 Since intensions are by definition not dependent on the choice of a particular world, it makes no sense to put a world-superscript on the intension-brackets. So don't ever write “ $\llbracket \dots \rrbracket_{\text{c}}^{w,g}$ ”; we'll treat that as undefined nonsense.

9 The definition here is simplified, in that it glosses over the fact that some expressions, in particular those that contain PRESUPPOSITION TRIGGERS, may fail to have an extension in certain worlds. In such a case, the intension has no extension to map such a world to. Therefore, the intension will have to be a partial function. So, the official, more “pedantic”, definition will have to be as follows: $\llbracket \alpha \rrbracket_{\text{c}}^g := \lambda w: \alpha \in \text{dom}(\llbracket \dots \rrbracket^{w,g}). \llbracket \alpha \rrbracket^{w,g}$.

one — whichever one it is, the extension or the intension of α — might then be called “*the* semantic value of α in the tree β ”.

It should be noted that the terminology of EXTENSION vs. INTENSION is time-honored but that the possible worlds interpretation thereof is more recent. The technical notion we are using is certainly less rich a notion of meaning than tradition assumed.¹⁰

STEP 8: SEMANTIC TYPES AND SEMANTICS DOMAINS. If we want to be able to feed the intensions to lexical items like *in the world of Sherlock Holmes*, we need to have the appropriate types in our system.

Recall that W is the set of all possible worlds. And recall that D is the set of all POSSIBLE INDIVIDUALS and thus contains all individuals existing in the actual world *plus* all individuals existing in any of the merely possible worlds.

We now expand the set of semantic types, to add intensions. Intensions are functions from possible worlds to all kinds of extensions. So, basically we want to add for any kind of extension we have in our system, a corresponding kind of intension, a function from possible worlds to that kind of extension.

We add a new clause, (22c), to the definition of semantic types:

- (22) SEMANTIC TYPES
- a. e and t are semantic types.
 - b. If σ and τ are semantic types, then $\langle \sigma, \tau \rangle$ is a semantic type.
 - c. If σ is a semantic type, then $\langle s, \sigma \rangle$ is a semantic type.
 - d. Nothing else is a semantic type.

We also add a fourth clause to the previous definition of semantic domains:

- (23) SEMANTIC DOMAINS
- a. $D_e = D$, the set of all possible individuals
 - b. $D_t = \{0, 1\}$, the set of truth-values
 - c. If σ and τ are semantic types, then $D_{\langle \sigma, \tau \rangle}$ is the set of all functions from D_σ to D_τ .
 - d. INTENSIONS: If σ is a type, then $D_{\langle s, \sigma \rangle}$ is the set of all functions from W to D_σ .

Clause (d) is the addition to our previous system of types. The functions of the schematic type $\langle s, \dots \rangle$ are intensions.¹¹ Here are some examples of intensions:

¹⁰ For example, Frege’s “modes of presentation” are not obviously captured by this possible worlds implementation of extension/intension.

¹¹ Note a curious feature of this set-up: there is no type s and no associated domain. This corresponds to the assumption that there are no expressions of English that take as their extension a possible world, that is, there are no pronouns or names referring to possible worlds. We will

The Port-Royal logicians distinguished EXTENSION from COMPREHENSION. Leibniz preferred the term INTENSION rather than COMPREHENSION. The notion probably goes back even further. See [Spencer \(1971\)](#) for some notes on this. The possible worlds interpretation is due to [Carnap \(1947\)](#).

- The intensions of sentences are of type $\langle s, t \rangle$, functions from possible worlds to truth values. These are usually called PROPOSITIONS. Note that if the function is total, then we can see the sentence as picking out a set of possible worlds, those in which the sentence is true. More often than not, however, propositions will be PARTIAL functions from worlds to truth-values, that is functions that fail to map certain possible worlds into either truth-value. This will be the case when the sentence contains a presupposition trigger, such as *the*. The famous sentence *The King of France is bald* has an intension that (at least in the analysis sketched in Heim & Kratzer) is undefined for any world where there fails to be a unique King of France.
- The intensions of one-place predicates are of type $\langle s, \langle e, t \rangle \rangle$, functions from worlds to set of individuals. These are usually called PROPERTIES.
- The intensions of expressions of type e are of type $\langle s, e \rangle$, functions from worlds to individuals. These are usually called INDIVIDUAL CONCEPTS.

STEP 9: A LEXICAL ENTRY FOR A SHIFTER. We are ready to formulate the lexical entry for *in the world of Sherlock Holmes*:¹²

$$(24) \quad \llbracket \text{in the world of Sherlock Holmes} \rrbracket^{w,g} = \\ \lambda p_{\langle s, t \rangle}. \text{ the world } w' \text{ as it is described in the Sherlock Holmes stories} \\ \text{is such that } p(w') = 1.$$

That is, *in the world of Sherlock Holmes* expects as its argument a function of type $\langle s, t \rangle$, a proposition. It yields the truth-value 1 iff the proposition is true in the world as it is described in the Sherlock Holmes stories.

All that's left to do now is to provide *in the world of Sherlock Holmes* with a proposition as its argument. This is the job of a new composition principle.

STEP 10: INTENSIONAL FUNCTIONAL APPLICATION. We add the new rule of Intensional Functional Application.

$$(25) \quad \text{INTENSIONAL FUNCTIONAL APPLICATION (IFA)} \\ \text{If } \alpha \text{ is a branching node and } \{\beta, \gamma\} \text{ the set of its daughters, then, for} \\ \text{any world } w \text{ and assignment } g: \text{ if } \llbracket \beta \rrbracket^{w,g} \text{ is a function whose domain} \\ \text{contains } \llbracket \gamma \rrbracket_e^g, \text{ then } \llbracket \alpha \rrbracket^{w,g} = \llbracket \beta \rrbracket^{w,g}(\llbracket \gamma \rrbracket_e^g).$$

actually question this assumption in a later chapter. For now, we will stay with this more conventional set-up.

¹² This is not yet the final semantics, see Section 1.3 for complications. One complication we will not even start to discuss is that obviously it is not a necessity that there are Sherlock Holmes stories in the first place and that the use of this operator *presupposes* that they exist; so a more fully explicit semantics would need to build in that presuppositional component. Also, note again the condensed notation: “ $\lambda p_{\langle s, t \rangle}. \dots$ ” stands for the fully official “ $\lambda p: p \in D_{\langle s, t \rangle}. \dots$ ”.

This is the crucial move. It makes space for expressions that want to take the intension of their sister as their argument and do stuff to it. Now, everything is in place. Given (24), the semantic argument of *in the world of Sherlock Holmes* will not be a truth-value but a proposition. And thus, *in the world of Sherlock Holmes* will be able to check the truth-value of its prejacent in various possible worlds. To see in practice that we have all we need, please do the following exercise.

EXERCISE 1.3: Calculate the conditions under which an utterance in a given possible world w_7 of the sentence *in the world of the Sherlock Holmes stories, a famous detective lives at 221B Baker Street* is true. \square

EXERCISE 1.4: What in our system prevents us from computing the extension of *Watson is slow*, for example, by applying the intension of *slow* to the extension of *Watson*? What in our system prevents us from computing the extension of *Watson is slow* by applying the intension of *slow* to the intension of *Watson*? \square

Please think about this exercise before looking at Section 1.4, which explores this issue.

EXERCISE 1.5: What is wrong with the following equation:

$$(26) \quad (\lambda x. x \text{ is slow in } w) (\text{Watson}) = \text{Watson is slow in } w.$$

[Hint: there is nothing wrong with the following:

$$(27) \quad (\lambda x. x \text{ is slow in } w) (\text{Watson}) = 1 \text{ iff Watson is slow in } w.] \square$$

1.3 Comments and Complications

1.3.1 Intensions All the Way?

We have seen that to adequately deal with expressions like *in the world of Sherlock Holmes*, we need an intensional semantics, one that gives us access to the extensions of expressions across the multitude of possible worlds. At the same time, we have kept the semantics for items like *and*, *every*, and *a* unchanged and extensional. This is not the only way one can set up an intensional semantics. The following exercise demonstrates this.

EXERCISE 1.6: Consider the following “intensional” meaning for *and*:

$$(28) \quad \llbracket \text{and} \rrbracket^{w,g} = \lambda p_{\langle s,t \rangle}. \lambda q_{\langle s,t \rangle}. p(w) = q(w) = 1.$$

With this semantics, *and* would operate on the intensions of the two conjoined sentences. In any possible world w , the complex sentence will be true iff the component propositions are both true of that world.

Compute the truth-conditions of the sentence *In the world of Sherlock Holmes, Holmes is quick and Watson is slow* both with the extensional meaning for *and*

given earlier and the intensional meaning given here. Is there any difference in the results? \square

There are then at least two ways one could develop an intensional system.

- (i) We could “generalize to the worst case” and make the semantics deliver intensions as *the* semantic value of an expression. Such systems are common in the literature (see [D. Lewis 1970b](#), [Max Cresswell 1973](#)).
- (ii) We could maintain much of the extensional semantics we have developed so far and extend it conservatively so as to account for non-extensional contexts.

We have chosen to pursue (ii) over (i), because it allows us to keep the semantics of extensional expressions simpler. The philosophy we follow is that we will only move to the intensional sub-machinery when triggered by an expression that creates a non-extensional context. As the exercise just showed, this is more a matter of taste than a deep scientific decision.

1.3.2 Why Talk about Other Worlds?

Why would natural language bother having such elaborate mechanisms to talk about other possible worlds? While having devices for spatial and temporal displacement (talking about Hamburg or what happened yesterday) seems eminently reasonable, talking about worlds other than the actual world seems only suitable for poets and the like. So, why?

The solution to this puzzle lies in a fact that our current semantics of the shifter *in the world of Sherlock Holmes* does not yet accurately capture: modal sentences have empirical content, they make CONTINGENT claims, claims that are true or false depending on the circumstances in the actual world.

Our example sentence *In the world of Sherlock Holmes, a famous detective lives at 221 Baker Street* is true in this world but it could easily have been false. There is no reason why Sir Arthur Conan Doyle could not have decided to locate Holmes’ abode on Abbey Road.

To see that our semantics does not yet capture this fact, notice that in the semantics we gave for *in the world of Sherlock Holmes*:

- (29) $\llbracket \text{in the world of Sherlock Holmes} \rrbracket^{w,g} =$
 $\lambda p_{\langle s,t \rangle}. \text{ the world } w' \text{ as it is described in the Sherlock Holmes stories}$
 $\text{is such that } p(w') = 1.$

there is no occurrence of w on the right hand side. This means that the truth-conditions for sentences with this shifter are world-independent. In other words, they are predicted to make non-contingent claims that are either true no-matter-what or false no-matter-what. This needs to be fixed.

The fix is obvious: what matters to the truth of our sentence is the content of the Sherlock Holmes stories as they are in the evaluation world. So, we need the following semantics for our shifter:

- (30) $\llbracket \text{in the world of Sherlock Holmes} \rrbracket^{w,g} =$
 $\lambda p_{\langle s,t \rangle}. \text{the world } w' \text{ as it is described in the Sherlock Holmes stories}$
in w is such that $p(w') = 1$.

We see now that sentences with this shifter do make a claim about the evaluation world: namely, that the Sherlock Holmes stories as they are in the evaluation world describe a world in which such-and-such is true. So, what is happening is that although it appears at first as if modal statements concern other possible worlds and thus couldn't really be very informative, they actually only talk about certain possible worlds, those that stand in some relation to what is going on at the ground level in the actual world. As a crude analogy, consider:

- (31) My grandmother is sick.

At one level this is a claim about my grandmother. But it is also a claim about me: namely that I have a grandmother who is sick. Thus it is with modal statements. They talk about possible worlds that stand in a certain relation to the actual world and thus they make claims about the actual world, albeit slightly indirectly.

1.3.3 The Worlds of Sherlock Holmes

So far, we have played along with colloquial usage in talking of *the* world of Sherlock Holmes. But it is important to realize that this is sloppy talk. D. Lewis (1978) writes:

[I]t will not do to follow ordinary language to the extent of supposing that we can somehow single out a single one of the worlds [as the one described by the stories]. Is the world of Sherlock Holmes a world where Holmes has an even or an odd number of hairs on his head at the moment when he first meets Watson? What is Inspector Lestrade's blood type? It is absurd to suppose that these questions about the world of Sherlock Holmes have answers. The best explanation of that is that the worlds of Sherlock Holmes are plural, and the questions have different answers at different ones.

The usual move at this point is to talk about the set of worlds "COMPATIBLE WITH the (content of) Sherlock Holmes stories in w ". We imagine that we ask of each possible world whether what is going on in it is compatible with the stories as they were written in our world. Worlds where Holmes lives on Abbey Road are not compatible. Some worlds where he lives at 221B Baker Street are compatible (again not all, because in some such worlds he is not a famous detective but

an obscure violinist). Among the worlds compatible with the stories are ones where he has an even number of hairs on his head at the moment when he first meets Watson and there are others where he has an odd number of hairs at that moment.

What the operator *in the world of Sherlock Holmes* expresses is that its complement is true throughout the worlds compatible with the stories. In other words, the operator *universally quantifies* over the compatible worlds. Our next iteration of the semantics for the operator is therefore this:

$$(32) \quad \llbracket \text{in the world of Sherlock Holmes} \rrbracket^{w,g} = \\ \lambda p_{\langle s,t \rangle}. \forall w' \text{ compatible with the Sherlock Holmes stories in } w: \\ p(w') = 1.$$

At a very abstract level, the way we parse sentences of the form *in the world of Sherlock Holmes*, ϕ is that both components, the *in*-phrase and the prejacent, determine sets of possible worlds and that the set of possible worlds representing the content of the fiction mentioned in the *in*-phrase is a subset of the set of possible worlds determined by the prejacent. We will see the same rough structure of relating sets of possible worlds in other intensional constructions.

This is where we will leave things. There is more to be said about fiction operators like *in the world of Sherlock Holmes*, but we will just refer to you to the relevant literature. In particular, one might want to make sense of Lewis' idea that a special treatment is needed for cases where the sentence makes a claim about things that are left open by the fiction (no truth-value, perhaps?). One also needs to figure out how to deal with cases where the fiction is internally inconsistent. In any case, for our purposes we're done with this kind of operator.

1.4 *Issues with an informal meta-language

Exercise 1.5 asks what is wrong with writing something like¹³

$$(33) \quad (\lambda x. x \text{ is slow in } w) (\text{Watson}) = \text{Watson is slow in } w.$$

Think about it. On the left hand side of the “=” sign is a meta-language expression consisting of a λ -expression (so some kind of function) applied to an individual (contributed by the meta-language name “Watson”). The function is a function from individuals to truth-values that will deliver the truth-value 1 iff the individual is slow in world w . So, what we have on the left hand side is the result of a function from individuals to truth-values applied to an individual. In other words, on the left hand side we have a truth-value, namely the truth-value 1 if Watson is slow in w and the truth-value 0 if Watson is not slow in w .

Starred sections are optional on a first pass.

¹³ Thanks to Magda Kaufmann, Angelika Kratzer, and Ede Zimmermann for discussion on the issues explored in this section.

Now, what do we have on the right hand side of the “=”? We have the meta-language sentence “Watson is slow in w ”. That is not nor does it contribute a truth-value. It is a statement of fact. Truth-values are not the same as statements of fact.

The proper thing to do is to write

$$(34) \quad (\lambda x. x \text{ is slow in } w) (\text{Watson}) = 1 \text{ iff Watson is slow in } w.$$

There are actually two ways to parse the statement in (34), both legitimate it appears.

On one parse, the major connective is the meta-language expression “iff”. On its left hand side is a meta-language statement (that applying the function to the individual Watson gives the truth-value 1) and on the right hand side of the “iff” we have another meta-language statement (that Watson is slow in w). So, the whole thing says that these two statements are equivalent: (i) that function applied to that individual gives us the truth-value 1, and (ii) that Watson is slow in w .

The other parse is perhaps more conspicuously represented as follows:

$$(35) \quad (\lambda x. x \text{ is slow in } w) (\text{Watson}) = \begin{cases} 1 & \text{if Watson is slow in } w \\ 0 & \text{if Watson is not slow in } w \end{cases}$$

Here, the “=” sign is the major connective. The left hand side is a meta-language expression that resolves to a truth-value and the right hand side as well contributes a truth-value: 1 if such and such and 0 if such and such.

H \mathcal{C} K, of course, introduced a convention that allowed meta-language statements to be used in a place where a truth-value was expected (p.37, (9)):

Read “[$\lambda\alpha: \phi. \gamma$]” as either (i) or (ii), whichever makes sense.

(i) “the function which maps every α such that ϕ to γ ”

(ii) “the function which maps every α such that γ to 1, if γ , and to 0 otherwise”

Since it never makes sense to map anything to a meta-language statement, no ambiguity will ever arise.

So, one might want to extend this leeway and use it in the case of (33) as well. We could say that in general, meta-language statements supply truth-values wherever that makes sense. In that case, (33) is just shorthand for (34).

Alternatively, one can introduce a new notation that indicates that a meta-language statement is being used to contribute a truth-value:

$$(36) \quad \vdash \alpha \dashv = \begin{cases} 1 & \text{if } \alpha \\ 0 & \text{if otherwise} \end{cases}$$

Lastly, one could abandon the H \mathcal{C} K informal meta-language approach altogether and introduce a rigidly formalized meta-language.

Is this weird? It turns out that natural language, not just our semi-formal meta-language, has conditionals that seem very similar: *I fear [the consequences if we fail]*. See Lasersohn 1996 for some discussion.

This is the approach of von Stechow 1991.

This is the approach Ede Zimmermann (pc) advocates and has been using in his classes.

These lecture notes will proceed to follow H&K's approach and will not introduce any further innovations. So, (33) is illicit and only (34) is acceptable.

1.5 Supplemental Readings

There is considerable overlap between this chapter and Chapter 12 of Heim & Kratzer's textbook:

Irene Heim & Angelika Kratzer. 1998. *Semantics in generative grammar*. Oxford: Blackwell.

Here, we approach intensional semantics from a different angle. It would probably be beneficial if you read H&K's Chapter 12 in addition to this chapter and if you did the exercises in there.

Come to think of it, some other ancillary reading is also recommended. You may want to look at relevant chapters in other textbooks:

David Dowty, Robert Wall & Stanley Peters. 1981. *Introduction to Montague semantics*. Kluwer. [Chapters 5 & 6].

L. T. F. Gamut. 1991. *Logic, language, and meaning*. Chicago University Press. [Volume II: Intensional Logic and Logical Grammar].

Gennaro Chierchia & Sally McConnell-Ginet. 2000. *Meaning and grammar: An introduction to semantics (2nd edition)*. MIT Press. [Chapter 5: Intensionality].

Thomas Ede Zimmermann & Wolfgang Sternefeld. 2013. *Introduction to semantics: An essential guide to the composition of meaning*. de Gruyter Mouton.

An encyclopedia article by Perry on possible worlds semantics:

John R. Perry. 1998. Semantics, possible worlds. In E. Craig (ed.), *Routledge encyclopedia of philosophy*. Preprint <http://www-csl.stanford.edu/~john/PHILPAPERS/possfld.pdf>. London: Routledge.

A couple of influential philosophical works on the metaphysics and uses of possible worlds:

Saul Kripke. 1980. *Naming and necessity*. Oxford: Blackwell.

David Lewis. 1986. *On the plurality of worlds*. Oxford: Blackwell.

An interesting paper on the origins of the modern possible worlds semantics for modal logic:

B. Jack Copeland. 2002. The genesis of possible worlds semantics. *Journal of Philosophical Logic* 31(2). 99–137. <http://dx.doi.org/10.1023/A:1015273407895>.

A personal history of formal semantics:

Barbara H. Partee. 2005. Reflections of a formal semanticist as of Feb 2005. ms. (longer version of introductory essay in 2004 book). http://people.umass.edu/partee/docs/BHP_Essay_Feb05.pdf.

A must read for students who plan to go on to becoming specialists in semantics, together with a handbook article putting it in perspective:

Richard Montague. 1973. The proper treatment of quantification in ordinary English. In Jaako Hintikka, Julius Moravcsik & Patrick Suppes (eds.), *Approaches to natural language*, 221–242. Reprinted in Portner & B. H. Partee (2002), pp. 17–34. Dordrecht: Reidel. http://www.blackwellpublishing.com/content/BPL/Images/Content_store/Sample_chapter/9780631215417/Portner.pdf.

Barbara H. Partee & Herman L.W. Hendriks. 1997. Montague grammar. In Johan van Benthem & Alice ter Meulen (eds.), *Handbook of logic and language*, 5–91. Elsevier.

To learn more about discourse about fiction, read Lewis:

David Lewis. 1978. Truth in fiction. *American Philosophical Quarterly* 15(1). Reprinted with postscripts in D. Lewis (1983), pp. 261–280, 37–46. <http://www.jstor.org/stable/20009693>.

Recent reconsiderations:

Andrea Bonomi & Sandro Zucchi. 2003. A pragmatic framework for truth in fiction. *Dialectica* 57(2). Preprint <http://filosofia.dipafilo.unimi.it/~bonomi/Pragmatic.pdf>, 103–120. <http://dx.doi.org/10.1111/j.1746-8361.2003.tb00259.x>.

Richard Hanley. 2004. As good as it gets: Lewis on truth in fiction. *Australasian Journal of Philosophy* 82(1). 112–128. <http://dx.doi.org/10.1080/713659790>.

Inconsistencies in fictions and elsewhere are discussed in:

Achille Varzi. 1997. Inconsistency without contradiction. *Notre Dame Journal of Formal Logic* 38(4). 621–638. <http://dx.doi.org/10.1305/ndjfl/1039540773>.

David Lewis. 1982. Logic for equivocators. *Notûs* 16(3). Reprinted in D. Lewis (1998, pp. 97–110), 431–441. <http://dx.doi.org/10.2307/2216219>.

Some other interesting work on stories and pictures and their content:

Jeff Ross. 1997. *The semantics of media* (Studies in Linguistics and Philosophy (SLAP) 64). Dordrecht: Kluwer.

Sandro Zucchi. 2001. Tense in fiction. In Carlo Cecchetto, Gennaro Chierchia & Maria Teresa Guasti (eds.), *Semantic interfaces: Reference, anaphora and aspect*, 320–355. CSLI Publications. <http://tinyurl.com/5ulwxwg>.

Ben Blumson. 2009. Pictures, perspective and possibility. *Philosophical Studies*. <http://dx.doi.org/10.1007/s11098-009-9337-2>.

Astonishingly, Lewis' doctrine of the reality of the plurality of possible worlds is being paralleled (pun absolutely intended) by theoretical physicists in a number of ways. There is a controversial "many worlds" interpretation of quantum mechanics, for example. Other terms found are the "multiverse" and "parallel universes". See for starters, Kai's blog entry on a popular book on the issue, <http://kaivonfintel.org/2011/01/25/many-worlds/>, MIT physics professor Max Tegmark's page on the topic, <http://space.mit.edu/home/tegmark/crazy.html>, and a Fresh Air interview with physicist Brian Greene, who just wrote a book called *The Hidden Reality: Parallel Universes and the Deep Laws of the Cosmos*: <http://www.npr.org/2011/01/24/132932268/a-physicist-explains-why-parallel-universes-may-exist>.

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CHAPTER TWO

PROPOSITIONAL ATTITUDES

With the basic framework in place, we now proceed to analyze a number of intensional constructions. We start with the basic possible worlds semantics for propositional attitude ascriptions. We talk briefly about the formal properties of accessibility relations.

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2.1 Hintikka's Idea

Expressions like *believe*, *know*, *doubt*, *expect*, *regret*, and so on are usually said to describe PROPOSITIONAL ATTITUDES, expressing relations between individuals (the attitude holder) and propositions (intensions of sentences).

The simple idea is that *George believes that Henry is a spy* claims that George believes of the proposition that Henry is a spy that it is true. Note that for the attitude ascription to be true it does not have to hold that Henry is actually a spy. But where — in which world(s) — does Henry have to be a spy for it be true that George believes that Henry is a spy? We might want to be inspired by the colloquial phrase “in the world according to George” and say that *George believes that Henry is a spy* is true iff in the world according to George's beliefs, Henry is a spy. We immediately recall from the previous chapter that we need to fix this idea up by making space for multiple worlds compatible with George's beliefs and by tying the truth-conditions to contingent facts about the evaluation world. That is, what George believes is different in different possible worlds.

The following lexical entry thus offers itself:

According to [Hintikka \(1969\)](#), the term PROPOSITIONAL ATTITUDE goes back to [Russell \(1940\)](#).

Of course, the possible worlds semantics for propositional attitudes was in place long before the extension to fiction contexts was proposed. Our discussion here has inverted the historical sequence for pedagogical purposes.

$$(I) \quad \llbracket \text{believe} \rrbracket^{w,g} = \lambda p_{\langle s,t \rangle}. \lambda x. \forall w' \text{ compatible with } x' \text{'s beliefs in } w: p(w') = 1.$$

It is important to realize the modesty of this semantics: we are not trying to figure out what belief systems are and particularly not what their internal workings are like. That is the job of psychologists (and philosophers of mind, perhaps). For our semantics, we treat the belief system as a black box that determines for each possible world whether it considers it possible that it is the world it is located in.

What is going on in this semantics? We conceive of George's beliefs as a state of his mind about whose internal structure we will remain agnostic, a matter left to other cognitive scientists. What we require of it is that it embody opinions about what the world he is located in looks like. In other words, if his beliefs are confronted with a particular possible world w' , they will determine whether that world may or may not be the world as they think it is. What we are asking of George's mental state is whether any state of affairs, any event, anything in w' is in contradiction with anything that George believes. If not, then w' is compatible with George's beliefs. For all George believes, w' may well be the world where he lives. Many worlds will pass this criterion, just consider as one factor that George is unlikely to have any precise opinions about the number of leaves on the tree in front of my house. George's belief system determines a set of worlds compatible with his beliefs: those worlds that are viable candidates for being the actual world, as far as his belief system is concerned.

Now, George believes a proposition iff that proposition is true in all of the worlds compatible with his beliefs. If there is just one world compatible with his beliefs where the proposition is not true, that means that he considers it possible that the proposition is not true. In such a case, we can't say that he believes the proposition. Here is the same story in the words of [Hintikka \(1969\)](#), the source for this semantics for propositional attitudes:



Jaakko Hintikka

My basic assumption (slightly simplified) is that an attribution of any propositional attitude to the person in question involves a division of all the possible worlds (...) into two classes: into those possible worlds which are in accordance with the attitude in question and into those which are incompatible with it. The meaning of the division in the case of such attitudes as knowledge, belief, memory, perception, hope, wish, striving, desire, etc. is clear enough. For instance, if what we are speaking of are (say) α 's memories, then these possible worlds are all the possible worlds compatible with everything he remembers. [...]

How are these informal observations to be incorporated into a more explicit semantical theory? According to what I have said, understanding attributions of the propositional attitude in question (...) means being able to make a distinction between two kinds of possible worlds, according to whether they are compatible with the relevant attitudes of the person in question. The semantical counterpart to this is of course a function which to a given individual person assigns a set of possible worlds.

However, a minor complication is in order here. Of course, the person in question may himself have different attitudes in the different worlds we are considering. Hence this function in effect becomes a relation which to a given individual and to a given possible world μ associates a number of possible worlds which we shall call the *ALTERNATIVES* to μ . The relation will be called the *alternativeness* relation. (For different propositional attitudes, we have to consider different alternativeness relations.)

EXERCISE 2.1: Let's adopt Hintikka's idea that we can use a function that maps x and w into the set of worlds w' compatible with what x believes in w . Call this function \mathcal{B} . That is,

$$(2) \quad \mathcal{B} = \lambda x. \lambda w. \{w' : w' \text{ is compatible with what } x \text{ believes in } w\}.$$

Using this notation, our lexical entry for *believe* would look as follows:

$$(3) \quad \llbracket \text{believe} \rrbracket^{w,g} = \lambda p_{\langle s,t \rangle}. \lambda x. \mathcal{B}(x)(w) \subseteq p.$$

We are here indulging in the usual sloppiness in treating p both as a function from worlds to truth-values and as the set characterized by that function.

Here now are two “alternatives” for the semantics of *believe*:

$$(4) \quad \text{ATTEMPT 1 (VERY WRONG)} \\ \llbracket \text{believe} \rrbracket^{w,g} = \lambda p \in D_{\langle s,t \rangle}. [\lambda x \in D. p = \mathcal{B}(x)(w)].$$

$$(5) \quad \text{ATTEMPT 2 (ALSO VERY WRONG)} \\ \llbracket \text{believe} \rrbracket^{w,g} = \lambda p \in D_{\langle s,t \rangle}. [\lambda x \in D. p \cap \mathcal{B}(x)(w) \neq \emptyset].$$

Explain why these do not adequately capture the meaning of *believe*. \square

EXERCISE 2.2: Follow-up: The semantics in (5) would have made *believe* into an existential quantifier of sorts: it would say that *some* of the worlds compatible with what the subject believes are such-and-such. You have argued (successfully, of course) that such an analysis is wrong for *believe*. But *are* there attitude predicates with such an “existential” meaning? Discuss some candidates. If you can't find any candidates that survive scrutiny, can you speculate why there might be no existential attitude predicates? [Warning: this is unexplored territory!] \square

We can also think of belief states as being represented by a function \mathcal{BS} , which maps an individual and a world into a set of propositions: those that the individual believes. From there, we could calculate the set of worlds compatible with an individual x 's beliefs in world w by retrieving the set of those possible worlds in which all of the propositions in $\mathcal{BS}(x)(w)$ are true: $\{w' : \forall p \in \mathcal{BS}(x)(w) : p(w') = 1\}$, which in set talk is simply the big intersection of all the propositions in the set: $\cap \mathcal{BS}(x)(w)$. Our lexical entry then would be:

\mathcal{BS} is meant to stand for 'belief state', not for what you might have thought!

$$(6) \quad \llbracket \text{believe} \rrbracket^{w,g} = \lambda p_{\langle s,t \rangle}. \lambda x. \cap \mathcal{BS}(x)(w) \subseteq p.$$

EXERCISE 2.3: Imagine that our individual x forms a new opinion. Imagine that we model this by adding a new proposition p to the pool of opinions. So, $\mathcal{BS}(x)(w)$ now contains one further element. There are now more opinions. What happens to the set of worlds compatible with x 's beliefs? Does it get bigger or smaller? Is the new set a subset or superset of the previous set of compatible worlds? \square

2.2 Accessibility Relations

Another way of reformulating Hintikka's semantics for propositional attitudes is via the notion of an ACCESSIBILITY RELATION. We talk of a world w' being accessible from w . Each attitude can be associated with such an accessibility relation. For example, we can introduce the relation $w\mathcal{R}_\alpha^B w'$ which holds iff w' is compatible with α 's belief state in w . We have then yet another equivalent way of specifying the lexical entry for *believe*:

$$(7) \quad \llbracket \text{believe} \rrbracket^{w,g} = \lambda p_{\langle s,t \rangle}. \lambda x. \forall w' : w\mathcal{R}_x^B w' \rightarrow p(w') = \mathbf{t}.$$

It is profitable to think of different attitudes (belief, knowledge, hope, regret, memory, ...) as corresponding to different accessibility relations. Recall now that the linguistic study of determiners benefitted quite a bit from an investigation of the formal properties of the relations between sets of individuals that determiners express. We can do the same thing here and ask about the formal properties of the accessibility relation associated with belief versus the one associated with knowledge, etc. The obvious properties to think about are reflexivity, transitivity, and symmetry.

2.2.1 Reflexivity

A relation is reflexive iff for any object in the domain of the relation we know that the relation holds between that object and itself. Which accessibility relations are reflexive? Take knowledge:

$$(8) \quad w\mathcal{R}_x^K w' \text{ iff } w' \text{ is compatible with what } x \text{ knows in } w.$$

We are asking whether for any given possible world w , we know that \mathcal{R}_x^K holds between w and w itself. It will hold if w is a world that is compatible with what we know in w . And clearly that must be so. Take our body of knowledge in w . The concept of knowledge crucially contains the concept of truth: what we know must be true. So if in w we know that something is the case then it must be the case in w . So, w must be compatible with all we know in w . \mathcal{R}_x^K is reflexive.

Kirill Shkolovsky (in class) asked why we call reflexivity, transitivity, and symmetry “formal” properties of relations. The idea is that certain properties are “formal” or “logical”, while others are more substantial. So, the fact that the relation “have the same birthday as” is symmetric seems a more formal fact about it than the fact that the relation holds between my daughter and my brother-in-law. Nevertheless, one of the most common ways of characterizing formal/logical notions (permutation-invariance, if you're curious) does not in fact make symmetry etc. a formal/logical notion. So, while intuitively these do seem to be formal/logical properties, we do not know how to substantiate that intuition. See MacFarlane (2005) for discussion.

We talk here about knowledge entailing (or even presupposing) truth but we do not mean to say that knowledge simply equals true belief. Professors Socrates and Gettier and their exegetes have further considerations.

Now, if an attitude X corresponds to a reflexive accessibility relation, then we can conclude from a Xs *that* p being true in w that p is true in w . This property of an attitude predicate is often called VERIDICALITY. It is to be distinguished from FACTIVITY, which is a property of attitudes which *presuppose* – rather than (merely) entail – the truth of their complement.

If we consider the relation \mathcal{R}_x^B pairing with a world w those worlds w' which are compatible with what x *believes* in w , we no longer have reflexivity: belief is not a veridical attitude. It is easy to have false beliefs, which means that the actual world is not in fact compatible with one's beliefs, which contradicts reflexivity. And many other attitudes as well do not involve veridicality/reflexivity: what we hope may not come true, what we remember may not be what actually happened, etc.

In modal logic, the correspondence between formal properties of the accessibility relation and the validity of inference patterns is well-studied. What we have just seen is that reflexivity of the accessibility relation corresponds to the validity of $\Box p \rightarrow p$. Other properties correspond to other characteristic patterns. Let's see this for transitivity and symmetry.

In modal logic notation:
 $\Box p \rightarrow p$. This pattern is sometimes called T or M, as is the corresponding system of modal logic.

The difference between *believe* and *know* in natural discourse is quite delicate, especially when one considers first person uses (*I believe the earth is flat* vs. *I know the earth is flat*).

2.2.2 *Transitivity

Transitivity of the accessibility relation corresponds to the inference $\Box p \rightarrow \Box \Box p$. The pattern seems not obviously wrong for knowledge: if one knows that p , doesn't one thereby know that one knows that p ? But before we comment on that, let's establish the formal correspondence between transitivity and that inference pattern. This needs to go in both directions.

In the literature on epistemic modal logic, the pattern is known as the KK THESIS or POSITIVE INTROSPECTION. In general modal logic, it is the characteristic axiom 4 of the modal logic system S_4 , which is a system that adds 4 to the previous axiom M/T. Thus, S_4 is the logic of accessibility relations that are both reflexive and transitive.

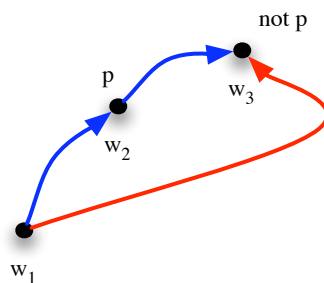


Figure 2.1: Transitivity

What does it take for the pattern to be valid? Assume that $\Box p$ holds for an arbitrary world w , i.e. that p is true in all worlds w' accessible from w . Now, the inference is to the fact that $\Box p$ again holds in any world w'' accessible from any of those worlds w' accessible from w . But what would prevent p from being false in some w'' accessible from some w' accessible from w ? That could only be prevented from happening if we knew that w'' itself is accessible from w as

well, because then we would know from the premiss that p is true in it (since p is true in *all* worlds accessible from w). Ah, but w'' (some world accessible from a world w' accessible from w) is only guaranteed to be accessible from w if the accessibility relation is transitive (if w' is accessible from w and w'' is accessible from w' , then transitivity ensures that w'' is accessible from w). This reasoning has shown that validity of the pattern requires transitivity. The other half of proving the correspondence is to show that transitivity entails that the pattern is valid.

The proof proceeds by reductio. Assume that the accessibility relation is transitive. Assume that (i) $\Box p$ holds for some world w but that (ii) $\Box\Box p$ doesn't hold in w . We will show that this situation cannot obtain. By (i), p is true in all worlds w' accessible from w . By (ii), there is some non- p world w'' accessible from some world w' accessible from w . But by transitivity of the accessibility relation, that non- p world w'' must be accessible from w . And since *all* worlds accessible from w are p worlds, w'' must be a p world, in contradiction to (ii). So, as soon as we assume transitivity, there is no way for the inference not to go through.

Now, do any of the attitudes have the transitivity property? It seems rather obvious that as soon as you believe something, you thereby believe that you believe it (and so it seems that belief involves a transitive accessibility relation). And in fact, as soon as you believe something, you believe that you *know* it. But one might shy away from saying that knowing something automatically amounts to knowing that you know it. For example, many are attracted to the idea that to know something requires that (i) that it is true, (ii) that you believe it, and (iii) that you are justified in believing it: the justified true belief analysis of knowledge. So, now couldn't it be that you know something, and thus (?) that you believe you know it, and thus that you believe that you are justified in believing it, but that you are not justified in believing that you are *justified* in believing it? After all, one's source of knowledge, one's reliable means of acquiring knowledge, might be a mechanism that one has no insight into. So, while one can implicitly trust (believe) in its reliability, and while it is in fact reliable, one might not have any means to have trustworthy beliefs about it. [Further worries about the KK Thesis are discussed by [Williamson \(2000\)](#).]

2.2.3 *Symmetry

What would the consequences be if the accessibility relation were symmetric? Symmetry of the accessibility relation \mathcal{R} corresponds to the validity of the following principle:

(9) Brouwer's Axiom:

$$\forall p \forall w : w \in p \rightarrow \left[\forall w' [w \mathcal{R} w' \rightarrow \exists w'' [w' \mathcal{R} w'' \& w'' \in p]] \right]$$

In modal logic notation:
 $p \rightarrow \Box\Diamond p$, known simply as B in modal logic. The system that combines T/M with B is often called Brouwer's System (B), after the mathematician L.E.J. Brouwer, not because he proposed it but because it was thought that it had some connections to his doctrines.

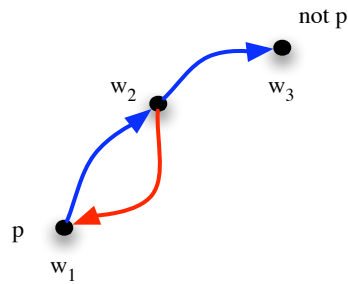


Figure 2.2: Symmetry



L.E.J. Brouwer

Here's the reasoning: Assume that R is in fact symmetric. Pick a world w in which p is true. Now, could it be that the right hand side of the inference fails to hold in w ? Assume that it does fail. Then, there must be some world w' accessible from w in which $\Diamond p$ is false. In other words, from that world w' there is no accessible world w'' in which p is true. But since R is assumed to be symmetric, one of the worlds accessible from w' is w and in w , p is true, which contradicts the assumption that the inference doesn't go through. So, symmetry ensures the validity of the inference.

The other way (validity of the inference requires symmetry): the inference says that from any p world we only have worlds accessible from which there is at least one accessible p world. But imagine that p is true in w but not true in any other world. So, the only way for the conclusion of the inference to hold automatically is to have a guarantee that w (the only p world) is accessible from any world accessible from it. That is, we need to have symmetry. QED.

To see whether a particular kind of attitude is based on a symmetric accessibility relation, we can ask whether Brouwer's Axiom is intuitively valid with respect to this attitude. If it is not valid, this shows that the accessibility relation can't be symmetric. In the case of a knowledge-based accessibility relation (epistemic accessibility), one can argue that *symmetry does not hold*:¹

The symmetry condition would imply that if something happens to be true in the actual world, then you know that it is compatible with your knowledge (Brouwer's Axiom). This will be violated by any case in which your beliefs are consistent, but mistaken. Suppose that while p is in fact true, you feel certain that it is false, and so think that you know that it is false. Since you think you know this, it is compatible with your knowledge that you know it. (Since we are assuming you are consistent, you can't both believe that you know it, and know that you do not). So it is compatible with your

¹ Thanks to Bob Stalnaker (pc to Kai von Fintel) for help with the following reasoning.

knowledge that you know that *not* p. Equivalently²: you don't know that you don't know that *not* p. Equivalently: you don't know that it's compatible with your knowledge that p. But by Brouwer's Axiom, since p is true, you would have to know that it's compatible with your knowledge that p. So if Brouwer's Axiom held, there would be a contradiction. So Brouwer's Axiom doesn't hold here, which shows that epistemic accessibility is not symmetric.

Game theorists and theoretical computer scientists who traffic in logics of knowledge often assume that the accessibility relation for knowledge is an equivalence relation (reflexive, symmetric, and transitive). But this is appropriate only if one abstracts away from any error, in effect assuming that belief and knowledge coincide. One striking consequence of working with an equivalence relation as the accessibility relation for knowledge is that one predicts the principle of NEGATIVE INTROSPECTION to hold:

- (IO) NEGATIVE INTROSPECTION (NI)
 If one doesn't know that p, then one knows that one doesn't know that p. ($\neg \Box p \rightarrow \Box \neg \Box p$).

This surely seems rather dubious: imagine that one strongly believes that p but that nevertheless p is false, then one doesn't know that p, but one doesn't seem to believe that one doesn't know that p, in fact one believes that one does know that p.

2.3 Supplemental Readings

A recent survey on attitudes:

Eric Swanson. 2009. Propositional attitudes. ms, prepared for *Semantics: An international handbook of meaning*, edited by Klaus von Heusinger, Claudia Maienbon, and Paul Portner. <http://tinyurl.com/swanson-attitudes>.

Further connections between mathematical properties of accessibility relations and logical properties of various notions of necessity and possibility are studied extensively in modal logic:

G.E. Hughes & M.J. Cresswell. 1996. *A new introduction to modal logic*. London: Routledge.

James Garson. 2008. Modal logic. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*. <http://plato.stanford.edu/entries/logic-modal/>,

² This and the following step rely on the duality of necessity and possibility: q is compatible with your knowledge iff you don't know that *not* q.

All one really needs to make NI valid is to have a EUCLIDEAN accessibility relation: any two worlds accessible from the same world are accessible from each other. It is a nice little exercise to prove this, if you have become interested in this sort of thing. Note that all reflexive and Euclidean accessibility relations are transitive and symmetric as well — another nice little thing to prove.

especially section 7 and 8, “Modal Axioms and Conditions on Frames”, “Map of the Relationships between Modal Logics”.

A thorough discussion of the possible worlds theory of attitudes, and some of its potential shortcomings, can be found in Bob Stalnaker’s work:

Robert Stalnaker. 1984. *Inquiry*. MIT Press.

Robert Stalnaker. 1999. *Context and content*. Oxford: Oxford University Press.

A quick and informative surveys about the notion of knowledge:

Matthias Steup. 2008. The analysis of knowledge. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Fall 2008. <http://plato.stanford.edu/archives/fall2008/entries/knowledge-analysis/>.

Linguistic work on attitudes has often been concerned with various co-occurrence patterns, particularly which moods (indicative or subjunctive or infinitive) occur in the complement and whether negative polarity items are licensed in the complement.

Mood licensing:

Paul Portner. 1997. The semantics of mood, complementation, and conversational force. *Natural Language Semantics* 5(2). 167–212. <http://dx.doi.org/10.1023/A:1008280630142>.

NPI-Licensing:

Nirit Kadmon & Fred Landman. 1993. *Any*. *Linguistics and Philosophy* 16(4). 353–422. <http://dx.doi.org/10.1007/BF00985272>.

Kai von Stechow. 1999. NPI licensing, Strawson entailment, and context dependency. *Journal of Semantics* 16(2). 97–148. <http://dx.doi.org/10.1093/jos/16.2.97>.

Anastasia Giannakidou. 1999. Affective dependencies. *Linguistics and Philosophy* 22(4). 367–421. <http://dx.doi.org/10.1023/A:1005492130684>.

There is some interesting work out of Amherst rethinking the way attitude predicates take their complements:

Angelika Kratzer. 2006. Decomposing attitude verbs. Handout from a talk honoring Anita Mittwoch on her 80th birthday at the Hebrew University of Jerusalem July 4, 2006. <http://semanticsarchive.net/Archive/DcwY2JkM/attitude-verbs2006.pdf>.

Keir Moulton. 2008. Clausal complementation and the *Wager*-class. *North East Linguistics Society (NELS)* 38. <http://sites.google.com/site/keirmoulton/Moultonnells2008wager.pdf>. <http://people.umass.edu/keir/Wager.pdf>.

Keir Moulton. 2009. *Natural selection and the syntax of clausal complementation*. University of Massachusetts at Amherst PhD thesis. http://scholarworks.umass.edu/open_access_dissertations/99/.

Tamina Stephenson in her MIT dissertation and related work explores the way attitude predicates interact with epistemic modals and taste predicates in their complements:

Tamina Stephenson. 2007a. Judge dependence, epistemic modals, and predicates of personal taste. *Linguistics and Philosophy* 30(4). 487–525. <http://dx.doi.org/10.1007/s10988-008-9023-4>.

Tamina Stephenson. 2007b. *Towards a theory of subjective meaning*. Massachusetts Institute of Technology PhD thesis. <http://semanticsarchive.net/Archive/2QxMjkoO/Stephenson-2007-thesis.pdf>.

Jon Gajewski in his MIT dissertation and subsequent work explores the distribution of the NEG-RAISING property among attitude predicates and traces it back to presuppositional components of the meaning of the predicates:

Jon Gajewski. 2005. *Neg-raising: Polarity and presupposition*. Massachusetts Institute of Technology PhD thesis. <http://dx.doi.org/1721.1/33696>.

Jon Gajewski. 2007. Neg-raising and polarity. *Linguistics and Philosophy*. <http://dx.doi.org/10.1007/s10988-007-9020-z>.

Interesting work has also been done on presupposition projection in attitude contexts:

Nicholas Asher. 1987. A typology for attitude verbs and their anaphoric properties. *Linguistics and Philosophy* 10(2). 125–197. <http://dx.doi.org/10.1007/BF00584317>.

Irene Heim. 1992. Presupposition projection and the semantics of attitude verbs. *Journal of Semantics* 9(3). 183–221. <http://dx.doi.org/10.1093/jos/9.3.183>.

Bart Geurts. 1998. Presuppositions and anaphors in attitude contexts. *Linguistics and Philosophy* 21(6). 545–601. <http://dx.doi.org/10.1023/A:1005481821597>.

CHAPTER THREE

MODALITY

We turn to modal auxiliaries and related constructions. The main difference from attitude constructions is that their semantics is more context-dependent. Otherwise, we are still quantifying over possible worlds.

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3.1 The Quantificational Theory of Modality

We will now be looking at modal auxiliaries like *may*, *must*, *can*, *have to*, etc. Most of what we say here should carry over straightforwardly to modal adverbs like *maybe*, *possibly*, *certainly*, etc. We will make certain syntactic assumptions, which make our work easier but which leave aside many questions that at some point deserve to be addressed.

3.1.1 Syntactic Assumptions

We will assume, at least for the time being, that a modal like *may* is a RAISING predicate (rather than a CONTROL predicate), i.e., its subject is not its own argument, but has been moved from the subject-position of its infinitival complement. So, we are dealing with the following kind of structure:

- (1) a. Ann may be smart.
 b. [Ann [λ_I [may [t_I be smart]]]]

Actually, we will be working here with the even simpler structure below, in which the subject has been reconstructed to its lowest trace position. (E.g., these could be generated by deleting all but the lowest copy in the movement chain.) We will be able to prove that movement of a name or pronoun never affects truth-conditions, so at any rate the interpretation of the structure in (1b) would be the same as of (2). As a matter of convenience, then, we will take the reconstructed structures, which allow us to abstract away from the (here irrelevant) mechanics of variable binding.

- (2) may [Ann be smart]

So, for now at least, we are assuming that modals are expressions that take a full sentence as their semantic argument.¹ Now then, what do modals mean?

3.1.2 Quantification over Possible Worlds

The basic idea of the possible worlds semantics for modal expressions is that they are quantifiers over possible worlds. Toy lexical entries for *must* and *may*, for example, would look like this:

- (3) $\llbracket \text{must} \rrbracket^{w,g} = \lambda p_{\langle s,t \rangle}. \forall w': p(w') = 1.$
 (4) $\llbracket \text{may} \rrbracket^{w,g} = \lambda p_{\langle s,t \rangle}. \exists w': p(w') = 1.$

This analysis is too crude (in particular, notice that it would make modal sentences non-contingent — there is no occurrence of the evaluation world on the right hand side!). But it does already have some desirable consequences that we will seek to preserve through all subsequent refinements. It correctly predicts a number of intuitive judgments about the logical relations between *must* and *may* and among various combinations of these items and negations. To start with some elementary facts, we feel that *must* ϕ entails *may* ϕ , but not vice versa:

¹ We will assume that even though *Ann be smart* is a non-finite sentence, this will not have any effect on its semantic type, which is that of a sentence, which in turn means that its semantic value is a truth-value. This is hopefully independent of the (interesting) fact that *Ann be smart* on its own cannot be used to make a truth-evaluable assertion.

The issue of raising vs. control will probably be taken up later. If you are eager to get started on it and other questions of the morphosyntax of modals, read the handout from an LSA class Sabine and Kai taught a few years ago: <http://web.mit.edu/fintel/lsaz20-class-2-handout.pdf>.

We will talk about reconstruction in more detail later.

This idea goes back a long time. It was famously held by Leibniz, but there are precedents in the medieval literature, see Knuuttila (2003). See Copeland (2002) for the modern history of the possible worlds analysis of modal expressions.

- (5) You must stay.
Therefore, you may stay. VALID
- (6) You may stay.
Therefore, you must stay. INVALID
- (7) a. You may stay, but it is not the case that you must stay.²
b. You may stay, but you don't have to stay. CONSISTENT

We judge *must* ϕ incompatible with its “inner negation” *must* [*not* ϕ], but find *may* ϕ and *may* [*not* ϕ] entirely compatible:

- (8) You must stay, and/but also, you must leave. (leave = not stay).
CONTRADICTORY
- (9) You may stay, but also, you may leave.
CONSISTENT

We also judge that in each pair below, the (a)-sentence and the (b)-sentences say the same thing.

- (10) a. You must stay.
b. It is not the case that you may leave.
You aren't allowed to leave.
(You may not leave.)³
(You can't leave.)
- (11) a. You may stay.
b. It is not the case that you must leave.
You don't have to leave.
You don't need to leave.
(You needn't leave.)

² The somewhat stilted *it is not the case*-construction is used in to make certain that negation takes scope over *must*. When modal auxiliaries and negation are together in the auxiliary complex of the same clause, their relative scope seems not to be transparently encoded in the surface order; specifically, the scope order is not reliably negation \succ modal. (Think about examples with *mustn't*, *can't*, *shouldn't*, *may not* etc. What's going on here? This is an interesting topic which we must set aside for now. See the references at the end of the chapter for relevant work.) With modal *main* verbs (such as *have to*), this complication doesn't arise; they are consistently inside the scope of clause-mate auxiliary negation. Therefore we can use (b) to (unambiguously) express the same scope order as (a), without having to resort to a biclausal structure.

³ The parenthesized variants of the (b)-sentences are pertinent here only to the extent that we can be certain that negation scopes over the modal. In these examples, apparently it does, but as we remarked above, this cannot be taken for granted in all structures of this form.

Given that *stay* and *leave* are each other's negations (i.e. $\llbracket \text{leave} \rrbracket^{w,g} = \llbracket \text{not stay} \rrbracket^{w,g}$, and $\llbracket \text{stay} \rrbracket^{w,g} = \llbracket \text{not leave} \rrbracket^{w,g}$), the LF-structures of these equivalent pairs of sentences can be seen to instantiate the following schemata:⁴

- (I2) a. $\text{must } \phi \equiv \text{not } [\text{may } [\text{not } \phi]]$
 b. $\text{must } [\text{not } \psi] \equiv \text{not } [\text{may } \psi]$
- (I3) a. $\text{may } \phi \equiv \text{not } [\text{must } [\text{not } \phi]]$
 b. $\text{may } [\text{not } \psi] \equiv \text{not } [\text{must } \psi]$

Our present analysis of *must*, *have-to*, ... as universal quantifiers and of *may*, *can*, ... as existential quantifiers straightforwardly predicts all of the above judgments, as you can easily prove.

More linguistic data regarding the “parallel logic” of modals and quantifiers can be found in Larry Horn’s dissertation (Horn 1972).

- (I4) a. $\forall x \phi \equiv \neg \exists \neg \phi$
 b. $\forall x \neg \phi \equiv \neg \exists x \phi$
- (I5) a. $\exists x \phi \equiv \neg \forall x \neg \phi$
 b. $\exists x \neg \phi \equiv \neg \forall x \phi$

3.2 Flavors of Modality

3.2.1 Contingency

We already said that the semantics we started with is too simple-minded. In particular, we have no dependency on the evaluation world, which would make modal statements non-contingent. This is not correct.

If one says *It may be snowing in Cambridge*, that may well be part of useful, practical advice about what to wear on your upcoming trip to Cambridge. It may be true or it may be false. The sentence seems true if said in the dead of winter when we have already heard about a Nor’Easter that is sweeping across New England. The sentence seems false if said by a clueless Australian acquaintance of ours in July.

The contingency of modal claims is not captured by our current semantics. All the *may*-sentence would claim under that semantics is that there is some possible world where it is snowing in Cambridge. And surely, once you have read Lewis’ quote in Chapter 1, where he asserts the existence of possible worlds with different physical constants than we enjoy here, you must admit that there have to be such worlds even if it is July. The problem is that in our semantics, repeated here

- (I6) $\llbracket \text{may} \rrbracket^{w,g} = \lambda p_{\langle s,t \rangle}. \exists w': p(w') = 1.$

4 In logicians’ jargon, *must* and *may* behave as DUALS of each other. For definitions of “dual”, see Barwise & Cooper (1981, p. 197) or Gamut (1991, vol.2,238).

there is no occurrence of w on the right hand side. This means that the truth-conditions for *may*-sentences are world-independent. In other words, they make non-contingent claims that are either true whatever or false whatever, and because of the plenitude of possible worlds they are more likely to be true than false. This needs to be fixed. But how?

Well, what makes *it may be snowing in Cambridge* seem true when we know about a Nor'Easter over New England? What makes it seem false when we know that it is summer in New England? The idea is that we only consider possible worlds COMPATIBLE WITH THE EVIDENCE AVAILABLE TO US. And since what evidence is available to us differs from world to world, so will the truth of a *may*-statement.

$$(17) \quad \llbracket \text{may} \rrbracket^{w,g} = \lambda p. \exists w' \text{ compatible with the evidence in } w: p(w') = 1.^5$$

$$(18) \quad \llbracket \text{must} \rrbracket^{w,g} = \lambda p. \forall w' \text{ compatible with the evidence in } w: p(w') = 1.$$

Let us consider a different example:

$$(19) \quad \text{You have to be quiet.}$$

Imagine this sentence being said based on the house rules of the particular dormitory you live in. Again, this is a sentence that could be true or could be false. Why do we feel that this is a contingent assertion? Well, the house rules can be different from one world to the next, and so we might be unsure or mistaken about what they are. In one possible world, they say that all noise must stop at 11pm, in another world they say that all noise must stop at 10pm. Suppose we know that it is 10:30 now, and that the dorm we are in has either one or the other of these two rules, but we have forgotten which. Then, for all we know, *you have to be quiet* may be true or it may be false. This suggests a lexical entry along these lines:

$$(20) \quad \llbracket \text{have-to} \rrbracket^{w,g} = \lambda p. \forall w' \text{ compatible with the rules in } w: p(w') = 1.$$

Again, we are tying the modal statement about other worlds down to certain worlds that stand in a certain relation to actual world: those worlds where the rules as they are here are obeyed.

A note of caution: it is very important to realize that the worlds compatible with the rules as they are in w are those worlds where nothing happens that violates any of the w -rules. This is not at all the same as saying that the worlds compatible with the rules in w are those worlds where the same rules are in force. Usually, the rules do not care what the rules are, unless the rules contain some kind of meta-statement to the effect that the rules have to be the way they are, i.e. that the rules cannot be changed. So, in fact, a world w' in which

⁵ From now on, we will leave off type-specifications such as that p has to be of type $\langle s, t \rangle$, whenever it is obvious what they should be and when saving space is aesthetically called for.

nothing happens that violates the rules as they are in w but where the rules are quite different and in fact what happens violates the rules as they are in w' is nevertheless a world compatible with the rules in w . For example, imagine that the only relevant rule in w is that students go to bed before midnight. Take a world w' where a particular student goes to bed at 11:30 pm but where the rules are different and say that students have to go to bed before 11 pm. Such a world w' is compatible with the rules in w (but of course not with the rules in w').

Apparently, there are different flavors of modality, varying in what kind of facts in the evaluation world they are sensitive to. The semantics we gave for *must* and *may* above makes them talk about evidence, while the semantics we gave for *have-to* made it talk about rules. But that was just because the examples were hand-picked. In fact, in the dorm scenario we could just as well have said *You must be quiet*. And, vice versa, there is nothing wrong with using *it has to be snowing in Cambridge* based on the evidence we have. In fact, many modal expressions seem to be multiply ambiguous.

Traditional descriptions of modals often distinguish a number of “readings”: EPISTEMIC, DEONTIC, ABILITY, CIRCUMSTANTIAL, DYNAMIC, . . . (Beyond “epistemic” and “deontic,” there is a great deal of terminological variety. Sometimes all non-epistemic readings are grouped together under the term ROOT MODALITY.) Here are some initial illustrations.

(21) EPISTEMIC MODALITY

A: Where is John?

B: I don't know. He *may* be at home.

(22) DEONTIC MODALITY

A: Am I allowed to stay over at Janet's house?

B: No, but you *may* bring her here for dinner.

(23) CIRCUMSTANTIAL/DYNAMIC MODALITY

A: I will plant the rhododendron here.

B: That's not a good idea. It *can* grow very tall.

How are *may* and *can* interpreted in each of these examples? What do the interpretations have in common, and where do they differ?

In all three examples, the modal makes an existentially quantified claim about possible worlds. This is usually called the MODAL FORCE of the claim. What differs is what worlds are quantified over. In EPISTEMIC modal sentences, we quantify over worlds compatible with the available evidence. In DEONTIC modal sentences, we quantify over worlds compatible with the rules and/or regulations. And in the CIRCUMSTANTIAL modal sentence, we quantify over the set of worlds which conform to the laws of nature (in particular, plant biology). What speaker B in (23) is saying, then, is that there are some worlds conforming to the laws of

nature in which this rhododendron grows very tall. (Or is this another instance of an epistemic reading? See below for discussion of the distinction between circumstantial readings and epistemic ones.)

How can we account for this variety of readings? One way would be to write a host of lexical entries, basically treating this as a kind of (more or less principled) ambiguity. Another way, which is preferred by many people, is to treat this as a case of context-dependency, as argued in seminal work by Kratzer (1977, 1978, 1981, 1991).

According to Kratzer, what a modal brings with it intrinsically is just a modal force, that is, whether it is an existential (possibility) modal or a universal (necessity) modal. What worlds it quantifies over is determined by context. In essence, the context has to supply a restriction to the quantifier. How can we implement this idea?

We encountered context-dependency before when we talked about pronouns and their referential (and E-Type) readings (H & K, chapters 9–11). We treated referential pronouns as free variables, appealing to a general principle that free variables in an LF need to be supplied with values from the utterance context. If we want to describe the context-dependency of modals in a technically analogous fashion, we can think of their LF-representations as incorporating or subcategorizing for a kind of invisible pronoun, a free variable that stands for a set of possible worlds. So we posit LF-structures like this:

$$(24) \quad [I' \ [I \text{ must } p_{\langle n, \langle s, t \rangle \rangle}] \ [VP \text{ you quiet}]]$$

$p_{\langle n, \langle s, t \rangle \rangle}$ here is a variable over (characteristic functions of) sets of worlds, which — like all free variables — needs to receive a value from the utterance context. Possible values include: the set of worlds compatible with the speaker's current knowledge; the set of worlds in which everyone obeys all the house rules of a certain dormitory; and many others. The denotation of the modal itself now has to be of type $\langle st, \langle st, t \rangle \rangle$ rather than $\langle st, t \rangle$, thus it will be more like a quantificational determiner rather than a complete generalized quantifier. Only after the modal has been combined with its covert restrictor do we obtain a value of type $\langle st, t \rangle$.

$$(25) \quad \begin{aligned} \text{a.} \quad & \llbracket \text{must} \rrbracket^{w,g} = \llbracket \text{have-to} \rrbracket^{w,g} = \llbracket \text{need-to} \rrbracket^{w,g} = \dots = \\ & \lambda p \in D_{\langle s, t \rangle}. \lambda q \in D_{\langle s, t \rangle}. \forall w \in W [p(w) = 1 \rightarrow q(w) = 1] \\ \text{b.} \quad & \llbracket \text{may} \rrbracket^{w,g} = \llbracket \text{can} \rrbracket^{w,g} = \llbracket \text{be-allowed-to} \rrbracket^{w,g} = \dots = \\ & \lambda p \in D_{\langle s, t \rangle}. \lambda q \in D_{\langle s, t \rangle}. \exists w \in W [p(w) = 1 \ \& \ q(w) = 1] \end{aligned}$$

in set talk: $p \subseteq q$

in set talk: $p \cap q \neq \emptyset$

On this approach, the epistemic, deontic, etc. “readings” of individual occurrences of modal verbs come about by a combination of two separate things. The lexical semantics of the modal itself encodes just a quantificational force, a *relation* between sets of worlds. This is either the subset-relation (universal quantification; necessity) or the relation of non-disjointness (existential quantifi-



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It is well-known that natural language quantification is in general subject to contextual restriction. See Stanley & Szabó (2000) for a recent discussion.

We are using the notation for variables of types other than e , introduced by Heim & Kratzer.

See p. 213. An index on a variable now is an ordered pair of a natural number and a type.

Q: Can you think of overt anaphoric expressions that are arguably of the type $\langle s, t \rangle$, a proposition?

cation; possibility). The covert variable next to the modal picks up a contextually salient set of worlds, and this functions as the quantifier's restrictor. The labels "epistemic", "deontic", "circumstantial" etc. group together certain conceptually natural classes of possible values for this covert restrictor.

Notice that, strictly speaking, there is not just one deontic reading (for example), but many. A speaker who utters

(26) You have to be quiet.

might mean: 'I want you to be quiet,' (i.e., you are quiet in all those worlds that conform to my preferences). Or she might mean: 'unless you are quiet, you won't succeed in what you are trying to do,' (i.e., you are quiet in all those worlds in which you succeed at your current task). Or she might mean: 'the house rules of this dormitory here demand that you be quiet,' (i.e., you are quiet in all those worlds in which the house rules aren't violated). And so on. So the label "deontic" appears to cover a whole open-ended set of imaginable "readings", and which one is intended and understood on a particular utterance occasion may depend on all sorts of things in the interlocutors' previous conversation and tacit shared assumptions. (And the same goes for the other traditional labels.)

3.2.2 Epistemic vs. Circumstantial Modality

Is it all context-dependency? Or do flavors of modality correspond to some sorts of signals in the structure of sentences? Read the following famous passage from Kratzer and think about how the two sentences with their very different modal meanings differ in structure:

Consider sentences (27) and (28):

(27) Hydrangeas can grow here.

(28) There might be hydrangeas growing here.

The two sentences differ in meaning in a way which is illustrated by the following scenario.

"Hydrangeas"

Suppose I acquire a piece of land in a far away country and discover that soil and climate are very much like at home, where hydrangeas prosper everywhere. Since hydrangeas are my favorite plants, I wonder whether they would grow in this place and inquire about it. The answer is (27). In such a situation, the proposition expressed by (27) is true. It is true regardless of whether it is or isn't likely that there are already hydrangeas in the country we are considering. All that matters is climate, soil, the special properties of hydrangeas, and the like. Suppose now that the country we are in

Quoted from Kratzer (1991). In Kratzer (1981), the hydrangeas were *Zwetschenbäume* 'plum trees'. The German word *Zwetsche*, by the way, is etymologically derived from the name of the city Damascus (Syria), the center of the ancient plum trade.

has never had any contacts whatsoever with Asia or America, and the vegetation is altogether different from ours. Given this evidence, my utterance of (28) would express a false proposition. What counts here is the complete evidence available. And this evidence is not compatible with the existence of hydrangeas.

(27) together with our scenario illustrates the pure CIRCUMSTANTIAL reading of the modal *can*. [...]. (28) together with our scenario illustrates the epistemic reading of modals. [...] circumstantial and epistemic conversational backgrounds involve different kinds of facts. In using an epistemic modal, we are interested in what else may or must be the case in our world given all the evidence available. Using a circumstantial modal, we are interested in the necessities implied by or the possibilities opened up by certain sorts of facts. Epistemic modality is the modality of curious people like historians, detectives, and futurologists. Circumstantial modality is the modality of rational agents like gardeners, architects, and engineers. A historian asks what might have been the case, given all the available facts. An engineer asks what can be done given certain relevant facts.

Consider also the very different prominent meanings of the following two sentences, taken from Kratzer as well:

- (29) a. Cathy can make a pound of cheese out of this can of milk.
 b. Cathy might make a pound of cheese out of this can of milk.

EXERCISE 3.1: Come up with examples of epistemic, deontic, and circumstantial uses of the necessity verb *have to*. Describe the set of worlds that constitutes the understood restrictor in each of your examples. □

3.2.3 Contingency Again

We messed up. If you inspect the context-dependent meanings we have on the table now for our modals, you will see that the right hand sides again do not mention the evaluation world w . Therefore, we will again have the problem of not making contingent claims, indirectly about the actual world. This needs to be fixed. We need a semantics that is both context-dependent and contingent.

The problem, it turns out, is with the idea that the utterance context supplies a *determinate set of worlds* as the restrictor. When I understand that you meant your use of *must*, in *you must be quiet*, to quantify over the set of worlds in which the house rules of our dorm are obeyed, this does not imply that you and I have to know or agree on which set exactly this is. That depends on what the house rules in our world actually happen to say, and this may be an open question at the current stage of our conversation. What we do agree on, if I have understood

your use of *must* in the way that you intended it, is just that it quantifies over *whatever set of worlds it may be* that the house rules pick out.

The technical implementation of this insight requires that we think of the context's contribution not as a set of worlds, but rather as a function which for each world it applies to picks out such a set. For example, it may be the function which, for any world w , yields the set $\{w' : \text{the house rules that are in force in } w \text{ are obeyed in } w'\}$. If we apply this function to a world w_1 , in which the house rules read “no noise after 10 pm”, it will yield a set of worlds in which nobody makes noise after 10 pm. If we apply the same function to a world w_2 , in which the house rules read “no noise after 11 pm”, it will yield a set of worlds in which nobody makes noise after 11 pm.

Suppose, then, that the covert restrictor of a modal predicate denotes such a function, i.e., its value is of type $\langle s, st \rangle$.

$$(30) \quad [{}_I [{}_I \text{ must } R_{\langle n, \langle s, st \rangle \rangle}] [{}_{VP} \text{ you quiet}]]$$

And the new lexical entries for *must* and *may* that will fit this new structure are these:

$$(31) \quad \begin{array}{ll} \text{a.} & \llbracket \text{must} \rrbracket^{w,g} = \llbracket \text{have-to} \rrbracket^{w,g} = \llbracket \text{need-to} \rrbracket^{w,g} = \dots = \\ & \lambda R \in D_{\langle s, st \rangle}. \lambda q \in D_{\langle s, t \rangle}. \forall w' \in W [R(w)(w') = 1 \rightarrow q(w') = 1] \\ \text{b.} & \llbracket \text{may} \rrbracket^{w,g} = \llbracket \text{can} \rrbracket^{w,g} = \llbracket \text{be-allowed-to} \rrbracket^{w,g} = \dots = \\ & \lambda R \in D_{\langle s, st \rangle}. \lambda q \in D_{\langle s, t \rangle}. \exists w' \in W [R(w)(w') = 1 \ \& \ q(w') = 1] \end{array}$$

in set talk: $(R(w) \subseteq q)$

in set talk: $(R(w) \cap q \neq \emptyset)$

Let us see now how this solves the contingency problem.

$$(32) \quad \text{Let } w \text{ be a world, and assume that the context supplies an assignment } g \text{ such that } g(R_{\langle I7, \langle s, st \rangle \rangle}) = \lambda w. \lambda w'. \text{ the house rules in force in } w \text{ are obeyed in } w'$$

$$\begin{aligned} \llbracket \text{must } R_{\langle I7, \langle s, st \rangle \rangle} \text{ you quiet} \rrbracket^{w,g} &= & (\text{IFA}) \\ \llbracket \text{must } R_{\langle I7, \langle s, st \rangle \rangle} \rrbracket^{w,g} (\lambda w' \llbracket \text{you quiet} \rrbracket^{w'}) &= & (\text{FA}) \\ \llbracket \text{must} \rrbracket^{w,g} (\llbracket R_{\langle I7, \langle s, st \rangle \rangle} \rrbracket^{w,g} (\lambda w' \llbracket \text{you quiet} \rrbracket^{w'})) &= & (\text{lex. entries } \textit{you}, \textit{quiet}) \\ \llbracket \text{must} \rrbracket^{w,g} (\llbracket R_{\langle I7, \langle s, st \rangle \rangle} \rrbracket^{w,g} (\lambda w'. \text{ you are quiet in } w')) &= & (\text{lex. entry } \textit{must}) \\ \forall w' \in W : \llbracket R_{\langle I7, \langle s, st \rangle \rangle} \rrbracket^{w,g} (w)(w') = 1 \rightarrow \text{you are quiet in } w' &= & (\text{pronoun rule}) \\ \forall w' \in W : g(R_{\langle I7, \langle s, st \rangle \rangle})(w)(w') = 1 \rightarrow \text{you are quiet in } w' &= & (\text{def. of } g) \\ \forall w' \in W [\text{the house rules in force in } w \text{ are obeyed in } w' &\rightarrow \text{you are quiet in } w'] \end{aligned}$$

As we see in the last line of (32), the truth-value of (30) depends on the evaluation world w .

EXERCISE 3.2: Describe two worlds w_1 and w_2 so that

$$\llbracket \text{must } R_{\langle I7, \langle s, st \rangle \rangle} \text{ you quiet} \rrbracket^{w_1, g} = 1 \text{ and } \llbracket \text{must } R_{\langle I7, \langle s, st \rangle \rangle} \text{ you quiet} \rrbracket^{w_2, g} = 0. \quad \square$$

You will of course recognize that functions of type $\langle s, st \rangle$ are simply a schönfinkelled version of the ACCESSIBILITY RELATIONS we introduced in the previous chapter.

EXERCISE 3.3: In analogy to the deontic relation $g(R_{\langle 17, \langle s, st \rangle \rangle})$ defined in (32), define an appropriate relation that yields an epistemic reading for a sentence like *You may be quiet*. \square

3.2.4 Iteration

Consider the following example:

- (33) You might have to leave.

What does this mean? Under one natural interpretation, we learn that the speaker considers it possible that the addressee is under the obligation to leave. This seems to involve one modal embedded under a higher modal. It appears that this sentence should be true in a world w iff some world w' compatible with what the speaker knows in w is such that every world w'' in which the rules as they are in w' are followed is such that you leave in w'' .

Assume the following LF:

- (34) $[I' [\text{might } R_{\langle 1, \langle s, st \rangle \rangle}] [VP [\text{have-to } R_{\langle 2, \langle s, st \rangle \rangle}] [IP \text{ you leave}]]]$

Suppose w is the world for which we calculate the truth-value of the whole sentence, and the context maps R_1 to the function which maps w to the set of all those worlds compatible with what is known in w . *might* says that some of those worlds are worlds w' that make the tree below *might* true. Now assume further that the context maps R_2 to the function which assigns to any such world w' the set of all those worlds in which the rules as they are in w' are followed. *have to* says that all of those worlds are worlds w'' in which you leave.

In other words, while it is not known to be the case that you have to leave, for all the speaker knows it might be the case.

EXERCISE 3.4: Describe values for the covert $\langle s, st \rangle$ -variable that are intuitively suitable for the interpretation of the modals in the following sentences:

- (35) As far as John's preferences are concerned, you *may* stay with us.
 (36) According to the guidelines of the graduate school, every PhD candidate *must* take 9 credit hours outside his/her department.
 (37) John *can* run a mile in 5 minutes.
 (38) This *has* to be the White House.
 (39) This elevator *can* carry up to 3000 pounds.

For some of the sentences, different interpretations are conceivable depending on the circumstances in which they are uttered. You may therefore have to sketch the utterance context you have in mind before describing the accessibility relation. \square

There is more to be said about which modals can embed under which other modals. See for some discussion the handout mentioned earlier: <http://web.mit.edu/fintel/ls220-class-2-handout.pdf>.

From now on, we will omit the type-designation of variables whenever we feel confident that their type is easy to figure out from the context.

EXERCISE 3.5: Collect two naturally occurring examples of modalized sentences (e.g., sentences that you overhear in conversation, or read in a newspaper or novel – not ones that are being used as examples in a linguistics or philosophy paper!), and give definitions of values for the covert $\langle s, st \rangle$ -variable which account for the way in which you actually understood these sentences when you encountered them. (If the appropriate interpretation is not salient for the sentence out of context, include information about the relevant preceding text or non-linguistic background.) \square

3.2.5 A technical variant of the analysis

In our account of the contingency of modalized sentences, we adopted lexical entries for the modals that gave them world-dependent extensions of type $\langle\langle s, st \rangle, \langle st, t \rangle\rangle$:

- (40) (repeated from earlier):
 For any $w \in W$: $\llbracket \text{must} \rrbracket^{w,g}$
 $\lambda R \in D_{\langle s, st \rangle} \cdot \lambda q \in D_{\langle st, t \rangle} \cdot \forall w' \in W [R(w)(w') = 1 \rightarrow q(w') = 1]$
 (in set talk: $\lambda R_{\langle s, st \rangle} \cdot \lambda q_{\langle st, t \rangle} \cdot (R(w) \subseteq q)$).

Unfortunately, this treatment somewhat obscures the parallel between the modals and the quantificational determiners, which have world-independent extensions of type $\langle et, \langle et, t \rangle \rangle$.

Let's explore an alternative solution to the contingency problem, which will allow us to stick with the world-independent type- $\langle st, \langle st, t \rangle \rangle$ -extensions that we assumed for the modals at first:

- (41) (repeated from even earlier):
 $\llbracket \text{must} \rrbracket^{w,g} = \lambda p \in D_{\langle s, st \rangle} \cdot \lambda q \in D_{\langle st, t \rangle} \cdot \forall w \in W [p(w) = 1 \rightarrow q(w) = 1]$
 (in set talk: $\lambda p \in D_{\langle s, st \rangle} \cdot \lambda q \in D_{\langle st, t \rangle} \cdot p \subseteq q$).

We posit the following LF-representation:

- (42) $[_I' [_I \text{ must } [_{R_{\langle 4, \langle s, st \rangle \rangle}} w^*]] [_{VP} \text{ you quiet}]]$

What is new here is that the covert restrictor is complex. The first part, $R_{\langle 4, \langle s, st \rangle \rangle}$, is (as before) a free variable of type $\langle s, st \rangle$, which gets assigned an accessibility relation by the context of utterance. The second part is a special terminal symbol which is interpreted as picking out the evaluation world:

- (43) For any $w \in W$: $\llbracket w^* \rrbracket^{w,g} = w$.⁶

Note that as soon as we're introducing an object language expression whose extension is a possible world, we will now have expressions of type s and should also introduce the domain of things of type s : $D_s = W$.

⁶ D. R. Dowty (1982) introduced an analogous symbol to pick out the evaluation *time*. We have chosen the star-notation to allude to this precedent.

When $R_{\langle 4, \langle s, st \rangle \rangle}$ and w^* combine (by Functional Application), we obtain a constituent whose extension is of type $\langle s, t \rangle$ (a proposition or set of worlds). This is the same type as the extension of the free variable p in the previous proposal, hence suitable to combine with the old entry for *must* (by FA). However, while the extension of p was completely fixed by the variable assignment, and did not vary with the evaluation world, the new complex constituent's extension depends on both the assignment and the world:

- (44) For any $w \in W$ and any assignment g :
- $$\llbracket R_{\langle 4, \langle s, st \rangle \rangle} (w^*) \rrbracket^{w, g} = g(\langle 4, \langle s, st \rangle \rangle)(w).$$

As a consequence of this, the extensions of the higher nodes I and I' will also vary with the evaluation world, and this is how we capture the fact that (42) is contingent.

Maybe this variant is more appealing. But for the rest of this chapter, we continue to assume the original analysis as presented earlier. In the next chapter on conditionals, we will however make crucial use of this way of formulating the semantics for modals. So, make sure you understand what we just proposed.

3.3 *Kratzer's Conversational Backgrounds

Angelika Kratzer has some interesting ideas on how accessibility relations are supplied by the context. She argues that what is really floating around in a discourse is a CONVERSATIONAL BACKGROUND. Accessibility relations can be computed from conversational backgrounds (as we shall do here), or one can state the semantics of modals directly in terms of conversational backgrounds (as Kratzer does).

A conversational background is the sort of thing that is identified by phrases like *what the law provides*, *what we know*, etc. Take the phrase *what the law provides*. What the law provides is different from one possible world to another. And what the law provides in a particular world is a *set of propositions*. Likewise, what we know differs from world to world. And what we know in a particular world is a set of propositions. The intension of *what the law provides* is then that function which assigns to every possible world the set of propositions p such that the law provides in that world that p . Of course, that doesn't mean that p holds in that world itself: the law can be broken. And the intension of *what we know* will be that function which assigns to every possible world the set of propositions we know in that world. Quite generally, conversational backgrounds are functions of type $\langle s, \langle st, t \rangle \rangle$, functions from worlds to (characteristic functions of) sets of propositions.

Now, consider:

- (45) (In view of what we know,) Brown must have murdered Smith.

The *in view of*-phrase may explicitly signal the intended conversational background. Or, if the phrase is omitted, we can just infer from other clues in the discourse that such an epistemic conversational background is intended. We will focus on the case of pure context-dependency.

How do we get from a conversational background to an accessibility relation? Take the conversational background at work in (45). It will be the following:

(46) $\lambda w. \lambda p. p$ is one of the propositions that we know in w .

This conversational background will assign to any world w the set of propositions p that in w are known by us. So we have a set of propositions. From that we can get the set of worlds in which all of the propositions in this set are true. These are the worlds that are compatible with everything we know. So, this is how we get an accessibility relation:

(47) For any conversational background f of type $\langle s, \langle st, t \rangle \rangle$, we define the corresponding accessibility relation R_f of type $\langle s, st \rangle$ as follows:
 $R_f := \lambda w. \lambda w'. \forall p [f(w)(p) = 1 \rightarrow p(w') = 1]$.

In words, w' is f -accessible from w iff all propositions p that are assigned by f to w are true in w' .

Kratzer uses the term MODAL BASE for the conversational background that determines the set of accessible worlds. We can be sloppy and use this term for a number of interrelated concepts:

- (i) the conversational background (type $\langle s, \langle st, t \rangle \rangle$),
- (ii) the set of propositions assigned by the conversational background to a particular world (type $\langle st, t \rangle$),
- (iii) the accessibility relation (type $\langle s, st \rangle$) determined by (i),
- (iv) the set of worlds accessible from a particular world (type $\langle s, t \rangle$).

Kratzer calls a conversational background (modal base) *REALISTIC* iff it assigns to *any* world a set of propositions that are all true in that world. The modal base *what we know* is realistic, the modal bases *what we believe* and *what we want* are not.

What follows are some (increasingly technical exercises) on conversational backgrounds.

EXERCISE 3.6: Show that a conversational background f is realistic iff the corresponding accessibility relation R_f (defined as in (47)) is reflexive. \square

EXERCISE 3.7: Let us call an accessibility relation *TRIVIAL* if it makes every world accessible from every world. R is *TRIVIAL* iff $\forall w \forall w': w' \in R(w)$. What would the conversational background f have to be like for the accessibility relation R_f to be trivial in this sense? \square

EXERCISE 3.8: The definition in (47) specifies, in effect, a function from $D_{\langle s, \langle st, t \rangle \rangle}$ to $D_{\langle s, st \rangle}$. It maps each function f of type $\langle s, \langle st, t \rangle \rangle$ to a unique function R_f of type $\langle s, st \rangle$. This mapping is not one-to-one, however. Different elements of $D_{\langle s, \langle st, t \rangle \rangle}$ may be mapped to the same value in $D_{\langle s, st \rangle}$.⁷

- Prove this claim. I.e., give an example of two functions f and f' in $D_{\langle s, \langle st, t \rangle \rangle}$ for which (47) determines $R_f = R_{f'}$.
- As you have just proved, if every function of type $\langle s, \langle st, t \rangle \rangle$ qualifies as a ‘conversational background’, then two different conversational backgrounds can collapse into the same accessibility relation. Conceivably, however, if we imposed further restrictions on conversational backgrounds (i.e., conditions by which only a proper subset of the functions in $D_{\langle s, \langle st, t \rangle \rangle}$ would qualify as conversational backgrounds), then the mapping between conversational backgrounds and accessibility relations might become one-to-one after all. In this light, consider the following potential restriction:

(48) Every conversational background f must be “closed under entailment”; i.e., it must meet this condition:

$$\forall w. \forall p [\cap f(w) \subseteq p \rightarrow p \in f(w)].$$

(In words: if the propositions in $f(w)$ taken together entail p , then p must itself be in $f(w)$.) Show that this restriction would ensure that the mapping defined in (47) will be one-to-one. \square

3.4 Supplementary Readings

The most important background readings for this chapter are the following two papers by Kratzer:

7 In this exercise, we systematically substitute sets for their characteristic functions. I.e., we pretend that $D_{\langle s, t \rangle}$ is the power set of W (i.e., elements of $D_{\langle s, t \rangle}$ are sets of worlds), and $D_{\langle st, t \rangle}$ is the power set of $D_{\langle s, t \rangle}$ (i.e., elements of $D_{\langle st, t \rangle}$ are sets of sets of worlds). On these assumptions, the definition in (47) can take the following form:

- (i) For any conversational background f of type $\langle s, \langle st, t \rangle \rangle$, we define the corresponding accessibility relation R_f of type $\langle s, st \rangle$ as follows:

$$R_f := \lambda w. \{w' : \forall p [p \in f(w) \rightarrow w' \in p]\}.$$

The last line of this can be further abbreviated to:

- (ii) $R_f := \lambda w. \cap f(w)$

This formulation exploits a set-theoretic notation which we have also used in condition (48) of the second part of the exercise. It is defined as follows:

- (iii) If S is a set of sets, then $\cap S := \{x : \forall Y [Y \in S \rightarrow x \in Y]\}.$

Angelika Kratzer. 1981. The notional category of modality. In Hans-Jürgen Eikmeyer & Hannes Rieser (eds.), *Words, worlds, and contexts: New approaches in word semantics* (Research in Text Theory 6), 38–74. Berlin: de Gruyter.

Angelika Kratzer. 1991. Modality. In Arnim von Stechow & Dieter Wunderlich (eds.), *Semantics: An international handbook of contemporary research*, 639–650. Berlin: de Gruyter. <https://udrive.oit.umass.edu/kratzer/kratzer-modality.pdf>.

Kratzer has been updating her classic papers for a volume of her collected work on modality and conditionals. These are very much worth studying: <http://semanticsarchive.net/Archive/Tc2NjArM/>.

A major new resource on modality is Paul Portner's book:

Paul Portner. 2009. *Modality*. Oxford University Press.

You might also profit from other survey-ish type papers:

Kai von Fintel. 2005. Modality and language. In Donald M. Borchert (ed.), *Encyclopedia of philosophy – second edition*. MacMillan. <http://mit.edu/fintel/fintel-2005-modality.pdf>.

Kai von Fintel & Anthony S. Gillies. 2007. An opinionated guide to epistemic modality. In Tamar Szabó Gendler & John Hawthorne (eds.), *Oxford studies in epistemology: Volume 2*, 32–62. Oxford University Press. <http://mit.edu/fintel/fintel-gillies-2007-ose2.pdf>.

Eric Swanson. 2008. Modality in language. *Philosophy Compass* 3(6). 1193–1207. <http://dx.doi.org/10.1111/j.1747-9991.2008.00177.x>.

Valentine Hacquard. 2009. Modality. ms, prepared for *Semantics: An international handbook of meaning*, edited by Klaus von Heusinger, Claudia Maienbon, and Paul Portner. http://ling.umd.edu/~hacquard/papers/HoS_Modality_Hacquard.pdf.

On the syntax of modals, there are only a few papers of uneven quality. Some of the more recent work is listed here. Follow up on older references from the bibliographies in these papers.

Rajesh Bhatt. 1997. Obligation and possession. In Heidi Harley (ed.), *Papers from the upenn/mit roundtable on argument structure and aspect*, vol. 32 (MIT Working Papers in Linguistics), 21–40. <http://people.umass.edu/bhatt/papers/bhatt-haveto.pdf>.

Susi Wurmbrand. 1999. Modal verbs must be raising verbs. *West Coast Conference on Formal Linguistics (WCCFL)* 18. 599–612. <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.35.7442&rep=rep1&type=pdf>.

Annabel Cormack & Neil Smith. 2002. Modals and negation in English. In Sjef Barbiers, Frits Beukema & Wim van der Wurff (eds.), *Modality and its interaction with the verbal system*, 133–163. Benjamins.

Jonny Butler. 2003. A minimalist treatment of modality. *Lingua* 113(10). 967–996. [http://dx.doi.org/10.1016/S0024-3841\(02\)00146-8](http://dx.doi.org/10.1016/S0024-3841(02)00146-8).

The following paper explores some issues in the LF-syntax of epistemic modals:

Kai von Stechow & Sabine Iatridou. 2003. Epistemic containment. *Linguistic Inquiry* 34(2). 173–198. <http://dx.doi.org/10.1162/00243890321663370>.

Valentine Hacquard's MIT dissertation is a rich source of cross-linguistic issues in modality, as is Fabrice Nauze's Amsterdam dissertation:

Valentine Hacquard. 2006. *Aspects of modality*. Massachusetts Institute of Technology PhD thesis. http://people.umass.edu/hacquard/hacquard_thesis.pdf.

Fabrice Nauze. 2008. *Modality in typological perspective*. Universiteit van Amsterdam PhD thesis. <http://www.illc.uva.nl/Publications/Dissertations/DS-2008-08.text.pdf>.

The semantics of epistemic modals has become a hot topic recently. Here are some of the main references:

Ian Hacking. 1967. Possibility. *The Philosophical Review* 76(2). 143–168. <http://dx.doi.org/10.2307/2183640>. <http://www.jstor.org/stable/2183640>.

Paul Teller. 1972. Epistemic possibility. *Philosophia* 2(4). 302–320. <http://dx.doi.org/10.1007/BF02381591>.

Keith DeRose. 1991. Epistemic possibilities. *The Philosophical Review* 100(4). 581–605. <http://dx.doi.org/10.2307/2185175>.

Andy Egan, John Hawthorne & Brian Weatherson. 2005. Epistemic modals in context. In Gerhard Preyer & Georg Peter (eds.), *Contextualism in philosophy: Knowledge, meaning, and truth*, 131–170. Oxford: Oxford University Press.

Andy Egan. 2007. Epistemic modals, relativism, and assertion. *Philosophical Studies* 133(1). 1–22. <http://dx.doi.org/10.1007/s11098-006-9003-x>.

John MacFarlane. 2006. Epistemic modals are assessment-sensitive. ms, University of California, Berkeley, forthcoming in an OUP volume on epistemic modals, edited by Brian Weatherson and Andy Egan. <http://sophos.berkeley.edu/macfarlane/epistmod.pdf>.

Tamina Stephenson. 2007a. Judge dependence, epistemic modals, and predicates of personal taste. *Linguistics and Philosophy* 30(4). 487–525. <http://dx.doi.org/10.1007/s10988-008-9023-4>.

John Hawthorne. 2007. Eavesdroppers and epistemic modals. ms, Rutgers University, to appear in the proceedings of the 2007 Sofia Conference in Mexico, in a supplement to *Noûs*.

Kai von Fintel & Anthony S. Gillies. 2008a. CIA leaks. *The Philosophical Review* 117(1). 77–98. <http://dx.doi.org/10.1215/00318108-2007-025>.

Kai von Fintel & Anthony S. Gillies. 2008b. *Might* made right. To appear in a volume on epistemic modality, edited by Andy Egan and Brian Weatherson, Oxford University Press. <http://mit.edu/fintel/fintel-gillies-2008-mm.pdf>.

A recent SALT paper by Pranav Anand and Valentine Hacquard tackles what happens to epistemic modals under attitude predicates:

Pranav Anand & Valentine Hacquard. 2008. Epistemics with attitude. *Semantics and Linguistic Theory (SALT)* 18. <http://dx.doi.org/1813/13025>.

Evidentiality is a topic closely related to epistemic modality. Some references:

Thomas Willett. 1988. A cross-linguistic survey of the grammaticalization of evidentiality. *Studies in Language* 12(1). 51–97.

Alexandra Y. Aikhenvald. 2004. *Evidentiality*. Oxford: Oxford University Press.

Hans Bernhard Drubig. 2001. On the syntactic form of epistemic modality. ms, Universität Tübingen. <http://www.sfb441.uni-tuebingen.de/b2/papers/DrubigModality.pdf>.

Eleanor M. Blain & Rose-Marie Déchaine. 2007. Evidential types: Evidence from Cree dialects. *International Journal of American Linguistics* 73(3). 257–291. <http://dx.doi.org/10.1086/521728>.

Eric McCready & Norry Ogata. 2007. Evidentiality, modality and probability. *Linguistics and Philosophy* 30(2). 147–206. <http://dx.doi.org/10.1007/s10988-007-9017-7>.

Peggy Speas. 2008. On the syntax and semantics of evidentials. *Language and Linguistics Compass* 2(5). 940–965. <http://dx.doi.org/10.1111/j.1749-818X.2008.00069.x>.

Kai von Fintel & Anthony S. Gillies. 2010. *Must* ... stay ... strong! *Natural Language Semantics* 18(4). 351–383. <http://dx.doi.org/10.1007/s11050-010-9058-2>.

Modals interact with disjunction and indefinites to generate so-called FREE CHOICE-readings, which are a perennial puzzle. Here is just a very small set of initial references:

Hans Kamp. 1973. Free choice permission. *Proceedings of the Aristotelian Society, New Series* 74. 57–74. <http://www.jstor.org/stable/4544849>.

- Thomas Ede Zimmermann. 2000. Free choice disjunction and epistemic possibility. *Natural Language Semantics* 8(4). 255–290. <http://dx.doi.org/10.1023/A:1011255819284>.
- Katrin Schulz. 2005. A pragmatic solution for the paradox of free choice permission. *Synthese* 147(2). 343–377. <http://dx.doi.org/10.1007/s11229-005-1353-y>.
- Maria Aloni. 2007. Free choice, modals, and imperatives. *Natural Language Semantics* 15(1). 65–94. <http://dx.doi.org/10.1007/s11050-007-9010-2>.
- Luis Alonso-Ovalle. 2006. *Disjunction in alternative semantics*. University of Massachusetts at Amherst PhD thesis. http://alonso-ovalle.net/index.php?page_id=28.
- Danny Fox. 2007. Free choice and the theory of scalar implicatures. In Uli Sauerland & Penka Stateva (eds.), *Presupposition and implicature in compositional semantics*, 537–586. New York: Palgrave Macmillan. http://web.mit.edu/linguistics/people/faculty/fox/free_choice.pdf.
- Robert van Rooij. 2006. Free choice counterfactual donkeys. *Journal of Semantics* 23(4). 383–402. <http://dx.doi.org/10.1093/jos/ffl004>.

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CHAPTER FOUR

CONDITIONALS

We integrate conditionals into the semantics of modal expressions that we are developing. We show that the material implication analysis and the strict implication analysis are inferior to the restrictor analysis. Our discussion will remain focussed on some simple questions and we refer you to the rich literature on conditionals for further topics.

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4.1 The Material Implication Analysis

Consider the following example:

- (1) If I am healthy, I will come to class.

The simplest analysis of such conditional constructions is the so-called MATERIAL IMPLICATION analysis,¹ which treats *if* as contributing a truth-function operating on the truth-values of the two component sentences (which are called the ANTECEDENT and CONSEQUENT — from Latin — or PROTASIS and APODOSIS — from Greek). The lexical entry for *if* would look as follows:

- (2) $\llbracket \text{if} \rrbracket = \lambda u \in D_t. \lambda v \in D_t. u = 0 \text{ or } v = 1.$

¹ Quoth the Stoic philosopher Philo of Megara: “a true conditional is one which does not have a true antecedent and a false consequent” (according to [Empiricus \(c. 200, pp. II, 110–112\)](#)).

Note that as a truth-functional connective, this *if* does not vary its denotation depending on the evaluation world. It's its arguments that vary with the evaluation world.

Applied to example in (1), this semantics would predict that the example is false just in case the antecedent is true, I am healthy, but the consequent false, I do not come to class. Otherwise, the sentence is true. We will see that there is much to complain about here. But one should realize that under the assumption that *if* denotes a truth-function, *this one* is the most plausible candidate.

Suber (1997) does a good job of persuading (or at least trying to persuade) recalcitrant logic students:

After saying all this, it is important to note that material implication does conform to some of our ordinary intuitions about implication. For example, take the conditional statement, *If I am healthy, I will come to class*. We can symbolize it: $H \supset C$.²

The question is: when is this statement false? When will I have broken my promise? There are only four possibilities:

H	C	$H \supset C$
T	T	?
T	F	?
F	T	?
F	F	?

- In case #1, I am healthy and I come to class. I have clearly kept my promise; the conditional is true.
- In case #2, I am healthy, but I have decided to stay home and read magazines. I have broken my promise; the conditional is false.
- In case #3, I am not healthy, but I have come to class anyway. I am sneezing all over you, and you're not happy about it, but I did not violate my promise; the conditional is true.
- In case #4, I am not healthy, and I did not come to class. I did not violate my promise; the conditional is true.

But this is exactly the outcome required by the material implication. The compound is only false when the antecedent is true and the consequence is false (case #2); it is true every other time.

Despite the initial plausibility of the analysis, it cannot be maintained. Consider this example:

2 The symbol \supset which Suber uses here is called the “horseshoe”. We have been using the right arrow \rightarrow as the symbol for implication. We think that this is much preferable to the confusing horseshoe symbol. There is an intimate connection between universal quantification, material implication, and the subset relation, usually symbolized as \subset , which is the other way round from the horseshoe. The horseshoe can be traced back to the notation introduced by Peano (1889), a capital C standing for ‘consequenza’ facing backwards. The C facing in the other (more “logical”) direction was actually introduced first by Gergonne (1817), but didn't catch on.

- (3) If there is a major earthquake in Cambridge tomorrow, my house will collapse.

If we adopt the material implication analysis, we predict that (3) will be false just in case there is indeed a major earthquake in Cambridge tomorrow but my house fails to collapse. This makes a direct prediction about when the negation of (3) should be true. A false prediction, if ever there was one:

- (4) a. It's not true that if there is a major earthquake in Cambridge tomorrow, my house will collapse.
b. \neq There will be a major earthquake in Cambridge tomorrow, and my house will fail to collapse.

Clearly, one might think that (4a) is true without at all being committed to what the material implication analysis predicts to be the equivalent statement in (4b). This is one of the inadequacies of the material implication analysis.

These inadequacies are sometimes referred to as the “paradoxes of material implication”. But that is misleading. As far as logic is concerned, there is nothing wrong with the truth-function of material implication. It is well-behaved and quite useful in logical systems. What is arguable is that it is not to be used as a reconstruction of what conditionals mean in natural language.

EXERCISE 4.1: Under the assumption that *if* has the meaning in (2), calculate the truth-conditions predicted for (5):

- (5) a. No student will succeed if he goofs off.
b. No student λx (if x goofs off, x will succeed)

State the predicted truth-conditions in words and evaluate whether they correspond to the actual meaning of (5). □

A problem that is not often raised for the material implication analysis is how badly it interacts with the analysis of modal expressions, once we look at sentences involving both a conditional clause and a modal. Consider:

- (6) If we are on Route 183, we might be in Lockhart now.
(7) If you keep this fern dry, it cannot grow.

We need to consider two possible LFs for these sentences, depending on whether wider scope is given to the modal or to the conditional clause. For example, in the margin you see LFs A and B for (6).

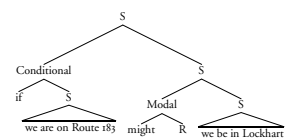


Figure 4.1: LF A for (6)

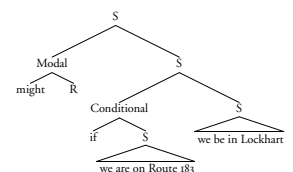


Figure 4.2: LF B for (6)

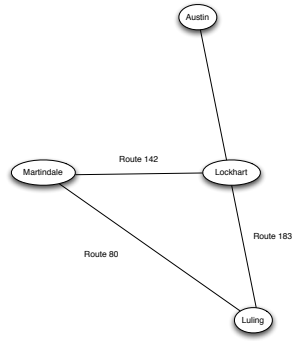


Figure 4.3: A schematic map of the relevant area in Texas

The reading for (6) we have in mind is an epistemic one; imagine for instance that (6) is uttered in a car by Mary to Susan, while Susan is driving and Mary is looking at a map. The information provided by the map, together with other background knowledge, constitutes the relevant context for the modal *might* here. The accessibility relation is roughly this:

- (8) $\lambda w. \lambda w'. w'$ is compatible with what the map says in w and what Mary knows about the geography of the relevant area in w .

Let's suppose (6) is uttered in the actual world w_0 and we are interested in its truth-value at this world. We now proceed to show that neither of the LFs A and B represent the intuitively natural meaning of (6) if we assume the material implication analysis of *if*.

Consider first LF A. There are two respects in which the predicted truth-conditions for this LF deviate from intuitive judgment. First, suppose that Susan and Mary are not on Route 183 in w_0 . Then (6) is predicted to be true in w_0 , regardless of the geographical facts, e.g. even if Lockhart is nowhere near Route 183. This is counterintuitive. Imagine the following quite sensible dialogue:

- (9) Mary: If we are on Route 183, we might be in Lockhart now.
 Susan (stops the car and looks at the map): You are wrong. Look here, Route 183 doesn't run anywhere near Lockhart.

If Mary concedes Susan's claim that Route 183 doesn't go through Lockhart, she has to also concede that her original assertion was false. It wouldn't do for her to respond: "I know that 183 runs about 10 miles east of Lockhart, but maybe we are not on Route 183, so I may still be right." Yet we predict that this should be a reasonable way for her to defend (6).

A second inadequacy is this: we predict that the truth of the consequent of (6) is a sufficient condition for the truth of (6) as a whole. If this were right, it would take very little for (6) to be true. As long as the map and the rest of Mary's knowledge in w_0 don't rule out the possibility that they are in Lockhart, *we might be in Lockhart* will be true in w_0 — regardless, once again, of whether Lockhart is anywhere near 183. It should therefore be reasonable for Mary to continue the dialogue in (9) with the rejoinder: "But how can you be so sure we are not in Lockhart?" According to intuitive judgment, however, this would not be a

pertinent remark and certainly would not help Mary defend (6) against Susan's objection.

Now let's look at LF B, where the modal has widest scope. Given the material implication analysis of *if*, this is predicted to mean, in effect: "It might be the case that we are either in Lockhart or not on Route 183". This truth-condition is also far too easy to satisfy: All it takes is that the map and the rest of Mary's knowledge in w_0 are compatible with Mary and Susan not being on Route 183, or that they are compatible with their being in Lockhart. So as long as it isn't certain that they are on Route 183, Mary should be justified in asserting (6), regardless, once again, of her information about the relative location of Lockhart and Route 183.

EXERCISE 4.2: Show that similar difficulties arise for the analysis of (7). \square

4.2 The Strict Implication Analysis

Some of the problems we encountered would go away if we treated *if* as introducing a modal meaning. The simplest way to do that would be to treat it as a universal quantifier over possible worlds. *If* p , q would simply mean that the set of p -worlds is a subset of the q -worlds. This kind of analysis is usually called STRICT IMPLICATION. The difference between *if* and *must* would be that *if* takes an overt restrictive argument. Here is what the lexical entry for *if* might look like:

$$(IO) \quad \llbracket \text{if} \rrbracket^{w,g} = \lambda p \in D_{\langle s,t \rangle}. \lambda q \in D_{\langle s,t \rangle}. \forall w': p(w') = 1 \rightarrow q(w') = 1.$$

(in set talk: $p \subseteq q$)

Applied to (3), we would derive the truth-conditions that (3) is true iff all of the worlds where there is a major earthquake in Cambridge tomorrow are worlds where my house collapses.

We immediately note that this analysis has the same problem of non-contingency that we faced with one of our early attempts at a quantificational semantics for modals like *must* and *may*. The obvious way to fix this here is to assume that *if* takes a covert accessibility function as one of its arguments. The antecedent clause then serves as an additional restrictive device. Here is the proposal:

$$(II) \quad \llbracket \text{if} \rrbracket^{w,g} = \lambda R \in D_{\langle s, \langle s,t \rangle \rangle}. \lambda p \in D_{\langle s,t \rangle}. \lambda q \in D_{\langle s,t \rangle}. \\ \forall w': (R(w)(w') = 1 \ \& \ p(w') = 1) \rightarrow q(w') = 1.$$

(in set talk: $R(w) \cap p \subseteq q$)

If we understand (3) as involving an epistemic accessibility relation, it would claim that among the worlds epistemically accessible from the actual world (i.e. the worlds compatible with what we know), those where there is a major earthquake in Cambridge tomorrow are worlds where my house collapses. This would appear to be quite adequate — although potentially traumatic to me.

EXERCISE 4.3: Can you come up with examples where a conditional is interpreted relative to a non-epistemic accessibility relation? □

EXERCISE 4.4: What prediction does the strict implication analysis make about the negated conditional in (4a)? □

What happens when we let this analysis loose on (6)? We again need to assess two LF's depending on the relative scope of *if* and *might*. Both LF's would have two covert variables over accessibility relations, one for *if* and one for *might*. Before we can assess the adequacy of the two candidate analyses, we need to decide what the contextually salient values for the accessibility relations might be. One would think that the epistemic accessibility relation that we have already encountered is the most likely value, and in fact for both variables.

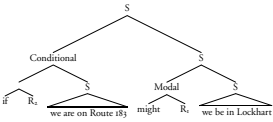


Figure 4.4: LF A' for (6)

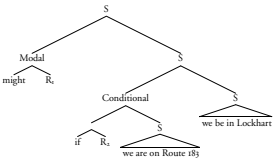


Figure 4.5: LF B' for (6)

Next, we need to consider the particular epistemic state that Mary is in. By assumption, Mary does not know where they are. Nothing in her visual environment helps her figure out where they are. She does see from the map that if they are on Route 183, one of the towns they might be in is Lockhart. But she doesn't know whether they are on Route 183. Even if they *are* on 183, she doesn't know that they are and her epistemic state would still be what it is: one of being lost.

Consider then LF A', with the modal in the scope of the conditional. Here, we derive the claim that all worlds w' compatible with what Mary knows in w and where they are on 183 are such that some world w'' compatible with what Mary knows in w' is such that they are in Lockhart. Is that adequate? Not really. We have just convinced ourselves that whether they are on 183 or not has no relevant influence on Mary's epistemic state, since she wouldn't know it either way. But that means that our analysis would predict that (6) is true as long as it is possible as far as Mary knows that they are in Lockhart. Whether they are on 183 or not doesn't change that. So, we would expect (6) to not be distinct in truth-value from something like:

(12) If we are on the Route 80, we might be in Lockhart.

But that is not right — Mary knows quite well that if they are on the Route 80, they cannot be in Lockhart.

Turning to LF B', with the modal having widest scope, doesn't help us either. Here, we would derive the claim that it is compatible with what Mary knows that from being on 183 it follows (according to what she knows) that they are in Lockhart. Clearly, that is not what (6) means. Mary doesn't consider it possible that if they are on 183, she knows that they are in Lockhart. After all, she's well aware that she doesn't know where they are.

4.3 *If*-Clauses as Restrictors

The problem we have encountered here with the interaction of an *if*-clause and the modal operator *might* is similar to others that have been noted in the literature. Most influentially, David Lewis in his paper "Adverbs of Quantification" showed how hard it is to find an adequate analysis of the interaction of *if*-clauses and ADVERBS OF QUANTIFICATION like *never*, *rarely*, *sometimes*, *often*, *usually*, *always*. Lewis proposed that in the cases he was considering, the adverb is the only operator at work and that the *if*-clause serves to restrict the adverb. Thus, it has much the same function that a common noun phrase has in a determiner-quantification.

The *if* of our restrictive *if*-clauses should not be regarded as a sentential connective. It has no meaning apart from the adverb it restricts. The *if* in *always if*..., ..., *sometimes if*..., ..., and the rest is on a par with the non-connective *and* in *between ... and* ..., with the non-connective *or* in *whether ... or* ..., or with the non-connective *if* in *the probability that ... if*.... It serves merely to mark an argument-place in a polyadic construction. D. Lewis 1975, p. 11

Building on Lewis' insight, Kratzer argued for a uniform treatment of *if*-clauses as restrictors. She claimed that

the history of the conditional is the story of a syntactic mistake. There is no two-place *if*... *then* connective in the logical forms of natural languages. *If*-clauses are devices for restricting the domains of various operators. (Kratzer 1986)

Let us repeat this:

(13) KRATZER'S THESIS

If-clauses are devices for restricting the domains of various operators.

Kratzer's Thesis gives a unified picture of the semantics of conditional clauses. Note that it is not meant to supplant previous accounts of the meaning of

conditionals. It just says that what those accounts are analyzing is not the meaning of *if* itself but the meaning of the operators that *if*-clauses restrict.

Let us see how this idea helps us with our Lockhart-sentence. The idea is to deny that there are two quantifiers over worlds in (6). Instead, the *if*-clause merely contributes a further restriction to the modal *might*. In effect, the modal is not quantifying over *all* the worlds compatible with Mary's knowledge but only over those where they are on Route 183. It then claims that at least some of those worlds are worlds where they are in Lockhart. We cannot anymore derive the problematic conclusion that it should also be true that if they are on the Route 80, they might be in Lockhart. In all, we have a good analysis of what (6) means.

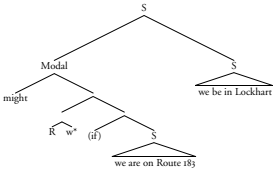


Figure 4.6: LF C for
(6)

What we don't yet have is a compositional calculation. What does it mean in structural terms for the *if*-clause to be restricting the domain of the modal? We will assume a structure as in LF C. Here, the *if*-clause is the sister to what used to be the covert set-of-worlds argument of the modal. As you can see, we have chosen the variant of the semantics for modals that was discussed in Section 3.2.5. The idea now is that the two restrictive devices work together: we just feed to the modal the *intersection* of (i) the set of worlds that are R-accessible from the actual world, and (ii) the set of worlds where they are on Route 183.

EXERCISE 4.5: To make the composition work, we need to be able to intersect the set of accessible worlds with the antecedent proposition. This could be done in two ways: (i) a new composition principle, which would be a slight modification of the PREDICATE MODIFICATION rule, (ii) give *if* a functional meaning that accomplishes the intersection. Formulate such a meaning for *if*.

Alternatively, we could do without the w^* device and instead give *if* a meaning that takes a proposition p and then modifies an accessibility relation to give a new accessibility relation, which is restricted to p -worlds. Formulate such a meaning for *if*. \square

What about cases like (3), now? Here there is no modal operator for the *if*-clause to restrict. Should we revert to treating *if* as an operator on its own? Kratzer proposes that we should not and that such cases simply involve covert modal operators.

Supplementary Readings

A short handbook article on conditionals:

Kai von Fintel. 2009. Conditionals. ms, prepared for *Semantics: An international handbook of meaning*, edited by Klaus von Heusinger, Claudia Maienborn, and Paul Portner. <http://mit.edu/fintel/fintel-2009-hsk-conditionals.pdf>.

Overviews of the philosophical work on conditionals:

Dorothy Edgington. 1995. On conditionals. *Mind* 104(414). 235–329. [10.1093/mind/104.414.235](https://doi.org/10.1093/mind/104.414.235).

Jonathan Bennett. 2003. *A philosophical guide to conditionals*. Oxford University Press.

A handbook article on the logic of conditionals:

Donald Nute. 1984. Conditional logic. In Dov Gabbay & Franz Guenther (eds.), *Handbook of philosophical logic. volume ii*, 387–439. Dordrecht: Reidel.

Three indispensable classics:

David Lewis. 1973. *Counterfactuals*. Oxford: Blackwell.

Robert Stalnaker. 1968. A theory of conditionals. In Nicholas Rescher (ed.), *Studies in logical theory* (American Philosophical Quarterly Monograph Series 2), 98–112. Oxford: Blackwell.

Robert Stalnaker. 1975. Indicative conditionals. *Philosophia* 5(3). 269–286. <http://dx.doi.org/10.1007/BF02379021>.

The Restrictor Analysis:

David Lewis. 1975. Adverbs of quantification. In Edward Keenan (ed.), *Formal semantics of natural language*, 3–15. Cambridge University Press.

Angelika Kratzer. 1986. Conditionals. *Chicago Linguistics Society* 22(2). 1–15. <https://udrive.oit.umass.edu/kratzer/kratzer-conditionals.pdf>.

The application of the restrictor analysis to the interaction of nominal quantifiers and conditionals:

Kai von Fintel. 1998. Quantifiers and ‘if’-clauses. *The Philosophical Quarterly* 48(191). 209–214. <http://dx.doi.org/10.1111/1467-9213.00095>. <http://mit.edu/fintel/www/qandif.pdf>.

Kai von Fintel & Sabine Iatridou. 2002. If and when *If*-clauses can restrict quantifiers. ms, MIT. <http://mit.edu/fintel/fintel-iatridou-2002-ifwhen.pdf>.

James Higginbotham. 2003. Conditionals and compositionality. *Philosophical Perspectives* 17(1). 181–194. <http://dx.doi.org/10.1111/j.1520-8583.2003.00008.x>.

Sarah-Jane Leslie. 2009. *If, unless, and quantification*. In Robert J. Stainton & Christopher Viger (eds.), *Compositionality, context and semantic values: Essays in honour of Ernie Lepore*, 3–30. Springer. http://dx.doi.org/10.1007/978-1-4020-8310-5_1.

Janneke Huitink. 2009b. Quantified conditionals and compositionality. ms, to appear in *Language and Linguistics Compass*. <http://user.uni-frankfurt.de/~huitink/compass-conditionals-final.pdf>.

Syntax of conditionals:

Kai von Fintel. 1994. *Restrictions on quantifier domains*. Amherst, MA: University of Massachusetts PhD thesis. <http://semanticsarchive.net/Archive/jA3N2IwN/fintel-1994-thesis.pdf>, Chapter 3: “Conditional Restrictors”

Sabine Iatridou. 1993. On the contribution of conditional *Then*. *Natural Language Semantics* 2(3). 171–199. <http://dx.doi.org/10.1007/BF01256742>.

Rajesh Bhatt & Roumyana Pancheva. 2006. Conditionals. In *The Blackwell companion to syntax*, vol. 1, 638–687. Blackwell. http://www-rcf.usc.edu/~pancheva/bhatt-pancheva_syncom.pdf.

A shifty alternative to the restrictor analysis:

Anthony S. Gillies. 2009. On truth-conditions for *if* (but not quite only *if*). *The Philosophical Review* 118(3). 325–349. <http://dx.doi.org/10.1215/00318108-2009-002>.

Anthony S. Gillies. 2010. Iffiness. *Semantics and Pragmatics* 3(4). 1–42. <http://dx.doi.org/10.3765/sp.3.4>.

The Belnap alternative:

Jr. Belnap Nuel D. 1970. Conditional assertion and restricted quantification. *Noûs* 4(1). 1–12. <http://dx.doi.org/10.2307/2214285>.

Jr. Belnap Nuel D. 1973. Restricted quantification and conditional assertion. In Hugues Leblanc (ed.), *Truth, syntax and modality: Proceedings of the Temple University conference on alternative semantics*, vol. 68 (Studies in Logic and the Foundations of Mathematics), 48–75. Amsterdam: North-Holland.

Kai von Fintel. 2007. *If*: The biggest little word. Slides from a plenary address given at the Georgetown University Roundtable, March 8, 2007. <http://mit.edu/fintel/gurt-slides.pdf>.

Janneke Huitink. 2008. *Modals, conditionals and compositionality*. Radboud Universiteit Nijmegen PhD thesis. <http://user.uni-frankfurt.de/~huitink/Huitink-dissertation.pdf>, Chapters 1 and 2 give a nice summary of what we’re covering in this class, while Chapter 5 is about the Belnap-method.

Janneke Huitink. 2009a. Domain restriction by conditional connectives. ms, Goethe-University Frankfurt. <http://semanticsarchive.net/Archive/zg2MDM4M/Huitink-domainrestriction.pdf>.

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CHAPTER FIVE

ORDERING

We have stressed throughout the previous two chapters that there are numerous parallels between quantification over ordinary individuals via determiner quantifiers and quantification over possible worlds via modal operators (including conditionals). Now, we turn to a phenomenon that (at least at first glance) appears to show that there are non-parallels as well: a sensitivity to an ORDERING of the elements in the domain of quantification. We first look at this in the context of simple modal sentences and then we look at conditionals.

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5.1 The Driveway

Consider a typical use of a sentence like (1).

- (1) John must pay a fine.

This is naturally understood in such a way that its truth depends both on facts about the law and facts about what John has done. For instance, it will be judged true if (i) the law states that driveway obstructors are fined, and (ii) John has obstructed a driveway. It may be false either because the law is different or because John's behavior was different.

What accessibility relation provides the implicit restriction of the quantifier *must* on this reading of (1)? A naïve attempt might go like this:

- (2) $\lambda w. \lambda w'. [\text{what happened in } w' \text{ up to now is the same as what happened in } w \text{ and } w' \text{ conforms to what the law in } w \text{ demands}]$.

The problem with (2) is that, unless there were no infractions of the law at all in w up to now, no world w' will be accessible from w . Therefore, (1) is predicted to follow logically from the premise that John broke some law. This does not represent our intuition about its truth conditions.

A better definition of the appropriate accessibility relation has to be more complicated:

- (3) $\lambda w. \lambda w'. [\text{what happened in } w' \text{ up to now is the same as what happened in } w \text{ and } w' \text{ conforms at least as well to what the law in } w \text{ demands as does any other world in which what happened up to now is the same as in } w]$.

(3) makes explicit that there is an important difference between the ways in which facts about John's behavior on the one hand, and facts about the law on the other, enter into the truth conditions of sentences like (1). Worlds in which John didn't do what he did are simply excluded from the domain of *must* here. Worlds in which the law isn't obeyed are not absolutely excluded. Rather, we restrict the domain to those worlds in which the law is obeyed as well as it can be, considering what has happened. We exclude only those worlds in which there are infractions above and beyond those that are shared by all the worlds in which John has done what he has done. The analysis of (1) thus crucially involves the notion of an ordering of worlds: here they are ordered according to how well they conform to what the law in w demands.

5.2 Kratzer's Solution: Doubly Relative Modality

Kratzer proposes that modal operators are sensitive to *two* context-dependent parameters: a set of accessible worlds (provided by an accessibility function computed from a conversational background, the MODAL BASE), and a partial ordering of the accessible worlds (computed from another conversational background, called the ORDERING SOURCE).

Let's see how the analysis applies to the previous example.

- The modal base will be a function that assigns to any evaluation world a set of propositions describing the relevant circumstances, for example, what John did. Since in our stipulated evaluation world John obstructed a driveway, the modal base will assign the proposition that John obstructed

a driveway to this world. The set of worlds accessible from the evaluation world will thus only contain worlds where John obstructed a driveway.

- The ordering source will be a function that assigns to any evaluation world a set of propositions \mathcal{P} whose truth is demanded by the law. Imagine that for our evaluation world this set of propositions contains (among others) the following two propositions: (i) nobody obstructs any driveways, (ii) anybody who obstructs a driveway pays a fine.
- The idea is now that such a set \mathcal{P} of propositions can be used to order the worlds in the modal base. For any pair of worlds w_1 and w_2 , we say that w_1 comes closer than w_2 to the ideal set up by \mathcal{P} (in symbols: $w_1 <_{\mathcal{P}} w_2$), iff the set of propositions from \mathcal{P} that are true in w_2 is a proper subset of the set of propositions from \mathcal{P} that are true in w_1 .
- For our simple example then, any world in modal base where John pays a fine will count as better than an otherwise similar world where he doesn't.
- Modals then make quantificational claims about the best worlds in the modal base (those for which there isn't a world that is better than them).
- In our case, (I) claims that in the best worlds (among those where John obstructed a driveway), he pays a fine.

More technically:

- (4) Given a set of worlds X and a set of propositions \mathcal{P} , define the STRICT PARTIAL ORDER $<_{\mathcal{P}}$ as follows:
 $\forall w_1, w_2 \in X: w_1 <_{\mathcal{P}} w_2$ iff $\{p \in \mathcal{P} : p(w_2) = 1\} \subset \{p \in \mathcal{P} : p(w_1) = 1\}$.
- (5) For a given strict partial order $<_{\mathcal{P}}$ on worlds, define the selection function $\max_{\mathcal{P}}$ that selects the set of $<_{\mathcal{P}}$ -best worlds from any set X of worlds:
 $\forall X \subseteq W: \max_{\mathcal{P}}(X) = \{w \in X: \neg \exists w' \in X: w' <_{\mathcal{P}} w\}$.
- (6) $\llbracket \text{must} \rrbracket^{w,g} = \lambda f_{\langle s, st \rangle} \cdot \lambda g_{\langle s, st \rangle} \cdot \lambda q_{\langle s, t \rangle} \cdot$
 $\forall w' \in \max_{g(w)}(\cap f(w)): q(w') = 1$.

TECHNICAL NOTE: This only works if we can in general assume that the $<_{\mathcal{P}}$ relation has minimal elements, that there always are accessible worlds that come closest to the \mathcal{P} -ideal, worlds that are better than any world they can be compared with via $<_{\mathcal{P}}$. It is possible, with some imagination, to cook up scenarios where this assumption fails. This problem has been discussed primarily in the area of the semantics of conditionals. There, Lewis presents relevant scenarios and argues that one shouldn't make this assumption, which he calls the Limit Assumption. Stalnaker, on the one other hand, defends the assumption against Lewis' arguments by saying that in actual practice, in actual natural language semantics and in actual modal/conditional reasoning, the assumption is eminently reasonable. Kratzer is persuaded by Lewis' evidence and does not make the Limit

Assumption; hence her semantics for modals is more convoluted than what we have in (5) and (6). I will side with Stalnaker, not the least because it makes life easier. For further discussion, see [D. Lewis \(1973, pp. 19–21\)](#) and [Stalnaker \(1984, Chapter 7, esp. pp. 140–142\)](#); [Pollock \(1976\)](#), [Herzberger \(1979\)](#), and [Warmbrod \(1982\)](#) argue for the Limit Assumption as well.

EXERCISE 5.1: In her handbook article [Kratzer 1991](#), Kratzer presents a number of examples of modal statements and sketches an analyses in terms of doubly relative modality. You should study her examples carefully. □

5.3 The Paradox of the Good Samaritan

[A. N. Prior \(1958\)](#) introduced the following “Paradox of the Good Samaritan”. Imagine that someone has been robbed and John is walking by. It is easy to conceive of a code of ethics that would make the following sentence true:

- (7) John ought to help the person who was robbed.

In our previous one-factor semantics for modals, we would have said that (7) says that in all of the deontically accessible worlds (those compatible with the code of ethics) John helps the person who was robbed. Prior’s point was that under such a semantics, something rather unfortunate holds. Notice that in all of the worlds where John helps the person who was robbed, someone was robbed in the first place. Therefore, it will be true that in all of the deontically accessible worlds, someone was robbed. Thus, (7) will entail:

- (8) It ought to be the case that someone was robbed.

It clearly would be good not make such a prediction.

The doubly-relative analysis of modality can successfully avoid this unfortunate prediction. We conceive of (7) as being uttered with respect to a circumstantial modal base that includes the fact that someone was robbed. Among those already somewhat ethically deficient worlds, the relatively best ones are all worlds where John helps the victim.

Note that we still have the problematic fact that among the worlds in the modal base, all are worlds where someone was robbed, and we would thus appear to still make the unfortunate prediction that (8) should be true. But this can now be fixed. For example, we could say that *ought* p is semantically defective if p is true throughout the worlds in the modal base. This could be a presupposition or some other ingredient of meaning. So, with respect to a modal base which pre-determines that someone was robbed, one couldn’t felicitously say (8).

Consequently, saying (8) would only be felicitous if a different modal base is intended, one that contains both *p* and non-*p* worlds. And given a choice between worlds where someone was robbed and worlds where nobody was robbed, most deontic ordering sources would probably choose the no-robbery worlds, which would make (8) false, as desired.

KRATZER'S VERSION OF THE SAMARITAN PARADOX [Kratzer \(1991\)](#) argues that the restrictor approach to deontic conditionals is the crucial ingredient in the solution to a conditional version of the Samaritan Paradox:

- (9) If a murder occurs, the jurors must convene.

Kratzer points out that if one tried to analyze (9) as a material implication embedded under deontic necessity, then one quickly runs into a problem. Surely, one wants the following to be a true statement about the law:

- (10) There must be no murder.

But this means that in the deontically accessible worlds, all of them have no murders occurring. Now, this means that in all of the deontically accessible worlds, any material implication of the form “if a murder occurs, *q*” will be true no matter what the consequent is since the antecedent will be false. Since that is an absurd prediction, (9) cannot be analyzed as material implication under deontic necessity. The combination of the restrictor approach to *if*-clauses and the doubly-relative theory of modals can rescue us from this problem. (10) is analyzed as the deontic necessity modal being restricted by the *if*-clause. The set of accessible worlds is narrowed down by the *if*-clause to only include worlds in which a murder occurs. The deontic ordering then identifies the best among those worlds and those are plausibly all worlds where the jurors convene.

5.4 Non-Monotonicity of Conditionals

The last case discussed takes us straight to the crucial role of the ordering of worlds in the semantics of conditionals, as we would of course expect under the analysis of *if*-clauses as restrictors of modal operators. In this arena, the discussion usually revolves around the failure of certain inference patterns, which one would expect a universal quantifier to validate. Here are the most important ones:

- (II) LEFT DOWNWARD MONOTONICITY (“DOWNWARD ENTAILINGNESS”)
Every *A* is a *B*. \rightarrow Every *A* & *C* is a *B*.

- (12) TRANSITIVITY
Every A is a B. Every B is a C. \rightarrow Every A is a C.
- (13) CONTRAPOSITION
Every A is a B. \rightarrow Every non-B is a non-A.

Conditionals were once thought to obey these patterns as well, known in conditional logic as STRENGTHENING THE ANTECEDENT, HYPOTHETICAL SYLLOGISM, and CONTRAPOSITION. But then spectacular counterexamples became known through the work of Stalnaker and Lewis.

- (14) FAILURE OF STRENGTHENING THE ANTECEDENT
- a. If I strike this match, it will light.
If I dip this match into water and strike it, it will light.
 - b. If John stole the earrings, he must go to jail.
If John stole the earrings and then shot himself, he must go to jail.
 - c. If kangaroos had no tails, they would topple over. If kangaroos had no tails but used crutches, they would topple over.
- (15) FAILURE OF THE HYPOTHETICAL SYLLOGISM (TRANSITIVITY)
- a. If Brown wins the election, Smith will retire to private life.
If Smith dies before the election, Brown will win the election.
If Smith dies before the election, Smith will retire to private life.
 - b. If Hoover had been a Communist, he would have been a traitor.
If Hoover had been born in Russia, he would have been a Communist.
If Hoover had been born in Russia, he would have been a traitor.
- (16) FAILURE OF CONTRAPOSITION
- a. If it rained, it didn't rain hard.
If it rained hard, it didn't rain.
 - b. (Even) if Goethe hadn't died in 1832, he would still be dead now.
If Goethe were alive now, he would have died in 1832.

The Goethe example is due to Kratzer.

Note that these cases involve examples of both “indicative” (epistemic) conditionals and counterfactual conditionals. It is sometimes thought that indicative conditionals are immune from these kinds of counterexamples, but it is clear that they are not. Also note that in (14b) we have a case of Failure of Strengthening the Antecedent with a deontic conditional. Deontic counterexamples to the other patterns seem harder to find.

The failure of these inference patterns again indicates that the semantics of modal operators (restricted by *if*-clauses) is more complicated than the simple

universal quantification we had previously been assuming. The basic idea of most approaches to this problem is this: the semantics of conditionals is more complicated than simple universal quantification. The conditional does not make a claim about simply every antecedent world, nor even about every contextually relevant antecedent world. Instead, in each of the conditional statements, only a particular subset of the antecedent worlds is quantified over. Informally, we can call those the “most highly ranked antecedent worlds”. Consider:

- (17) If I had struck this match, it would have lit.
 If I had dipped this match into water and struck it, it would have lit.

According to the Stalnaker-Lewis account, this inference is semantically invalid. The premise merely claims that the most highly ranked worlds in which I strike this match are such that it lights. No claim is made about the most highly ranked worlds in which I first dip this match into water and then strike it. Strengthening the Antecedent will only be safe if it is additionally known that the strengthened antecedent is instantiated among the worlds that verify the original antecedent.

The other fallacies receive similar treatments. Transitivity (Hypothetical Syllogism) fails for the new non-monotonic quantifier because even if all the most highly rated p-worlds are q-worlds and all the most highly rated q-worlds are r-worlds, we are not necessarily speaking about the same q-worlds (the q-worlds that p takes us to may be rather remote ones). So in the Hoover-example, we get the following picture: The most highly ranked p-worlds in which Hoover was born in Russia (but where he retains his level of civic involvement), are all q-worlds in which he becomes a Communist. On the other hand, the most highly ranked q-worlds in which he is a Communist (but retaining his having been born in the United States and being a high level administrator) are all r-worlds in which he is a traitor. However, the most highly ranked p-worlds do not get us to the most highly ranked q-worlds, so the Transitive inference does not go through.

Contraposition fails because the fact that the most highly rated p-worlds are q-worlds does not preclude a situation where the most highly rated non q-worlds are also p-worlds. The most highly rated p-worlds in which Goethe didn't die in 1832 are all q-worlds where he dies nevertheless (well) before the present. But of course, the most highly rated (in fact, all) non-q-worlds (where he is alive today) are also p-worlds where he didn't die in 1832.

In the conditionals literature, the ordering of worlds is usually given directly as an evaluation parameter. The typical gloss is that the ordering ranks possible worlds based on how *similar* they are to the evaluation world. Kratzer developed an alternative where the ordering is computed from a set of propositions true

in the evaluation world. D. Lewis (1981) showed that ordering semantics and premise semantics are largely notational variants.

Supplementary Readings

The central readings for this chapter are two papers by Kratzer:

Angelika Kratzer. 1991. Modality. In Arnim von Stechow & Dieter Wunderlich (eds.), *Semantics: An international handbook of contemporary research*, 639–650. Berlin: de Gruyter. <https://udrive.oit.umass.edu/kratzer/kratzer-modality.pdf>.

Angelika Kratzer. 1981. The notional category of modality. In Hans-Jürgen Eikmeyer & Hannes Rieser (eds.), *Words, worlds, and contexts: New approaches in word semantics* (Research in Text Theory 6), 38–74. Berlin: de Gruyter.

Some work that discusses and uses Kratzer's two factor semantics for modals:

Anette Frank. 1996. *Context dependence in modal constructions*. Universität Stuttgart PhD thesis. <http://www.cl.uni-heidelberg.de/~frank/papers/header.pdf>.

Kai von Fintel & Sabine Iatridou. 2005. What to do if you want to go to Harlem: Anankastic conditionals and related matters. ms, MIT. <http://mit.edu/fintel/fintel-iatridou-2005-harlem.pdf>.

Kai von Fintel & Sabine Iatridou. 2008. How to say *ought* in Foreign: The composition of weak necessity modals. In Jacqueline Guéron & Jacqueline Lecarme (eds.), *Time and modality* (Studies in Natural Language and Linguistic Theory 75), 115–141. Springer. <http://dx.doi.org/10.1007/978-1-4020-8354-9>.

Some work that discusses whether non-monotonicity could be or might have to be relegated to a dynamic pragmatic component of meaning:

Kai von Fintel. 2001. Counterfactuals in a dynamic context. In Michael Kenstowicz (ed.), *Ken Hale: A life in language*, 123–152. MIT Press. <http://mit.edu/fintel/fintel-2001-counterfactuals.pdf>.

Kai von Fintel. 1999. NPI licensing, Strawson entailment, and context dependency. *Journal of Semantics* 16(2). 97–148. <http://dx.doi.org/10.1093/jos/16.2.97>.

Anthony S. Gillies. 2007. Counterfactual scorekeeping. *Linguistics and Philosophy* 30(3). 329–360. <http://dx.doi.org/10.1007/s10988-007-9018-6>.

Schlenker explored whether the apparent non-monotonicity in conditional is paralleled in quantification over individuals:

Philippe Schlenker. 2004. Conditionals as definite descriptions (A referential analysis). *Research on Language and Computation* 2(3). 417–462. <http://dx.doi.org/10.1007/s11168-004-0908-2>.

CHAPTER SIX

BASICS OF TENSE AND ASPECT

We explore an analysis of tense that treats tenses as intensional operators manipulating a time parameter of evaluation. The treatment is formally quite parallel to the treatment of modals in Chapter 3. We touch on many basic questions about tense and aspect, without exploring them fully.

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6.1 A First Proposal for Tense

Tense logic, or temporal logic, is a branch of logic first developed by the aptly named Arthur Prior in a series of works, in which he proposed treating tense in a way that is formally quite parallel to the treatment of modality discussed in Chapter 3. Since tense logic (and modal logic) typically is formulated at a high level of abstraction regarding the structure of sentences, it doesn't concern itself with the internal make-up of "atomic" sentences and thus treats tenses as sentential operators (again, in parallel to the way modal operators are typically treated in modal logic). We will implement a version of Prior's tense logic in our framework.

See the Stanford Encyclopedia of Philosophy entry on temporal logic: <http://plato.stanford.edu/entries/logic-temporal/> and the website for Prior studies: <http://www.prior.aau.dk/index2.htm>.

The first step is to switch to a version of our intensional semantic system where instead of a world parameter, the evaluation function is sensitive to a time parameter (and a variable assignment). Eventually, we will want to deal with the full complexity and relativize the evaluation function to both worlds and times, but for now, we will just relativize to times. The composition principles developed in Chapter 1 will be adopted *mutatis mutandis*. Predicates will now have lexical entries that incorporate their sensitivity to time:

$$(1) \quad \llbracket \text{tired} \rrbracket^{t,g} = \lambda x \in D. x \text{ is tired at } t.^1$$

It is customary in the literature to introduce a new basic type for times; for now, we will recycle the designation *s* as the type for times. Then, for example, the intension of sentence will again be of type $\langle s, t \rangle$, but now that would be a temporal proposition, a function from times to truth-values.

In this framework, we can now formulate a very simple-minded first analysis of the present and past tenses and the future auxiliary *will*. As for (LF) syntax let's assume that (complete matrix) sentences are TPs, headed by T (for “tense”). There are two morphemes of the functional category T, namely *PAST* (past tense) and *PRES* (present tense). The complement of T is an MP or a VP. MP is headed by M (for “modal”). Morphemes of the category M include the modal auxiliaries *must*, *can*, etc., which we talked about in previous chapters, the semantically vacuous *do* (in so-called “do-support” structures), and the future auxiliary *will*. Evidently, this is a semantically heterogeneous category, grouped together solely because of their common syntax (they are all in complementary distribution with each other). The complement of M is a VP. When the sentence contains none of the items in the category M, we assume that MP isn't projected at all; the complement of T is just a VP in this case. We thus have LF-structures like the following. (The corresponding surface sentences are given below, and we won't be explicit about the derivational relation between these and the LFs. Assume your favorite theories of syntax and morphology here.)

- (2) [TP Mary [T' PRES [VP t [V' be tired]]]]
= Mary is tired.
- (3) [TP Mary [T' PAST [VP t [V' be tired]]]]
= Mary was tired.
- (4) [TP Mary [T' PRES [MP t [M' woll [VP t [V' be tired]]]]]]
= Mary will be tired.

¹ We remain vague for now about what we mean by “times” (points in time? time intervals?). This will need clarification. We will also see the need to clarify what we mean by “at” in the metalanguage in this entry and others.

When we have proper name subjects, we will pretend for simplicity that they are reconstructed somehow into their VP-internal base position. (We will talk more about reconstruction later on.)

What are the meanings of *PRES*, *PAST*, and *will*? For *PRES*, the simplest assumption is actually that it is semantically vacuous. This means that the interpretation of the LF in (2) is identical to the interpretation of the bare VP *Mary be tired*:

- (5) For any time t :
 $\llbracket \text{PRES (Mary be tired)} \rrbracket^t = \llbracket \text{Mary be tired} \rrbracket^t = 1$ iff Mary is tired at t .

Does this adequately capture the intuitive truth-conditions of the sentence *Mary is tired*? It does if we make the following general assumption:

- (6) An utterance of a sentence (= LF) ϕ that is made at a time t counts as true iff $\llbracket \phi \rrbracket^t = 1$ (and as false if $\llbracket \phi \rrbracket^t = 0$).

This assumption ensures that (unembedded) sentences are, in effect, interpreted as claims about the time at which they are uttered (“utterance time” or “speech time”). If we make this assumption and we stick to the lexical entries we have adopted, then we are driven to conclude that the present tense has no semantic job to do. A tenseless VP *Mary be tired* would in principle be just as good as (2) to express the assertion that Mary is tired at the utterance time. Apparently it is just not well-formed as an unembedded structure, but this fact must be attributed to principles of syntax rather than semantics.

What about *PAST*? When a sentence like (3) *Mary was tired* is uttered at a time t , then what are the conditions under which this utterance is judged to be true? A quick (and perhaps ultimately wrong) answer is: an utterance of (3) at t is true iff there is some time before t at which Mary is tired. This suggests the following entry:

- (7) For any time t :
 $\llbracket \text{PAST} \rrbracket^t = \lambda p \in D_{\langle s, t \rangle}. \exists t' \text{ before } t : p(t') = 1$

So, the past tense seems to be an existential quantifier over times, restricted to times before the utterance time.

For *will*, we can say something completely analogous:

- (8) For any time t :
 $\llbracket \text{will} \rrbracket^t = \lambda p \in D_{\langle s, t \rangle}. \exists t' \text{ after } t : p(t') = 1$

Apparently, *PAST* and *will* are semantically alike, even mirror images of each other, though they are of different syntactic categories. The fact that *PAST* is the topmost head in its sentence, while *will* appears below *PRES*, is due to the fact that syntax happens to require a T-node in every complete sentence. Semantically, this has no effect, since *PRES* is vacuous.

Both (7) and (8) presuppose that the set T comes with an intrinsic order. For concreteness, assume that the relation ‘precedes’ (in symbols: $<$) is a strict linear order on T.² The relation ‘follows’, of course, can be defined in terms of ‘precedes’ (t follows t’ iff t’ precedes t).

There are many things wrong with this simple analysis. We will not have time here to diagnose most of the problems, much less correct them. But let’s see a couple of things that work out OK and let’s keep problems and remedies for later.

6.1.1 *former*

There is a brief discussion on p. 72 of H&K about the inadequacy of an extensional semantics for the adjective *former* as in

- (9) John is a former teacher.

We can now write a semantics for *former*. While there are a bunch of people who are currently teachers, there are others that aren’t now teachers but were at some previous time. The latter are the ones that the predicate *former teacher* should be true of. In other words, *former teacher* is a predicate that is true of individuals just in case the predicate *teacher* was true of them at some previous time (and is not true of them now). So, *former* needs to be an intensional operator that “displaces” the evaluation of time of its complement from “now” to some previous time. To be able to do that, it needs to take the intension of its complement as its argument. This suggests the following lexical entry:

- (10) $\llbracket \text{former} \rrbracket = \lambda f \in D_{\langle s, \langle e, t \rangle \rangle} . \lambda x . [f(t)(x) = o \ \& \ \exists t' \text{ before } t : f(t')(x) = i]$.

2 Definition: A relation R is a strict linear order on a set S iff it has the following four properties:

- (i) $\forall x \forall y \forall z ((Rxy \ \& \ Ryz) \rightarrow Rxz)$ “Transitivity”
- (ii) $\forall x (\neg Rxx)$ “Irreflexivity”
- (iii) $\forall x \forall y (Rxy \rightarrow \neg Ryx)$ “Asymmetry”, and
- (iv) $\forall x \forall y (x \neq y \rightarrow (Rxy \vee Ryx))$ “Connectedness”

EXERCISE 6.1: H&K on p.72 mention the adjective *alleged* in one breath with *former*. Formulate a lexical entry for *alleged* as used in *John is an alleged murderer*. [This will use our original intensional system with a world parameter] □

6.1.2 Some Time Adverbials

At least to a certain extent, we can also provide a treatment of temporal adverbials such as:

(11) Mary was tired on February 1, 2001.

The basic idea would be that phrases like *on February 1, 2001* are propositional modifiers. Propositions are the intensions of sentences. At this point, propositions are functions from times to truth-values. Propositional modifiers take a proposition and return a proposition with the addition of a further condition on the time argument.

(12) $\llbracket \text{on February 1, 2001} \rrbracket^t$
 $= \lambda p \in D_{\langle s, t \rangle}. [p(t) = 1 \ \& \ t \text{ is part of Feb 1, 2001}]$

(13) $[T' \text{ PAST } [_{VP} [_{VP} \text{ Mary } [_V \text{ be tired}]]] [_{PP} \text{ on February 1, 2001}]]]$

An alternative would be to treat *on February 1, 2001* as a “sentence” by itself, whose intension then would be a proposition.

(14) $\llbracket \text{on February 1, 2001} \rrbracket^t = 1 \text{ iff } t \text{ is part of February 1, 2001}$

(15) $\llbracket \text{on} \rrbracket^t = \lambda x. t \text{ is part of } x$

To make this work, we would then have to devise a way of combining two tenseless sentences (*Mary be tired* and *on February 1, 2001*) into one. We could do this by positing a silent *and* or by introducing a new composition rule (“Propositional Modification?”).

Let’s not spend time on such a project.

EXERCISE 6.2: Imagine that *Mary was tired on February 1, 2001* is not given the LF in (13) but this one:

(16) $[T' [_{T'} \text{ PAST } [_{VP} \text{ Mary } [_V \text{ be tired}]]] [_{PP} \text{ on February 1, 2001}]]]$

What would the truth-conditions of this LF be? Does this result correspond at all to a possible reading of this sentence (or any other analogous sentence)? If not, how could we prevent such an LF from being produced? □

EXERCISE 6.3: When a quantifier appears in a tensed sentence, we might expect two scope construals. Consider a sentence like this:

(I7) Every professor (in the department) was a teenager in the Sixties.

We can imagine two LFs:

(I8) PAST [[every professor be a teenager] [in the sixties]]

(I9) [every professor] λ_2 [PAST [[t₂ be a teenager] [in the sixties]]

Describe the different truth-conditions which our system assigns to the two LFs.

Is the sentence ambiguous in this way?

If not this sentence, are there analogous sentences that do have the ambiguity?□

EXERCISE 6.4: The following entry for *every* makes it a time-insensitive item:

(20) $\llbracket \text{every} \rrbracket^t = \lambda f \in D_{\langle e, t \rangle} . \lambda g \in D_{\langle e, t \rangle} . \forall x [f(x) = 1 \rightarrow g(x) = 1]$

Consider now two possible variants (we have underlined the portion where they differ):

(21) $\llbracket \text{every} \rrbracket^t = \lambda f \in D_{\langle e, t \rangle} . \lambda g \in D_{\langle e, t \rangle} . \forall x \text{ at } t [f(x) = 1 \rightarrow g(x) = 1]$

(22) $\llbracket \text{every} \rrbracket^t = \lambda f \in D_{\langle e, t \rangle} . \lambda g \in D_{\langle e, t \rangle} . \forall x [f(x) = 1 \text{ at } t \rightarrow g(x) = 1 \text{ at } t]$

Does either of these alternative entries make sense? If so, what does it say? Is it equivalent to our official entry? Could it lead to different predictions about the truth-conditions of English sentences?□

6.1.3 A Word of Caution

Compare the semantics given for *former* and the one for *PAST*:

(23) $\llbracket \text{former} \rrbracket = \lambda f \in D_{\langle s, \langle e, t \rangle \rangle} . \lambda x . [f(t)(x) = 0 \ \& \ \exists t' \text{ before } t : f(t')(x) = 1].$

(24) $\llbracket \text{PAST} \rrbracket^t = \lambda p \in D_{\langle s, t \rangle} . \exists t' \text{ before } t : p(t') = 1$

Notice that these entries have an interesting consequence:

- (25) a. John is a former teacher.
b. John was a teacher.

The two sentences in (25) differ in their truth-conditions. The sentence in (25a) can only be true if John is not a teacher anymore while this is not part of the truth-conditions of the sentence in (25b). To see that this analysis is in fact correct, consider this:

- (26) Last night, John was reading a book about tense.
 a. !! The authors are former Italians.
 b. The authors were Italian.

Consider the past tense in (26b). It is not (necessarily) interpreted as claiming that the authors are not Italian anymore. But this is in fact required by (26a).

There are some cases where it seems that the past tense does trigger inferences that one would not expect from the lexical entry that we gave. Surely, if I tell you *My cousin John was a teacher* you will infer that he isn't a teacher anymore. In fact, you may even infer that he is not alive anymore. One promising approach that tries to reconcile a semantics like ours with the possibility of stronger inferences in some contexts is based on pragmatic considerations, see [Musan 1997](#).

Examples like the one in (26) are problematic for widely held naive conceptions of what the past tense means. One often hears that *PAST* expresses the fact that “the time of the reported situation precedes the speech time”. If this were to mean that the time of the book's authors being Italian precedes the speech time, this would presumably wrongly predict that they would have to be not Italian anymore for the sentence to be true (or usable).

6.2 Are Tenses Referential?

Our semantics for the past tense treats it essentially as an existential quantifier over times (albeit in the meta-language), the same way we treated possibility modals as existential quantifiers over (accessible) worlds. This seems quite adequate for examples like (27), which seem to display the expected quantified meaning:

- (27) John went to a private school.

All we learn from (27) is that at some point in the past, whenever it was that John went to school, he went to a private school.

Partee in her famous paper “Some structural analogies between tenses and pronouns in English” ([B. H. Partee 1973](#)) presented an example where tense appears to act more “referentially”:

- (28) I didn't turn off the stove.

“When uttered, for instance, halfway down the turnpike, such a sentence clearly does not mean either that there exists some time in the past at which I did not turn off the stove or that there exists no time in the past at which I turned off the stove. The sentence clearly refers to a particular time — not a particular instant, most likely, but a definite interval whose identity is generally clear from the extralinguistic context, just as the identity of the *he* in [*He shouldn't be in here*] is clear from the context.”

Partee here is arguing that neither of the two plausible LFs derivable in our current system correctly captures the meaning of (28). Given that the sentence contains a past tense (which we have treated as an existential quantifier over past times) and a negation, we need to consider two possible scopings of the two operators:

- (29) a. PAST NEG I turn off the stove.
- b. NEG PAST I turn off the stove.

EXERCISE 6.5: Show that neither LF in (29) captures the meaning of (28) correctly.□

At this point, we will not develop Partee's analysis in formally explicit detail. If tenses refer to times, it would be easiest to give up on the treatment of times as evaluation parameters and move to a system where times are object language arguments of time-sensitive expressions. We will see a system of that nature later on.

In a commentary on Partee's paper at the same conference it was presented at, Stalnaker pointed out that the Priorean theory can in fact deal with (28), if one allows the existential quantifier over times to be contextually restricted to times in the salient interval of Partee leaving her house — since natural language quantifiers are typically subject to contextual restrictions, this is not a problematic assumption. (Note that Partee formulated her observation in quite a circumspect way: “The sentence refers to a particular time”; Stalnaker's suggestion is that the reference to a particular time is part of the restriction to the quantifier over times expressed by tense, rather than tense itself being a referring item (of type *s*).)

EXERCISE 6.6: Assuming a restricted existential quantification à la Stalnaker, which of the LFs in (29) captures the meaning of (28) correctly?□

Ogihara (1995) argues that the restricted existential quantification view is in fact superior to Partee's analysis, since Partee's analysis needs an existential quantifier anyway. Note that it is clear that the time being referred to is a protracted interval (the time during which Partee was leaving her house). But the sentence is not interpreted as saying that this interval is not a time at which she turned off her stove, which would have to be a fairly absurd turning-off-of-the-stove (turning

B. H. Partee 1984 adopts an existential quantifier analysis.

off the stove only takes a moment and doesn't take up a significant interval). Instead, the sentence says that *in* that salient interval there is no time at which she turned off the stove. Clearly, we do need an existential quantifier in there somewhere and the Priorean theory provides one.³ Ogihara makes the point with the following example:

- (30) John: Did you see Mary?
 Bill: Yes, *I saw her*, but I don't remember exactly when.

The question and answer in this dialogue concern the issue of whether Bill saw Mary at *some* time in a contextually salient interval.

6.3 The Need for Intervals

We have just seen a reason to recognize that natural language can talk not just about moments of time but also about intervals (connected sets of moments), which is a fairly trivial fact; after all, what does *the year 2010* refer to if not an interval of time? We have to go even farther, though. It can be shown that we need the time parameter of the evaluation function to be able to be an interval. Consider the tenseless clause *John build a house* and consider a situation where John starts building a house (the only house he has ever built) on April 1, 2009 and finishes building it on April 1, 2010. Now, which times do we want to be times *at which* “John build a house” is true? If we allow the clause to be true at moments during the building, we would make it true at other times during the building (the ones after the first times) that *John built a house*, but that is wrong. So, the time(s) at which “John build a house” cannot be before April 1, 2010. And clearly, times after April 1, 2010 cannot be times at which “John build a house” is true. So, perhaps, the only time at which “John build a house” is true is the moment on April 1, 2010 when he finishes building the house? But then we would incorrectly predict that on the day before, when he has already been building the house for almost a year, we can truthfully say that *John will build a house*. So, no moment of time can be the time at which “John build a house” is true. The solution is that the time at which “John build a house” is true is exactly the interval that starts with the first moment of the building project and ends with the last nail hammered into the wall. Then, we can say before April 1, 2009 that John will build a house and after April 1, 2010, that John built a house.

What can we say during the building of the house, though? The English present tense is not correctly used in this circumstance:

³ Clearly, the alternative is to say that the existential quantifier is not expressed by tense but comes from somewhere else, perhaps aspect, perhaps in the lexical meaning of *turn off*. We will not pursue those options here.

(31) !!John builds a house.

Our analysis may be read as predicting this fact. Assume that for an unembedded clause, the time parameter is set to be the speech time. But what is the speech time? Perhaps, it is the exact interval it takes to utter the particular clause being evaluated. If so, an example like (31) can only possibly be true if the speech interval exactly coincides with the reported event, here the building of the house. That is, the speaker of (31) would have to ensure that she starts speaking at the very first moment of John's building the house, continues speaking rather slowly, and then finishes speaking with the very last nail. It is intriguing to note that sentences like (31) become acceptable in situations where a sentence is conceived of as exactly coinciding the event being reported, namely play-by-play sports commentary ("He passes the ball to Messi").⁴

What English needs to do instead is to use the progressive:

(32) John is building a house.

(32) expresses that the speech time is included in an interval of John building a house. Elements that connect the evaluation time to the time at which a predicate holds are usually called *aspectual* operators or simply *aspects*. The English progressive then is an aspectual expression. We will look closer at its meaning in a little while.

6.4 Aktionsarten

We can distinguish predicates with respect to their temporal profile. The traditional classification has four categories:

- *accomplishment* predicates
- *achievement* predicates
- *activity* predicates
- *stative* predicates

Accomplishment predicates (*build a house*, *cross the street*) describe an event that has a defined beginning and end (*telos*, 'goal') and takes some amount of time to finish. Achievement predicates (*reach the summit*, *notice the problem*) also have a *telos* but are conceived of as describing an instantaneous event. Accomplishment predicates and achievement predicates constitute the class of *telic* predicates.

4 We cannot go into this fascinating topic further here, but there is much more to explore about the peculiar nature of (31). M. Bennett & B. Partee (1978) assume that the speech time is a moment and use that assumption to derive the nature of (31). Ejerhed (1974) calls the typical use of (31), the "voyeur present"; see also Cooper 1986.

Activity predicates (*run, dance*) describe events that are not conceived of as having a defined goal. Stative predicates (*be in New York, know French*) describe states that are true of intervals. The difference between activity predicates and stative predicates is often said to turn on whether there is an agent being active in the described event.

— Read [Rothstein 2004](#), Chapter 1, pp. 1–35 —

6.5 The Progressive

— Read [Portner 1998](#) —

6.6 Tense in Embedded Clauses

What happens to the time-sensitivity of the verb in a tenseless clause? Consider ECM complements to verbs of believing:

- (33) John believed it to be raining.

Evidently, there is some kind of dependency of the time reference in the lower clause and the higher clause. The simplest approach in our framework would be to have *believe* pass down its evaluation time to the lower clause and to assume that the lower clause doesn't have a tense operator. Then, whatever time *believe* is being interpreted at would be the same time that the lower verb would be evaluated at.

- (34) $\llbracket \text{believe} \rrbracket^{w,t} = \lambda p_{\langle s,t \rangle} . \lambda x . p(w', t)$, for all worlds w' compatible with what x believes in w at t .

Together with the rest of the system, we predict that (33) will be true iff there is a past time t such that it is raining at t in all worlds which conform to what John believes at t , which seems adequate. Unfortunately, it only *seems* adequate.

Consider these four worlds:

- w_1 rain at 4am, John awake at 4am
- w_2 rain at 4am, John awake at 5am
- w_3 rain at 5am, John awake at 4am
- w_4 rain at 5am, John awake at 5am

Assume that in all four worlds, John wakes up, has no idea what time it is, hears a dripping noise, and says to himself “it is raining (now)”. Which worlds conform

to what John believes at 4am in w_1 ? In which worlds is it raining at 4am? Are the former a subset of the latter? No!

Consider a variant of the story. Everything is the same as above, except that John wakes up, thinks it is 5am and says to himself: “It was raining at 4am.” Fact: Sentence (33) is not a true report of John’s beliefs in w_1 in this story. Why not? There is a description, viz. *4am*, which in fact picks out the time of John’s thinking, and under which he ascribes rain to that time.

Conclusion: Sentence (33) unambiguously means that there is a past time t such that John at t ascribes rain to t under the description “now”. We need to capture this but the proposal encapsulated in (34) doesn’t achieve this.

The solution: *believe* (and other attitude verbs, or perhaps the complementizer they select) controls not just the world parameter of its prejacent but also the time parameter.

- (35) $\llbracket \text{believe} \rrbracket^{w,t} = \lambda p_{\langle s,t \rangle} . \lambda x . p(w', t'),$ for all worlds w' and t' such that for all that x can tell in w at t , x might be located in w' at t' .

On this analysis, (33) means essentially that John located himself at a raining time. This is intuitively correct.

— More on tense in tensed complement clauses —

Supplementary Readings

A nice and gentle introduction to some of the issues discussed in this chapter comes from Ogihara:

Toshiyuki Ogihara. 2007. Tense and aspect in truth-conditional semantics. *Lingua* 117(2). 392–418. <http://dx.doi.org/10.1016/j.lingua.2005.01.002>.

Partee’s seminal paper is a must read:

Barbara H. Partee. 1973. Some structural analogies between tenses and pronouns in English. *The Journal of Philosophy* 70(18). 601–609. <http://dx.doi.org/10.2307/2025024>.

Musan’s work on the pragmatic effects of tense:

Renate Musan. 1997. Tense, predicates, and lifetime effects. *Natural Language Semantics* 5(3). 271–301. <http://dx.doi.org/10.1023/A:1008281017969>.

The three essential works on the progressive:

David R. Dowty. 1977. Toward a semantic analysis of verb aspect and the english ‘imperfective’ progressive. *Linguistics and Philosophy* 1(1). 45–77. <http://dx.doi.org/10.1007/BF00351936>.

Fred Landman. 1992. The progressive. *Natural Language Semantics* 1(1). 1–32. <http://dx.doi.org/10.1007/BF02342615>.

Paul Portner. 1998. The progressive in modal semantics. *Language* 74(4). 760–787. <http://dx.doi.org/10.2307/417002>.

The first chapter of Susan Rothstein’s book on lexical aspect gives a nice overview of Aktionsarten/aspectual classes:

Susan Rothstein. 2004. *Structuring events: A study in the semantics of lexical aspect* (Explorations in Semantics). Blackwell. <http://tinyurl.com/rothstein-aktionsarten>, Chapter 1: “Verb Classes and Aspectual Classification”, pp. 1–35, available online at <http://tinyurl.com/rothstein-aktionsarten>.

Concise statements of some of the issues surrounding dependent tenses:

Arnim von Stechow. 1995. On the proper treatment of tense. *Semantics and Linguistic Theory (SALT)* 5. <http://www2.sfs.uni-tuebingen.de/~arnim10/Aufsaeetze/SALT95.pdf>.

Arnim von Stechow. 2009. Tenses in compositional semantics. To be published in Wolfgang Klein (ed) *The Expression of Time in Language*. <http://www2.sfs.uni-tuebingen.de/~arnim10/Aufsaeetze/Approaches.pdf>.

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CHAPTER SEVEN

DPS AND SCOPE IN MODAL CONTEXTS

We discuss ambiguities that arise when DPs occur in modal contexts.

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7.1 *Specific vs. Non-specific as a Scope Ambiguity*

When a DP appears inside the clausal or VP complement of a modal predicate¹, there is often a so-called *specific-non-specific* ambiguity. A classic example is (1), which contains the DP *a plumber* inside the infinitive complement of *want*.

- (1) John wants to marry a plumber.

According to the non-specific reading, every possible world in which John gets what he wants is a world in which there is a plumber whom he marries. According to the specific reading, there is a plumber in the actual world whom John marries in every world in which he gets what he wants. We can imagine situations in which one of the readings is true and the other one false.

For example, suppose John thinks that plumbers make ideal spouses, because they can fix things around the house. He has never met one so far, but he definitely wants to marry one. In this scenario, the non-specific reading is true,

¹ We will be using the terms “modal operator” and “modal predicate” in their widest sense here, to include modal auxiliaries (“modals”), modal main verbs and adjectives, attitude predicates, and also modalizing sentence-adverbs like *possibly*.

but the specific reading is false. What all of John's desire-worlds have in common is that they have a plumber getting married to John in them. But it's not the same plumber in all those worlds. In fact, there is no particular individual (actual plumber or other) whom he marries in every one of those worlds.

For a different scenario, suppose that John has fallen in love with Robin and wants to marry Robin. Robin happens to be a plumber, but John doesn't know this; in fact, he wouldn't like it and might even call off the engagement if he found out. Here the specific reading is true, because there is an actual plumber, viz. Robin, who gets married to John in every world in which he gets what he wants. The non-specific reading is false, however, because the worlds which conform to John's wishes actually do not have him marrying a plumber in them. In his favorite worlds, he marries Robin, who is not a plumber in those worlds.

When confronted with this second scenario, you might, with equal justification, say 'John wants to marry a plumber', or 'John *doesn't* want to marry a plumber'. Each can be taken in a way that makes it a true description of the facts – although, of course, you cannot assert both in the same breath. This intuition fits well with the idea that we are dealing with a genuine ambiguity.²

Let's look at another example:

- (2) John believes that your abstract will be accepted.

Here the relevant DP in the complement clause of the verb *believe* is *your abstract*. Again, we detect an ambiguity, which is brought to light by constructing different scenarios.

2. In much of the literature the specific/non-specific contrast is also called *de dicto/de re*. As of the Spring 2015 edition of these notes, following discussions with Angelika Kratzer, we do not employ that terminology. But one might wonder what was behind the Latin terminology "de re" (lit.: 'of the thing') and "de dicto" (lit.: 'of what is said')? Apparently, the term "de dicto" is to indicate that on this reading, the *words* which I, the speaker, am using to describe the attitude's content, are the same (at least as far as the relevant DP is concerned) as the words that the subject herself would use to express her attitude. Indeed, if we asked the John in our example what he wants, then in the first scenario he'd say "marry *a plumber*", but in the second scenario he would not use these words. The term "de re", by contrast, indicates that there is a common *object* (here: Robin) whom I (the speaker) am talking about when I say "a plumber" in my report and whom the attitude holder would be referring to if he were to express his attitude in his own words. E.g., in our second scenario, John might say that he wanted to marry "Robin", or "this person here" (pointing at Robin). He'd thus be referring to the same *person* that I am calling "a plumber", but wouldn't use that same description.

Don't take this "definition" of the terms too seriously, though! The terminology is much older than any precise truth-conditional analysis of the two readings, and it does not, in hindsight, make complete sense.

- (i) John's belief may be about an abstract that he reviewed, but since the abstract is anonymous, he doesn't know who wrote it. He told me that there was a wonderful abstract about subjacency in Hindi that is sure to be accepted. I know that it was your abstract and inform you of John's opinion by saying (2). This is the specific reading. In the same situation, the non-specific reading is false: Among John's belief worlds, there are many worlds in which *your abstract will be accepted* is not true or even false. For all he knows, you might have written, for instance, that terrible abstract about Antecedent-Contained Deletion, which he also reviewed and is positive will be rejected.
- (ii) For the other scenario, imagine that you are a famous linguist, and John doesn't have a very high opinion about the fairness of the abstract selection process. He thinks that famous people never get rejected, however the anonymous reviewers judge their submissions. He believes (correctly or incorrectly – this doesn't matter here) that you submitted a (unique) abstract. He has no specific information or opinion about the abstract's content and quality, but given his general beliefs and his knowledge that you are famous, he nevertheless believes that your abstract will be accepted. This is the non-specific reading. Here it is true in all of John's belief worlds that you submitted a (unique) abstract and it will be accepted. The specific reading of (2), though, may well be false in this scenario. Suppose – to flesh it out further – the abstract you actually submitted is that terrible one about ACD. That one surely doesn't get accepted in every one of John's belief worlds. There may be some where it gets in (unless John is certain it can't be by anyone famous, he has to allow at least the possibility that it will get in despite its low quality). But there are definitely also belief-worlds of his in which it doesn't get accepted.

We have taken care here to construct scenarios that make one of the readings true and the other false. This establishes the existence of two distinct readings. We should note, however, that there are also many possible and natural scenarios that simultaneously support the truth of *both* readings. Consider, for instance, the following third scenario for sentence (2).

- (iii) John is your adviser and is fully convinced that your abstract will be accepted, since he knows it and in fact helped you when you were writing it. This is the sort of situation in which both the non-specific and the specific reading are true. It is true, on the one hand, that the sentence *your abstract will be accepted* is true in every one of John's belief worlds (non-specific reading). And on the other hand, if we ask whether the abstract which you actually wrote will get accepted in each of John's belief worlds, that is likewise true (specific reading).

In fact, this kind of “doubly verifying” scenario is very common when we look at actual uses of attitude sentences in ordinary conversation. There may even be many cases where communication proceeds smoothly without

either the speaker or the hearer making up their minds as to which of the two readings they intend or understand. It doesn't matter, since the possible circumstances in which their truth-values would differ are unlikely and ignorable anyway. Still, we *can* conjure up scenarios in which the two readings come apart, and our intuitions about those scenarios do support the existence of a semantic ambiguity.

In the paraphrases by which we have elucidated the two readings of our examples, we have already given away the essential idea of the analysis that we will adopt: We will treat non-specific/specific ambiguities as ambiguities of *scope*. The non-specific readings, it turns out, are the ones which we predict without further ado if we assume that the position of the DP at LF is within the modal predicate's complement. (That is, it is either *in situ* or QRed within the complement clause.) For example:

- (3) John wants [[a plumber]_i [PRO₂ to marry t_i]]
 (4) John believes [the abstract-by-you will-be-accepted]

To obtain the specific readings, we apparently have to QR the DP to a position above the modal predicate, minimally the VP headed by *want* or *believe*.

- (5) [a plumber]_i [John wants [PRO₂ to marry t_i]]
 (6) [the abstract-by-you]_i [John believes t_i will-be-accepted]]

EXERCISE 7.1: Calculate the interpretations of the four structures in (3)–(6), and determine their predicted truth-values in each of the (types of) possible worlds that we described above in our introduction to the ambiguity.

Some assumptions to make the job easier: (i) Assume that (3) and (5) are evaluated with respect to a variable assignment that assigns John to the number 2. This assumption takes the place of a worked out theory of how controlled PRO is interpreted. (ii) Assume that *abstract-by-you* is an unanalyzed one-place predicate. This takes the place of a worked out theory of how genitives with a non-possessive meaning are to be analyzed. □

7.2 Raised subjects

In the examples of specific-non-specific ambiguities that we have looked at so far, the surface position of the DP in question was inside the modal predicate's clausal or VP-complement. We saw that if it stays there at LF, a non-specific reading results, and if it covertly moves up above the modal operator, we get a specific reading. In the present section, we will look at cases in which a DP that

is superficially *higher* than a modal operator can still be read non-specific. In these cases, it is the specific reading which we obtain if the LF looks essentially like the surface structure, and the non-specific reading for which we apparently have to posit a non-trivial covert derivation.

7.2.1 Examples of non-specific readings for raised subjects

Suppose I come to my office one morning and find the papers and books on my desk in different locations than I remember leaving them the night before. I say:

- (7) Somebody must have been here (since last night).

On the assumptions we have been making, *somebody* is base-generated as the subject of the VP *be here* and then moved to its surface position above the modal. So (7) has the following S-structure, which is also an interpretable LF.

- (8) somebody [λ_2 [[must R] [t_2 have-been-here]]]

What does (8) mean? The appropriate reading for *must* here is epistemic, so suppose the variable R is mapped to the relation [$\lambda w. \lambda w'. w'$ is compatible with what I believe in w]. Let w_o be the utterance world. Then the truth-condition calculated by our rules is as follows.

- (9) $\exists x[x \text{ is a person in } w_o \ \& \ \forall w'[w' \text{ is compatible with what I believe in } w_o \rightarrow x \text{ was here in } w']]$

But this is not the intended meaning. For (9) to be true, there has to be a person who in every world compatible with what I believe was in my office. In other words, all my belief-worlds have to have one and the same person coming to my office. But this is not what you intuitively understood me to be saying about my belief-state when I said (7). The context we described suggests that I do not know (or have any opinion about) which person it was that was in my office. For all I know, it might have been John, or it might have been Mary, or it have been this stranger here, or that stranger there. In each of my belief-worlds, somebody or other was in my office, but no one person was there in all of them. I do not believe of anyone in particular that he or she was there, and you did not understand me to be saying so when I uttered (7). What you did understand me to be claiming, apparently, was not (9) but (10).

- (10) $\forall w'[w' \text{ is compatible with what I believe in } w_o \rightarrow \exists x [x \text{ is a person in } w' \ \& \ x \text{ was here in } w']]$

In other words – to use the terminology we introduced in the last section – the DP *somebody* in (7) appears to have a non-specific reading.

How can sentence (7) have the meaning in (10)? The LF in (8), as we saw, means something else; it expresses a specific reading, which typically is false when (7) is uttered sincerely. So there must be another LF. What does it look like and how is it derived? One way to capture the intended reading, it seems, would be to generate an LF that's essentially the same as the underlying structure we posited for (7), i.e., the structure *before* the subject has raised:

- (11) [IP e [I' [must R] [somebody have-been-here]]]

(11) means precisely (10) (assuming that the unfilled Spec-of-IP position is semantically vacuous), as you can verify by calculating its interpretation by our rules. So is (11) (one of) the LF(s) for (7), and what assumption about syntax allow it to be generated? Or are there other – perhaps less obvious, but easier to generate – candidates for the non-specific LF-structure of (7)?

Before we get into these question, let's look at a few more examples. Each of the following sentences, we claim, has a non-specific reading for the subject, as given in the accompanying formula. The modal operators in the examples are of a variety of syntactic types, including modal auxiliaries, main verbs, adjectives, and adverbs.

- (12) Everyone in the class may have received an A.
 $\exists w'[w' \text{ conforms to what I believe in } w \ \& \ \forall x[x \text{ is in this class in } w' \rightarrow x \text{ received an A in } w']]$
- (13) At least two semanticists have to be invited.
 $\forall w'[w' \text{ conforms to what is desirable in } w \rightarrow \exists_2 x [x \text{ is a semanticist in } w' \ \& \ x \text{ is invited in } w']]$
- (14) Somebody from New York is expected to win the lottery.
 $\forall w'[w' \text{ conforms to what is expected in } w \rightarrow \exists x[x \text{ is a person from NY in } w' \ \& \ x \text{ wins the lottery in } w']]$
- (15) Somebody from New York is likely to win the lottery.
 $\forall w'[w' \text{ is as likely as any other world, given I know in } w \rightarrow \exists x[x \text{ is a person from NY in } w' \ \& \ x \text{ wins the lottery in } w']]$ ³

3 Hopefully the exact analysis of the modal operators *likely* and *probably* is not too crucial for the present discussion, but you may still be wondering about it. As you see in our formula, we are thinking of *likely* (*probably*) as a kind of epistemic necessity operator, i.e., a universal quantifier over a set of worlds that is somehow determined by the speaker's knowledge. (We are focussing on the "subjective probability" sense of these words. Perhaps there is also an "objective

- (16) One of these two people is probably infected.
 $\forall w'[w' \text{ is as likely as any other world, given what I know in } w$
 $\rightarrow \exists x[x \text{ is one of these two people \& } x \text{ is infected in } w']]$

To bring out the intended non-specific reading of the last example (to pick just one) imagine this scenario: We are tracking a dangerous virus infection and have sampled blood from two particular patients. Unfortunately, we were sloppy and the blood samples ended up all mixed up in one container. The virus count is high enough to make it quite probable that one of the patients is infected but because of the mix-up we have no evidence about which one of them it may be. In this scenario, (16) appears to be true. It would not be true under a specific reading, because neither one of the two people is infected in every one of the likely worlds.

A word of clarification about our empirical claim: We have been concentrating on the observation that non-specific readings are *available*, but have not addressed the question whether they are the *only* available readings or coexist with equally possible specific readings. Indeed, some of the sentences in our list appear to be ambiguous: For example, it seems that (14) could also be understood to claim that there is a particular New Yorker who is likely to win (e.g., because he has bribed everybody). Others arguably are not ambiguous and can only be read non-specific. This is what von Stechow & Iatridou (2003) claim about sentences

probability” reading that is circumstantial rather than epistemic.) What is the difference then between *likely* and e.g. epistemic *must* (or *necessary* or *I believe that*)? Intuitively, ‘it is likely that p’ makes a weaker claim than ‘it must be the case that p’. If both are universal quantifiers, then, it appears that *likely* is quantifying over a smaller set than *must*, i.e., over only a proper subset of the worlds that are compatible with what I believe. The difference concerns those worlds that I cannot strictly rule out but regard as remote possibilities. These worlds are included in the domain for *must*, but not in the one for *likely*. For example, if there was a race between John and Mary, and I am willing to bet that Mary won but am not completely sure she did, then those worlds where John won are remote possibilities for me. They are included in the domain of *must*, and so I will not say that Mary *must* have won, but they are not in the domain quantified over by *likely*, so I do say that Mary is *likely* to have won.

This is only a very crude approximation, of course. For one thing, probability is a gradable notion. Some things are more probable than others, and where we draw the line between what’s probable and what isn’t is a vague or context-dependent matter. Even *must*, *necessary* etc. arguably don’t really express complete certainty (because in practice there is hardly anything we are completely certain of), but rather just a very high degree of probability. For more discussion of *likely*, *necessary*, and other graded modal concepts in a possible worlds semantics, see e.g. Kratzer 1981, 1991.

A different approach may be that *likely* quantifies over the same set of worlds as *must*, but with a weaker, less than universal, quantificational force. I.e., ‘it is likely that p’ means something like p is true in *most* of the worlds conforming to what I know. A *prima facie* problem with this idea is that presumably every proposition is true in infinitely many possible worlds, so how can we make sense of cardinal notions like ‘more’ and ‘most’ here? But perhaps this can be worked out somehow.

like (12). They note that if (12) also allowed a specific reading, it should be possible to make coherent sense of (17).

(17) Everyone in the class may have received an A. But not everybody did.

In fact, (17) sounds contradictory, which they show is explained if only the non-specific reading is permitted by the grammar. They conjecture that this is a systematic property of epistemic modal operators (as opposed to deontic and other types of modalities). Epistemic operators always have widest scope in their sentence.

So there are really two challenges here for our current theory. We need to account for the existence of non-specific readings, and also for the absence, in at least some of our examples, of specific readings. We will be concerned here exclusively with the first challenge and will set the second aside. We will aim, in effect, to set up the system so that all sentences of this type are in principle ambiguous, hoping that additional constraints that we are not investigating here will kick in to exclude the specific readings where they are missing.

To complicate the empirical picture further, there are also examples where raised subjects are unambiguously specific. Such cases have been around in the syntactic literature for a while, and they have received renewed attention in the work of Lasnik and others. To illustrate just one of the systematic restrictions, negative quantifiers like *nobody* seem to permit only surface scope (i.e., wide scope) with respect to a modal verb or adjective they have raised over.

(18) Nobody from New York is likely to win the lottery.

(18) does not have a non-specific reading parallel to the one for (15) above, i.e., it cannot mean that it is likely that nobody from NY will win. It can only mean that there is nobody from NY who is likely to win. This too is an issue that we set aside.

In the next couple of sections, all that we are trying to do is find and justify a mechanism by which the grammar is capable to generate both specific and non-specific readings for subjects that have raised over modal operators. It is quite conceivable, of course, that the nature of the additional constraints which often exclude one reading or the other is ultimately relevant to this discussion and that a better understanding of them may undermine our conclusions. But this is something we must leave for further research.

7.2.2 Syntactic “Reconstruction”

Given that the non-specific reading of (7) we are aiming to generate is equivalent to the formula in (10), an obvious idea is that there is an LF which is essentially the pre-movement structure of this sentence, i.e., the structure prior to the raising of the subject above the operator. There are a number of ways to make such an LF available.

One option, most defended in Sauerland & Elbourne (2002), is to assume that the raising of the subject can happen in a part of the derivation which only feeds PF, not LF. In that case, the subject simply stays in its underlying VP-internal position throughout the derivation from DS to LF. (Recall that quantifiers are interpretable there, as they generally are in subject positions.)

Another option is a version of the so-called Copy Theory of movement introduced in Chomsky (1993). This assumes that movement generally proceeds in two separate steps, rather than as a single complex operation as we have assumed so far. Recall that in H&K, it was stipulated that every movement effects the following four changes:

- (i) a phrase α is deleted,
- (ii) an index i is attached to the resulting empty node (making it a so-called trace, which the semantic rule for “Pronouns and Traces” recognizes as a variable),
- (iii) a new copy of α is created somewhere else in the tree (at the “landing site”), and
- (iv) the sister-constituent of this new copy gets another instance of the index i adjoined to it (which the semantic rule of Predicate Abstraction recognizes as a binder index).

If we adopt the Copy Theory, we assume instead that there are three distinct operations:

“Copy”: Create a new copy of α somewhere in the tree, attach an index i to the original α , and adjoin another instance of i to the sister of the new copy of α . (= steps (ii), (iii), and (iv) above)

“Delete Lower Copy”: Delete the original α . (= step (i) above)

“Delete Upper Copy”: Delete the new copy of α and both instances of i .

The Copy operation is part of every movement operation, and can happen anywhere in the syntactic derivation. The Delete operations happen at the end of the LF derivation and at the end of the PF deletion. We have a choice of applying either Delete Lower Copy or Delete Upper Copy to each pair of copies, and we can make this choice independently at LF and at PF. (E.g., we can do Copy in the common part of the derivation and then Delete Lower Copy at LF and Delete Upper Copy at PF.) If we always choose Delete Lower Copy at LF, this system generates exactly the same structures and interpretations as the

one from $H \not\sim K$. But if we exercise the Delete Upper Copy option at LF, we are effectively undoing previous movements, and this gives us LFs with potentially new interpretations. In the application we are interested in here, we would apply the Copy step of subject raising before the derivation branches, and then choose Delete Lower Copy at PF but Delete Upper Copy at LF. The LF will thus look as if the raising never happened, and it will straightforwardly get the desired non-specific reading.

If the choice between the two Delete operations is generally optional, we in principle predict ambiguity wherever there has been movement. Notice, however, first, that the two structures will often be truth-conditionally equivalent (e.g. when the moved phrase is a name), and second, that they will not always be both interpretable. (E.g., if we chose Delete Upper Copy after QRing a quantifier from object position, we'd get an uninterpretable structure, and so this option is automatically ruled out.) Even so, we predict lots of ambiguity. Specifically, since raised subjects are always interpretable in both their underlying and raised locations, we predict all raising structures where a quantificational DP has raised over a modal operator (or over negation or a temporal operator) to be ambiguous. As we have already mentioned, this is not factually correct, and so there must be various further constraints that somehow restrict the choices. (Similar comments apply, of course, to the option we mentioned first, of applying raising only on the PF-branch.)

Yet another solution was first proposed by [May \(1977a\)](#): May assumed that QR could in principle apply in a “downward” fashion, i.e., it could adjoin the moved phrase to a node that doesn't contain its trace. Exercising this option with a raised subject would let us produce the following structure, where the subject has first raised over the modal and then QRed below it.

(19) $t_j \lambda_i [\text{must-R} [\text{someone } \lambda_j [t_i \text{ have been here}]]]$

As it stands, this structure contains at least one free variable (the trace t_j) and can therefore not possibly represent any actual reading of this sentence. May further assumes that traces can in principle be deleted, when their presence is not required for interpretability. This is not yet quite enough, though to make (19) interpretable, at least not within our framework of assumptions, for (20) is still not a candidate for an actual reading of (7).

(20) $\lambda_i [\text{must-R} [\text{someone } \lambda_j [t_i \text{ have been here}]]]$

We would need to assume further that the topmost binder index could be deleted along with the unbound trace, and also that the indices i and j can be the same, so that the raising trace t_j is bound by the binding-index created by QR. If these

things can be properly worked out somehow, then this is another way to generate the non-specific reading. Notice that the LF is not exactly the same as on the previous two approaches, since the subject ends up in an adjoined position rather than in its original argument position, but this difference is obviously without semantic import.

What all of these approaches have in common is that they place the burden of generating the non-specific reading for raised subjects on the syntactic derivation. Somehow or other, they all wind up with structures in which the subject is lower than it is on the surface and thereby falls within the scope of the modal operator. They also have in common that they take the modal operator (here the auxiliary, in other cases a main predicate or an adverb) to be staying put. I.e., they assume that the non-specific readings are not due to the modal operator being covertly higher than it seems to be, but to the subject being lower. Approaches with these features will be said to appeal to “syntactic reconstruction” of the subject.⁴

7.2.3 Some Alternatives to Syntactic Reconstruction

Besides (some version of) syntactic reconstruction, there are many other ways in which one try to generate non-specific readings for raised subjects. Here are some other possibilities that have been suggested and/or readily come to mind. We will see that some of them yield exactly the non-specific reading as we have been describing it so far, whereas others yield a reading that is very similar but not quite the same. We will confine ourselves to analyses which involve no or only minor changes to our system of syntactic and semantic assumptions. Obviously, if departed from these further, there would be even more different options, but even so, there seem to be quite a few.

1. RAISING THE MODAL OPERATOR, VARIANT I: NO TRACE Conceivably, an LF for the non-specific reading of (7) might be derived from the S-structure (= (8)) by covertly moving *must* (and its covert *R*-argument) up above the subject. This would have to be a movement which leaves no (semantically non-vacuous) trace. Given our inventory of composition rules, the only type that the trace could have to make the structure containing it interpretable would be the type of the moved operator itself (i.e. $\langle st, t \rangle$). If it had that type, however, the movement would be semantically inconsequential, i.e., the structure would mean exactly the same as (8). So this would not be a way to provide an LF for the non-specific

4 This is a very broad notion of “reconstruction”, where basically any mechanism which puts a phrase at LF in a location nearer to its underlying site than its surface site is called “reconstruction”. In some of the literature, the term is used more narrowly. For example, May’s downward QR is sometimes explicitly contrasted with genuine reconstruction, since it places the quantifier somewhere else than exactly where it has moved from.

reading. If there was no trace left however (and also no binder index introduced), we indeed would obtain the non-specific reading.

EXERCISE 7.2: Prove the claims we just made in the previous paragraph. Why is no type for the trace other than $\langle st, t \rangle$ possible? Why is the movement semantically inert when this type is chosen? How does the correct intended meaning arise if there is no trace and binder index? \square

2. RAISING THE MODAL OPERATOR, VARIANT 2: TRACE OF TYPE S [Requires slightly modified inventory of composition rules. Derives an interpretation that is not quite the same as the non-specific reading we have assumed so far. Rather, it is a “narrow-Q, R-de-re” interpretation in the sense discussed in the next chapter.]

3. HIGHER TYPE FOR TRACE OF RAISING, VARIANT 1: TYPE $\langle et, t \rangle$ [Before reading this section, read and do the exercise on p.212/3 in H&K]

So far in our discussion, we have taken for granted that the LF which corresponds to the surface structure, viz. (8), gives us the specific reading. This, however, is correct only on the tacit assumption that the trace of raising is a variable of type e . If it is part of our general theory that all variables, or at least all interpretable binder indices (hence all bound variables), in our LFs are of type e , then there is nothing more here to say. But it is not *prima facie* obvious that we must or should make this general assumption, and if we don't, then the tree in (8) is not really one single LF, but the common structure for many different ones, which differ in the type chosen for the trace. Most of the infinitely many semantic types we might assign to this trace will lead to uninterpretable structures, but there turns out to be one other choice besides e that works, namely $\langle et, t \rangle$:

(21) somebody $\lambda_{2, \langle et, t \rangle}$ [[must R] [$t_{2, \langle et, t \rangle}$ have-been-here]]

(21) is interpretable in our system, but again, as above, the predicted interpretation is not exactly the non-specific reading as we have been describing it so far, but a “narrow-Q, R-de-re” reading.

EXERCISE 7.3: Using higher-type traces to “reverse” syntactic scope-relation is a trick which can be used quite generally. It is useful to look at a non-intensional example as a first illustration. (22) contains a universal quantifier and a negation, and it is scopally ambiguous between the readings in (a) and (b).

(22) Everything that glitters is not gold.
 a. $\forall x[x \text{ glitters} \rightarrow \neg x \text{ is gold}]$ “surface scope”
 b. $\neg \forall x[x \text{ glitters} \rightarrow x \text{ is gold}]$ “inverse scope”

We could derive the inverse scope reading for (22) by generating an LF (e.g. by some version of syntactic reconstruction") in which the *every*-DP is below *not*. Interestingly, however, we can also derive this reading if the *every*-DP is in its raised position above *not* but its trace has the type $\langle\langle e, t \rangle, t\rangle$.

Spell out this analysis. (I.e., draw the LF and show how the inverse-scope interpretation is calculated by our semantic rules.) \square

EXERCISE 7.4: Convince yourself that there are no other types for the raising trace besides e and $\langle et, t \rangle$ that would make the structure in (8) interpretable. (At least not if we stick exactly to our current composition rules.) \square

4. HIGHER TYPE FOR TRACE OF RAISING, VARIANT 2: TYPE $\langle s, \langle et, t \rangle \rangle$ If we want to get *exactly* the non-specific reading that results from syntactic reconstruction out of a surface-like LF of the form (8), we must use an even higher type for the raising trace, namely $\langle s, \langle\langle e, t \rangle, t \rangle \rangle$, the type of the intension of a quantifier. As you just proved in the exercise, this is not possible if we stick to exactly the composition rules that we have currently available. The problem is in the VP: the trace in subject position is of type $\langle s, \langle\langle e, t \rangle, t \rangle \rangle$ and its sister is of type $\langle e, t \rangle$. These two cannot combine by either FA or IFA, but it works if we employ another variant of functional application.⁵

(23) *Extensionalizing Functional Application* (EFA)

If α is a branching node and $\{\beta, \gamma\}$ the set of its daughters, then, for any world w and assignment g :

if $\llbracket \beta \rrbracket^{w,g}$ is a function whose domain contains $\llbracket \gamma \rrbracket^{w,g}$,
then $\llbracket \alpha \rrbracket^{w,g} = \llbracket \beta \rrbracket^{w,g}(w)(\llbracket \gamma \rrbracket^{w,g})$.

EXERCISE 7.5: Calculate the truth-conditions of (8) under the assumption that the trace of the subject quantifier is of type $\langle s, \langle\langle e, t \rangle, t \rangle \rangle$. \square

CAN WE CHOOSE BETWEEN ALL THESE OPTIONS? Two of the methods we tried derived readings in which the raised subject's *quantificational determiner* took scope below the world-quantifier in the modal operator, but the raised subject's

⁵ Notice that the problem here is kind of the mirror image of the problem that led to the introduction of "Intensional Functional Application" in H&K, ch. 12. There, we had a function looking for an argument of type $\langle s, t \rangle$, but the sister node had an extension of type t . IFA allowed us to, in effect, construct an argument with an added "s" in its type. This time around, we have to get rid of an "s" rather than adding one; and this is what EFA accomplishes.

So we now have three different "functional application"-type rules altogether in our system: ordinary FA simply applies $\llbracket \beta \rrbracket^w$ to $\llbracket \gamma \rrbracket^w$; IFA applies $\llbracket \beta \rrbracket^w$ to $\lambda w'. \llbracket \gamma \rrbracket^{w'}$; and EFA applies $\llbracket \beta \rrbracket^w(w)$ to $\llbracket \gamma \rrbracket^w$. At most one of them will be applicable to each given branching node, depending on the type of $\llbracket \gamma \rrbracket^w$.

Think about the situation. Might there be other variant functional application rules?

restricting NP still was evaluated in the utterance world (or the evaluation world for the larger sentence, whichever that may be). It is difficult to assess whether these readings are actually available for the sentences under consideration, and we will postpone this question to a later section. We would like to argue here, however, that even if these readings are available, they cannot be the *only* readings that are available for raised subjects besides their wide-scope readings. In other words, even if we allowed one of the mechanisms that generated these sort of hybrid readings, we would still need another mechanism that gives us, for at least some examples, the “real” non-specific readings that we obtain e.g. by syntactic reconstruction. The relevant examples that show this most clearly involve DPs with more descriptive content than *somebody* and whose NPs express clearly contingent properties.

(24) A neat-freak must have been here.

If I say this instead of our original (7) when I come to my office in the morning and interpret the clues on my desk, I am saying that every world compatible with my beliefs is such that someone who is a neat-freak *in that world* was here in that world. Suppose there is a guy, Bill, whom I know slightly but not well enough to have an opinion on whether or not he is neat. He may or not be, for all I know. So there are worlds among my belief worlds where he is a neat-freak and worlds where he is not. I also don’t have an opinion on whether he was or wasn’t the one who came into my office last night. He did in some of my belief worlds and he didn’t in others. I am implying with (24), however, that if Bill isn’t a neat-freak, then it wasn’t him in my office. I.e., (24) *is* telling you that, even if I have belief-worlds in which Bill is a slob and I have belief-worlds in which (only) he was in my office, I do not have any belief-worlds in which Bill is a slob *and* the only person who was in my office. This is correctly predicted if (24) expresses the “genuine” non-specific reading in (25), but not if it expresses the “hybrid” reading in (26).

(25) $\forall w'[w' \text{ is compatible with what I believe in } w_o \rightarrow$
 $\exists x[x \text{ is a neatfreak in } w' \text{ and } x \text{ was here in } w']]$

(26) $\forall w'[w' \text{ is compatible with what I believe in } w_o \rightarrow$
 $\exists x[x \text{ is a neatfreak in } w_o \text{ and } x \text{ was here in } w']]$

We therefore conclude the mechanisms 2 and 3 considered above (whatever there merits otherwise) cannot supplant syntactic reconstruction or some other mechanism that yields readings like (25).

This leaves only the first and fourth options that we looked at as potential competitors to syntactic reconstruction, and we will focus the rest of the discussion on how we might be able to tease apart the predictions that these mechanisms imply from the ones of a syntactic reconstruction approach.

additional mechanism of semantic reconstruction via high-type traces is then at best redundant.

Tentative conclusion: Syntactic reconstruction (some version of it) provides the best account of non-specific readings for raised subjects.

CHAPTER EIGHT

BEYOND *de re* — *de dicto*: THE THIRD READING

In this chapter, we will see that quantificational noun phrases in the scope of a modal operator can receive a reading where their restrictive predicate is not interpreted in the worlds introduced by the modal operator (which is what happens in de re readings as well) while at the same time their quantificational force takes scope below the modal operator (which is what happens in de dicto readings as well). This seemingly paradoxical situation might force whole-sale revisions to our architecture. We discuss the standard solution (which involves supplying predicates with world-arguments) and some alternatives.

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8.1 A Problem: Additional Readings and Scope Paradoxes

Janet Dean Fodor discussed examples like (1) in her dissertation (1970).

- (1) Mary wanted to buy a hat just like mine.

Fodor observes that (1) has three readings, which she labels “specific *de re*,” “non-specific *de re*,” and “non-specific *de dicto*.”

- (i) On the “specific *de re*” reading, the sentence says that there is a particular hat which is just like mine such that Mary has a desire to buy it. Say, I am walking along Newbury Street with Mary. Mary sees a hat in a display window and wants to buy *it*. She tells me so. I don’t reveal that I have one just like it. But later I tell *you* by uttering (1).
- (ii) On the “non-specific *de dicto*” reading, the sentence says that Mary’s desire was to buy some hat or other which fulfills the description that it is just like mine. She is a copycat.
- (iii) On the “non-specific *de re*” reading, finally, the sentence will be true, e.g., in the following situation: Mary’s desire is to buy some hat or other, and the only important thing is that it be a Red Sox cap. Unbeknownst to her, my hat is one of those as well.

The existence of three different readings appears to be problematic for the scopal account of *de re-de dicto* ambiguities that we have been assuming. It seems that our analysis allows just two semantically distinct types of LFs: Either the DP *a hat just like mine* takes scope below *want*, as in (2), or it takes scope above *want*, as in (3).

- (2) Mary wanted [[a hat-just-like-mine]_i [PRO to buy t_i]]

- (3) [a hat-just-like-mine]_i [Mary wanted [PRO to buy t_i]]

In the system we have developed so far, (2) says that in every world *w'* in which Mary gets what she wants, there is something that she buys in *w'* that’s a hat in *w'* and like my hat in *w'*. This is Fodor’s “non-specific *de dicto*” reading. (3), on the other hand, says that there is some thing *x* which is a hat in the actual world and like my hat in the actual world, and Mary buys *x* in every one of her desire worlds. That is Fodor’s “specific *de re*.” But what about the “non-specific *de re*”? To obtain this reading, it seems that we would have to evaluate the predicate *hat just like mine* in the actual world, so as to obtain its actual extension (in the scenario we have sketched, the set of all Red Sox caps). But the existential quantifier expressed by the indefinite article in the *hat*-DP should not take scope

over the modal operator *want*, but below it, so that we can account for the fact that in different desire-worlds of Mary's, she buys possibly different hats.

There is a tension here: one aspect of the truth-conditions of this reading suggests that the DP *a hat just like mine* should be *outside* of the scope of *want*, but another aspect of these truth-conditions compels us to place it *inside* the scope of *want*. We can't have it both ways, it would seem, which is why this has been called a "scope paradox"

Another example of this sort, due to Bäuerle (1983), is (4):

- (4) Georg believes that a woman from Stuttgart loves every member of the VfB team.

Bäuerle describes the following scenario: Georg has seen a group of men on the bus. This group happens to be the VfB team (Stuttgart's soccer team), but Georg does not know this. Georg also believes (Bäuerle doesn't spell out on what grounds) that there is some woman from Stuttgart who loves every one of these men. There is no particular woman of whom he believes that, so there are different such women in his different belief-worlds. Bäuerle notes that (4) can be understood as true in this scenario. But there is a problem in finding an appropriate LF that will predict its truth here. First, since there are different women in different belief-worlds of Georg's, the existential quantifier *a woman from Stuttgart* must be inside the scope of *believe*. Second, since (in each belief world) there aren't different women that love each of the men, but one that loves them all, the *a*-DP should take scope over the *every*-DP. If the *every*-DP is in the scope of the *a*-DP, and the *a*-DP is in the scope of *believe*, then it follows that the *every*-DP is in the scope of *believe*. But on the other hand, if we want to capture the fact that the men in question need not be VfB-members in Georg's belief-worlds, the predicate *member of the VfB team* needs to be outside of the scope of *believe*. Again, we have a "scope paradox".

Before we turn to possible solutions for this problem, let's have one more example:

- (5) Mary hopes that a friend of mine will win the race.

This again seems to have three readings. In Fodor's terminology, the DP *a friend of mine* can be "non-specific *de dicto*," in which case (5) is true iff in every world where Mary's hopes come true, there is somebody who is my friend and wins. It can also have a "specific *de re*" reading: Mary wants John to win, she doesn't know John is my friend, but I can still report her hope as in (5). But there is a third option, the "non-specific *de re*" reading. To bring out this rather exotic reading, imagine this: Mary looks at the ten contestants and says *I hope one of the*

three on the right wins - they are so shaggy - I like shaggy people. She doesn't know that those are my friends. But I could still report her hope as in (5).

8.2 The Standard Solution: Overt World Variables

The scope paradoxes we have encountered can be traced back to a basic design feature of our system of intensional semantics: the relevant “evaluation world” for each predicate in a sentence is strictly determined by its LF-position. All predicates that occur in the (immediate) scope of the same modal operator must be evaluated in the same possible worlds. E.g. if the scope of *want* consists of the clause *a friend of mine (to) win*, then every desire-world w' will be required to contain an individual that wins in w' and is also my friend *in* w' . If we want to quantify over individuals that are my friends in the actual world (and not necessarily in all the subject's desire worlds), we have no choice but to place *friend of mine* outside of the scope of *want*. And if we want to accomplish this by means of QR, we must move the entire DP *a friend of mine*.

Not every kind of intensional semantics constrains our options in this way. One way to visualize what we might want is to write down an LF that looks promising:

- (6) Mary wanted _{w_o} [$\lambda w'$ [a hat-just-like-mine _{w_o}] λx_I [PRO to buy _{w'} x_I]]

We have annotated each predicate with the world in which we wish to evaluate it. w_o is the evaluation world for the entire sentence and it is the world in which we evaluate the predicates *want* and *hat-just-like-mine*. The embedded sentence contributes a function from worlds to truth-values and we insert an explicit λ -operator binding the world where the predicate *buy* is evaluated. The crucial aspect of (6) is that the world in which *hat-just-like-mine* is evaluated is the matrix evaluation world and not the same world in which its clause-mate predicate *buy* is evaluated. This LF thus looks like it might faithfully capture Fodor's third reading.

Logical forms with overt world variables such as (6) are in fact the standard solution to the problem presented by the third reading. Let us spell out some of the technicalities. Later, we will consider a couple of alternatives.

8.2.1 Semantic Values

In this new system, we do *not* relativize the interpretation function to a possible world. As in the old extensional system, the basic notion is just “ $\llbracket \alpha \rrbracket$,” i.e., “the semantic value of α ”. (Or “ $\llbracket \alpha \rrbracket^g$,” “the semantic value of α under assignment g ”, if α contains free variables.) However, semantic values are no longer always extensions; some of them still are, but others are intensions. Here are some

representative examples of the types of semantic values for various kinds of words.

8.2.2 Lexical entries

- (7) a. $\llbracket \text{smart} \rrbracket = \lambda w \in D_s. \lambda x \in D_e. x \text{ is smart in } w$
 b. $\llbracket \text{likes} \rrbracket = \lambda w \in D_s. \lambda x \in D_e. \lambda y \in D_e. y \text{ likes } x \text{ in } w$
 c. $\llbracket \text{teacher} \rrbracket = \lambda w \in D_s. \lambda x \in D_e. x \text{ is a teacher in } w$
 d. $\llbracket \text{friend} \rrbracket = \lambda w \in D_s. \lambda x \in D_e. \lambda y \in D_e. y \text{ is } x\text{'s friend in } w$
- (8) a. $\llbracket \text{believe} \rrbracket = \lambda w \in D_s. \lambda p \in D_{\langle s, t \rangle}. \lambda x \in D.$
 $\quad \forall w' [w' \text{ conforms to what } x \text{ believes in } w \rightarrow p(w') = 1]$
 b. $\llbracket \text{must} \rrbracket = \lambda w \in D_s. \lambda R \in D_{\langle s, st \rangle}. \lambda p \in D_{\langle s, t \rangle}.$
 $\quad \forall w' [R(w)(w') = 1 \rightarrow p(w') = 1]$
- (9) a. $\llbracket \text{Ann} \rrbracket = \text{Ann}$
 b. $\llbracket \text{and} \rrbracket = \lambda u \in D_t. [\lambda v \in D_t. u = v = 1]$
 c. $\llbracket \text{the} \rrbracket = \lambda f \in D_{\langle e, t \rangle}. \exists! x. f(x) = 1. \text{ the } y \text{ such that } f(y) = 1.$
 d. $\llbracket \text{every} \rrbracket = \lambda f \in D_{\langle e, t \rangle}. \lambda g \in D_{\langle e, t \rangle}. \forall x [f(x) = 1 \rightarrow g(x) = 1]$

The entries in (9) (for words whose extensions are constant across worlds) have stayed the same; their semantic values are still extensions. But the ones for predicates (ordinary ones and modal ones) in (7) and (8) have changed; these items now have as their semantic values what used to be their *intensions*.

8.2.3 Composition Rules

We abolish the special rule of Intensional Functional Application (IFA)¹ and go back to our old inventory of Functional Application, λ -Abstraction, and Predicate Modification².

8.2.4 Syntax

What we have at this point does not allow us to interpret even the simplest syntactic structures. For instance, we can't interpret the tree in (10).

- (10) $[_{VP} \text{ John leave}]$

¹ We also abolish the Extensional Functional Application rule (EFA), if we had that one (see section 7.2.3 “Semantic Reconstruction”).

² Actually, PM requires a slightly revised formulation: $\llbracket \alpha \beta \rrbracket^g = \lambda w \in D_s. \lambda x \in D_e. \llbracket \alpha \rrbracket^g(w)(x) = \llbracket \beta \rrbracket^g(w)(x) = 1$. But we will not be concerned with the compositional interpretation of modifier-structures here, so you won't be needing this rule.

The verb's type is $\langle s, et \rangle$, so it's looking for a sister node which denotes a *world*. *John*, which denotes an individual, is not a suitable argument.

We get out of this problem by positing more abstract syntactic structures (at the LF level). Specifically, we assume that there is a set of covert “world pronouns” which are generated as sisters to all lexical predicates in LF-structures. Officially, the variable would be a pair of an index and the type *s*. Inofficially, we will use “*w*” with a subscripted index, with the understanding that the “*w*” indicates we are dealing with a variable of type *s*. So, the syntax would generate something like (11):

(11) [John [leave w_{12}]]

The sentence would then obviously have an assignment-dependent extension (a truth-value), depending on what world the variable assignment assigns to the world variable with index 12. In our intensional system of Chapter 1 — 5, we were assuming the following principle:

(12) An utterance of a sentence (=LF) ϕ in world w is true iff $\llbracket \phi \rrbracket^w = 1$.

To achieve the same in our new system, we would have to ensure that the variable assignment assign the utterance world to the free world variable(s) in the sentence. Another possibility, which we will adopt here, is to introduce a variable binder on top of the sentence. We will assume the following kind of syntactic structure at LF:

(13) $[\lambda w_{12} [\text{John} [\text{leave } w_{12}]]]$

The sentence now has as its extension what used to be its intension, a proposition. The principle of utterance truth now is this:

(14) An utterance of a sentence (=LF) ϕ in world w is true iff $\llbracket \phi \rrbracket(w) = 1$.

Now, we have to look at more complex sentences. First, a simple case of embedding. The sentence is *John wants to leave*, which now as an LF like this:

(15) $[\lambda w_o [\text{John} [[\text{wants } w_o [[\lambda w_1 [\text{PRO} [\text{leave } w_1]]]]]]]]$

EXERCISE 8.1: Calculate the semantic value of (15). □

Next, look at an example involving a complex subject, such as *the teacher left*.

(16) $[\lambda w_o [[\text{the} [\text{teacher } w_o]] [\text{left } w_o]]]$

The verb will need a world argument as before. The noun *teacher* will likewise need one, so that *the* can get the required argument of type $\langle e, t \rangle$ (not $\langle s, et \rangle$!). If we co-index the two world variables, we derive as the semantic value for (16) what its intension would have been in old system. But nothing we have said forces us to co-index the two world variables, which is what will allow us to derive the third reading for relevant examples.

Consider what happens when the sentence contains both a modal operator and a complex DP in its complement.

(17) Mary wants a friend of mine to win.

There are now three predicates that need world arguments. Furthermore, there will be two λ -operators binding world variables. We can now represent the three readings (to make the structures more readable, we'll leave off most of the bracketing and start writing the world arguments as subscripts to the predicates):

- (18) a. non-specific *de dicto*:
 λw_o Mary wants _{w_o} [λw_i a friend-of-mine _{w_i} leave _{w_i}]
 b. specific *de re*:
 λw_o [a friend-of-mine _{w_o}] λx_3 Mary wants _{w_o} [λw_i x_3 leave _{w_i}]
 c. non-specific *de re*:
 λw_o Mary wants _{w_o} [λw_i a friend-of-mine _{w_o} leave _{w_i}]

In this new framework, then, we have a way of resolving the apparent “scope paradoxes” and of acknowledging Fodor’s point that there are two separate distinctions to be made when DPs interact with modal operators. First, there is the scopal relation between the DP and the operator; the DP may take wider scope (Fodor’s “specific” reading) or narrower scope (“non-specific” reading) than the operator. Second, there is the choice of binder for the world-argument of the DP’s restricting predicate; this may be cobound with the world-argument of the embedded predicate (Fodor’s “*de dicto*”) or with the modal operator’s own world-argument (“*de re*”). So the *de re-de dicto* distinction in the sense of Fodor is not *per se* a distinction of scope; but it has a principled connection with scope in one direction: Unless the DP is within the modal operator’s scope, the *de dicto* option (= co-binding the world-pronoun with the embedded predicate’s world-argument) is in principle unavailable. (Hence “specific” implies “*de re*”, and “*de dicto*” implies “non-specific”.) But there is no implication in the other direction: if the DP has narrow scope w.r.t. to the modal operator, either the local or the long-distance binding option for its world-pronoun is in principle available. Hence “non-specific” readings may be either “*de re*” or “*de dicto*”.

For the sake of clarity, we should introduce a different terminology than Fodor's. The labels "specific" and "non-specific" especially have been used in so many different senses by so many different people that it is best to avoid them altogether. So we will refer to Fodor's "specific readings" and "non-specific readings" as "wide-quantification readings" and "narrow-quantification readings", or "narrow-Q/wide-Q readings" for short. For the distinction pertaining to the interpretation of the restricting NP, we will keep the terms "*de re*" and "*de dicto*", but will amplify them to "restrictor-*de re*" and "restrictor-*de dicto*" ("R-*de re*" / "R-*de dicto*").

EXERCISE 8.2: For DPs with extensions of type *e* (specifically, DPs headed by the definite article), there is a truth-conditionally manifest R-*de re*/R-*de dicto* distinction, but no truth-conditionally detectable wide-Q/narrow-Q distinction. In other words, if we construct LFs analogous to (18a-c) above for an example with a definite DP, we can always prove that the first option (wide scope DP) and the third option (narrow scope DP with distantly bound world-pronoun) denote identical propositions. In this exercise, you are asked to show this for the example in (19).

(19) John believes that your abstract will be accepted. □

8.2.5 The Need for a Binding Theory for World Variables

One could in principle imagine some indexings of our LFs that we have not considered so far. The following LF indexes the predicate of the complement clause to the matrix λ -operator rather than to the one on top of its own clause.

(20) λw_o John wants _{w_o} [λw_i PRO leave _{w_o}]

Of course, the resulting semantics would be pathological: what John would be claimed to stand in the wanting relation to is a set of worlds that is either the entire set *W* of possible worlds (if the evaluation world is one in which John leaves) or the empty set (if the evaluation world is one in which John doesn't leave). Clearly, the sentence has no such meaning. Do we need to restrict our system to not generate such an LF? Perhaps not, if the meaning is so absurd that the LF would be filtered out by some overarching rules distinguishing sense from nonsense.

But the problem becomes real when we look at more complex examples. Here is one discussed by Percus in important work (Percus 2000):

(21) Mary thinks that my brother is Canadian.

Since the subject of the lower clause is a type *e* expression, we expect at least two readings: *de dicto* and *de re*, cf. Exercise 8.2. The two LFs are as follows:

- (22) a. *de dicto*
 λw_o Mary thinks_{*w_o*} [(that) λw_i my brother_{*w_i*} (is) Canadian_{*w_i*}]
 b. *de re*
 λw_o Mary thinks_{*w_o*} [(that) λw_i my brother_{*w_o*} (is) Canadian_{*w_i*}]

But as Percus points out, there is another indexing that might be generated:

- (23) λw_o Mary thinks_{*w_o*} [(that) w_i my brother_{*w_i*} (is) Canadian_{*w_o*}]

In (23), we have co-indexed the main predicate of the lower clause with the matrix λ -operator and co-indexed the nominal predicate *brother* with the embedded λ -operator. That is, in comparison with the *de re* reading in (22b), we have just switched around the indices on the two predicates in the lower clause.

Note that this LF will not lead to a pathological reading. So, is the predicted reading one that the sentence actually has? No. For the *de re* reading, we can easily convince ourselves that the sentence does have that reading. Here is Percus' scenario: "My brother's name is Allon. Suppose Mary thinks Allon is not my brother but she also thinks that Allon is Canadian." In such a scenario, our sentence can be judged as true, as predicted if it can have the LF in (22b). But when we try to find evidence that (23) is a possible LF for our sentence, we fail. Here is Percus:

If the sentence permitted a structure with this indexing, we would take the sentence to be true whenever there is some *actual* Canadian who *Mary thinks* is my brother — even when this person is not my brother in actuality, and *even when Mary mistakenly thinks that he is not Canadian*. For instance, we would take the sentence to be true when Mary thinks that Pierre (the Canadian) is my brother and naturally concludes — since she knows that *I* am American — that Pierre too is American. But in fact we judge the sentence to be *false* on this scenario, and so there must be something that makes the indexing in (23) impossible.

Percus then proposes the following descriptive generalization:

- (24) GENERALIZATION X: The situation pronoun that a verb selects for must be coindexed with the nearest λ above it.³

³ Percus works with situation pronouns rather than world pronouns, an immaterial difference for our purposes here.

We expect that there will need to be a lot of work done to understand the deeper sources of this generalization. For fun, we offer the following implementation (devised by Irene Heim).

8.2.6 Two Kinds of World Pronouns

We distinguish two syntactic types of world-pronouns. One type, *w-PRO*, behaves like relative pronouns and *PRO* in the analysis of H&K, ch. 8.5 (pp. 226ff.): it is semantically vacuous itself, but can move and leave a trace that is a variable. The only difference between *w-PRO* and *PRO* is that the latter leaves a variable of type *e* when it moves, whereas the former leaves a variable of type *s*. The other type of world-pronoun, *w-pro*, is analogous to bound-variable personal pronouns, i.e., it is itself a variable (here of type *s*). Like a personal pronoun, it can be coindexed with the trace of an existing movement chain.

With this inventory of world-pronouns, we can capture the essence of Generalization X by stipulating that *w-pro* is only generated in the immediate scope of a determiner (i.e., as sister to the determiner's argument). Everywhere else where a world-pronoun is needed for interpretability, we must generate a *w-PRO* and move it. This (with some tacit assumptions left to the reader to puzzle over) derives the result that the predicates inside nominals can be freely indexed but that the ones inside predicates are captured by the closest λ -operator.

As we said, there is plenty more to be explored in the Binding Theory for world pronouns. The reader is referred to the paper by Percus and the references he cites.

8.2.7 Excursus: Semantic reconstruction for *de dicto* raised subjects?

Let us look back at the account of *de dicto* readings of raised subjects that we sketched earlier in Section 7.2.3. We showed that you can derive such readings by positing a high type trace for the subject raising, a trace of type $\langle s, \langle et, t \rangle \rangle$. Before the lower predicate can combine with the trace, the semantic value of the trace has to be extensionalized by being applied to the lower evaluation world (done via the EFA composition principle). Upstairs the raised subject has to be combined with the λ -abstract (which will be of type $\langle \langle s, \langle et, t \rangle \rangle, t \rangle$) via its intension.

We then saw recently discovered data suggesting that syntactic reconstruction is actually what is going on. This, of course, raises the question of why semantic reconstruction is unavailable (otherwise we wouldn't expect the data that we observed).

Fox (2000, p. 171, fn. 41) mentions two possible explanations:

- (i) “traces, like pronouns, are always interpreted as variables that range over individuals (type e)”,
- (ii) “the semantic type of a trace is determined to be the lowest type compatible with the syntactic environment (as suggested in Beck (1996))”.

In this excursus, we will briefly consider whether our new framework has something to say about this issue. Let’s figure out what we would have to do in the new framework to replicate the account in the section on semantics reconstruction.

Downstairs, we would have a trace of type $\langle s, \langle et, t \rangle \rangle$. To calculate its extension, we do not need recourse to a special composition principle, but can simply give it a world-argument (co-indexed with the abstractor resulting from the movement of the w -PRO in the argument position of the lower verb).

Now, what has to happen upstairs? Well, there we need the subject to be of type $\langle s, \langle et, t \rangle \rangle$, the same type as the trace, to make sure that its semantics will enter the truth-conditions downstairs. But how can we do this?

We need the DP *somebody from New York* to have as its semantic value an intension, the function from any world to the existential quantifier over individuals who are people from New York in that world. This is actually hard to do in our system. It *would* be possible if (i) the predicate(s) inside the DP received w -PRO as their argument, and if (ii) that w -PRO were allowed to moved to adjoin to the DP. If we manage to rule out at least one of the two preconditions on principled grounds, we would have derived the impossibility of semantic reconstruction as a way of getting *de dicto* readings of raised subjects.

- (i) may be ruled out by the Binding Theory for world pronominals, when it gets developed.
- (ii) may be ruled out by principled considerations as well. Perhaps, world-abstractors are only allowed at sentential boundaries. See Larson (2002) for some discussion of recalcitrant cases, one of which is the object position of so-called intensional transitive verbs, the topic of another section.

8.3 Alternatives to Overt World Variables

We presented (a variant of) what is currently the most widely accepted solution to the scope paradoxes, which required the use of non-locally bound world-variables. There are some alternatives, one of which is to some extent a “notational variant”, the others involved syntactic scoping after all.

8.3.1 Indexed Operators

It is possible to devise systems where predicates maintain the semantics we originally gave them, according to which they are sensitive to a world of evaluation

parameter. The freedom needed to account for the third reading and further facts would be created by assuming more sophisticated operators that shift the evaluation world. Here is a toy example:

- (25) Mary wants [a [ACTUALLY_o friend-of-mine] leave]

The idea is that the ACTUALLY “temporarily” shifts the evaluation world back to what it was “before” the abstraction over worlds triggered by *want* happened.

This kind of system can be spelled out in as much detail as the world-variable analysis. Max Cresswell (1990) proves that the two systems are equivalent in their expressive power. The decision is therefore a syntactic one. Does natural language have a multitude of indexed world-shifters or a multitude of indexed world-variables? Cresswell suspects the former, as did Kamp (1971) who wrote:

I of course exclude the possibility of symbolizing the sentence by means of explicit quantification over moments. Such a symbolization would certainly be possible; and it would even make the operators P and F superfluous. Such symbolizations, however, are a considerable departure from the actual form of the original sentences which they represent — which is unsatisfactory if we want to gain insight into the semantics of English. Moreover, one can object to symbolizations involving quantification over such abstract objects as moments, if these objects are not explicitly mentioned in the sentences that are to be symbolized.

There is some resistance to world-time variables because they are not phonetically realized. But in an operator-based system, we’ll have non-overt operators all over the place. So, there is no a priori advantage for either system. We will stick with the more transparent LFs with world variables.

8.3.2 Scoping After All?

Suppose we didn’t give up our previous framework, in which the evaluation-world for any predicate was strictly determined by its LF-position. It turns out that there is a way (actually, two ways) to derive Fodor’s non-specific *de re* reading in that framework after all.

Recall again what we need. We need a way to evaluate the restrictive predicate of a DP with respect to the higher evaluation world while at the same time interpreting the quantificational force of the DP downstairs in its local clause. We saw that if we move the DP upstairs, we get the restriction evaluated upstairs but we also have removed the quantifier from where it should exert its force. And if we leave the DP downstairs where its quantificational forces is felt, its

restriction is automatically evaluated down there as well. That is why Fodor's reading is paradoxical for the old framework. In fact, though there is no paradox.

- Way 1 Raise the DP upstairs but leave a $\langle\langle e, t \rangle, t\rangle$ trace. This way the restriction is evaluated upstairs, then a quantifier extension is calculated, and that quantifier extension is transmitted to trace position. This is just what we needed.
- Way 2 Move the NP-complement of a quantificational D independently of the containing DP.⁴ Then we could generate three distinct LFs for a sentence like *Mary wants a friend of mine to win*: two familiar ones, in which the whole DP *a friend of mine* is respectively inside and outside the scope of *want*, plus a third one, in which the NP *friend of mine* is outside the scope of *want* but the remnant DP *a* [_{NP} *t*] has been left behind inside it:

$$(26) \quad [[_{NP} \text{f-o-m}] \lambda_I [\text{Mary} [\text{want} [[_{DP} a \text{ } t_{\langle e, t \rangle, I}] \text{win}]]]]$$

EXERCISE 8.3: Convince yourself that this third LF represents the narrow-quantification, restrictor-*de re* reading (Fodor's "non-specific *de re*"). □

We have found, then, that it is in principle possible after all to account for narrow-Q R-*de re* readings within our original framework of intensional semantics.

EXERCISE 8.4: In (26), we chose to annotate the trace of the movement of the NP with the type-label $\langle e, t \rangle$, thus treating it as a variable whose values are predicate-extensions (characteristic functions of sets of individuals). As we just saw, this choice led to an interpretable structure. But was it our only possible choice? Suppose the LF-structure were exactly as in (26), except that the trace had been assigned type $\langle s, et \rangle$ instead of $\langle e, t \rangle$. Would the tree still be interpretable? If yes, what reading of the sentence would it express? □

EXERCISE 8.5: We noted in the previous section about the world-pronouns framework that there was a principled reason why restrictor-*de dicto* readings necessarily are narrow-quantification readings. (Or, in Fodor's terms, why there is no such thing as a "specific *de dicto*" reading.) In that framework, this was simply a consequence of the fact that bound variables must be in the scope of their binders. What about the alternative account that we have sketched in the present section? Does this account also imply that R-*de dicto* readings are necessarily narrow-Q? □

4 Something like this was proposed by Groenendijk & Stokhof (1982) in their treatment of questions with *which*-DPs.

8.4 Scope, Restrictors, and the Syntax of Movement

To conclude our discussion of the ambiguities of DPs in the complements of modal operators, let us consider some implications for the study of LF-syntax. This will be very inconclusive.

Accepting the empirical evidence for the existence of narrow-Q R-*de re* readings which are truth-conditionally distinct from both the wide-Q R-*de re* and the narrow-Q R-*de dicto* readings, we are facing a choice between two types of theories. One theory, which we have referred to as the “standard” one, uses a combination of DP-movement and world-pronoun binding; it maintains that wide-quantification readings really do depend on (covert) syntactic movement, but *de re* interpretations of the restrictor do not. The other theory, which we may dub the “scopal” account, removes the restrictor from the scope of the modal operator, either by QR (combined with an $\langle et, t \rangle$ type trace) or by movement of the NP-restrictor by itself.

In order to adjudicate between these two competing theories, we may want to inquire whether the R-*de re* — *de dicto* distinction exhibits any of the properties that current syntactic theory would take to be diagnostic of movement. This is a very complex enterprise, and the few results to have emerged so far appear to be pointing in different directions.

We have already mentioned that it is questionable whether NPs that are complements to D can be moved out of their DPs. Even if it is possible, we might expect this movement to be similar to the movement of other predicates, such as APs, VPs, and predicative NPs. Such movements exist, but — as discussed by Heycock, Fox, and the sources they cite — they typically have no effect on semantic interpretation and appear to be obligatorily reconstructed at LF. The type of NP-movement required by the purely scopal theory of R-*de re* readings would be exceptional in this respect.

Considerations based on the locality of uncontroversial instances of QR provide another reason to doubt the plausibility of the scopal theory. May (1977b) argued, on the basis of examples like (27), that quantifiers do not take scope out of embedded tensed clauses.

- (27) a. Some politician will address every rally in John’s district.
 b. Some politician thinks that he will address every rally in John’s district.

While in (27a) the universal quantifier can take scope over the existential quantifier in subject position, this seems impossible in (27b), where the universal

quantifier would have to scope out of its finite clause. Therefore, May suggested, we should not attribute the *de re* reading in an example like our (28) to the operation of QR.

- (28) John believes that your abstract will be accepted.

As we saw above, the standard theory which appeals to non-locally bound world-pronouns does have a way of capturing the *de re* reading of (28) without any movement, so it is consistent with May's suggestion. The purely scopal theory would have to say something more complicated in order to reconcile the facts about (27) and (28). Namely, it might have to posit that DP-movement is finite-clause bound, but NP-movement is not. Or, in the other version, it would have to say that QR can escape finite clauses but only if it leaves a $\langle et, t \rangle$ type trace.

Both theories, by the way, have a problem with the fact that May's finite-clause-boundedness does not appear to hold for all quantificational DPs alike. If we look at the behavior of *every*, *no*, and *most*, we indeed can maintain that there is no DP-movement out of tensed complements. For example, (29) could mean that Mary hopes that there won't be any friends of mine that win. Or it could mean (with suitable help from the context) that she hopes that there is nobody who will win among those shaggy people over there (whom I describe as my friends). But it cannot mean merely that there isn't any friend of mine who she hopes will win.

- (29) Mary hopes that no friend of mine will win.

So (30) has R-*de dicto* and R-*de re* readings for *no friend of mine*, but no wide-quantification reading where the negative existential determiner *no* takes matrix scope. Compare this with the minimally different infinitival complement structure, which does permit all three kinds of readings.

- (30) Mary expects no friend of mine to win.

However, indefinite DPs like *a friend of mine*, *two friends of mine* are notoriously much freer in the scope options for the existential quantification they express. For instance, even the finite clause in (31) seems to be no impediment to a reading that is not only R-*de re* but also wide-quantificational (i.e., it has the existential quantifier over individuals outscoping the universal world-quantifier).

- (31) Mary hopes that a friend of mine will win.

The peculiar scope-taking behavior of indefinites (as opposed to universal, proportional, and negative quantifiers) has recently been addressed by a number of authors (Abusch 1994, Reinhart 1997, Winter 1997, Matthewson 1999, Kratzer 1998), and there are good prospects for a successful theory that generates even the *wide-Q* R-*de re* readings of indefinites without any recourse to non-local DP-movement. You are encouraged to read these works, but for our current purposes here, all we want to point out is that, with respect to the behavior of indefinites, neither of the two theories we are trying to compare seems to have a special advantage over the other. This is because wide-Q readings result from DP-movement according to *both* theories.

As we mentioned in the previous chapter, a number of recent papers have been probing the connection between *de dicto* readings and the effects of Binding Condition C applying at LF. These authors have converged on the conclusion that DPs which are read as *de dicto* behave w.r.t. Binding Theory as if they are located below the relevant modal predicate at LF, and DPs that are read as *de re* (i.e., wide-Q, R-*de re*) behave as if they are located above. It is natural to inquire whether the same kind of evidence could also be exploited to determine the LF-location of the NP-part of a DP which is read as narrow-quantificational but restrictor-*de re*. If this acted for Condition C purposes as if it were below the attitude verb, it would confirm the standard theory (non-locally bound world-pronouns), whereas if it acted as if it was scoped out, we'd have evidence for the scopal account. Sharvit (1998) constructs some of the relevant examples and reports judgments that actually favor the scopal theory.⁵ For example, she observes that (32a) does allow the narrow-Q, R-*de re*-reading indicated in (32b).

- (32) a. How many students who like John_i does he_i think every professor talked to?
 b. For which n does John think that every professor talked to n people in the set of students who actually like John?

More research is required to corroborate this finding.

As a final piece of potentially relevant data, consider a contrast in Marathi recently discussed by Bhatt (1999).

- (33) [ji bai kican madhe ahe]_i Ram-la watte ki [[t_i [ti
 REL woman kitchen in is Ram thinks that that woman
 bai]_i] kican madhe nahi]
 kitchen in not is
 'Ram thinks that the woman who is in the kitchen is not in the kitchen'

⁵ Sharvit's own conclusion, however, is not that her data supports the purely scopal theory.

- (34) Ram-la watte ki [[ji bai kican madhe ahe]_i [[t_i [ti bai]_i] kican madhe nahi]]
 Ram thinks that REL woman kitchen in is that woman kitchen in not is
 ‘Ram thinks that the woman who is in the kitchen is not in the kitchen’

The English translation of both examples has two readings: a (plausible) *de re* reading, on which Ram thinks of the woman who is actually in the kitchen that she isn’t, and an (implausible) *de dicto* reading, on which Ram has the contradictory belief that he would express by saying: “the woman in the kitchen is not in the kitchen”. The Marathi sentence (33) also allows these two readings, but (34) unambiguously expresses the implausible *de dicto* reading. Bhatt’s explanation invokes the assumption that covert movement in Hindi cannot cross a finite clause boundary. In (33), where the correlative clause has moved overtly, it can stay high or else reconstruct at LF, thus yielding either reading. But in (34), where it has failed to move up overtly, it must also stay low at LF, and therefore can only be *de dicto*. What is interesting about this account is that it crucially relies on a scopal account of the R-*de re*-R-*de dicto* distinction. (Recall that with type-e DPs like definite descriptions, there *is* no additional wide/narrow-Q ambiguity.) If the standard theory with its non-locally bindable world-pronouns were correct, we would not expect the constraint that blocks covert movement in (34) to affect the possibility of a *de re* reading.

In sum, then, the evidence appears to be mixed. Some observations appear to favor the currently standard account, whereas others look like they might confirm the purely scopal account after all. Much more work is needed.

8.5 A Recurring Theme: Historical Overview

To recap, the main shape of the phenomenon discussed in this chapter is that the intensional parameter (time, world) with respect to which the predicate restricting a quantifier is interpreted can be distinct from the one that is introduced by the intensional operator that immediately scopes over the quantifier. The crucial cases have the character of a “scope paradox”. This discovery is one that has been made repeatedly in the history of semantics. It has been made both in the domain of temporal dependencies and in the domain of modality. Here are some of the highlights of that history.⁶

1. The *now*-operator

A. Prior (1968) noticed a semantic problem with the adverb *now*. The main early researchers that addressed the problem were Kamp (1971) and Vlach

⁶ Some of this history can be found in comments throughout Cresswell’s book (Max Cresswell 1990), which also contains additional references

(1973). A good survey was prepared by van Benthem (1977). Another early reference is Saarinen (1979). The simplest scope paradox examples looked like this:

- (35) One day all persons now alive will be dead.

While for this example one could say that *now* is special in always having access to the utterance time, other examples show that an unbounded number of times need to be tracked. It became clear in this work that whether one uses a multitude of indexed *now* and *then*-operators or allows variables over times is a syntactic and not a deep semantic question.

2. The *actually*-operator

The modal equivalent of the Prior-Kamp scope paradox sentence is:

- (36) It might have been that everyone actually rich was poor.

Crossley & Humberstone (1977) discuss such examples. Double-indexed systems of modal logic were studied by Segerberg (1973) and Åqvist (1973). See also work by D. Lewis (1970a), van Inwagen (1980), and Hazen (1979). Indexed *actually*-operators are discussed by A. N. Prior & Fine (1977), Peacocke (1978), and Forbes (1983, 1985, 1989).

3. The time of nominal predicates

There is quite a bit of work that argues that freedom in the time-dependency of nominals even occurs when there is no apparent space for temporal operators. Early work includes Enç (1981, 1986). But see also Ejerhed (1980). More recently Musan's dissertation (Musan 1995) is relevant.

- (37) Every fugitive is back in custody.

4. Tense in Nominals

There is some syntactic work on tense in nominals, see for example Wiltschko (2003).

5. The Fodor-Reading

Examples similar to the ones from Fodor and Bäuerle that we used at the beginning of this chapter are discussed in many places (Ioup 1977, Hellan 1978, Abusch 1994, Bonomi 1995, Farkas 1997). The point that all these authors have made is that the NP-predicate restricting a quantifier may be evaluated in the actual world, even when that quantifier clearly takes scope below a modal predicate.

Heim (Heim 1991) gives an example like this:

- (38) Every time it could have been the case that the player on the left was on the right instead.

Here, *the player on the left* must be evaluated with respect to the actual world. But it is inside a tensed clause, which — as we saw earlier — is usually considered a scope island for quantifiers.

6. Explicit World Variables

Systems with explicit world/time variables were introduced by Tichy (1971) and Gallin (1975). A system (Ty2) with overt world-variables is used by Groenendijk & Stokhof in their dissertation on the semantics of questions. See also Zimmermann (1989) on the expressive power of that system.

7. Movement

The idea of getting the third reading via some kind of syntactic scoping has not been pursued much. But there is an intriguing idea in a paper by Bricker (1989), cited by Max Cresswell (1990, p. 76). Bricker formalizes a sentence like *Everyone actually rich might have been poor* as follows:

$$(39) \quad \exists X(\forall y(Xy \equiv \text{rich } y) \& \diamond \forall y(Xy \rightarrow \text{poor } y))$$

This is apparently meant to be interpreted as ‘there is a plurality X all of whose members are rich and it might have been the case that all of the members of X are poor’. This certainly looks like somehow a syntactic scoping of the restrictive material inside the universal quantifier out of the scope of the modal operator has occurred.

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