Rutgers Linguistics Workshop on Mixed Effects Models — Mixed effects linear regression —

09/16/2016
Judith Degen
Stanford University

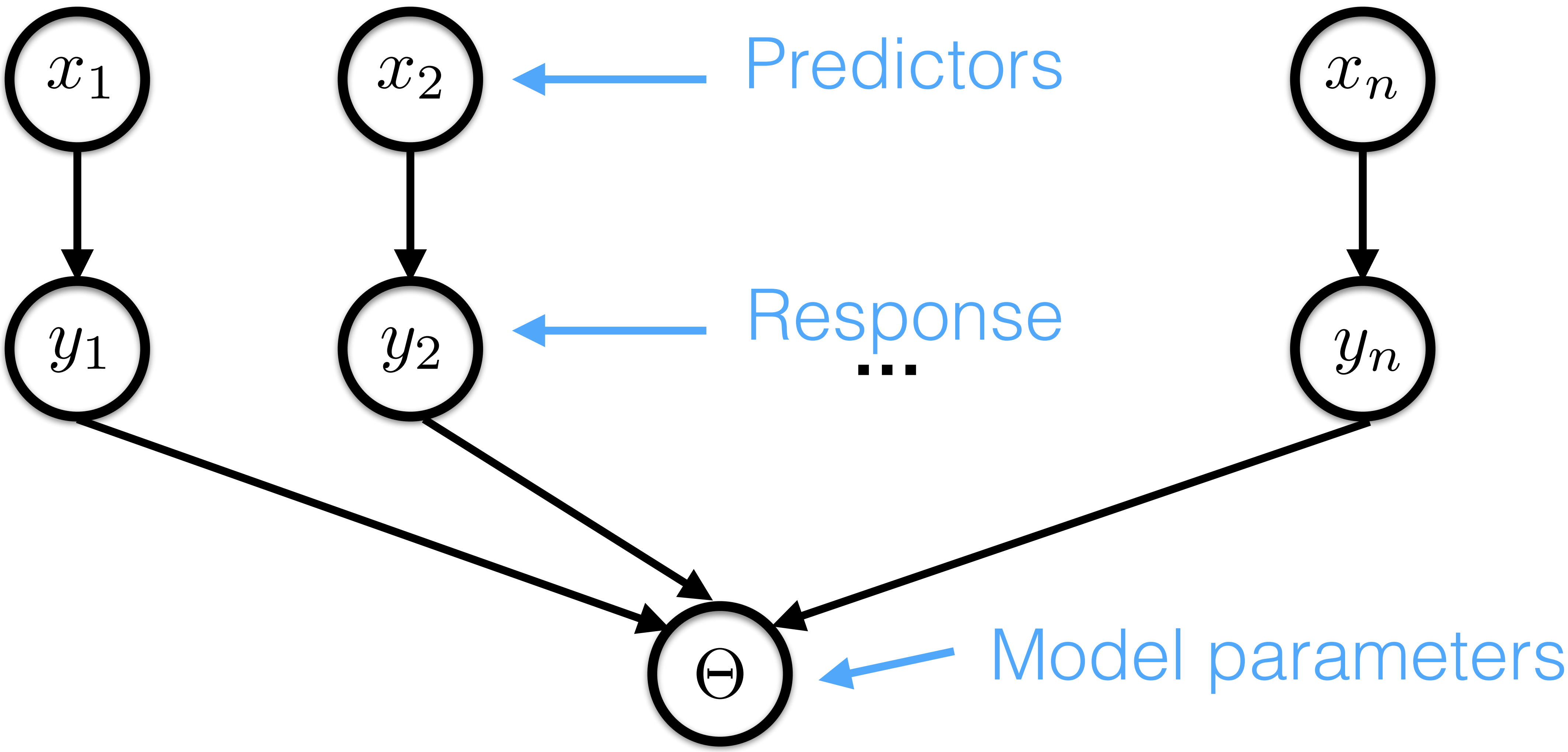
Generalized Linear Mixed Models

Experiments typically have more than one **participant** and more than one **item**
—> violation of the assumption of independence!

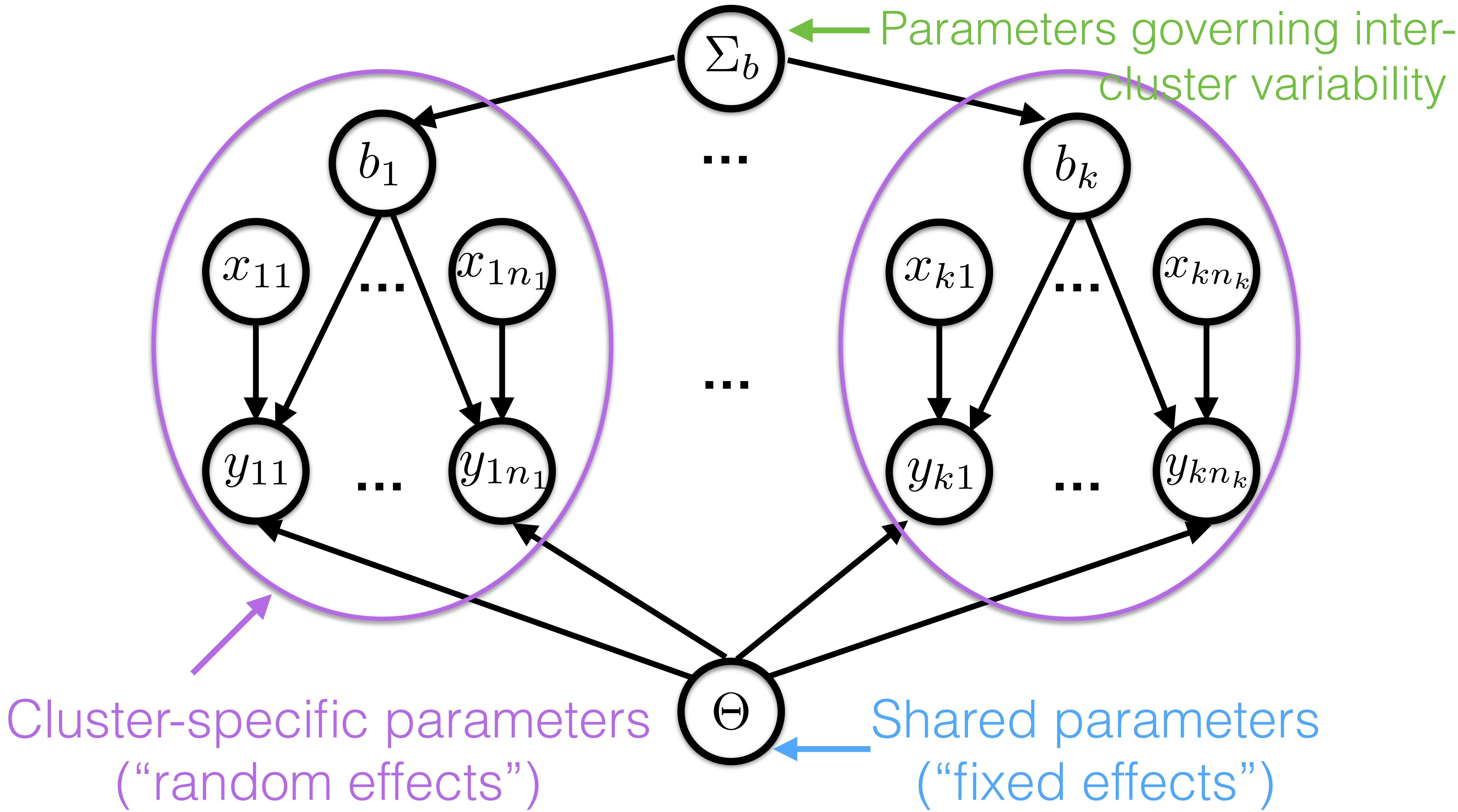
We would like to take these idiosyncrasies into account, and maybe even investigate them.

Solution: Generalized Linear Mixed or Multilevel Models (a.k.a. hierarchical, mixed-effects)

Recall: Generalized Linear Model



Generalized Linear Mixed Model



Mixed linear model

…back to our lexical decision experiment.

A number of predictors seem to affect RTs:

- frequency
- family size
- native language
- interactions

Additionally, different participants may have

- different overall decision speeds
- differing sensitivity to predictors

We want to draw inferences about all of this at the same time!

Mixed linear model

Let's allow each participant i to have idiosyncratic differences in response times:

$$RT_{ij} = \alpha + \beta x_{ij} + \overbrace{b_i}^{\sim N(0,\sigma_b)} + \epsilon_{ij}$$

- Idea: model distribution of participant differences as deviation from grand mean
- Mixed models approximate deviation by fitting a normal distribution
    - with mean 0
    - only additional parameter to fit is variance

Let's translate this into R!

Interpretation of the output

Random effects estimates

```
Random effects:
 Groups   Name        Variance Std.Dev.
 Subject  (Intercept) 0.02373  0.1541
 Residual             0.03274  0.1809
Number of obs: 1659, groups:  Subject, 21
```

Fixed effects estimates (same)

```
Fixed effects:
             Estimate Std. Error t value
(Intercept)  6.588778   0.037737   174.6
Frequency   -0.042872   0.003485   -12.3
```

2 notable differences:

1. random effects estimates
2. no more p-values!

Mixed models with one intercept

Let’s look at the by-subject adjustments to the intercept: Best Unbiased Linear Predictors (BLUPs)

BLUPs are *not* fitted parameters. Only one degree of freedom added to the model. BLUPs are estimated a posteriori based on the fitted model.

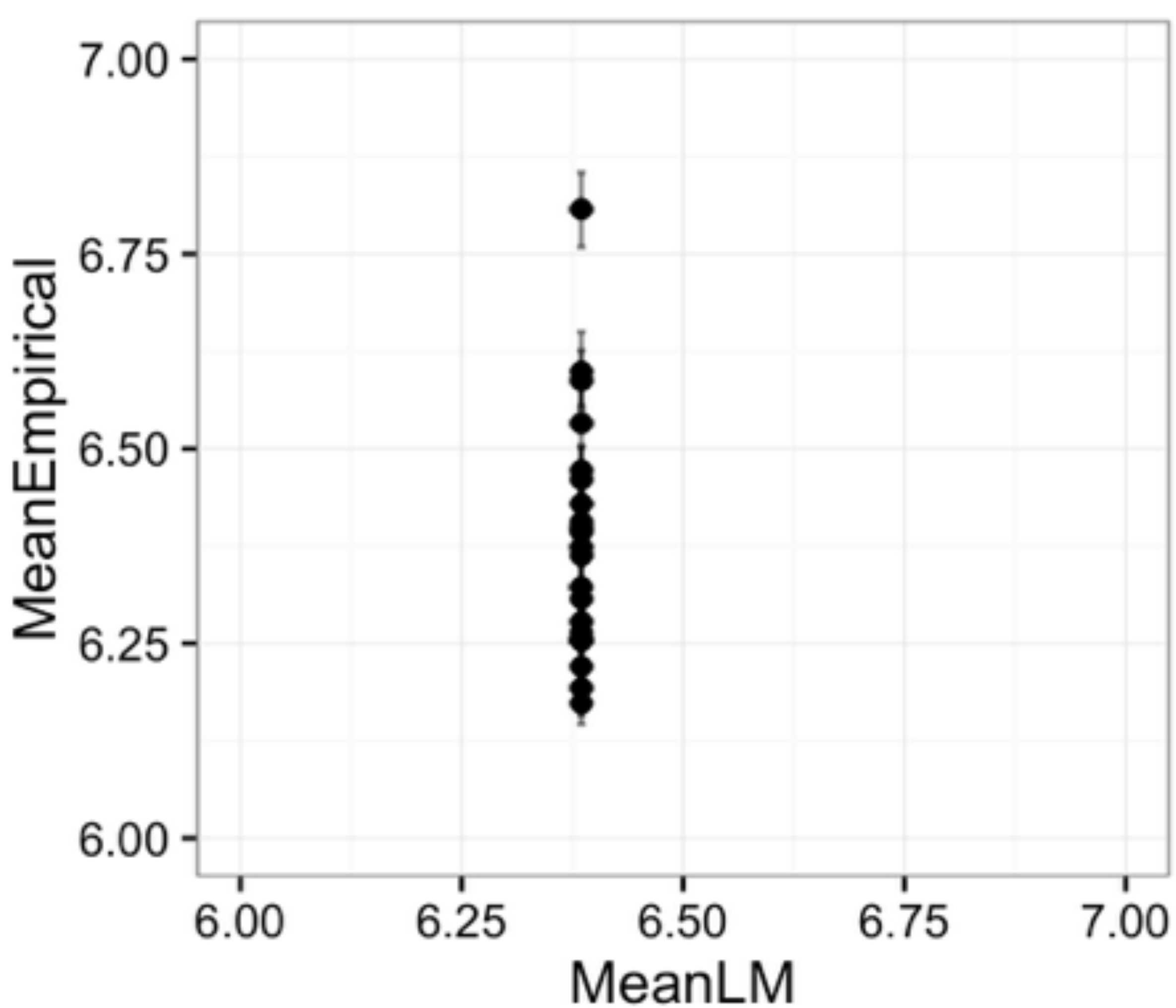
$$P(b_i|\hat{\alpha}, \hat{\beta}, \hat{\sigma_b}, \hat{\sigma_\epsilon}, X, Y)$$

The BLUPs are chosen to maximize this probability.

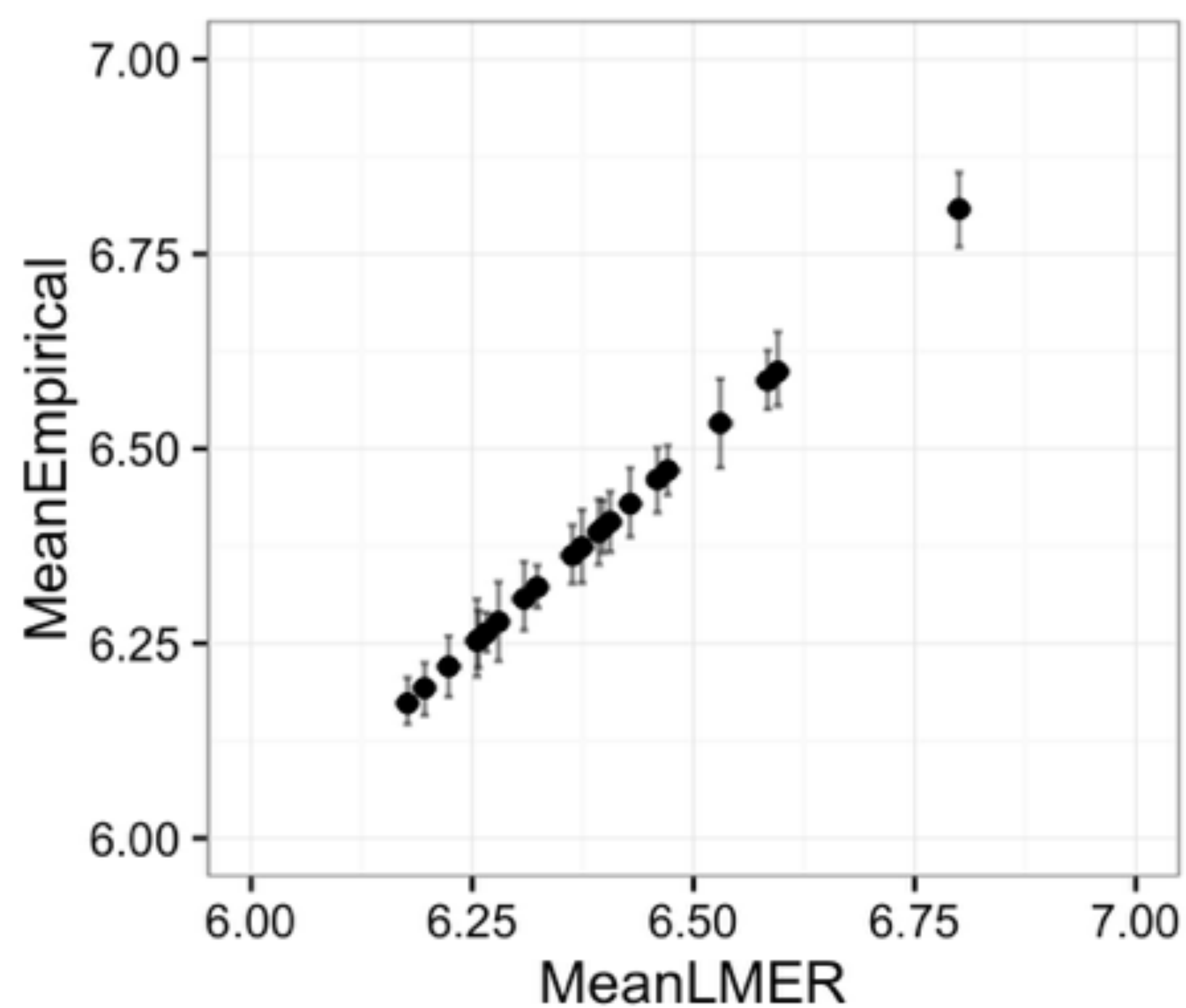
The effect

Predicted by-subject means

Linear model



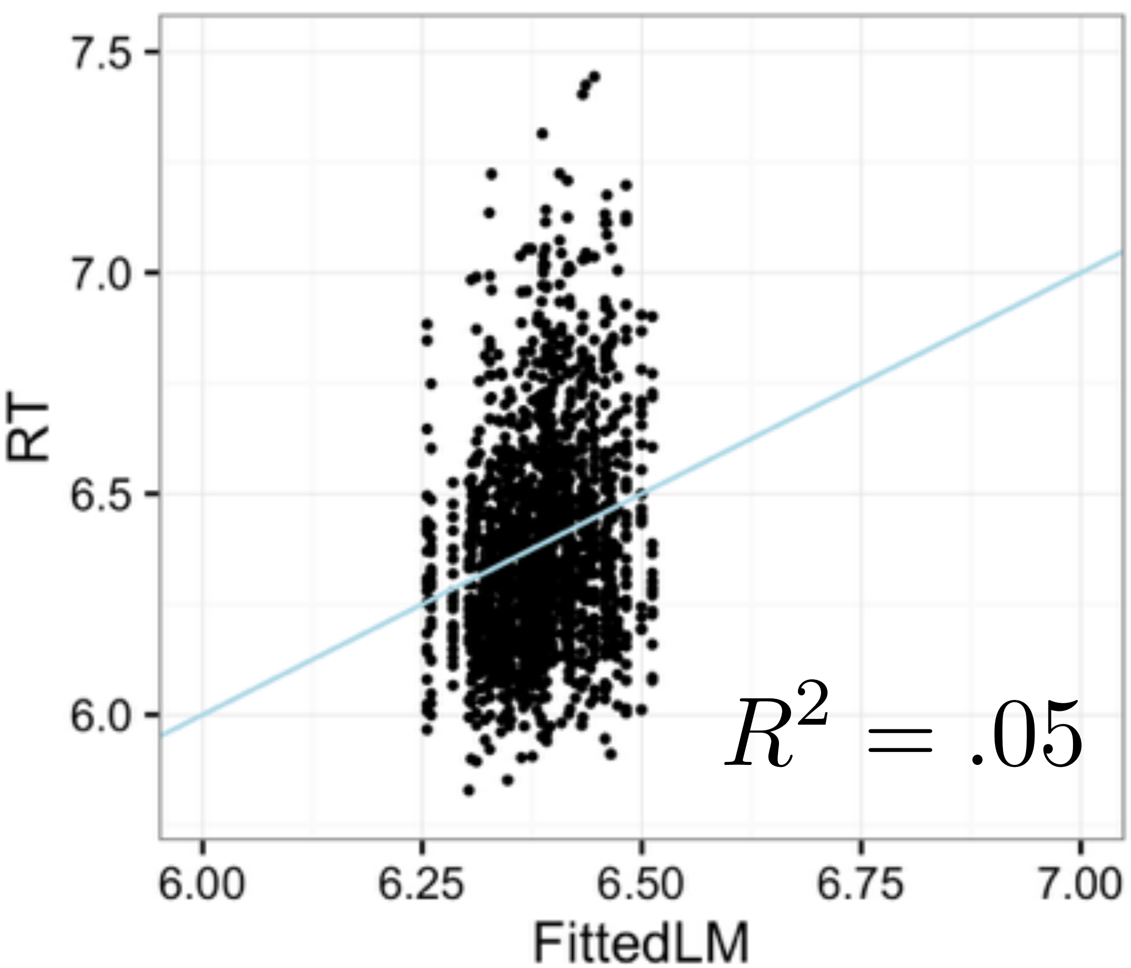
Mixed linear model



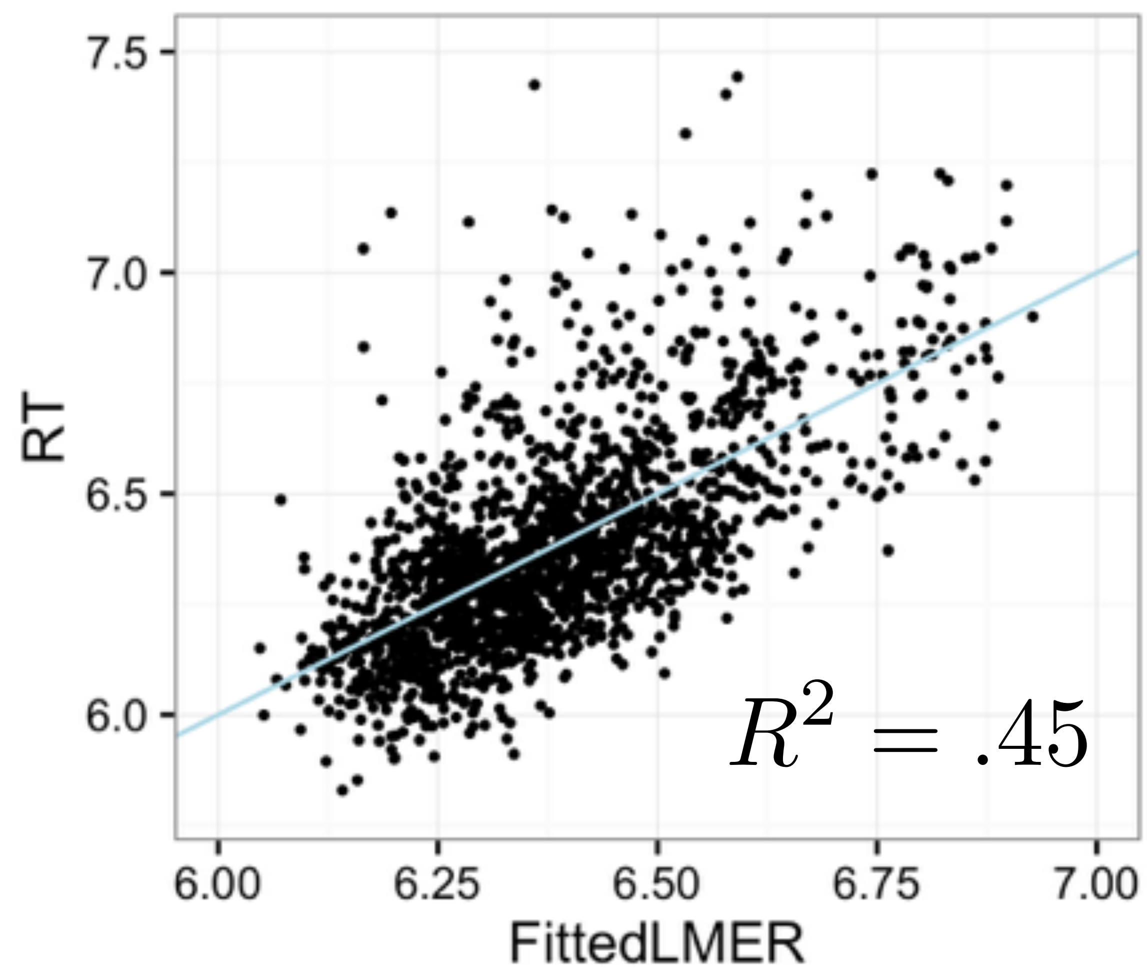
The effect

Predicted RTs

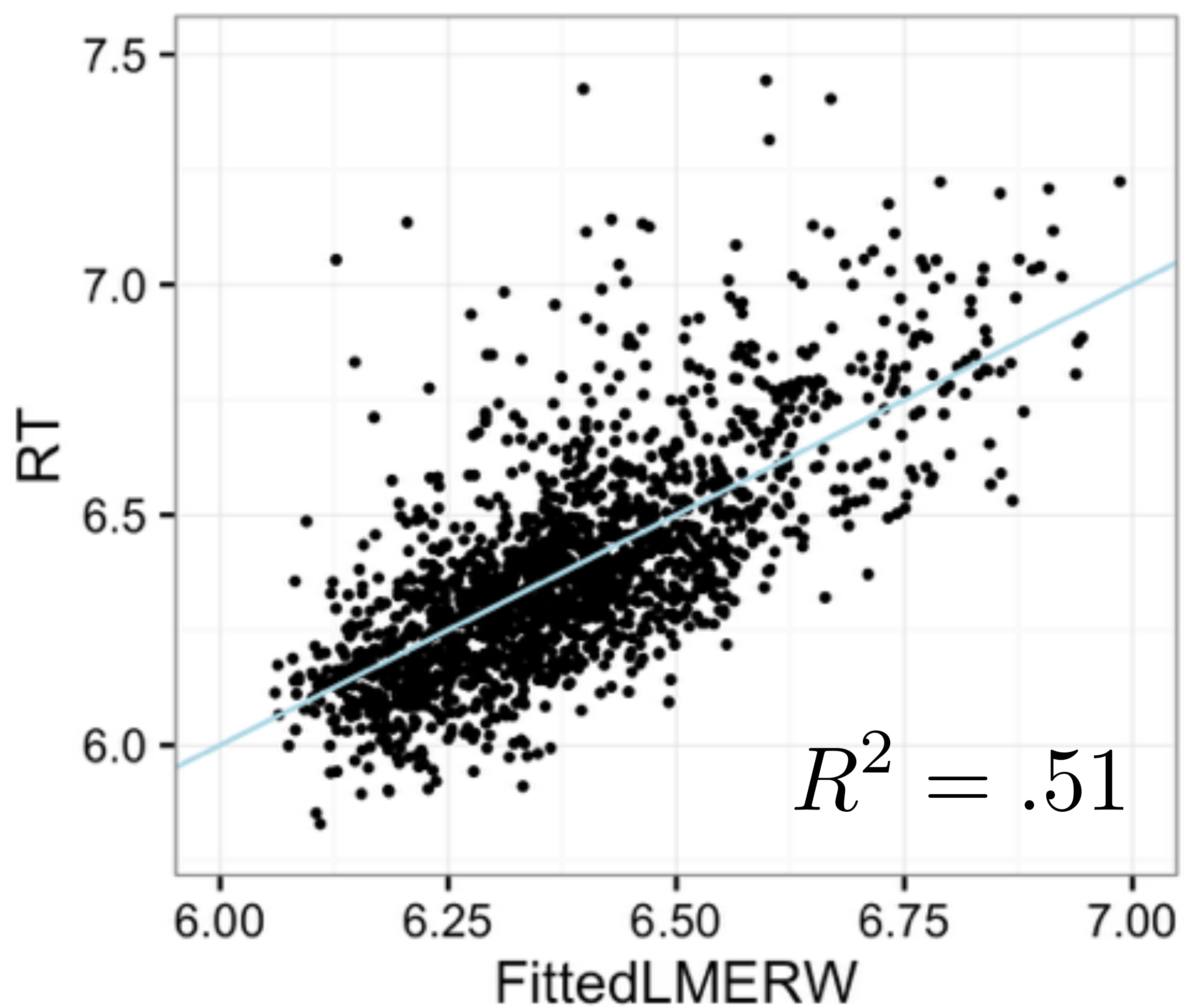
Linear model



Mixed linear model



A second random intercept



Mixed models with random slopes

Not just the intercept, but **any of the slopes** (of the predictors) may differ between individuals.

For example, subjects may show different sensitivity to frequency:

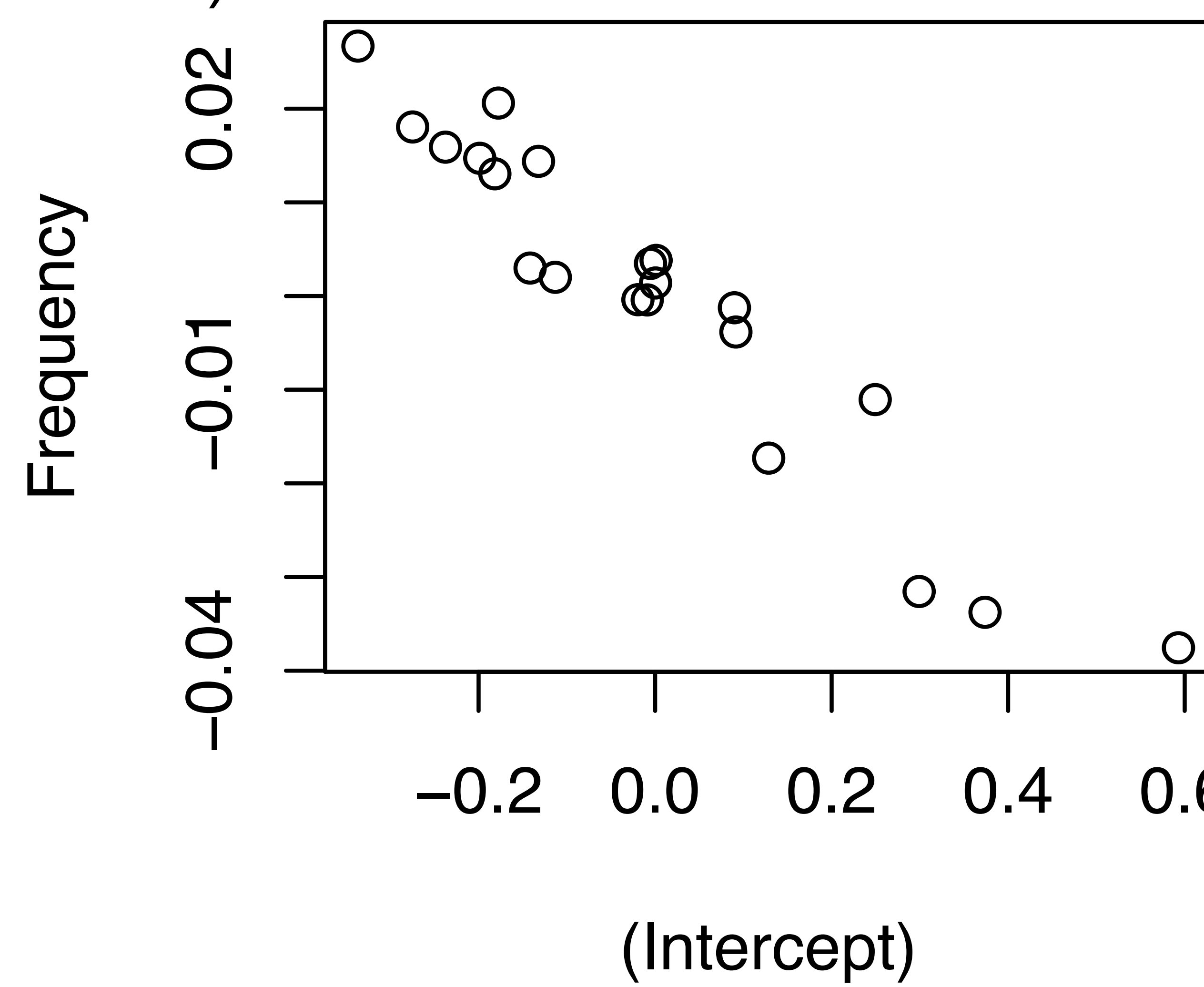
```
> head(ranef(m)$Subject)
    (Intercept)   Frequency
A1 -0.113081677 0.002001991
A2 -0.237501826 0.015898141
A3 -0.005238541 0.003482887
C  -0.132056269 0.014382981
D   0.001134412 0.003810049
I  -0.141643431 0.002989325
```

The BLUPs of the random slope reflect the by-subject adjustments to the overall frequency effect.

Random intercept, slope, and covariance

Random effects (e.g., intercepts and slopes) may be correlated.
By default, R fits these covariances, introducing additional degrees of freedom (parameters).

What do correlations between random effects mean?



Where did our p-values go?

- t-value anti-conservative, so no p-values!
- there are many different options for computing p-values if you really need them. You can check these in R directly by saying `?pvalues`
- some reasonably easy to use ones for **fixed effects**:
    - likelihood ratio tests via anova
    - lmerTest package (uses Satterthwaite or Kenward-Roger approximations)