Beyond the exponential family

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Away from the exponential family

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- `mgcv` has expanded to cover many new families
- Lets you model a much wider range of scenarios with smooths

What we’ll cover

- “Counts”: Negative binomial and Tweedie distributions
- Modelling proportions with the Beta distribution
- Robust regression with the Student's t distribution
- Ordered and unorderd categorical data
- Multivariate normal data
- Modelling exta zeros with zero-inflated and adjusted families

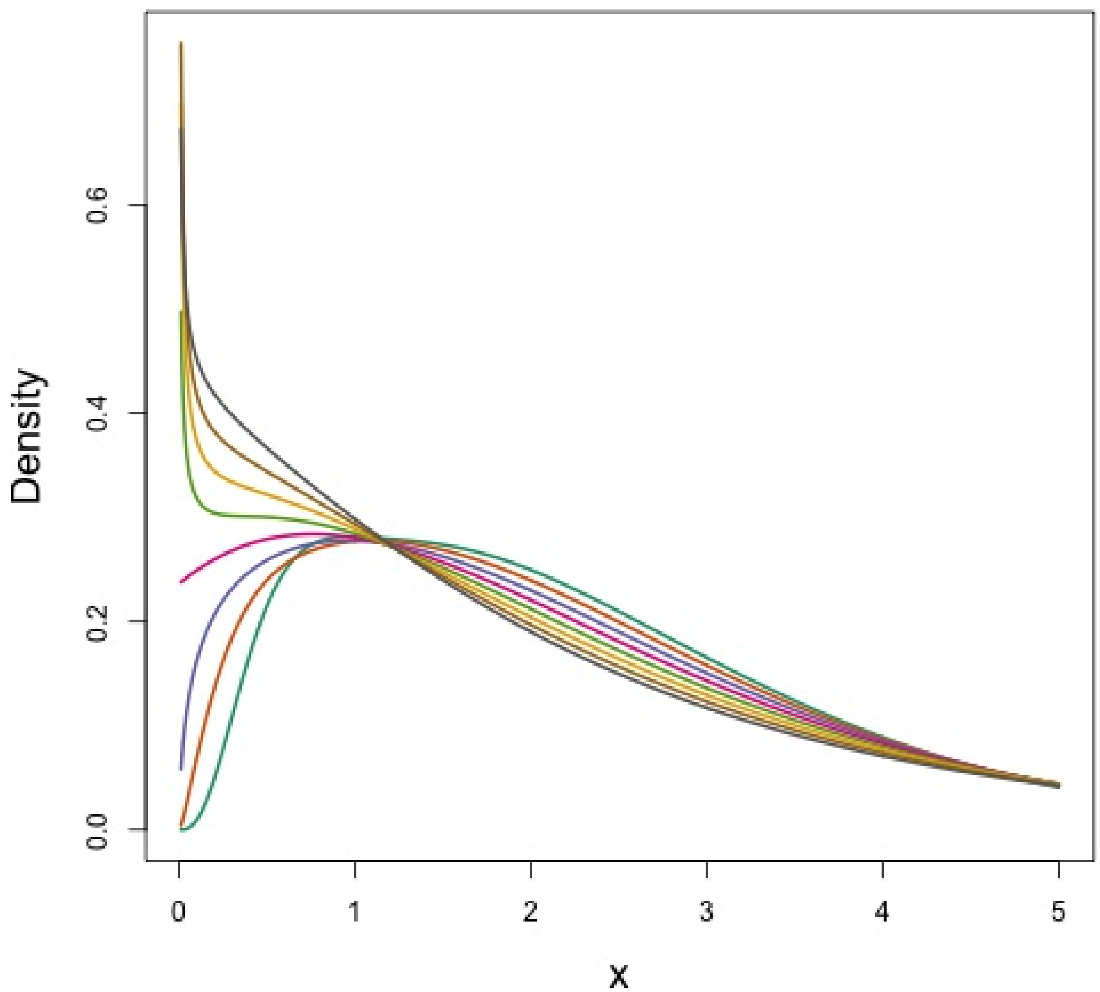
- *NOTE*: All the distributions we're covering here have their own quirks. Read the help files carefully before using them!

Modelling ”counts”

Counts and count-like things

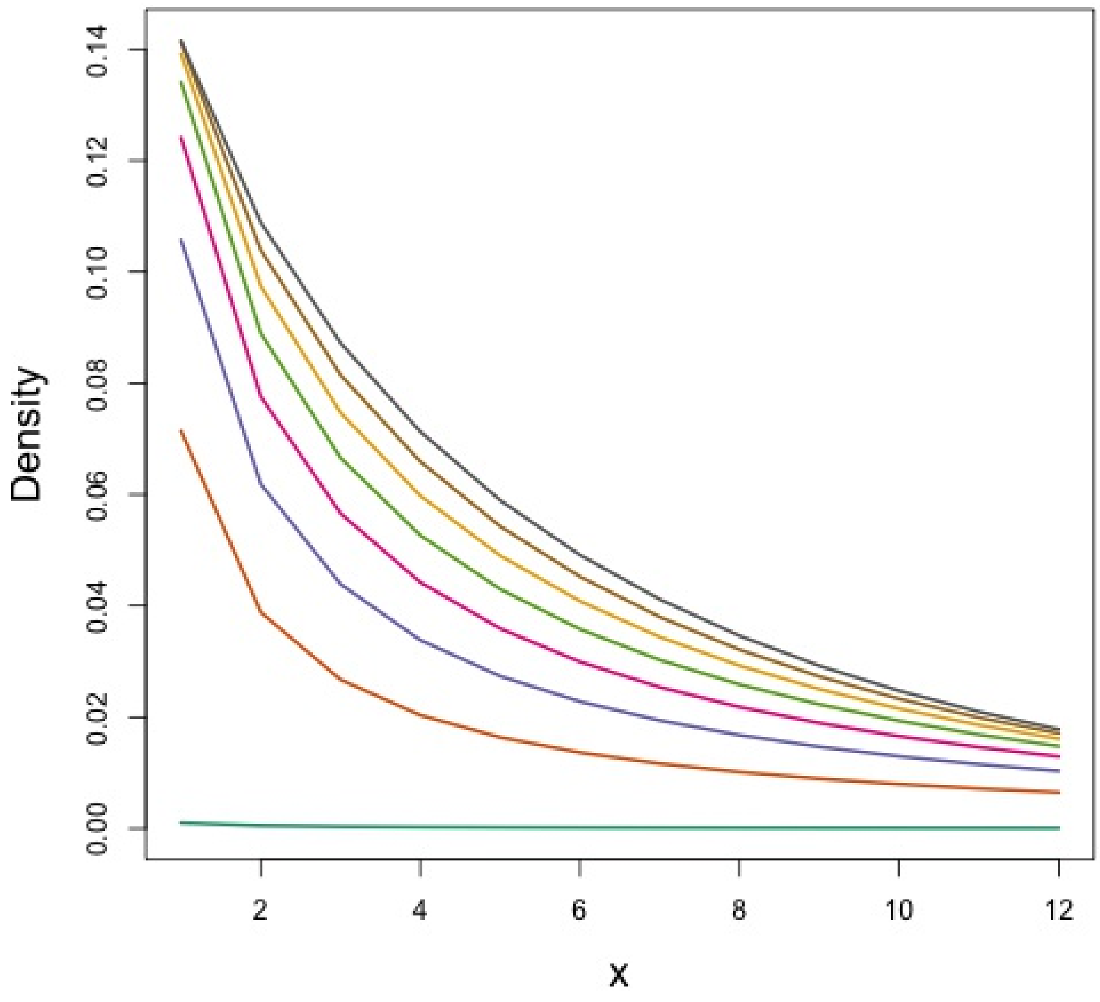
- Response is a count (not always integer)
- Often, it's mostly zero (that's complicated)
- Could also be catch per unit effort, biomass etc
- Flexible mean-variance relationship

Tweedie distribution



- $\mathrm{Var}(\mathrm{count}) = \varphi(\mathrm{count})^q$
- Common distributions are sub-cases:
  - $q = 1 \Rightarrow$ Poisson
  - $q = 2 \Rightarrow$ Gamma
  - $q = 3 \Rightarrow$ Normal
- We are interested in $1 < q < 2$
- (here $q = 1.2, 1.3, \ldots, 1.9$)
- `tw()`

Negative binomial



- $\text{Var}(\text{count}) = (\text{count}) + \varkappa(\text{count})^2$
- Estimate $\varkappa$
- Is quadratic relationship a "strong" assumption?
- Similar to Poisson: $\text{Var}(\text{count}) = (\text{count})$
- nb()

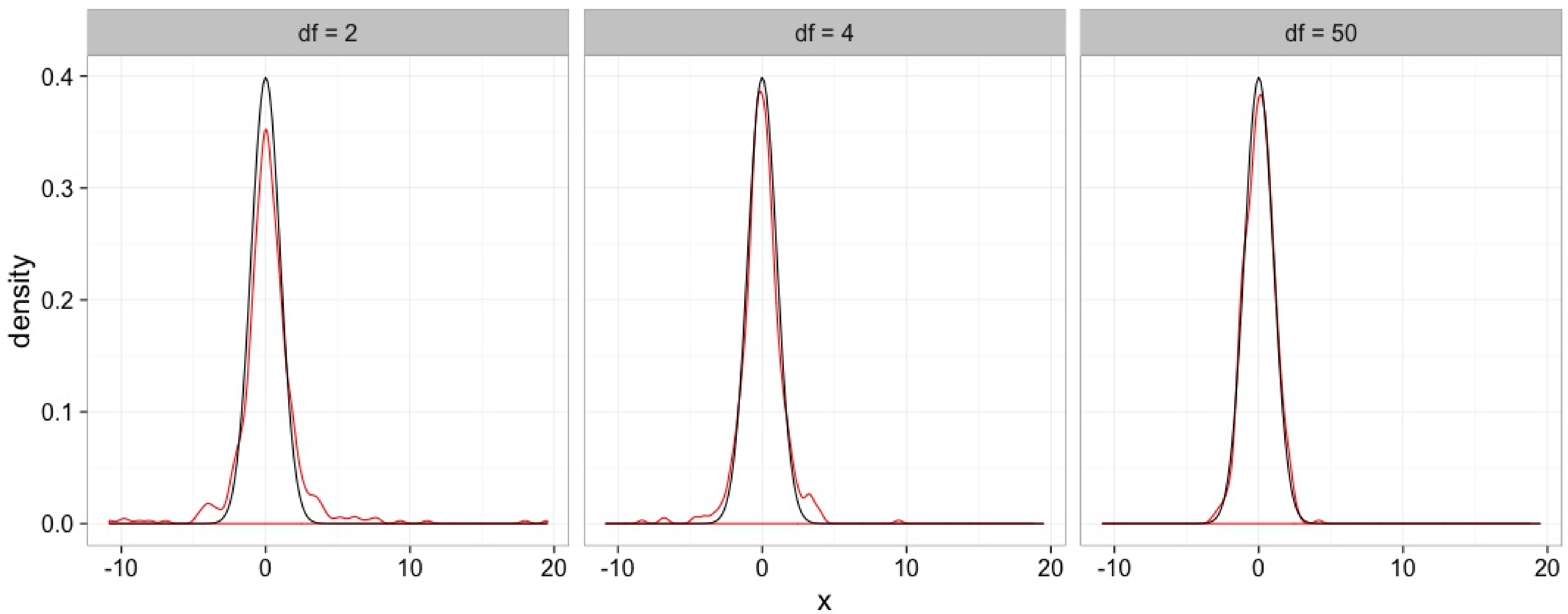
Modelling proportions

The Beta distribution

Modelling outliers

The student-t distribution

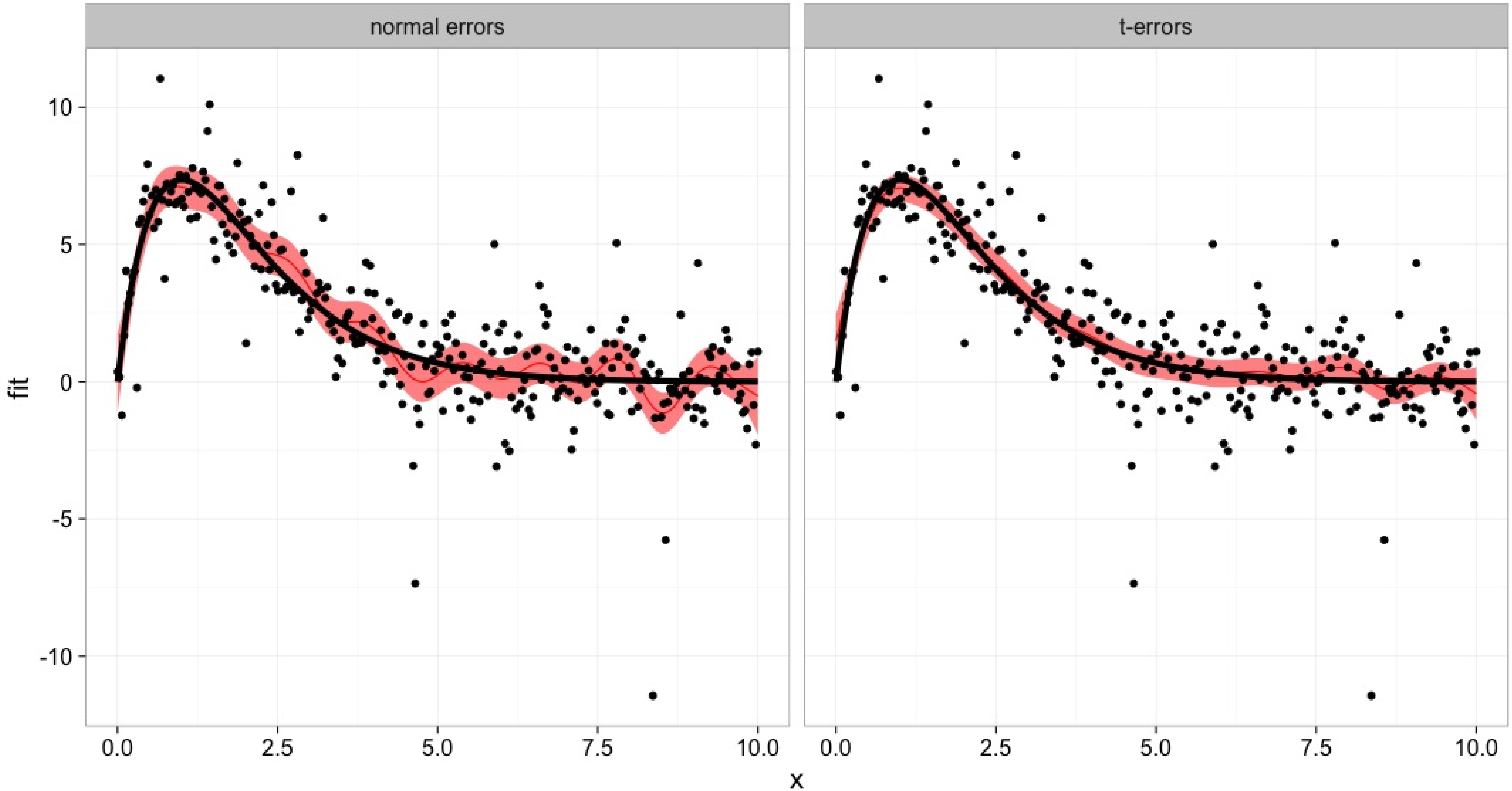
- Models continuous data w/ longer tails than normal
- Far less sensitive to outliers
- Has one extra parameter: df.
- bigger df: t dist approaches normal



The student-t distribution: Usage

```
set.seed(4)
n=300
dat = data.frame(x=seq(0,10,length=n))
dat$f = 20*exp(-dat$x)*dat$x
dat$y  = 1*rt(n,df = 3) + dat$f
norm_mod =  gam(y~s(x,k=20), data=dat,
family=gaussian(link="identity"))
t_mod = gam(y~s(x,k=20), data=dat, family=scat(link="identity"))
```

The student-t distribution: Usage



The student-t distribution: Usage

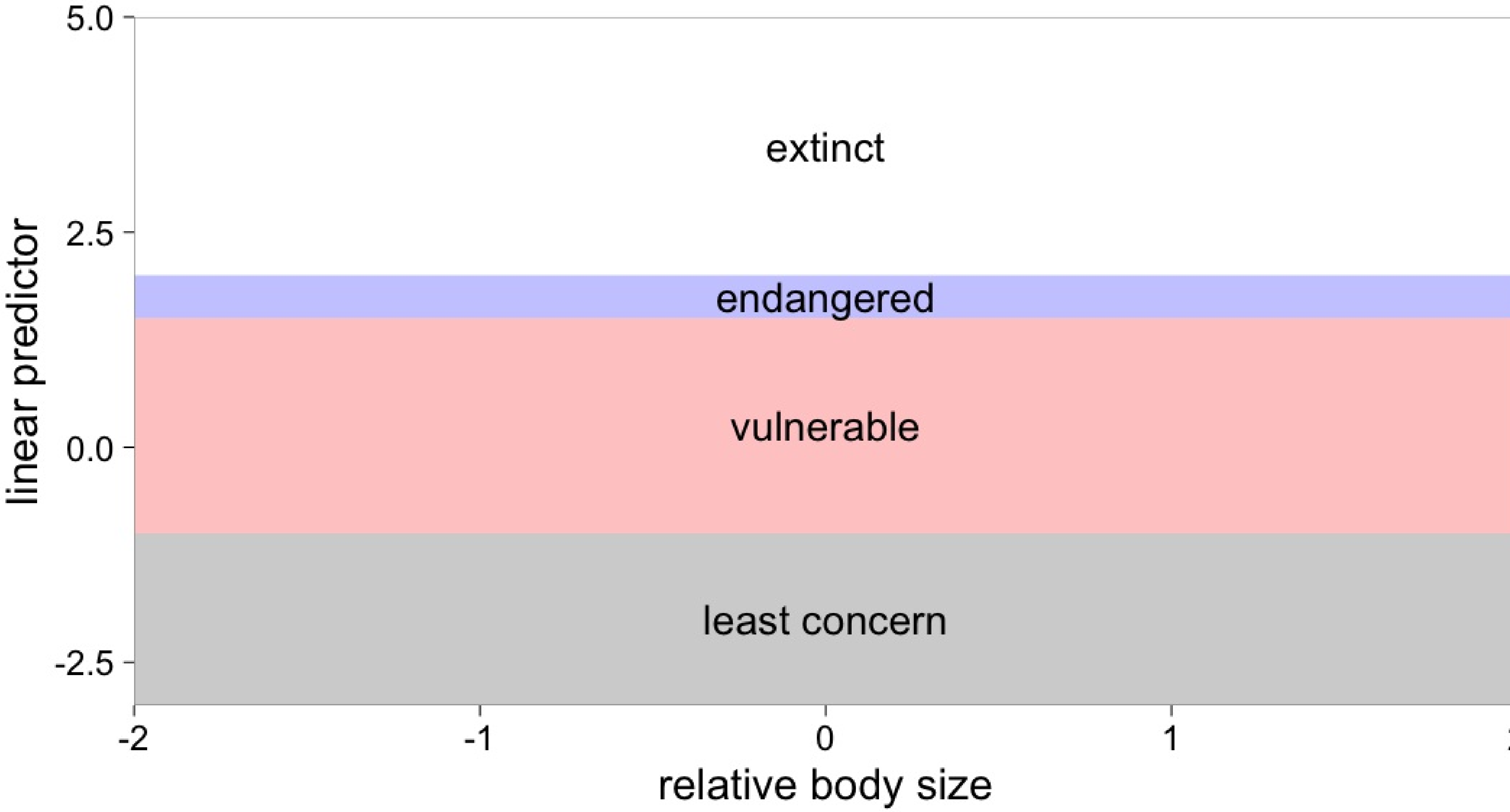
```
Family: Scaled t(2.976,0.968)
Link function: identity

Formula:
y ~ s(x, k = 20)

Parametric coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  2.02664    0.06853   29.57   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Approximate significance of smooth terms:
       edf Ref.df Chi.sq p-value
s(x) 13.27  15.71   1221  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R-sq.(adj) =  0.695   Deviance explained = 63.1%
-REML = 546.75  Scale est. = 1         n = 300
```

Modelling multi-dimensional data

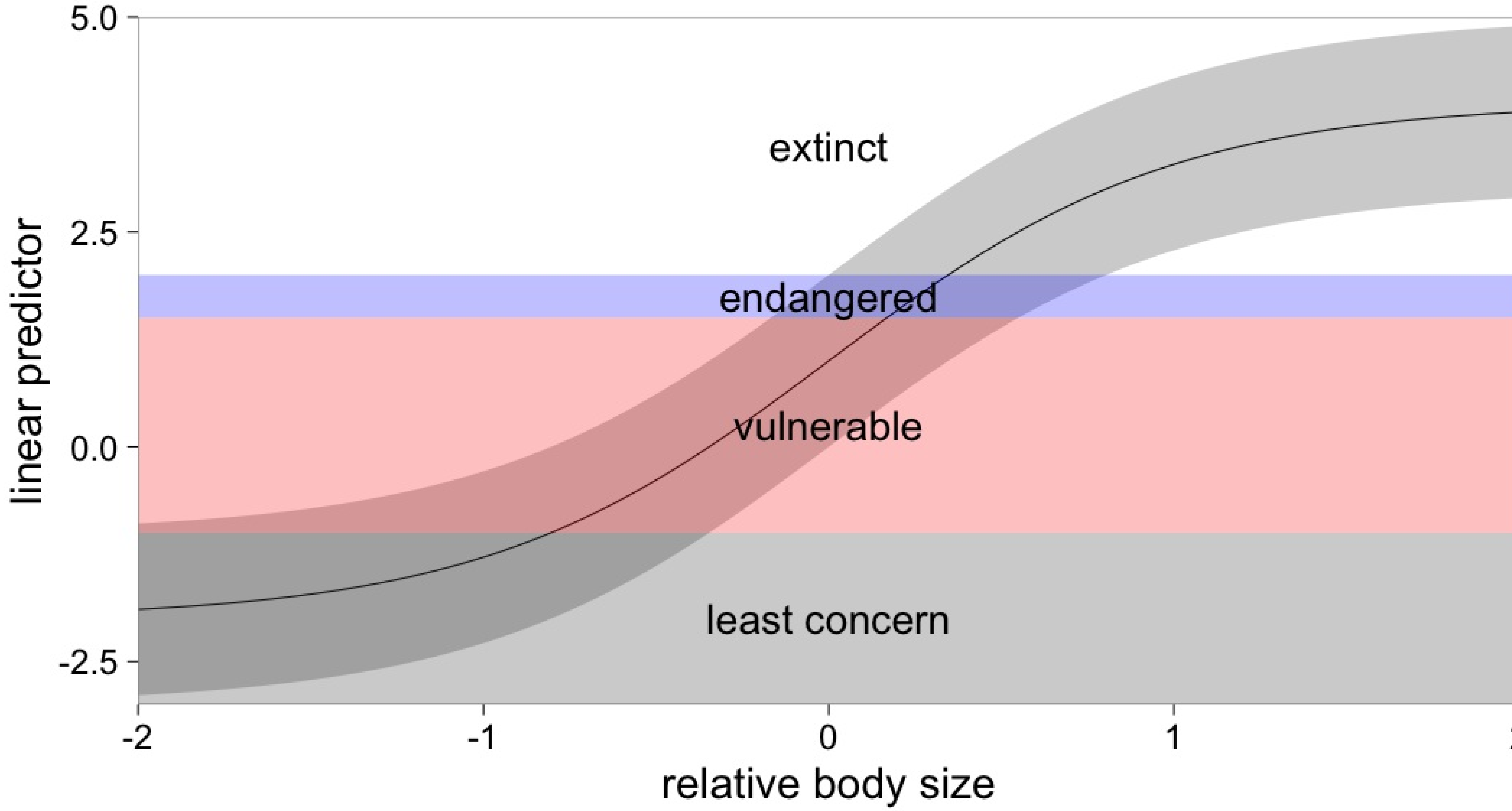
Ordered categorical data

- Assumes data are in discrete categories, and categories fall in order
- e.g.: conservation status: “least concern”, “vulnerable”, “endangered”, “extinct”
- fits a linear latent model using covariates, w/ threshold for each level
- First cut-off always occurs at -1

Ordered categorical data

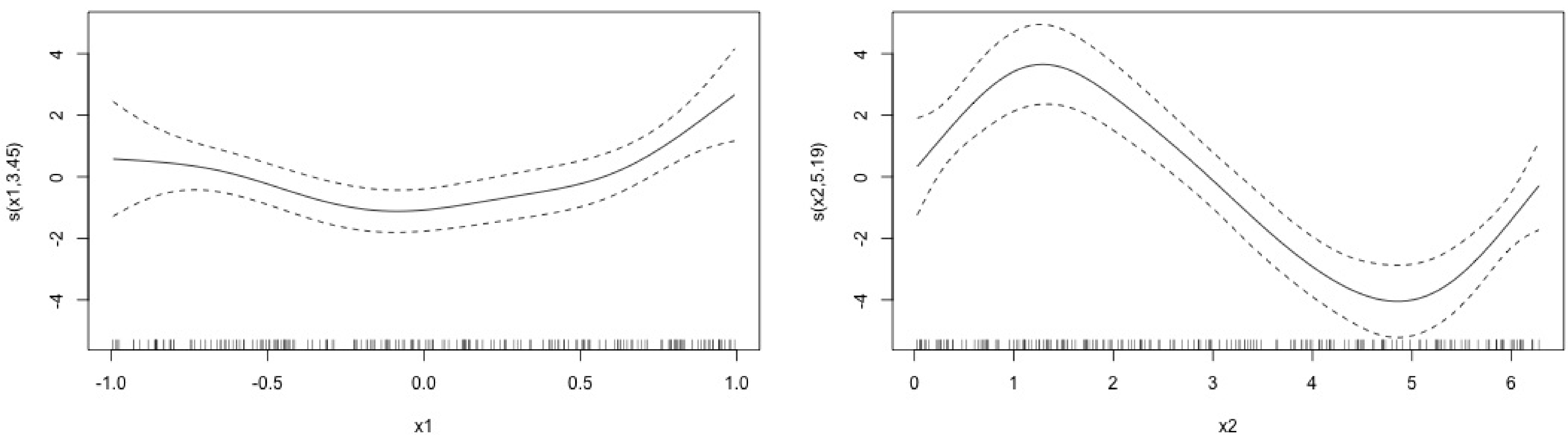


Ordered categorical data



Using ocat

```
n= 200
dat = data.frame(x1 = runif(n,-1,1),x2=2*pi*runif(n))
dat$f = dat$x1^2 + sin(dat$x2)
dat$y_latent = dat$f + rnorm(n,dat$f)
dat$y = ifelse(dat$y_latent<0,1, ifelse(dat$y_latent<0.5,2,3))
ocat_model = gam(y~s(x1)+s(x2), family=ocat(R=3),data=dat)
plot(ocat_model,page=1)
```



Using ocat

```
summary(ocat_model)
```

```

Family: Ordered Categorical(-1,-0.09)
Link function: identity

Formula:
y ~ s(x1) + s(x2)

Parametric coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)   0.5010     0.2792   1.794   0.0727 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Approximate significance of smooth terms:
        edf Ref.df Chi.sq  p-value
s(x1) 3.452  4.282  18.67  0.00133 **
s(x2) 5.195  6.270  84.34 1.09e-15 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Deviance explained = 57.7%
-REML =  97.38  Scale est. = 1         n = 200
```

Using ocat

```
Error in eval(expr, envir, enclos) : invalid subscript type
'double'
```