How do we represent data?

Jeffrey Leek, Assistant Professor of Biostatistics
Johns Hopkins Bloomberg School of Public Health

How do we write about data?

- Each data point is usually represented by a capital letter.
  - H for height, W for weight.
- If there are more than one data point of the same type we use subscripts.
  - H_1, H_2, H_3 for three different people's heights.
- Sometimes it is more compact to write X_1 for height and X_2 for weight.
- Then we need another subscript for the individual data point
  - X_{11} for the height of the first person.
- Y representes general outcomes and X general covariates.
- In this course we will try to use informative letters when possible.

Randomness

- Variables like X and Y are called *random variables* because we expect them to be *random* in some way.
- In general, randomness is a hard thing to define
- In this class a variable may be random because
  - It represents an incompletely measured variable
  - It represents a sample drawn from a population using a random mechanism.
- Once we are talking about a specific value of a variable we have observed it isn't random anymore, we write these values with lower case letters x, y, etc.
- We write $X = x$ or $X = 1$ to indicate we have observed a specific value x or 1.

Randomness and measurement

- A coin flip is commonly considered random
- But it can be modeled by deterministic equations
  - Dynamical bias in the coin toss (Diaconis, Holmes and Montgomery SIAM Review 2007)
  - Modeled the tossing as a dynamical system
  - Showed that a coin is more likely to land on the side it started
  - Did experiments that demonstrated it was a 51% chance
- Some have taken it a bit farther making predictable coin flipping machines based on physical properties.

Distributions

- In statistical modeling, random variables like X are assumed to be samples from a *distribution*
- A distribution tells us the possible values of X and the probabilities for each value.
- Probability is the chance something will happen and is abbreviated Pr
- The probabilities must all be between 0 and 1.
- The probabilities must add up to 1.
- An example:
  - Let's flip a coin and allow X to represent whether it is heads or tails
  - $X = 1$ if it is heads and $X = 0$ if it is tails
  - We expect that about 50% of the time it will be heads.
  - The distribution can then be written Pr($X = 1$)=0.5 and Pr($X = 0$)=0.5

Continuous versus discrete distributions

- *discrete* distributions specify probabilities for discrete values
  - Qualitative variables are discrete
  - So are variables that take on all values 0,1,2,3...
- *continuous* distributions specify probabilities for ranges of values
  - Quantitative variables are often assumed to be continuous
  - But we might only see specific values

Parameters

- Distributions are defined by a set of fixed values called *parameters*.
- *parameters* are sometimes represented by Greek letters like μ, σ, τ.
- Distributions are written as letters with the parameters in parentheses like $N(\mu, \sigma)$ or $Poisson(\lambda)$.
- $X \sim N(\mu, \sigma)$ means that X has the $N(\mu, \sigma)$ distribution.

The three most important parameters

- If X is a random variable, the mean of that random variable is written $E[X]$
  - Stands for expected value
  - Measures the "center" of a distribution
- The variance of that random variable is written $Var[X]$
  - Measures how "spread out" a distribution is
  - Measurement is in (units of X)^2
- The standard deviation is written $SD[X] = \sqrt{Var[X]}$
  - Also measures how "spread out" a distribution is
  - Measurement is in units of X

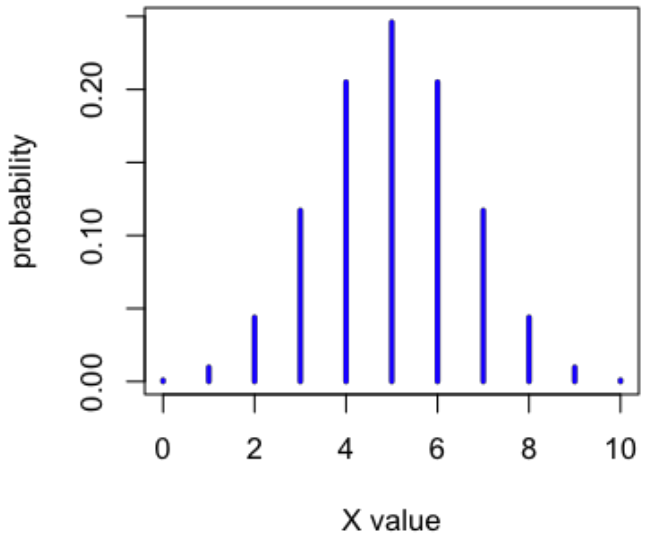
Conditioning

- The variables X are considered to be random
- The parameters are considered to be fixed values
- Sometimes we want to talk about a case where one of the random variables is fixed
- To indicate what is fixed, we *condition* using the symbol "|""
  - $X|\mu$ means that X is a random variable with fixed parameter μ
  - $Y|X = 2$ means Y is the random variable Y when X is fixed at 2.

Example distribution: Binomial

Binomial distribution: $Bin(n, p)$

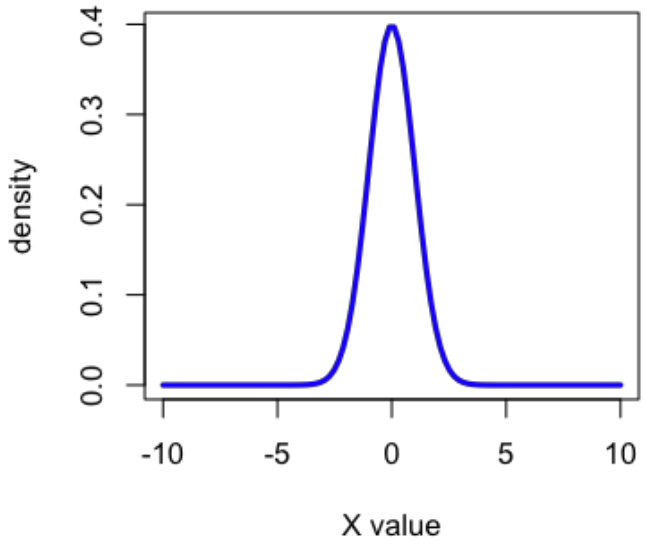
- $X \sim Bin(10, 0.5)$



Example distribution: Normal

Normal Distribution: $N(\mu, \sigma)$

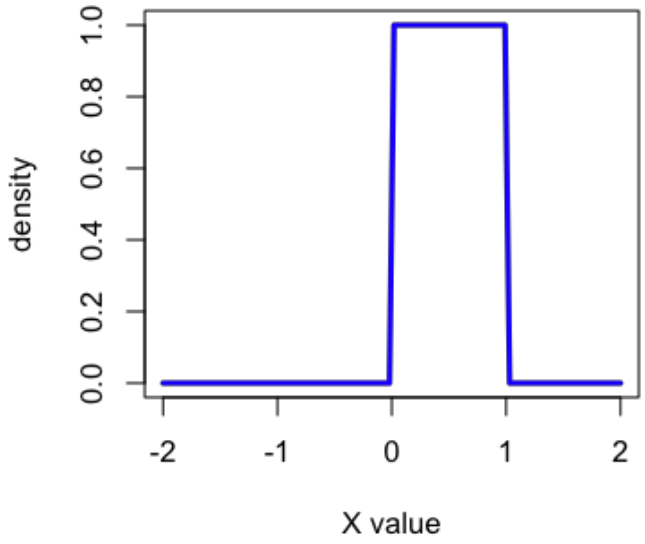
- $X \sim N(0, 1)$



Example distribution: Uniform

Uniform distribution: $U(\alpha, \beta)$

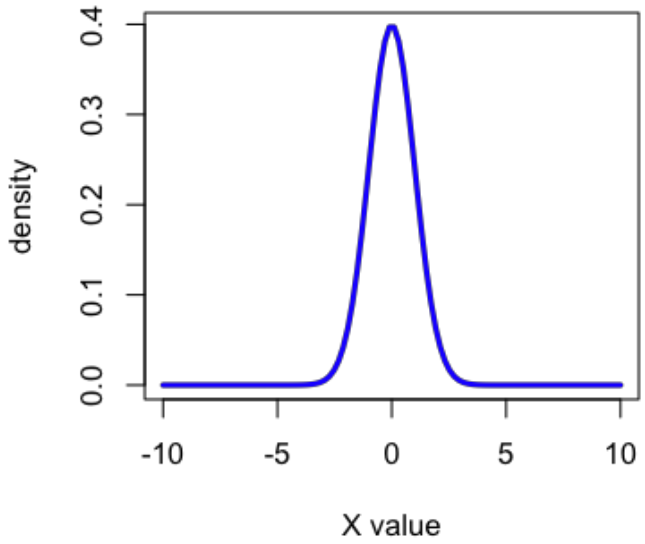
- $X \sim U(0, 1)$



Changing parameters

Normal Distribution: $N(\mu, \sigma)$

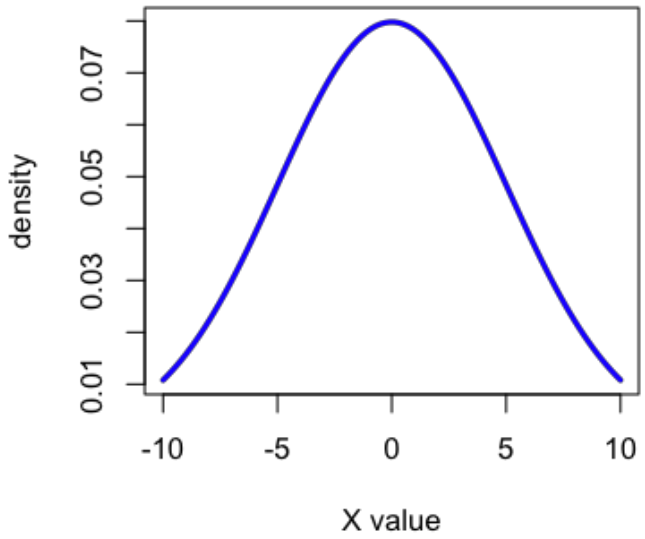
- $X \sim N(0, 1)$, $E[X] = \mu = 0$, $Var[X] = \sigma^2 = 1$



Changing parameters: the variance

Normal Distribution: $N(\mu, \sigma)$

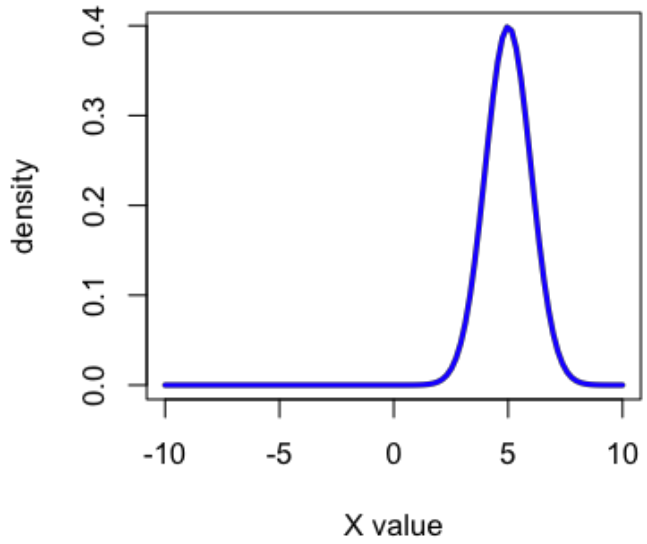
- $X \sim N(0, 5)$, $E[X] = \mu = 0$, $Var[X] = \sigma^2 = 25$



Changing parameters: the mean

Normal Distribution: $N(\mu, \sigma)$

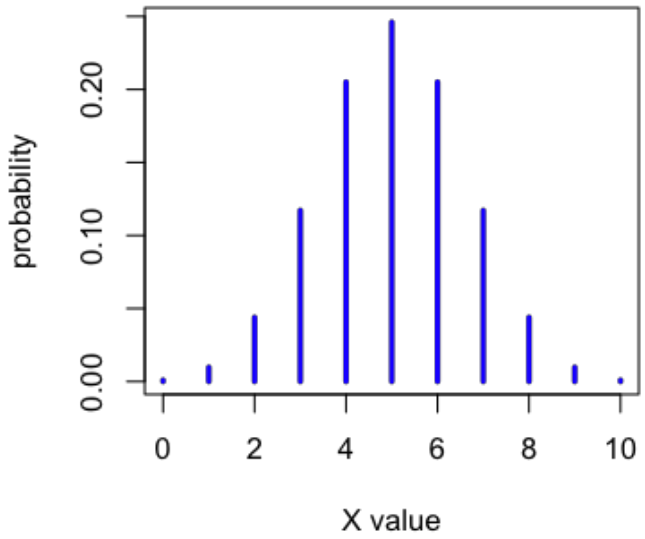
- $X \sim N(5, 1)$, $E[X] = \mu = 5$, $Var[X] = \sigma^2 = 1$



Example distribution: Binomial

Binomial distribution: $Bin(n, p)$

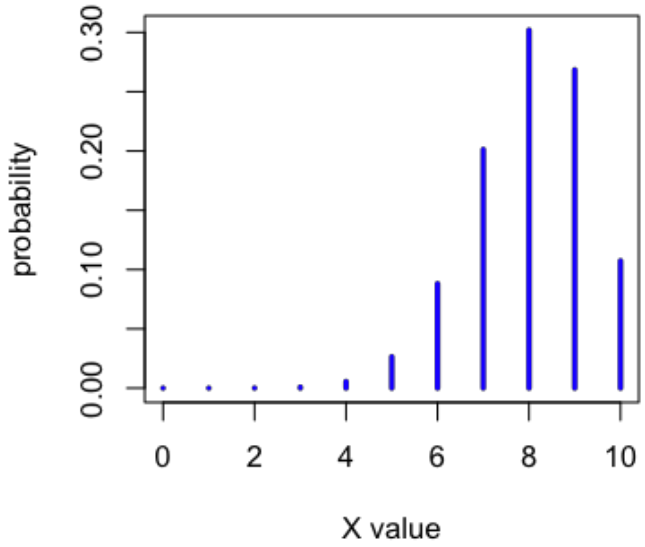
- $X \sim Bin(10, 0.5)$, $E[X] = n \times p = 5$, $Var[X] = n \times p \times (1 - p) = 2.5$



Changing parameters: both mean and variance

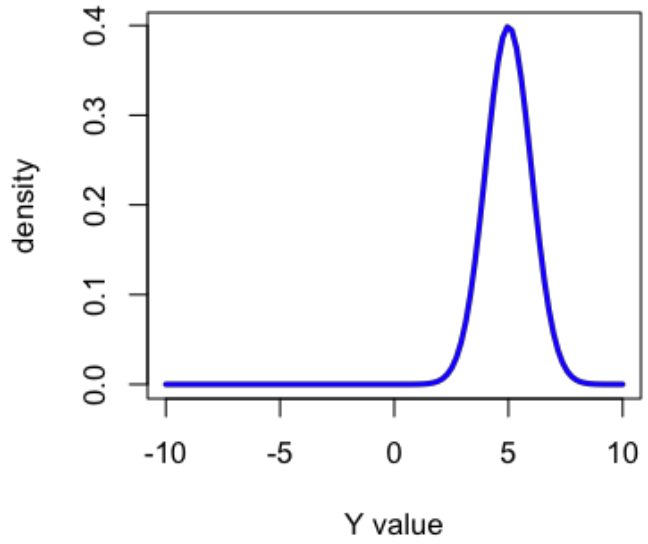
Binomial distribution: $Bin(n, p)$

- $X \sim Bin(10, 0.8)$, $E[X] = n \times p = 8$, $Var[X] = n \times p \times (1 - p) = 1.6$



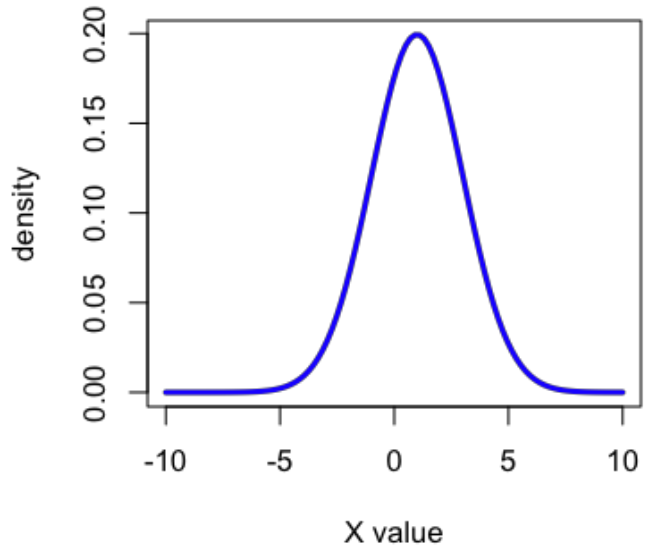
Conditioning

- Suppose $Y \sim N(X, 1)$ and $X \sim N(0, 1)$, then the distribution of $Y|X = 5$ is



Conditioning

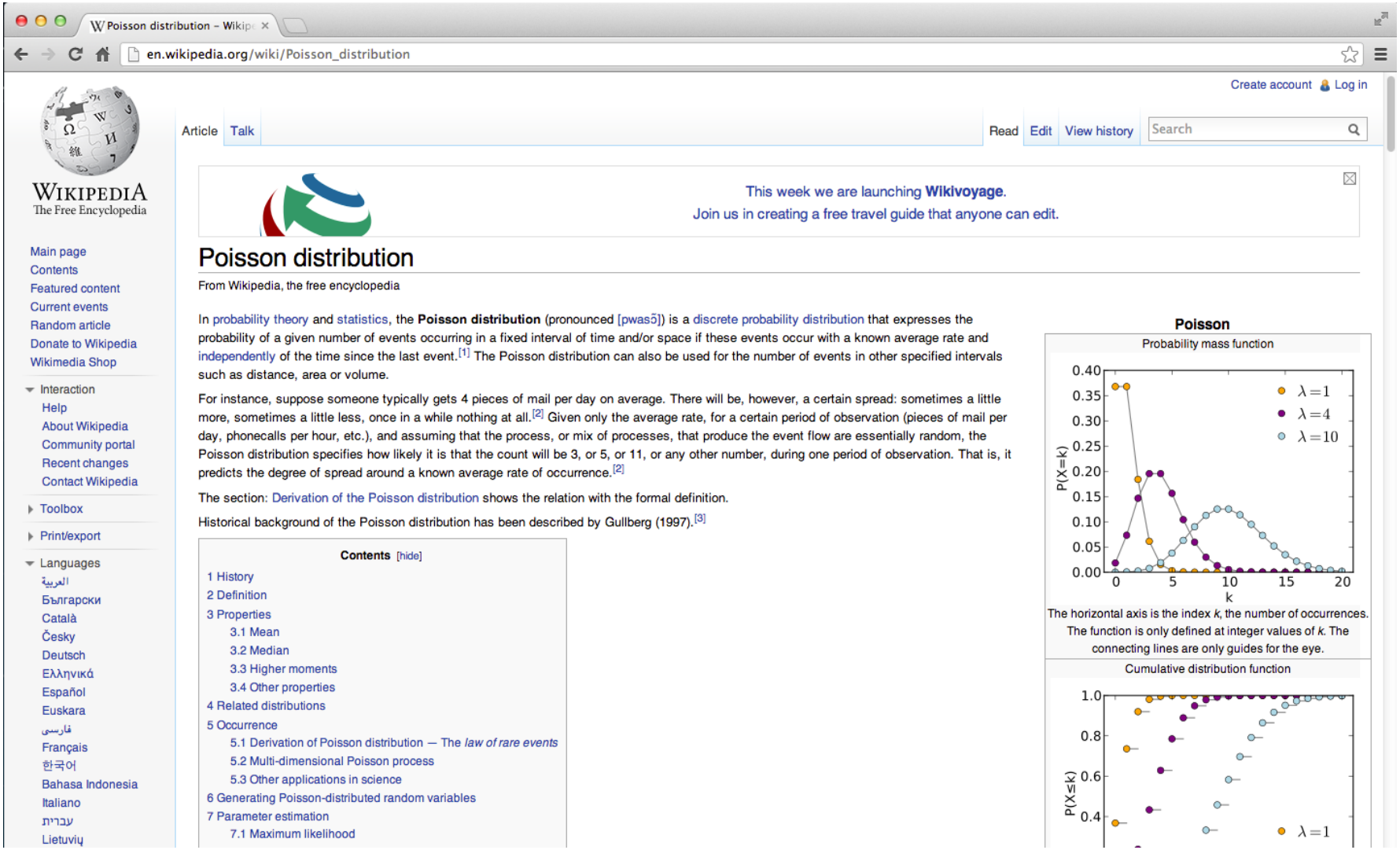
- Suppose $Y \sim N(X, 1)$ and $X \sim N(0, 1)$, then the distribution of Y is



http://en.wikipedia.org/wiki/Law_of_total_variance

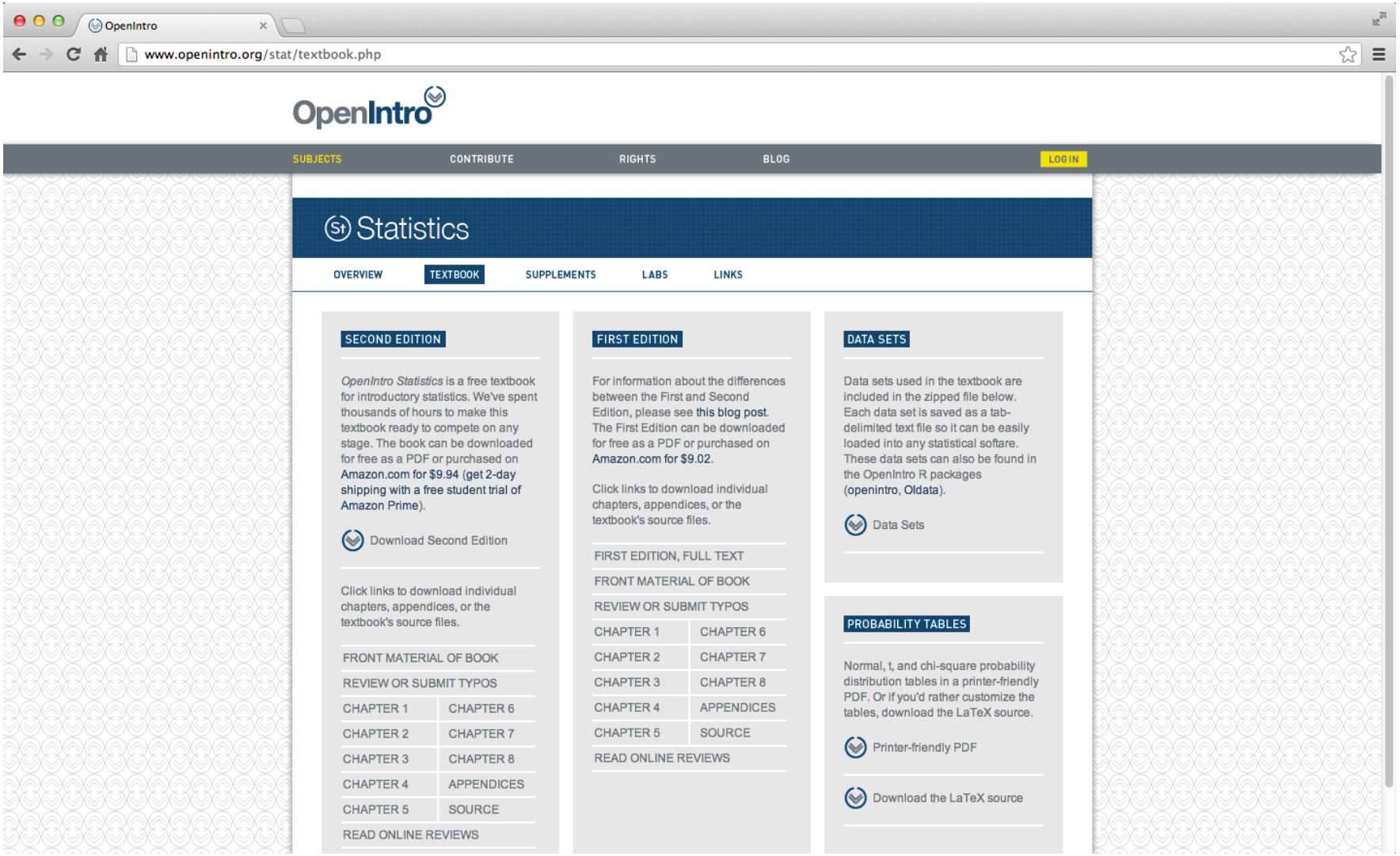
http://en.wikipedia.org/wiki/Law_of_total_expectation

Learning more about a specific distribution



http://en.wikipedia.org/wiki/Poisson_distribution

Learning more about representing data



http://www.openintro.org/stat/textbook.php