Game Theory

Strategic interaction of self-interested people

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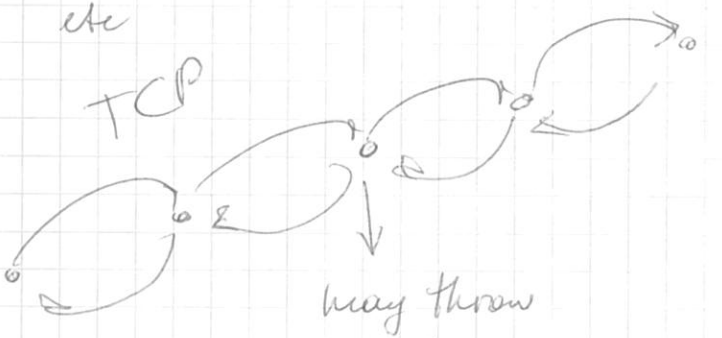
economics, sociale studies, networking etc



backoff - if messages are started to be lost, network & connection slows down it order to get all msgs delivered

Game - interaction b/w 2 or more people

Outcome depends on what everybody does

different players are happy with different outcomes

Some questions:

- what actions a game player takes?
- all users behave in the same way?
- global behavioral pattern? (for designers)
- if number changed, how behavior will change
- if they can communicate, what effect?
- repetitions (finite, infinite?)
- does it matter if the opponent is rational?

Self-interested agent

- they have opinions/preferences

utility function - math measure how much A likes a situation?

Utility function

- why single dimention?
- uncertain -
  now it can be measured

Players - who are making decisions
(companies, governments, etc)

Actions - what Act. Players can take?
(how much to buy, etc)
(how to vote?)

Pay offs what motivates people?

Representation of games

Normal Form (Matrix, Strategic, etc)
Lists payoffs & players can get

Extensive

poker?

includes timing (chess, etc)

represented as a tree

what players know when they move

Normal Form

Finite, n-person normal form game:

$$\langle N, A, u \rangle$$

Players: $N = \{1, \ldots, n\}$ – finite set of n

Actions:

A_i – actions for player i

$a = (a_1 \ldots a_n) \in A = \underbrace{A_1 \times \ldots \times A_n}$

action profile

Utility (payoff) function

for i player:

$u_i : A \to \mathbb{R}$ how they evaluate the outcome of a game?

$u = \underbrace{(u_1 \ldots u_n)}$

profile of utility

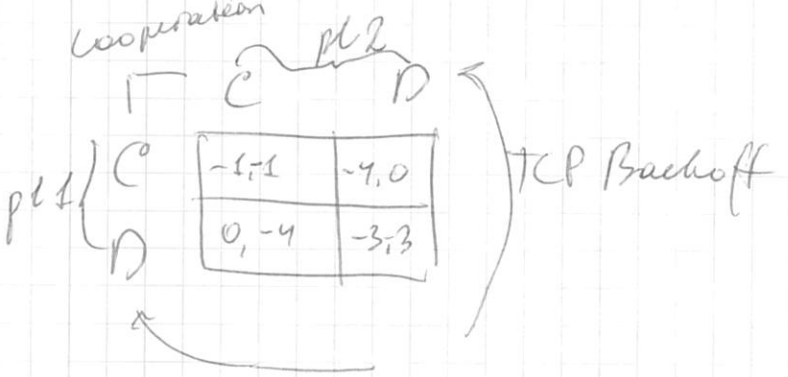
2 player game – matrix

row-player 1 – represents actions $a_1 \in A_1$

col-player 2 – actions $a_2 \in A_2$

~~row~~

cells listing utility/payoff values for each player



Large Collective Action

$N = \{1 \dots 10\,000\,000\}$

(revolt, vote, etc)

$A_i = \{Revolt, Not\}$

$u_i = 1$ if $\sum_j > 2mln$

-1 $\sum_j < 2mln$

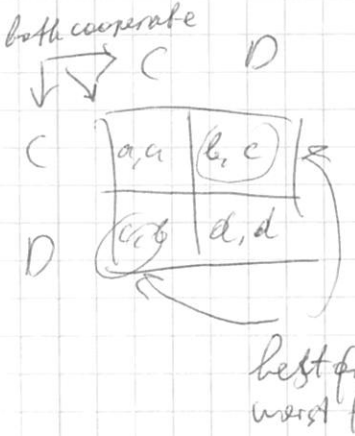
here payoffs depends on what other players are doing

Examples

Prisoner's dilemma

C - coordinate

D - miscoordinate



$c > a > d > b$

Pure competitions

$a \in A$, $u_1(a) + u_2(a) = c$

zero-sum games
constant games

Matching Pennies

	Heads	Tails	one wants match
Heads	1, -1	-1, 1	another mismatch
Tails	-1, 1	1, -1	

	Rock	Papers	Scissors
R	0 0	-1 1	1 -1
P	1 -1	0 0	-1 1
S	-1 1	1, -1	0 0

Games of Cooperation

all agens have the same interests

$$\forall a \in A \; \forall i,j \; u_i(a) = u_j(a)$$

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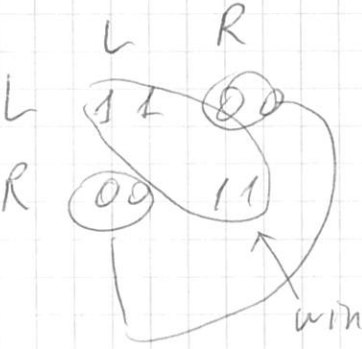
a single payoff per all

noncooperative

side of the road



2 people walking against each other

both sides loose

	B	F
B	2,1	0,0
F	0,0	1,2

battle of sexes

both want to go on the same movies

Nash equilibrium intro.

Beauty contest game

- you have a stock and the price is rising
- you think the price is too high
  ⇓
  you want to sell it
- the price should be reaching a peak (almost before peak)
- just before other investors do it

- How players act?

Stylized version

- each player names an integer between ~~the players who names~~ 1 and 100
- the player who names one closest to the 2/3 of the avg with the price, others get nothing
- Ties are broken randomly

you have to predict by picture who will win the beauty contest.

Strategic reasoning

- what will other players do?
- what should I do in response?

- Each player best responds to the others!

Nash Equilibrium

they choose the optimal response, giving them the change of winning

So I believe that everybody acts rationally

Unique Nash Equilibrium = 1 best

Nash E.

- consistent bost of actions

  each player maximizes his/her payoff

- self-consistent / stable
- nobody has an incentive to deviate from their action

Should we expect equilibria to be played?

non-equilibria?

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non-stable

We should expect non-equilibria to be vanish over time

Best Response and Nash Equilibrium

if you know what others are going to do it would be easier to pick your own action

let $a_{-i} = \langle a_1 \dots a_{i-1} \overset{\text{no } a_i}{\checkmark} a_{i+1} \dots a_n \rangle$

$a = (a_{-i}, a_i)$

a_{-i}: everybody except you

a: entire action profile

Your best response

$a_i^* \in BR(a_{-i})$ iff $\forall a_i \in A_i$,

$u_i(a_i^*, a_{-i}) \geq u_i(a_i, a_{-i})$

better than anything else

BR – best response

But you don't know what others will do

Ideal: look for stable action profiles

So, Nash Equilibrium is

$a = \langle a_1 \dots a_n \rangle$ if $\forall i, a_i \in BR(a_{-i})$

"pure strategy"

Prisoner's dilemma

	C	D (Defect)
C	-1 -1	-4 0
D	0 -4	-3 -3

if prisoners cooperate, both get slight punishment

if one doesn't cooperate, only he doesn't get punishment

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so both decide not to cooperate

↗

this is nash eq.

L-R game

	L	R
L	(1 1)	0 0
R	0 0	(1 1)

Nash Eq.

best strategy - do the same as the other party

Battle of the sexes

	B	F
B	(2, 1)	0, 0
F	0, 0	(1, 2)

two pure strategy Nash Eq.

best response – to go to the move selected by the other party

Matching Pennies

	H	T
H	(1, -1)	-1, (1)
T	-1, 1	(1, -1)

0-sum game

here also best to see what other party picks up

if p1 picks H, p2 picks T
(p2 gets 1 in this case, not -1)

and vise-versa

no pure strategy (NE), ~~because what~~ cycling goes on cycle

if p1 picks H → p2 picks T
p2 pick T → p1 picks H

p1 picks T → p2 picks H
p2 picks H → p1 picks T

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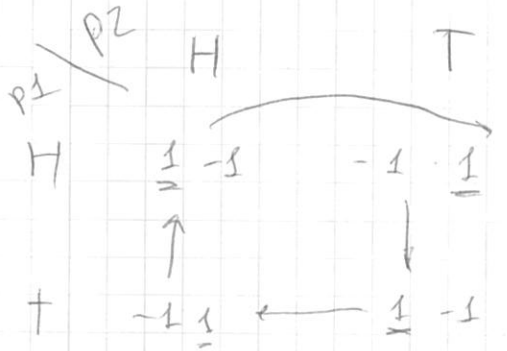


best responses are leading us to a cycle

Dominant Strategies

"Strategy" means choosing some action

s_i and s_i' - two strategies for pl. i

S_{-i} all possible strategy profiles for other players

s_i **strictly dominates** s_i'

if $\forall s_{-i} \in S_{-i}$, $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$

($\forall s_{-i} \in S_{-i}$: for every action others could take)

($u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$: player i gets more if he uses s_i)

for every action others could take

player i gets more if he plays s_i rather than s_i'

⇕ smth inbetween

s_i **weakly dominates** s_i'

if $\forall s_{-i} \in S_{-i}$, $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$

$=$ not strict

~~(so if s_i dominates s_j, it's certainly a good idea to play s_i not s_j).~~

so if s_i dominates s_j, it's a good idea to play it

if s_i dominates all other s,
it is dominant

in a strategy profile if everybody plays their dominant strategy,
it is a Nash Eq.

	C	D
C	-1, -1	-4, 0
D	0, -4	-3, -3

prisoner dilemma

0 is better than -1, so p1 plays d.

so does p2, and

both have strictly dominant strategies

Pareto Optimality

to see as an outside observer.
(not a player)

Can some outcomes of a game be said to be better than others?

You cannot say that

- one agent's interests are more important than others'
- you don't know how much is the outcome (unknown currency, say)

Idea: sometimes there is one outcome O which is good for every agent as another outcome O',

and there's an agent who strictly prefers O to O'

O
in this case it's better than O'

$\Downarrow$

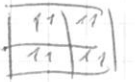
O pareto-dominates O'

Pareto Optimality –

Outcome O^* is pareto-optimal if there is no other outcome that pareto-dominates it

– Can a game have more than one Pareto-optimal outcomes?

it is possible.



– Does every game have at least one Pareto-optimal outcome?

yes, every game has to have a P-O outcome

we cannot have cycles for pareto-dominance

	L	R
L	(1,1)	0,0
R	0,0	(1,1)

	B	F
B	(2,1)	0,0
F	0,0	(1,2)

	H	T
H	(1,-1)	(-1,1)
T	(-1,1)	(1,-1)

every outcome is pareto-optimal

⇓

generally true for zero-sum games

	C	D
C	(-1, -1)	(-4, 0)
D	(0, -4)	(-3, -3)

DD is dominated by CC (CC is better)

~~Dilemma~~ Paradox of Prisoner's Dilemma:

the Nash Eq. is the only non-Pareto-optimal outcome

	C	D
C	✱	a
D	x	#

✱ – good from social perspective

– good from player's point of view

add-ons

TCP backoff game

when TCP is correctly implemented, it has "backoff mechanism"

if rates at which sender sends data causes congestions, the sender reduces the rate for a while, until a jam is subsided

Defective implementation doesn't back off

C - use correct implementation
D - ~~you~~ use a defect one

you \ other	C	D
C	1,1	4,0 ← D experiences no delay, C - 4 ms
D	0,4	3,3

3,3 → more packet losses

you both want to minimize these delays

Matching Pennies

- zero-sum game
- each chooses independently if to orient a penny to Heads or Tails
- if both chooses the same orientation, p1 wins, p2 loses
- if both players chooses different orientation p2 wins, p1 loses

		non-matching player: H	non-matching player: T
matching player	H	1, -1	-1, 1
matching player	T	-1, 1	1, -1

Coordination Game

- 2 players meet at a passage,
- they want to get through
- they need to choose, whether to go left or right

	L	R
L	1,1	0,0
R	0,0	1,1

Battle of the Sexes

- 2 players - a husband and a wife
- 2 options – Ballet or Football
- they want to go together

	F	B
F	1,2	0,0
B	0,0	2,1

Keynet Beauty Contest

- 7 people choose from 1 to 100
- winner – who picked up a value closest to 2/3 of avg
- ties broken at random

8 5 12 8 11 27 (9)

↑ avg

$\frac{2}{3}$ avg = 7.62

8 is the closest

one of the students with 8 won