Feedback — Problem Set 2

You submitted this homework on **Thu 17 Jan 2013 4:48 PM CET**. You got a score of **5.00** out of **5.00**.

Question 1

Mixed Strategy Nash Equilibrium

1\ 2	Left	Right
Left	4,2	5,1
Right	6,0	3,3

Find a mixed strategy Nash equilibrium where player 1 randomizes over the pure strategy Left and Right with probability p for Left. What is p?

Your Answer		Score	Explanation
b) 3/4	✓	1.00	
Total		1.00 / 1.00	

Question Explanation

(b) is true.

- In a mixed strategy equilibrium in this game both players must mix and so 2 must be indifferent between Left and Right.
- Left gives 2 an expected payoff: $2p + 0(1 - p)$
- Right gives 2 an expected payoff: $1p + 3(1 - p)$
- Setting these two payoffs to be equal leads to $p = 3/4$.

Question 2

Comparative Statics

1\ 2	Left	Right
Left	x,2	0,0

Right	0,0	2,2

In a mixed strategy Nash equilibrium where player 1 plays Left with probability p and player 2 plays Left with probability q. How do p and q change as X is increased ($X > 1$)?

Your Answer		Score	Explanation
a) p is the same, q decreases.	✓	1.00	
Total		1.00 / 1.00	

Question Explanation

(a) is true.

- In a mixed strategy equilibrium, 1 and 2 are each indifferent between Left and Right.
- For p:
  - Left gives 2 an expected payoff: $2p$
  - Right gives 2 an expected payoff: $2(1-p)$
  - These two payoffs are equal, thus we have $p = 1/2$.
- For q: setting the Left expected payoff equal to the Right leads to $Xq = 2(1-q)$, thus $q = 2/(X+2)$, which decreases in X.

Question 3

Employment

- There are 2 firms, each advertising an available job opening.
- Firms offer different wages: $w_1 = 4$ and $w_2 = 6$.
- There are two unemployed workers looking for jobs. They simultaneously apply to either of the firms.
  - If only one worker applies to a firm, then he/she gets the job
  - If both workers apply to the same firm, the firm hires a worker at random and the other worker remains unemployed (and receives a payoff of 0).

Find a mixed strategy Nash Equilibrium where p is the probability that worker 1 applies to firm 1 and q is the probability that worker 2 applies to firm 1.

Your Answer		Score	Explanation
d) $p = q = 1/5$.	✓	1.00	

Total	1.00 / 1.00

Question Explanation

(d) is correct.

- In a mixed strategy equilibrium, worker 1 and 2 must be indifferent between applying to firm 1 and 2.
- For a given p, worker 2's indifference condition is given by $2p + 4(1-p) = 6p + 3(1-p)$.
- Similarly, for a given q, worker 1's indifference condition is given by $2q + 4(1-q) = 6q + 3(1-q)$.
- Both conditions are satisfied when $p = q = 1/5$.

Question 4

Treasure

- A king is deciding where to hide his treasure, while a pirate is deciding where to look for the treasure.
- The payoff to the king from successfully hiding the treasure is 5 and from having it found is 2.
- The payoff to the pirate from finding the treasure is 9 and from not finding it is 4.
- The king can hide it in location X, Y or Z.

Suppose the pirate has two pure strategies: inspect both X and Y (they are close together), or just inspect Z (it is far away). Find a mixed strategy Nash equilibrium where p is the probability the treasure is hidden in X or Y and $1-p$ that it is hidden in Z (treat the king as having two strategies) and q is the probability that the pirate inspects X and Y:

Your Answer		Score	Explanation
a) $p = 1/2$, $q = 1/2$;	✓	1.00	
Total		1.00 / 1.00	

Question Explanation

(a) is true.

- There is no pure strategy equilibrium, so in a mixed strategy equilibrium, both players are indifferent among their strategies.

- For p:
  - Inspecting X \& Y gives pirate a payoff: $9p + 4(1-p)$
  - Inspecting Z gives pirate a payoff: $4p + 9(1-p)$
  - These two payoffs are equal, thus we have $p = 1/2$.
- For q: indifference for the king requires that $5q + 2(1-q) = 2q + 5(1-q)$, thus $q = 1/2$.

Question 5

Treasure

- A king is deciding where to hide his treasure, while a pirate is deciding where to look for the treasure.
- The payoff to the king from successfully hiding the treasure is 5 and from having it found is 2.
- The payoff to the pirate from finding the treasure is 9 and from not finding it is 4.
- The king can hide it in location X, Y or Z.

Suppose instead that the pirate can investigate any two locations, so has three pure strategies: inspect XY or YZ or XZ. Find a mixed strategy Nash equilibrium where the king mixes over three locations (X, Y, Z) and the pirate mixes over (XY, YZ, XZ). The following probabilities (king), (pirate) form an equilibrium:

Your Answer		Score	Explanation
d) (1/3, 1/3, 1/3), (1/3, 1/3, 1/3);	✓	1.00	
Total		1.00 / 1.00	

Question Explanation

(d) is true.

- Check (a):
  - Pirate inspects (XY, YZ, XZ) with prob (4/9, 4/9, 1/9);
  - Y is inspected with prob 8/9 while X (or Z) is inspected with prob 5/9;
  - King prefers to hide in X or Z, which contradicts the fact that in a mixed strategy equilibrium, king should be indifferent.
- Similarly, you can verify that (b) and (c) are not equilibria in the same way.
- In (d), every place is chosen by king and inspected by pirate with equal probability and they are indifferent between all strategies.