Problem Set 6

The **due date** for this homework is **Mon 25 Feb 2013 8:59 AM CET**.

Question 1

Core

- Three players together can obtain 1 to share, any two players can obtain 0.8, and one player by herself can obtain zero.
- Then, $N = 3$ and $v(1) = v(2) = v(3) = 0$, $v(1,2) = v(2,3) = v(3,1) = 0.8$, $v(1,2,3) = 1$.

Which allocation is in the core of this coalitional game?

- a) (0,0,0);
- b) (0.4, 0.4, 0);
- c) (1/3, 1/3, 1/3);
- d) The core is empty;

Question 2

Buyers and Sellers

- There is a market for an indivisible good with B buyers and S sellers.
- Each seller has only one unit of the good and has a reservation price of 0.
- Each buyer wants to buy only one unit of the good and has a reservation price of 1.
- Thus $v(C) = min(B_C, S_C)$ where B_C and S_C are the number of buyers and sellers in coalition C (and so, for instance, $v(i) = 0$ for any single player, and $v(i,j) = 1$ if i, j are a pair of a buyer and seller).

If the number of buyers and sellers is $B = 2$ and $S = 1$, respectively, which allocations are in the core? [There might be more than one]

- a) Each seller receives 1 and each buyer receives 0.
- b) Each seller receives 0 and each buyer receives 1.
- c) Each seller receives 1/2 and each buyer receives 1/2.

Question 3

Buyers and Sellers

- There is a market for an indivisible good with B buyers and S sellers.
- Each seller has only one unit of the good and has a reservation price of 0.
- Each buyer wants to buy only one unit of the good and has a reservation price of 1.
- Thus $v(C) = min(B_C, S_C)$ where B_C and S_C are the number of buyers and sellers in coalition C (and so, for instance, $v(i) = 0$ for any single player, and $v(i, j) = 1$ if i, j are a pair of a buyer and seller).

Now assume that competition among sellers increases, so that $B = 2$ and $S = 2$. Which allocations are in the core? [There might be more than one]

- [ ] a) Each seller receives 1 and each buyer receives 0.
- [ ] b) Each seller receives 0 and each buyer receives 1.
- [ ] c) Each seller receives 1/2 and each buyer receives 1/2.

Question 4

Core and Shapley Value

- The instructor of a class allows the students to collaborate and write up together a particular problem in the homework assignment.
- Points earned by a collaborating team are divided among the students in any way they agree on.
- There are exactly three students taking the course, all equally talented, and they need to decide which of them if any should collaborate.
- The problem is so hard that none of them working alone would score any points. Any two of them can score 4 points together. If all three collaborate, they can score 6 points.

Which allocations are in the core of this coalitional game?

- ○ a) (0,0,0);
- ○ b) (2, 2, 0);
- ○ c) (2, 2, 2);
- ○ d) The core is empty;

Question 5

Core and Shapley Value

- The instructor of a class allows the students to collaborate and write up together a particular problem in the homework assignment.
- Points earned by a collaborating team are divided among the students in any way they agree on.
- There are exactly three students taking the course, all equally talented, and they need to decide which of them if any should collaborate.
- The problem is so hard that none of them working alone would score any points. Any two of them can score 4 points together. If all three collaborate, they can score 6 points.

What is the Shapley value of each player?

- a) $\phi = (0, 0, 0)$
- b) $\phi = (2, 0, 2)$
- c) $\phi = (1/3, 1/3, 1/3)$
- d) $\phi = (2, 2, 2)$

Question 6

Production

- There is a single capitalist (c) and a group of 2 workers ($w1$ and $w2$).
- The production function is such that total output is 0 if the firm (coalition) is composed only of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).
- The production function satisfies:
  - $F(c \cup w1) = F(c \cup w2) = 3$
  - $F(c \cup w1 \cup w2) = 4$

Which allocations are in the core of this coalitional game? [There might be more than one]

- a) $x_c = 2$, $x_{w1} = 1$, $x_{w2} = 1$;
- b) $x_c = 2.5$, $x_{w1} = 0.5$, $x_{w2} = 1$;
- c) $x_c = 4$, $x_{w1} = 0$, $x_{w2} = 0$;

Question 7

Production

- There is a single capitalist (c) and a group of 2 workers ($w1$ and $w2$).
- The production function is such that total output is 0 if the firm (coalition) is composed only

of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).
- The production function satisfies:
  - $F(c \cup w1) = F(c \cup w2) = 3$
  - $F(c \cup w1 \cup w2) = 4$

What is the Shapley value of the capitalist?

- a) 3;
- b) 4;
- c) 7/3;
- d) 7;

Question 8

Production

- There is a single capitalist (c) and a group of 2 workers ($w1$ and $w2$).
- The production function is such that total output is 0 if the firm (coalition) is composed only of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).
- The production function satisfies:
  - $F(c \cup w1) = F(c \cup w2) = 3$
  - $F(c \cup w1 \cup w2) = 4$

What is the Shapley value of each worker?

- a) 1;
- b) 5/6;
- c) 3/4;
- d) 1/2;

Question 9

Production

- There is a single capitalist (c) and a group of 2 workers ($w1$ and $w2$).
- The production function is such that total output is 0 if the firm (coalition) is composed only of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).

- The production function satisfies:
  - $F(c \cup w1) = F(c \cup w2) = 3$
  - $F(c \cup w1 \cup w2) = 4$

True or False: If there was an additional 3^{rd} worker that is completely useless (i.e., his marginal contribution is 0 in every coalition), then the sum of the Shapley Values of the capitalist and the first two workers will remain unchanged.

- a) True;
- b) False;

In accordance with the Honor Code, I certify that my answers here are my own work.

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