Week 3

Beyond the Nash E

- doing for yourself as much as possible
- hurting other player as much as possible
- being a Nash Eq.

All turned out to coincide

Coordinated equilibrium - fairness without miscoordination

Strictly Dominated Strategies & Iterative Removal

Rationality - players maximize their payoffs

What if all players know this?
And they know that I know?

A Strictly dominated st never a best reply

Remove this strategy

Iterate over again untill all dominated strategies are removed

Iterated Removal of Strictly Dominated Strategies

Strictly dominated strategy

no matter what others do,
strategy a_i is always better than ~~others~~
~~any~~ some others

Ex 1

	L	C	R
U	3,0	2,1	0,0
M	1,1	1,1	5,0
D	0,1	4,2	0,1

← look for SD strat and remove

R is str. dominated by C

⇓

remove it

⇓

	L	C
U	3,0	2,1
M	1,1	1,1
C	0,1	4,2

no str. domination for col-player

~~M dominates strictly~~

M is strictly dominated by U

⇓

	L	C
U	3,0	2,1
D	0,1	4,2

L is strictly dominated

⇓

	C
~~U~~	~~2,1~~
D	4,2

$\Rightarrow$

	C
D	4,2

unique
Nash Eq.
(Coincidence)

<u>Ex. 2</u>

	L	C	~~R~~
U	3,1	0,1	~~0,0~~
~~M~~	~~1,1~~	~~1,1~~	~~5,0~~
D	0,1	4,1	~~0,0~~

1. R is dominated (by L/C)
2. no strict domination for row

$\Downarrow$

3. playing $\frac{1}{2}, \frac{1}{2}$ on U and D gives strictly dominant strategy,

(M is dominated by the mixed strategy that selects U and D with ½ / ½)

$\Downarrow$

	L	C
U	3,1	0,1
D	0,1	4,1

much simpler, much easier to analyze

Iterative Removal reserves NE.

- can be a preprocessing step
- order of removal doesn't matter
  (if strict dominance)

Games that are solvable by this technique are
dominance solvable

<u>Weakly dominanted strategies:</u>

is as good as or better than
all the other strategies

Can remove them also, but

- they could be best reply

	L	R
U	1, 1	2, 1
D	1, .	3,

NE

D dominates
but depending on
what are the other player
payoffs are, it can
be a
best responce

- order of removal matter

- at ~~least~~ least one E is preserved

- Beauty Contest Game - can be used there

Summary

- players maximize their payoffs
- don't play strictly dominated strategies
- NE ~~are~~ are a subset of what remains
- we see such behavior in ~~reta~~ reality.

Application

- 2 pigs in a cage
  (one pig dominates other)

they need to press a lever in order to get food

food and lever are ~~at~~ at opposite sides

$\Downarrow$

since there are 2 pigs in a cage, if one presses the lever, the other gets the food

10 units of food. Typical split:

- large gets first, then 1, 9
  (1 for small, 9 for big)
- small gets first: 4, 6
- at the same time: 3, 7

- pressing the lever costs 2 units of energy

$\Downarrow$

S (small) \ L (large)	Press	Wait
Press	1, 5	-1, 9
Wait	4, 4	0, 0

$\Downarrow$

for the small: waiting str. dominates $\Rightarrow$

$\Downarrow$ remove dominated

S \ L	Press
Wait	4, 4

Maxmin strategies

Player's i maxmin strategy is a strategy that maximizes i's worst-case payoff in situations where others play strategies causing the greatest harm to i

$\Downarrow$

(it maximizes the minimal outcome)

The maxmin value (safety level) of the game for player i – guaranteed minimal payoff.

Definition

maxmin strategy for i iff

$$\arg\max_{s_i} \min_{s_{-i}} u(s_1, s_2)$$

maxmin value – $\max_{s_i} \min_{s_{-i}} u(s_1, s_2)$

maximizes minimum

Minmax Strategy

i's minmax strategy against player -i
a strategy that minimizes the maximal value of -i.

Def:

in a 2-pl game, the minmax strategy for i against -i is

$$\arg\min_{s_i}\max_{s_{-i}}(s_i, s_{-i})$$

i's minmax value against -i

$$\min_{s_i}\max_{s_{-i}}(s_i, s_{-i})$$

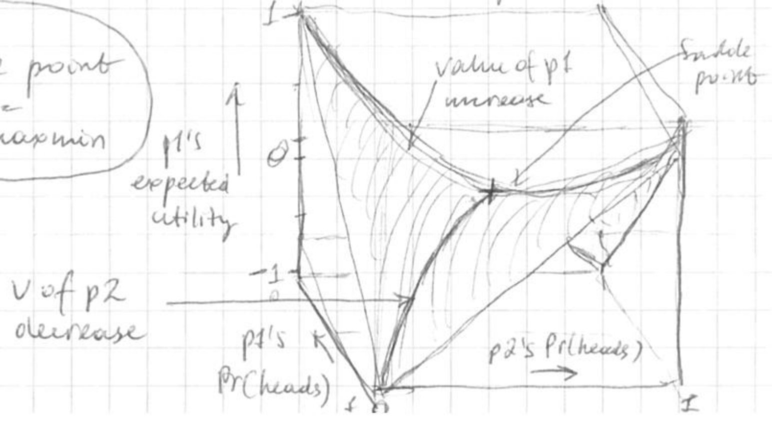
Minmax Theorem, von Neumann

> In any finite, 2-pl, zero-sum game, in any NE each player receives a payoff that is equal to both his maxmin value and his minmax value

~~1. Each~~

Saddle point
minmax = maxmin

Matching Pennies



Computing Minmax

For 2 players game minmax is solvable with LP

minimize U_1^* (value of the game)

subject to $\sum_{k \in A_2} u_1(a_1^j, a_2^k) \cdot S_2^k \leq U_1^*$ $\quad \forall j \in A_1$

for each action p1 may consider

$$\sum_{k \in A_2} S_2^k = 1$$

$S_2^k \geq 0 \quad \forall k \in A_2$

I want to find a strategy S_2 so for any strategy it won't exceed U_1^*

In words: I want to find U_1^* such as for any action p1 can take, p2 will find a strategy S_2 p1 won't get pay off bigger than U_1^*

Correlated Equilibrium: Intuition

	B	F
B	2,1	0,0
F	0,0	1,2

Battle of the Sexes

NE { B,B , F,F - NE
randomize 1/3 2/3 , 2/3 , 1/3

50/50 split between
(F,F) and (B,B)
⇓
not stable, but they may miscoordinate

Traffic game

car1 \ car2	go	wait
go	-10, -10	1,0
wait	0,1	-1, -1

Pure NE

Traffic light - a fair randomizing device that decides;

tells agent to go or to wait

Benefits:

- negative outcomes avoided
- fairness is achieved
- the total sum can exceed the NE

We can use the same idea for the battle of the Sexes

(flip a coin and go either to ballet or to football)

$\Downarrow$

Correlated Equilibrium

a randomized assignment of action recommendations to agents such as nobody wants to deviate

Built-in quizzes

Example. 1

Iterative Removal

p1 \ p2	u	m	d
U	2,1	5,3	3,1
~~M~~	~~0,7~~	~~2,10~~	~~0,0~~
~~D~~	~~5,0~~	~~1,1~~	~~2,4~~

Which of the strategies survives the process of iterative removal of strictly dominated strategies?

1. u is dominated by m
2. D is dominated by U
3. d is dominated by m
4. M is dominated by U

$\Downarrow$

(U, m) survives

Example 2

Maxmin

p1 \ p2	Movie	Home
Movie	3, 0	1, 2
Home	2, 1	0, 3

Which is maxmin strategy? (for player 1)

$$S_1 = \arg \max_{s_1' \in S_1} \min_{s_2 \in S_2} u_1(s_1', s_2)$$

Play Movie

Regardless of player 1's strategy,
choosing Home by player 2 minimizes
1's payoff

- if 1 plays Movie, 1 gets $\underline{\underline{3}}$
  when 2 plays Movies
  $\underline{\underline{1}}$ when 2 plays Home $\quad \underline{1 < 3}$

- if 1 plays Home
  1 gets $\underline{\underline{2}}$ when 2 plays Movie
  1 gets $\underline{\underline{0}}$ when 2 plays Home
  $\underline{0 < 2}$

Given 2 plays home, to minimize 1's payoff,
1 plays movie to maximize its payoff