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Calculating

© Battle of the sexes

p1 \ p2	p B	$1-p$ F
B	2, 1	0, 0
F	0, 0	1, 2

p2 plays B with p
p2 plays F with $1-p$

$u_1(B)$ utility of player 1 to play B
given that player 2 plays B and F with $(p, 1-p)$

$u_1(F)$ - utility of p1 to play F
given that p2 plays B and F with $(p, 1-p)$

$u_1(B)$: p1 plays B

$$\underbrace{\underset{\text{payoff p1 gets when p2 plays B}}{2} * \underset{\text{probability that p2 plays B}}{p}}_{\text{p2 plays B}} + \underbrace{\underset{\text{payoff p1 gets when p2 plays F}}{0} * \underset{\text{probability that p2 plays F}}{(1-p)}}_{\text{p2 plays F}}$$

$u_1(F)$: p1 plays F

$$\underbrace{0 * p}_{\text{p2 plays B}} + \underbrace{1 * (1-p)}_{\text{p2 plays F}}$$

By making p_2 indifferent, let's find his p:

$$u_1(B) = u_2(F)$$
$$\Downarrow$$
$$p = \frac{1}{3}$$

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② predator vs prey

		p	1 − p
	pred \ prey	Active	Passive
q	Active	2, −5	3, −6
1 − q	Passive	3, −2	−1, 0

prey plays A with p
prey plays P with 1 − p

pred plays A with q
pred plays P with 1 − q

predator plays A :
prey A: p, gets 2
prey P: 1 − p, gets 3

predator plays P:
prey A: p, gets 3
prey P, 1 − p, gets −1

$p \cdot 2 + (1-p) \cdot 3 = p \cdot 3 + (1-p)(-1)$

$\Downarrow$

$p = \frac{4}{5}$

predator plays A with $\frac{4}{5}$
plays P with $1/5$

prey plays A
pred A: q, gets −5
pred P: 1 − q, gets −2

prey plays P
pred A: q, gets −6
pred P: 1 − q, gets 0

$q \cdot (-5) + (1-q) \cdot (-2) =$
$= q(-6) + (1-q) \cdot 0$

$\Downarrow$

$q = 2/3$

prey plays A with $2/3$
plays P with $1/3$

Problem Set 2

The **due date** for this homework is **Sun 27 Jan 2013 8:59 PM CET**.

Question 1

p1 plays L with p:

consider p2:

Mixed Strategy Nash Equilibrium

	1\ 2	Left	Right
p	Left	4,2	5,1
$1-p$	Right	6,0	3,3

p2 plays L

$u_2(L) =$
$= 2p + 0(1-p)$

p2 plays R

$u_2(R) = 1 \cdot p + 3(1-p)$

Find a mixed strategy Nash equilibrium where player 1 randomizes over the pure strategy Left and Right with probability p for Left. What is p?

- a) 1/4
- ✓ b) 3/4
- c) 1/2
- d) 2/3

$u_2(L) = u_2(R)$
$2p = p + 3(1-p)$
$2p = p + 3 - 3p$
$2p = 3 - 2p$
$4p = 3, \; p = \frac{3}{4}$

Question 2

p2 plays L with q

Comparative Statics

P1 plays L with p

	1\ 2	Left (q)	Right ($1-q$)
p	Left	x,2	0,0
$1-p$	Right	0,0	2,2

p1: $\underbrace{x \cdot q + 0(1-q)}_{L} = \underbrace{0 \cdot 2(1-q)}_{R}$

$xq = 2(1-q)$
$x \cdot q = 2 - 2q$
$q(x+2) = 2$
$q = \frac{2}{2+x}$

the more x the less q

In a mixed strategy Nash equilibrium where player 1 plays Left with probability p and player 2 plays Left with probability q. How do p and q change as X is increased ($X > 1$)?

- ✓ a) p is the same, q decreases.
- b) p increases, q increases.
- c) p decreases, q decreases.

p2: $\underbrace{2p + 0}_{L} = \underbrace{0 + 2(p-1)}_{R}$

doesn't depend on x, remains the same

d) p is the same, q increases.

Question 3

Employment

- There are 2 firms, each advertising an available job opening.
- Firms offer different wages: $w_1 = 4$ and $w_2 = 6$.
- There are two unemployed workers looking for jobs. They simultaneously apply to either of the firms.
    - If only one worker applies to a firm, then he/she gets the job
    - If both workers apply to the same firm, the firm hires a worker at random and the other worker remains unemployed (and receives a payoff of 0).

Find a mixed strategy Nash Equilibrium where p is the probability that worker 1 applies to firm 1 and q is the probability that worker 2 applies to firm 1.

a) $p = q = 1/2$;

b) $p = q = 1/3$;

c) $p = q = 1/4$;

✓ d) $p = q = 1/5$.

$p: w1 \to I$ $\quad q: w2 \to I$

	w1 \ w2	q I	1-q II
p	I	4,0 0,4	4,6
1-p	II	6,4	6,0 0,6 3½

w1:

$2q + 4 - 4q = 6q + 3 - 3q$

$-2q + 4 = 3q + 3$

$1 = 5q, \; q = 1/5$

w2:

$2p + 4 - 4p = 6p + 3 - 3p$

$p = 1/5$

Question 4

Treasure

- A king is deciding where to hide his treasure, while a pirate is deciding where to look for the treasure.
- The payoff to the king from successfully hiding the treasure is 5 and from having it found is 2.
- The payoff to the pirate from finding the treasure is 9 and from not finding it is 4.
- The king can hide it in location X, Y or Z.

Suppose the pirate has two pure strategies: inspect both X and Y (they are close together), or just inspect Z (it is far away). Find a mixed strategy Nash equilibrium where p is the probability the treasure is hidden in X or Y and $1 - p$ that it is hidden in Z (treat the king as having two strategies) and q is the probability that the pirate inspects X and Y:

pirate \ king	p: XY	1-p: Z
q: XY	9 2	4 5
1-q: Z	4 5	9 2

a) $p = 1/2, q = 1/2$;

b) $p = 4/9, q = 2/5$;

c) $p = 5/9, q = 3/5$;

d) $p = 2/5, q = 4/9$;

pirate: look in XY = look in Z

$9p + 4(1-p) = 4p + 9(1-p)$

$5p + 4 = -5p + 9$, $10p = 5$, $p = 1/2$

king: hide in XY = hide in Z

$2q + 5(1-q) = 5q + 2(1-q)$

$-3q + 5 = 3q + 2$, $6q = 3$, $q = 1/2$

Question 5

Treasure

- A king is deciding where to hide his treasure, while a pirate is deciding where to look for the treasure.
- The payoff to the king from successfully hiding the treasure is 5 and from having it found is 2.
- The payoff to the pirate from finding the treasure is 9 and from not finding it is 4.
- The king can hide it in location X, Y or Z.

Suppose instead that the pirate can investigate any two locations, so has three pure strategies: inspect XY or YZ or XZ. Find a mixed strategy Nash equilibrium where the king mixes over three locations (X, Y, Z) and the pirate mixes over (XY, YZ, XZ). The following probabilities (king), (pirate) form an equilibrium:

d) (1/3, 1/3, 1/3), (1/3, 1/3, 1/3);

b) (4/9, 4/9, 1/9), (1/3, 1/3, 1/3);

c) (1/3, 1/3, 1/3), (2/5, 2/5, 1/5);

a) (1/3, 1/3, 1/3), (4/9, 4/9, 1/9);

☐ In accordance with the Honor Code, I certify that my answers here are my own work.

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(5)

$$p_1 + p_2 + p_3 = 1$$

$p \backslash k$		X	Y	Z
q_1	XY	9 2	9 2	4 5
$+ q_2$	YZ	4 5	9 2	9 2
$+ q_3$	XZ	9 2	4 5	9 2
$= 1$				

<u>pirate</u>: look in XY $\quad$ look in YZ $\quad$ look in XY

$$9p_1 + 9p_2 + 4p_3 = 4p_1 + 9p_2 + 9p_3 = 9p_1 + 4p_2 + 9p_3$$

$$p_1 + p_2 + p_3 = 1.$$

$$\left(\begin{array}{ccc|c} 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right) \Rightarrow p_1 = p_2 = p_3 = \frac{1}{3}$$

<u>king</u>: hide in X $\quad$ hide in Y $\quad$ hide in Z

$$2q_1 + 5q_2 + 2q_3 = 2q_1 + 2q_2 + 5q_3 = 5q_1 + 2q_2 + 2q_3$$

$$q_1 + q_2 + q_3 = 1$$

$$\left(\begin{array}{ccc|c} 0 & 1 & -1 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right) \Rightarrow q_1 = q_2 = q_3 = 1/3$$