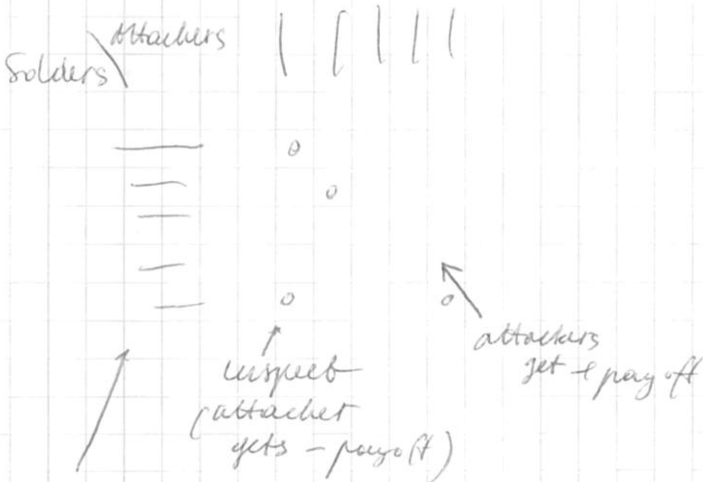
Week 2

Mixed-Strategy Nash Equilibrium

1. Mixed Strategies

Roads:



randomized!

mix-strategy.

	H	T
H	1,-1	-1,1
T	-1,1	1,-1

not a good idea to play determenistic game

↓

so another player will alway want to change

Idea: Confuse the opponent by playing randomly (flip a coin!)

Strategy S_i for agent i is any probability distribution over the action A_i.

pure strategy: only one action is played with positive probability

mixed strategy: more than one action is played with positive probability

↓

these actions are called the support of the mixed strategy

(H & T)

Let the set of all strategies for i be S_i

all ~~strategies~~ strategy profiles $= S = \underbrace{S_1 \times \dots \times S_n}_{\text{Cartesian product}}$

~~Since~~ ~~Sie~~ Since now, it's probability, old utility function makes no sence

Expected payoff

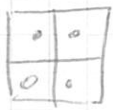
$$u_i(s) = \sum_{a \in A} u_i(a) \Pr(a|s)$$

sum over all ~~actions~~ cells of the games

$$\Pr(a|s) = \prod_{j \in N} s_j(a_j)$$

probability that I get to a using ~~profile~~ a strategy s

sum over all cells of the game and for each, multiply it's probability on the possible payoff



	0.5	0.5
0.5	0.25	
0.5		

Best Response

$S_i^* \in BR(s_{-i})$ iff $\forall s_i \in S$, $u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i})$

(infinitive here)

Strategy profile

$S = \langle S_1 \dots S_n \rangle$ is a Nash Equilibrium,

iff $\forall i$, $s_i \in BR(s_{-i})$

pure strategy Nash Equilibrium

Theorem Nash

Every finite game has a Nash Equilibrium

(mixed strategy)

$\Downarrow$

has a finite number of players,
finite number of actions $\Rightarrow$
finite number of utility functions

		0.5	0.5
		H	T
0.5	H	1, -1	-1 1
0.5	T	-1 1	1 -1

mixed strategy NE here — to play randomly, 50/50

Coordination game

		0.5	0.5
		L	R
0.5	L	1 1	0 0
0.5	R	0 0	1 1

both players should randomize 50/50

↓

Nash Eq.

Prisoner's Dilemma

	C	D
C	-1 -1	-4 0
D	0 -4	-3 -3

only a pure ~~mixe~~ strategy NE,

~~not~~ doesn't have Mixed Str. NE.

How to compute?

~~Battle of the sexes~~

It's hard to compute the NE, but it's easy when you ~~have~~ can guess the support

↓

Set of pure strategies, which occur with positive probability

Battle of the Sexes

	p B	$1-p$ F
B	2,1	0 0
F	0 0	1 0

p $\quad$ $1-p$

for player 2

if player I best-responds with a mixed strategy,

player 2 must make him indifferent between F and B

↓

he himself plays mixed strategy
So it's a best response

if he's not indifferent, i.e. increase the probability on B and decrease on F,

So he will want to play a mixed strategy if it's the same for him to play either B or F

$$u_1(B) = u_1(F)$$

player 1 plays B

given

$$2p + 0(1-p)$$

(he gets 2 when plays B with probability p, and 0 when plays F with probability $1-p$)

$$2p + 0(1-p) = 0p \underset{q}{+} 1(1-p)$$

(0 when plays B,
1 when plays F)

$$p = \frac{1}{3}$$

So player 2 can be indifferent if only $p = 1/3$

Likewise for player 1

	B	F	
B	2 1	0 0	q
F	0 0	1 2	$1-q$

p1 wants also make p2 indifferent

$$U_2(B) = U_1(F)$$

$$q + 0(1-q) = 0q + 2(1-q)$$

$$q = \frac{2}{3}$$

Thus the mixed strategies $\left(\frac{2}{3}, \frac{1}{3}\right)\left(\frac{1}{3}, \frac{2}{3}\right)$ are a Nash Equilibrium

Interpreting Mixed Strategy Equilibria

What does it mean to play a mixed strategy?

- Randomize to confuse your opponents (consider the matching pennies)
- Randomize when uncertain about the other's actions (consider the battle of the sexes)
- it's a concise description of what might happen in repeated play ("like statistics")

~~mixed~~ Predator vs Prey

a competition between 2 animals who each can choose strategies of either being passive or active.

Payoffs are based on survival probabilities and caloric expenditure.

mixed strategy game:

Prey / Pred	p Active	$1-p$ Passive	
Active	2, -5	3, -6	q
Passive	3, -2	-1, 0	$1-q$

What p and q are a mixed-strategy equilibrium?

predator:

Payoffs should be equal since the p wants to be indifferent

$$\underbrace{2p + 3(1-p)}_{\text{when playing active}} = \underbrace{3p - (1-p)}_{\text{when playing passive}}$$

$$p = \frac{4}{5}$$

prey:

$$\underbrace{-5q - 2(1-q)}_{\text{when active}} = \underbrace{-6q + 0(1-q)}_{\text{when passive}}$$

$$q = \frac{2}{3}$$

~~Hardness beyond~~ 2x2 games

PPAD

Examples

Soccer penalty kicks

unpredictable
how to adjust?

Kicker / Goalie

		Goalie	
		Left $p=1/2$	Right $1/2$
Kicker	Left $1/2$	0, 1	1, 0
	Right $1/2$	1, 0	0, 1

(similar to matching pennies)

equilibrium is $\frac{1}{2}$

b) Now we'll consider a kicker who sometimes misses on the right

		G	
		L	R
		p	$1-p$
K	q L	0, 1	1, 0
	$1-q$ R	.75, .25	0, 1

$1-p = .075p$

↑ misses $\frac{1}{4}$ of the time

$p = \frac{4}{7}$.

Goalie should go left with $\frac{4}{7}$, and right with $\frac{3}{7}$

$q + 0.25(1-q) = 1-q$

$q = .75(1+q)$, $q = \frac{3}{7}$

So:

		G	
		4/7 L	3/7 R
K	3/7 L	0, 1	1, 0
	4/7 R	.75, .25	0, 1

kicker kicks more to the Right (his weakness!) (because the Goalie also made adjustment)

- In a mixed eq, the goalie's strategy must have the kicker indifferent

- p - prob. goalie goes left;
  kicker indifferent: $(1-p) 1 = p \cdot 0.75$,
  $$p = 4/7$$

- Goalie goes left more often than Right ($4/7$ to $3/7$), kicker goes Right more frequently

$\Downarrow$

- Goalie's strategy adjusts, and the kicker adjusts to kick more to their weak side!

Real data:

		G p_g L	G $1-p_g$ R
p_k K	L	.58 .42	.95 .05
$1-p_k$	R	.93 .07	.70 .30

$p_g = .42$

$p_k = .38$

		.42 L	.58 R
.38	L	.58 .42	.95 .05
.62	R	.93 .07	.70 .30

	G_L	G_R	K_L	K_R
Calc	.42	.58	.38	.62
Actual	.42	.58	.40	.60

↑

almost exactly Nash Eq!

Players randomize over time

1 \ 2	L	R
T	2, 2	0, 2
B	1, 2	3, 3

Which of the following are Nash equilibria of the game

1. (T, L) and (B, R) – pure strategy Nash Equilibria

(no single player would be better off by defecting from the selected strategy)

2. mixed equilibria : q for 1 (T) and p for 2 (L)

$2p + 0(1-p) = 1p + 3(1-p)$, $p = 3/4$

$2q + 2(1-q) = 0q + 3(1-q)$, $q = 1$.