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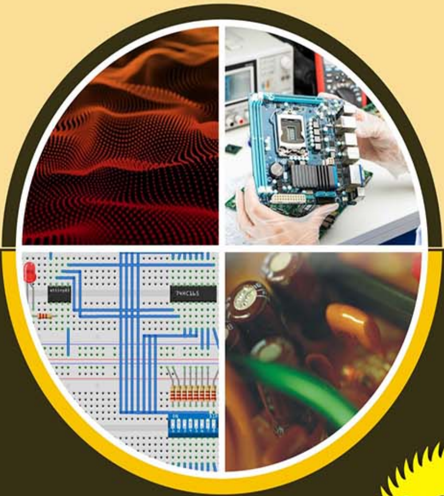

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Network Analysis & Synthesis

By

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KEC303	**Network Analysis and Synthesis**	**3L:0T:0P**	**3 Credits**

Unit	Topics	Lectures
I	Node and mesh analysis, matrix approach of network containing voltage & current sources and reactances, source transformation and duality.	8
II	Network theorems: Superposition, reciprocity, Thevenin's, Norton's, Maximum power transfer, compensation and Tallegen's theorem as applied to A.C. circuits.	8
III	Trigonometric and exponential Fourier series: Discrete spectra and symmetry of waveform, steady state response of a network to non-sinusoidal periodic inputs, power factor, effective values, Fourier transform and continuous spectra, three phase unbalanced circuit and power calculation.	8
IV	Laplace transforms and properties: Partial fractions, singularity functions, waveform synthesis, analysis of RC, RL, and RLC networks with and without initial conditions with Laplace transforms evaluation of initial conditions.	8
V	Transient behaviour, concept of complex frequency, driving points and transfer functions poles and zeros of immittance function, their properties, sinusoidal response from pole-zero locations, convolution theorem and two four port network and interconnections, behaviour of series and parallel resonant circuits, introduction to band pass, low pass, high pass and band reject filters.	8

Text/Reference Books

1. Franklin F. Kuo, "Network Analysis and Synthesis," Wiley India Education, 2nd Ed., 2006.
2. Van, Valkenburg, "Network analysis," Pearson, 2019.
3. Sudhakar, A., Shyammohan, S. P., "Circuits and Network," Tata McGraw-Hill New Delhi, 1994.
4. A William Hayt, "Engineering Circuit Analysis," 8th Edition, McGraw-Hill Education.
5. A. Anand Kumar, "Network Analysis and Synthesis," PHI publication, 2019.

Course Outcomes:

At the end of this course students will demonstrate the ability to:

1. Understand basics electrical circuits with nodal and mesh analysis.
2. Appreciate electrical network theorems.
3. Apply Laplace transform for steady state and transient analysis.
4. Determine different network functions.
5. Appreciate the frequency domain techniques.

1 UNIT

Node and Mesh Analysis

CONTENTS

PART-1

Nodal Analysis.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.1. **Discuss nodal analysis.**

Answer

1. In general, in a N node circuit, one of the nodes is chosen as reference or datum node, then it is possible to write $N - 1$ nodal equations by assuming $N - 1$ node voltages.
2. Each node in a circuit can be assigned a number or a letter. The node voltage is the voltage of a given node with respect to one particular node, called the reference node, which we assume at zero potential.
3. In the circuit shown in Fig. 1.1.1, node 3 is assumed as the reference node.
4. The voltage at node 1 is the voltage at that node with respect to node 3. Similarly, the voltage at node 2 is the voltage at that node with respect to node 3.
5. Applying Kirchhoff's current law at node 1,

$$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$

where V_1 and V_2 are the voltages at node 1 and 2, respectively.

$$V_1\left[\frac{1}{R_1} + \frac{1}{R_2}\right] - V_2\left[\frac{1}{R_2}\right] = I_1 \quad ...(1.1.1)$$

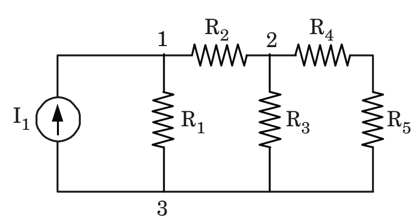


Fig. 1.1.1.

6. Similarly, at node 2,

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_4 + R_5} = 0$$

$$-V_1\left[\frac{1}{R_2}\right] + V_2\left[\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4 + R_5}\right] = 0 \qquad ...(1.1.2)$$

7. From the eq. (1.1.1) and eq. (1.1.2), we can find the voltages at each node.

Que 1.2. **For the circuit shown in Fig. 1.2.1, find the voltage V_x using nodal analysis.**

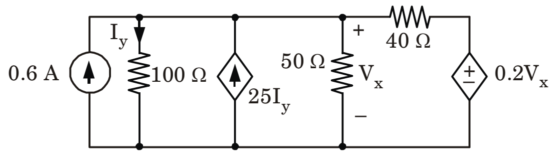


Fig. 1.2.1.

Answer

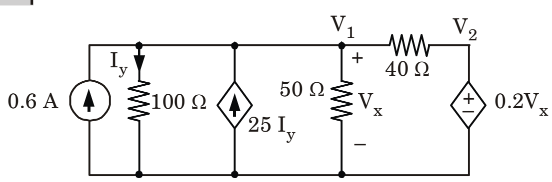


Fig. 1.2.2.

1. By KCL at node 1,

$$-0.6 + I_y + \frac{V_x}{50} - 25I_y + \frac{V_1 - V_2}{40} = 0 \qquad ...(1.2.1)$$

2. By KCL at node 2,

$$V_2 = 0.2\,V_x \qquad ...(1.2.2)$$

and other constraint equation,

$$I_y = \frac{V_x}{100} \text{ and } V_1 = V_x \qquad ...(1.2.3)$$

3. Putting value of I_y, V_1 and V_2 in eq. (1.2.1),

$$-0.6 + \frac{V_x}{100} + \frac{V_x}{50} - \frac{25V_x}{100} + \frac{V_1 - V_2}{40} = 0$$

$$-120 + 2V_x + 4V_x - 50V_x + 5V_x - 5 \times 0.2\,V_x = 0$$

$$\therefore \qquad V_x = \frac{120}{-40} = -3 \text{ volt}$$

Que 1.3. **For the circuit shown in Fig. 1.3.1, determine the voltage v using nodal analysis.**

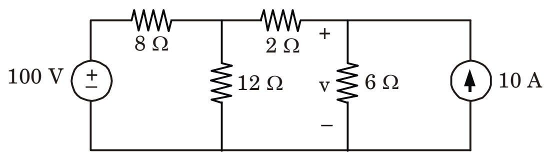


Fig. 1.3.1.

Answer

1. Let the node voltage be V_1 and V_2. Here, $V_2 = v$

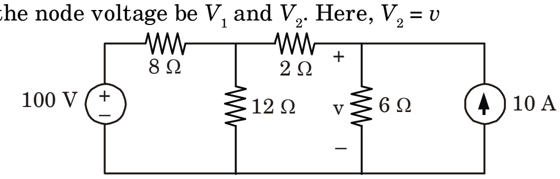


Fig. 1.3.2.

2. By KCL at node 1,

$$\frac{V_1 - 100}{8} + \frac{V_1}{12} + \frac{V_1 - V_2}{2} = 0$$

$$17V_1 - 12v = 300 \quad ...(1.3.1)$$

3. By KCL at node 2,

$$\frac{V_2 - V_1}{2} + \frac{V_2}{6} - 10 = 0$$

$$-3V_1 + 4v = 60 \quad ...(1.3.2)$$

4. After solving eq. (1.3.1) and eq. (1.3.2), we get

$$v = 60 \text{ V}$$

Que 1.4. **Using nodal analysis, determine the power supplied by the 8 V voltage source in Fig. 1.4.1.**

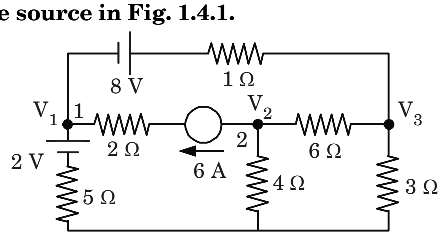


Fig. 1.4.1.

Answer

1. Nodal equations for the circuit of Fig. 1.4.1, at node 1 as

$$\frac{V_1 - 2}{5} + \frac{V_1 + 8 - V_3}{1} - 6 = 0$$

$$6V_1 - 5V_3 = -8 \quad ...(1.4.1)$$

2. The KCL equation at node 2 is

$$6 + \frac{V_2}{4} + \frac{V_2 - V_3}{6} = 0 \text{ or } 5V_2 - 2V_3 = -72 \qquad ...(1.4.2)$$

3. The KCL equation at node 3 is

$$\frac{V_3 - V_2}{6} + \frac{V_3}{3} + \frac{V - 8 - V_1}{1} = 0 \text{ or } -6V_1 - V_2 + 9V_3 = 48 \qquad ...(1.4.3)$$

4. Solving the eq. (1.4.1), eq. (1.4.2) and eq. (1.4.3), we get
   $V_1 = 4.592$ V; $V_2 = -11.555$ V; $V_3 = 7.111$ V
5. Current through 1 Ω resistance and 8 V voltage source

$$= \frac{V_1 + 8 - V_3}{1} = \frac{4.592 + 8 - 7.111}{1} = 5.481 \text{ A}$$

6. Power supplied by the 8 V source = 8 × 5.481 = 43.848 W.

Que 1.5. **Write nodal equations for the circuit shown in Fig. 1.5.1, and find the power supplied by the 20 V source.**

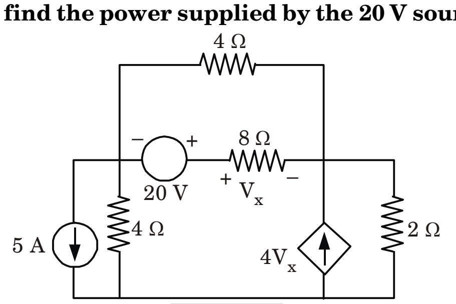


Fig. 1.5.1.

Answer

1. Consider nodes 1 and 2 and node voltages V_1 and V_2 as shown in Fig. 1.5.2.

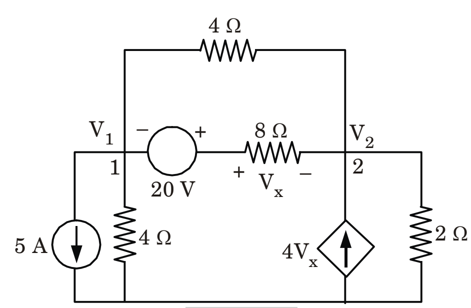


Fig. 1.5.2.

2. The KCL equation at node 1 is :

$$5 + \frac{V_1}{4} + \frac{V_1 - V_2}{4} + \frac{V_1 + 20 - V_2}{8} = 0$$

$$0.625V_1 - 0.375V_2 = -7.5 \quad \text{...(1.5.1)}$$

3. The KCL equation at node 2 is :

$$\frac{V_2 - V_1}{4} + \frac{V_2 - 20 - V_1}{8} + \frac{V_2}{2} - 4V_x = 0$$

$$-0.375V_1 + 0.875\,V_2 - 4V_x = 2.5 \quad \text{...(1.5.2)}$$

4. Substituting, $V_x = V_1 + 20 - V_2$ in eq. (1.5.2)

$$-4.375V_1 + 4.875V_2 = 82.5 \quad \text{...(1.5.3)}$$

5. Solving eq. (1.5.1) and eq. (1.5.3), we have the node voltages, $V_1 = -4.02$ V and $V_2 = 13.39$ V.
6. The current delivered by the 20 V source is

$$\frac{V_1 + 20 - V_2}{8} = \frac{-4.02 + 20 - 13.39}{8} = 0.324 \text{ A}$$

7. The power supplied by the 20 V source is

$$P_s = 20 \times 0.324 = 6.48 \text{ W}$$

Que 1.6. **Explain the concept of supernode analysis.**

OR

How do you apply nodal analysis when there is only a voltage source in a branch between two nodes ?

Answer

1. Suppose any of the branches in the network connected between two nodes has only a voltage source, then it is slightly difficult to apply nodal analysis.
2. An alternative way to overcome this difficulty is to apply the supernode technique.
3. In this method, the two adjacent nodes that are connected by a voltage source are reduced to a single larger node called supernode and then the equations are formed by applying KCL.
4. This is explained with the help of Fig. 1.6.1. Node 4 is the reference node and nodes 2 and 3 form the supernode.

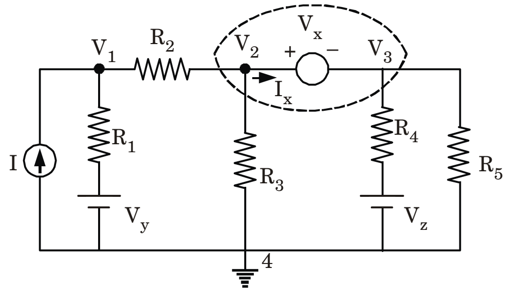


Fig. 1.6.1.

5. KCL at node 1 is :

$$I = \frac{V_1 - V_y}{R_1} + \frac{V_1 - V_2}{R_2} \quad ...(1.6.1)$$

6. KCL at supernode 2, 3 is :

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_3 - V_z}{R_4} + \frac{V_3}{R_5} = 0 \quad ...(1.6.2)$$

7. The constraint equation is :

$$V_2 - V_x = V_3$$

$$V_2 = V_3 + V_x \quad ...(1.6.3)$$

8. After solving eq. (1.6.1), eq. (1.6.2) and eq. (1.6.3), V_1, V_2 and V_3 can be calculated.

Que 1.7. **Determine the current in the 5 Ω resistor for the circuit shown in Fig. 1.7.1.**

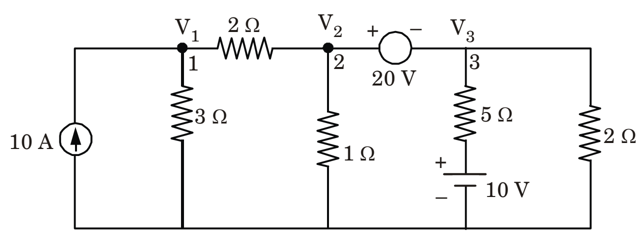


Fig. 1.7.1.

Answer

1. At node 1

$$10 = \frac{V_1}{3} + \frac{V_1 - V_2}{2}$$

$$V_1\left[\frac{1}{3}+\frac{1}{2}\right]-\frac{V_2}{2}-10=0$$

$$0.83\,V_1 - 0.5\,V_2 - 10 = 0 \qquad ...(1.7.1)$$

2. At node 2 and 3, the supernode equation is

$$\frac{V_2-V_1}{2}+\frac{V_2}{1}+\frac{V_3-10}{5}+\frac{V_3}{2}=0$$

$$\frac{-V_1}{2}+V_2\left[\frac{1}{2}+1\right]+V_3\left[\frac{1}{5}+\frac{1}{2}\right]=2$$

$$-0.5\,V_1 + 1.5\,V_2 + 0.7\,V_3 - 2 = 0 \qquad ...(1.7.2)$$

3. The voltage between nodes 2 and 3 is given by

$$V_2 - V_3 = 20 \qquad ...(1.7.3)$$

4. Solving eq. (1.7.1), eq. (1.7.2) and eq. (1.7.3), we obtain

$$V_3 = -\,8.42\text{ V}$$

5. The current in the 5 Ω resistor = $\dfrac{V_3-10}{5}$

$$=\frac{-\,8.42-10}{5}=-\,3.68\text{ A (current towards nodes 3)}$$

Que 1.8. **Find the current i_a, using nodal analysis in the circuit shown in Fig. 1.8.1.**

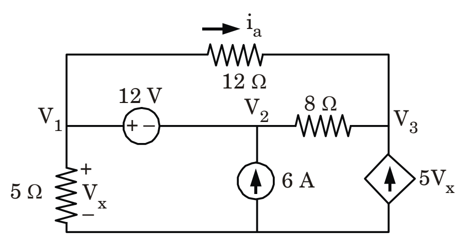


Fig. 1.8.1.

Answer

1. Considering the supernode in Fig. 1.8.1, the supernode equation is

$$\frac{V_1}{5}+\frac{V_1-V_3}{12}+\frac{V_2-V_3}{8}-6=0$$

$$0.283V_1 + 0.125V_2 - 0.2083V_3 = 6 \qquad ...(1.8.1)$$

2. The KCL equation at node 3 is

$$\frac{V_3 - V_1}{12} + \frac{V_3 - V_2}{8} - 5V_x = 0$$

$$-5.083V_1 - 0.125V_2 + 0.208V_3 = 0 \quad \text{...(1.8.2)}$$

3. The constraint equation is $V_1 = V_2 + 12$ ...(1.8.3)
4. Solving eq. (1.8.1), eq. (1.8.2) and eq. (1.8.3), we have

$$V_1 = -1.25 \text{ V}$$
$$V_2 = -13.25 \text{ V}$$
$$V_3 = -38.5 \text{ V}$$

5. $$i_a = \frac{V_1 - V_3}{12} = \frac{-1.25 - (-38.5)}{12} = 3.104 \text{ A}$$

PART-2

Mesh Analysis.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.9. **Explain mesh analysis briefly.**

Answer

1. Mesh analysis is applicable only for planar networks. For non-planar circuits, mesh analysis is not applicable.
2. A circuit is said to be planar, if it can be drawn on a plane surface without crossovers. A non-planar circuit cannot be drawn on a plane surface without a crossover.
3. Fig. 1.9.1(a) is a planar circuit and Fig. 1.9.1(b) is a non-planar circuit.

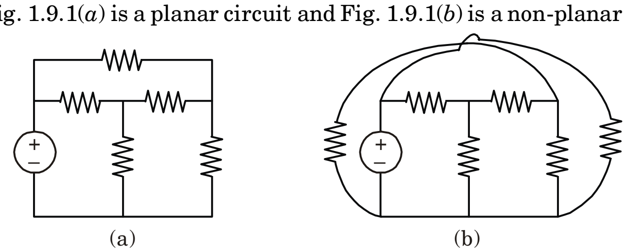


Fig. 1.9.1.

4. A mesh is defined as a loop which does not contain any other loops within it.

5. To apply mesh analysis, our first step is to check whether the circuit is planar or not and the second is to select mesh currents.
6. Finally, writing Kirchhoff's voltage law equations in terms of unknowns and solving them leads to the final solution.
7. In circuit shown in Fig. 1.9.2, there are two loops *abefa* and *bcdeb* in the network.

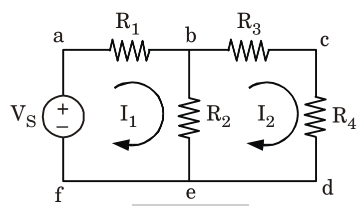


Fig. 1.9.2.

8. Let us assume loop currents I_1 and I_2 with directions as indicated in the Fig. 1.9.2.
9. Considering the loop *abefa* alone, we observe that current I_1 is passing through R_1, and $(I_1 - I_2)$ is passing through R_2. By applying Kirchhoff's voltage law, we can write

$$V_s = I_1R_1 + R_2(I_1 - I_2)$$

$$I_1(R_1 + R_2) - I_2 R_2 = V_s \quad ...(1.9.1)$$

10. Similarly, if we consider the second mesh *bcdeb*, the current I_2 is passing through R_3 and R_4, and $(I_2 - I_1)$ is passing through R_2. By applying Kirchhoff's voltage law around the second mesh, we have

$$R_2 (I_2 - I_1) + R_3 I_2 + R_4 I_2 = 0$$

$$- I_1 R_2 + (R_2 + R_3 + R_4)I_2 = 0 \quad ...(1.9.2)$$

11. By solving the eq. (1.9.1) and eq. (1.9.2), we can find the currents I_1 and I_2.
12. If we observe Fig. 1.9.2, the circuit consists of five branches and four nodes, including the reference node. The number of mesh currents is equal to the number of mesh equations.
13. In general, if we have B number of branches and N number of nodes including the reference node then the number of linearly independent mesh equations $M = B - (N - 1)$.

Que 1.10. **Write the mesh current equations in the circuit shown in Fig. 1.10.1, and determine the currents.**

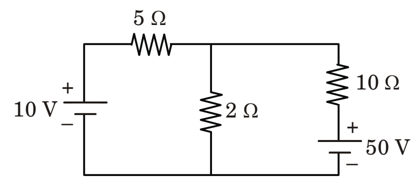


Fig. 1.10.1.

Answer

1. Assume two mesh currents in the direction as indicated in Fig. 1.10.2.

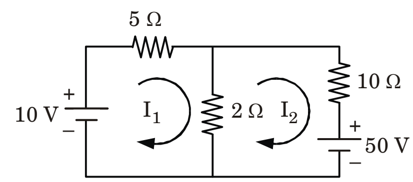


Fig. 1.10.2.

2. The mesh current equations are

$$5I_1 + 2(I_1 - I_2) = 10 \qquad ...(1.10.1)$$

$$10I_2 + 2(I_2 - I_1) + 50 = 0 \qquad ...(1.10.2)$$

3. We can rearrange the eq. (1.10.1) and eq. (1.10.2) as

$$7I_1 - 2I_2 = 10 \qquad ...(1.10.3)$$

$$-2I_1 + 12I_2 = -50 \qquad ...(1.10.4)$$

4. By solving the eq. (1.10.3) and eq. (1.10.4) then we have

$$I_1 = 0.25 \text{ A, and } I_2 = -4.125 \text{ A}$$

5. Here, the current in the second mesh, I_2, is negative; that is the actual current I_2 flows opposite to the assumed direction of current in the circuit of Fig. 1.10.2.

Que 1.11.

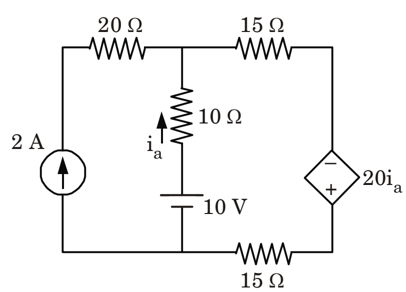


Fig. 1.11.1.

i. Use the mesh current method to solve for i_a in the circuit shown in Fig. 1.11.1.

ii. Find the power delivered by the independent current and voltage sources.

iii. Find the power delivered by the dependent voltage source.

iv. Show that the power delivered by the sources is equal to the power dissipated by the resistors.

Answer

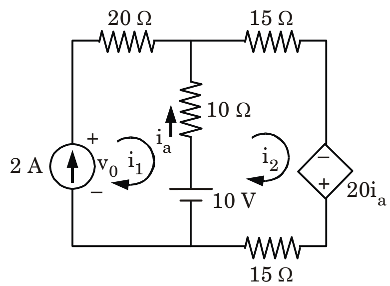


Fig. 1.11.2.

i. The circuit with loop currents marked is shown in Fig. 1.11.2. It has two loops. Loop 1 has an independent current source of 2 A. Hence $i_1 = 2$ A.

$\therefore$ KVL equation for loop 2, we get

$$10(i_2 - 2) + 15i_2 - 20i_a + 15i_2 - 10 = 0$$

$$i_a = i_2 - i_1 = i_2 - 2$$

$$\therefore\ 10(i_2 - 2) + 15i_2 - 20(i_2 - 2) + 15i_2 - 10 = 0,$$

$$20i_2 = -10$$

$$i_2 = -0.5 \text{ A}$$

$$\therefore \quad i_a = i_2 - 2 = -0.5 - 2 = -2.5 \text{ A}$$

ii. Applying KVL to mesh 1, we get

$$-v_0 + 20i_1 + 10(-i_a) + 10 = 0,$$

$$v_0 = 20 \times 2 + 10(2.5) + 10 = 75 \text{ V}$$

So power delivered by 2 A current source = 75 ×2 = 150 W

Power delivered by 10 V source = $10i_a = 10 \times (-2.5) = -25$ W

The negative sign implies that power is absorbed by the 10 V source.

iii. Power delivered by the dependent voltage source of $20i_a$ is

$$20i_a \times i_2 = 20(-2.5)(-0.5) = 25 \text{ W}$$

iv. So net power delivered by all sources = 150 – 25 + 25 = 150 W

Power dissipated by resistors $= i_1^2 \times 20 + i_a^2 \times 10 + i_2^2 \times 15 + i_2^2 \times 15$

$$= 2^2 \times 20 + (-2.5)^2 \times 10 + (-0.5)^2 \times 30 = 150 \text{ W}$$

So Power delivered by sources = Power dissipated by resistors.

Que 1.12. **Using mesh analysis, determine what value of v_s in Fig. 1.12.1 will cause $v_x = 0$.**

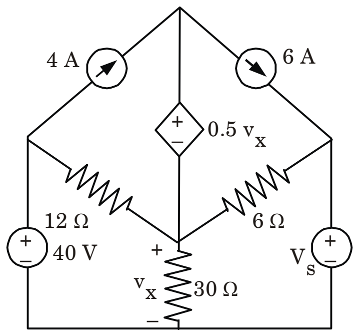


Fig. 1.12.1.

Answer

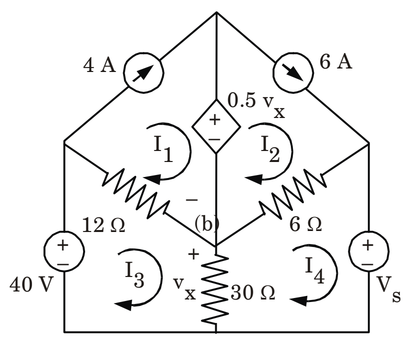


Fig. 1.12.2.

1. In Fig. 1.12.2, loop current $I_1 = 4$ A and loop current $I_2 = 6$ A.
2. KVL equations around mesh 3

   $-40 + 12(I_3 - I_1) + v_x = 0$
3. It is given that $v_x = 0$. Therefore,

   $$-40 + 12(I_3 - 4) = 0$$
   $$12I_3 = 88$$
   $$I_3 = 88/12 = 7.333 \text{ A}$$
4. Also $v_x = 30(I_3 - I_4) = 0$

   $\therefore$ $I_3 = I_4 = 7.333$ A
5. Applying KVL to mesh 4, we get

$$-v_x + 6(I_4 - I_2) + v_s = 0$$

$$v_s = -6(I_4 - I_2) = -6(7.33 - 6) = -8 \text{ V}$$

Que 1.13. How do you apply mesh analysis when there is a current source in a branch common to two loops ?

OR

Explain concept of supermesh analysis.

Answer

1. Suppose any of the branches in the network which is common to two loops has a current source, then it is slightly difficult to apply mesh analysis straight forward.
2. One way to overcome this difficulty is by applying the supermesh technique.
3. A supermesh is constituted by two adjacent loops that have a common current source, that is, a supermesh is a combination of two adjacent meshes ignoring the common branch in which current source is present.
4. Consider the network shown in Fig. 1.13.1. Here the current source I is in the common boundary for the two meshes 1 and 2. Let the voltage across the current source I be V_x.

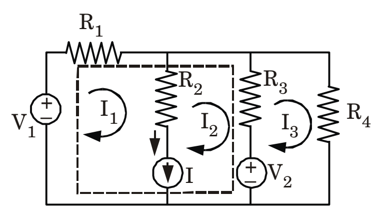


Fig. 1.13.1. Circuit with supermesh.

5. Using supermesh for Fig. 1.13.1,
   $I_1R_1 + (I_2 - I_3)R_3 + V_2 - V_1 = 0$ ...(1.13.1)
6. Constraint equation $I_1 - I_2 = I$ ...(1.13.2)
7. For loop 3, $(I_3 - I_2)R_3 + I_3R_4 - V_2 = 0$ ...(1.13.3)
8. Using eq. (1.13.1), eq. (1.13.2) and eq. (1.13.3), currents I_1, I_2 and I_3 can be calculated.

Que 1.14. In the circuit shown in Fig. 1.14.1, find the power delivered by the 10 V source and the voltage across the 4 Ω resistor using mesh analysis.

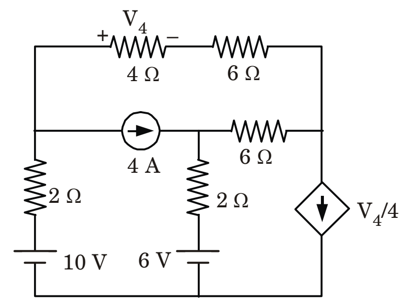


Fig. 1.14.1.

Answer

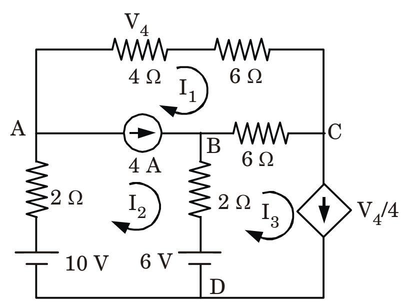


Fig. 1.14.2.

1. Since the 4 A current source is in a branch common to meshes 1 and 2, we use the supermesh techniques.
2. There is a current source in mesh 3,

   $\therefore \qquad I_3 = V_4/4.$
3. Also $I_1 = V_4/4.$

   So $\qquad I_1 = I_3.$
4. Also $\qquad I_2 - I_1 = 4$

   $I_2 = I_1 + 4$
5. The combined supermesh equation is :

   $4I_1 + 6I_1 + 6(I_1 - I_3) + 2(I_2 - I_3) + 6 - 10 + 2I_2 = 0,$

   $8I_1 + 2I_2 - 4I_3 = 2 \qquad (I_2 = I_1 + 4)$

   $\therefore\ 8I_1 + 2(I_1 + 4) - 4I_1 = 2$

   $I_1 = -1$ A

   $\therefore I_3 = -1$ A and $I_2 = -1 + 4 = 3$ A
6. The voltage across the 4 Ω resistor = $4I_1 = 4 \times (-1) = -4$ V

7. Power delivered by the 10 V source, $P_d = 10I_2 = 10 \times 3 = 30$ W

PART-3

Matrix Approach of Network Containing Voltage and Current Sources and Reactance.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.15. **Using mesh analysis, determine the voltage V_s which gives a voltage of 50 V across the 10 Ω resistor as shown in Fig. 1.15.1.**

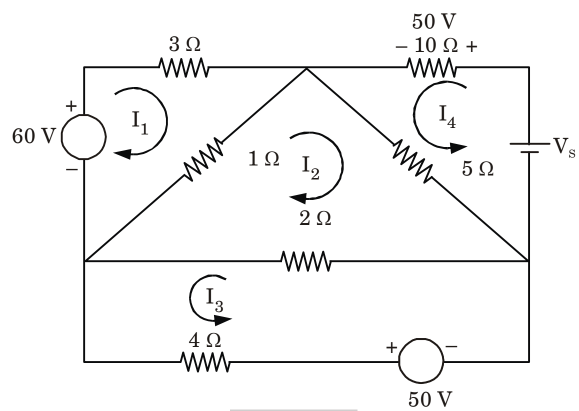


Fig. 1.15.1.

Answer

1. Since the voltage across the 10 Ω resistor is 50 V, the current passing through it is $I_4 = 50/10 = 5$ A.
2. From Fig. 1.15.1, we can form four equations in terms of the currents I_1, I_2, I_3 and I_4, as

$$4I_1 - I_2 = 60 \quad ...(1.15.1)$$

$$-I_1 + 8I_2 - 2I_3 + 5I_4 = 0 \quad ...(1.15.2)$$

$$-2I_2 + 6I_3 = 50 \quad ...(1.15.3)$$

$$5I_2 + 15I_4 = V_s \quad ...(1.15.4)$$

3. Solving the eq. (1.15.1), (1.15.2), (1.15.3) and (1.15.4) using Cramer's rule, we get

$$I_4 = \frac{\begin{vmatrix} 4 & -1 & 0 & 60 \\ -1 & 8 & -2 & 0 \\ 0 & -2 & 6 & 50 \\ 0 & 5 & 0 & V_s \end{vmatrix}}{\begin{vmatrix} 4 & -1 & 0 & 0 \\ -1 & 8 & -2 & 5 \\ 0 & -2 & 6 & 0 \\ 0 & 5 & 0 & 15 \end{vmatrix}}$$

$$\Delta = 4\begin{vmatrix} 8 & -2 & 5 \\ -2 & 6 & 0 \\ 5 & 0 & 15 \end{vmatrix} + 1\begin{vmatrix} -1 & -2 & 5 \\ 0 & 6 & 0 \\ 0 & 0 & 15 \end{vmatrix}$$

$$= 4\{8(90) + 2(-30) + 5(-30)\} + 1\{-1(90)\}$$

$$= 1950$$

$$\Delta_4 = 4\begin{vmatrix} 8 & -2 & 0 \\ -2 & 6 & 50 \\ 5 & 0 & V_S \end{vmatrix} + 1\begin{vmatrix} -1 & -2 & 0 \\ 0 & 6 & 50 \\ 0 & 0 & V_S \end{vmatrix} - 60\begin{vmatrix} -1 & 8 & -2 \\ 0 & -2 & 6 \\ 0 & 5 & 0 \end{vmatrix}$$

$$= 4\{8(6V_s) + 2(-2V_s - 250)\} + 1\{-1(6V_s)\} - 60\{-1(-30)\}$$

$$= 170V_s - 3800$$

$$I_4 = \frac{170V_s - 3800}{1950} = 5 \text{ A} \qquad [\because I_4 = 5 \text{ A}]$$

$$\therefore \quad V_s = \frac{1950 \times 5 + 3800}{170} = 79.7 \text{ V}$$

Que 1.16. **Determine the voltages at each node for the circuit shown in Fig. 1.16.1.**

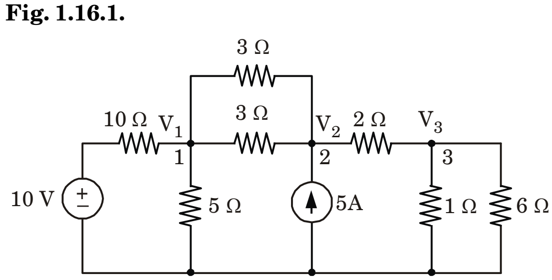


Fig. 1.16.1.

Answer

1. At node 1, assuming that all currents are leaving, we have

$$\frac{V_1 - 10}{10} + \frac{V_1 - V_2}{3} + \frac{V_1}{5} + \frac{V_1 - V_2}{3} = 0$$

$$V_1\left[\frac{1}{10} + \frac{1}{3} + \frac{1}{5} + \frac{1}{3}\right] - V_2\left[\frac{1}{3} + \frac{1}{3}\right] = 1$$

$$0.96\ V_1 - 0.66\ V_2 = 1 \qquad \text{...(1.16.1)}$$

2. At node 2, assuming that all currents are leaving except the current from current source, we have

$$\frac{V_2 - V_1}{3} + \frac{V_2 - V_1}{3} + \frac{V_2 - V_3}{2} = 5$$

$$-V_1\left[\frac{2}{3}\right] + V_2\left[\frac{1}{3} + \frac{1}{3} + \frac{1}{2}\right] - V_3\left[\frac{1}{2}\right] = 5$$

$$-0.66\ V_1 + 1.16\ V_2 - 0.5V_3 = 5 \qquad \text{...(1.16.2)}$$

3. At node 3, assuming all currents are leaving, we have

$$\frac{V_3 - V_2}{2} + \frac{V_3}{1} + \frac{V_3}{6} = 0$$

$$-0.5\ V_2 + 1.66\ V_3 = 0 \qquad \text{...(1.16.3)}$$

4. Applying Cramer's rule, we get

$$V_1 = \frac{\begin{vmatrix} 1 & -0.66 & 0 \\ 5 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}} = \frac{7.154}{0.887} = 8.06\ \text{V}$$

5. Similarly,

$$V_2 = \frac{\begin{vmatrix} 0.96 & 1 & 0 \\ -0.66 & 5 & -0.5 \\ 0 & 0 & 1.66 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}} = \frac{9.06}{0.887} = 10.2\ \text{V}$$

$$V_3 = \frac{\begin{vmatrix} 0.96 & -0.66 & 1 \\ -0.66 & 1.16 & 5 \\ 0 & -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}} = \frac{2.73}{0.887} = 3.07\ \text{V}$$

Que 1.17. **Determine the mesh current I_1 in the circuit shown in Fig. 1.17.1.**

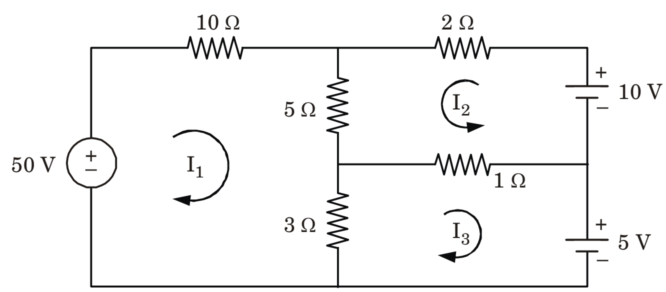


Fig. 1.17.1.

Answer

1. From the circuit in Fig. 1.17.1, we can form the following three mesh equations

$$10I_1 + 5(I_1 + I_2) + 3(I_1 - I_3) = 50 \quad ...(1.17.1)$$

$$2I_2 + 5(I_2 + I_1) + 1(I_2 + I_3) = 10 \quad ...(1.17.2)$$

$$3(I_3 - I_1) + 1(I_3 + I_2) = -5 \quad ...(1.17.3)$$

2. Rearranging the eq. (1.17.1), (1.17.2) and (1.17.3), we get

$$18I_1 + 5I_2 - 3I_3 = 50$$

$$5I_1 + 8I_2 + I_3 = 10$$

$$-3I_1 + I_2 + 4I_3 = -5$$

3. According to Cramer's rule

$$I_1 = \frac{\begin{vmatrix} 50 & 5 & -3 \\ 10 & 8 & 1 \\ -5 & 1 & 4 \end{vmatrix}}{\begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix}} = \frac{1175}{356} \text{ A}$$

$$I_1 = 3.3 \text{ A}$$

Que 1.18. **Determine the power dissipation in the 4 Ω resistor of the circuit shown in Fig. 1.18.1 by using mesh analysis.**

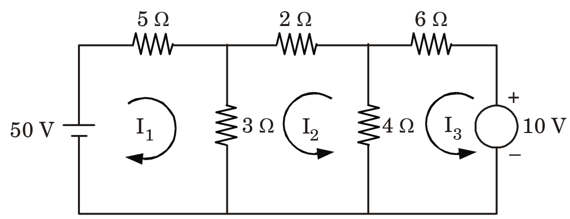


Fig. 1.18.1.

Answer

1. Power dissipated in the 4 Ω resistor is $P_4 = 4(I_2 - I_3)^2$
2. By using mesh analysis, we can find the currents I_2 and I_3.
3. From the given circuit in Fig. 1.18.1, we can obtain three mesh equations in terms of I_1, I_2 and I_3

$$8I_1 + 3I_2 = 50 \quad ...(1.18.1)$$

$$3I_1 + 9I_2 - 4I_3 = 0 \quad ...(1.18.2)$$

$$-4I_2 + 10I_3 = 10 \quad ...(1.18.3)$$

4. By solving the eq. (1.18.1), (1.18.2) and (1.18.3), we can find I_2 and I_3.

$$I_2 = \frac{\begin{vmatrix} 8 & 50 & 0 \\ 3 & 0 & -4 \\ 0 & 10 & 10 \end{vmatrix}}{\begin{vmatrix} 8 & 3 & 0 \\ 3 & 9 & -4 \\ 0 & -4 & 10 \end{vmatrix}} = \frac{-1180}{502} = -2.35 \text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 8 & 3 & 50 \\ 3 & 9 & 0 \\ 0 & -4 & 10 \end{vmatrix}}{\begin{vmatrix} 8 & 3 & 0 \\ 3 & 9 & -4 \\ 0 & -4 & 10 \end{vmatrix}} = \frac{30}{502} = 0.06 \text{ A}$$

5. The current in the 4 Ω resistor $= (I_2 - I_3)$

$$= (-2.35 - 0.06) \text{ A} = -2.41 \text{ A}$$

6. Therefore, the power dissipated in the 4 Ω resistor,

$$P_4 = (2.41)^2 \times 4 = 23.23 \text{ W}$$

Que 1.19. **Use nodal analysis to find the power dissipated in the 6 Ω resistor for the circuit shown in Fig. 1.19.1.**

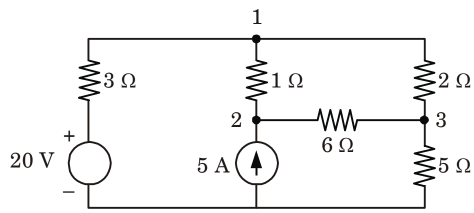


Fig. 1.19.1.

Answer

1. Assume voltage V_1, V_2 and V_3 at nodes 1, 2, and 3 as shown in Fig. 1.19.1.
2. By applying current law at node 1, we have

$$\frac{V_1 - 20}{3} + \frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{2} = 0$$

$$1.83V_1 - V_2 - 0.5V_3 = 6.67 \quad \text{...(1.19.1)}$$

3. At node 2,

$$\frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{6} = 5 \text{ A}$$

$$- V_1 + 1.167V_2 - 0.167V_3 = 5 \quad \text{...(1.19.2)}$$

4. At node 3,

$$\frac{V_3 - V_1}{2} + \frac{V_3 - V_2}{6} + \frac{V_3}{5} = 0$$

$$- 0.5\, V_1 - 0.167\, V_2 + 0.867\, V_3 = 0 \quad \text{...(1.19.3)}$$

5. Applying Cramer's rule to eq. (1.19.1), (1.19.2) and (1.19.3), we have

$$V_2 = \frac{\Delta_2}{\Delta}$$

where
$$\Delta = \begin{vmatrix} 1.83 & -1 & -0.5 \\ -1 & +1.167 & -0.167 \\ -0.5 & -0.167 & 0.867 \end{vmatrix} = 0.47$$

$$\Delta_2 = \begin{vmatrix} 1.83 & 6.67 & -0.5 \\ -1 & 5 & -0.167 \\ -0.5 & 0 & 0.867 \end{vmatrix} = 13.02$$

$$\therefore \qquad V_2 = \frac{13.02}{0.47} = 27.70 \text{ V}$$

6. Similarly,

$$V_3 = \frac{\Delta_3}{\Delta}$$

$$\Delta_3 = \begin{vmatrix} 1.83 & -1 & 6.67 \\ -1 & +1.167 & 5 \\ -0.5 & -0.167 & 0 \end{vmatrix} = 9.03$$

$$\therefore \qquad V_3 = \frac{9.03}{0.47} = 19.22 \text{ V}$$

7. The current in the 6 Ω resistor is

$$I_6 = \frac{V_2 - V_3}{6} = \frac{27.70 - 19.22}{6} = 1.41 \text{ A}$$

8. The power absorbed or dissipated = $I_6^2 R_6$

$$= (1.41)^2 \times 6$$
$$= 11.99 \text{ W}$$

PART-4

Source Transformation and Duality.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.20. **Discuss source transformation technique for solving network.**

Answer

1. Any practical voltage source consists of an ideal voltage source in series with an internal resistance. Similarly, a practical current source consists of an ideal current source in parallel with an internal resistance as shown in Fig. 1.20.1.
2. R_v and R_1 represent the internal resistances of the voltage source V_s, and current source I_s, respectively.

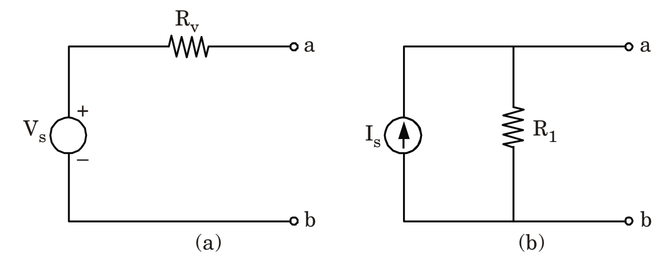


Fig. 1.20.1.

3. Any source, be it a current source or a voltage source, drives currents through its load resistance, and the magnitude of the current depends on the value of the load resistance.
4. Fig. 1.20.2(*a*) and (*b*) represent a practical voltage source and a practical current source respectively connected to the same load resistance R_L.

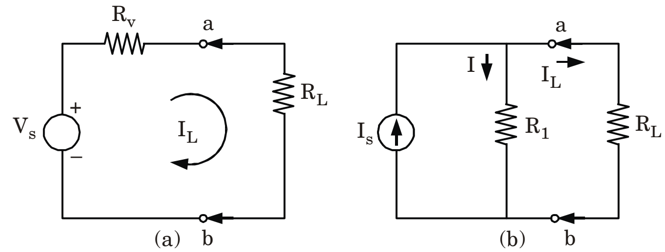


Fig. 1.20.2.

5. From Fig. 1.20.2(*a*), the load voltage can be calculated by using Kirchhoff's voltage law as

$$V_{ab} = V_s - I_L R_v$$

Open circuit voltage $V_{oc} = V_s$

Short circuit current $I_{sc} = \dfrac{V_s}{R_v}$

6. From Fig. 1.20.2(*b*)

$$I_L = I_s - I = I_S - \frac{V_{ab}}{R_1}$$

Open circuit voltage $V_{oc} = I_s R_1$

Short circuit current $I_{sc} = I_s$

7. Here two sources (voltage and current source) are said to be equal, if they produce equal amounts of current and voltage when they are connected to identical load resistance.
8. Therefore, by equating the open circuit voltages and short circuit currents of the two sources we obtain

$$V_{oc} = I_s R_1 = V_s$$

$$I_{sc} = I_s = \frac{V_s}{R_v}$$

9. If follows that $R_1 = R_v = R_s$

$\therefore$ $V_s = I_s R_s$

where R_s is the internal resistance of the voltage or current source.

10. Therefore, any practical voltage source, having an ideal voltage V_s and internal series resistance R_s can be replaced by a current source $I_s = V_s / R_s$ in parallel with an internal resistance R_s.
11. The reverse transformation is also possible. Thus, a practical current source in parallel with an internal resistance R_s can be replaced by a voltage source $V_s = I_s R_s$ in series with an internal resistance R_s.

Que 1.21. **Determine the equivalent voltage source for the current source shown in Fig. 1.21.1.**

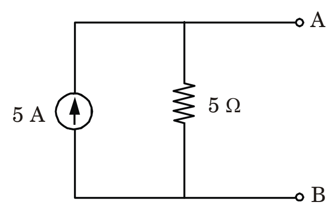


Fig. 1.21.1.

Answer

1. The voltage across terminals A and B is equal to 25 V.
2. Since the internal resistance for the current source is 5 Ω, the internal resistance of the voltage source is also 5 Ω.
3. The equivalent voltage source is shown in Fig. 1.21.2.

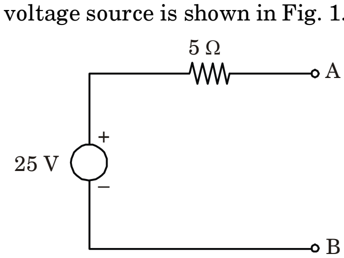


Fig. 1.21.2.

Que 1.22. **Determine the equivalent current source for the voltage source shown in Fig. 1.22.1.**

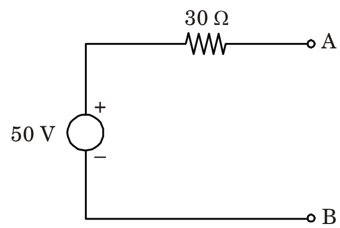


Fig. 1.22.1.

Answer

1. The short circuit current at terminals A and B is equal to

$$I = \frac{50}{30} = 1.66 \text{ A}$$

2. Since the internal resistance for the voltage source is 30 Ω, the internal resistance of the current source is also 30 Ω.
3. The equivalent current source is shown in Fig. 1.22.2.

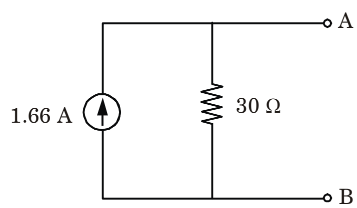


Fig. 1.22.2.

Que 1.23. **Using source transformation, find the power delivered by the 50 V voltage source in the circuit shown in Fig. 1.23.1.**

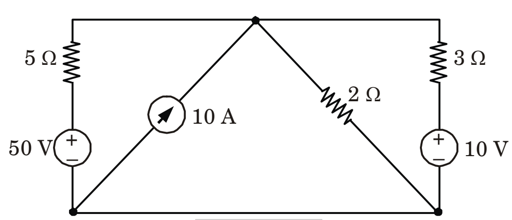


Fig. 1.23.1.

Answer

1. The current source in the circuit in Fig. 1.23.1 can be replaced by a voltage source as shown in Fig. 1.23.2.

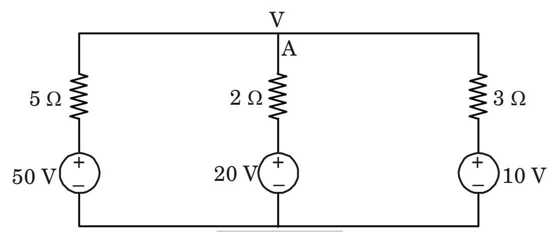


Fig. 1.23.2.

2. Apply KCL at node A,

$$\frac{V-50}{5}+\frac{V-20}{2}+\frac{V-10}{3}=0$$

$$V[0.2 + 0.5 + 0.33] = 23.33$$

$$V = \frac{23.33}{1.03} = 22.65 \text{ V}$$

3. The current delivered by the 50 V voltage source = $(50 - V) / 5$

$$= \frac{50-22.65}{5} = 5.47 \text{ A}$$

4. Hence, the power delivered by the 50 V voltage source,

$$= 50 \times 5.47 = 273.5 \text{ W}$$

Que 1.24. **What do you understand by the term 'duality' ? Discuss the procedure to find out the dual of a given network having both voltage and current sources.**

OR

Draw the dual of the network shown in Fig. 1.24.1.

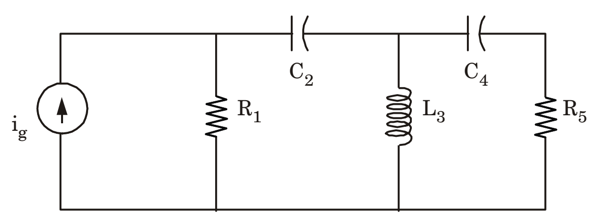


Fig. 1.24.1.

OR

What do you mean by "duality of graph of the network" ? Also mention its utilities and drawbacks.

Answer

Duality :

1. Duality is a transformation in which currents and voltages are interchanged. Two phenomena are said to be dual if they are described by equations of the same mathematical form.
2. There are a number of similarities and analogies between the two circuit analysis techniques based on loop-current method and node voltage method.
3. The principle quantities and concepts involved in these two methods based on KVL and KCL are dual of each other with voltage variables substituted by current variables, independent loop by independent node-pair, etc. This similarity is termed as 'principle of duality'.

Dual quantities and concepts :

S. No.	Quantities	Dual
1.	Current	Voltage
2.	Resistance	Conductance
3.	Inductance	Capacitance
4.	Impedance	Admittance
5.	Reactance	Susceptance
6.	Branch current	Branch voltage
7.	Mesh or loop	Node or node-pair
8.	Mesh current or loop current	Node voltage or node-pair voltage
9.	Link	Tree branch
10.	Link current	Tree branch voltage
11.	Tree branch current	Link voltage
12.	Tie-set	Cut-set
13.	Short-circuit	Open-circuit
14.	Parallel paths	Series paths

Construction of dual of a network :

1. A dot is placed inside each independent loop of the given network; these dots correspond to the non-reference nodes of the dual network.
2. A dot is placed outside the network; this dot corresponds to the datum node.

3. All internal dots are connected by dashed lines crossing the common branches.
4. All internal dots are connected to the external dot by dashed lines crossing all external branches.
5. Now replace all the crossed branch element by its dual element.

Utility of dual networks : It makes very easy to solve complex network problems.

Limitations of dual network :

1. Power has no dual due to its non-linearity.
2. Even when linearity applies a circuit element may not have a dual like mutual inductance.

Duality of network is shown in Fig. 1.24.3.

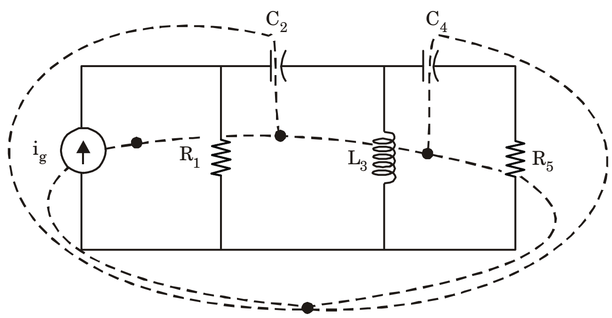


Fig. 1.24.2.

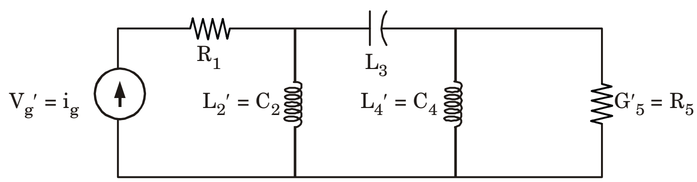


Fig. 1.24.3.

Que 1.25. Find the dual of the network shown in Fig. 1.25.1.

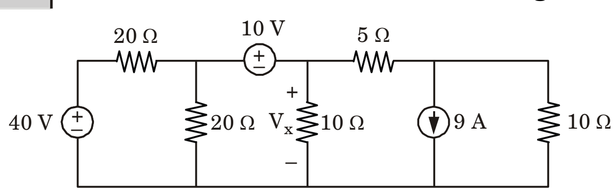


Fig. 1.25.1.

Answer

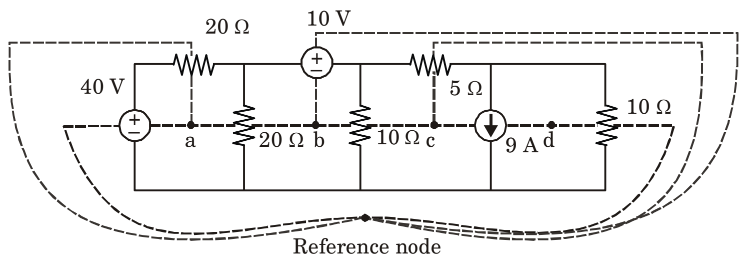


Fig. 1.25.2.

1. A node is placed inside each loop. In given Fig. 1.25.2, there are four loops.
2. An extra node is placed outside the network.
3. All the nodes are joined through element of original network, transversing only one element at a time.
4. For each element transversed in original network, dual element is connected.

   Dual network is as follows :

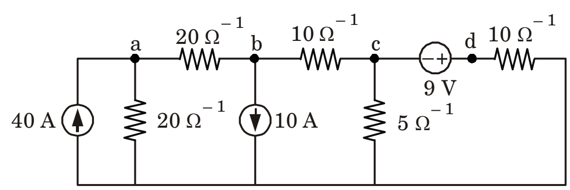


Fig. 1.25.3.

Que 1.26. **Draw the dual of the circuit shown in Fig. 1.26.1.**

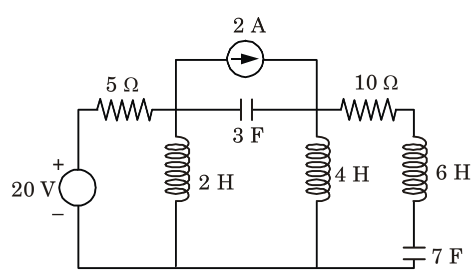
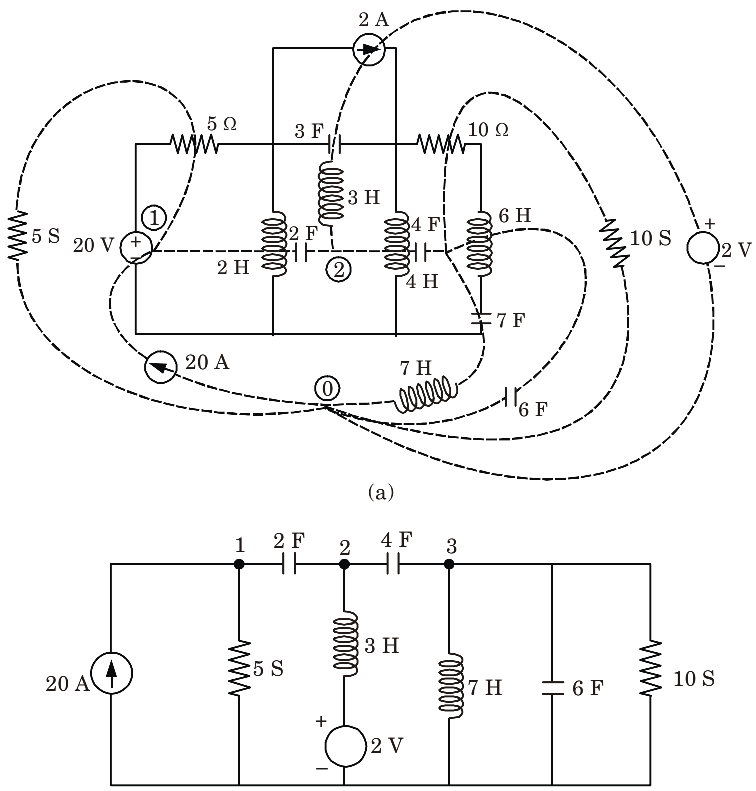


Fig. 1.26.1.

Answer



(a)

(b)

Fig. 1.26.2. (a) Development of the dual network, (b) Redrawn dual network.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. For the circuit shown in Fig. 1, find the voltage V_x using nodal analysis.

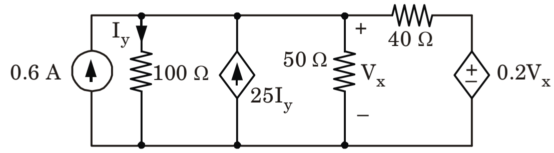


Fig. 1.

Ans. Refer Q. 1.2.

Q. 2. Determine the current in the 5 Ω resistor for the circuit shown in Fig. 2.

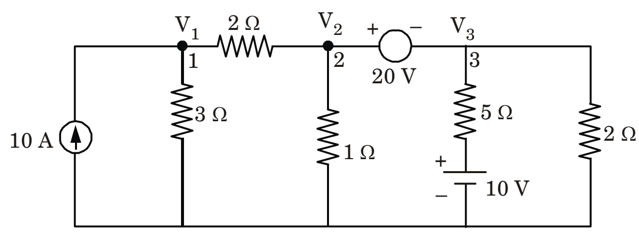


Fig. 2.

Ans. Refer Q. 1.7.

Q. 3. Explain mesh analysis briefly.

Ans. Refer Q. 1.9.

Q. 4. How do you apply mesh analysis when there is a current source in a branch common to two loops ?

Ans. Refer Q. 1.13.

Q. 5. In the circuit shown in Fig. 3, find the power delivered by the 10 V source and the voltage across the 4 Ω resistor using mesh analysis.

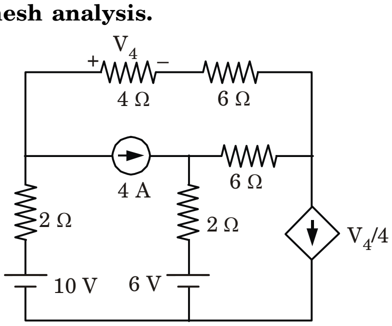


Fig. 3.

Ans. Refer Q. 1.14.

Q. 6. Determine the mesh current I_1 in the circuit shown in Fig. 4.

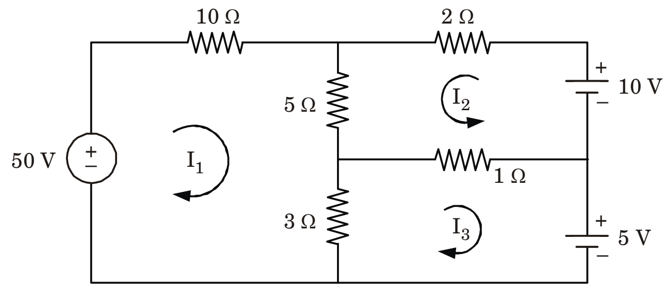


Fig. 4.

Ans. Refer Q. 1.17.

Q. 7. Discuss source transformation technique for solving network.

Ans. Refer Q. 1.20.

Q. 8. What do you understand by the term 'duality' ? Discuss the procedure to find out the dual of a given network having both voltage and current sources.

Ans. Refer Q. 1.24.

Q. 9. Draw the dual of the circuit shown in Fig. 5.

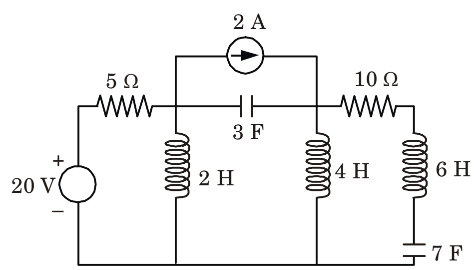


Fig. 5.

Ans. Refer Q. 1.26.

☺☺☺

2
UNIT

Network Theorems

CONTENTS

PART-1

Superposition Theorem.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.1. **State and explain superposition theorem.**

Answer

Statement :

If a number of voltage or current sources are acting simultaneously in a linear network, the resultant current in any branch is the algebraic sum of the currents that would be produced in it, when each source acts alone replacing all other independent sources by their internal resistances.

Explanation :

1. To apply superposition theorem in Fig. 2.1.1(*a*), let us first take the source V_1 alone at first replacing V_2 by short circuit [Fig. 2.1.1(*b*)]
2. Here,

$$I_1' = \frac{V_1}{\dfrac{R_2 R_3}{R_2 + R_3} + R_1}$$

$$I_2' = I_1' \frac{R_3}{R_2 + R_3}$$

and $I_3' = I_1' - I_2'$

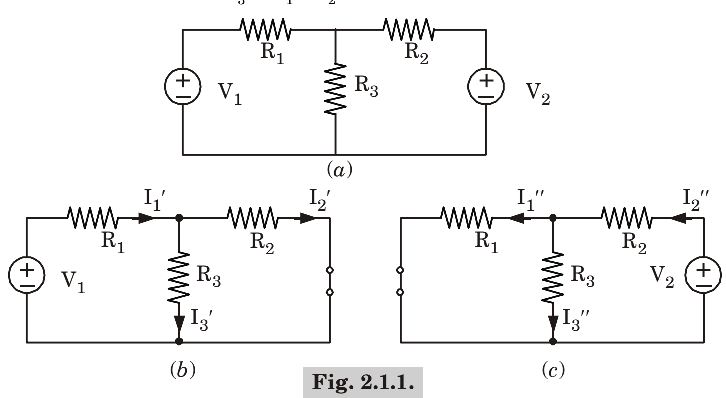


Fig. 2.1.1.

3. Next, removing V_1 by short circuit, let the circuit be energized by V_2 only Fig. 2.1.1(c).

Here, $$I_2'' = \frac{V_2}{\dfrac{R_1 R_3}{R_1 + R_3} + R_2} \quad \text{and} \quad I_1'' = I_2'' \frac{R_3}{R_1 + R_3}$$

also, $$I_3'' = I_2'' - I_1''.$$

4. As per superposition theorem,

$$I_3 = I_3' + I_3''$$
$$I_2 = I_2' - I_2''$$
$$I_1 = I_1' - I_1''$$

5. It may be noted that during application of superposition, the direction of currents calculated for each source should be taken care.

Que 2.2. **Explain why "superposition theorem" is not applicable for power verifications of a given network ? Explain the following :**

i. Linearity principle.

ii. Homogeneity principle.

Answer

A. Superposition theorem is applicable only when we are applying it to linear variable such as voltage and current, whereas power, $P = I^2R$, is a non-linear variable and hence superposition theorem cannot be applied.

B. Linearity principle :

1. A circuit is said to be linear if it obeys the law of superposition.
2. According to superposition theorem, current I_1 due to n independent voltage sources is given by,

$$I_1 = I_1' + I_1'' + I_1''' + \ldots$$

where I_1' is component of I_1 due to source 1 when all other sources are inactive. I_1'' is component of I_1 due to source 2 alone when all other sources are inactive and so on.

3. This is nothing but additive property of a linear network. Hence network must be linear for the application of superposition theorem. This is the principle of linearity.

C. Principle of homogeneity :

1. In the result of a superposition theorem, if we multiply each component of I_1 by a constant α then the total response I_1 also gets multiplied by the same constant.

if $$I_1 = I_1' + I_1'' + I_1''' + \ldots$$

then $$\alpha I_1 = \alpha I_1' + \alpha I_1'' + \alpha I_1''' + \ldots$$

This is called principle of homogeneity.

Que 2.3. **Consider the circuit shown in Fig. 2.3.1.**

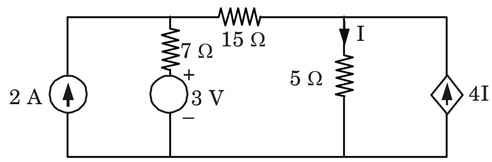


Fig. 2.3.1.

Obtain the voltage across each current source using superposition theorem.

Answer

1. If 2 A source current is acting alone, then

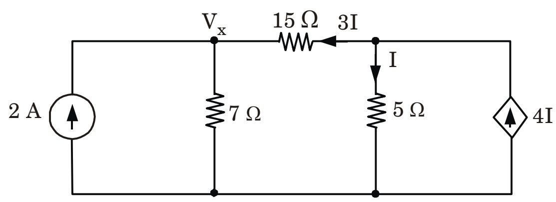


Fig. 2.3.2.

The equations are: $$3I = \frac{5I - V_x}{15} \quad ...(2.3.1)$$

and $$\frac{V_x}{7} = 2 + \frac{5I - V_x}{15} \quad ...(2.3.2)$$

$$\frac{V_x}{7} = \frac{30 + 5I - V_x}{15}$$

$$15V_x = 210 + 35I - 7V_x$$

$$22V_x = 210 + 35I$$

$$35I = 22V_x - 210$$

$$I = \frac{22V_x - 210}{35} \quad ...(2.3.3)$$

2. Putting the value I in eq. (2.3.1)

$$3 \times \frac{22V_x - 210}{35} = \frac{5 \times \frac{22V_x - 210}{35} - V_x}{15}$$

$$\frac{66V_x - 630}{35} = \frac{22V_x - 210 - 7V_x}{15 \times 7}$$

$$\frac{66V_x - 630}{35} = \frac{15V_x - 210}{105}$$

$$\frac{66V_x - 630}{7} = \frac{15V_x - 210}{21}$$

$$1386V_x - 13230 = 105V_x - 1470$$
$$V_x = 9.18 \text{ V}$$

3. Putting value of V_x in eq. (2.3.3)

Now, $$I = \frac{22V_x - 210}{35} = -0.23 \text{ A} \quad ...(2.3.4)$$

4. Now if 3 V source is acting alone, then the equations are :

$$\frac{V_y}{5} = I' \quad ...(2.3.5)$$

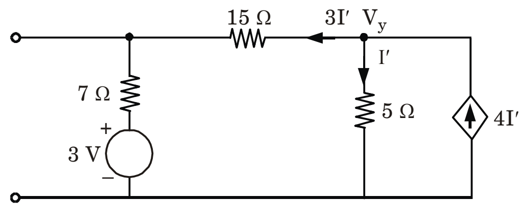


Fig. 2.3.3.

$$\frac{V_y - 3}{22} = 3I' \quad ...(2.3.6)$$

5. Solving eq. (2.3.5) and eq. (2.3.6),

$$\frac{5I' - 3}{22} = 3I'$$
$$5I' - 3 = 66I'$$
$$61I' = -3$$
$$I' = -3 / 61 = -0.05 \text{ A}$$

6. The voltage across $4I$ current source = $(-0.23 - 0.05) \times 5 = -1.4$ V
7. The voltage across 2 A current source = $7(2 + 3I + 3I') + 3$
$= 7(2 - 0.69 - 0.15) + 3 = 11.12$ V

Que 2.4. **Determine the current in capacitor by the principle of superposition of the network shown in Fig. 2.4.1.**

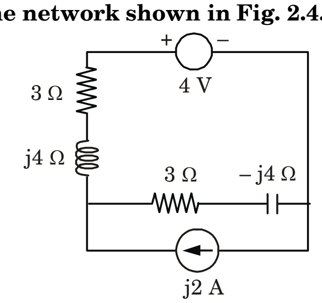


Fig. 2.4.1.

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Answer

1. When the voltage source is acting along as shown in Fig. 2.4.2, the current in the capacitor branch is

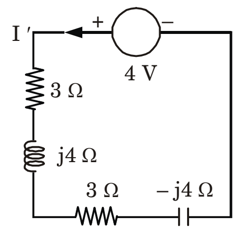


Fig. 2.4.2.

$$I' = \frac{4}{(3+j4)+(3-j4)} = \frac{2}{3}\text{ A}$$

2. When the current source is acting alone as shown in Fig. 2.4.3, the current in the capacitor branch is

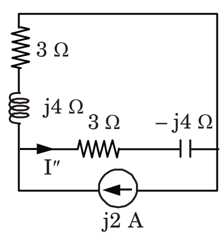


Fig. 2.4.3.

$$I'' = j2 \times \frac{(3+j4)}{(3+j4)+(3-j4)} = \left(-\frac{4}{3}+j1\right)\text{ A}$$

3. Using superposition principle, the total current when both the sources are acting simultaneously, is

$$I = (I'' + I'') = \left(\frac{2}{3}-\frac{4}{3}+j1\right) = \left(-\frac{2}{3}+j1\right)$$

$$= 1.2\angle 123.7\text{ A}$$

Que 2.5. **Calculate the current *I* shown in Fig. 2.5.1 by using superposition theorem.**

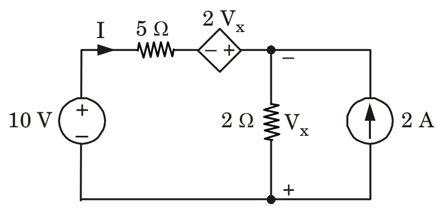


Fig. 2.5.1.

Answer

1. i. When the 10 V voltage source is acting alone.

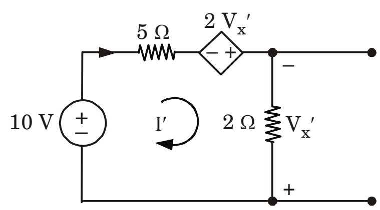


Fig. 2.5.2. Voltage source acting alone.

ii. By KVL,

$$5I' - 2V_x' + 2I' = 10 \text{ with } V_x' = -2I'$$
$$7I' + 4I' = 10$$
$$I' = \frac{10}{11} \text{ A}$$

2. i. When 2 A current source is acting alone.

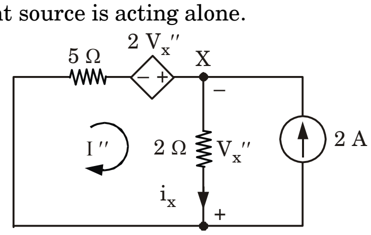


Fig. 2.5.3. Current source acting alone.

ii. By KCL at the node (X)

$$2 = I_x - I'' = -\frac{V_x''}{2} - I'' \quad \text{...(2.5.1)}$$

iii. But loop analysis in the left loop gives

$$5i'' - 3V_x'' = 0$$
$$I'' = \frac{3}{5}V_x'' \quad \text{...(2.5.2)}$$

iv. Putting value of I'' in eq. (2.5.1),

$$2 = -\frac{V_x''}{2} - \frac{3}{5}V_x''$$

$$V_x'' = -\frac{20}{11}$$

$$I'' = +\frac{3}{5} \times (-20)$$

$$I'' = -\frac{12}{11}\text{ A}$$

3. So, by the superposition theorem total current, when both the sources are acting simultaneously, is

$$I = (I' + I'') = \left(\frac{10}{11} - \frac{12}{11}\right) = -\frac{2}{11}\text{ A}$$

Que 2.6. **Calculate the voltage V across the resistor R by using the superposition theorem.**

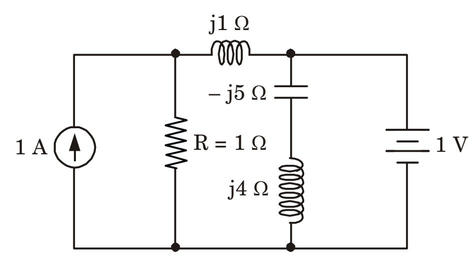


Fig. 2.6.1.

Answer

1. When the 1 A current source is acting alone.

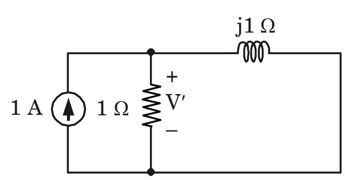


Fig. 2.6.2.

the voltage across the resistor $R = 1\ \Omega$ is,

$$V' = \frac{j}{1+j}\text{ V}$$

2. When the 1 V voltage source is acting alone.

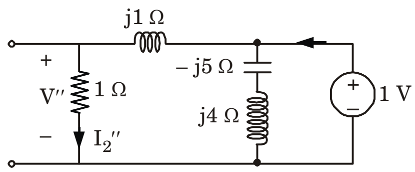


Fig. 2.6.3.

the current through the resistor

$$I_2'' = \frac{1}{1+j}\,\text{A}$$

and hence, the voltage across the resistor $R = 1\ \Omega$ is

$$V'' = I_2'' \times 1\ \frac{1}{1+j}\,\text{A} = \frac{1}{1+j}\,V$$

3. So, by the superposition theorem, total voltage across the resistor when both the sources are acting simultaneously is,

$$V = (V' + V'') = \frac{j}{1+j} + \frac{1}{1+j} = 1\ \text{V}$$

PART-2

Reciprocity Theorem.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.7. **State reciprocity theorem in AC network.**

Answer

1. For a linear, time invariant, bilateral network, the ratio of input to the output remains constant in a reciprocal network with respect to an interchange between the points of application of input and measurement of output.

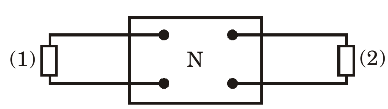


Fig. 2.7.1.

2. Consider a reciprocal network N, as shown in Fig. 2.7.1. Now, apply a voltage source of emf E_1 in branch 1 which produces current I_2 in branch 2 as shown in Fig. 2.7.2.

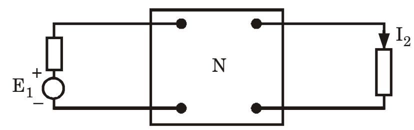


Fig. 2.7.2.

3. If the E_2 emf voltage source applied in branch 2 of network then the current I_1 is produced in branch 1 as shown in Fig. 2.7.3.

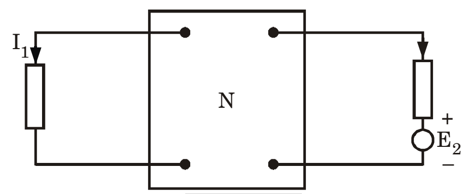


Fig. 2.7.3.

4. According to reciprocity theorem,

$$\frac{E_1}{I_2} = \frac{E_2}{I_1}$$

Que 2.8. **Consider the network shown in Fig. 2.8.1.**

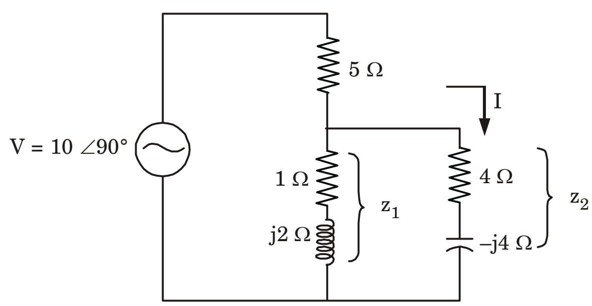


Fig. 2.8.1.

Verify reciprocity theorem for *V* and *I* for the network.

Answer

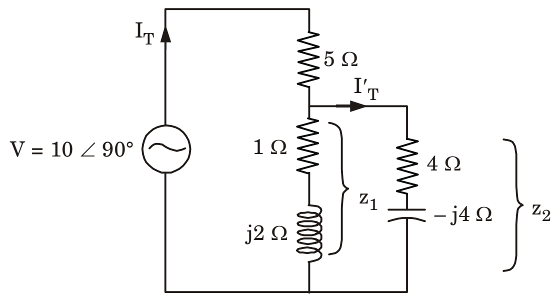


Fig. 2.8.2.

1. $$1 + 2j = z_1 = 2.23 \angle 63.43° \ \Omega$$
$$4 - 4j = z_2 = 5.65 \angle -45° \ \Omega$$

2. $$z_1 + z_2 = 5 - 2j = 5.385 \angle -21.80° \ \Omega$$

3. $$z_1 \,||\, z_2 = \frac{z_1 z_2}{z_1 + z_2} = \frac{2.23\angle 63.43° \times 5.65\angle -45°}{5.385\angle -21.80°}$$
$$= 2.34 \angle 40.23° = (1.786 + j1.511)\ \Omega$$

4. $$Z_{eq} = 5 + [z_1 \,||\, z_2] = 5 + 1.786 + j1.511$$
$$= 6.786 + j1.511 = 6.952 \angle 12.55° \ \Omega$$

5. Total current, $$I_T = \frac{10\angle 90^\circ}{6.952\angle 12.55^\circ} = 1.438 \angle 77.45° \text{ A}$$

6. Current through z_2, $$I = I_T \times \frac{z_1}{z_1 + z_2} = \frac{1.438\angle 77.45° \times 2.23\angle 63.43°}{5.385\angle -21.80°}$$
$$= 0.59 \angle 162.7° \text{ A}$$

7. Putting the voltage source in z_2 branch, 5 Ω and z_1 are in parallel,

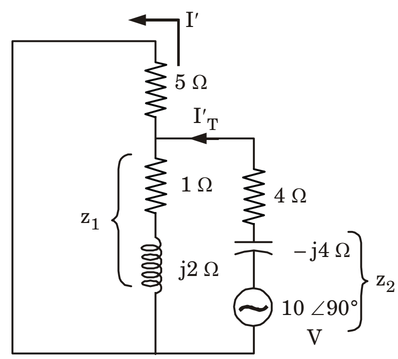


Fig. 2.8.3.

so, $$z_3 = \frac{5 \times 2.23\angle 63.43^\circ}{6.324\angle 18.43^\circ} \quad [5 + z_1 = 6.324 \angle 18.43^\circ]$$

$$= 1.763 \angle 45^\circ = 1.246 + 1.246j\ \Omega$$

8. This impedance is in series with z_2.

$$Z_{eq} = 1.246 + 1.246j + 4 - 4j = 5.924 \angle -27.70^\circ\ \Omega$$

9. Total current, $$I_T' = \frac{10\angle 90^\circ}{5.924\angle -27.70^\circ} = 1.68 \angle 117.7^\circ \text{ A}$$

10. Current through 5 Ω, $$I' = I_T' \times \frac{z_1}{5 + z_1} = \frac{1.68\angle 117.7^\circ \times 2.23\angle 63.43^\circ}{6.324\angle 18.43^\circ}$$

$$= 0.59 \angle 162.7^\circ \text{ A}$$

11. Since $I = I'$, therefore reciprocity theorem is verified.

Que 2.9. **Using reciprocity theorem and superposition theorem simultaneously, obtain current *I* in the network of Fig. 2.9.1, if 10 V source acting alone produces a current of 2 A in resistor *R*.**

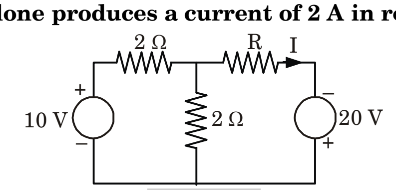


Fig. 2.9.1.

Answer

1. It is given that in Fig. 2.9.2(a), the 10 V source alone produces a current $I' = 2$ A in the resistor R.

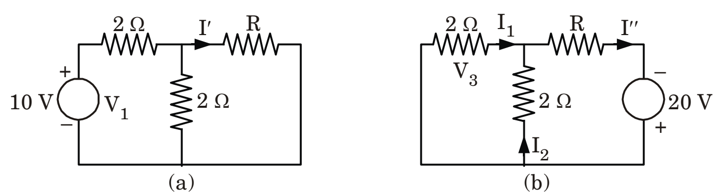


Fig. 2.9.2. Circuit with only (a) 10 V source, (b) 20 V source.

2. Now, let the 20 V source be acting alone with the 10 V source reduced to zero as shown in Fig. 2.9.2 (b).
3. Comparing Fig. 2.9.2 (a) and (b), we can observe that network in Fig. 2.9.2 (b) is obtained by interchanging the source and response in the network Fig. 2.9.2 (a).
4. Hence, using reciprocity theorem.

$$\frac{V_1}{I'} = \frac{V_2}{I_1} \quad \therefore I_1 = V_2 \times \frac{I'}{V_1} = 20 \times \frac{2}{10} = 4 \text{ A}$$

$\therefore \quad V_3 = 2 \times 4 = 8 \text{ V and } I_2 = \frac{V_3}{2} = \frac{8}{2} = 4 \text{ A}$

$\therefore \quad I'' = I_1 + I_2 = 4 + 4 = 8 \text{ A}$

5. By superposition theorem, current $I = I' + I'' = 2 + 8 = 10$ A

Que 2.10. **Verify the reciprocity theorem in the circuit shown in Fig. 2.10.1.**

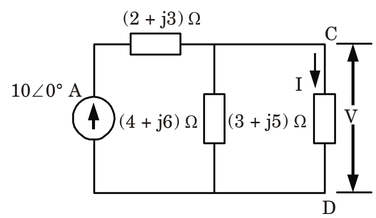


Fig. 2.10.1.

Answer

1. i. The excitation and response are in original position as shown in Fig. 2.10.1.

ii. The voltage across the terminals CD is :

$$V = I \times (3 + j5)$$

iii. $I = 10 \angle 0° \times \dfrac{4 + j6}{4 + j6 + 3 + j5} = 5.53 \angle -1.22° = (5.529 - j0.12)$ A

iv. $V = 5.53 \angle -1.22° \times (3 + j5) = 5.53 \angle -1.22° \times 5.83 \angle 59.03°$

$$= 32.34 \angle 57.81° \text{ V}$$

2. i. The positions of excitation and response are interchanged as shown in Fig. 2.10.2.

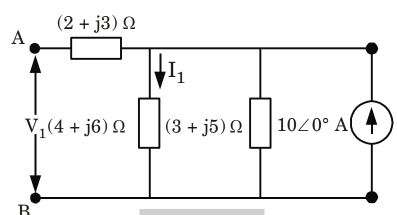


Fig. 2.10.2.

ii. Since terminals AB are open, no current flows through $(2 + j3)\ \Omega$ and V_1 is the voltage across $(4 + j6)\ \Omega$.

iii. Voltage across $(4 + j6)\ \Omega$,

$$V_1 = I_1 \times (4 + j6)$$

iv. $I_1 = 10 \angle 0° \times \dfrac{3 + j5}{(3 + j5) + (4 + j6)} = 10 \angle 0° \times \dfrac{5.83\angle 59.03°}{13.04\angle 57.52°}$

$$= 4.47\angle 1.51° \text{ A}$$

v. $V_1 = I_1 \times (4 + j6) = 4.47 \angle 1.51° \times 7.21\angle 56.3° = 32.20\angle 57.81° \text{ V}$

3. In both the cases, the ratio of current to voltage is same.

$$\frac{I}{V} = \frac{I_1}{V_1}$$

Hence the reciprocity theorem is verified.

PART-3

Thevenin's Theorem.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.11. **State and prove Thevenin's theorem.**

Answer

Statement :

1. A linear active bilateral network can be replaced at any two of its terminals by an equivalent voltage source (Thevenin's voltage source), V_{oc}, in series with an equivalent impedance (Thevenin's impedance), Z_{Th}.

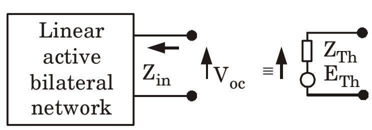
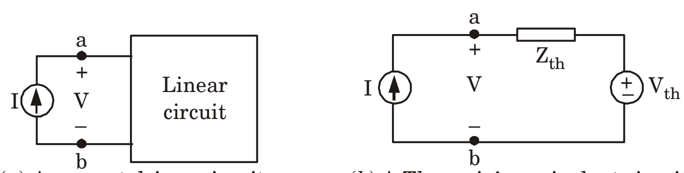


Fig. 2.11.1. Thevenin's theorem.

2. Here, V_{oc} is the open circuit voltage between the two terminals under the action of all sources and initial conditions, and Z_{Th} is the impedance obtained across the terminals with all sources removed by their internal impedance and initial conditions reduced to zero.

Proof :

1. We consider a linear active circuit of Fig. 2.11.2(*a*). An external current source is applied through the terminals *ab*, where we have access to the circuit.



(*a*) A current driven circuit. (*b*) A Thevenin's equivalent circuit.

Fig. 2.11.2.

2. We have to prove that the $v - i$ relation at terminals ab of Fig. 2.11.2(a), is identical with that of the Thevenin's equivalent circuit of Fig. 2.11.2(b).
3. For simplicity, we assume that the circuit contains two independent voltage sources V_{s1} and V_{s2} and two independent current sources I_{s1} and I_{s2}.
4. Considering the contribution due to each independent source including the external one, the voltage at ab, V, is given by superposition theorem,

$$V = K_0 I + K_1 V_{s1} + K_2 V_{s2} + K_3 I_{s1} + K_4 I_{s2}$$

where, K_0, K_1, K_2, K_3, K_4 are constants.

or, $$V = K_0 I + P_0 \quad ...(2.11.1)$$

where, $$P_0 = K_1 V_{s1} + K_2 V_{s2} + K_3 I_{s1} + K_4 I_{s2}$$

= total contribution due to internal independent sources.

5. To evaluate the constants K_0 and P_0 of eq. (2.11.1), two conditions are :

i. When the terminals a and b are open-circuited,

$$I = 0, \quad \text{and } V = V_{oc} = V_{th}$$

Putting in eq. (2.11.1), $V_{th} = V_{oc} = P_0$

$$V_{th} = P_0$$

ii. When all the internal sources are turned off,

$P_0 = 0$ and the equivalent impedance is Z_{th}

From eq. (2.11.1), $V = K_0 I$

or, $$\frac{V}{I} = K_0 = Z_{th}$$

$$K_0 = Z_{th}$$

6. Thus, substituting the values of K_0 and P_0, the $v - i$ relation becomes,

$$V = Z_{th} I + V_{th} \quad ...(2.11.2)$$

Eq. (2.11.2) represents the $v - i$ relationship of Fig. 2.11.2(b). So, theorem is proved.

Que 2.12. **Give the steps to determine the Thevenin's/Norton's equivalent circuit.**

Answer

1. The portion of the network across which the Thevenin's or Norton's equivalent circuit is to be found out is removed from the network.

2. a. The open-circuit voltage (V_{oc} or I_N) is calculated keeping all the sources at their normal values.

   b. The short-circuit current (I_{sc} or I_N) flowing from one terminal to the other is calculated keeping all the sources at their normal values.

3. **Calculation of Z_{th} or Y_N :**

   Case I : When the circuit contains only independent sources :

a. All voltage sources are short-circuited.

b. All current sources are open-circuited.

c. Equivalent impedance or admittance is calculated looking back to the circuit with respect to the two terminals.

   Case II : When the circuit contains both dependent and independent sources :

a. Open-circuit voltage (V_{oc}) is calculated with all sources alive.

b. Short-circuit current (I_{sc}) is calculated with all sources alive.

c. Thevenin's impedance is obtained as, $Z_{th} = \dfrac{V_{oc}}{I_{sc}} = \dfrac{1}{Y_N}$

   Case III : When the circuit contains only dependent sources :

a. In this case, $V_{oc} = 0$.

b. We connect a test voltage (or current) source at the terminals a and b and the current flowing through ab (voltage drop between the terminals ab) is calculated.

   c. Thevenin's impedance is obtained as, $Z_{th} = \dfrac{V_{test}}{I_{test}} = \dfrac{1}{Y_N}$

4. Finally, Thevenin's equivalent circuit is obtained by placing V_{oc} in series with Z_{th} and Norton's equivalent is obtained by placing I_{sc} in parallel with Y_N.

Que 2.13. **Using Thevenin's theorem, find the current through load impedance Z_L shown in the Fig. 2.13.1.**

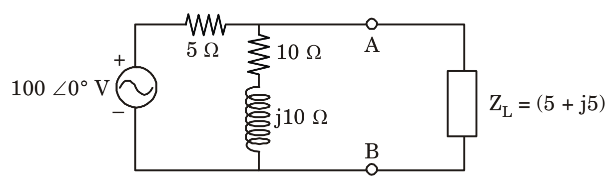


Fig. 2.13.1.

AKTU 2013-14, Marks 05

Answer

1. Calculation of Z_{Th} :

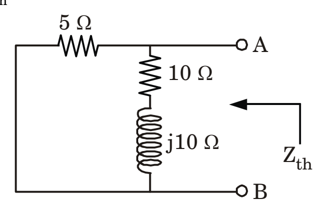


Fig. 2.13.2.

$$Z_{Th} = 5 \,||\, (10 + j10) = \frac{5 \times (10 + j10)}{5 + 10 + j10}$$

$$Z_{Th} = \frac{10(1 + j)}{3 + j2}\,\Omega$$

2. Calculation of V_{Th} :

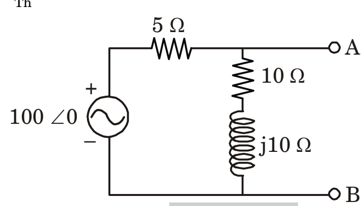


Fig. 2.13.3.

$$V_{Th} = \frac{(10 + j10)\,100}{15 + j10}$$

$$V_{Th} = \frac{200(1 + j)}{3 + j2}\,\text{V}$$

3. Then Thevenin's equivalent circuit is,

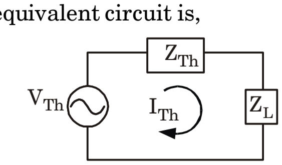


Fig. 2.13.4.

4. Current, $$I_{Th} = \frac{V_{Th}}{Z_{Th} + Z_L}$$

$$= \frac{\dfrac{200(1 + j)}{3 + j2}}{\dfrac{10(1 + j)}{3 + j2} + 5 + j5}$$

$$= \frac{200(1+j)}{10(1+j)+5(1+j)(3+j2)}$$

$$= \frac{200}{10+5(3+j2)} = \frac{200}{25+j10}$$

$$\therefore \qquad I_{Th} = \frac{40}{5+j2} = (6.9 - j2.75)\text{ A}$$

Que 2.14. **Find current through 5 Ω resistor using Thevenin's theorem.**

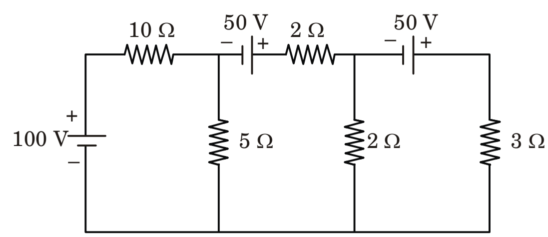


Fig. 2.14.1.

AKTU 2017-18, Marks 07

Answer

Determination of Thevenin's resistance R_{Th} :

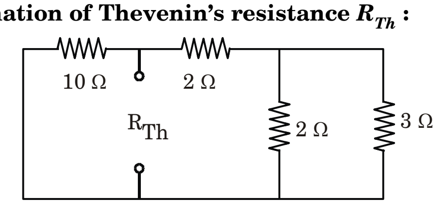


Fig. 2.14.2.

$$R_{Th} = 10 \parallel (2 + (2 \parallel 3))$$

$$= 10 \parallel \left(2 + \frac{2\times 3}{2+3}\right) = 10 \parallel \left(2+\frac{6}{5}\right)$$

$$= 10 \parallel \left(\frac{16}{5}\right) = \frac{10 \times \frac{16}{5}}{10 + \frac{16}{5}} = \frac{32\times 5}{66}$$

$$R_{Th} = \frac{80}{33}\,\Omega$$

Determination of Thevenin's voltage V_{Th} :

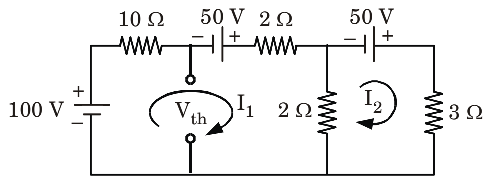


Fig. 2.14.3.

i. Applying KVL in loop 1

$$10I_1 - 50 + 2I_1 + 2(I_1 - I_2) = 100$$

$$14I_1 - 2I_2 = 150 \quad ....(2.14.1)$$

ii. Applying KVL in loop 2

$$3I_2 + 2(I_2 - I_1) = 50$$

$$5I_2 - 2I_1 = 50$$

$$-2I_1 + 5I_2 = 50 \quad ...(2.14.2)$$

iii. Solving both eq. (2.14.1) and (2.14.2), we get

$$I_1 = \frac{425}{33} \quad \text{and} \quad I_2 = \frac{500}{33}$$

iv. So,

$$V_{th} = 100 - 10\,I_1 = 100 - 10 \times \frac{425}{33}$$

$$V_{th} = \frac{-950}{33}\ \text{V}$$

The Thevenin's equivalent circuit is shown in Fig. 2.14.4 :

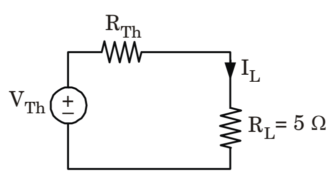


Fig. 2.14.4.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{-950/33}{\frac{80}{33} + 5} = \frac{-950}{245}$$

$$\therefore \quad I_L = -3.87\ \text{A}$$

Que 2.15. **Find the current through 1 Ω resistor shown in Fig. 2.15.1 using Thevenin's method.**

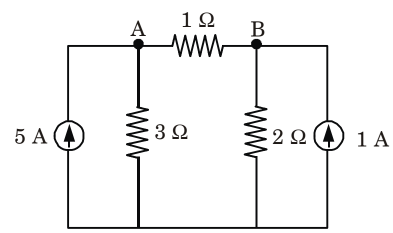


Fig. 2.15.1.

AKTU 2018-19, Marks 07

Answer

a. Calculation of V_{Th} :

1. Load R_L is open circuited.

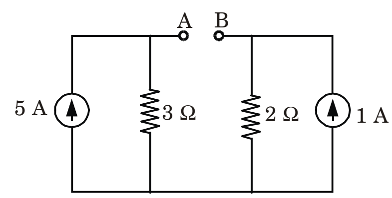


Fig. 2.15.2.

2. Converting current source to voltage source,

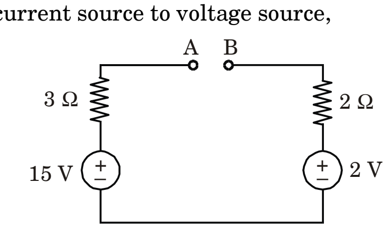


Fig. 2.15.3.

$$V_{Th} = V_{AB} = 15 - 2 = 13 \text{ V}$$

b. Calculation of R_{Th} : To calculate R_{Th}, the current source and load terminal are open circuited.

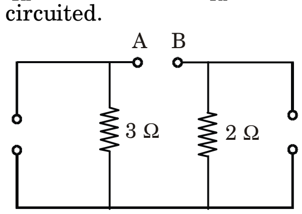


Fig. 2.15.4.

$$R_{Th} = R_{AB} = 5\ \Omega$$

c. **Thevenin's equivalent network :**

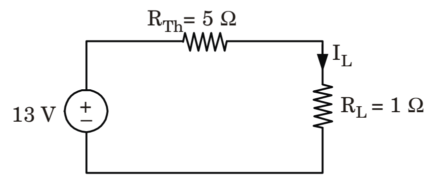


Fig. 2.15.5.

Current through 1 Ω resistor is

$$I_L = \frac{13}{6} = 2.166\ \text{A}$$

PART-4

Norton's Theorems.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.16. **State and prove Norton's theorem.**

Answer

Statement :

1. A linear active bilateral network can be replaced at any two of its terminals, by an equivalent current source (Norton's current source), I_{sc}, in parallel with an equivalent admittance (Norton's admittance), Y_N.
2. Here, I_{sc} is the short-circuit current flowing from one terminal to the other under the action of all sources and initial conditions, and Y_N is the admittance obtained across the terminals with all sources removed by their internal impedance and initial conditions reduced to zero.

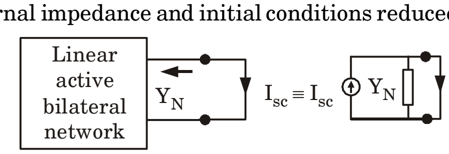


Fig. 2.16.1. Norton's theorem.

Proof :

1. We consider a linear active circuit of Fig. 2.16.2(*a*). An external voltage source is applied through the terminals *ab*, where we have access to the circuit.

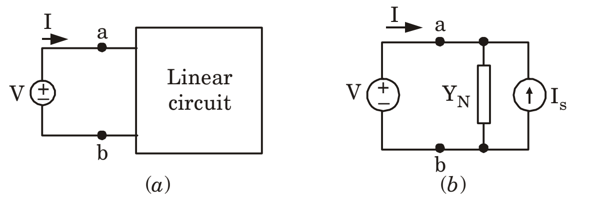


Fig. 2.16.2. (*a*) A voltage-driven circuit (*b*) Norton's equivalent circuit.

2. We have to prove that the $v - i$ relation at terminals ab of Fig. 2.16.2(*a*) is identical with that of the Norton's equivalent circuit of Fig. 2.16.2(*b*).
3. For simplicity, we assume that the circuit contains two independent voltage sources V_{s1} and V_{s2} and two independent current sources I_{s1} and I_{s2}.
4. Considering the contribution due to each independent source including the external one, the current entering at a, I, is, given by superposition theorem,

$$I = K_0V + K_1V_{s1} + K_2V_{s2} + K_3I_{s1} + K_4I_{s2}$$

where, K_0, K_1, K_2, K_3, K_4 are constants.

or, $$I = K_0V + P_0 \quad ...(2.16.1)$$

where, $$P_0 = K_1V_{s1} + K_2V_{s2} + K_3I_{s1} + K_4I_{s2}$$

= Total contribution due to internal independent sources.

5. To evaluate the constants K_0 and P_0 of eq. (2.16.1), two conditions are :

i. When the terminals a and b are short-circuited.

$V = 0$, and $I = -I_{sc} = I_N$

From eq. (2.16.1), $-I_{sc} = P_0 \Rightarrow \quad I_{sc} = -P_0$

ii. When all the internal sources are turned off

$P_0 = 0$ and the equivalent admittance is Y_N.

From eq. (2.16.1), $I = K_0V$

or, $$\frac{I}{V} = K_0 = Y_N \Rightarrow K_0 = Y_N$$

6. Thus, substituting the values of K_0 and P_0, the $v - i$ relation becomes,

$$I = VY_N - I_N \quad ...(2.16.2)$$

Eq. (2.16.2) represents the $v - i$ relationship of Fig. 2.16.2(*b*). So, Norton's theorem is proved.

Que 2.17. **Explain why "Thevenin's theorem" and "Norton's theorem" are dual to each other ? Consider the circuit shown in Fig. 2.17.1.**

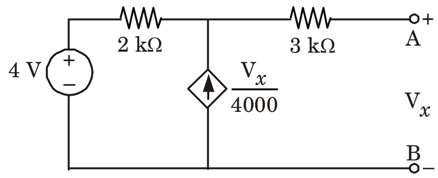


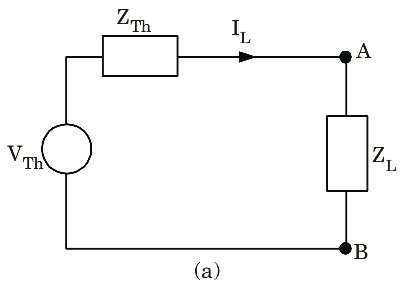
Fig. 2.17.1.

i. Find the Thevenin's equivalent circuit of Fig. 2.17.1 across *AB*-terminals.

ii. Find the Norton's equivalent circuit from Thevenin's equivalent circuit of Fig. 2.17.1 across *AB*-terminals.

Answer

1. Thevenin's theorem replaces a complex network by a voltage source V_{Th} in series with an impedance Z_{Th}, and Norton's theorem replaces a complex network by a current source I_N in parallel with an impedance Z_N as shown in Fig. 2.17.2 (*a*) and (*b*) respectively.



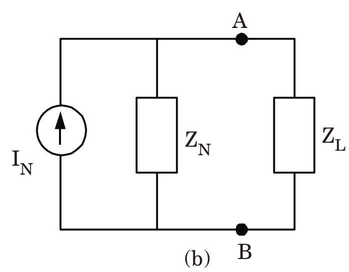


Fig. 2.17.2. (a) Thevenin's equivalent circuit, (b) Norton's equivalent circuit.

2. In Fig. 2.17.2 (*a*),

$$V_{AB} = \frac{Z_L}{Z_{Th} + Z_L} V_{Th} \qquad ...(2.17.1)$$

and

$$I_L = \frac{V_{Th}}{Z_{Th} + Z_L} \qquad ...(2.17.2)$$

3. In Fig. 2.17.2 (*b*),

$$V_{AB} = \frac{Z_N Z_L}{Z_N + Z_L} I_N \qquad ...(2.17.3)$$

and

$$I_L = \frac{Z_N}{Z_N + Z_L} I_N \qquad ...(2.17.4)$$

4. Let $Z_{Th} = Z_N$ and $I_N = V_{Th}/Z_{Th}$. Putting these values in eq. (2.17.3) and eq. (2.17.4), then

$$V_{AB} = \frac{Z_{Th}\, Z_L}{Z_{Th} + Z_L} \frac{V_{Th}}{Z_{Th}} = V_{Th} \frac{Z_L}{Z_{Th} + Z_L} \qquad ...(2.17.4)$$

Also
$$I_L = \frac{Z_{Th}}{Z_{Th} + Z_L} \frac{V_{Th}}{Z_{Th}} = \frac{V_{Th}}{Z_{Th} + Z_L} \qquad ...(2.17.5)$$

5. From eq. (2.17.1), (2.17.2), (2.17.4), and (2.17.5), we conclude that V_{AB} and I_L are same in both the cases. Thus Norton's and Thevenin's theorem are dual of each other and $Z_N = Z_{TH}$ and $I_N = V_{Th}/Z_{Th}$.

Numerical :

1. From the given circuit, $V_x = V_{\text{Th}}$
2. Using KVL, $V_x = \left(\frac{V_x}{4000}\right) \times 2000 + 4$

$$V_x \left[1 - \frac{1}{2}\right] = 4$$

$$V_x = +8 \text{ V}$$

So $V_{\text{Th}} = 8$ V

3. To find I_{sc}, let us short circuit the terminal *AB*

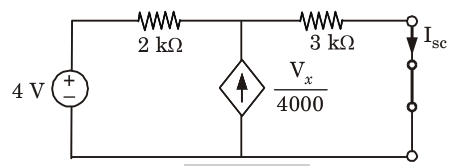
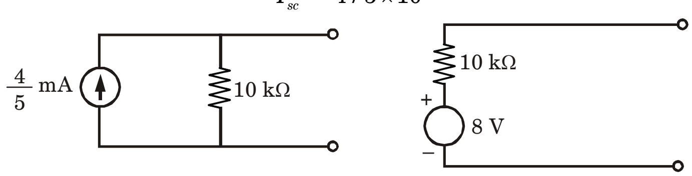


Fig. 2.17.3.

$$V_x = 0$$

$$I_{\text{sc}} = \frac{4}{5 \times 10^3} = \frac{4}{5} \text{ mA}$$

$$\therefore \quad R_{\text{Th}} = \frac{V_{\text{Th}}}{I_{sc}} = \frac{8}{4/5 \times 10^{-3}} = 10 \text{ k}\Omega$$



Norton's equivalent circuit   Thevenin's equivalent circuit

Fig. 2.17.4.

Que 2.18. **For the circuit shown in Fig. 2.18.1, determine Norton's equivalent circuit between output terminals *AB*.**

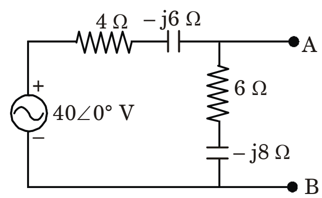


Fig. 2.18.1.

Answer

Determination of I_N :

1. To find the Norton's equivalent current source, short circuited output terminals AB is shown in Fig. 2.18.2 and the short circuit current is I_N.

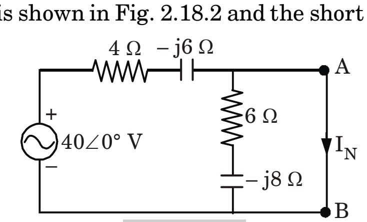


Fig. 2.18.2.

2. $$I_N = \frac{40\angle 0°}{4 - j6} = \frac{40\angle 0°}{7.2\angle -56.31°} = 5.56\angle 56.31° \text{ A}$$

Determination of Z_N :

1. To find the Norton's equivalent impedance Z_N, replace the voltage source by a short circuit, and find the impedance looking into terminals AB, as shown in Fig. 2.18.3.

2. $$Z_N = (6 - j8) \,||\, (4 - j6) = \frac{(6 - j8)(4 - j6)}{(6 - j8) + (4 - j6)} = 4.19\angle 305.06° \ \Omega$$

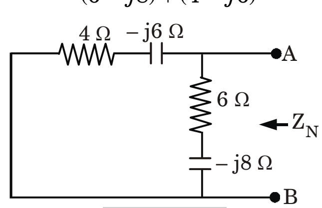


Fig. 2.18.3.

Norton's equivalent circuit : Connect I_N in parallel with Z_N across terminals AB

The Norton's equivalent circuit is shown in Fig. 2.18.4.

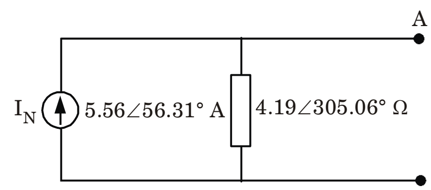


Fig. 2.18.4.

Que 2.19. **Find the Norton equivalent circuit of the circuit in Fig. 2.19.1 at terminals *a-b*.**

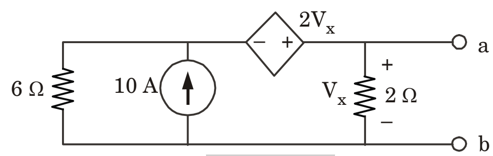


Fig. 2.19.1.

AKTU 2014-15, Marks 05

Answer

1. To get Norton's equivalent circuit, we have to find I_{sc} and R_N. First to get I_{sc}, short circuiting terminal a and b, as shown in Fig. 2.19.2.

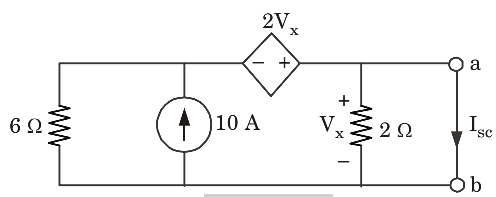


Fig. 2.19.2.

2. Converting current source to voltage source, we get Fig. 2.19.3.

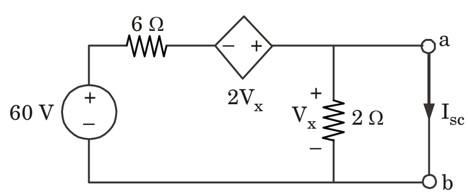


Fig. 2.19.3.

3. As current must follow through short circuit path, thus

$$V_x = 0,$$

Therefore, $I_{sc} = \frac{60}{6} = 10 \text{ A}$

4. Now to find R_N, connecting a 1 V source across *a-b* and replacing active source with their internal resistances as shown in Fig. 2.19.4.

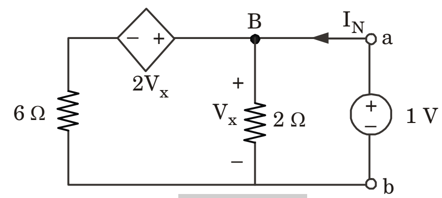


Fig. 2.19.4.

5. Applying KCL at *B*, $V_x = 1$ V

$$I_N = \frac{1}{2} + \frac{(1-2)}{6} = 2/6 = 1/3 \text{ A}$$

and $$R_N = \frac{1}{I_N} = 3\ \Omega$$

6. Thus, the Norton equivalent circuit is shown in Fig. 2.19.5.

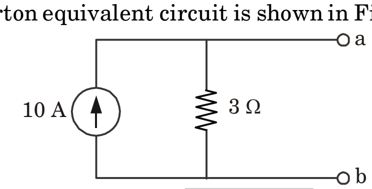


Fig. 2.19.5.

Que 2.20. **Find the current in the 5 Ω resistance for the circuit shown in given Fig. 2.20.1 by Norton's theorem.**

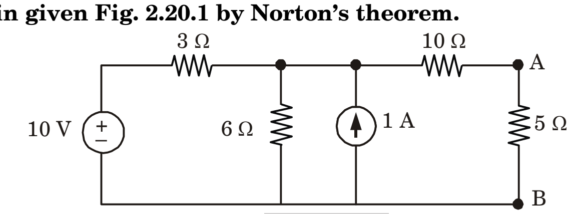


Fig. 2.20.1.

Answer

1. Let us first remove the 5 Ω resistor and short the *A-B* terminals. Assuming the voltage to be positive at node 1 (in Fig. 2.20.2) nodal analysis gives

$$\frac{V-10}{3} + \frac{V}{6} - 1 + \frac{V}{10} = 0$$

$$0.33V + 0.17V + 0.1V = 4.33$$
$$V = 7.22 \text{ V}$$

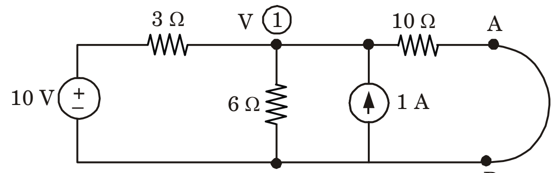


Fig. 2.20.2.

2. Applying KVL in the right most loop of Fig. 2.20.2.

$$-V + I_{sc} \times 10 = 0$$
$$-7.22 + 10I_{sc} = 0$$
$$I_{sc} = I_N = 0.722 \text{ A}$$

3. To find Norton's equivalent resistance through *A*-*B*, 5 Ω resistor is removed and all the constant sources are deactivated as in Fig. 2.20.3.

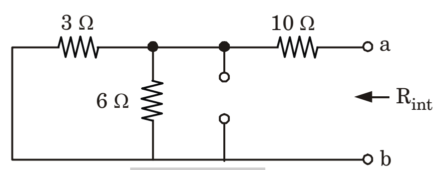


Fig. 2.20.3.

$$R_{int} = \frac{3 \times 6}{3 + 6} + 10 = 12\ \Omega$$

4. Norton's equivalent circuit is shown in Fig. 2.20.4.

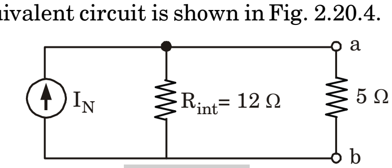


Fig. 2.20.4.

5. Now Norton's equivalent current through 5 Ω resistor

$$I_{5\Omega} = I_N \frac{R_{int}}{R_{int} + 5} = 0.722 \times \frac{12}{12 + 5} = 0.5096 \text{ A} = 509.6 \text{ mA}$$

PART-5

Maximum Power Transfer Theorem.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.21. **State and prove the maximum power transfer theorem applied to the AC circuits.** **AKTU 2017-18, Marks 07**

OR

State and prove maximum power transfer theorem with example.

AKTU 2018-19, Marks 07

Answer

Consider a simple network as shown in Fig. 2.21.1. There are three possible cases for load impedance Z_L.

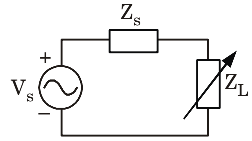


Fig. 2.21.1. A simple network.

Case I : When the load impedance is a variable resistance.

1. $$I_L = \frac{V_s}{Z_s + Z_L} = \frac{V_s}{R_s + jX_s + R_L}$$

$$|I_L| = \frac{|V_s|}{\sqrt{(R_s + R_L)^2 + X_s^2}}$$

2. The power delivered to the load is

$$P_L = |I_L|^2 R_L = \frac{|V_s|^2 R_L}{(R_s + R_L)^2 + X_s^2}$$

3. For power P_L to be maximum,

$$\frac{dP_L}{dR_L} = 0$$

$$|V_s|^2 \left[\frac{\{(R_s + R_L)^2 + X_s^2\} - 2R_L(R_s + R_L)}{[(R_s + R_L)^2 + X_s^2]^2}\right] = 0$$

$$(R_s + R_L)^2 + X_s^2 - 2R_L(R_s + R_L) = 0$$

$$R_s^2 + 2R_sR_L + R_L^2 + X_s^2 - 2R_LR_s - 2R_L^2 = 0$$

$$R_s^2 + X_s^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2 + X_s^2$$

$$R_L = \sqrt{R_s^2 + X_s^2} = |Z_s|$$

4. Hence, load resistance R_L should be equal to the magnitude of the source impedance for maximum power transfer.

Case II : When the load impedance is a complex impedance with variable resistance and variable reactance.

1. $$I_L = \frac{V_s}{Z_s + Z_L}$$

$$|I_L| = \frac{|V_s|}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

2. The power delivered to the load is

$$P_L = |I_L|^2 R_L = \frac{|V_s|^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2} \quad ...(2.21.1)$$

3. For maximum value of P_L, denominator of the eq. (2.21.1) should be small, *i.e.*, $X_L = -X_s$

$$P_L = \frac{|V_s|^2 R_L}{(R_s + R_L)^2} \quad ...(2.21.2)$$

4. Differentiating eq. (2.21.2) for P_L w.r.t R_L and equating to zero, we get

$$\frac{dP_L}{dR_L} = |V_s|^2 \left[\frac{(R_s + R_L)^2 - 2R_L(R_s + R_L)}{(R_s + R_L)^2}\right]$$

$$(R_s + R_L)^2 - 2R_L(R_s + R_L) = 0$$

$$R_s{}^2 + R_L{}^2 + 2R_sR_L - 2R_LR_s - 2R_L{}^2 = 0$$

$$R_s^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2$$

$$R_L = R_s$$

5. Hence, load resistance R_L should be equal to the resistance R_s and load reactance X_L should be equal to negative value of source reactance.
6. Load impedance for maximum power transfer is :

$$Z_L = Z_s^* = R_s - jX_s$$

i.e., load impedance must be the complex conjugate of the source impedance.

Case III : When the load impedance is a complex impedance with variable resistance and fixed reactance.

1. $$I_L = \frac{V_s}{Z_s + Z_L}$$

$$|I_L| = \frac{|V_s|}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

2. The power delivered to the load is

$$P_L = |I_L|^2 R_L = \frac{|V_s|^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

3. For maximum power,

$$\frac{dP_L}{dR_L} = 0$$

$$|V_s|^2 \left[\frac{(R_s + R_L)^2 + (X_s + X_L)^2 - 2R_L(R_s + R_L)}{\{(R_s + R_L)^2 + (X_s + X_L)^2\}} \right] = 0$$

$$(R_s + R_L)^2 + (X_s + X_L)^2 - 2R_L(R_s + R_L) = 0$$

$$R_s^2 + 2R_sR_L + R_L^2 + (X_s + X_L)^2 - 2R_LR_s - 2R_L^2 = 0$$

$$R_s^2 + (X_s + X_L)^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2 + (X_s + X_L)^2$$

$$R_L = \sqrt{R_s^2 + (X_s + X_L)^2}$$

$$= |R_s + j(X_s + X_L)|$$

$$= |(R_s + jX_s) + jX_L|$$

$$= |Z_s + jX_L|$$

4. Hence, load resistance R_L should be equal to the magnitude of the impedance $Z_s + jX_L$, *i.e.*, $|Z_s + jX_L|$ for maximum power transfer.

Que 2.22. **What should be the value of R_L so the maximum power can be transferred from the source to R_L for the given Fig. 2.22.1 ?**

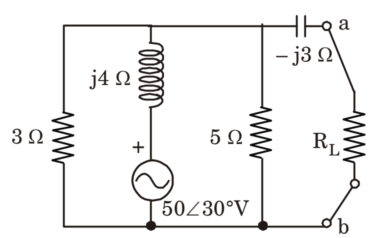


Fig. 2.22.1.

AKTU 2015-16, Marks 10

Answer

To satisfy maximum power transfer, we have to find internal impedance of the circuit.

1. **Finding internal impedance :**

i. Voltage source is removed by short circuit since it does not have any internal resistance.

ii. Load terminal is opened.

2. **Reduced network :**

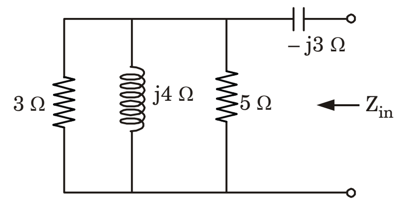


Fig. 2.22.2.

3. Component 3 Ω, $j4$ Ω and 5 Ω are in parallel to each other.
   Then, equivalent of these = Z_1

$$\frac{1}{Z_1} = \frac{1}{3} + \frac{1}{j4} + \frac{1}{5}$$

$$\frac{1}{Z_1} = \frac{8}{15} + \frac{1}{j4}$$

$$\frac{1}{Z_1} = \frac{j32 + 15}{j60}$$

$$\therefore \quad Z_1 = \frac{j60}{j32 + 15} = \frac{j60 \times (15 - j32)}{15^2 + 32^2}$$

$$= 1.54 + j0.72\ \Omega$$

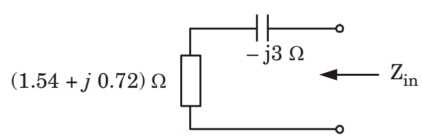


Fig. 2.22.3.

4. $$Z_{in} = Z_1 - j3$$
$$= 1.54 + j0.72 - j3$$
$$= (1.54 - j2.28)\ \Omega$$

5. Then, $R_L = 2.75\ \Omega$

Que 2.23. **Find the value of *R* in the circuit of Fig. 2.23.1 such that maximum power transfer takes place. What is the amount of this power ?**

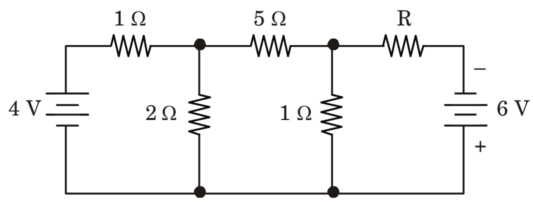


Fig. 2.23.1.

Answer

1. The Thevenin's equivalent resistance of the network shown in Fig. 2.23.2 can be calculated as,

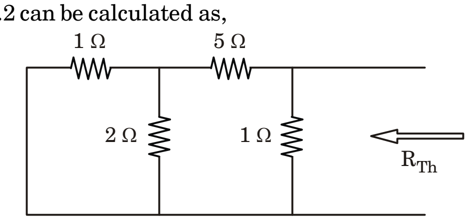


Fig. 2.23.2.

$$R_{Th} = \left[\left(\frac{1\times 2}{1+2}\right)+5\right] \parallel 1 = \left(\frac{2}{3}+5\right)||1 = \frac{\frac{17}{3}\times 1}{\frac{17}{3}+1} = \frac{17}{20}\ \Omega$$

2. The open circuit voltage of the network is shown in Fig. 2.23.3, can be calculated as

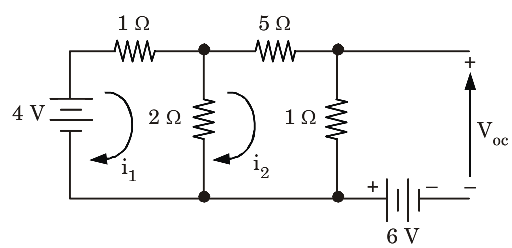


Fig. 2.23.3.

3. Removing the resistance R,

$\therefore$ $$3i_1 - 2i_2 = 4 \quad \text{...(2.23.1)}$$

and $$-2i_1 + 8i_2 = 0 \quad \text{...(2.23.2)}$$

Solving eq. (2.23.1) and (2.23.2), we get

$$i_2 = \frac{2}{5}\ \text{A}$$

$\therefore \qquad 1 \times i_2 + 6 = V_{oc}$

$$V_{oc} = \left(6 + \frac{2}{5}\right) = \frac{32}{5} \text{ V}$$

5. For maximum power transfer, $R = R_{Th} = \dfrac{17}{20} = 0.85\ \Omega$

Maximum power, $P_{\max} = \dfrac{V_{oc}^2}{4R} = 12$ W

PART-6

Compensation and Tellegen's Theorem as Applied to AC Circuits.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.24. Give the statement of compensation theorem. Also prove it for linear network.

Answer

Statement : In a linear time variant network, when the resistance (R) of an uncoupled branch, carrying a current (I), is changed by (ΔR), the current in all the branches would change and can be obtained by assuming that an ideal voltage source (V_c) has been connected in series with ($R + \Delta R$), when all other sources in the network are replaced by their internal resistance.

Proof :

1. Let us assume a load R_L be connected to a DC source network whose Thevenin's equivalent gives V_0 as the Thevenin's voltage and R_{Th} as Thevenin's resistance as evident from Fig. 2.24.1.

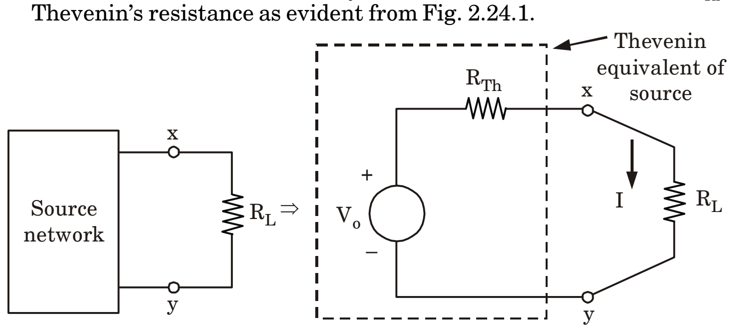
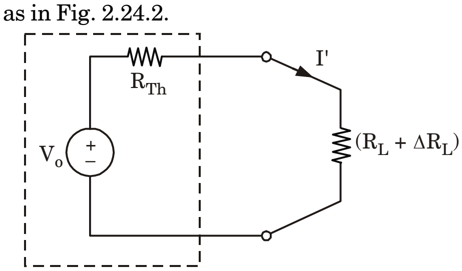


Fig. 2.24.1.

2. Let the load resistance R_L be changed to $(R_L + \Delta R_L)$. Since the rest of the circuit remains unchanged, the Thevenin's equivalent network remains the same as in Fig. 2.24.2.



Thevenin's equivalent of source

Fig. 2.24.2.

Here $$I' = \frac{V_o}{R_{Th} + (R_L + \Delta R_L)}$$

3. The change of current being termed as ΔI.

We find, $$\Delta I = I' - I$$

$$= \frac{V_o}{R_{Th} + (R_L + \Delta R_L)} - \frac{V_o}{R_{Th} + R_L}$$

$$= -\left(\frac{V_o}{R_{Th} + R_L}\right)\frac{\Delta R_L}{R_{Th} + R_L + \Delta R_L}$$

$$= -\frac{I\Delta R_L}{R_{Th} + R_L + \Delta R_L} = \frac{(-V_c)}{R_{Th} + R_L + \Delta R_L}$$

4. Thus it has been proved that change of branch resistance, branch current is changed and the change is equivalent to an ideal compensating voltage source in series with the branch opposing the original current.

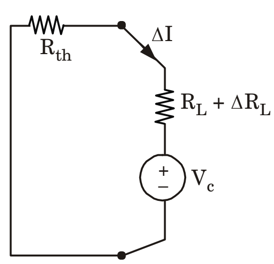


Fig. 2.24.3.

Que 2.25. **State and prove Tellegen's theorem.**

Answer

Tellegen's Theorem :

Tellegen's theorem states that for a network of n elements and e nodes, if a set of current passing through various elements be i_1, i_2,...., i_n satisfying KCL and its set of voltages be V_1, V_2,...., V_n satisfying KVL for every loop then Tellegen's theorem is

$$\sum_{k=1}^{n} V_k i_k = 0$$

Proof :

1. Let i_{pq} (= i_k) = K_{th} branch through current.

   V_K = Voltage drop in branch $K = V_p - V_q$, where V_p and V_q are the respective node voltages at p and q nodes.

   We have, $V_K\, i_{pq} = (V_p - V_q)\, i_{pq} = V_K\, i_K$ ...(2.25.1)

   $V_K\, i_K = (V_q - V_p)\, i_{qp}$, obviously, $i_{qp} = - i_{pq}]$ ...(2.25.2)

   Summing eq. (2.25.1) and (2.25.2)

$$2\, V_K\, i_K = (V_p - V_q)\, i_{pq} + (V_q - V_p)\, i_{qp}$$

$$V_K\, i_K = \frac{1}{2}\, [(V_p - V_q)\, i_{pq} + (V_q - V_p)\, i_{qp}]$$

$$\sum_{k=1}^{n} V_k i_k = \frac{1}{2} \sum_{p=1}^{n} \sum_{q=1}^{n} (V_p - V_q) i_{pq}$$

$$= \frac{1}{2} \sum_{p=1}^{n} V_p \left(\sum_{q=1}^{n} i_{pq} \right) - \frac{1}{2} \sum_{q=1}^{n} V_q \left(\sum_{p=1}^{n} i_{pq} \right)$$

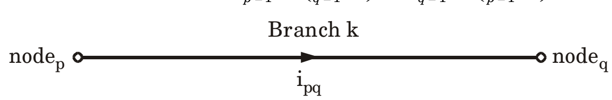


Fig. 2.25.1.

3. Following Kirchhoff's current law, the algebraic sum of current at each node is equal to zero.

   i.e.
$$\sum_{p=1}^{n} i_{pq} = 0$$

$$\sum_{q=1}^{n} i_{qp} = 0$$

   With this finally we get $\sum_{k=1}^{n} V_k i_k = 0$

   Thus, it has been observed that the sum of power delivered to a closed network is zero.

Que 2.26. **Consider the network shown in Fig. 2.26.1.**

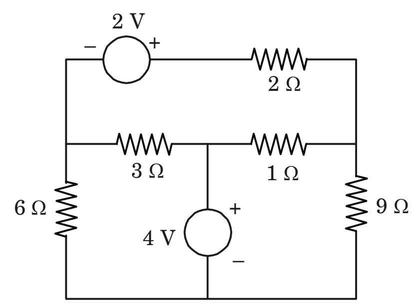


Fig. 2.26.1.

Verify the Tellegen's theorem for the network shown in Fig. 2.26.1.

Answer

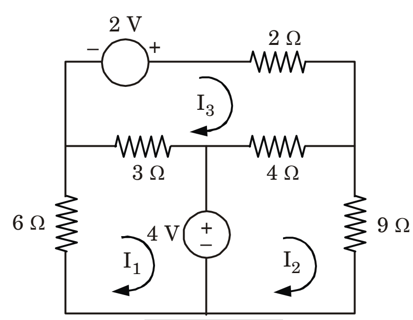


Fig. 2.26.2.

1. Applying KVL in loop 1

$$6I_1 + 3(I_1 - I_3) + 4 = 0$$

$$9I_1 - 3I_3 = -4 \quad ...(2.26.1)$$

2. Applying KVL in loop 2

$$-4 + 4(I_2 - I_3) + 9I_2 = 0$$

$$13I_2 - 4I_3 = 4 \quad ...(2.26.2)$$

3. Applying KVL in loop 3

$$0 = -2 + 2I_3 + 4(I_3 - I_2) + 3(I_3 - I_1)$$

$$9I_3 - 4I_2 - 3I_1 = 2 \quad ...(2.26.3)$$

Solving eq. (2.26.1), (2.26.2) and (2.26.3)

$$I_1 = -0.351 \text{ A}$$

$$I_2 = 0.394 \text{ A}$$

$$I_3 = 0.28 \text{ A}$$

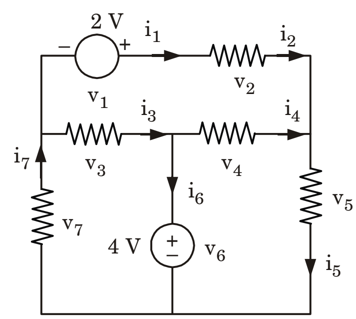


Fig. 2.26.3.

4. For Tellegen's theorem, $\Sigma v_k i_k = 0$

$$v_1 = 2\text{ V},\ i_1 = 0.28\text{ A}$$
$$v_2 = 2 \times 0.28 = 0.56\text{ V},\ i_2 = 0.28\text{ A}$$
$$v_3 = 3 \times (I_1 - I_3)$$
$$= 3 \times (-0.631) = -1.893\text{ V}$$
$$i_3 = I_1 - I_3 = -0.631\text{ A}$$
$$v_4 = 4(I_2 - I_3) = 4 \times 0.114 = 0.456\text{ V}$$
$$I_4 = 0.114\text{ A}$$
$$v_5 = 9 \times I_2 = 3.546\text{ V}$$
$$i_5 = 0.394\text{ A}$$
$$v_6 = 4\text{ V}$$
$$i_6 = I_1 - I_2 = -0.745\text{ A}$$
$$v_7 = 6 \times I_1 = -2.106\text{ V}$$
$$i_7 = I_1 = -0.351\text{ A}$$
$$\Sigma v_k i_k = v_1 i_1 + v_2 i_2 + v_3 i_3 + v_4 i_4 + v_5 i_5 + v_6 i_6 + v_7 i_7$$
$$= 2(-0.28) + (0.56)(0.28) + (-1.893)(-0.631)$$
$$+ (+0.456)(+0.114) + (3.546)(0.394) + 4(-0.745)$$
$$+ (-2.106)(-0.351)$$
$$= 0$$

Hence Tellegen's theorem is verified.

Que 2.27. **State and explain "compensation theorem" and "Tellegen's theorem" in circuit theory. Also mention their significances and limitations. What are the advantages and disadvantages of "Tellegen's and reciprocity theorems".**

Answer

Compensation theorem : Refer Q. 2.24, Page 2–34C, Unit-2.

Significance of compensation theorem :

This theorem is used to calculate the incremental changes in the voltages and currents in the branches of a circuit due to change of impedance in one branch.

Limitations of compensation theorem :

1. This theorem is not applicable to circuits with only dependent sources.
2. This theorem is not applicable to circuits with non-linear element.

Tellegen's theorem : Refer Q. 2.25, Page 2–35C, Unit-2.

Significance of Tellegen's theorem :

This theorem implies that the power delivered by independent sources of the network must be equal to the sum of the power absorbed (dissipated or stored) in all other elements in the network.

Limitations of Tellegen's theorem :

The only limitation is that the voltages V_r satisfy KVL and the currents I_r satisfy KCL.

Advantages of reciprocity theorem :

This theorem is applicable to the networks comprising of linear, time-invariant, bilateral, passive elements such as ordinary resistors, inductors, capacitors and transformers.

Disadvantages of reciprocity theorem :

1. This theorem is inapplicable to unilateral networks, such as networks comprising of electron tubes or other control devices.
2. This theorem is inapplicable to circuits with time varying elements.
3. This theorem is inapplicable to circuits with dependent sources.

Advantages of Tellegen's theorem :

1. This theorem is applicable for any lumped network having elements which are linear or non-linear, active or passive, time varying or time-invariant.
2. This theorem is completely independent of the nature of elements and is only concerned with graph of the network.

Disadvantages of Tellegen's theorem :

1. This theorem is not concerned with the type of circuit elements.
2. This theorem is only based on the two Kirchhoff's law.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Explain why "superposition theorem" is not applicable for power verifications of a given network ? Explain the following :

i. Linearity principle.

ii. Homogeneity principle.

Ans. Refer Q. 2.2.

Q. 2. Determine the current in capacitor by the principle of superposition of the network shown in Fig. 1.

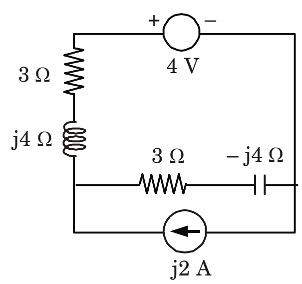


Fig. 1.

Ans. Refer Q. 2.4.

Q. 3. State reciprocity theorem in AC network.

Ans. Refer Q. 2.7.

Q. 4. Using Thevenin's theorem, find the current through load impedance Z_L shown in the Fig. 2.

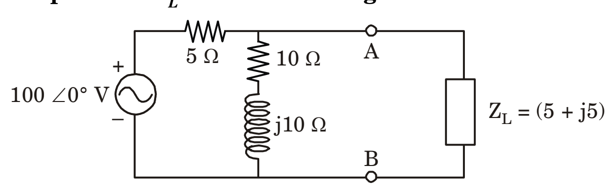


Fig. 2.

Ans. Refer Q. 2.13.

Q. 5. Find the Norton equivalent circuit of the circuit in Fig. 3 at terminals *a-b*.

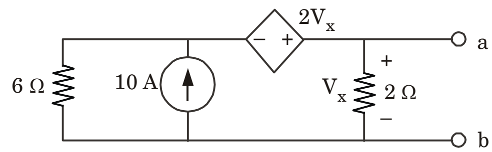


Fig. 3.

Ans. Refer Q. 2.19.

Q. 6. State and prove the maximum power transfer theorem applied to the AC circuits.

Ans. Refer Q. 2.21.

Q. 7. What should be the value of R_L so the maximum power can be transferred from the source to R_L for the given Fig. 4 ?

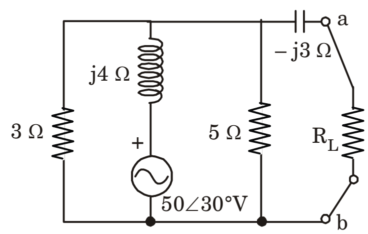


Fig. 4.

Ans. Refer Q. 2.22.

Q. 8. Give the statement of compensation theorem. Also prove it for linear network.

Ans. Refer Q. 2.24.

Q. 9. State and prove Tellegen's theorem.

Ans. Refer Q. 2.25.

Q. 10. State and explain "compensation theorem" and "Tellegen's theorem" in circuit theory. Also mention their significances and limitations. What are the advantages and disadvantages of "Tellegen's and reciprocity theorems" ?

Ans. Refer Q. 2.27.

☺☺☺

3
UNIT

Fourier Series

CONTENTS

PART-1

Discrete Spectra and Symmetry of Waveform.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.1. Briefly explain the three forms of Fourier series.

Answer

i. **Trigonometric Fourier series :** The trigonometric Fourier series representation of a periodic signal $x(t)$ with fundamental period T_0 is given by :

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

where, $$\omega_0 = \frac{2\pi}{T_0}$$

a_0, a_k and b_k are the Fourier coefficients given by :

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\, dt$$

$$a_n = \frac{2}{T_0}\int_{T_0} x(t) \cos n\omega_0 t\, dt$$

$$b_n = \frac{2}{T_0}\int_{T_0} x(t) \sin n\omega_0 t\, dt$$

ii. **Harmonic Fourier series :** Another form of Fourier series representation of a real periodic signal $x(t)$ with fundamental period T_0 is :

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos (n\omega_0 t - \theta_n),$$

where, $$\omega_0 = \frac{2\pi}{T_0}$$

$$c_0 = \text{DC component}$$

$$c_n \cos (k\omega_0 t - \theta_n) = n^{\text{th}} \text{ harmonic component of } x(t)$$

$$c_0 = a_0, \quad c_n = \sqrt{a_n^{\,2} + b_n^{\,2}}, \quad \theta_n = \tan^{-1}\frac{b_n}{a_n}$$

iii. Exponential Fourier series :

1. The exponential Fourier series is the most widely used form of Fourier series.
2. In this, the function $x(t)$ is expressed as a weighted sum of the complex exponential functions.

$$x(t) = C_0 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} C_n \, e^{jn\omega_0 t}$$

where,
$$C_n = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) e^{-jn\omega_0 t} \, dt$$

$$C_0 = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) \, dt$$

Que 3.2. **Discuss waveform symmetries.**

OR

Define odd and even function. Also find Fourier coefficient for odd and even function.

Answer

1. Even symmetry :

i. A function $f(t)$ is said to have even symmetry if $f(t) = f(-t)$.

ii. Even function shows even symmetry.

iii. Even nature is preserved on addition of a constant.

iv. Sum of even functions remains even.

Example :

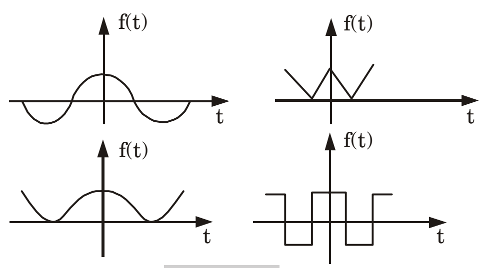


Fig. 3.2.1.

Fourier constants for even function :

$$a_0 = \frac{2}{T} \int_{0}^{T/2} f(t) \, dt$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega t\, dt$$

$$b_n = 0$$

2. Odd symmetry :

i. A function $f(t)$ is said to have odd symmetry, if $f(t) = -f(-t)$.

ii. If $f(t) = -f(-t)$, function is an odd function.

iii. Addition of a constant removes odd nature of the function.

iv. Sum of odd functions remains odd.

Example :

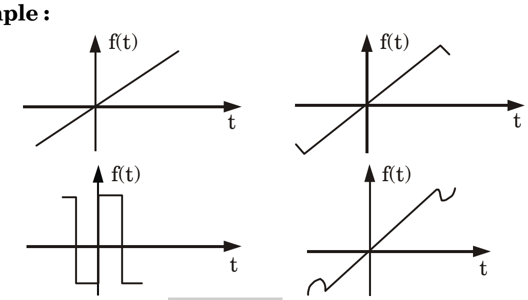


Fig. 3.2.2.

Fourier constants for odd function :

$$a_0 = 0$$

$$a_n = 0$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega t\, dt$$

3. Half wave symmetry :

A periodic function is said to have half wave symmetry if,

$$f(t) = -f(t \pm T/2)$$

Example :

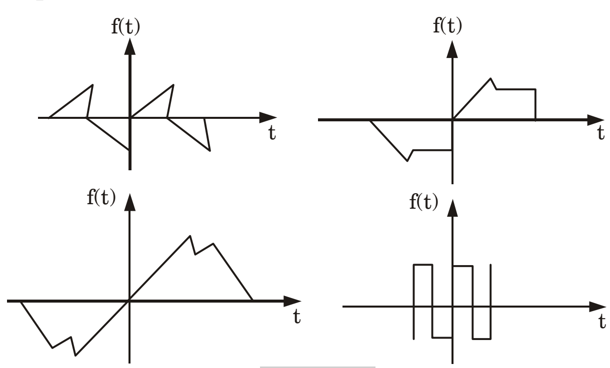


Fig. 3.2.3.

Fourier constants for half wave symmetry :

***n* odd**	***n* even**
$a_0 = 0$	$a_0 = 0$
$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega t\, dt$	$a_n = 0$
$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega t\, dt$	$b_n = 0$

4. **Quarter wave symmetry :** If signal has following both properties, it is said to have quarter wave symmetry :

i. It is half wave symmetric.

ii. It has symmetry (odd or even) about the quarter period point.

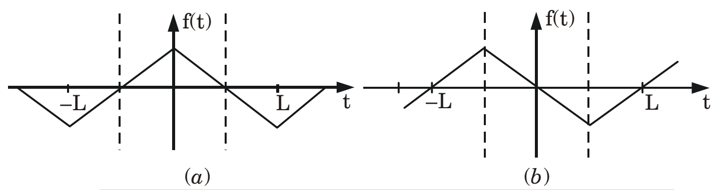


Fig. 3.2.4. (*a*) Even signal with Quarter wave symmetry. (*b*) Odd signal with Quarter wave symmetry.

Que 3.3. **Find the exponential Fourier series representation of the infinite square wave shown in Fig. 3.3.1.**

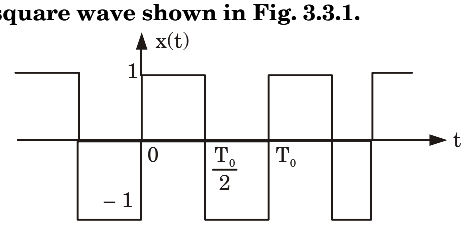


Fig. 3.3.1.

Answer

1.
$$x(t) = \begin{cases} 1 & ; \text{ for } 0 \le t \le T_0/2 \\ -1 & ; \text{ for } \frac{T_0}{2} \le t < T_0 \end{cases}$$

2.
$$C_0 = \frac{1}{T_0}\left[\int_0^{T_0/2} 1\, dt - \int_{T_0/2}^{T_0} 1\, dt\right] = 0$$

3.
$$C_n = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\left[\int_0^{T_0/2} 1e^{-jn\omega_0 t} + \int_{T_0/2}^{T_0} (-1)e^{-jn\omega_0 t}dt\right]$$

$$= \frac{1}{T_0}\left\{\left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_0^{T_0/2} - \left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_{T_0/2}^{T_0}\right\}$$

$$= \frac{e^{-jn\omega_0 T_0/2} - e^0}{-jn\omega_0 T_0} - \frac{e^{-jn\omega_0 T_0} - e^{-jn\omega_0 T_0/2}}{-jn\omega_0 T_0} \quad ...(3.3.1)$$

4. Since $\omega_0 = \frac{2\pi}{T_0}$, $\omega_0 T_0 = 2\pi$, eq. (3.3.1) will be

$$C_n = \frac{e^{-jn\pi} - 1}{-jn2\pi} - \frac{e^{-jn2\pi} - e^{-jn\pi}}{-jn2\pi} = \frac{e^{-jn\pi} - 1 - e^{-jn2\pi} + e^{-jn\pi}}{-jn2\pi}$$

$$e^{-jn2\pi} = \cos(2\pi n) - j\sin(2\pi n) = 1 \text{ for all } n.$$

5. Hence
$$C_n = \frac{2e^{-jn\pi} - 2}{-jn2\pi} = \frac{1 - e^{-jn\pi}}{jn\pi}$$

6. $1 - e^{-jn\pi} = 1 - [\cos n\pi - j\sin n\pi] = 1 - \cos n\pi$, since $\sin n\pi = 0$

$$= \begin{cases} 2 & ; \text{ for } n = 1, 3, 5, ...\text{odd} \\ 0 & ; \text{ for } n = 0, 2, 4, 6, ...\text{even} \end{cases}$$

7.
$$C_n = \begin{cases} \frac{2}{jn\pi} & ; \text{ for } n = 1, 3, 5, ...\text{odd} \\ 0 & ; \text{ for } n = 0, 2, 4, 6, ...\text{even} \end{cases}$$

8. Hence, exponential Fourier series is

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} = \sum_{n=1,3,5,...} \frac{2}{jn\pi} e^{jn\omega_0 t}$$

Que 3.4. **Find the exponential form of Fourier series for a triangular wave of maximum value 1 and time period 2 seconds.**

Answer

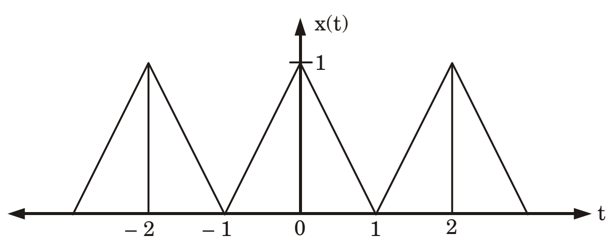


Fig. 3.4.1.

1. $$x(t) = \begin{cases} t+1 \;;\; -1 < t < 0 \\ -t+1 \;;\; 0 < t < 1 \end{cases}$$

2. $$A_0 = C_0 = \frac{1}{2}$$

3. $$C_n = \frac{1}{T}\int_0^T x(t)e^{-jn\omega_0 t}\,dt$$

$$= \frac{1}{2}\left(\int_{-1}^{0}(t+1)e^{-jn\omega_0 t}\,dt + \int_0^1 -(t-1)e^{-jn\omega_0 t}\,dt\right)$$

$$= \frac{1}{2}\left(\int_{-1}^{0} te^{-jn\omega_0 t}\,dt + \int_{-1}^{0} e^{-jn\omega_0 t}\,dt + \int_0^1 -te^{-jn\omega_0 t}\,dt + \int_0^1 e^{-jn\omega_0 t}\,dt\right)$$

$$= \frac{1}{2}\left(\int_{-1}^{0} te^{-jn\omega_0 t}dt - \int_0^1 te^{-jn\omega_0 t}dt\right) + \frac{1}{2}\int_{-1}^{1} e^{-jn\omega_0 t}\,dt$$

$$= \frac{1}{2}\left\{\left[\frac{e^{-jn\omega_0 t}(-jn\omega_0 t - 1)}{(-jn\omega_0)^2}\right]_{-1}^{0} - \left[\frac{e^{-jn\omega_0 t}(-jn\omega_0 t - 1)}{(-j\omega_n)^2}\right]_0^1\right\} + 0$$

$$\left[\because \int te^{-nt}\,dt = \frac{e^{-nt}(-nt-1)}{(-n)^2}\right]$$

$$= \frac{1}{2}\left[\frac{-1}{-(n\omega_0)^2} + \frac{e^{jn\omega_0}(jn\omega_0 - 1)}{(n\omega_0)^2}\right] - \frac{1}{2}\left[\frac{e^{-jn\omega_0}(-j\omega n_0 - 1)}{-(n\omega_0)^2} - \frac{1}{(n\omega_0)^2}\right]$$

$$= \frac{1}{2}\left[\frac{2}{(n\omega_0)^2} + jn\omega_0\left(\frac{e^{jn\omega_0} - e^{-jn\omega_0}}{(n\omega_0)^2}\right) - \left(\frac{e^{jn\omega_0} + e^{-jn\omega_0}}{(n\omega_0)^2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{2}{(n\omega_0)^2} + \frac{(2j \times jn\omega_0)}{(n\omega_0)^2}\sin n\omega_0 - 2\cos n\omega_0\right]$$

$$= \frac{1 - \cos n\omega_0}{(n\omega_0)^2}$$

$$\therefore \quad C_n = \begin{cases} 0 \;;\; \text{for even } n \\ \dfrac{2}{(n\omega_0)^2} \;;\; \text{for odd } n \end{cases}$$

4. $$x(t) = \frac{1}{2} + \sum_{n=\text{odd}}^{\infty} \frac{2}{(n\omega_0)^2} e^{jn\omega_0 t}$$

where, $\omega_0 = \dfrac{2\pi}{T} = \dfrac{2\pi}{2} = \pi$

Que 3.5. **Write a short note on fourier spectrum.**

Answer

1. The Fourier spectrum of a periodic signal $x(t)$ is a plot of its Fourier coefficients versus frequency ω.
2. It is in two parts :
   a. Amplitude spectrum and
   b. Phase spectrum.
3. The plot of the amplitude of Fourier coefficients versus frequency is known as the amplitude spectra and the plot of phase of Fourier coefficients versus frequency is known as phase spectra.
4. The Fourier spectrum exists only at discrete frequencies $n\omega$, where $n = 0,1, 2,....$ Hence it is known as discrete spectrum or line spectrum.
5. The magnitude spectrum is symmetrical about the vertical axis passing through the origin and the phase spectrum is antisymmetrical about the vertical axis passing through the origin.
6. So the magnitude spectrum exhibits even symmetry and the phase spectrum exhibits odd symmetry.

Que 3.6. **Obtain the exponential Fourier series for the waveform shown in Fig. 3.6.1. Also draw the frequency spectrum.**

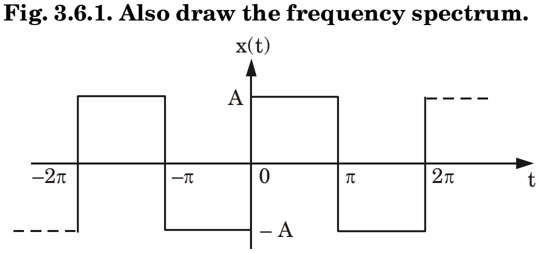


Fig. 3.6.1.

Answer

1. The periodic waveform shown in Fig. 3.6.1 can be expressed as :

$$x(t) = \begin{cases} A\,; & 0 \le t \le \pi \\ -A\,; & \pi \le t \le 2\pi \end{cases}$$

Fundamental frequency

$$\omega = \frac{2\pi}{2\pi} = 1$$

2. $$C_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{2\pi}\left[\int_0^{\pi} A\,dt + \int_{\pi}^{2\pi} -A\,dt\right] = 0$$

3. $$C_n = \frac{1}{T}\int_0^T x(t)\,e^{-jn\omega t}\,dt$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} Ae^{-jnt}\,dt + \int_{\pi}^{2\pi} -Ae^{-jnt}\,dt\right] = \frac{A}{2\pi}\left[\left(\frac{e^{-jnt}}{-jn}\right)_0^{\pi} - \left(\frac{e^{-jnt}}{-jn}\right)_{\pi}^{2\pi}\right]$$

$$= \frac{-A}{j2n\pi}[(e^{-jn\pi} - e^0) - (e^{-j2n\pi} - e^{-jn\pi})]$$

$$\therefore \quad C_n = \begin{cases} -j\dfrac{2A}{n\pi} \ ; \ \text{for odd } n \\ 0 \ ; \ \text{for even } n \end{cases}$$

4. $\therefore \quad x(t) = C_0 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} C_n\, e^{jn\omega t} = \sum_{n=-\infty}^{\infty} -j\frac{2A}{n\pi} e^{jnt}$; for odd n

5. $\therefore \quad C_0 = 0,\ C_1 = C_{-1} = \dfrac{2A}{\pi}$

$$C_3 = C_{-3} = \frac{2A}{3\pi}, C_5 = C_{-5} = \frac{2A}{5\pi}$$

The frequency spectrum is shown in Fig. 3.6.2.

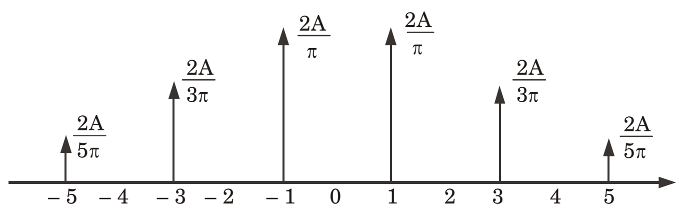


Fig. 3.6.2. Frequency spectrum.

Que 3.7. **Determine the exponential Fourier series for the waveform shown in Fig. 3.7.1. Also draw the frequency spectrum.**

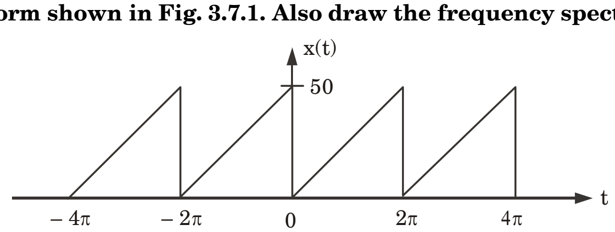


Fig. 3.7.1.

Answer

1. The waveform shown in Fig. 3.7.1 can be expressed as $x(t) = \dfrac{50}{2\pi}t$. The period $T = 2\pi$. Therefore, fundamental frequency $\omega = 2\pi/2\pi = 1$.

2. $$C_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{2\pi}\int_0^{2\pi}\frac{50}{2\pi}\,t\,dt = \frac{50}{(2\pi)^2}\left[\frac{t^2}{2}\right]_0^{2\pi} = \frac{50}{2} = 25$$

3. $$C_n = \frac{1}{T}\int_0^T x(t)\,e^{-jn\omega t}\,dt = \frac{1}{2\pi}\int_0^{2\pi}\frac{50}{2\pi}\,t\,e^{-jnt}\,dt$$

$$= \frac{50}{4\pi^2}\left[\left(\frac{t\,e^{-jnt}}{-jn}\right)_0^{2\pi} - \int_0^{2\pi}\frac{e^{-jnt}}{-jn}\,dt\right]$$

$$= \frac{50}{4\pi^2}\left\{-\frac{1}{jn}\left[2\pi e^{-j2n\pi} - 0 + \left(\frac{e^{-jnt}}{jn}\right)_0^{2\pi}\right]\right\}$$

$$= \frac{50}{4\pi^2}\left\{-\frac{1}{jn}\left[(2\pi\,(1) + \frac{1}{jn}\,(e^{-j2n\pi} - e^0)\right]\right\}$$

$$= \frac{50}{4\pi^2}\left[-\frac{1}{jn}\left(2\pi + \frac{1}{jn}(1-1)\right)\right] = j\,\frac{50}{2\pi n} = j\,\frac{25}{\pi n}$$

4. $$x(t) = 25 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} j\,\frac{25}{\pi n}\,e^{jnt} = 25 + j\,\frac{25}{\pi}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty}\frac{e^{jnt}}{n}$$

is the exponential Fourier series. The Fourier spectrum is shown in Fig. 3.7.2.

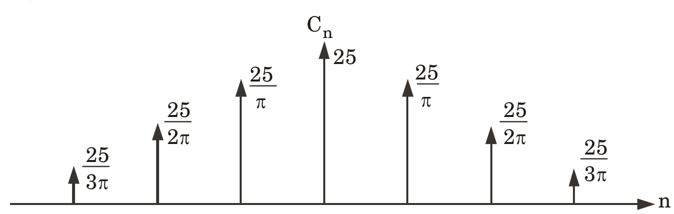


Fig. 3.7.2. Frequency spectrum.

PART-2

Effective Values.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.8. **Derive expression of effective values of fourier-series.**

Answer

1. If $$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$

   by definition, the effective or rms value can be written as

   $$F_{\text{rms}} = \left\{\frac{1}{T}\int_0^T\left[a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)\right]^2 dt\right\}^{1/2}$$

   $$= \left\{a_0^2 + \frac{1}{2}\left[a_1^2 + a_2^2 + a_3^2 + ... + b_1^2 + b_2^2 + b_3^2 + ...\right]\right\}^{1/2}$$

   $$= \left[c_0^2 + \frac{1}{2}\left(c_1^2 + c_2^2 + c_3^2 + ...\right)\right]^{1/2}$$

   $$c_n^2 = a_n^2 + b_n^2 \text{ and } c_0 = a_0$$

   where c_0 is the DC component and c_1, c_2, c_3, ... are the amplitudes of the harmonics.
2. In general, if the voltage and current are given by

   $$v(t) = V_0 + \Sigma V_n \sin(n\omega t + \phi_n)$$

   and $$i(t) = I_0 + \Sigma I_n \sin(n\omega t + \theta_n),$$
3. Their effective values are

   $$V_{\text{rms}} = \left[V_0^2 + \frac{1}{2}(V_1^2 + V_2^2 + V_3^2 + ...)\right]^{1/2} = \left[V_0^2 + V_1'^2 + V_2'^2 + V_3'^2 + ...\right]^{1/2}$$

   and $$I_{\text{rms}} = \left[I_0^2 + \frac{1}{2}(I_1^2 + I_2^2 + I_3^2 + ...)\right]^{1/2} = \left[I_0^2 + I_1'^2 + I_2'^2 + I_3'^2 + ...\right]^{1/2}$$

   where the rms values of the harmonic components are

   $$I_t' = \frac{I_t}{\sqrt{2}}, V_t' = \frac{V_t}{\sqrt{2}}; i = 1, 2, ...$$

PART-3

Power Factor.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.9. **Derive the expression of power and power factor for fourier series.**

Answer

Power :

1. In general, if the voltage and current are given by

$$v(t) = V_0 + \Sigma V_n \sin (n\omega t + \phi_n)$$
$$i(t) = I_0 + \Sigma I_n \sin (n\omega t + \theta_n)$$

2. The general expression for average power is

$$P = \frac{1}{T}\int_0^T v(t)\, i(t)\, dt$$
$$= \frac{1}{T}\int_0^T [V_0 + \Sigma V_n \sin (n\omega t + \phi_n)]\ [I_0 + \Sigma I_n \sin (n\omega t + \theta_n)]\, dt$$
$$= V_0 I_0 + \frac{1}{2}\Sigma [V_n I_n \cos(\phi_n - \theta_n)]$$
$$= V_0 I_0 + \Sigma(V'_n I'_n \cos \psi_n)$$

where, $\psi_n = \phi_n - \theta_n$ and $V'_n = V_n / \sqrt{2}\ and\ I'_n = I_n / \sqrt{2}$

3. The total average power is the sum of the harmonic powers. The power components result only from the corresponding harmonics of voltage and current.

Power factor :

1. The power factor is defined as the ratio of the power to the volt-ampere.
2. The volt-ampere is itself a product of the effective values of voltage and current

$$V'I' = (V_0^2 + V'^2_1 + V'^2_2 + V'^2_3 + ...)^{1/2}\ (I_0 + I'^2_1 + I'^2_2 + I'^2_3 + ...)^{1/2}$$

3. Power factor, $\Delta = \dfrac{V_0 I_0 + (V'_n I'_n \cos \psi_n)}{[(V_0^2 + \Sigma V'^2_n)\,(I_0^2 + \Sigma I'^2_n)]^{1/2}}$

Que 3.10. **In a two-element series network, voltage $v(t)$ is applied, which is given as**

$$v(t) = 50 + 50 \sin 5000t + 30 \sin 10000t$$

The resulting current is given as

$$i(t) = 11.2 \sin (5000t + 63.4°) + 10.6 \sin (10000t + 45°)$$

Determine the network elements and the power dissipated in the circuit.

Answer

1. In the expression of current $i(t)$, the DC term is missing though it is present in the applied voltage $v(t)$. Hence, in the series network, there must be a capacitor which blocks the DC components.

2. Again, from the expression of $i(t)$, we can observe that current is leading by an angle less than 90°. Hence, the conclusion is the presence of a resistive element R in series with the capacitor C.
3. The impedance at 5 kHz (fundamental frequency) is

$$Z_5 = \frac{50}{11.2} = 4.47\ \Omega = (R^2 + X_c^2)^{1/2} \qquad ...(3.10.1)$$

4. Similarly, at second harmonic *i.e.*, at 10 kHz, is

$$Z_{10} = \frac{30}{10.6} = 2.83\ \Omega = (R^2 + X_c^2/4)^{1/2} \qquad ...(3.10.2)$$

5. Solving eq. (3.10.1) and eq. (3.10.2), we get

$$R = 2\ \Omega \text{ and } C = 50\ \mu\text{F}$$

6. The power in the circuit is

$$P = \frac{1}{2}(V_n I_n \cos \theta_n)$$

$$= \frac{1}{2}(50 \times 11.2 \times \cos 63.4° + 30 \times 10.6 \times \cos 45°)$$

$$= 238 \text{ W}$$

PART-4

Steady State Response of a Network to Non-Sinusoidal Periodic Inputs.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.11. **Discuss steady state response of network for periodic signal.**

Answer

1. The voltage (periodic) is $v(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega t - \phi_n)$

   We want to find out the steady state current, $i(t)$.
2. Phasors corresponding to terms in right-hand side are,

$$V_0 = A_0 e^{j0} \text{ and } V_n = A_n e^{-j\phi_n}$$

3. Let, $Z(j\omega)$ = Impedance phasor of the network at any frequency ω. So, the current phasors are,

$$I_0 = \frac{V_0}{Z(j0)} = \frac{A_0 e^{j0}}{Z(j0)} = |I_0| e^{j0}$$

$$I_n = \frac{V_n}{Z(j\omega)} = \frac{A_n e^{-j\phi_n}}{Z(j\omega)} = |I_n| e^{-j\alpha_n}$$

4. By superposition principle, the net current phasor is $i(t) = I_0 + I_1 + I_2 + ...$
   So, transforming from frequency domain to time domain,

$$i(t) = I_0 + \sum_{n=1}^{\infty} |I_n| \cos(n\omega t - \alpha_n)$$

Que 3.12. **For the circuit of Fig. 3.12.1(*a*), determine the periodic response *i*(*t*) corresponding to the forcing function shown in Fig. 3.12.1(*b*) if *i*(0) = 0.**

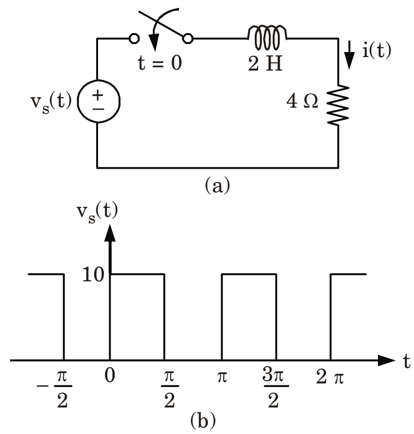


Fig. 3.12.1.

Answer

1. The forcing function has a fundamental frequency $\omega_0 = 2$ rad/s, and its Fourier series may be written down as

$$v_s(t) = 5 + \frac{20}{\pi} \sum_{n=1(\text{odd})}^{\infty} \frac{\sin 2nt}{n}$$

2. We will find the forced response for the n_{th} harmonic by working in the frequency domain. Thus,

$$v_{sn}(t) = \frac{20}{n\pi} \sin 2nt$$

and $$V_{sn} = \frac{20}{n\pi} \angle -90° = -j\frac{20}{n\pi}$$

3. The impedance offered by the *RL* circuit at this frequency is

$$Z_n = 4 + j(2n)2 = 4 + j4n$$

and thus the component of the forced response at this frequency is

$$I_{fn} = \frac{V_{sn}}{Z_n} = \frac{-j5}{n\pi(1+jn)}$$

4. Transforming to time domain, we have

$$i_{fn} = \frac{5}{n\pi}\frac{1}{\sqrt{1+n^2}}\cos(2nt - 90° - \tan^{-1} n)$$

$$= \frac{5}{\pi(1+n^2)}\left(\frac{\sin 2nt}{n} - \cos 2nt\right)$$

5. Since the response to the DC component is simply $5V / 4\Omega = 1.25$ A, the forced response may be expressed as the summation

$$i_f(t) = 1.25 + \frac{5}{\pi}\sum_{n=1(\text{odd})}^{\infty}\left[\frac{\sin 2nt}{n(1+n^2)} - \frac{\cos 2nt}{1+n^2}\right]$$

6. The natural response of the circuit is the single exponential term [characterizing the single pole of the transfer function, $I_f / V_s = 1/(4+2s)$]

$$i_n(t) = Ae^{-2t}$$

7. The complete response is therefore

$$i(t) = i_f(t) + i_n(t)$$

8. Letting $t = 0$, we find A using $i(0) = 0$,

$$A = -i_f(0) = -1.25 + \frac{5}{\pi}\sum_{n=1(\text{odd})}^{\infty}\frac{1}{1+n^2}$$

9. The sum of the first 5 terms of $\Sigma\, 1/(1+n^2)$ is 0.671, the sum of the first 10 terms is 0.695, the sum of the first 20 terms is 0.708, and the exact sum is 0.720 to three significant figures.

Thus, $$A = -1.25 + \frac{5}{\pi}(0.720) = -0.104$$

10. $$i(t) = -0.104e^{-2t} + 1.25 + \frac{5}{\pi}\sum_{n=1(\text{odd})}^{\infty}\left[\frac{\sin 2nt}{n(1+n^2)} - \frac{\cos 2nt}{1+n^2}\right]$$

Que 3.13. **The voltage $v(t)$ having the waveform shown in Fig. 3.13.1 is applied to the circuit shown in Fig. 3.13.2. Determine the current $i(t)$ using Fourier series.**

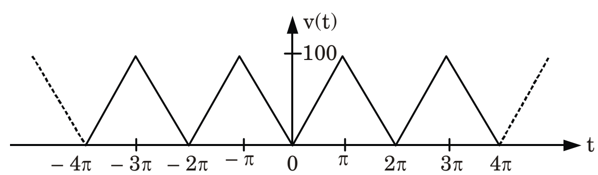


Fig. 3.13.1.

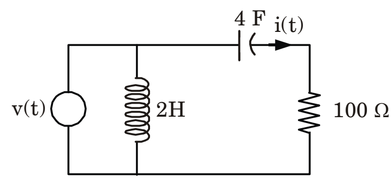


Fig. 3.13.2.

Answer

1. The given waveform of Fig. 3.13.1 is periodic with period $T = 2\pi$. Choose one cycle of the waveform from $-\pi$ to π. Let

$$t_0 = -\pi,\ t_0 + T = \pi$$

2. Then Fundamental frequency $\omega_o = \dfrac{2\pi}{T} = \dfrac{2\pi}{2\pi} = 1$

3. The waveform is described by

$$x(t) = \begin{cases} -\dfrac{100}{\pi}t\,; & \text{for } -\pi \le t \le 0 \\ \dfrac{100}{\pi}t\,; & \text{for } 0 \le t \le \pi \end{cases}$$

4. The waveform has even symmetry

$\therefore$
$$a_0 = \frac{2}{T}\int_0^{T/2} x(t)dt$$

$$a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos n\omega_0 t\,dt \quad \text{and } b_n = 0$$

5. Now,
$$a_0 = \frac{2}{T}\int_0^{T/2} x(t)\,dt = \frac{2}{2\pi}\int_0^{\pi}\frac{100}{\pi}t\,dt$$

$$= \frac{100}{\pi^2}\int_0^{\pi} t\,dt = \frac{100}{\pi^2}\left(\frac{t^2}{2}\right)_0^{\pi}$$

$$= \frac{100}{\pi^2}\left(\frac{\pi^2}{2}\right) = \frac{100}{2} = 50$$

6.
$$a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos n\omega_0 t\,dt = \frac{4}{2\pi}\int_0^{\pi}\frac{100}{\pi}t\cos nt\,dt$$

$$= \frac{200}{\pi^2}\left[\left(\frac{t\sin nt}{n}\right)_0^{\pi} - \int_0^{\pi}\frac{\sin nt}{n}dt\right]$$

$$= \frac{200}{\pi^2}\left[(\pi\sin n\pi - 0) + \left(\frac{\cos nt}{n^2}\right)_0^{\pi}\right] = \frac{200}{\pi^2 n^2}(\cos n\pi - \cos 0)$$

$$= \frac{200}{n^2\pi^2}[(-1)^n - 1]$$

$$a_n = \begin{cases} -\dfrac{400}{n^2\pi^2}; & \text{for odd } n \\ 0; & \text{for even } n \end{cases}$$

7. The Trigonometric Fourier series is

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$= 50 + \sum_{n=\text{odd}}^{\infty} \frac{-400}{n^2\pi^2} \cos nt$$

$$= 50 - \frac{400}{\pi^2} \sum_{n=\text{odd}}^{\infty} \frac{\cos nt}{n^2}$$

8. Since the capacitor in the circuit shown in Fig. 3.13.2 does not allow the DC current, the DC term in the input can be neglected and only harmonic components are to be considered.

$$v(t) = -\frac{400}{\pi^2} \sum_{n=\text{odd}}^{\infty} \frac{\cos nt}{n^2}$$

9. So the given network can be redrawn as shown in Fig. 3.13.3.

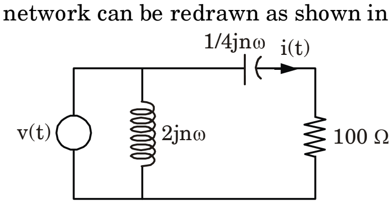


Fig. 3.13.3.

$$\therefore \qquad i(t) = \frac{v(t)}{100 + \left(\dfrac{1}{4\,jn\omega}\right)} = \frac{v(t)}{100 - \dfrac{j}{4n\omega}}$$

$$= \frac{4n\omega\, v(t)}{400n\omega - j} = \frac{4n\omega\, v(t)}{\sqrt{1 + 160000\, n^2\omega^2}} \angle -\tan^{-1}\left(\frac{1}{400n\omega}\right) \text{A}$$

PART-5

Fourier Transform and Continuous Spectra.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.14. **Derive the expression of Fourier transform from Fourier series.**

Answer

1. Exponential form of the Fourier series :

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \quad ...(3.14.1)$$

where $$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_0 t}dt \quad ...(3.14.2)$$

and $$\omega_0 = \frac{2\pi}{T} \quad ...(3.14.3)$$

2. We now let $T \to \infty$

   and thus, from eq. (3.14.3), ω_0 must become vanishingly small. We represent this limit by a differential

$$\omega_0 \to d\omega$$

Thus $$\frac{1}{T} = \frac{\omega_0}{2\pi} \to \frac{d\omega}{2\pi} \quad ...(3.14.4)$$

3. Finally, the frequency of any "harmonic" $n\omega_0$ must now correspond to the general frequency variable which describes the continuous spectrum. In other words, n must tend to infinity as ω_0 approaches zero, so that the product is finite

$$n\omega_0 \to \omega \quad ...(3.14.5)$$

4. Multiplying by T on each side of eq. (3.14.2) and putting $n\omega_0 \to \omega$; $T \to \infty$ we get

$$c_n T \to \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt \quad ...(3.14.6)$$

5. The right-hand side eq. (3.14.6) is a function of ω (and not of t), and we represent it by $F(\omega)$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt \quad ...(3.14.7)$$

6. Now let us apply the limiting process to eq. (3.14.1). We begin by multiplying and dividing the summation by T,

$$f(t) = \sum_{n=-\infty}^{\infty} c_n\, T e^{jn\omega_0 t}\, \frac{1}{T}$$

   next replacing $c_n T$ with the new quantity $F(\omega)$, and then making use of expressions (3.14.4) and (3.14.5).

   In the limit, the summation becomes an integral, and

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega \quad ...(3.14.8)$$

7. Eq. (3.14.7) and (3.14.8) are collectively called the Fourier transform pair. The function $F(\omega)$ is the Fourier transform of $f(t)$ and $f(t)$ is the inverse Fourier transform of $F(\omega)$.

Que 3.15. **Use the Fourier transform to obtain the continuous spectrum of the single rectangular pulse Fig. 3.15.1.**

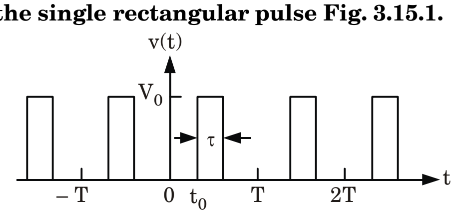


Fig. 3.15.1.

Answer

1. The pulse is described by

$$f(t) = \begin{cases} V_0\,; & t_0 < t < t_0 + \tau \\ 0\,; & t < t_0 \text{ and } t > t_0 + \tau \end{cases}$$

2. The Fourier transform of $f(t)$ is

$$F(\omega) = \int_{t_0}^{t_0+\tau} V_0 e^{-j\omega t}dt \; = \; V_0\tau \frac{\sin \frac{1}{2}\omega\tau}{\frac{1}{2}\omega\tau} e^{-j\omega(t_0 + \tau/2)}$$

The value of $F(0)$ is $V_o I$.

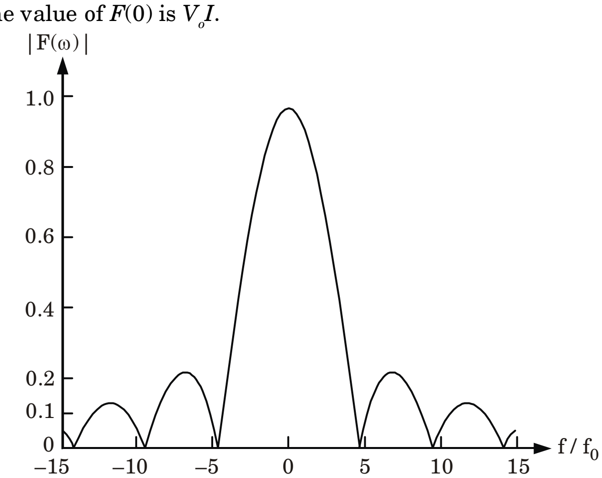


Fig. 3.15.2.

3. Plot of $|F(\omega)|$ corresponding to the pulse with $v_0 = 1$, $T = 1$ and $t_0 = 0$.

Que 3.16. What are the different properties of Fourier transform ?

Answer

1. **Linearity :**

If $$x_1(t) \xleftrightarrow{\text{FT}} X_1(\omega)$$

and $$x_2(t) \xleftrightarrow{\text{FT}} X_2(\omega)$$

then according to this property,

$$a_1x_1(t) + a_2x_2(t) \xleftrightarrow{\text{FT}} a_1X_1(\omega) + a_2X_2(\omega)$$

2. **Symmetry or duality property :**

If $$x(t) \xleftrightarrow{\text{FT}} X(\omega)$$

then, $$X(t) \xleftrightarrow{\text{FT}} 2\pi x(-\omega)$$

3. **Time-shifting :** This property states that a shift in the time-domain by an amount of a is equivalent to multiplication by $e^{-j\omega a}$ in the frequency-domain.

If $$x(t) \xleftrightarrow{\text{FT}} X(\omega)$$

then, $$x(t-a) \xleftrightarrow{\text{FT}} e^{-j\omega a}\, X(\omega)$$

4. **Frequency-shifting :** This property states that a shift in frequency domain by an amount 'ω_0' is equivalent to multiplication by '$e^{j\omega_0 t}$' to time-domain function $x(t)$.

If $$x(t) \xleftrightarrow{\text{FT}} X(\omega)$$

then, $$e^{j\omega_0 t}x(t) \xleftrightarrow{\text{FT}} X(\omega - \omega_0)$$

5. **Differentiation in time-domain :**

If $$x(t) \xleftrightarrow{\text{FT}} X(\omega)$$

then, $$\frac{dx(t)}{dt} \xleftrightarrow{\text{FT}} j\omega \,.\, X(\omega)$$

6. **Differentiation in frequency-domain :**

If $$x(t) \xleftrightarrow{\text{FT}} X(\omega)$$

then, $$t.x(t) \xleftrightarrow{\text{FT}} j\frac{d}{d\omega}\, X(\omega)$$

7. **Time-scaling :** This property states that time compression of a signal results in its spectral (frequency-domain) expansion, whereas, time expansion of a signal results in its spectral (frequency-domain) compression.

If $$x(t) \xleftrightarrow{\text{FT}} X(\omega)$$

then, $$x(at) \xleftrightarrow{\text{FT}} \frac{1}{|a|}X(\omega / a)$$

where $a \to$ real constant.

8. Time-integration :

If $$x(t) \xleftrightarrow{\text{FT}} X(\omega)$$

then, $$\int_{-\infty}^{t} x(t)\, dt \xleftrightarrow{\text{FT}} \frac{1}{j\omega} X(\omega)$$

Assuming all initial conditions equal to zero.

9. Time-reversal :

If $$x(t) \xleftrightarrow{\text{FT}} X(\omega)$$

then, $$x(-t) \xleftrightarrow{\text{FT}} X(-\omega)$$

10. Conjugation :

If $$x(t) \xleftrightarrow{\text{FT}} X(\omega)$$

then, $$x^*(t) \xleftrightarrow{\text{FT}} X^*(-\omega)$$

11. Multiplication property (frequency-convolution) :

If $$x_1(t) \xleftrightarrow{\text{FT}} X_1(\omega)$$

and $$x_2(t) \xleftrightarrow{\text{FT}} X_2(\omega)$$

then, $$x_1(t) \,.\, x_2(t) \xleftrightarrow{\text{FT}} \frac{1}{2\pi}\left[X_1(\omega) \otimes X_2(\omega)\right]$$

12. Convolution (time-convolution) :

If $$x_1(t) \xleftrightarrow{\text{FT}} X_1(\omega)$$

and $$x_2(t) \xleftrightarrow{\text{FT}} X_2(\omega)$$

then, $$x_1(t) \otimes x_2(t) \xleftrightarrow{\text{FT}} X_1(\omega) \,.\, X_2(\omega)$$

Que 3.17. Find the Fourier transform of the following and sketch the magnitude and phase spectrum of $x(t) = e^{-2t}\, u(t)$.

Answer

1. The Fourier transform

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{0}^{\infty} e^{-2t}\, e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-(2+j\omega)t}\, dt$$

$$= -\frac{1}{(2+j\omega)}\left[e^{-(2+j\omega)t}\right]_0^{\infty} = \frac{1}{(2+j\omega)}$$

2. Now magnitude and phase are as follows :

$$|X(\omega)| = \frac{1}{\sqrt{4+\omega^2}}$$

$$\angle X(\omega) = -\tan^{-1}\frac{\omega}{2}$$

3. The amplitude spectrum $|X(\omega)|$ and phase spectrum $\angle X(\omega)$ are shown in Fig. 3.17.1 (*a*) and (*b*) respectively.

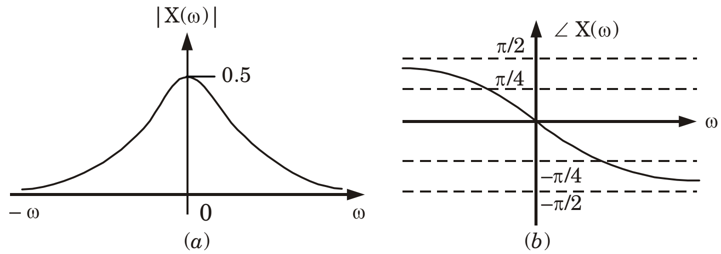


Fig. 3.17.1. (*a*) Magnitude spectrum (*b*) Phase spectrum.

Que 3.18. **Determine the Fourier transform of the rectangular pulse :**

$$x(t) = \text{rect}\left(\frac{t}{\tau}\right) = \begin{cases} 1 & \text{for } \dfrac{-\tau}{2} < t < \dfrac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$$

OR

Find the continuous-time Fourier transform of the gate rectangular signal. Also plot its magnitude response.

Answer

1. The Fourier transform is given by,

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \text{rect}\left(\frac{t}{\tau}\right) e^{-j\omega t}\, dt = \int_{-\tau/2}^{\tau/2} 1 . e^{-j\omega t}\, dt$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-\tau/2}^{\tau/2} = \left[\frac{e^{-j\omega\tau/2} - e^{+j\omega\tau/2}}{-j\omega}\right]$$

$$= \left[\frac{-2j\sin\left(\dfrac{\omega\tau}{2}\right)}{-j\omega}\right] = \frac{2\sin\left(\dfrac{\omega\tau}{2}\right)}{\omega} = \frac{\sin\left(\dfrac{\omega\tau}{2}\right)}{\omega/2} = \tau . \frac{\sin\left(\dfrac{\omega\tau}{2}\right)}{(\omega\tau/2)}$$

$$= \tau \text{ sinc } (\omega\tau/2)$$

2. Since, sinc $(x) = 0$ for $x = \pm n\pi$

   Therefore, sinc $(\omega\tau/2) = 0$ for $(\omega\tau/2) = \pm n\pi$

or $$\omega = \frac{\pm 2n\pi}{\tau} \qquad n = 1, 2, 3, \ldots\ldots\ldots$$

or $$\omega = \frac{\pm 2\pi}{\tau}, \frac{\pm 4\pi}{\tau}, \frac{\pm 6\pi}{\tau}, \frac{\pm 8\pi}{\tau}, \ldots..$$

3. The plot of the $X(\omega)$ is given in Fig. 3.18.1.

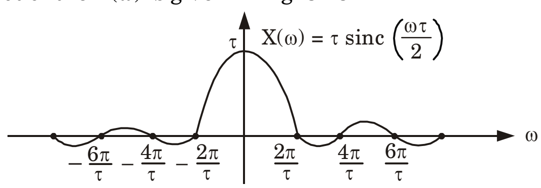


Fig. 3.18.1.

PART-6

Three Phase Unbalanced Circuit and Power Calculation.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.19. **Discuss three phase unbalanced circuits.**

Answer

1. A three phase circuit is said to be unbalanced, if the loads connected across the three phases have different magnitudes and power factors, *i.e.*, the loads connected across the three phases are not identical to each other.
2. The phase current in delta and the phase currents in star differ in unbalanced loading. In unbalanced star, current flows in neutral wire

$$I_N = I_R + I_Y + I_B$$

3. There are three cases of unbalanced loads :

i. Unbalanced delta-connected load.

ii. Unbalanced four-wire star-connected load.

iii. Unbalanced three-wire star-connected load.

Que 3.20. **Given expressions for phase currents and line currents of unbalanced delta-connected load.**

Answer

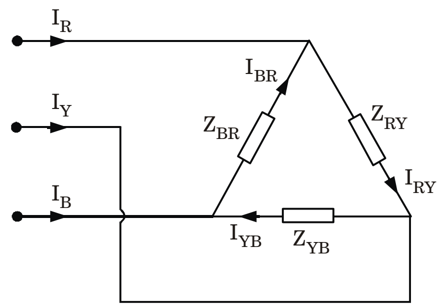


Fig. 3.20.1. Unbalanced Δ-connected load.

1. Fig. 3.20.1 shows an unbalanced delta-load connected to a balanced three-phase supply.
2. Voltage across the load is independent of the nature of the load and is equal to the line voltage of the supply.
3. The current in each load phase is equal to the line voltage divided by the impedance of that phase. The line current will be the phasor difference of the corresponding phase currents taking V_{RY} as the reference phasor.
4. For a delta-connected load, let

$$V_L = V_{ph}$$
$$V_{RY} = V_{ph}\angle 0°$$
$$V_{YB} = V_{ph}\angle -120°$$
$$V_{BR} = V_{ph}\angle -240°$$

Phase currents are given by

$$I_{RY} = \frac{V_{RY}}{Z_{RY}}$$

$$I_{YB} = \frac{V_{YB}}{Z_{YB}}$$

$$I_{BR} = \frac{V_{BR}}{Z_{BR}}$$

Line currents are given by

$$I_R = I_{RY} - I_{BR}$$
$$I_Y = I_{YB} - I_{RY}$$
$$I_B = I_{BR} - I_{YB}$$

Que 3.21. **A three-phase supply with a line voltage of 400 V has an unbalanced delta-connected as shown in Fig. 3.21.1. Determine**

a. Phase current,

b. Line current if phase sequence is *RYB*.

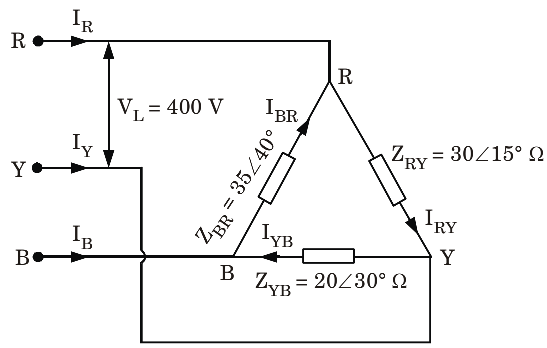


Fig. 3.21.1.

Answer

a. Phase current $I_{RY} = \dfrac{V_{RY}}{Z_{RY}} = \dfrac{400\angle 0°}{30\angle 15°} = 13.33\ \angle -15°$ A

$$I_{YB} = \frac{V_{YB}}{Z_{YB}} = \frac{400\angle -120°}{20\angle 30°} = 20\ \angle -150° \text{ A}$$

$$I_{BR} = \frac{V_{BR}}{Z_{BR}} = \frac{400\angle -240°}{35\angle 40°} = 11.428\ \angle 80° \text{ A}$$

b. Line current

$I_R = I_{RY} - I_{BR} = 13.33\ \angle -15° - 11.428\ \angle 80° = 10.891 - j14.7$
$= 18.295\ \angle -53.46°$ A

$I_Y = I_{YB} - I_{RY} = 20\ \angle -150° - 13.33\ \angle -15° = -30.195 - j6.55$
$= 30.897\ \angle 192.24°$ A

$I_B = I_{BR} - I_{YB} = 11.428\ \angle 80° - 20\ \angle -150° = 19.304 + j21.25$
$= 28.7\ \angle +47.75°$ A

Que 3.22. **Explain unbalanced four-wire star connected load.**

Answer

1. Fig. 3.22.1 shows an unbalanced three-phase star-connected load connected to a balanced three-phase, four-wire supply. It is the simplest case of an unbalanced load because of the presence of the neutral wire.

2. The neutral point N of the load is connected to the neutral point O of the supply. So, the star points of the supply O and the load N are at the same potential.
3. It means that the voltage across each load impedance is equal to the phase voltage of the supply. However the current in each phase will be different.

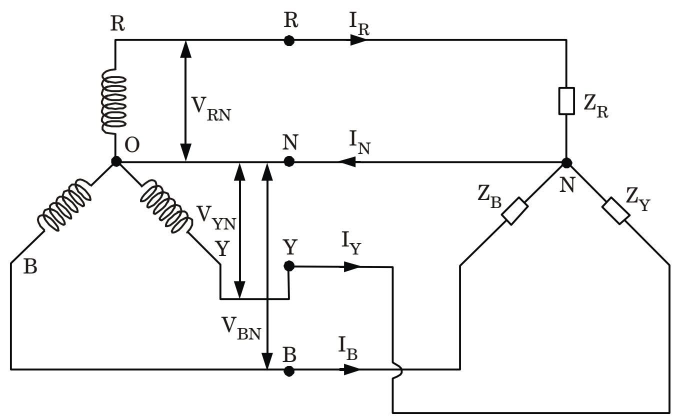


Fig. 3.22.1. Three-phase supply feeding unbalanced four-wire star connected load.

4. For a star-connected load, $V_L = \sqrt{3}\, V_{ph}$
5. Phase voltages are

$$V_{RN} = V_{ph} \angle 0°$$
$$V_{YN} = V_{ph} \angle -120°$$
$$V_{BN} = V_{ph} \angle -240°$$

6. Phase currents are

$$I_R = \frac{V_{RN}}{Z_R}\ ;\ I_Y = \frac{V_{YN}}{Z_Y}\ ;\ I_B = \frac{V_{BN}}{Z_B}$$

7. For a star-connected load, phase currents are equal to the line currents. The current in neutral wire is given by

$$I_N = I_R + I_Y + I_B$$

Que 3.23. **A three-phase 440 V, four-wire system of Fig. 3.23.1, has a star-connected load with Z_R = (8 + *j*12) Ω, Z_Y = (18 + *j*14) Ω, Z_B = (6 + *j*10) Ω. Find the line currents and current through neutral conductor.**

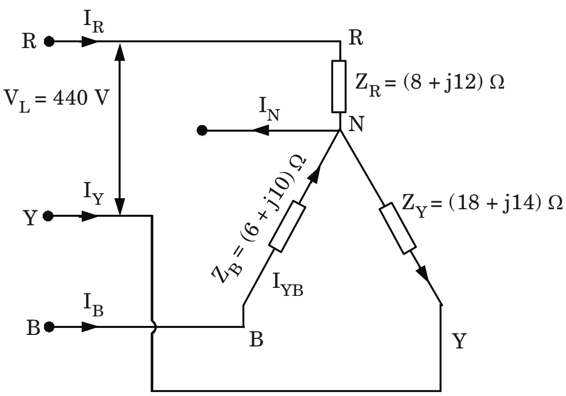


Fig. 3.23.1.

Answer

1. For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254 \text{ V}$$

2. The phase sequence is assumed to be $R - Y - B$.

$$V_{RN} = 254 \angle 0° \text{ V}$$
$$V_{YN} = 254 \angle - 120° \text{ V}$$
$$V_{BN} = 254 \angle - 240° \text{ V}$$

3. Phase currents

$$I_R = \frac{V_{RN}}{Z_R} = \frac{254\angle 0°}{14.42\angle 56.31°} = 17.6 \angle - 56.31° \text{ A}$$

$$I_Y = \frac{V_{YN}}{Z_Y} = \frac{254\angle - 120°}{22.8\angle 37.87°} = 11.14 \angle - 157.87° \text{ A}$$

$$I_B = \frac{V_{BN}}{Z_B} = \frac{254\angle - 240°}{11.66\angle 59°} = 21.78 \angle - 299° \text{ A}$$

4. In a star-connected load, line currents are equal to the phase current.

$\therefore I_{LR} = I_R = 17.6\angle - 56.31°$ A;

$I_{LY} = I_Y = 11.14\angle - 157.87°$ A;

$I_{LB} = I_B = 21.78\angle - 299°$ A

5. Current through neutral conductor,

$I_N = I_R + I_Y + I_B = 17.6\angle - 56.31° + 11.14\angle - 157.87° + 21.78\angle - 299°$
$= 10.05\angle 1.364°$ A

Que 3.24. **Discuss unbalanced three wire star connected load.**

Answer

1. Fig. 3.24.1 shows as unbalanced three-wire star-connected load.
2. If neutral point N of the load is not connected to the neutral point O of the supply, a voltage exists between the supply neutral point and load neutral point.
3. The load phase voltage is not equal to the supply phase voltage, and they are not only unequal in magnitude, but also subtend an angle other than 120° with one another.
4. The magnitude of each phase voltage depends upon the individual phase load. The phase voltage of the load is not $1/\sqrt{3}$ of the line voltage.

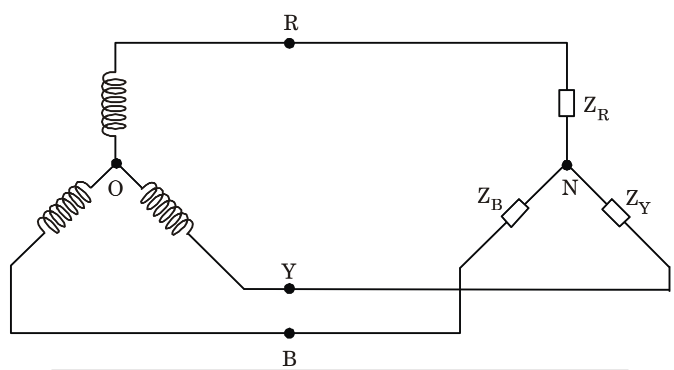


Fig. 3.24.1. Unbalanced three-wire, star connected load.

5. The star connected loads are replaced by an equivalent delta-connected load.

In this
$$Z_{RY} = Z_R + Z_Y + \frac{Z_R Z_Y}{Z_B}$$
$$Z_{YB} = Z_Y + Z_B + \frac{Z_Y Z_B}{Z_R}$$
$$Z_{BR} = Z_B + Z_R + \frac{Z_B Z_R}{Z_Y}$$

6. The problem is then solved as an unbalanced delta-connected load. The line currents so calculated are equal to line currents of the original star-connected load.

Que 3.25. **Discuss two wattmeter method of power calculation in three phase system.**

Answer

1. In this method, the current coil of wattmeter W_1 is inserted in phase R and that of wattmeter W_2 is inserted in phase Y and the pressure coils of W_1 and W_2 are connected across R and B, and Y and B respectively.
2. Fig. 3.25.1(*a*) shows the schematic diagram and Fig. 3.25.1(*b*) shows the phasor diagram.

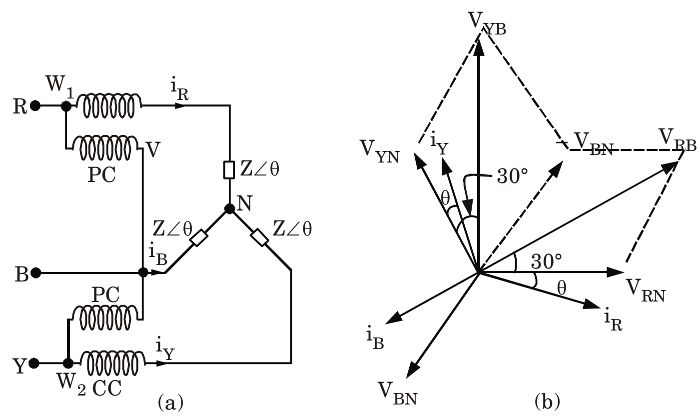


Fig. 3.25.1. Two-wattmeter method : (a) Schematic diagram, (b) Phasor diagram.

3. Now, the sum of the instantaneous powers measured by W_1 and W_2 is given by

$$\begin{aligned} W &= W_1 + W_2 \\ &= i_R V_{RB} + i_Y V_{YB} \\ &= i_R(V_{RN} - V_{BN}) + i_Y(V_{YN} - V_{BN}) \\ &= i_R V_{RN} + i_Y V_{YN} - V_{BN}(i_R + i_Y) \\ &= i_R V_{RN} + i_Y V_{YN} + i_B V_{BN} \qquad (\because \; i_R + i_Y + i_B = 0) \end{aligned}$$

Therefore, wattmeter gives total instantaneous power in the three-phase circuits.

4. Fig. 3.25.1(*b*) is for a balanced load, and the current in each phase lags the corresponding phase voltage by angle θ.
5. From the phasor diagram in Fig. 3.26.1(*b*), we can see that current I_R through the current coil of W_1 lags the line voltage V_{RB} by an angle of 30° + θ. Similarly, the current I_Y through the current coil W_2 lags the line voltage V_{YB} by an angle of 30° – θ.

6. Each wattmeter measures the product of effective value of the current through its current coil, effective value of the voltage across its pressure coil and the cosine of the angle between them.

$\therefore \quad W_1 = V_L I_L \cos(30° + \theta)$

and $\quad W_2 = V_L I_L \cos(30° - \theta)$

$$W_1 + W_2 = V_L I_L \cos(30° + \theta) + V_L I_L \cos(30° - \theta)$$
$$= V_L I_L [\cos(30° + \theta) + \cos(30° - \theta)]$$
$$= V_L I_L (2 \cos 30° \cos\theta) = \sqrt{3}\ V_L I_L \cos\theta$$
$$= \text{Total three phase power}$$

7. Similarly, $W_2 - W_1 = V_L I_L [\cos(30° - \theta) - \cos(30° + \theta)]$
$= V_L I_L \sin\theta$

Measurement of power factor by two-wattmeter method :

$$\therefore \quad \frac{W_2 - W_1}{W_2 + W_1} = \frac{V_L I_L \sin\theta}{\sqrt{3} V_L I_L \cos\theta} = \frac{\tan\theta}{\sqrt{3}}$$

$$\therefore \quad \tan\theta = \sqrt{3}\left(\frac{W_2 - W_1}{W_2 + W_1}\right)$$

$$\therefore \quad \theta = \tan^{-1}\left[\sqrt{3}\left(\frac{W_2 - W_1}{W_2 + W_1}\right)\right]$$

$$\text{Power factor} = \cos\theta = \cos\left\{\tan^{-1}\left[\sqrt{3}\left(\frac{W_2 - W_1}{W_2 + W_1}\right)\right]\right\}$$

Que 3.26. **Two wattmeters are used to measure power in a three-phase balanced load. Find the power factor, if**

a. Two reading are equal and positive,

b. two reading are equal and opposite and

c. one wattmeter reads zero.

Answer

a. Power factors if two readings are equal and positive

$$W_1 = W_2$$

$$\tan\theta = \sqrt{3}\left(\frac{W_2 - W_1}{W_2 + W_1}\right) = \sqrt{3} \times 0 = 0°$$

$$\therefore \quad \theta = \tan^{-1}(0) = 0°$$

$$\text{Power factor} = \cos\theta = \cos 0 = 1$$

b. Power factor, if two readings are equal and opposite.

$$W_1 = -W_2$$

$$\tan\theta = \sqrt{3}\left(\frac{W_2 - (-W_2)}{W_2 - W_2}\right) = \sqrt{3} \times \frac{2W_2}{0} = \infty$$

$\therefore$ $\theta = \tan^{-1}(\infty) = 90°$

Power factor = $\cos\theta = \cos 90° = 0$

c. Power factor, if one wattmeter reads zero

$W_1 = 0$

$$\tan\theta = \sqrt{3}\left(\frac{W_2 - 0}{W_2 + 0}\right) = \sqrt{3}$$

$\therefore$ $\theta = \tan^{-1}(\sqrt{3}) = 60°$

Power factor = $\cos\theta = \cos 60° = 0.5$.

Que 3.27. **Discuss different methods of measuring three-phase power.**

Answer

a. **Three-wattmeter method :**

1. This method is used for balanced as well as unbalanced loads.
2. Three wattmeters one in each phase are inserted in the star-connected or delta-connected load. Each wattmeter will give the power consumed in that phase.
3. For balanced load, $W_1 = W_2 = W_3$

   For unbalanced load, $W_1 \neq W_2 \neq W_3$
4. Total power, $W_1 + W_2 + W_3$

b. **One-wattmeter method :**

1. This method is used for balanced load only. When the load is balanced, total power is given by

$$P = 3V_{ph}I_{ph}\cos\theta$$

2. Hence, one wattmeter is used to measure power in one phase. The wattmeter reading is then multiplied by three to obtain three-phase power. The load may be star or delta connected.

c. **Two wattmeter method :** Refer Q. 3.25, Page 3–29C, Unit-3.

Que 3.28. **Two wattmeters *A* and *B* give readings as 6000 W and 2000 W respectively of power measurement in a three-phase, three-wire balanced load system.**

a. **Calculate the power and power factor, (i) when both meters read direct, and (ii) when the second meter *B* reads in the reverse. The voltage of the circuit is 440 V.**

b. What is the value of the capacitance which must be introduced in each phase so that the whole of the power will appear on wattmeter *A*. The frequency of supply is 50 Hz.

Answer

a. 1. When both meters read direct :

i. $$W_2 = 6000 \text{ W and } W_1 = 2000 \text{ W}$$

ii. $$\text{Total power} = W_2 + W_1 = 6000 + 2000 = 8000 \text{ W}$$

iii. $$\tan\theta = \sqrt{3}\left(\frac{W_2 - W_1}{W_2 + W_1}\right) = \sqrt{3}\left(\frac{6000 - 2000}{6000 + 2000}\right) = 0.866$$

$$\theta = \tan^{-1}(0.866) = 40.9°$$

iv. $\therefore$ $$\text{Power factor} = \cos(40.9°) = 0.756$$

2. When the second meters *B* reads in reverse :

i. $$W_2 = 6000 \text{ W and } W_1 = 2000 \text{ W}$$

ii. $$\text{Total power} = W_2 + (-W_1) = 6000 - 2000 = 4000 \text{ W}$$

iii. $$\tan\theta = \sqrt{3}\left(\frac{W_2 - W_1}{W_2 + W_1}\right) = \sqrt{3}\left(\frac{6000 - (-2000)}{6000 - 2000}\right) = 3.464$$

$$\theta = \tan^{-1}(3.464) = 73.9°$$

$$\text{Power factor} = \cos(73.9°) = 0.277 \text{ (lagging)}$$

b. Capacitance *C* to be introduced in each phase :

i. The reading of wattmeter *B* will be zero, and whole of the power would appear on wattmeter *A* when power factor $\cos\theta = 0.5$ or $\theta = 60°$. Therefore, the power factor is to be improved from 0.277 to 0.5.

ii. Load current per phase,

$$I_{ph} = I_L = \frac{P}{\sqrt{3}\, V_L \cos\theta} = \frac{4000}{\sqrt{3} \times 440 \times 0.277} = 18.95 \text{ A}$$

iii. Load impedance/phase, $Z_{ph} = \dfrac{V_{ph}}{I_{ph}} = \dfrac{440/\sqrt{3}}{18.95} = 13.4\ \Omega$

iv. Load resistance/phase, $R_{ph} = Z_{ph} \cos\theta = 13.4 \times 0.277 = 3.71\ \Omega$

v. Load reactance/phase, $X_{ph} = Z_{ph} \sin\theta = 13.4 \times \sin(\cos^{-1} 0.277) = 12.87\ \Omega$

vi. Since there is no change of resistance, the reactance per phase when $\theta = 60°$ is given by

$$X_{ph}' = R_{ph} \tan 60° = 3.71 \times \sqrt{3} = 6.426\ \Omega$$

vii. Capacitive reactance introduced in each phase

$$X_C = X_{ph} - X_{ph}' = 12.87 - 6.426 = 6.44\ \Omega$$

viii. Capacitance introduced in each phase,

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 6.426} = 495.35\ \mu\text{F}$$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Discuss waveform symmetries.

Ans. Refer Q. 3.2.

Q. 2. Obtain the exponential Fourier series for the waveform shown in Fig. 1. Also draw the frequency spectrum.

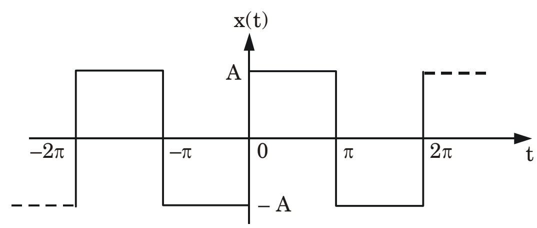


Fig. 1.

Ans. Refer Q. 3.6.

Q. 3. Determining the exponential Fourier series for the waveform shown in Fig. 2.

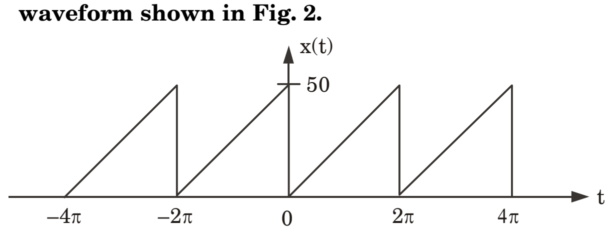


Fig. 2.

Ans. Refer Q. 3.7.

Q. 4. Derive expression of effective values of fourier-series.

Ans. Refer Q. 3.8.

Q. 5. For the circuit of Fig. 3(*a*), determine the periodic response *i*(*t*) corresponding to the forcing function shown in Fig. 3(*b*) if *i*(0) = 0.

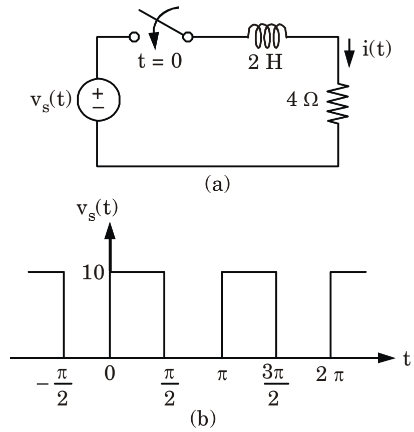


Fig. 3.

Ans. Refer Q. 3.12.

Q. 6. Derive the expression of Fourier transform from Fourier series.

Ans. Refer Q. 3.14.

Q. 7. What are the different properties of Fourier transform ?

Ans. Refer Q. 3.16.

Q. 8. Discuss three phase unbalanced circuits.

Ans. Refer Q. 3.19.

Q. 9. Discuss two wattmeter method of power calculation in three phase system.

Ans. Refer Q. 3.25.

☺☺☺

4
UNIT

Laplace Transform

CONTENTS

PART-1

Laplace Transform and Properties.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.1. **What do you understand by "Laplace Transform" ? Also mention its advantages and disadvantages. Enlist applications of Laplace transform.**

Answer

Laplace Transform :

1. Laplace transform of a time function $f(t)$ is defined as :

$$F(s) = L[f(t)] = \int_{-\infty}^{\infty} f(t)e^{-st}dt \quad \text{...(4.1.1)}$$

where, s is a complex variable and is equal to $\sigma + j\omega$.

2. The Laplace transform as defined in eq. (4.1.1) with $-\infty$ as the lower limit for the integral is called the 'two-sided' or 'bilateral' Laplace transform.
3. If the lower limit is changed to 0, we get the 'one-sided' or 'unilateral' Laplace transform.
4. Hence, we define the one-sided Laplace transform as :

$$F(s) = L[f(t)] = \int_{0}^{\infty} f(t)e^{-st}dt$$

Advantages :

1. Laplace transform represents continuous time signals in terms of complex exponentials, *i.e.*, e^{-st}. Hence Laplace transform can be used to analyze the signals or functions which are not absolutely integrable.
2. Continuous time systems are also analyzed more effectively using Laplace transform. Laplace transform can be applied to the analysis of unstable systems also.

Disadvantages :

1. The integral representation of Laplace domain is complicated.
2. Unsuitable for the purpose of data processing in random vibrations.

Applications of Laplace Transform :

1. It is a technique mainly utilized in engineering purposes for system modeling in which a large differential equation must be solved.
2. It can also be used to solve differential equations and is used extensively in electrical engineering.
3. It is used in electrical circuits for the analysis of linear time-invariant systems.

Que 4.2. Write properties of Laplace transform.

Answer

i. Multiplication by a constant :

Let $F(s) \rightarrow$ Laplace Transform of $f(t)$

$k \rightarrow$ Multiplication factor

$$L\{k\, f(t)\} = kF(s)$$

ii. Sum and Difference :

Let $F_1(s) \rightarrow$ Laplace transform of $f_1(t)$

$F_2(s) \rightarrow$ Laplace transform of $f_2(t)$

$$L\{f_1(t) \pm f_2(t)\} = F_1(s) \pm F_2(s)$$

iii. Time Shifting :

If $F(s) \rightarrow$ Laplace transform of $f(t)$

Then for $t_0 \geq 0$

$$L\{f(t - t_0)\} = F(s)\; e^{-st_0}$$

iv. Frequency Shifting :

If $F(s) \rightarrow$ Laplace transform of $f(t)$

Then $L\{f(t)\; e^{s_0 t}\} = F(s - s_0)$

v. Time Differentiation Property :

If $F(s) \rightarrow$ Laplace transform of $f(t)$

Then $$L\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0^-)$$

and $$L\left\{\frac{d^n f(t)}{dt^n}\right\} = s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) - \ldots - f^{n-1}(0^-)$$

vi. Time Integration Property :

If $F(s) \rightarrow$ Laplace transform of $f(t)$

$$L\left\{\int_0^t f(\tau)\, d\tau\right\} = \frac{F(s)}{s}$$

vii. Frequency Differentiation :

$$L\{t\, f(t)\} = -\frac{d\, F(s)}{ds}$$

viii. Frequency Integration :

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(s)\, ds$$

ix. Scaling :

$$L\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$$

Que 4.3. **Write the Laplace transforms of :**

i. Unit impulse
ii. Unit step
iii. Unit ramp and
iv. Parabolic functions.

Answer

i. Unit impulse :

$$f(t) = \delta(t)$$

$$L\{f(t)\} = F(s) = \int_0^{\infty} \delta(t)\, e^{-st}\, dt$$

$$F(s) = e^{-st}\big|_{t=0} = 1 \qquad [\because\ \delta(t) = 1 \text{ only } t = 0]$$

ii. Unit Step :

$$f(t) = u(t)$$

$$L\{f(t)\} = F(s) = \int_0^{\infty} u(t)\, e^{-st}\, dt$$

$$= \int_0^{\infty} e^{-st}\, dt \qquad [\because\ u(t) = 1 \text{ for } t > 0]$$

$$= \left.\frac{e^{-st}}{-s}\right|_{t=0}^{t=\infty}$$

$$F(s) = \frac{1}{s}$$

iii. Unit ramp :

$$f(t) = r(t) = t\, u(t)$$

$$F(s) = L\{t\, u(t)\} = \int_0^{\infty} t\, e^{-st}\, dt = \left[t \left.\frac{e^{-st}}{-s}\right|_0^{\infty} - \int_0^{\infty} 1\, \frac{e^{-st}}{-s}\, dt \right]$$

$$= [0 - 0] + \frac{1}{s} \int_0^{\infty} e^{-st}\, dt = -\frac{1}{s^2}\, e^{-st}\Big|_0^{\infty} = \frac{1}{s^2}$$

iv. Parabolic function :

$$f(t) = kt^2\, u(t)$$

$$F(s) = L\{kt^2\, u(t)\} = \int_0^{\infty} k\, t^2\, e^{-st}\, dt$$

$$= k\left[t^2 \left.\frac{e^{-st}}{-s}\right|_0^{\infty} - \int_0^{\infty} 2t\, \frac{e^{-st}}{-s}\, dt \right]$$

$$= k[0-0] + \frac{2k}{s}\int_0^{\infty} t\, e^{-st}\, dt$$

$$= \frac{2k}{s}\left[\left.\frac{t\, e^{-st}}{-s}\right|_0^{\infty} - \int_0^{\infty} 1\, \frac{e^{-st}}{-s}\, dt \right]$$

$$= \frac{2k}{s}\,[0-0] + \frac{2k}{s^2}\int_0^{\infty} e^{-st}\, dt$$

$$= -\frac{2k}{s^2}\left.\frac{e^{-st}}{s}\right|_0^{\infty} = \frac{2k}{s^3}$$

Que 4.4. **Find Laplace transform of following functions :**

i. e^{at} **ii.** $u(t)$
iii. $\sin \omega t$ **iv.** $\cos \omega t$
v. $\sinh at$ **vi.** $\cosh at$
vii. $e^{-at} \sin \omega t$ **viii.** $e^{-at} \cos \omega t$
ix. $e^{-at} \cosh bt$ **x.** $e^{-at} \sinh bt$
xi. t^n **xii.** $Ae^{-at} \sin (bt + \theta)$

Answer

i.
$$L\{e^{at}\} = \int_0^{\infty} e^{at} e^{-st} dt = \int_0^{\infty} e^{-(s-a)t} dt = \frac{1}{s-a}$$

ii.
$$L\{u(t)\} = \int_0^{\infty} u(t)\, e^{-st} dt = \int_0^{\infty} e^{-st} dt = \frac{1}{s}$$

iii.
$$L\{\sin \omega t\} = \frac{1}{2j} L\left\{e^{j\omega t} - e^{-j\omega t}\right\}$$

$$= \frac{1}{2j}\left(\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right) = \frac{\omega}{s^2+\omega^2}$$

iv.
$$L\{\cos \omega t\} = \frac{1}{2} L\left\{e^{j\omega t} + e^{-j\omega t}\right\}$$

$$= \frac{1}{2}\left(\frac{1}{s-j\omega} + \frac{1}{s+j\omega}\right) = \frac{s}{s^2+\omega^2}$$

v. $$L\{\sinh at\} = \frac{1}{2}L\left[e^{at} - e^{-at}\right] = \frac{1}{2}\left[L\{e^{at}\} - L\{e^{-at}\}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right] = \frac{a}{s^2 - a^2}$$

vi. $$L\{\cosh at\} = \frac{1}{2}L\left[e^{at} + e^{-at}\right] = \frac{1}{2}\left[L\left\{e^{at}\right\} + L\left\{e^{-at}\right\}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-a} + \frac{1}{s+a}\right] = \frac{s}{s^2 - a^2}$$

vii. $$L\left\{e^{-at}\sin \omega t\right\} = L\left[e^{-at}\left\{\frac{1}{2j}(e^{j\omega t} - e^{-j\omega t})\right\}\right]$$

$$= L\left[\frac{1}{2j}\left[e^{-(a-j\omega)t} - e^{-(a+j\omega)t}\right]\right]$$

$$= \frac{1}{2j}\left[\frac{1}{(s+a)-j\omega} - \frac{1}{(s+a)+j\omega}\right] = \frac{\omega}{(s+a)^2 + \omega^2}$$

viii. $$L\left\{e^{-at}\cos \omega t\right\} = \frac{1}{2}L\left[\left\{e^{-(a-j\omega)t} + e^{-(a+j\omega)t}\right\}\right]$$

$$= \frac{1}{2}\left[\frac{1}{(s+a)-j\omega} + \frac{1}{(s+a)+j\omega}\right] = \frac{s+a}{(s+a)^2 + \omega^2}$$

ix. $$L\left\{e^{-at}\cosh bt\right\} = L\left[e^{-at}\left(\frac{e^{bt} + e^{-bt}}{2}\right)\right]$$

$$= \frac{1}{2}L\left[e^{(b-a)t} + e^{-(b+a)t}\right] = \frac{1}{2}L\left[e^{-(a-b)t} + e^{-(a+b)t}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s+(a-b)} + \frac{1}{s+(a+b)}\right] = \frac{s+a}{(s+a)^2 - b^2}$$

x. $$L\{e^{-at}\sinh bt\} = L\left[e^{-at}\left(\frac{e^{bt} - e^{-bt}}{2}\right)\right] = \frac{1}{2}L\left[e^{-(a-b)t} - e^{-(a+b)t}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s+(a-b)} - \frac{1}{s+(a+b)}\right] = \frac{b}{(s+a)^2 - b^2}$$

xi. $$L[t^n] = \int_0^\infty (t^n)\, e^{-st} dt$$

Integrating by parts

Let
$$u = t^n$$
$$dv = e^{-st}\, dt$$
$$du = nt^{n-1}$$
$$v = \int e^{-st}\, dt = \frac{-e^{-st}}{s}$$

$$L[t^n] = \int_0^\infty u\,dv = uv\Big|_0^\infty - \int_0^\infty v\,du$$

$$= -\frac{t^n}{s}\left[e^{-st}\right]_0^\infty + \frac{n}{s}\int_0^\infty t^{n-1}e^{-st}dt$$

$$= \frac{n}{s}\int_0^\infty t^{n-1}e^{-st}dt = \frac{n}{s}L\left[t^{n-1}\right] = \frac{n}{s}\left(\frac{n-1}{s}\right)L\left[t^{n-2}\right]$$

$$= \frac{n}{s}\times\frac{(n-1)}{s}\times\frac{(n-2)}{s}.....\frac{1}{s}L\left[t^0\right]$$

$$= \frac{\lfloor n}{s^n}L\left[u(t)\right] = \frac{\lfloor n}{s^{n+1}}$$

xii. $L[Ae^{-at}\sin(bt+\theta)]$

As, $L[A\sin(bt+\theta)] = A\,L\left[\sin bt\cos\theta + \sin\theta\cos bt\right]$

$$= A\left[\frac{b\cos\theta}{s^2+b^2} + \frac{s\sin\theta}{s^2+b^2}\right]$$

So, $L\left[Ae^{-at}\sin(bt+\theta)\right] = A\dfrac{b\cos\theta}{(s+a)^2+b^2} + A\dfrac{(s+a)\sin\theta}{(s+a)^2+b^2}$

$$= A\left[\frac{b\cos\theta + (s+a)\sin\theta}{(s+a)^2+b^2}\right]$$

Que 4.5. **What is inverse Laplace transform ? Calculate inverse Laplace Transform of e^{-5s} $U(s)$.**

Answer

Inverse Laplace Transform :

It is used to convert frequency-domain signal $F(s)$ to the time-domain signal $f(t)$. It is given as,

$$f(t) = L^{-1}[F(s)]$$

$$f(t) = \frac{1}{2\pi}\int_{\sigma-j\omega}^{\sigma+j\omega} F(s)e^{st}ds$$

Numerical :

$$f(t) = \frac{1}{2\pi}\int_0^\infty e^{-5s}e^{st}\,ds = \frac{1}{2\pi}\int_0^\infty e^{(-5+t)s}\,ds$$

$$= \frac{1}{2\pi}\left[\frac{e^{(-5+t)s}}{t-5}\right]_0^\infty = \frac{1}{2\pi}\left(\frac{1}{5-t}\right)$$

$$f(t) = \frac{1}{2\pi(5-t)}$$

PART-2

Partial Fractions.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.6. **Discuss the different methods of partial fractions.**

Answer

Real roots :

1. Consider the function

$$F(s) = \frac{N(s)}{(s - s_0)(s - s_1)(s - s_2)} \quad ...(4.6.1)$$

where s_0, s_1, and s_2 are distinct, real roots, and the degree of $N(s) < 3$.

2. Expanding $F(s)$ we have

$$F(s) = \frac{K_0}{s - s_0} + \frac{K_1}{s - s_1} + \frac{K_2}{s - s_2} \quad ...(4.6.2)$$

3. Let us first obtain the constant K_0. We proceed by multiplying both sides of the equation by $(s - s_0)$ to give

$$(s - s_0)F(s) = K_0 + \frac{(s - s_0)K_1}{s - s_1} + \frac{(s - s_0)K_2}{s - s_2} \quad ...(4.6.3)$$

4. If we let $s = s_0$ in Eq. (4.6.3), we obtain

$$K_0 = (s - s_0)\, F(s)\big|_{s = s_0} \quad ...(4.6.4)$$

5. Similarly, we see that the other constants can be evaluated through the general relation

$$K_i = (s - s_i)\, F(s)\big|_{s = s_i} \quad ...(4.6.5)$$

Complex roots :

1. Suppose $F(s)$ is given by

$$F(s) = \frac{N(s)}{D_1(s)(s - \alpha - j\beta)(s - \alpha + j\beta)}$$

$$= \frac{K_1}{s - \alpha - j\beta} + \frac{K_2}{s - \alpha + j\beta} + \frac{N_1(s)}{D_1(s)} \quad ...(4.6.6)$$

where N_1 / D_1 is the reminder term.

2. Using eq.(4.6.5), we have

$$K_1 = \frac{N(\alpha + j\beta)}{2j\beta D_1(\alpha + j\beta)}$$

$$K_2 = \frac{N(\alpha - j\beta)}{-2j\beta D_1(\alpha - j\beta)}$$

where we assume that $s = \alpha \pm j\beta$ are not zeros of $D_1(s)$.

3. It can be shown that the constants K_1 and K_2 associated with conjugate roots are themselves conjugate. Therefore, if we denote K_1 as

$$K_1 = A + jB \quad ...(4.6.7)$$

then $$K_2 = A - jB = K_1^* \quad ...(4.6.8)$$

4. If we denote the inverse transform of the complex conjugate terms as $f_1(t)$, we see that

$$f_1(t) = L^{-1}\left[\frac{K_1}{s-\alpha-j\beta} + \frac{K_1^*}{s-\alpha+j\beta}\right]$$

$$= e^{\alpha t}(K_1 e^{j\beta t} + K_1^* e^{-j\beta t})$$

$$= 2e^{\alpha t}(A \cos \beta t - B \sin \beta t) \quad ...(4.6.9)$$

Multiple roots :

1. Suppose we are given the function

$$F(s) = \frac{N(s)}{(s - s_0)^n D_1(s)} \quad ...(4.6.10)$$

with multiple roots of degree n at $s = s_0$.

2. The partial fraction expansion of $F(s)$ is

$$F(s) = \frac{K_0}{(s-s_0)^n} + \frac{K_1}{(s-s_0)^{n-1}} + \frac{K_2}{(s-s_0)^{n-2}} + ... + \frac{K_{n-1}}{s-s_0} + \frac{N_1(s)}{D_1(s)} \quad ...(4.6.11)$$

where $N_1(s) / D_1(s)$ represents the remaining terms of the expansion.

3. $$K_0 = (s - s_0)^n F(s)\big|_{s=s_0} \quad ...(4.6.12)$$

4. $$K_1 = \frac{d}{ds} F_1(s)\bigg|_{s=s_0} \quad ...(4.6.13)$$

On the same basis

$$K_2 = \frac{1}{2}\frac{d^2}{ds^2} F_1(s)\bigg|_{s=s_0} \quad ...(4.6.14)$$

5. In general

$$K_j = \frac{1}{j!}\frac{d^j}{ds^j} F_1(s)\bigg|_{s=s_0} \; ; j = 0, 1, 2, ..., n-1 \quad ...(4.6.15)$$

Que 4.7. **Find the partial fraction expansion for**

$$\boldsymbol{F(s) = \frac{s^2 + 2s - 2}{s(s+2)(s-3)}}$$

Answer

1. $$F(s) = \frac{s^2 + 2s - 2}{s(s+2)(s-3)} = \frac{K_0}{s} + \frac{K_1}{s+2} + \frac{K_2}{s-3} \quad ...(4.7.1)$$

2. Using,

$$K_0 = sF(s)\big|_{s=0}$$

$$= \left.\frac{s^2 + 2s - 2}{(s+2)(s-3)}\right|_{s=0} = \frac{1}{3}$$

3. $$K_1 = \left.\frac{s^2 + 2s - 2}{s(s-3)}\right|_{s=-2} = -\frac{1}{5} \quad ...(4.7.2)$$

4. $$K_2 = \left.\frac{s^2 + 2s - 2}{s(s+2)}\right|_{s=3} = \frac{13}{15}$$

Que 4.8. **Find the inverse Laplace transform of the following function :**

$$\boldsymbol{F(s) = \frac{s+1}{s^3 + 4s^2 + 4s + 4}}$$

Answer

1. $$F(s) = \frac{s+1}{s^3 + 4s^2 + 6s + 4}$$

$$= \frac{s+1}{(s+2)(s^2 + 2s + 2)} = \frac{A}{s+2} + \frac{Bs + C}{s^2 + 2s + 2}$$

$$= \frac{A(s^2 + 2s + 2) + (Bs + C)(s+2)}{(s+2)(s^2 + 2s + 2)}$$

$$= \frac{(A + B)s^2 + (2A + 2B + C)s + 2A + 2C}{(s+2)(s^2 + 2s + 2)}$$

Comparing the numerators of LHS and RHS, we get

$A + B = 0 \therefore A = -B$

$2A + 2B + C = 1 \therefore C = 1$

$2A + 2C = 1 \therefore 2A = 1 - 2 = -1$ or $A = -1/2 \therefore B = 1/2$

$$\therefore \quad X(s) = \frac{-\frac{1}{2}}{s+2} + \frac{\frac{1}{2}s + 1}{s^2 + 2s + 2} = \frac{-\frac{1}{2}}{s+2} + \frac{\frac{1}{2}(s+1) + \frac{1}{2}}{(s+1)^2 + 1^2}$$

$$= \frac{-1/2}{s+2} + \frac{1}{2}\frac{(s+1)}{(s+1)^2+1^2} + \frac{1}{2}\frac{1}{(s+1)^2+1^2}$$

Taking inverse Laplace transform on both sides, we get

$$f(t) = -\frac{1}{2}e^{-2t}u(t) + \frac{1}{2}e^{-t}\cos tu(t) + \frac{1}{2}e^{-t}\sin tu(t)$$

Que 4.9. **Consider the function**

$$\boldsymbol{F(s) = \frac{s-2}{s(s+1)^3}}$$

Find the partial fraction expression.

Answer

1. We represent in expanded form as

$$F(s) = \frac{K_0}{(s+1)^3} + \frac{K_1}{(s+1)^2} + \frac{K_2}{s+1} + \frac{A}{s}$$

2. The constant A for the simple root at $s = 0$ is

$$A = s\,F(s)\big|_{s=0} = -2$$

3. To obtain the constants for the multiple roots we first find $F_1(s)$.

$$F_1(s) = (s+1)^3\,F(s) = \frac{s-2}{s}$$

4. Using the general formula for the multiple root expansion, we obtain

$$K_0 = \frac{1}{0!}\frac{d^0}{ds^0}\left(\frac{s-2}{s}\right)\bigg|_{s=-1} = 3$$

$$K_1 = \frac{1}{1!}\frac{d}{ds}\left(\frac{s-2}{s}\right)\bigg|_{s=-1} = \frac{2}{s^2}\bigg|_{s=-1} = 2$$

$$K_2 = \frac{1}{2!}\frac{d}{ds}\left(\frac{2}{s^2}\right)\bigg|_{s=-1} = \left(-\frac{2}{s^3}\right)\bigg|_{s=-1} = 2$$

so that

$$F(s) = \frac{3}{(s+1)^3} + \frac{2}{(s+1)^2} + \frac{2}{s+1} - \frac{2}{s}$$

Que 4.10. **Find the inverse Laplace transform of the following function :**

$$\boldsymbol{F(s) = \frac{1+e^{-2s}}{s^2\,(s+1)}}$$

Answer

1. $$F(s) = \frac{1+e^{-2s}}{s^2\,(s+1)} = \frac{1}{s^2\,(s+1)} + \frac{e^{-2s}}{s^2\,(s+1)}$$

$$= F_1(s) + F_2(s) \quad \ldots(4.10.1)$$

$$F_1(s) = \frac{1}{s^2 (s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1}$$

$$C = (s+1) F_1(s)\Big|_{s=-1} = (s+1) \frac{1}{s^2 (s+1)}\Big|_{s=-1}$$

$$= \frac{1}{s^2 (s+1)}\Big|_{s=-1}$$

$$= 1$$

$$B = s^2 F_1(s)\Big|_{s=0} = s^2 \frac{1}{s^2 (s+1)}\Big|_{s=0} = \frac{1}{s+1}\Big|_{s=0}$$

$$= 1$$

$$A = \frac{1}{1!} \frac{d}{ds} [s^2 F_1(s)]\Big|_{s=0} = \frac{d}{ds}\left[s^2 \frac{1}{s^2(s+1)}\right]\Bigg|_{s=0}$$

$$= \frac{(s+1)\,0 - 1(1)}{(s+1)^2}\Big|_{s=0}$$

$$= -1$$

$$\therefore \quad F_1(s) = -\frac{1}{s} + \frac{1}{s^2} + \frac{1}{s+1} \quad \text{...(4.10.2)}$$

3. Taking inverse Laplace transform on both sides, of eq. (4.10.2) we get

$$f_1(t) = -u(t) + tu(t) + e^{-t} u(t) = [t - 1 + e^{-t}]\, u(t) \quad \text{...(4.10.3)}$$

4. Now, from the time shift theorem, we know that

$$f_2(t) = L^{-1}\left[\frac{e^{-2s}}{s^2 (s+1)}\right] = L^{-1}\left[\frac{1}{s^2 (s+1)}\right]\Bigg|_{t \to (t-2)}$$

$$= f_1(t)\ \Big|_{t \to t-2} = [t - 1 + e^{-t}]\Big|_{t \to (t-2)}$$

$$= [(t-3) + e^{-(t-2)}]\, u(t-2)$$

$$\therefore \quad f(t) = [t - 1 + e^{-t}]\, u(t) + [(t-3) + e^{-(t-2)}] u(t-2) \quad \text{...(4.10.4)}$$

PART-3

Singularity Functions, Waveform Synthesis.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.11. Discuss briefly singularity functions.

Answer

1. Certain special forcing functions which are discontinuous or have discontinuous derivatives are called singularity functions.
2. The commonly used singularity functions are :
   i. Step function
   ii. Ramp function
   iii. Impulse function

i. Unit step (Step function) : Signals which start at time $t = 0$ and have magnitude of unity are called unit step signals.

They are represented by a unit step function $u(t)$.

They are defined mathematically as :

$$u(t) = \begin{cases} 1\,; & t \geq 0 \\ 0\,; & t < 0 \end{cases}$$

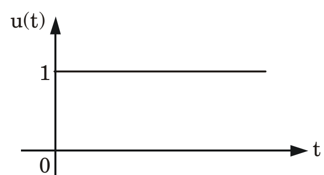


Fig. 4.11.1. Unit step.

ii. Unit ramp (Ramp function) : Signals which start from zero and are linear in nature with a constant slope m are called unit ramp signals.

They are represented by a unit ramp function $r(t)$.

They are defined mathematically as :

$$r(t) = \begin{cases} mt\,; & t \geq 0 \\ 0\,; & t < 0 \end{cases}$$

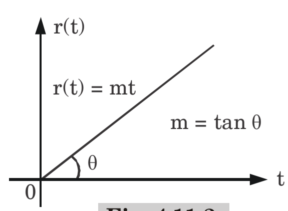


Fig. 4.11.2.

iii. Unit impulse (Impulse function) : Signals which act for very small time but have large amplitude are called unit impulse functions.

They are represented by $\delta(t)$.
They are defined mathematically as,

$$\delta(t) = \begin{cases} 0 ; & t \neq 0 \\ 1 ; & t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

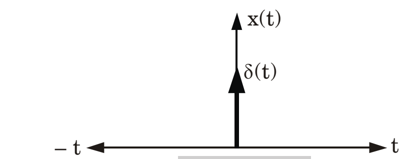


Fig. 4.11.3.

Que 4.12. **Discuss the basic operations applied on signals.**

Answer

i. Time shifting :

1. Mathematically, the time shifting of a continuous-time signal $x(t)$ can be represented by

$$y(t) = x(t - T) \quad ...(4.12.1)$$

2. The time shifting of a signal may result in time delay or time advance.
3. In eq. (4.12.1) if T is positive the sifting is to the right and then the shifting delays the signal, and if T is negative the shift is to the left and then the shifting advances the signal.
4. An arbitrary signal $x(t)$, its delayed version and advanced version are shown in Fig. 4.12.1(a), (b) and (c).

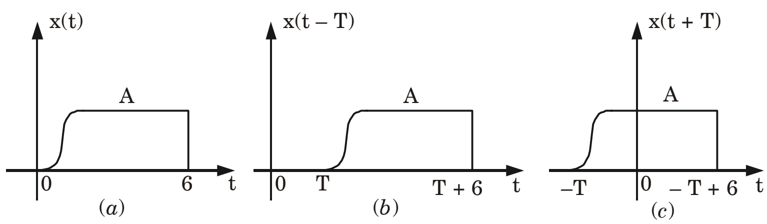


Fig. 4.12.1. (a) Signal, (b) Its delayed version, (c) Its time advanced version.

ii. Time reversal :

1. Time reversal, also called time folding of a signal $x(t)$ can be obtained by folding the signal about $t = 0$. This operation is very useful in convolution. It is denoted by $x(-t)$.
2. It is obtained by replacing the independent variable t by $(-t)$. Folding is also called as the reflection of the signal about the time origin $t = 0$.

3. Fig. 4.12.2(a) shows an arbitrary signal $x(t)$, and Fig. 4.12.2(b) shows its reflection $x(-t)$.

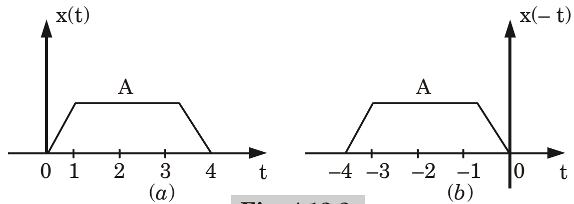


Fig. 4.12.2.

iii. Amplitude scaling :

1. The amplitude scaling of a continuous-time signal $x(t)$ can be represented by $y(t) = Ax(t)$
where A is a constant.

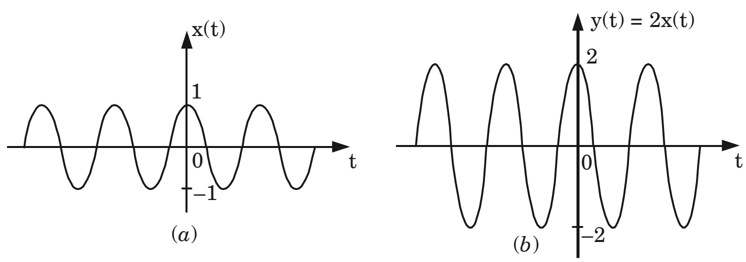


Fig. 4.12.3. Plots of (a) x(t) = cos ωt, (b) y(t) = 2x(t).

iv. Time scaling :

1. Time scaling may be time expansion or time compression.
2. The time scaling of a signal $x(t)$ can be accomplished by replacing t by at in it. Mathematically, it can be expressed as :
$y(t) = x(at)$
3. If $a > 1$, it results in time compression by a factor a and if $a < 1$, it results in time expansion by a factor a because with that transformation a point at at in signal $x(t)$ becomes a point at t in $y(t)$.

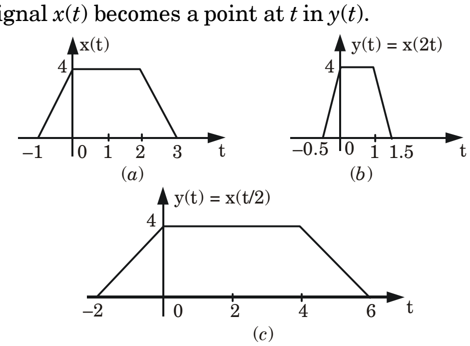


Fig. 4.12.4. (a) Original signal, (b) Compressed signal, (c) Enlarged signal.

v. Signal addition :

1. The sum of two continuous-time signals $x_1(t)$ and $x_2(t)$ can be obtained by adding their values at every instant of time.
2. Similarly, the subtraction of one continuous-time signal $x_2(t)$ from another signal $x_1(t)$ can be obtained by subtracting the value of $x_2(t)$ from that of $x_1(t)$ at every instant.
3. Consider two signals $x_1(t)$ and $x_2(t)$ shown in Fig. 4.12.5(*a*) and (*b*).

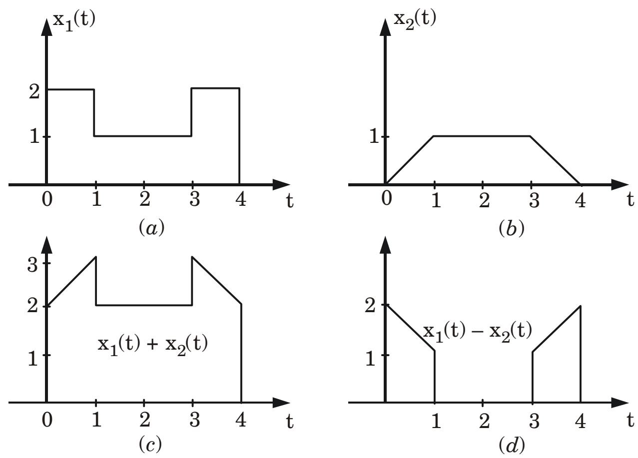


Fig. 4.12.5. Addition and subtraction of continuous-time signals.

4. Fig. 4.12.5(*c*) is the addition of $x_1(t)$ and $x_2(t)$ and Fig. 4.12.5(*d*) is the subtraction of $x_2(t)$ from $x_1(t)$.

vi. Signal multiplication :

1. The multiplication of two continuous-time signals can be performed by multiplying their values at every instant.
2. Two continuous-time signals $x_1(t)$ and $x_2(t)$ shown in Fig. 4.12.6(*a*) and (*b*) are multiplied to obtain $x_1(t)\, x_2(t)$ shown in Fig. 4.12.6(*c*).

   For $0 \le t \le 1$ $\quad x_1(t) = 2$ and $x_2(t) = 1$

   Hence $\quad x_1(t)\, x_2(t) = 2 \times 1 = 2$

   For $1 \le t \le 2$ $\quad x_1(t) = 1$ and $x_2(t) = 1 + (t - 1)$

   Hence $\quad x_1(t)\, x_2(t) = (1)\,[1 + (t - 1)] = 1 + (t - 1)$

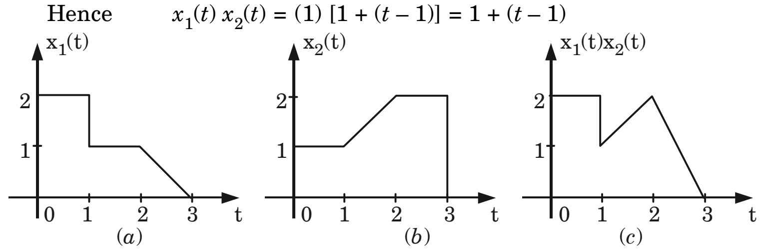


Fig. 4.12.6. Multiplication of continuous-time signals.

Que 4.13. **Synthesize a triangular waveform shown in Fig. 4.13.1 in terms of ramp and step signals.**

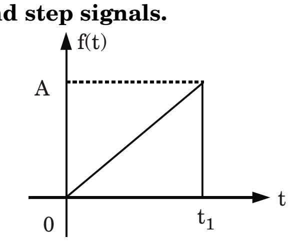


Fig. 4.13.1.

Answer

1. The waveform can be synthesized into three following waveforms as shown in Fig. 4.13.2.

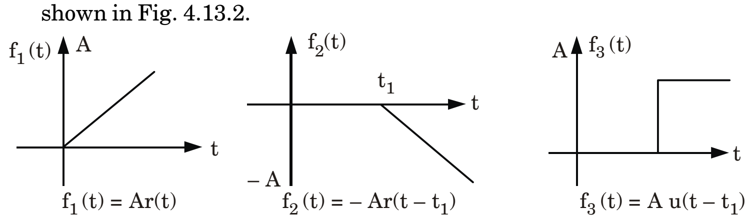


Fig. 4.13.2.

2.
$$\begin{aligned} f(t) &= f_1(t) + f_2(t) + f_3(t) \\ &= Ar(t) - Ar(t - t_1) - Au(t - t_1) \\ &= At\,u(t) - A(t - t_1)\,u(t - t_1) - Au\,(t - t_1) \\ &= At\,u(t) - Au(t - t_1)\,\{t - t_1 + 1\} \\ &= A\,[t\,u\,(t) - (t - t_1 + 1)\,u(t - t_1)] \end{aligned}$$

Que 4.14. **Find the Laplace transform of the waveform shown in Fig. 4.14.1.**

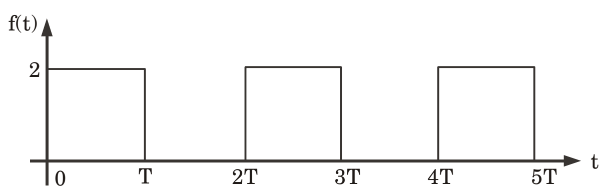


Fig. 4.14.1.

Answer

1. The waveform in Fig. 4.14.1 can be synthesized as shown in Fig. 4.14.2.

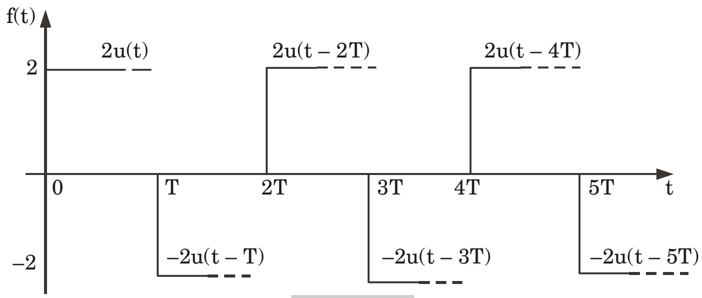


Fig. 4.14.2.

2. From Fig. 4.16.2, waveform can be expressed mathematically in terms of singular functions as

   $f(t) = 2u(t) - 2u(t - T) + 2u(t - 2T) - 2u(t - 3T) + 2u(t - 4T) - 2u(t - 5T)$ + ...

3. We know that,

$$L[u(t)] = \frac{1}{s} \text{ and } L[u(t - t_0)] = \frac{e^{-t_0 s}}{s}. \text{ Therefore,}$$

$$F(s) = \frac{2}{s} - 2\frac{e^{-Ts}}{s} + 2\frac{e^{-2Ts}}{s} - 2\frac{e^{-3Ts}}{s} + 2\frac{e^{-4Ts}}{s} - 2\frac{e^{-5Ts}}{s}$$

$$= \frac{2}{s}\left[1 - e^{-Ts} + e^{-2Ts} - e^{-3Ts} + e^{-4Ts} - e^{-5Ts} + ...\right]$$

$$= \frac{2}{s}\left[1 + e^{-Ts}\right]^{-1} = \frac{2}{s}\left[\frac{1}{1 + e^{-Ts}}\right]$$

Que 4.15. **Find the Laplace transform of the waveform shown in Fig. 4.15.1.**

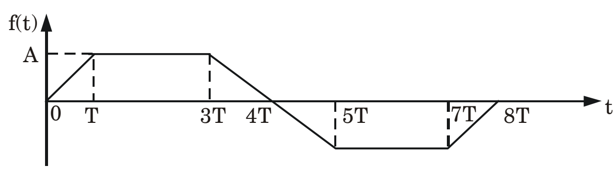


Fig. 4.15.1.

Answer

1. The waveform in Fig. 4.15.1 can be synthesized as shown in Fig. 4.15.2.

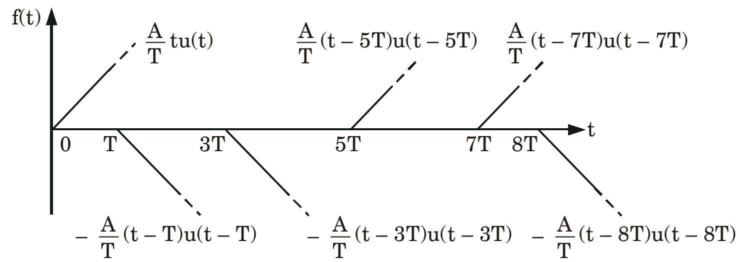


Fig. 4.15.2.

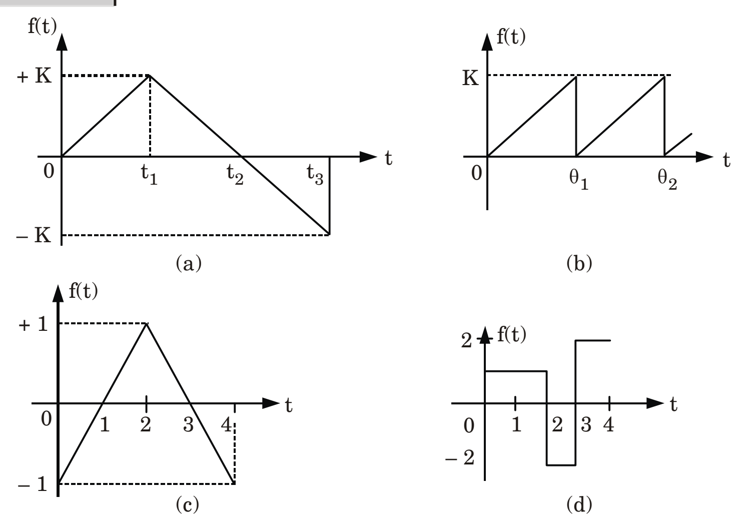
2. Waveform can be expressed mathematically in terms of singular functions as

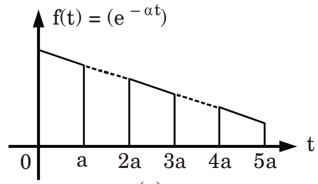
$$f(t) = \frac{A}{T}tu(t) - \frac{A}{T}(t-T)u(t-T) - \frac{A}{T}(t-3T)u(t-3T) + \frac{A}{T}(t-5T)u(t-5T) + \frac{A}{T}(t-7T)u(t-7T) - \frac{A}{T}(t-8T)u(t-8T)$$

$$\therefore \quad f(s) = \frac{A}{T}\left[\frac{1}{s^2} - \frac{e^{-Ts}}{s^2} - \frac{e^{-3Ts}}{s^2} + \frac{e^{-5Ts}}{s^2} + \frac{e^{-7Ts}}{s^2} - \frac{e^{-8Ts}}{s^2}\right]$$

$$= \frac{A}{Ts^2}[1 - e^{-Ts} - e^{-3Ts} + e^{-5Ts} + e^{-7Ts} - e^{-8Ts}]$$

Que 4.16. **Express the following waveforms by standard signals :**





(e)

Fig. 4.16.1.

Answer

i. $f(t) = Kt\,u(t) - 2K(t - t_1)\,u(t - t_1) + K(t - t_2)\,u(t - t_2) - K(t - t_2)$
$[-u(t - t_2) + u(t - t_3)]$

ii. $f(t) = t\,[u(t) - u(t - \theta_1)] + (t - \theta_1)\,[u(t - \theta_1) - u(t - \theta_2)]$
$= tu(t) - \theta_1\,u(t - \theta_1) - (t - \theta_1)\,u(t - \theta_2)$

iii. $f(t) = (t - 1)\,[u(t) - u(t - 2)] + (-1)(t - 3)\,[u(t - 2) - u(t - 4)]$
$= (t - 1)u(t) - 2(t - 2)u(t - 2) + (t - 3)\,u(t - 4)$

iv. $f(t) = 1\,[u(t) - u(t - 2)] + (-2)\,[u(t - 2) - u(t - 3)] + 2\,[u(t - 3)]$
$= u(t) - 3u(t - 2) + 4u(t - 3)$

v. $f(t) = e^{-\alpha t}\,[u(t) - u(t - a)] + e^{-\alpha t}\,(u(t - 2a) - u(t - 3a)]$
$+ e^{-\alpha t}\,[u(t - 4a) - u(t - 5a)]$.

PART-4

Analysis of RC, RL and RLC Network with and Without Initial Condition with Laplace Transform.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.17. **Discuss step response of series RL network.**

Answer

1. Fig. 4.17.1 shows a network consisting of resistance (R) and inductance (L). They are connected in series. It is connected to a DC supply E_0.
2. The network connected with a switch as shown in Fig. 4.17.1. The switch is closed at $t = 0$.

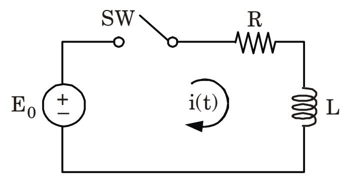


Fig. 4.17.1.

3. Using loop equations we get

$$Ri\,(t) + L\,\frac{di(t)}{dt} = E_0 \qquad ...(4.17.1)$$

4. Taking Laplace transform of both sides

$$L\;[sI\,(s) - i(0^-)] + RI(s) = \frac{E_0}{s} \qquad ...(4.17.2)$$

5. The current through an inductor cannot change instantaneously, and the current before $t = 0$ is zero.

$$\therefore \qquad i(0^-) = 0 \qquad ...(4.17.3)$$

6. Using the conditions of eq. (4.17.3) in eq. (4.17.2), we get

$$Ls\,I\,(s) + RI(s) = \frac{E_0}{s}$$

$$(sL + R)I(s) = \frac{E_0}{s}$$

$$I(s) = \frac{E_0}{s\,(sL + R)} = \frac{E_0}{L}\,\frac{1}{s\left(s + \frac{R}{L}\right)} \qquad ...(4.17.4)$$

7. Putting $R/L = \alpha$, we get from eq. (4.17.4)

$$I(s) = \frac{E_0}{L}\,\frac{1}{s(s+\alpha)}$$

$$= \frac{E_0}{L}\,\frac{1}{\alpha}\left\{\frac{1}{s} - \frac{1}{s+\alpha}\right\} \qquad ...(4.17.5)$$

8. Taking inverse Laplace transform of both sides of eq. (4.17.5), we get

$$i(t) = \frac{E_0}{L} \times \frac{L}{R}\,\{1 - e^{-\alpha t}\}$$

$$i(t) = \frac{E_0}{R}\left\{1 - e^{-\left(\frac{R}{L}\right)t}\right\} \qquad ...(4.17.6)$$

Eq. (4.17.6) is the complete solution of eq. (4.17.1).

9. It may be noted here that E_0/R is the steady state response while $\frac{E_0}{R}\,e^{-\left(\frac{R}{L}\right)t}$ is the transient response.

Que 4.18. **Determine the current $i(t)$ for $t \geq 0$ if initial current $i(0^-) = 2$ A for the circuit in Fig. 4.18.1.**

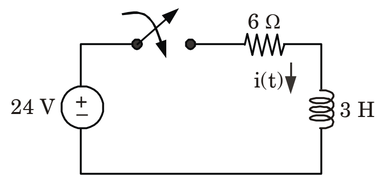


Fig. 4.18.1.

Answer

1. The transformed version of Fig. 4.18.1 is shown in Fig. 4.18.2.

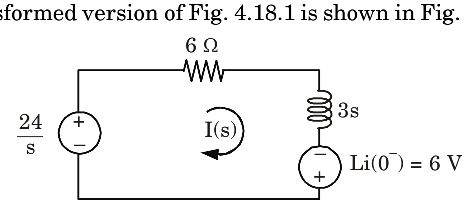


Fig. 4.18.2.

2. KVL around the single loop in Fig. 4.18.2, we have

$$\frac{24}{s} = 6I(\mathrm{s}) + 3sI(s) - 6$$

$$3I(s)[s+2] = \frac{24}{s} + 6 = \frac{6s+24}{s}$$

$$I(s) = \frac{6[s+4]}{3s\,(s+2)} = \frac{2(s+4)}{s(s+2)} = \frac{4}{s} - \frac{2}{s+2}$$

3. Taking inverse Laplace transform, we get

$$i(t) = (4 - 2e^{-2t})u(t)$$

Que 4.19. **A series *RL* circuit with $R = 20\ \Omega$ and $L = 10$ H has a constant voltage $V = 40$ V applied at $t = 0$ as shown in Fig. 4.19.1. Determine the current i, the voltage across resistor and the voltage across the inductor.**

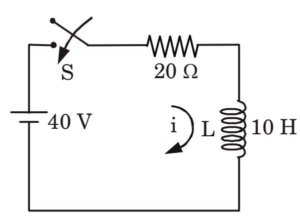


Fig. 4.19.1.

Answer

1. By applying KVL

$$10\frac{di}{dt} + 20\,i = 40$$

$$\frac{di}{dt} + 2i = 4 \qquad ...(4.19.1)$$

2. The general solution for a linear differential equation is

$$i = ce^{-pt} + e^{-pt}\int ke^{pt}dt$$

where $p = 2, k = 4$

$\therefore$ $i = ce^{-2t} + 2 \qquad ...(4.19.2)$

3. At $t = 0$ the switch is closed. Since the inductor never allows to a sudden change in current,

so at $t = 0^+, i = 0$

$\therefore$ $0 = c + 2$

$c = -2$

4. Substituting the value of c in the current eq. (4.19.2), we have

$$i = 2\,(1 - e^{-2t})\text{ A}$$

5. Voltage across resistor

$$V_R = iR = 2\,(1 - e^{-2t}) \times 20 = 40\,(1 - e^{-2t})\text{ V}$$

6. Voltage across inductor

$$V_L = \frac{L\,di}{dt}$$

$$= 10 \times \frac{d}{dt}2(1 - e^{-2t}) = 20 \times 2e^{-2t}$$

$$= 40\,e^{-2t}\text{ V}$$

Que 4.20. **Explain step response of series RC circuit.**

Answer

1. Fig. 4.20.1 shows a network consisting of resistance (R) and capacitance (C). They are connected in series. It is connected to a DC supply E_0.

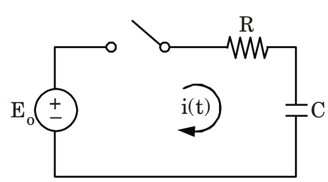


Fig. 4.20.1.

2. The network is connected to a switch as shown in Fig. 4.20.1. The switch was initially kept open. The switch is closed at $t = 0$.
3. Using loop equations, we get

$$Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = E_0$$

Applying Laplace transform on both sides

$$\frac{1}{C}\left[\frac{I(s)}{s} + \frac{q(0^-))}{s}\right] + RI(s) = \frac{E_0}{s} \qquad \text{...(4.20.1)}$$

4. Let $q(0^-))$ is charge on the capacitor just before closing the switch. If the capacitor was initially uncharged then $q(0^-)) = 0$.
5. Hence eq. (4.20.1) becomes

$$\frac{1}{C}\frac{I(s)}{s} + RI(s) = \frac{E_0}{s}$$

$$\therefore \quad I(s)\left\{\frac{1 + C\,Rs}{Cs}\right\} = \frac{E_0}{s}$$

$$I(s) = \frac{E_0\,C}{1 + C\,Rs} = \frac{E_0 / R}{s + \dfrac{1}{RC}} \qquad \text{...(4.20.2)}$$

6. Taking inverse Laplace transform of eq. (4.20.2)

$$i(t) = \frac{E_0}{R}\,e^{-\left(\frac{1}{RC}\right)t}$$

Que 4.21. **Determine the current $i(t)$ for $t \geq 0$ if $v_C(0) = 2$ V for the circuit shown in Fig. 4.21.1.**

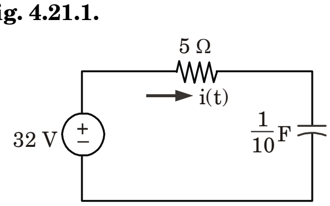


Fig. 4.21.1.

Answer

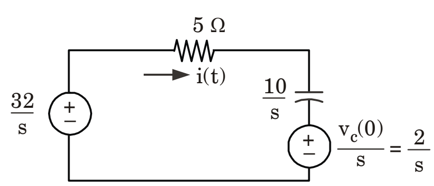


Fig. 4.21.2.

1. KVL around the single loop in Fig. 4.21.2, we have

$$\frac{32}{s} = 5I(s) + \frac{10}{s}I(s) + \frac{2}{s}$$

$$I(s)\left[5+\frac{10}{s}\right]=\frac{30}{s}$$

$$I(s)=\frac{30}{5s+10}=\frac{6}{s+2}$$

2. Taking inverse Laplace transform, we get

$$i(t)= 6\,e^{-2t}$$

Que 4.22. **Determine the voltage $v_C(t)$ and the current $i_c(t)$ for $t \geq 0$ for the circuit shown in Fig. 4.22.1.**

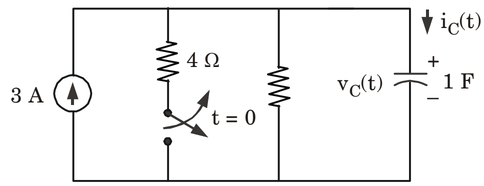


Fig. 4.22.1.

Answer

1. The switch is opened at $t = 0$. So at $t = 0^-$, the switch was closed.
2. Let us assume that steady state has been reached at $t = 0^-$. So we have to find the initial voltage across the capacitor.
3. For DC, the capacitor acts as open and the circuit shown in Fig. 4.22.2 results for $t = 0^-$.

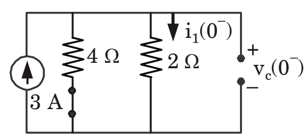


Fig. 4.22.2.

$$i_1(0^-) = \frac{3\times 4}{4+2} = 2\text{ A}$$

$\therefore$ $$v_0(0^-) = 2i_1(0^-) = 2 \times 2 = 4\text{ V}$$

4. Since the voltage across a capacitor cannot change instantaneously,

$$v_c(0^-) = v_c(0) = v_0(0^+) = 4\text{ V}$$

5. Fig. 4.22.3 represents frequency domain equivalent circuit.

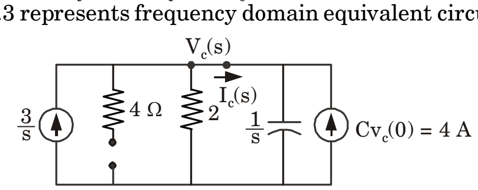


Fig. 4.22.3.

$$\frac{V_c(s)}{2} + \frac{V_c(s)}{1/s} = \frac{3}{s} + 4$$

$$V_c(s)\left[\frac{1}{2} + s\right] = \frac{3}{s} + 4$$

$$V_c(s) = \frac{3+4s}{s} \Big/ \frac{1+2s}{2} = \frac{2(3+4s)}{s(1+2s)} = \frac{3+4s}{s\left(s+\frac{1}{2}\right)}$$

6. Taking inverse Laplace transform, we

$$v_c(t) = \left[6 - 2e^{-\frac{1}{2}t}\right]u(t)\text{ V}$$

7. Also
$$I_c(s) = \frac{V_c(s)}{1/s} - 4$$

$$= \frac{\dfrac{3+4s}{s\left(s+\frac{1}{2}\right)}}{1/s} - 4 = \frac{3+4s}{s+\frac{1}{2}} - 4 = \frac{1}{s+\frac{1}{2}}$$

8. Taking inverse Laplace transform, we get

$$i_c(s) = e^{-\frac{1}{2}t}\,u(t)\text{ A}$$

Que 4.23. **Discuss step response of series RLC network.**

Answer

1. Fig. 4.23.1 shows a network consisting of resistance (R), capacitance (C) and inductance (L) are connected in series.
2. It is provided with a switch as shown in Fig. 4.23.1. Initially switch was kept open. Switch was closed at $t = 0$.

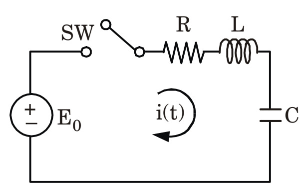


Fig. 4.23.1.

3. Applying loop equation, we get

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(t)dt = E_0 \qquad ...(4.23.1)$$

4. Taking Laplace transformation of both sides of eq. (4.23.1)

$$L\{sI(s) - i(0^+)\} + RI(s) + \frac{1}{C}\left\{\frac{I(s)}{s} + \frac{q(0^+)}{s}\right\} = \frac{E_0}{s} \quad ...(4.23.2)$$

5. At $t = 0^+$, inductance acts as previous current source and capacitance acts as previous voltage source.

$\therefore \quad i(0^+) = 0$ and $q(0^+) = 0$

6. Hence, eq. (4.23.2) reduces to

$$sL\,I(s) + RI(s) + \frac{I(s)}{Cs} = \frac{E_0}{s}$$

$$I(s)\left\{sL + R + \frac{1}{Cs}\right\} = \frac{E_0}{s}$$

$$I(s)\left\{s^2 L + Rs + \frac{1}{C}\right\} = E_0$$

7. If a and b are the roots of equation $s^2 L + Rs + 1 / C = 0$

We have $s^2L + Rs + 1 / C = (s - \alpha)(s - \beta)$

where
$$\alpha = \frac{-R + \sqrt{R^2 - 4\frac{L}{C}}}{2L}$$

$$\beta = \frac{-R - \sqrt{R^2 - 4\frac{L}{C}}}{2L}$$

8. $\therefore$
$$I(s) = \frac{E_0}{(s-\alpha)(s-\beta)}$$

$$= \frac{E_0}{(\alpha-\beta)}\left\{\frac{1}{(s-\alpha)} - \frac{1}{(s-\beta)}\right\} \quad ...(4.23.3)$$

9. Taking inverse Laplace transform of both sides, we get

$$i(t) = \frac{E_0}{\alpha - \beta}\{e^{\alpha t} - e^{\beta t}\}$$

Que 4.24. **Consider a series RL circuit shown in Fig. 4.24.1. The switch is closed at time $t = 0$, find the current $i(t)$ using Laplace transform.**

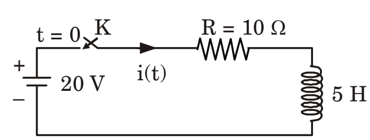


Fig. 4.24.1.

Answer

1. Initial condition, $i(0) = 0$

2. Using KVL,

$$R\,i(t) + 5\frac{di}{dt} = 20$$

3. Taking Laplace transform,

$$10I(s) + 5[sI(s) - i(0)] = \frac{20}{s}$$

$$10I(s) + 5sI(s) = \frac{20}{s} \qquad (\because i(0) = 0)$$

$$I(s) = \frac{20/s}{10+5s} = \frac{20}{s(5s+10)} = \frac{4}{s(s+2)}$$

4. Using partial fraction,

$$I(s) = \frac{2}{s} - \frac{2}{s+2}$$

5. Taking inverse Laplace,

$$i(t) = L^{-1}[I(s)] = L^{-1}\left[\frac{2}{s} - \frac{2}{s+2}\right]$$

$$= 2(1 - e^{-2t})$$

Que 4.25. **An R-L-C series circuit is as shown in Fig. 4.25.1. The switch is moved from position 1 to 2 at $t = 0$. Initially it remained in position 1 for a long time. The initial current at ($t = 0^-$) in the inductor is 2 A and the voltage across the capacitor at that instant is 4 volts. Find the expression for the inductor current $i(t)$ for $t > 0$.**

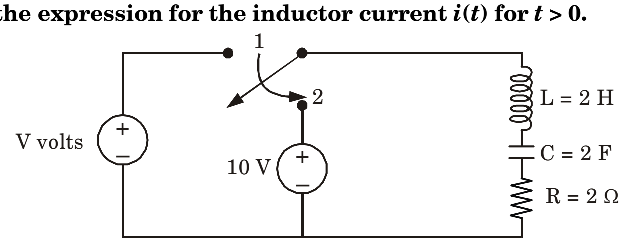


Fig. 4.25.1.

Answer

1. For switch position 2,

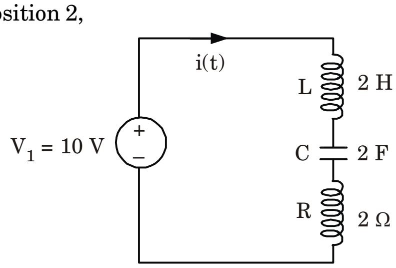


Fig. 4.25.2.

2. Using KVL

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,dt + V_C(0^-) = V_1$$

3. Taking Laplace transform,

$$RI(s) + L\,sI(s) - Li(0^-) + \frac{1}{C}\frac{I(s)}{s} + \frac{V_C(0^-)}{s} = \frac{V_1}{s}$$

$$\left[R + Ls + \frac{1}{Cs}\right]I(s) - Li(0^-) + \frac{V_C(0^-)}{s} = \frac{V_1}{s}$$

$$\left[2 + 2s + \frac{1}{2s}\right]I(s) = \frac{10}{s} + 4 - \frac{4}{s}$$

$$I(s) = \frac{8s+12}{4\left(s+\frac{1}{2}\right)^2}$$

4. $\therefore$

$$i(t) = L^{-1}\{I(s)\} = \frac{1}{4}L^{-1}\left[\frac{8s+12}{\left(s+\frac{1}{2}\right)^2}\right] = 2L^{-1}\left[\frac{s+\frac{3}{2}}{\left(s+\frac{1}{2}\right)^2}\right]$$

$$= 2L^{-1}\left[\frac{s+\frac{1}{2}+1}{\left(s+\frac{1}{2}\right)^2}\right] = 2L^{-1}\left[\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2} + \frac{1}{\left(s+\frac{1}{2}\right)^2}\right]$$

$$= 2L^{-1}\left[\frac{1}{\left(s+\frac{1}{2}\right)} + \frac{1}{\left(s+\frac{1}{2}\right)^2}\right] = 2e^{-\frac{1}{2}t} + 2te^{-\frac{1}{2}t} \text{ A}$$

Que 4.26. **In the circuit shown in Fig. 4.26.1, determine the current $i(t)$ when the switch is at position 2. The switch S is moved from position 1 to position 2 at $t = 0$. Initially the switch has been at position 1 for a long time.**

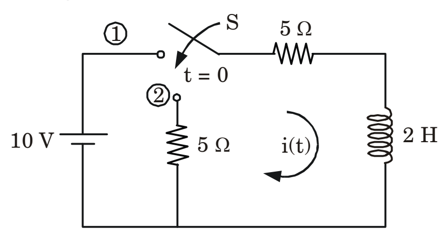


Fig. 4.26.1.

Answer

1. When switch S is in position 1,
   Steady state current
   $$i(0^-) = 10 / 5 = 2 \text{ A}$$
2. When switch is at position 2, for $t > 0$, we have
   $$2\frac{di(t)}{dt} + 5i(t) + 5i(t) = 0$$
   $$2\frac{di(t)}{dt} + 10i(t) = 0$$
   $$\frac{di(t)}{dt} + 5i(t) = 0$$
3. Taking Laplace transform, we obtain
   $s\,I(s) - i(0^-) + 5\,I(s) = 0$
   $$s\,I(s) - 2 + 5\,I(s) = 0$$
   $$I(s) = \frac{2}{s+5}$$
4. Taking inverse Laplace transform, we get
   $$i(t) = 2e^{-5t}\,u(t)$$

PART-5

Evaluation of Initial Conditions.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.27. Discuss procedure for evaluating initial condition.

Answer

1. Draw the equivalent network at = 0^-. Before switching action takes place, *i.e.*, for $t = -\infty$ to $t = 0^-$, the network is under steady-state conditions. Hence, find the current flowing through the inductors $i_L(0^-)$ and voltage across the capacitor $v_C(0^-)$.
2. Draw the equivalent network at $t = 0^+$, *i.e.*, immediately after switching. Replace all the inductors with open circuits or with current sources $i_L(0^+)$ and replace all capacitors by short circuits or voltage sources $v_C(0^+)$. Resistors are kept as it is in the network.
3. Initial voltages or currents in the network are determined from the equivalent network at $t = 0^+$.

4. Initial conditions *i.e.*, $\frac{di}{dt}(0^+)$, $\frac{dv}{dt}(0^+)$, $\frac{d^2i}{dt^2}(0^+)$, $\frac{d^2v}{dt^2}(0^+)$ are determined by writing integro-differential equations for the network $t > 0$. *i.e.*, after the switching action by making use of initial conditions.

Que 4.28. **In the network shown in Fig. 4.28.1, switch *K* is closed at $t = 0$ with the capacitor uncharged. Find the values of i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t = 0^+$, for element values as follows ; $V = 100$ V, $R = 1000\ \Omega$, C =1 µF.**

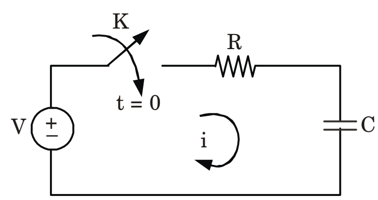


Fig. 4.28.1.

Answer

1. At $t = 0^-$, switch K is open.
   Here current in the circuit will be zero. It is given that capacitor is initially uncharged. So voltage across capacitor is zero.

$$\therefore \quad v_C(0^-) = 0 = v_C(0^+) \quad \ldots(4.28.1)$$

2. For all $t \geq 0^+$, switch K is closed.
   Applying KVL,

$$i\,R + \frac{1}{C}\int_{-\infty}^{t} i\,dt = V$$

$$\therefore \quad i\,R + \frac{1}{C}\int_{-\infty}^{0} i\,dt + \frac{1}{C}\int_{0}^{t} i\,dt = V \quad \ldots(4.28.2)$$

3. Here, $\frac{1}{C}\int_{-\infty}^{0} i\,dt$ represents initial voltage on capacitor. From eq. (4.28.1) it is equal to zero.
4. Hence eq. (4.28.1) becomes

$$i\,R + \frac{1}{C}\int_{0}^{t} i\,dt = V \quad \ldots(4.28.3)$$

5. At $t = 0^+$, eq. (4.28.3) becomes

$$i(0^+)\,R + \frac{1}{C}\int_0^0 i\,dt = V$$

$\therefore$ $$i(0^+)\,(1000) + 0 = 100$$

$\therefore$ $$i(0^+) = 0.1\text{ A}$$

6. Differentiating eq. (4.28.3) with respect to t,

$$R\frac{di}{dt} + \frac{1}{C}i = 0 \qquad ...(4.28.4)$$

7. At $t = 0^+$, eq. (4.28.4) becomes,

$$R\frac{di}{dt}(0^+) + \frac{1}{C}i(0^+) = 0$$

$$(1000)\frac{di}{dt}(0^+) + \left(\frac{1}{1\times10^{-6}}\right)(0.1) = 0$$

$\therefore$ $$\frac{di}{dt}(0^+) = -100\text{ A/sec}$$

8. Differentiating eq. (4.28.4) with respect to t,

$$R\frac{d^2i}{dt^2} + \frac{1}{C}\frac{di}{dt} = 0 \qquad ...(4.28.5)$$

9. At $t = 0^+$, eq. (4.28.4) becomes

$$R\frac{d^2i}{dt^2}(0^+) + \frac{1}{C}\frac{di}{dt}(0^+) = 0$$

$$(1000)\frac{d^2i}{dt^2}(0^+) + \frac{1}{1\times10^{-6}}(-100) = 0$$

$\therefore$ $$\frac{d^2i}{dt^2}(0^+) = 10^5\text{ A / sec}^2$$

Que 4.29. **The switch is closed $t = 0$. Find values of i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t = 0^+$. Assume initial current of inductor to be zero.**

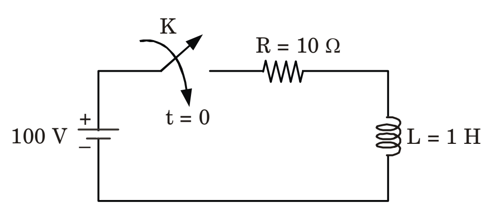


Fig. 4.29.1.

Answer

1. At $t = 0^-$, switch K is open. Here current in the circuit is zero. Hence, current through inductor is given by,

$$i(0^-) = 0 = i(0^+) \quad ...(4.29.1)$$

Because current through inductor cannot change instantaneously.

2. For all $t \geq 0^+$, switch K is closed. The circuit can be drawn as shown in Fig. 4.29.2.

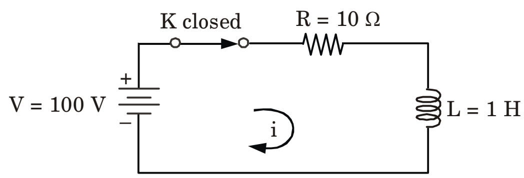


Fig. 4.29.2.

3. Applying KVL,

$$i\,R + L\frac{di}{dt} = V \quad ...(4.29.2)$$

4. At $t = 0^+$, eq. (4.29.2) becomes,

$$i(0^+)\,R + L\frac{di}{dt}(0^+) = V$$

Substituting values

$$(0)\,R \;+ 1\frac{di}{dt}(0^+) = 100$$

$$\therefore \quad \frac{di}{dt}(0^+) = 100 \text{ A / sec}$$

5. Differentiating eq. (4.29.2) with respect to t,

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} = 0 \quad ...(4.29.3)$$

6. At $t = 0^+$, eq. (4.29.3) becomes,

$$R\frac{di}{dt}(0^+) + L\frac{d^2i}{dt^2}(0^+) = 0$$

$$\frac{d^2i}{dt^2}(0^+) = -\frac{R}{L}\frac{di}{dt}(0^+)$$

$$= -\frac{10}{1}(100)$$

$$\therefore \quad \frac{d^2i}{dt^2}(0^+) = -1000 \text{ A/sec}^2$$

Que 4.30. **In the circuit shown in Fig. 4.30.1, V = 10 V, R = 10 Ω, L = 1 H, C = 10 μF, and $v_c(0) = 0$. Find $i(0^+)$, $di/dt(0^+)$, and $d^2i/dt^2(0^+)$.**

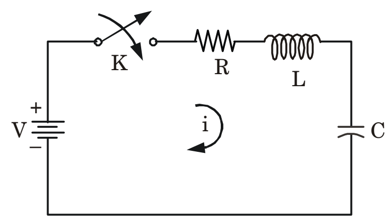


Fig. 4.30.1. RLC network.

Answer

1. Using the Kirchhoff voltage law,

$$V = L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt \qquad \text{...(4.30.1)}$$

2. At $t = 0^-$,

$$i(0^+) = 0$$

3. The last term in eq. (4.30.1), $(1/C)\int i\,dt$, represents the voltage across the capacitor, which is zero at $t = 0$.
4. Eq. (4.30.1) becomes $t = 0^+$,

$$V = L\frac{di}{dt}(0^+) + R(0) + 0$$

$$\frac{di}{dt}(0^+) = \frac{V}{L} = 10 \text{ amp / sec}$$

5. Differentiating eq. (4.30.1)

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0 \qquad \text{...(4.30.2)}$$

$$\frac{d^2i}{dt^2}(0^+) = -\frac{R}{L}\frac{di}{dt}(0^+) = -100 \text{ amp / sec}^2$$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Write properties of Laplace transform.

Ans. Refer Q. 4.2.

Q. 2. Find the partial fraction expansion for

$$F(s) = \frac{s^2 + 2s - 2}{s(s+2)(s-3)}$$

Ans. Refer Q. 4.7.

Q. 3. Discuss briefly singularity functions.

Ans. Refer Q. 4.11.

Q. 4. Express the following waveforms by standard signals :

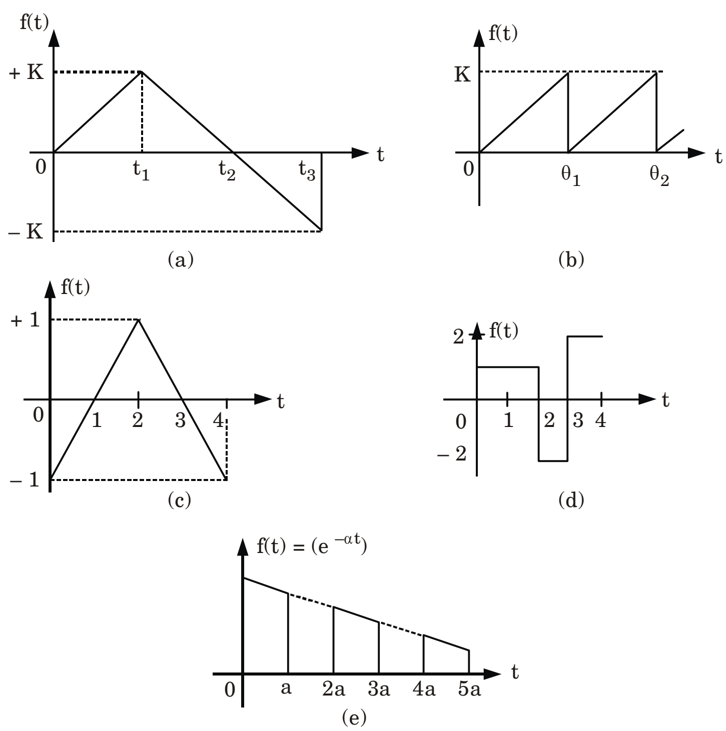


Fig. 1.

Ans. Refer Q. 4.16.

Q. 5. Discuss step response of series RL network.

Ans. Refer Q. 4.17.

Q. 6. Discuss step response of series RLC network.

Ans. Refer Q. 4.23.

Q. 7. An R-L-C series circuit is as shown in Fig. 2. The switch is moved from position 1 to 2 at $t = 0$. Initially it remained in position 1 for a long time. The initial current at ($t = 0^-$) in the inductor is 2 A and the voltage across the capacitor at that instant is 4 volts.
Find the expression for the inductor current $i(t)$ for $t > 0$.

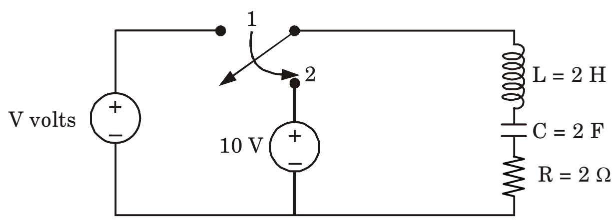


Fig. 2.

Ans. Refer Q. 4.25.

Q. 8. Discuss procedure for evaluating initial condition.

Ans. Refer Q. 4.27.

Q. 9. In the circuit shown in Fig. 3, V = 10 V, R = 10 Ω, L = 1 H, C = 10 μF, and $v_c(0) = 0$. Find $i(0^+)$, $di/dt(0^+)$, and $d^2i/dt^2(0^+)$.

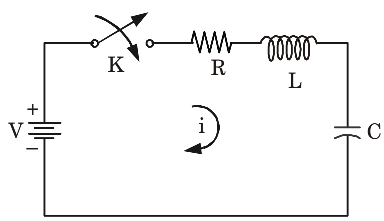


Fig. 3. RLC network.

Ans. Refer Q. 4.30.

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5 UNIT

Transient Behaviour

CONTENTS

PART-1

Transient Behaviour.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.1. **Discuss transient behaviour of electrical circuits.**

Answer

1. In a network containing energy storage elements, with change in excitation, the currents and voltages change from one state to other state.
2. The behaviour of the voltage or current when it is changed from one state to another is called the transient state.
3. The time taken for the circuit to change from one steady state to another steady state is called the transient time.
4. The application of KVL and KCL to circuits containing energy storage elements results in differential, rather than algebraic, equations.
5. When we consider a circuit containing storage elements which are independent of the sources, the response depends upon the nature of the circuit and is called the natural response.
6. Storage elements deliver their energy to the resistances. Hence the response changes with time, gets saturated after some time, and is referred to as the transient response.
7. When we consider sources acting on a circuit, the response depends on the nature of the source or sources. This response is called forced response.
8. In other words, the complete response of a circuit consists of two parts : the forced response and the transient response.

PART-2

Concept of Complex Frequency.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.2. **Explain the concept of complex frequency.**

Answer

1. Complex frequency is a generalised frequency whose real part σ describes growth or decay of the amplitudes of signals and whose imaginary part $j\omega$ is the angular frequency.
2. This complex frequency is applicable for cisoidal signals where
$$r(t) = Ae^{j\omega t}$$
3. The angular frequency ω can be taken as a velocity at the end of the phasor $r(t)$ [since the velocity is also at right angles to the phasor.]
4. Next, let us consider a general case when the velocity (symbolised as s) is inclined with an angle θ as shown in Fig. 5.2.1(*b*).
5. Here $$s = -\sigma + j\omega$$
[s is composed of a component ω at right angles to r and another component $-\sigma$ parallel to r. $-\sigma$ component reduces magnitude of r as it rotates counter clockwise towards origin]
6. $\therefore$ $Re\,[r(t)] = Ae^{-\sigma t}\cos\omega t$
$Im\,[r(t)] = Ae^{-\sigma t}\sin\omega t$ [as shown in Fig. 5.2.2(*a*) and (*b*)]

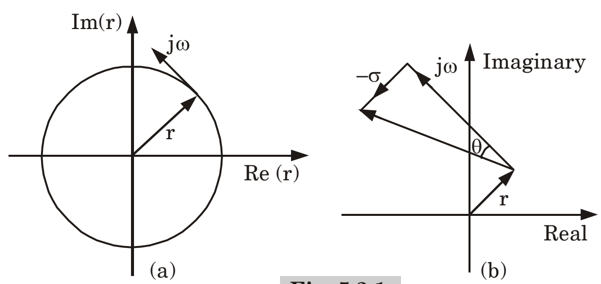

Fig. 5.2.1.

7. On the other hand, if $s = \sigma + j\omega$ as shown in Fig. 5.2.2(*c*), the phasor increases exponentially in magnitude.
8. Then, in general, for a cisoidal signal,
$r(t) = Ae^{st} = Ae^{(\sigma + j\omega)t}$
9. If σ is +ve, signal amplitude increases,
σ is zero, sinusoid is undamped,
σ is –ve sinusoid is damped,
$j\omega$ is zero, signal is exponential,
$j\omega = 0 = \sigma$, signal is constant.

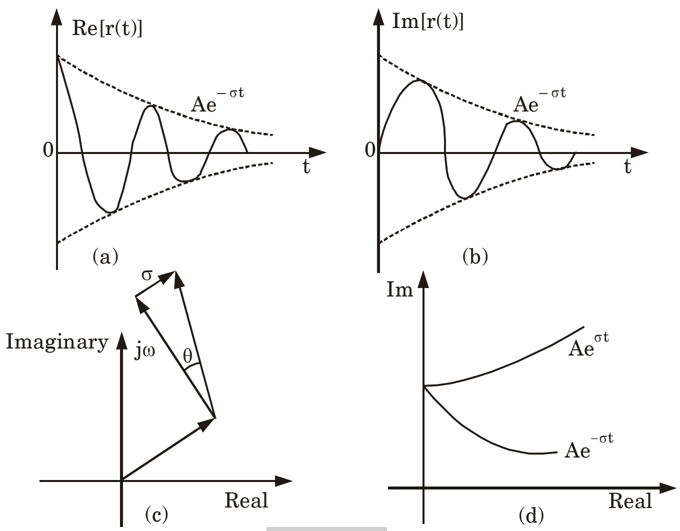

Fig. 5.2.2.

PART-3

Driving Points and Transfer Functions.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.3. **Explain the following terms :**

i. Network functions
ii. Driving point functions
iii. Transfer functions

Answer

i. **Network functions :** The network function $H(s)$ of a linear, time-invariant network is defined as the ratio of the Laplace transform of the response to the Laplace transform of the excitation, while all the initial conditions are zero.

$$\text{Network function} = \frac{\text{L (Response with zero initial conditions)}}{\text{L (Excitation)}}$$

$$H(s) = \left.\frac{Y(s)}{X(s)}\right|_{\text{all initial conditions} = 0}$$

ii. Driving point function : When the excitation and response are defined at a single port, these network functions are called driving point function.

There are two types of driving point function :

a. Driving point impedance function : It is defined as the ratio of Laplace transform of a voltage at any port to the Laplace transform of a current at the same port.

$$Z_{11}(s) = \frac{V_{11}(s)}{I_{11}(s)}$$

b. Driving point admittance function : Driving point admittance function is defined as the reciprocal of driving point impedance function.

$$Y_{11}(s) = \frac{1}{Z_{11}(s)} = \frac{I_{11}(s)}{V_{11}(s)}$$

iii. Transfer function : Transfer function, in general, relates a quantity at one port to a quantity at another port. It is used to describe networks which have at least two ports. The transfer function may have the following forms :

a. Voltage transfer function : This is the ratio of voltage at one port to voltage at another port.

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)};\ G_{12}(s) = \frac{V_1(s)}{V_2(s)}$$

b. Current transfer function : This is the ratio of the current at one port to the current at another port.

$$\alpha_{21}(s) = \frac{I_2(s)}{I_1(s)};\ \alpha_{12}(s) = \frac{I_1(s)}{I_2(s)}$$

c. Transfer impedance function : This is the ratio of voltage at one port to current at another port.

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)};\ Z_{12}(s) = \frac{V_1(s)}{I_2(s)}$$

d. Transfer admittance function : This is the ratio of current at one port to voltage at another port.

$$Y_{21}(s) = \frac{I_2(s)}{V_1(s)};\ Y_{12}(s) = \frac{I_1(s)}{V_2(s)}$$

Que 5.4. **Explain properties of driving point function.**

Answer

1. The coefficients in the polynomials $P(s)$ and $Q(s)$ of network functions $N(s) = \dfrac{P(s)}{Q(s)}$ must be real and positive.
2. Poles and zeros must be conjugate; if imaginary or complex.
3. The real part of all poles and zeros must be negative or zero. If the real part is zero then the pole or zero must be simple.
4. The polynomial $P(s)$ and $Q(s)$ may not have missing terms between that of highest and lowest degree, unless all even or all odd terms are missing.
5. The degrees of $P(s)$ and $Q(s)$ may differ by either zero or one only.
6. The terms of lowest degree in $P(s)$ and $Q(s)$ may differ in degree by one at most.

Que 5.5. **Explain properties of transfer function.**

Answer

1. The coefficients in polynomials $P(s)$ and $Q(s)$ of $N(s) = \dfrac{P(s)}{Q(s)}$ must be real and those for $Q(s)$ must be positive.
2. Poles and zeros must be conjugate; if imaginary or complex.
3. The real part of poles must be negative or zero. If the real part is zero, then the pole must be simple.
4. The polynomial $Q(s)$ may not have any missing terms between that of highest and lowest degree, unless all even or all odd terms are missing.
5. The polynomial $P(s)$ may have terms missing between the terms of lowest and highest degree, and some of the coefficients may be negative.
6. The degree of $P(s)$ may be as small as zero, independent of the degree of $Q(s)$.

Que 5.6. **Consider the network shown in Fig. 5.6.1. Determine the transfer function.**

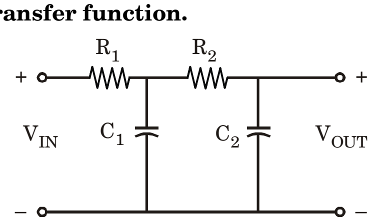

Fig. 5.6.1.

Answer

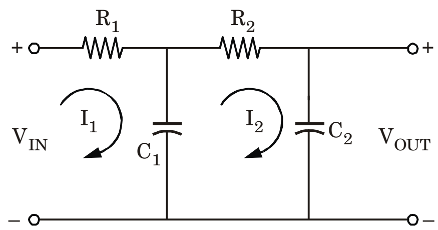

Fig. 5.6.2.

1. In 1st loop

$$V_{IN} = I_1(s)R_1 + (I_1(s) - I_2(s))\ \frac{1}{sC_1}$$

$$= I_1(s)R_1 + \frac{I_1(s)}{sC_1} - \frac{I_2(s)}{sC_1} = I_1\left(R_1 + \frac{1}{sC_1}\right) - \frac{I_2}{sC_1} \qquad ...(5.6.1)$$

2. In 2nd loop

$$\frac{1}{sC_1}(I_2(s) - I_1(s)) + \left(R_2 + \frac{1}{sC_2}\right) I_2 = 0$$

$$\frac{I_2}{sC_1} - \frac{I_1}{sC_1} + R_2 I_2 + \frac{I_2}{sC_2} = 0 \qquad ...(5.6.2)$$

3. $$V_{OUT} = \frac{I_2}{sC_2}$$

4. From eq. (5.6.2) $\quad I_1 = sC_1 I_2\left(\frac{1}{sC_1} + R_2 + \frac{1}{sC_2}\right) = I_2\left(1 + sC_1R_2 + \frac{C_1}{C_2}\right)$

5. Substituting value of I in eq. (5.6.1)

$$I_2\left(1 + sC_1R_2 + \frac{C_1}{C_2}\right)\left(R_1 + \frac{1}{sC_1}\right) - \frac{I_2}{sC_1} = V_{IN}$$

$$V_{IN} = I_2\left(R_1 + sC_1R_1R_2 + R_2 + \frac{C_1R_1}{C_2} + \frac{1}{sC_2}\right)$$

6. But $\quad I_2 = V_{OUT}\, sC_2$

$$V_{IN} = V_{OUT}\, sC_2\ \left(R_1 + sC_1R_1R_2 + R_2 + \frac{C_1R_1}{C_2} + \frac{1}{sC_2}\right)$$

7. $$\frac{V_{OUT}}{V_{IN}} = \frac{1}{sC_2\left(R_1 + sC_1R_1R_2 + R_2 + \frac{C_1R_1}{C_2} + \frac{1}{sC_2}\right)}$$

$$= \frac{1}{s^2C_1C_2R_1R_2 + s(R_1C_2 + R_2C_2 + R_1C_1) + 1}$$

PART-4

Poles and Zeros of Immitance Functions.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.7. **Explain poles and zeros of network functions.**

Answer

1. Any network function may be expressed in the form of a quotient of polynomial in s.

$$N(s) = \frac{P(s)}{Q(s)} = \frac{a_0 s^n + a_1^{n-1} s^{n-1} + ... + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + ... + b_{m-1} s + b_m} \quad ...(5.7.1)$$

where coefficients a_0 to a_n and b_0 to b_m are real and positive for a network containing passive elements only and containing no controlled sources.

$$N(s) = H \frac{(s - Z_1)(s - Z_2)....(s - Z_n)}{(s - P_1)(s - P_2)....(s - P_m)} \quad ...(5.7.2)$$

H is a constant and called scale factor.

2. When the variable s has value equal to any of roots Z_1, Z_2 .... Z_n, the network function $N(s)$ becomes zero. Hence these complex frequencies Z_1, Z_2, ... Z_n, are called the zeros of the network function.
3. When the variable s has any of the values P_1, P_2, ... P_m, the network function $N(s)$ becomes infinite. Hence these complex frequencies P_1, P_2 ..... P_m, are called the poles of the network function.
4. Partial fraction of eq. (5.7.2) is

$$N(s) = \frac{K_1}{s - P_1} + \frac{K_2}{s - P_2} + ... + \frac{K_m}{s - P_m}$$

5. For the stability of the network, the poles should lie only in left half and not in right half of s-plane.

Que 5.8. **Obtain the relationship between various pole locations in s-plane and stability.**

Answer

i. Poles on the negative real axis :

1. If the network has a simple pole on the negative real axis,

$$F(s) = \frac{K}{s+\alpha}$$

2. Corresponding impulse response for $t > 0$

$$f(t) = Ke^{-\alpha t}$$

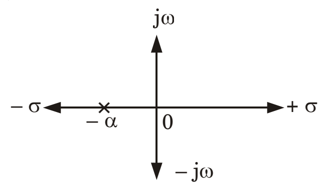

Fig. 5.8.1.

3. As t increases, the value of $f(t)$ decreases.

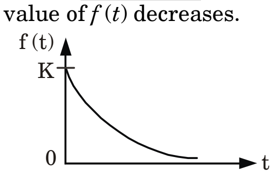

Fig. 5.8.2.

As the time t tends to increase, the response $f(t)$ approaches zero hence the system is stable.

ii. Complex poles in the left half of the *s*-plane :

1. Let the transfer function has a complex conjugate poles at $s = -\alpha \pm j\beta$ as shown in Fig. 5.8.3.

$$F(s) = \frac{K_1}{s+\alpha-j\beta} + \frac{K_1}{s+\alpha+j\beta}$$

The time response $\quad f(t) = L^{-1}\{F(s)\}$

$$= L^{-1}\left[\frac{2K_1(s+\alpha)}{(s+\alpha)^2+\beta^2}\right] = 2K_1\, e^{-\alpha t}\cos\beta t$$

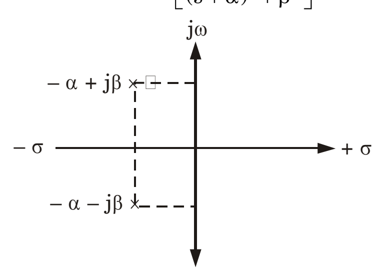

Fig. 5.8.3.

3. The time response is shown in Fig. 5.8.4.

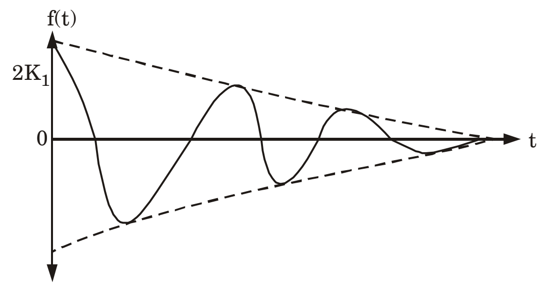

Fig. 5.8.4.

As t increases, $f(t)$ tends to zero. Hence system is stable.

iii. Poles on the positive real axis :

1. Let the transfer function has a simple pole on positive real axis at $s = \alpha$, as shown in Fig. 5.8.5.

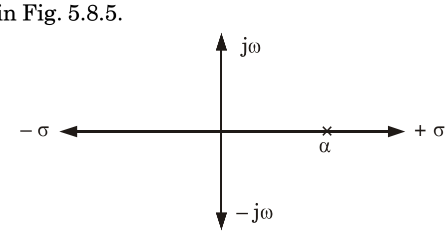

Fig. 5.8.5.

$$F(s) = \frac{K}{s - \alpha}$$

2. The time response $f(t) = Ke^{\alpha t}$

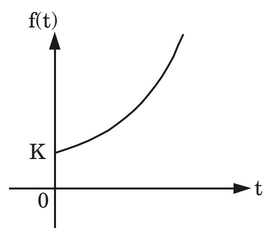

Fig. 5.8.6.

As time t increases, $f(t)$ increases exponentially as shown in Fig. 5.8.6. Hence the system is unstable.

iv. Complex poles in the right half of the *s*-plane :

1. Let the transfer function has a complex conjugate pole at $s = \alpha \pm j\beta$ as shown in Fig. 5.8.7.

$$F(s) = \frac{K}{s - \alpha + j\beta} + \frac{K}{s - \alpha - j\beta}$$

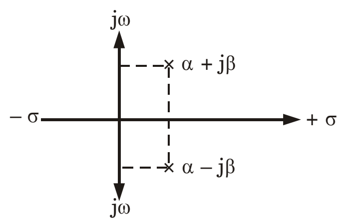

Fig. 5.8.7.

2. Time response

$$f(t) = L^{-1}\left[\frac{K}{s-\alpha+j\beta}+\frac{K}{s-\alpha-j\beta}\right] = L^{-1}\left\{\frac{2K(s-\alpha)}{(s-\alpha)^2+\beta^2}\right\}$$

$$f(t) = 2K\, e^{\alpha t} \cos \beta t$$

As t increases, $f(t)$ increases sinusoidally and exponentially. Hence system is unstable.

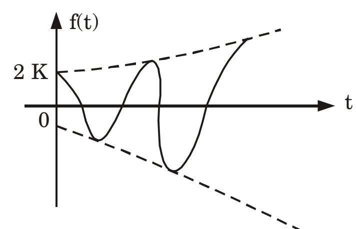

Fig. 5.8.8.

v. Pole at the origin :

1. Consider a pole at origin,

$$F(s) = K/s$$

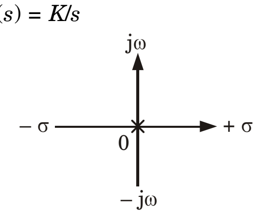

Fig. 5.8.9.

2. Time domain response $f(t) = K$.

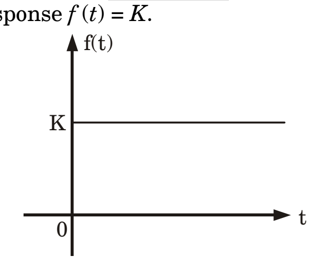

Fig. 5.8.10.

Hence system is marginally stable.

3. If there is multiple order poles at origin,

i.e., $$F(s) = \frac{K}{s^r}$$

then in time domain

$$f(t) = L^{-1}\left[\frac{K}{s^r}\right] = \frac{Kt^{r-1}}{r!}$$

Hence the system is unstable.

vi. Poles on $j\omega$ axis :

1. If the network has complex poles on $j\omega$ axis as shown in Fig. 5.8.11.

$$F(s) = \frac{2Ks}{s^2+\beta^2} = \frac{K}{s+j\beta} + \frac{K}{s-j\beta}$$

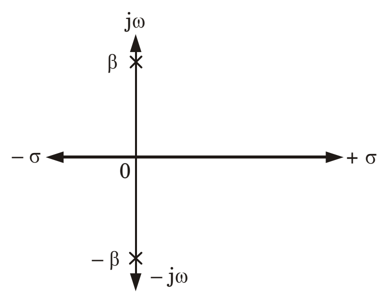

Fig. 5.8.11.

2. The time response of given network is

$$f(t) = L^{-1}\{F(s)\} = L^{-1}\left[\frac{K}{s+j\beta} + \frac{K}{s-j\beta}\right]$$

$$= L^{-1}\left\{\frac{2Ks}{s^2+\beta^2}\right\} = 2K\cos\beta t$$

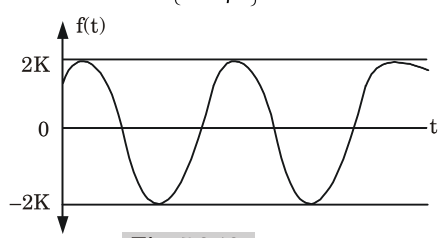

Fig. 5.8.12.

Hence the system is marginally stable.

PART-5

Sinusoidal Response from Pole Zero Locations.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.9. **Explain time response and stability from pole-zero diagram.**

Answer

Time response :

1. For the given network function, a pole-zero plot can be drawn which gives useful information regarding the critical frequencies.
2. The time domain response can also be obtained from pole-zero plot of a network function.
3. Consider an array of poles as shown is Fig. 5.9.1.

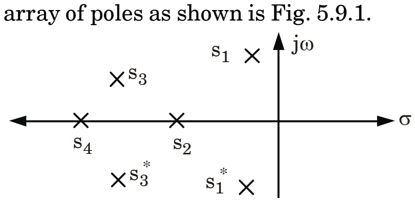

Fig. 5.9.1.

4. In Fig. 5.9.1, s_1 and s_3 are complex conjugate poles, whereas s_2 and s_4 are real poles. The quadratic function is

   $s^2 + 2\delta\,\omega_n s + \omega_n^2 = 0$ for $\delta > 1$

   where, δ is the damping ratio and ω_n is the undamped natural frequency.

$$s_2, s_4 = -\delta\omega_n \pm \omega_n\sqrt{\delta^2 - 1}\ ;\ \delta > 1$$

5. For these poles, the time domain response is given by

$$i(t) = k_2 e^{s2t} + k_4 e^{s4t}$$

6. The response due to pole s_4 dies faster compared to that of s_2 as shown in Fig. 5.9.2.

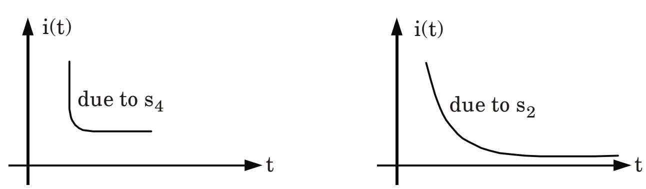

Fig. 5.9.2.

7. s_1 and s_3 constitute complex conjugate poles. If the poles are complex conjugate, then quadratic function is

$$s^2 + 2\delta\,\omega_n\,s + \omega_n^{\,2} = 0 \text{ for } \delta < 1$$

The roots are $s_1, s_1^* = -\delta\omega_n \pm j\omega_n\sqrt{1-\delta^2}$ for $\delta < 1$

8. For these poles, the time domain response is given by

$$i(t) = k_1 e^{-\delta\omega_n t + j(\omega_n\sqrt{1-\delta^2})t} + k_1^{\,*}\, e^{-\delta\omega_n t - j(\omega_n\sqrt{1-\delta^2})t}$$

$$= ke^{-\delta\omega_n t}\sin\,(\omega_n\sqrt{1-\delta^2})t\,.$$

Stability :

1. For a linear system to be stable, all of its poles must have negative real parts, *i.e.*, they must lie within left-half of the s-plane.
2. A system having one or more poles lying on the imaginary axis of s-plane has non-decaying oscillatory components in its natural response, and is defined as a marginally stable system.
3. The location of the poles in the s-plane therefore defines the n components in the homogenous response and stability of the system as :

i. A real pole $p_i = -\sigma$ in the left-half of the s-plane defines an exponentially decaying component, $K\,e^{-\sigma t}$, in the homogenous response. The rate of decay is determined by the pole location; poles far from the origin in the left-half plane correspond to components that decay rapidly, while poles near the origin correspond to slowly decaying components. In this case, the system is a stable system.

ii. A pole at the origin $p_i = 0$ defines a component that is constant in amplitude and defined by the initial conditions. In this case, the system is marginally stable system.

iii. A real pole in the right-half s-plane corresponds to an exponentially increasing component $Ke^{\sigma t}$ in the homogenous response; thus defining the system to be unstable. In this case, the system is an unstable system.

iv. A complex conjugate pole-pair $(\sigma \pm j\omega)$ in the left-half of the s-plane combine to generate a response component that is decaying sinusoid of the form $A\,e^{-\sigma t}\sin\,(\omega t + \phi)$ where, A and ϕ are determined by the initial conditions. The rate of decay is specified by σ, the frequency of oscillation is determined by ω. The system is a stable system.

v. An imaginary pole-pair, *i.e.*, a pole-pair lying on the imaginary axis, $\pm j\omega$, generates an oscillatory component with constant amplitude determined by the initial conditions. The system is marginally stable system.

vi. A complex pole-pair in the right-half plane generates an exponentially increasing component. The system is an unstable system.

Que 5.10. **A two terminal network consisting of a coil having inductance L and resistance R shunted by a capacitor C. The poles**

and zeros of the driving point impedance function $Z(s)$ of this network are shown in Fig. 5.10.1. Find the values of L, R and C.

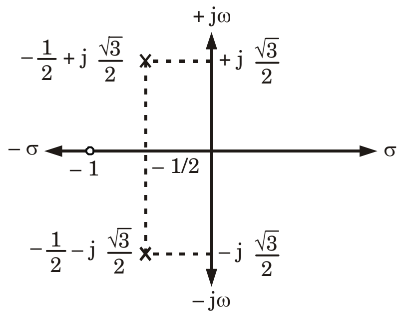

Fig. 5.10.1.

Answer

1.
$$Z(s) = \frac{(sL+R)\times\frac{1}{sC}}{sL+R+\frac{1}{sC}} = \frac{\frac{sL+R}{sC}}{\frac{s^2LC+RsC+1}{sC}}$$

$$= \frac{sL+R}{s^2LC+RCs+1} \qquad ...(5.10.1)$$

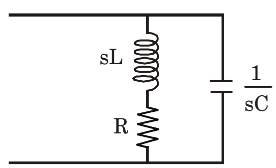

Fig. 5.10.2.

Given zeros are at –1

Poles at $\frac{-1}{2} + j\frac{\sqrt{3}}{2}$ and $\frac{-1}{2} - j\frac{\sqrt{3}}{2}$

2. Transfer function
$$= \frac{s-(-1)}{\left[s-\left(\frac{-1}{2}+j\frac{\sqrt{3}}{2}\right)\right]\left[s-\left(\frac{-1}{2}-j\frac{\sqrt{3}}{2}\right)\right]}$$

$$= \frac{s+1}{\left(s+\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)\left(s+\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{s+1}{s^2+\frac{1}{4}+s+\frac{3}{4}} = \frac{s+1}{s^2+s+1} \qquad ...(5.10.2)$$

3. Comparing eq. (5.10.1) and (5.10.2), we get

$$R = 1\ \Omega, L = 1\text{ H and } C = 1\text{ F}$$

PART-6

Convolution Theorem.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.11. **State and prove convolution theorem.**

Answer

Statement : Let $F_1(s)$ and $F_2(s)$ be the Laplace transform of functions $f_1(t)$ and $f_2(t)$ respectively, then the inverse Laplace transform of product $F_1(s).F_2(s)$ may be obtained from the convolution of $f_1(t)$ and $f_2(t)$ as given by the equation :

$$f(t) = L^{-1}\left[F_1(s).F_2(s)\right] = \int_0^t f_1(t-\tau).f_2(\tau)d\tau = f_1(t) \otimes f_2(t)$$

$$F_1(s).F_2(s) = L\{f_1(t) \otimes f_2(t)\}$$

Proof :

1. By the definition

$$L[f_1(t) \otimes f_2(t)] = \int_0^\infty e^{-st}\left[\int_0^t f_1(t-\tau)f_2(\tau)d\tau\right]dt$$

2. From the definition of the shifted step function

$$u(t-\tau) = \begin{cases} 1 & ; \quad \tau > t \\ 0 & ; \quad \tau \le t \end{cases}$$

3. As we know

$$\int_0^t f_1(t-\tau)f_2(\tau)d\tau = \int_0^\infty f_1(t-\tau)u(t-\tau)f_2(\tau)d\tau$$

$$L[f_1(t) \otimes f_2(t)] = \int_0^\infty e^{-st}\int_0^\infty f_1(t-\tau)u(t-\tau)f_2(\tau)d\tau\, dt$$

4. Let $x = t - \tau$, $\quad e^{-st} = e^{-s(x+\tau)}$

then

$$L\left[f_1(t) \otimes f_2(t)\right] = \int_0^\infty f_1(x)u(x)\int_0^\infty f_2(\tau)e^{-s\tau}\ e^{-sx}d\tau\, dx$$

$$= \int_0^\infty f_1(x)u(x)e^{-sx}dx\int_0^\infty f_2(\tau)e^{-s\tau}d\tau$$

$$= F_1(s).\ F_2(s)$$

Que 5.12. **Find the convolution of two rectangular pulses of width T and amplitude A.**

Answer

1. A rectangular pulse of width T and amplitude A is shown in Fig. 5.12.1.

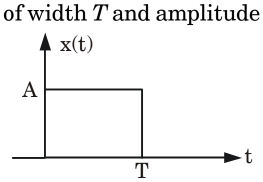

Fig. 5.12.1.

2. We can write $x(t) = A\,[u(t) - u(t-T)]$. Laplace transform of this pulse can be determined as,

$$X(s) = A\left[\frac{1}{s} - e^{-sT}\,\frac{1}{s}\right] = \frac{A}{s}[1 - e^{-sT}] \qquad ...(5.12.1)$$

3. Now let $y(t) = x(t) \otimes x(t)$

$$Y(s) = X(s) \cdot X(s) = \frac{A^2}{s^2}[1 - e^{-sT}]^2$$

$$Y(s) = \frac{A^2}{s^2}[1 + e^{-2sT} - 2e^{-sT}] \qquad ...(5.12.2)$$

4. Taking inverse Laplace transform of eq. (5.12.2) we get

$$y(t) = A^2\,[tu(t) + (t-2T)\,u\,(t-2T) - 2\,(t-T)\,u\,(t-T)]$$
$$= A^2\,[tu(t) - 2\,(t-T)\,u\,(t-T) + (t-2T)\,u\,(t-2T)]$$

PART-7

Two Port Network and Interconnections.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.13. **Discuss linear time-invariant two-port network.**

Answer

1. A two-port network is illustrated in Fig. 5.13.1.

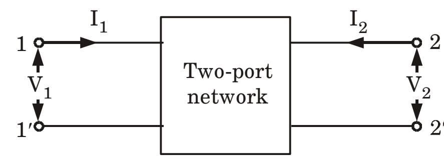

Fig. 5.13.1. Two-port network with standard reference directions for voltages and currents.

2. By analogy with transmission networks, one of the ports (normally the port labelled 1-1′) is called the input port 1, while the other (labelled port 2-2′) is termed the output port 2.
3. The port variables are port currents and port voltages, with the standard references shown in Fig. 5.13.1.
4. External networks that may be connected at the input and output ports are called terminations.
5. In order to describe the relationships among the port voltages and currents of a linear multi-port, as many linear equations are required as there are ports.
6. Thus, for a two-port, two linear equations are required among the four variables.
7. However, the choice of two 'independent' and two 'dependent' variables is a matter of convenience in a given application.

Que 5.14. **Explain in detail the interconnection of all two-port networks.**

Answer

i. **Series connection :**
Series connection of two port network N_a and N_b with open circuit parameters Z_a and Z_b.

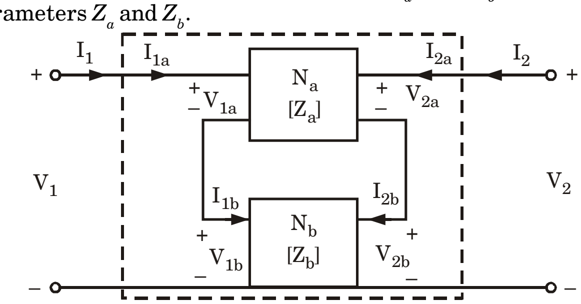

Fig. 5.14.1.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where, $Z_{11} = Z_{11a} + Z_{11b}$

$$Z_{12} = Z_{12a} + Z_{12b}$$
$$Z_{21} = Z_{21a} + Z_{21b}$$
$$Z_{22} = Z_{22a} + Z_{22b}$$

Z-parameter matrix of a series two-port network is the sum of the Z-parameter matrices of each individual two port network connected in series.

ii. Parallel connection :

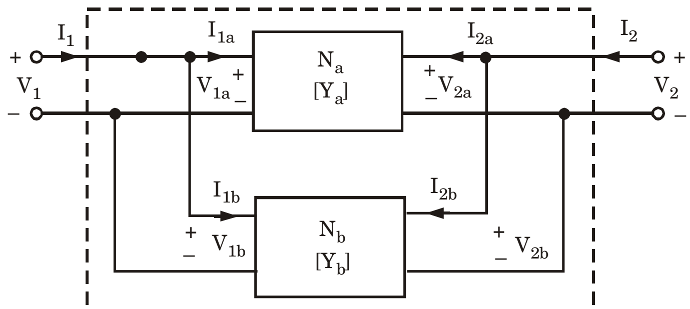

Fig. 5.14.2.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where,
$$Y_{11} = Y_{11a} + Y_{11b}$$
$$Y_{12} = Y_{12a} + Y_{12b}$$
$$Y_{21} = Y_{21a} + Y_{21b}$$
$$Y_{22} = Y_{22a} + Y_{22b}$$

Y-parameters matrix of parallel connected two-port network is the sum of the Y-parameter matrices of each individual two-port network.

iii. Cascade connection :

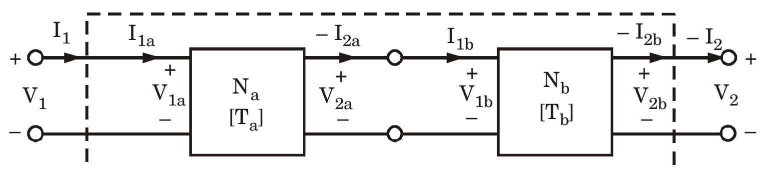

Fig. 5.14.3.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where,
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

$$A = A_a A_b + B_a C_b$$
$$B = A_a B_b + B_a D_b$$
$$C = C_a A_b + D_a C_b$$
$$D = C_a B_b + D_a D_b$$

T matrix for cascade connected two-port network is the matrix product of T-parameter of each individual two-port network.

iv. Series-parallel connection :

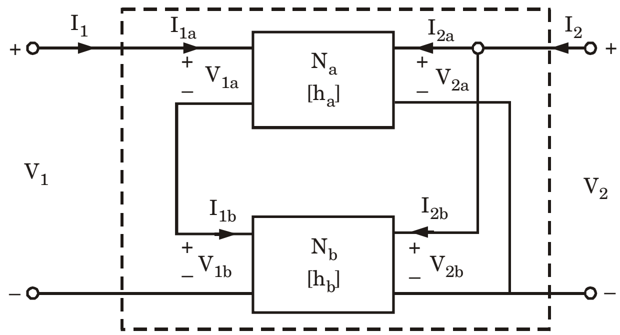

Fig. 5.14.4.

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

where,
$$h_{11} = h_{11a} + h_{11b}$$
$$h_{12} = h_{12a} + h_{12b}$$
$$h_{21} = h_{21a} + h_{21b}$$
$$h_{22} = h_{22a} + h_{22b}$$

The h-parameters matrix for series-parallel connected two-port network is the sum of individual h-parameter of each two-port network connected.

v. Parallel-series connection :

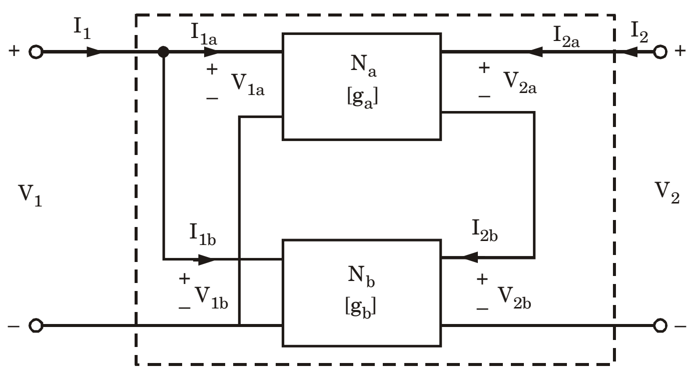

Fig. 5.14.5.

$$\begin{bmatrix} I_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

where,
$$g_{11} = g_{11a} + g_{11b}$$

$$g_{12} = g_{12a} + g_{12b}$$
$$g_{21} = g_{21a} + g_{21b}$$
$$g_{22} = g_{22a} + g_{22b}$$

The g-parameter matrix for parallel-series connected two-port network is sum of g-parameter matrices of each individual two-port network connected.

PART-8

Behaviour of Series and Parallel Resonant Circuits.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.15. **Explain resonance in a series RLC circuit with the help of impedance v/s frequency diagram and derive an expression for resonant frequency. Write properties and applications of series resonance circuit.**

Answer

A.

1. Consider an AC circuit containing a resistance R, inductance L and a capacitance C connected in series, as shown in Fig. 5.15.1.
2. Impedance of the circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$
3. If resonant frequency is denoted by f_r, then

$$X_L = \omega L = 2\pi f_r L$$

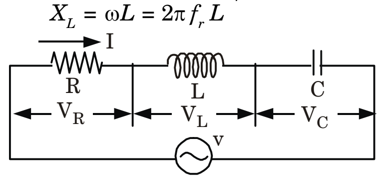

Fig. 5.15.1.

and $$X_C = \frac{1}{2\pi f_r C}$$

4. Since for resonance $X_L = X_C$

$\therefore$ $$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_r = \sqrt{1/LC} \qquad ...(5.15.1)$$

5. From eq. (5.15.1) it is obvious that the value of resonance frequency depends on the parameters of the two energy-storing elements.

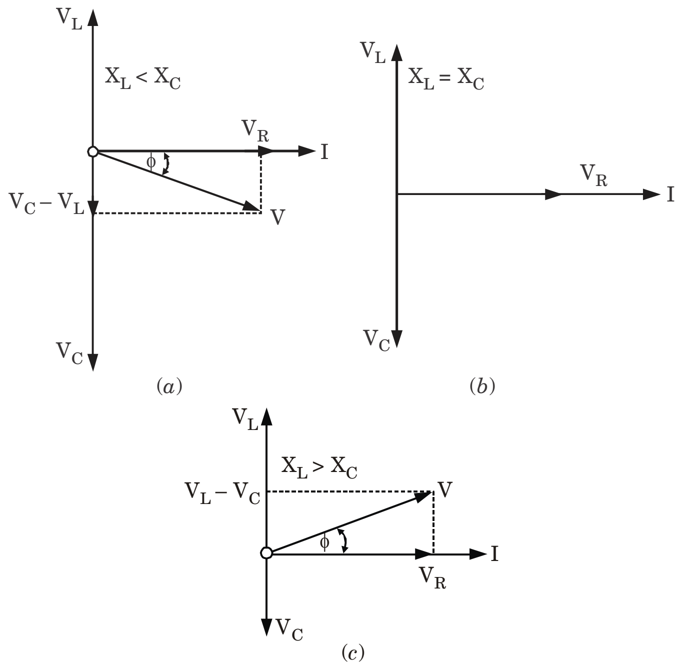

Fig. 5.15.2. Phasor diagram.

B. Property : At resonance,

1. Net reactance is zero, *i.e.*, $X = 0$.
2. Impedance of the circuit, $Z = R$.
3. The current flowing through the circuit is maximum and in phase with the applied voltage. The magnitude of the current will be equal to V/R.
4. The voltage drop across the inductance is equal to the voltage drop across capacitance.
5. The power factor is unity.

C. Impedance *v/s* frequency diagram :

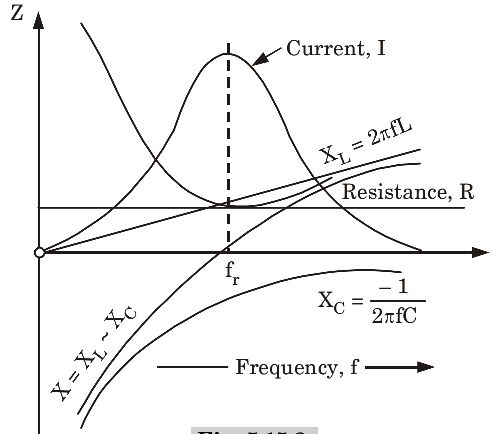

Fig. 5.15.3.

D. Applications of resonance :

1. Resonance circuits are used in tuning applications for radio and TV.
2. These circuits are also used in oscillators.

Que 5.16. **Derive the expression of half power frequencies in terms of resonant frequency f_r. Also derive the expression for bandwidth.**

Answer

A. Derivation of half power frequencies and bandwidth :

1. Consider resonance curve as shown in Fig. 5.16.1.

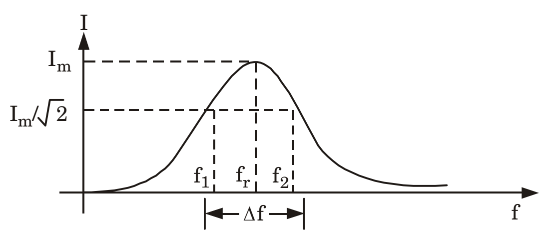

Fig. 5.16.1.

2. Cut off frequency or half power frequency is the frequency where current in the circuit is $1/\sqrt{2}$ times to maximum value of the current.

   $\therefore$ At $f_1, X_L < X_C$ whereas at $f_2, X_C < X_L$

3. Impedance at resonance (f_r) frequency

$$Z = R = \frac{V}{I_m}$$

At f_1 impedance, $Z_1 = \dfrac{V}{I_1} = \dfrac{V}{I_m/\sqrt{2}} = \sqrt{2}\,\dfrac{V}{I_m}$

$$Z_1 = \sqrt{2}\,R \quad ...(5.16.1)$$

Similarly at f_2, $Z_2 = \dfrac{V}{I_2} = \dfrac{V}{I_m/\sqrt{2}} = \sqrt{2}\,\dfrac{V}{I_m}$

$$Z_2 = \sqrt{2}\,R \quad ...(5.16.2)$$

4. At f_1, impedance

$$Z_1 = \sqrt{R^2 + (X_L - X_C)^2}$$

5. Let $X_L - X_C = X$

$\therefore$ $$Z_1 = \sqrt{R^2 + X^2} \quad ...(5.16.3)$$

6. From eq. (5.16.1) and (5.16.3),

$\therefore$ $$\sqrt{2}\,R = \sqrt{R^2 + X^2}$$

$\therefore$ $$R = X$$

7. But since $X_L < X_C$, X is negative.

$\therefore$ $$R = -X$$

$$-R = X_L - X_C$$

$$-R = \omega_1 L - \frac{1}{\omega_1 C}$$

$\therefore$ $$\omega_1^2 LC - 1 = -RC\omega_1$$

$$\omega_1^2 LC + RC\omega_1 - 1 = 0$$

$$\omega_1^2 + \frac{RC}{LC}\,\omega_1 - \frac{1}{LC} = 0$$

$$\omega_1^2 + \left(\frac{R}{L}\right)\omega_1 - \frac{1}{LC} = 0 \quad ...(5.16.4)$$

$\therefore$ $$\omega_1 = \frac{-\dfrac{R}{L} \pm \sqrt{\dfrac{R^2}{L^2} + \dfrac{4}{LC}}}{2}$$

i.e., $$\omega_1 = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

8. Let $$\frac{R}{2L} = \alpha \quad ...(5.16.5)$$

And $$\frac{1}{\sqrt{LC}} = \omega_r \quad ...(5.16.6)$$

$\therefore$ $$\omega_1 = -\alpha \pm \sqrt{\alpha^2 + \omega_r^2} \quad ...(5.16.7)$$

9. Similarly at f_2,

$$R = X$$

$$X_L - X_C = R$$

$$\omega_2 L - \frac{1}{\omega_2 C} = R$$

$$\omega_2^2 LC - 1 - RC\omega_2 = 0$$

$$\omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} = 0$$

$$\omega_2 = \frac{\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - 4\left(-\frac{1}{LC}\right)}}{2}$$

$$\omega_2 = \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

From eq. (5.16.5) and (5.16.6)

$$\omega_2 = \alpha \pm \sqrt{\alpha^2 + \omega_r^2} \quad \text{...(5.16.8)}$$

10. $$\Delta\omega = \omega_2 - \omega_1 = 2\alpha = 2\left(\frac{R}{2L}\right) = \frac{R}{L}$$

11. $\therefore$ **Bandwidth,** $\Delta f = \dfrac{\Delta\omega}{2\pi} = \dfrac{R}{2\pi L}$

$\therefore$ From Fig. 5.16.1 $f_1 = f_r - \dfrac{\Delta f}{2}$ ...(5.16.9)

and $f_2 = f_r + \dfrac{\Delta f}{2}$ ...(5.16.10)

eq. (5.16.9) and (5.16.10) are the expressions for upper and lower cut off frequencies, respectively.

Que 5.17. Explain quality factor. Also give relationship between bandwidth and quality factor of the circuit.

Answer

A. Quality factor :

1. The Q-factor of an RLC series circuit is the voltage magnification that the circuit produces at resonance.
2. Since current at resonance is maximum, supply voltage, $V = I_{max} R$
3. Voltage across inductance or capacitance $= I_{max} X_L = I_{max} X_C$
4. Voltage magnification $= \dfrac{I_{max} X_L}{I_{max} R} = \dfrac{X_L}{R}$

$$= \frac{\omega_r L}{R}$$

Q factor at resonance,

$$Q_r = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R} = \frac{2\pi L}{R} \times \frac{1}{2\pi\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\because \qquad f_r = \frac{1}{2\pi\sqrt{LC}}$$

B. Relation between bandwidth and Quality Factor :

Q factor is also defined as the ratio of resonant frequency to bandwidth, *i.e.*,

$$Q_r = \frac{f_r}{\Delta f} = \frac{\dfrac{1}{2\pi\sqrt{LC}}}{R/2\pi L} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

where, $\Delta f = \dfrac{R}{2\pi L}$ = Bandwidth.

Que 5.18. **Derive as expression for parallel resonance and mention its salient features.**

Answer

A. Derivation :

1. Consider a coil in parallel with a condenser, as shown in Fig. 5.18.1.
2. Let the coil be of resistance R ohms and inductance L henrys and the condenser of resistance R ohms and capacitance C farads.

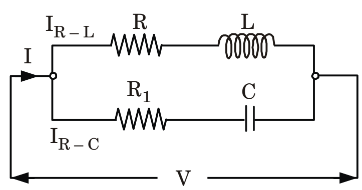

Fig. 5.18.1.

3. Such a circuit is said to be in electrical resonance when the reactive (or wattless) component of line current becomes zero. The frequency at which this happens is known as resonant frequency.
4. Circuit will be in electrical resonance if reactive component of R-L branch current, $I_{R-L} \sin \phi_{R-L}$ = Reactive component of R-C branch current, $I_{R-C} \sin \phi_{R-C}$.
5. Now since $I_{R-L} = \dfrac{V}{\sqrt{R^2 + (\omega_r L)^2}}$

$$\text{and } \sin \phi_{R-L} = \frac{X_L}{Z_{R-L}} = \frac{\omega_r L}{\sqrt{R^2 + (\omega_r L)^2}}$$

$$I_{R-C} = \frac{V}{Z_{R-C}} = \frac{V}{\sqrt{R_1^2 + \left(\dfrac{1}{\omega_r C}\right)^2}}$$

and $\sin \phi_{R-C} = \dfrac{X_C}{Z_{R-C}} = \dfrac{1/\omega_r C}{\sqrt{R_1^2 + \left(\dfrac{1}{\omega_r C}\right)^2}}$

$$\therefore \frac{V}{\sqrt{R^2 + (\omega_r L)^2}} \times \frac{\omega_r L}{\sqrt{R^2 + (\omega_r L)^2}} = \frac{V}{\sqrt{R_1^2 + \left(\dfrac{1}{\omega_r C}\right)^2}} \times \frac{1/\omega_r C}{\sqrt{R_1^2 + \left(\dfrac{1}{\omega_r C}\right)^2}}$$

$$\frac{\omega_r L}{R^2 + (\omega_r L)^2} = \frac{1/\omega_r C}{R_1^2 + (1/\omega_r C)^2}$$

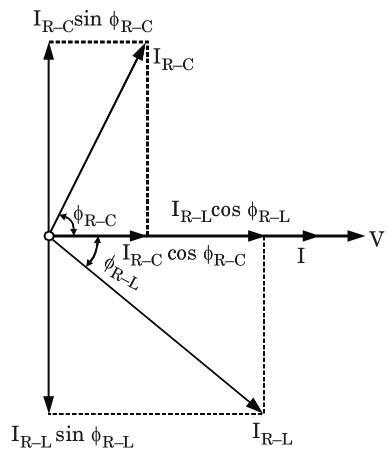

Fig. 5.18.2.

$$\frac{\omega_r L}{R^2 + \omega_r^2 L^2} = \frac{\omega_r C}{\omega_r^2 R_1^2 C^2 + 1}$$

$$L(\omega_r^2 R_1^2 C^2 + 1) = C(R^2 + \omega_r^2 L^2)$$

$$\omega_r^2 L C(R_1^2 C - L) = CR^2 - L$$

$$\omega_r = \frac{1}{\sqrt{LC}} \sqrt{\frac{CR^2 - L}{CR_1^2 - L}}$$

Resonant frequency, $f_r = \dfrac{1}{2\pi} \omega_r$

$$= \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{CR^2 - L}{CR_1^2 - L}} \quad ...(5.18.1)$$

6. If resistance of capacitor is negligible, *i.e.*, $R_1 = 0$, as is usually the case,

Resonant frequency, $f_r = \dfrac{1}{2\pi\sqrt{LC}}\sqrt{1-\dfrac{CR^2}{L}} = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}-\dfrac{R^2}{L^2}}$ ...(5.18.2)

7. If resistance of coil R is zero

Resonant frequency, $f_r = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}}$ ...(5.18.3)

B. Features of current or parallel resonance :

1. Net susceptance is zero.
2. The admittance is equal to conductance.
3. Reactive or wattless component of line current is zero, hence circuit power factor is unity.
4. Line current is minimum and is equal to $\dfrac{V}{L/CR}$ in magnitude and is in phase with the applied voltage.
5. Frequency is equal to $\dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}-\dfrac{R^2}{L^2}}\ Hz.$

Que 5.19. Derive the quality factor of the parallel RLC circuit at resonance.

Answer

1. Consider a current excited RLC parallel network

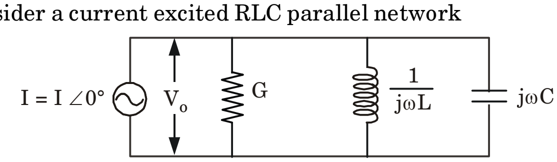

Fig. 5.19.1.

2. Let, $i(t) = I_m \cos \omega_o t$
3. At resonance condition, the currents in inductance and capacitance cancel themselves out and the circuit current I flow in the conductance.
4. The corresponding voltage is (at resonance)

$$v(t) = \frac{i(t)}{G} = \frac{I_m}{G}\cos \omega_o t$$

5. The instantaneous energy stored in the capacitance is

$$w_C(t) = \frac{1}{2}Cv^2 = \frac{I_m^2 C}{2G^2}\cos^2 \omega_o t$$

6. The instantaneous energy stored in the inductor is

$$w_L(t) = \frac{1}{2}Li^2 = \frac{1}{2}L\left(\frac{1}{L}\int_0^L v\,dt\right)^2$$

$$= \frac{I_m^2 C}{2G^2} \sin^2 \omega_o t$$

7. Total instantaneous energy stored in C and L is

$$w(t) = w_C(t) + w_L(t) = \frac{I_m^2 C}{2G^2} \cos^2 \omega_o t + \frac{I_m^2 C}{2G^2} \sin^2 \omega_o t$$

$$= \frac{I_m^2 C}{2G^2}$$

8. Average power dissipated by the conductance

$$P_G = \frac{I_m^2 C}{2G^2}$$

9. Energy dissipated in one cycle

$$P_{GT} = \frac{1}{f_0} \frac{I_m^2}{2G} = \frac{2\pi}{\omega_o} \frac{I_m^2}{2G}$$

10. Quality factor, $Q_0 = 2\pi \left[\dfrac{\text{Maximum energy stored per period}}{\text{Total energy lost per period}}\right]$

$$Q_0 = 2\pi \left[\left(\frac{I_m^2 C}{2G^2}\right) \div \left(\frac{2\pi}{\omega_o} \frac{I_m^2}{2G}\right)\right]$$

$$Q_0 = \frac{\omega_o C}{G} = \omega_o RC$$

PART-9

Introduction to Band Pass, Low Pass, High Pass and Band Reject Filter.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.20. Define low pass, high pass, band pass and band reject filters.

Answer

a. **Low pass filter :** These filters reject all frequencies above a specified value called the cut-off frequency. The attenuation characteristic of an ideal LP filter is shown in Fig. 5.20.1.

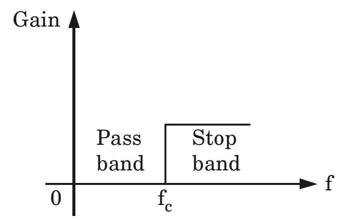

Fig. 5.20.1.

b. **High pass filter :** These filters reject all frequencies below the cut-off frequency. The attenuation characteristic of a high pass filter is shown in Fig. 5.20.2.

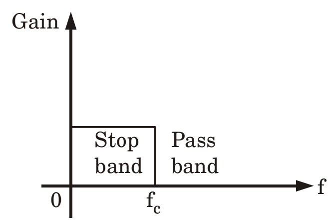

Fig. 5.20.2.

c. **Band pass filter :** A band pass filter passes or allows transmission of a band of frequencies and reject all frequencies beyond this band. As shown in Fig. 5.20.3.

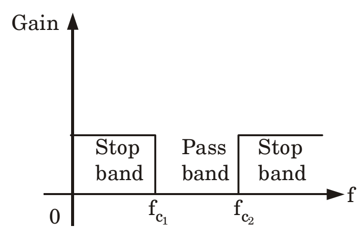

Fig. 5.20.3.

d. **Band reject filter :** A band stop filter rejects or disallows transmission of a limited band of frequencies but allows transmission of all other frequencies and is shown in Fig. 5.20.4.

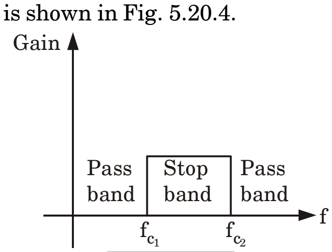

Fig. 5.20.4.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Explain the concept of complex frequency.
Ans. Refer Q. 5.2.

Q. 2. Explain the following terms :
i. Network functions
ii. Driving point functions
iii. Transfer functions
Ans. Refer Q. 5.3.

Q. 3. Obtain the relationship between various pole locations in *s*-plane and stability.
Ans. Refer Q. 5.8.

Q. 4. State and prove convolution theorem.
Ans. Refer Q. 5.11.

Q. 5. Explain in detail the interconnection of all two-port networks.
Ans. Refer Q. 5.14.

Q. 6. Explain resonance in a series RLC circuit with the help of impedance v/s frequency diagram and derive an expression for resonant frequency. Write properties of series resonance circuit.
Ans. Refer Q. 5.15.

Q. 7. Explain quality factor. Also give relationship between bandwidth and quality factor of the circuit.
Ans. Refer Q. 5.17.

Q. 8. Derive the quality factor of the parallel RLC circuit at resonance.
Ans. Refer Q. 5.19.

Q. 9. Define low pass, high pass, band pass and band reject filters.
Ans. Refer Q. 5.20.

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1 UNIT

Node and Mesh Analysis (2 Marks Questions)

1.1. Write Kirchhoff's law.

Ans. **Kirchhoff's current law (KCL) :** The algebraic sum of the branch currents at a node is zero at all instant of time.

$$\Sigma i = 0$$

Kirchhoff's voltage law (KVL) : The algebraic sum of all branch voltages around any closed loop of a network is zero at all instant of time.

$$\Sigma V = 0$$

1.2. What is duality and write any two limitations of dual network ?

Ans. Duality is a transformation in which currents and voltages are interchanged. Two phenomena are said to be dual if they are described by equations of the same mathematical form.

Limitations :

1. Power has no dual network due to its non-linearity.
2. Even when linearity applies, a circuit element may not have a dual network like mutual inductance.

1.3. Define ideal voltage and current source.

Ans. **Ideal voltage source :** A constant voltage source is an ideal source element capable of supplying any current at a given voltage. If the internal resistance of a voltage source is zero, the terminal voltage is equal to the voltage across the source and is independent of the amount of load current.

Ideal current source : A source that supplies a constant current to a load even if its impedance varies is called ideal current source. The current supplied by such a source should remain constant irrespective of the load impedance.

1.4. What information is obtained from the loop ?

Ans. Algebraic sum of potential differences is obtained from loop.

1.5. Define node.

Ans. A node being defined as a connection of two or more circuit elements.

1.6. Explain mesh in electrical network.

Ans. A mesh is defined as a closed path around a circuit that does not contain any other closed path within it.

1.7. What are the utilities of duality of network ?

Ans. It makes very easy to solve complex network problems.

1.8. What is a loop ?

Ans. A loop is a closed path drawn starting at a node and tracing the path such that we return to the original node without passing an intermediate node more than once.

1.9. When is mesh analysis preferred over nodal analysis ?

Ans. Mesh analysis is preferred over nodal analysis :

1. When the circuit contains only voltage sources or a large number of voltage sources.
2. When the circuit contains fewer meshes than nodes.
3. When several currents are to be determined.

1.10. When is nodal analysis preferred over mesh analysis ?

Ans. Nodal analysis is preferred over mesh analysis :

1. When the circuit contains only current sources or more current sources.
2. When the circuit contains fewer nodes than meshes.
3. When node voltage are required.
4. When the network is non-planar only nodal analysis can be used.

1.11. If a circuit has *B* branches and *N* nodes including reference node, how many meshes are there ?

Ans. Number of meshes = Branches – (Nodes – 1),

$$M = B - (N - 1)$$

1.12. What is planar circuit ?

Ans. A planar circuit is a circuit which can be drawn on a plane surface without crossovers.

1.13. What is non-planar circuit ?

Ans. A non-planar circuit is a circuit which cannot be drawn on a plane surface without crossovers.

1.14. When are two networks said to be duals of each other ?

Ans. Two networks are said to be duals of each other, if they are governed by the same set of equations.

1.15. Write pairs of dual terms for electrical networks.

Ans. Some dual terms for electrical circuits are : current-voltage, resistance-conductance, inductance-capacitance, loop-node, series-parallel, open-short, KCL-KVL.

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2 UNIT

Network Theorems (2 Marks Questions)

2.1. State superposition theorem.

Ans. The response in any element of a linear bilateral *RLC* network containing more than one independent voltage and current source is the algebraic sum of the response produced by the source each acting alone.

2.2. Write the application of superposition theorem with its statement. **AKTU 2016-17, Marks 02**

Ans.

1. This theorem is valid for all types of linear circuits having time-varying or time-invariant elements.
2. This theorem is used to find the current or voltage in a branch when the circuit has a large number of independent sources.

2.3. Give limitations of superposition theorem.

Ans.

1. Not applicable for the network containing non-linear elements or unilateral elements.
2. Not applicable for the non-linear parameters such as power.
3. This theorem is not valid for power relationship.

2.4. Write the statement of Thevenin's theorem.

AKTU 2015-16, Marks 02

Ans. Any two terminal linear network containing sources (generators) and impedances can be replaced with an equivalent circuit consisting of a voltage source V_{th} in series with impedance Z_{th}. V_{th} is the open-circuit voltage between the terminals of the network and Z_{th} is the impedance measured between the terminals of the network with all energy sources eliminated.

2.5. What are the limitations of Thevenin's theorem ?

Ans.

1. This theorem is inapplicable to loads which are magnetically coupled to other branch in the circuit.

2. This theorem is inapplicable for non-linear and unilateral networks.
3. This theorem is inapplicable for active load.
4. The load should not contain any dependent source.

2.6. State the reciprocity theorem and write its advantages.

Ans. It states that in any linear time-varying network, the ratio of response to excitation remains same for an interchange of the position of excitation and response in the network.

Advantages :

1. Applicable to the network comprising of linear, time-invariant, bilateral, passive elements, and transformers.
2. To apply this theorem, we have to consider only the zero-state response by taking all the initial conditions to be zero.

2.7. Mention the important points of Norton's theorem.

Ans.

1. Applicable to any linear, bilateral, active network.
2. Inapplicable to non-linear and unilateral networks.
3. This theorem is inapplicable for active load.

2.8. What do you understand by maximum power transfer theorem ?

Ans. Maximum power is absorbed by one network from another connected to it at two terminals, when the impedance of one is the complex conjugate of the other.

2.9. What do you understand by compensation theorem ?

Ans. In any linear, bilateral, active network, if any branch carrying a current I has its impedance Z changed by an amount δZ, the resulting changes that occur in the branches are same as those which would have been caused by the injection of a voltage source of $-I\delta Z$ in the modified branch.

2.10. State Tellegen's theorem.

Ans. Tellegen's theorem states that for a network of n elements and e nodes, if a set of current passing through various elements be $i_1, i_2, \ldots, i_e$ satisfying KCL and its set of voltages be $V_1, V_2, \ldots, V_e$ satisfying KVL for every loop, then Tellegen's theorem is

$$\sum_{k=1}^{e} V_k i_k = 0$$

2.11. What is the condition for maximum power transfer in network ? Also mention any two applications of maximum power transfer theorem.

Ans. Condition for maximum power transfer : Maximum power will be transferred when the load resistance is equal to the source resistance *i.e.*,

$$R_s = R_L$$

Applications of maximum power transfer theorem :

1. In communication system.
2. In car engines.

2.12. Where is compensation theorem mainly used ?

Ans. In bridge and potentiometer circuits.

2.13. What is the limitation of reciprocity theorem ?

Ans. It is applicable only to single source networks and not multi source networks. Also the network should not have any time varying elements and dependent sources.

2.14. Why superposition principle cannot be applied directly to find power ?

Ans. Superposition principle cannot be applied directly to find power because current and power are not linearly related.

2.15. If a circuit contains no independent energy sources, what are the values of V_{Th} and I_N ?

Ans. Both V_{Th} and I_N are zero, and the circuit has only the equivalent Thevenin resistance.

2.16. What is the efficiency of the circuit under the condition of maximum power transfer ?

Ans. Under maximum power transfer condition, the efficiency of the circuit is 50 %.

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3
UNIT

Fourier Series (2 Marks Questions)

3.1. What is Fourier series ?

Ans. The representation of signals over a certain interval of time in terms of the linear combination of orthogonal functions is called Fourier series.

3.2. Write Dirichlet's conditions for the existence of Fourier series.

Ans. The conditions under which a periodic signal can be represented by a Fourier series are known as Dirichlet's conditions. They are as follows :

In each period,

i. The function $x(t)$ must be a single valued function.

ii. The function $x(t)$ has only a finite number of maxima and minima.

iii. The function $x(t)$ has a finite number of discontinuities.

iv. The function $x(t)$ is absolutely integrable over one period, that is $\int_0^T |x(t)|\, dt < \infty$.

3.3. Discuss even symmetry and odd symmetry.

Ans. A function $x(t)$ is said to have even symmetry, if $x(t) = x(-t)$.
A function $x(t)$ is said to have odd symmetry, if $x(-t) = -x(t)$.

3.4. What is half wave symmetry ?

Ans. A function $x(t)$ is said to have half wave symmetry, if

$$x(t) = -x\left(t \pm \frac{T}{2}\right)$$

When it exists, only odd harmonics will be present.

3.5. What do you understand by Fourier transform ?

Ans. Fourier transform is a transformation technique which transforms signals from the continuous-time domain into frequency domain and vice versa.

3.6. Mention the merits of Fourier transform.

Ans.

1. The original time function can be uniquely recovered from it.
2. Convolution integrals can be evaluated using the Fourier transform.

3.7. Differentiate between Fourier series and Fourier transform.

Ans.

S. No.	Fourier series	Fourier transform
1.	The representation of a signal over a certain interval of time in terms of linear combination of orthogonal functions is called Fourier series.	Fourier transform is a transformation technique which transforms signals from the continuous time domain to corresponding frequency domain and vice-versa.
2.	It is only applicable for periodic signals.	It is applicable for both periodic and non-periodic signals.
3.	The spectrum is discrete in nature.	The spectrum is continuous in nature.

3.8. Give the applications of Fourier transform.

Ans.

1. Analysis of LTI systems
2. Cryptography
3. Signal analysis
4. Signal processing

3.9. What is frequency spectrum ?

Ans. The plot of $|H(\omega)|$ versus ω is known as magnitude spectrum and the plot of $\angle H(\omega)$ versus ω is known as phase spectrum. The amplitude spectrum and phase spectrum together is called the frequency response.

3.10. What are the cases of unbalanced load in a three-phase system ?

Ans.

1. Unbalanced delta-connected load
2. Unbalanced three-wire, star-connected load
3. Unbalanced four-wire, star-connected load

3.11. Find the Fourier transform of $x(t - t_0)$.

Ans. Let the function is $x(t - t_0)$.

$$\therefore \quad F[x(t-t_0)] = \int_{-\infty}^{\infty} x(t-t_0)\, e^{-j\omega t}\, dt$$

Let $t - t_0 = p$

$$\therefore \quad F[x(t - t_0)] = \int_{-\infty}^{\infty} x(p)\, e^{-j\omega(p + t_0)}\, dp$$

$$= e^{-j\omega t_0} \int_{-\infty}^{\infty} x(p)\, e^{-j\omega p}\, dp = e^{-j\omega t_0}\, X(\omega)$$

$$\therefore \quad F(t - t_0) = e^{-j\omega t_0}\, X(\omega)$$

3.12. What are the methods to solve three-phase, three-wire, star-connected unbalanced loads ?

Ans.

1. Star-delta transformation
2. Millman's method.

3.13. Which method is used to measure the power in balanced and unbalanced load ?

Ans. Both three-wattmeter method and two-wattmeter method are used to measure power in both balanced and unbalanced loads, whereas one-wattmeter method is used to measure power in only balanced loads.

3.14. How are powers in a three-phase balanced load calculated ?

Ans. Active power, $P = 3V_{ph}I_{ph} \cos\theta = \sqrt{3}\ V_L I_L \cos\theta$

Reactive power, $Q = 3V_{ph}I_{ph} \sin\theta = \sqrt{3}\ V_L I_L \sin\theta$

Apparent power, $S = 3V_{ph}I_{ph} = \sqrt{3}\ V_L I_L$

3.15. What is the relation between line and phase voltages in a star-connected system ?

Ans. $$V_L = \sqrt{3}\ V_{ph}$$

3.16. What is the relation between line and phase current in a delta-connected system ?

Ans. $$I_L = \sqrt{3}\ I_{ph}$$

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4 UNIT

Laplace Transform (2 Marks Questions)

4.1. Give the advantages of Laplace transform.

Ans.

1. Signals which are not convergent in Fourier transform are convergent in Laplace transform.
2. Convolution in time domain can be obtained by multiplication in s-domain.
3. Integro-differential equations of a system can be converted into simple algebraic equations. So LTI systems can be analysed easily using Laplace transforms.

4.2. Write the disadvantages of Laplace transform.

Ans.

1. The integral representation of Laplace domain is complicated.
2. Frequency response of the system cannot be drawn or estimated. Instead only the pole-zero plot can be drawn.
3. $s = j\omega$ is used only for sinusoidal steady-state analysis.

4.3. Compare Laplace transform and Fourier transform.

Ans.

S. No.	Fourier transform	Laplace transform
1.	It does not have any convergence factor.	It has a convergence factor.
2.	It cannot be used to analyse unstable systems.	It can be used to analyse even unstable system.

4.4. Mention the applications of Laplace transform.

Ans.

1. For system modeling.
2. Used to solve differential equations.
3. Used in electrical circuits for the analysis of linear time invariant systems.

4.5. How are resistance (R), inductance (L) and capacitance (C) transformed into s-domain ?

Ans. R remains as R in s-domain,
L transforms to Ls in s-domain,
C transforms to $1 / Cs$ in s-domain.

4.6. What are the basic elements in electrical circuits ?

Ans.
1. Resistor,
2. Inductor and
3. Capacitor.

4.7. Write the *v-i* relationship in the case of
i. Pure resistance,
ii. Pure inductance
iii. Pure capacitance.

Ans.
i. $v(t) = i(t)R$

ii. $v(t) = L\dfrac{di(t)}{dt}$

iii. $v(t) = \dfrac{1}{C}\int i(t)dt$

4.8. Define the transfer function of a system.

Ans. The transfer function of a system is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input when the initial conditions are neglected.

4.9. State initial value theorem.

Ans. The initial value theorem states that $\lim_{t \to 0} f(t) = f(0) = \lim_{s \to \infty} s\,F(s)$.

4.10. State final value theorem.

Ans. The final value theorem states that $\lim_{t \to \infty} f(t) = f(\infty) = \lim_{s \to 0} s\,F(s)$.

4.11. When does the Laplace transform of a function *f*(*t*) exist ?

Ans. The Laplace transform of a function $f(t)$, *i.e.*, $F(s)$ exists only if $\int_{-\infty}^{\infty} |f(t)e^{-st}|\,dt < \infty$. That means, $f(t)\ e^{-\sigma t}$ must be absolutely integrable.

4.12. What is the relation between Laplace transform and Fourier transform ?

Ans. The relation between the Laplace transform and the Fourier transform is that the Laplace transform of $f(t)$, *i.e.*, $F(s)$ is the Fourier transform of $f(t)e^{-\sigma t}$ or fourier transform is the Laplace transform evaluated along the imaginary axis of s-plane, *i.e.*, $F(j\omega) = F(s)\big|_{s = j\omega}$.

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5 UNIT

Transient Behaviour (2 Marks Questions)

5.1. Write system stability condition.

Ans.
1. For a linear system to be stable, all of its poles must have negative real parts, *i.e.*, they must lie within the left-half of the s-plane.
2. A system having one or more poles lying on the imaginary axis of the s-plane has non-decaying oscillatory components in its natural response, and is defined as a marginally stable system.

5.2. What are the advantages of transfer function ?

Ans.
1. It gives simple mathematical algebraic equation.
2. It gives poles and zeros of the system directly.
3. Stability of the system can be determined easily.
4. The output of the system for any input can be determined easily.

5.3. What are the disadvantages of transfer function ?

Ans.
1. It is applicable for LTI system.
2. It does not take initial condition into account.
3. Applicable only for single input and single output.
4. Controllability and observability cannot be determined.

5.4. What are poles and zeros ?

Ans. Poles are critical complex frequencies at which the network function has infinite value and zeros are critical complex frequencies at which the network function has zero value.

5.5. What do complex frequencies corresponding to system poles and excitation poles reveal ?

Ans. Complex frequencies corresponding to system poles yields the system's natural or transient response and depends on the system function while complex frequencies corresponding to excitation poles reveal the forced or steady response and depends on the driving force.

5.6. For a two-port network define the driving point functions and transfer functions

Ans. Driving point functions :

1. Driving point impedances are $Z_{11}(s) = \dfrac{V_1(s)}{I_1(s)}$ and $Z_{22}(s) = \dfrac{V_2(s)}{I_2(s)}$
2. Driving point admittances are $Y_{11}(s) = \dfrac{I_1(s)}{V_1(s)}$ and $Y_{22}(s) = \dfrac{I_2(s)}{V_2(s)}$

Transfer functions :

1. Voltage gains are $G_{21}(s) = \dfrac{V_2(s)}{V_1(s)}$ and $G_{12}(s) = \dfrac{V_1(s)}{V_2(s)}$
2. Current gains are $\alpha_{21}(s) = \dfrac{I_2(s)}{I_1(s)}$ and $\alpha_{12}(s) = \dfrac{I_1(s)}{I_2(s)}$
3. Transfer impedances are $Z_{21}(s) = \dfrac{V_2(s)}{I_1(s)}$ and $Z_{12}(s) = \dfrac{V_1(s)}{I_2(s)}$
4. Transfer admittances are $Y_{21}(s) = \dfrac{I_2(s)}{V_1(s)}$ and $Y_{12}(s) = \dfrac{I_1(s)}{V_2(s)}$

5.7. What is a filter ?

Ans. A filter is a frequency selective network that freely passes the desired bands of frequencies, while almost totally suppressing all other bands.

5.8. How are filters classified ?

Ans. Filters are classified into four common type low-pass, high-pass, band-pass and band-elimination.

5.9. What is a low-pass filter ?

Ans. A low-pass filter is one which passes without attenuation all frequencies up to the cut-off frequency f_c and attenuates all other frequencies greater than f_c.

5.10. What is a high-pass filter ?

Ans. A high-pass filter is one which attenuates all frequencies below a designated cut-off frequency f_c and passes all frequencies above f_c.

5.11. What is a band-pass filter ?

Ans. A band-pass filter is one which passes all frequencies between two designated cut-off frequencies and attenuates all other frequencies.

5.12. What is a band-stop filter ?

Ans. A band-stop filter is one which passes all frequencies lying outside a certain range, while it attenuates all frequencies between the two designated frequencies.

5.13. Why z-parameters are called open circuit impedance parameters ?

Ans. z-parameters are called open circuit impedance parameters because they are defined under open circuit conditions and have the units of impedance.

5.14. Why *ABCD*-parameters are called transmission parameters ?

Ans. *ABCD*-parameters are called transmission parameters because they are widely used in transmission line theory and cascade network.

5.15. Where are hybrid parameters mostly used ?

Ans. Hybrid parameters are extensively used in transistor circuits.

5.16. What is the transmission matrix of a cascade of two-part networks ?

Ans. The transmission matrix of a cascade of two-port networks is the product of transmission matrices of the individual two-port networks.

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B.Tech.

(SEM. III) ODD SEMESTER THEORY EXAMINATION, 2019-20

NETWORK ANALYSIS AND SYNTHESIS

Time : 3 Hours **Max. Marks : 100**

Note : Attempt **all** sections. If require any missing data; then choose suitably.

SECTION-A

1. Attempt **all** questions in brief : **(2 × 10 = 20)**

a. Explain the concept of complex frequency.

b. Define 'Transfer function' of a network.

c. State two properties of the *R-C* driving point impedance function.

d. Find the laplace transform of

$$X(t) = e^{-at} \sin \omega_o t$$

e. Find current in 10 ohm resistor as shown in Fig. 1.

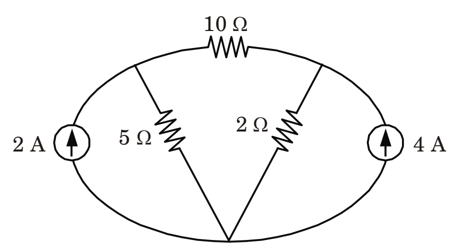


Fig. 1.

f. Draw the dual circuit of parallel *RLC* circuit with current source.

g. What are the dependent and independent terms in the *Z*-parameter ?

h. State compensation theorem.

i. Give examples of active and passive elements in a network.

j. Draw the frequency response curve of parallel resonance *R-L-C* circuit.

SECTION-B

2. Attempt any **three** questions in brief : **(10 × 3 = 30)**

a. Find *Y* and *Z* parameters of the network as shown in Fig. 2.

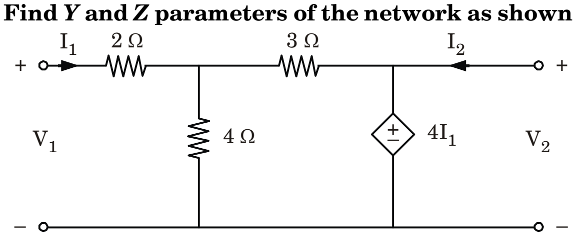


Fig. 2.

b. Explain low pass filter, high pass filter, band pass filter, band reject filter.

c. Find the current i_2 for $t > 0$ in the circuit as shown in Fig. 3.

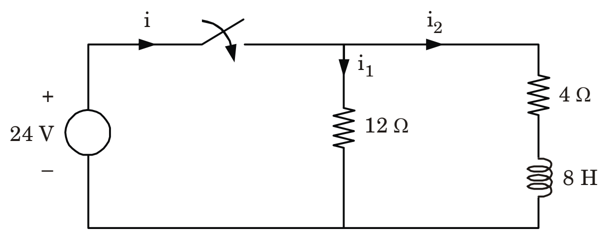


Fig. 3.

d. Explain maximum power transfer theorem related to AC circuits.

e. Calculate the inverse laplace transform $h(t)$ of given transfer function.

$$H(s) = \frac{s^2 + 5s - 9}{(s+1)(s^2 - 2s + 10)}$$

SECTION-C

3. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. A series *R-L* circuit has constant voltage *V* applied at $t = 0$. At what time does $V_R = V_L$ happens.

b. A periodic waveform whose one period is shown in Fig. 4. Determine the trigonometric fourier series coefficients.

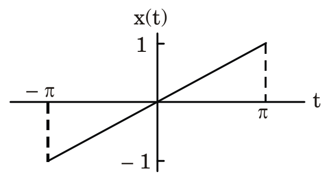


Fig. 4.

4. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Calculate the inverse laplace transform using convolution integral.

$$F(s) = \frac{1}{(s+a)(s+b)}$$

b. For the continuous time periodic signal

$$x(t) = 1 + \cos\frac{2\pi}{3}t + 4\cos\frac{5\pi}{3}t$$

Determine the fundamental frequency ω_0 and exponential fourier series coefficients.

5. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. For the given circuit in Fig. 5, the value of given voltage V_0 across 4 ohm resistance.

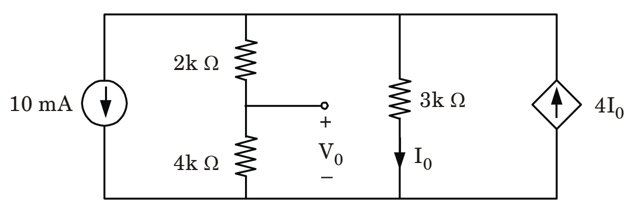


Fig. 5.

b. Calculate the fourier transform of cos $\omega_0 t$. Also sketch its spectrum.

6. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Calculate the short circuit admittance parameter of the given circuit in Fig. 6.

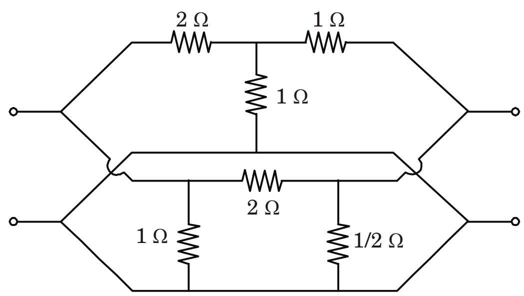


Fig. 6.

b. Prove that for a symmetric network $Z_{11} = Z_{22}$, where Z_{11} and Z_{22} are Z-parameters.

7. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Calculate the impedance $Z(s)$, if driving point impedance $Z(s)$, of a network has pole-zero location as shown in Fig. 7. Also $Z(0) = 3$.

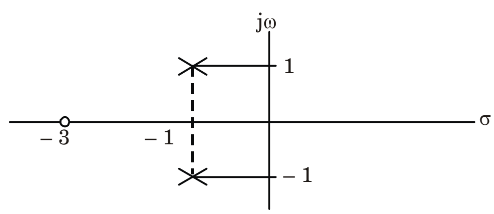


Fig. 7.

b. A practical DC current source provides 20 kW to a 50 load and 20 kW to a 200 load. Calculate the maximum power that can draw from it.

☺☺☺

SOLUTION OF PAPER (2019-20)

Note : Attempt **all** sections. If require any missing data; then choose suitably.

SECTION-A

1. Attempt **all** questions in brief : **(2 × 10 = 20)**

a. Explain the concept of complex frequency.

Ans. Complex frequency is a generalised frequency whose real part σ describes growth or decay of the amplitudes of signals and whose imaginary part $j\omega$ is the angular frequency.

b. Define 'Transfer function' of a network.

Ans. The transfer function of a system is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input when the initial conditions are neglected.

c. State two properties of the *R-C* driving point impedance function.

Ans. Properties of the *R-C* driving point impedance function are :

1. Poles and zeros lie on the negative real axis.
2. Poles and zeros must alternate on the negative real axis.

d. Find the laplace transform of

$$X(t) = e^{-at} \sin \omega_o t$$

Ans. Given, $X(t) = e^{-at} \sin \omega_o t$

Taking laplace transform,

$$X(s) = \frac{\omega_o}{(s+a)^2 + \omega_o^2}$$

e. Find current in 10 ohm resistor as shown in Fig. 1.

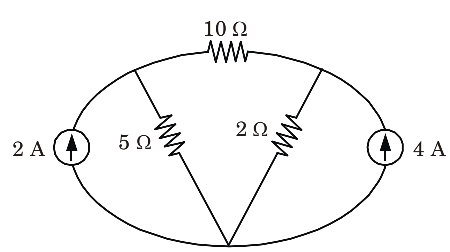


Fig. 1.

Ans.

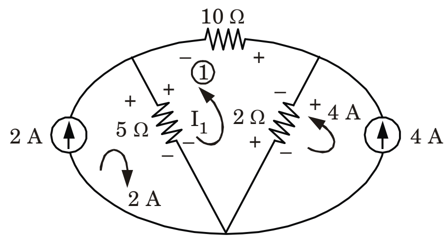


Fig. 2.

Apply KVL in loop 1

$$5(I_1 + 2) + 2(I_1 - 4) + 10I_1 = 0$$

$$5I_1 + 10 + 2I_1 - 8 + 10I_1 = 0$$

$$17I_1 = -2$$

$$I_1 = -\frac{2}{17} = -0.117 \text{ A}$$

f. Draw the dual circuit of parallel *RLC* circuit with current source.

Ans.

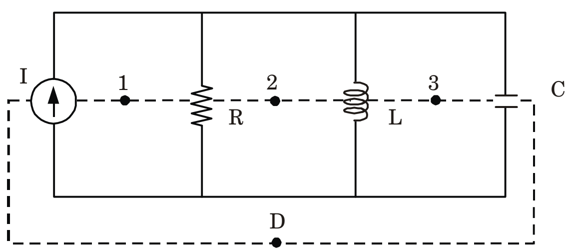


Fig. 3.

Dual network,

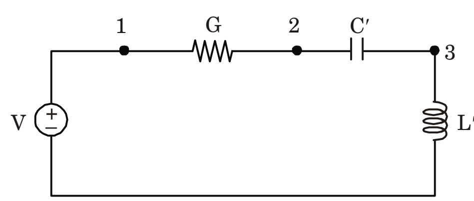


Fig. 4.

g. What are the dependent and independent terms in the *Z*-parameter ?

Ans. Dependent terms in the Z-parameter are V_1 and V_2.
Independent terms in the Z-parameter are I_1 and I_2.

h. State compensation theorem.

Ans. In any linear, bilateral, active network, if any branch carrying a current I has its impedance Z changed by an amount δZ, the resulting changes that occur in the branches are same as those which would have been caused by the injection of a voltage source of $-I\delta Z$ in the modified branch.

i. Give examples of active and passive elements in a network.

Ans. **Examples :**

A. Active elements :
1. Voltage source
2. Current source.

B. Passive elements :
1. Resistor
2. Inductor
3. Capacitor.

j. Draw the frequency response curve of parallel resonance *R-L-C* circuit.

Ans.

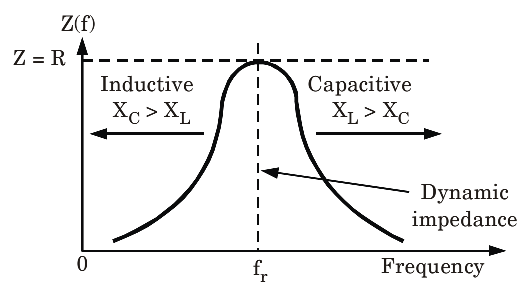


Fig. 5. Frequency response of parallel resonance *R-L-C* circuit.

SECTION-B

2. Attempt any **three** questions in brief : **(10 × 3 = 30)**

a. Find *Y* and *Z* parameters of the network as shown in Fig. 6.

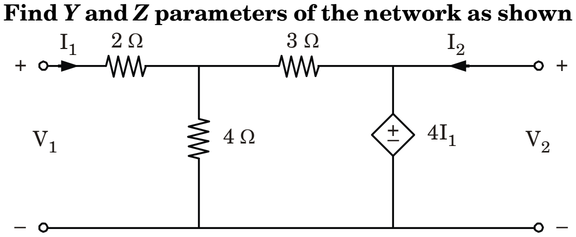


Fig. 6.

Ans.

1. Redrawing the network as shown in Fig. 7.
2. The loop equations are

$$V_1 = 2I_1 + 4(5I_1 + I_2)$$

$$V_1 = 22I_1 + 4I_2 \quad ...(1)$$

and $V_2 = 3(4I_1 + I_2) + 4(5I_1 + I_2)$

or $V_2 = 32I_1 + 7I_2 \quad ...(2)$

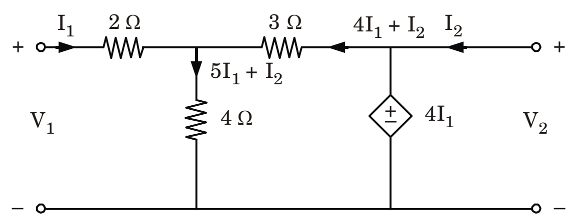


Fig. 7.

3. For Z-parameter, the equations are :

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad ...(3)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad ...(4)$$

4. On comparing eq. (1), (2), (3) and (4) then we get

$$Z_{11} = 22, Z_{12} = 4$$

$$Z_{21} = 32, Z_{22} = 7$$

5. For Y-parameter,

$$\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$$

$$= 22 \times 7 - 4 \times 32 = 26$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{7}{26} \, \Omega^{-1}$$

$$Y_{12} = -\frac{Z_{12}}{\Delta Z} = -\frac{4}{26} = -\frac{2}{13} \, \Omega^{-1}$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z} = -\frac{32}{26} = -\frac{16}{13} \, \Omega^{-1}$$

$$Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{22}{26} = \frac{11}{13} \, \Omega^{-1}$$

b. Explain low pass filter, high pass filter, band pass filter, band reject filter.

Ans.

a. Low pass filter : These filters reject all frequencies above a specified value called the cut-off frequency. The attenuation characteristic of an ideal LP filter is shown in Fig. 8.

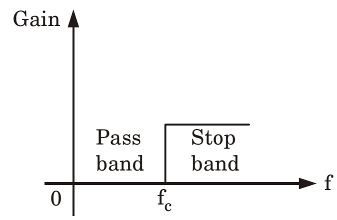


Fig. 8.

b. High pass filter : These filters reject all frequencies below the cut-off frequency. The attenuation characteristic of a high pass filter is shown in Fig. 9.

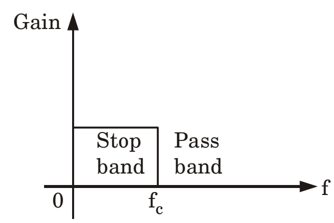


Fig. 9.

c. Band pass filter : A band pass filter passes or allows transmission of a band of frequencies and reject all frequencies beyond this band. As shown in Fig. 10.

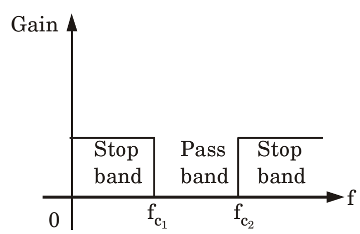


Fig. 10.

d. Band reject filter : A band stop filter rejects or disallows transmission of a limited band of frequencies but allows transmission of all other frequencies and is shown in Fig. 11.

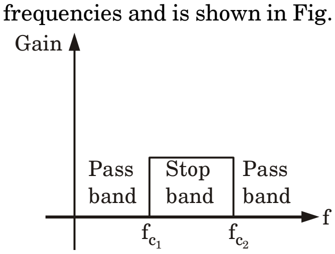


Fig. 11.

c. Find the current i_2 for $t > 0$ in the circuit as shown in Fig. 12.

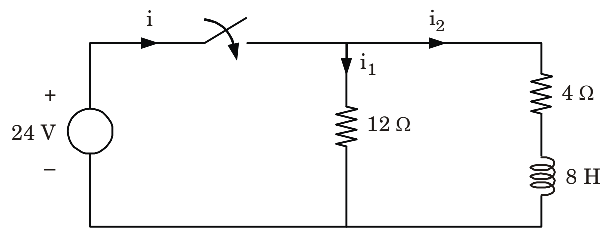


Fig. 12.

Ans. Case 1 : At $t < 0$ switch will be open circuited at that time $i_L(0^-) = 0$
Case 2 : At $t > 0$

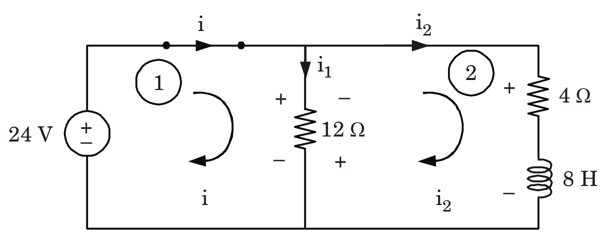


Fig. 13.

1. Apply KVL in loop (1) and (2)

$$-24 + 12\,i_1 = 0$$

$$-24 + 12(i - i_2) = 0 \quad (\because i_1 = i_2 - i) \qquad ...(1)$$

$$12(-i_1) + 4i_2 + 8\frac{di_2}{dt} = 0$$

$$12(i_2 - i) + 4i_2 + 8\frac{di_2}{dt} = 0 \qquad ...(2)$$

2. Taking laplace transform of eq. (1)

$$-\frac{24}{s} + 12I(s) - 12I_2(s) = 0$$

$$\frac{-24 + 12sI(s) - 12s\,I_2(s)}{s} = 0$$

$$12sI(s) - 12s\,I_2(s) = 24 \qquad ...(3)$$

3. Taking laplace transform of eq. (2)

$$12I_2(s) - 12I(s) + 4I_2(s) + 8[sI_2(s) - 0] = 0$$

$$16I_2 - 12I(s) + 8sI_2(s) = 0$$

$$(16 + 8s)\,I_2(s) - 12I(s) = 0 \qquad ...(4)$$

4. Adding eq. (3) and (4)

$$12s\,I(s) - 12s\,I_2(s) = 24$$

$$-12s\,I(s) + (16 + 8s)s\,I_2(s) = 0$$

$$[(16 + 8s)s - 12s]I_2\,(s) = 24$$

$$(16s + 8s^2 - 12s)I_2(s) = 24$$
$$(4s + 8s^2)I_2(s) = 24$$
$$I_2(s) = \frac{24}{(4s + 8s^2)}$$
$$= \frac{24}{8s \times \left[s + \frac{1}{2}\right]}$$
$$I_2(s) = \frac{3}{s\left[s + \frac{1}{2}\right]}$$

5. Taking partial fraction

$$I_2(s) = \frac{3}{s\left(s + \frac{1}{2}\right)} = \frac{A}{s} + \frac{B}{s + \frac{1}{2}}$$
$$I_2(s) = \frac{6}{s} - \frac{6}{s + \frac{1}{2}}$$

6. Taking inverse laplace transform,

$$i_2(t) = 6 - 6e^{-1/2t} = 6[1 - e^{-1/2t}]$$

d. Explain maximum power transfer theorem related to AC circuits.

Ans. Consider a simple network as shown in Fig. 14. There are three possible cases for load impedance Z_L.

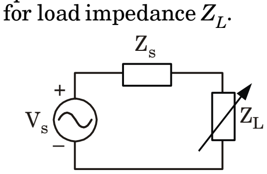


Fig. 14. A simple network.

Case I : When the load impedance is a variable resistance.

1. $$I_L = \frac{V_s}{Z_s + Z_L} = \frac{V_s}{R_s + jX_s + R_L}$$
$$|I_L| = \frac{|V_s|}{\sqrt{(R_s + R_L)^2 + X_s^2}}$$

2. The power delivered to the load is

$$P_L = |I_L|^2 R_L = \frac{|V_s|^2 R_L}{(R_s + R_L)^2 + X_s^2}$$

3. For power P_L to be maximum,

$$\frac{dP_L}{dR_L} = 0$$

$$|V_s|^2 \left[\frac{\{(R_s + R_L)^2 + X_s^2\} - 2R_L(R_s + R_L)}{[(R_s + R_L)^2 + X_s^2]^2} \right] = 0$$

$$(R_s + R_L)^2 + X_s^2 - 2R_L(R_s + R_L) = 0$$

$$R_s^2 + 2R_sR_L + R_L^2 + X_s^2 - 2R_LR_s - 2R_L^2 = 0$$

$$R_s^2 + X_s^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2 + X_s^2$$

$$R_L = \sqrt{R_s^2 + X_s^2} = |Z_s|$$

4. Hence, load resistance R_L should be equal to the magnitude of the source impedance for maximum power transfer.

Case II : When the load impedance is a complex impedance with variable resistance and variable reactance.

1. $$I_L = \frac{V_s}{Z_s + Z_L}$$

$$|I_L| = \frac{|V_s|}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

2. The power delivered to the load is

$$P_L = |I_L|^2 R_L = \frac{|V_s|^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2} \quad ...(1)$$

3. For maximum value of P_L, denominator of the eq. (1) should be small, *i.e.*, $X_L = -X_s$

$$P_L = \frac{|V_s|^2 R_L}{(R_s + R_L)^2} \quad ...(2)$$

4. Differentiating eq. (2) for P_L with respect to R_L and equating to zero, we get

$$\frac{dP_L}{dR_L} = |V_s|^2 \left[\frac{(R_s + R_L)^2 - 2R_L(R_s + R_L)}{(R_s + R_L)^2} \right]$$

$$(R_s + R_L)^2 - 2R_L(R_s + R_L) = 0$$

$$R_s^2 + R_L^2 + 2R_sR_L - 2R_LR_s - 2R_L^2 = 0$$

$$R_s^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2$$

$$R_L = R_s$$

5. Hence, load resistance R_L should be equal to the resistance R_s and load reactance X_L should be equal to negative value of source reactance.
6. Load impedance for maximum power transfer is :

$$Z_L = Z_s^* = R_s - jX_s$$

i.e., load impedance must be the complex conjugate of the source impedance.

Case III : When the load impedance is a complex impedance with variable resistance and fixed reactance.

1. $$I_L = \frac{V_s}{Z_s + Z_L}$$

$$|I_L| = \frac{|V_s|}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

2. The power delivered to the load is

$$P_L = |I_L|^2 R_L = \frac{|V_s|^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

3. For maximum power,

$$\frac{dP_L}{dR_L} = 0$$

$$|V_s|^2 \left[\frac{(R_s + R_L)^2 + (X_s + X_L)^2 - 2R_L(R_s + R_L)}{\{(R_s + R_L)^2 + (X_s + X_L)^2\}}\right] = 0$$

$$(R_s + R_L)^2 + (X_s + X_L)^2 - 2R_L(R_s + R_L) = 0$$

$$R_s^2 + 2R_sR_L + R_L^2 + (X_s + X_L)^2 - 2R_LR_s - 2R_L^2 = 0$$

$$R_s^2 + (X_s + X_L)^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2 + (X_s + X_L)^2$$

$$R_L = \sqrt{R_s^2 + (X_s + X_L)^2}$$

$$= |R_s + j(X_s + X_L)|$$

$$= |(R_s + jX_s) + jX_L|$$

$$= |Z_s + jX_L|$$

4. Hence, load resistance R_L should be equal to the magnitude of the impedance $Z_s + jX_L$, *i.e.*, $|Z_s + jX_L|$ for maximum power transfer.

e. Calculate the inverse laplace transform $h(t)$ of given transfer function.

$$\boldsymbol{H(s) = \frac{s^2 + 5s - 9}{(s+1)(s^2 - 2s + 10)}}$$

Ans.

1. Given, $H(s) = \dfrac{s^2 + 5s - 9}{(s+1)(s^2 - 2s + 10)}$

2. By taking partial fraction,

$$H(s) = \frac{s^2 + 5s - 9}{(s+1)(s^2 - 2s + 10)} = \frac{A}{s+1} + \frac{B}{s - 1 - 3i} + \frac{C}{s - 1 - 3i}$$

$$A = \left.\frac{s^2 + 5s - 9}{(s^2 - 2s + 10)}\right|_{s=-1} = -\frac{13}{13} = -1$$

$$B = \left.\frac{s^2 + 5s - 9}{(s+1)(s-1+3i)}\right|_{s=1+3i} = 1 - \frac{1}{2}i$$

$$C = \left.\frac{s^2 + 5s - 9}{(s+1)(s-1-3i)}\right|_{s=1-3i} = 1 + \frac{1}{2}i$$

$$\therefore \quad C = B^* = 1 + \frac{1}{2}i$$

$$H(s) = -\frac{1}{(s+1)} + \frac{1-\frac{1}{2}i}{(s-1-3i)} + \frac{1+\frac{1}{2}i}{(s-1+3i)}$$

$$H(s) = \frac{-1}{s+1} + \frac{s-1+3i-\frac{1}{2}si+\frac{1i}{2}+\frac{3}{2}+s-1-3i+\frac{1}{2}si-\frac{1}{2}i+\frac{3}{2}}{(s-1)^2+9}$$

$$= \frac{-1}{s+1} + \frac{2s-2+3}{(s-1)^2+3^2}$$

$$= \frac{-1}{s+1} + \frac{2(s-1)}{(s-1)^2+3^2} + \frac{3}{(s-1)^2+3^2}$$

3. Taking inverse laplace,

$$h(t) = -e^{-t} + 2e^{t}\cos 3t + 3e^{t}\sin 3t$$

SECTION-C

3. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. A series *R-L* circuit has constant voltage *V* applied at *t* = 0. At what time does $V_R = V_L$ happens.

Ans.

1. The current in an *RL* circuit is a continuous function, starting at zero in this case, and reaching the final value *V/R*. Thus, for $t > 0$

$$i = \frac{V}{R}(1 - e^{-t/\tau})$$

and $\quad v_R = Ri = V(1 - e^{-t/\tau})$

where $\tau = L/R$ is the time constant of the circuit. Since $v_R + v_L = V$, the two voltages will be equal when

$$v_R = \frac{1}{2}V$$

$$V(1 - e^{-t/\tau}) = \frac{1}{2}V$$

$$e^{-t/\tau} = \frac{1}{2}$$

$$\frac{t}{\tau} = \ln 2$$

that is, when $t = 0.693\tau$.

Note : This time is independent of V.

b. A periodic waveform whose one period is shown in Fig. 15. Determine the trigonometric fourier series coefficients.

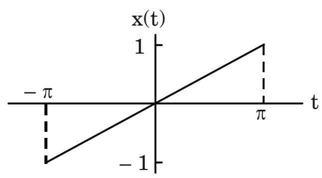


Fig. 15.

Ans.

1. The waveform shown in Fig. 15 can be expressed as $x(t) = t / \pi$. The time period, $T = 2\pi$.
2. The trigonometric fourier series representation is,

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\,\omega_0 t + b_n \sin n\,\omega_0 t)$$

where $\omega_0 = \frac{2\pi}{T_0}$

a_0, a_n and b_k are the fourier coefficient.

3.
$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\, dt$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{t}{\pi}\, dt$$

$$a_0 = \frac{1}{2\pi^2}[t]_{-\pi}^{\pi} = \frac{1}{2\pi^2}[\pi - (-\pi)]$$

$$= \frac{1}{2\pi^2} \times 2\pi = \frac{1}{\pi}$$

4.
$$a_n = \frac{2}{T_0}\int_{T_0} x(t) \cos n\,\omega_0 t\, dt$$

where $\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2\pi} = 1$

$$a_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} \frac{t}{\pi} \cos nt\, dt$$

$$a_n = \frac{1}{\pi^2}\int_{0}^{2\pi} t \cos nt\, dt$$

$$a_n = \frac{1}{\pi^2}\left[\left[-t\,\frac{\sin n\,t}{n}\right]_0^{2\pi} - \int_0^{2\pi} 1 \times \left(\frac{-\sin nt}{n}\right) dt\right]$$

$$= \frac{1}{\pi^2}\left[-\left[\frac{2\pi \sin 2n\,\pi - 0}{n}\right] + \left[\frac{\cos nt}{n^2}\right]_0^{2\pi}\right]$$

$$= \frac{1}{\pi^2}\left[-\frac{2\pi \sin 2n\,\pi}{n} + \frac{\cos 2n\pi}{n^2} - \frac{1}{n^2}\right]$$

$$a_n = \frac{1}{\pi^2}\left[0 + \frac{1}{n^2} - \frac{1}{n^2}\right] = 0$$

5. $$b_n = \frac{2}{T_0}\int_{T_0} x(t) \sin n\,\omega_0 t\,dt$$

$$b_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} \frac{t}{\pi} \sin nt\,dt$$

$$b_n = \frac{2}{2\pi^2}\int_0^{2\pi} t \sin nt\,dt$$

$$= \frac{1}{\pi^2}\left[\left[\frac{-t\cos nt}{n}\right]_0^{2\pi} - \int_0^{2\pi} \frac{\cos nt}{n}\,dt\right]$$

$$= \frac{1}{\pi^2}\left[\left[\frac{-2\pi\cos 2\pi\,n}{n} - 0\right] - \left[\frac{-\sin nt}{n^2}\right]_0^{2\pi}\right]$$

$$= \frac{1}{\pi^2}\left[\frac{-2\pi\cos 2\pi\,n}{n} + \frac{\sin 2\pi n}{n^2} - \frac{\sin 0}{n^2}\right]$$

$$b_n = \frac{1}{\pi^2} \times \frac{-2\pi}{n} = \frac{-2}{n\pi} = \frac{-2}{\pi n}$$

4. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Calculate the inverse laplace transform using convolution integral.

$$\boldsymbol{F(s) = \frac{1}{(s+a)(s+b)}}$$

Ans.

1. Given, $F(s) = \dfrac{1}{(s+a)(s+b)}$

2. Here, $F_1(s) = \dfrac{1}{(s+a)}$, $F_2(s) = \dfrac{1}{(s+b)}$

   $\therefore$ $f_1(t) = e^{-at}; f_2(t) = e^{-bt}$

3. By convolution integral,

$$= \int_0^t f_1(t-\tau) f_2(\tau)\,d\tau$$

$$= \int_0^t e^{-a(t-\tau)} e^{-b\tau}\, d\tau = e^{-at}\int_0^t e^{(a-b)\tau}\, d\tau$$

$$= e^{-at}\left[\frac{e^{(a-b)\tau}}{a-b}\right]_0^t$$

$$= e^{-at}\left[\frac{e^{(a-b)t}-1}{a-b}\right] = \frac{e^{-at}-e^{-bt}}{b-a}$$

b. For the continuous time periodic signal

$$\boldsymbol{x(t) = 1 + \cos\frac{2\pi}{3}t + 4\cos\frac{5\pi}{3}t}$$

Determine the fundamental frequency ω_0 and exponential fourier series coefficients.

Ans.

1. Given,
$$x(t) = 1 + \cos\frac{2\pi}{3}t + 4\cos\frac{5\pi}{3}t$$
$$x(t) = 1 + \frac{1}{2}e^{j(2\pi/3)t} + \frac{1}{2}e^{-j(2\pi/3)t} + 4\left[\frac{1}{2}e^{j(5\pi/3)t} + \frac{1}{2}e^{-j(5\pi/3)t}\right]$$
$$x(t) = 1 + \frac{1}{2}e^{j(2\pi/3)t} + \frac{1}{2}e^{-j(2\pi/3)t} + 2e^{j5(2\pi/6)t} + 2e^{-j5(2\pi/6)t} \quad ...(1)$$
2. From eq. (1), we may conclude that the fundamental frequency of $x(t)$ is $2\pi/6 = \pi/3$.
3. Thus, the exponential fourier series coefficients are :
$$C_0 = 1,\ C_2 = C_{-2} = \frac{1}{2},\ C_5 = C_{-5} = 2$$

5. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. For the given circuit in Fig. 16, the value of given voltage V_0 across 4 ohm resistance.

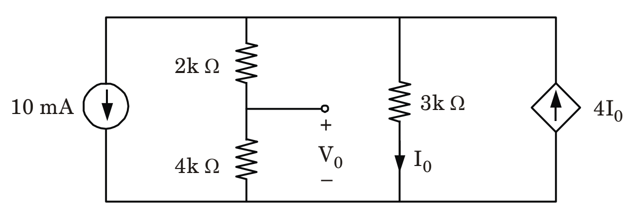


Fig. 16.

Ans.

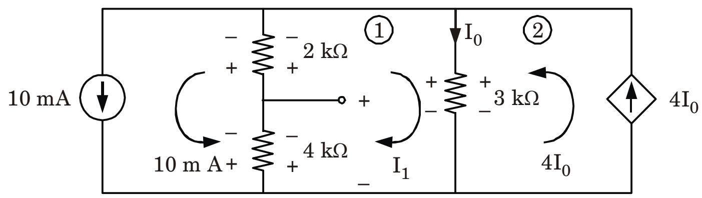


Fig. 17.

1. Apply KVL in loop (1)

$$4(I_1 + 10) + 2(I_1 + 10) + 3(I_1 + 4I_0) = 0$$

$$9I_1 + 12\,I_0 = -\,60 \qquad \text{...(1)}$$

2. Apply KVL in loop (2)

$$3(4I_0 + I_1) = 0$$

$$12I_0 + 3I_1 = 0 \qquad \text{...(2)}$$

3. Solving eq. (1) and (2) then we get,

$$I_1 = -\,10 \text{ Amp}$$

$$I_0 = 2.5 \text{ Amp}$$

4. Voltage across 4 kΩ,

$$= 4(10 + I_1) = 4[10 - 10] = 0 \text{ V}$$

b. Calculate the fourier transform of cos $\omega_0 t$. Also sketch its spectrum.

Ans. Given $x(t) = \cos \omega_0 t$

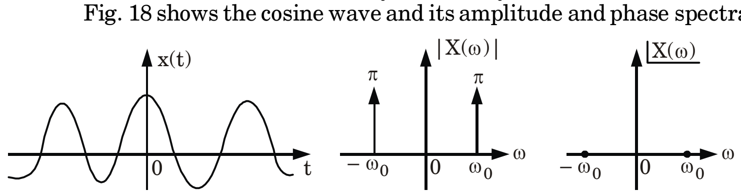
Then, $X(\omega) = F[x(t)] = F[\cos \omega_0 t] = \left[\frac{1}{2}(e^{j\omega_0 t} + e^{-j\omega_0 t})\right]$

$$= \frac{1}{2}[F(e^{j\omega_0 t}) + F(e^{-j\omega_0 t})]$$

$$= \frac{1}{2}[2\pi\delta(\omega - \omega_0) + 2\pi\delta(\omega + \omega_0)]$$

$$= \pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$$

Fig. 18 shows the cosine wave and its amplitude and phase spectra.



(a) Cosine wave (b) Its magnitude spectrum (c) Its phase spectrum

Fig. 18.

6. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Calculate the short circuit admittance parameter of the given circuit in Fig. 19.

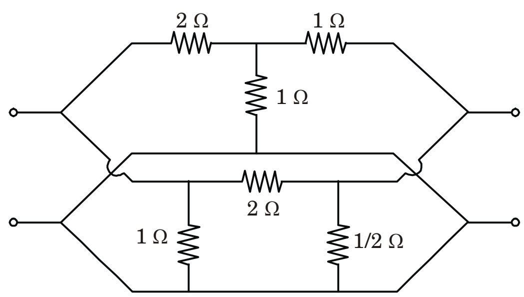


Fig. 19.

Ans.

1. Let top network is A and bottom network is B, then

$$Y_A = \left[Z_A^{-1}\right] = \begin{bmatrix} 2+1 & 1 \\ 1 & 1+1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{bmatrix}$$

and

$$Y_B = \begin{bmatrix} 1+2 & -2 \\ -2 & \dfrac{1}{2}+2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 \\ -2 & 5/2 \end{bmatrix}$$

2. Therefore, the required Y-parameters are

$$Y = Y_A + Y_B = \begin{bmatrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -2 & 5/2 \end{bmatrix}$$

$$= \begin{bmatrix} 17/5 & -11/5 \\ -11/5 & 31/5 \end{bmatrix}$$

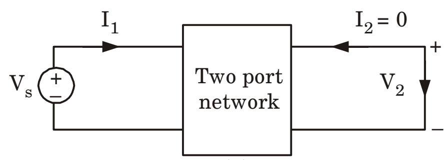
b. Prove that for a symmetric network $Z_{11} = Z_{22}$, where Z_{11} and Z_{22} are Z-parameters.

Ans.

1. A two port network is said to be symmetrical if the ports can be interchanged without changing the port voltages and currents.

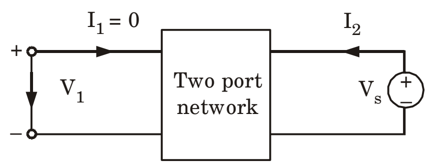
2. Mathematically,

$$\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}$$



(a)

$(V_1 = V_s, I_1 = I_1, I_2 = 0, V_2 = V_2)$



(b)

$(V_2 = V_s, I_2 = I_2, I_1 = 0, V_1 = V_1)$

Fig. 20. Determination the condition for symmetry.

Condition for symmetry :

From Fig. 20(a), $V_1 = V_s, I_1 = I_1, I_2 = 0, V_2 = V_2$.

$$V_s = Z_{11}I_1$$

$$\left.\frac{V_s}{I_1}\right|_{I_2=0} = Z_{11}$$

From Fig 20(b), $V_2 = V_s, I_2 = I_2, I_1 = 0, V_1 = V_1$.

$$V_s = Z_{22}I_2$$

$$\left.\frac{V_s}{I_2}\right|_{I_1=0} = Z_{22}$$

From the definition of symmetry, $\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}$ leads to

$$Z_{11} = Z_{22}$$

7. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Calculate the impedance $Z(s)$, if driving point impedance $Z(s)$, of a network has pole-zero location as shown in Fig. 21. Also $Z(0) = 3$.

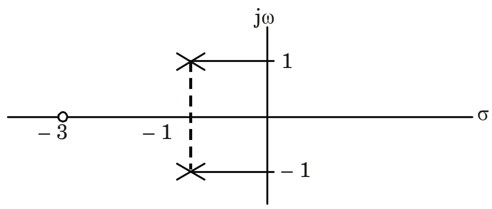


Fig. 21.

Ans.

1. For any pole-zero plot, function $Z(s)$ is given by,

$$Z(s) = k\frac{N(s)}{D(s)}$$

2. From the pole-zero plot of Fig. 21, we have
   zeroes $s = -3$
   Therefore, $N(s) = (s + 3)$
3. Poles $s = -1 \pm j$
   Therefore, $D(s) = (s + 1 + j)(s + 1 - j)$
4. $$Z(s) = k\frac{(s+3)}{(s+1+j\omega)(s+1-j\omega)} = \frac{k(s+3)}{(s+1)^2+1}$$
5. Given, $Z(0) = 3$

$$Z(0) = \frac{k(0+3)}{(0+1)^2+1} = 3 = \frac{3k}{2} = 3 \quad \Rightarrow \quad k = 2$$

6. $$Z(s) = \frac{2(s+3)}{(s+1)^2+1}$$

b. A practical DC current source provides 20 kW to a 50 load and 20 kW to a 200 load. Calculate the maximum power that can draw from it.

Ans.

1. Let us assume i current pass through the circuit.
   The circuit shown in Fig. 22.

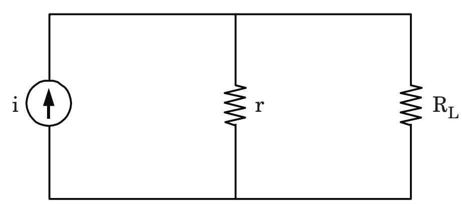


Fig. 22.

2. For 50 Ω load, power is,

$$\left(\frac{ir}{r+50}\right)^2 50 = 20 \times 1000 \quad ...(1)$$

3. For 200 Ω load, power is,

$$\left(\frac{ir}{r+200}\right)^2 200 = 20 \times 1000 \quad ...(2)$$

4. On comparing eq. (1) and (2) then we get

$$\frac{(r+50)^2}{50} = \frac{(r+200)^2}{200}$$

$$4(r+50)^2 = (r+200)^2 \quad ...(3)$$

5. After solving the eq. (3) then we get

$$r = 100\ \Omega$$
$$i = 30\ \text{A}$$

6. $$P_{\max} = \frac{30^2 \times 100}{4} = 22.5\ \text{kW}$$

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