

# Image Transformation Techniques

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## 1. Introduction

The choice of a particular transform in a given application depends on the amount of reconstruction error that can be tolerated and the computational resources available. Compression is achieved during the quantization of the transformed coefficients not during the transformation step. Image modeling or transformation is aimed at the exploitation of statistical characteristics of the image (i.e. high correlation, redundancy).

Some transform techniques are:

**Fourier Transform (FFT, DFT, WFT)**

**Discrete Cosine Transform (DCT)**

**Walsh-Hadamard Transform (WHT)**

**Wavelet Transform (CWT, DWT, FWT)**

For Fourier Transform and DCT basis images are fixed i.e. they are input independent and sinusoidal (cosines and sines) in nature. Provides frequency view i.e. provide frequency information and temporal information is lost in transformation process.

WHT is non-sinusoidal in nature and easy to implement.(Frequency domain)

Wavelet Transforms provides time-frequency view i.e. provides both frequency as well as temporal (localization) information. Wavelets give time-scale viewpoint and exhibits multiresolution characteristics. Fourier is good for periodic or stationary signals but Wavelet is good for transients i.e. for non-stationary data. Localization property allows wavelets to give efficient representation of transients.

## 2. Fourier Transform

Since the Fourier Transform is widely used in analyzing and interpreting signals and images, I will first have a survey on it prior to going further to the Wavelet Transform. The tool which converts a spatial (real space) description of an image into one in terms of its frequency components is called the **Fourier transform**. Through Fourier Transform, it is possible to compose a signal by superposing a series of sine and cosine functions. These sine and cosine functions are known as basis functions (Figure 2.2.1) and are mutually orthogonal. The transform decomposes the signal into the basis functions, which means that it determines the contribution of each basis function in the structure of the original signal. These individual contributions are called the Fourier coefficients. Reconstruction of the original signal from its Fourier coefficients is accomplished by multiplying each basis function with its corresponding coefficient and adding them up together, i.e. a linear superposition of the basis functions.

### Fourier Analysis and Orthogonality

Fourier analysis is one of the most widely used tools in spectral analysis. The basis for this analysis is the Fourier Integral, which computes the amplitude spectral density  $F(\omega)$  of a time-domain signal  $f(t)$ .

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \exp(-j\omega t) dt$$

$F(\omega)$  is actually complex, so one obtains the amplitude spectral density  $A(f)$  and phase spectral density  $\phi(f)$  as a function of frequency. Another way of looking at the Fourier transform is that it answers the question: what continuous distribution of sine waves

$A(f)\cos(j\omega t + \phi(f))$  when added together on a continuous basis best represents the original time signal? We call these distributions the amplitude and phase spectral densities (or spectra). Complex exponentials are popular basis functions because in many engineering and science problems, the relevant signals are sinusoidal in nature. It is noticed that when signals are not sinusoidal in nature, a wide spectrum of the basis function is needed in order to represent the time signal accurately. An important property of any family of basis functions  $\psi(t)$  is that it is orthogonal.

The basis functions in the Fourier Transform are  $\psi(t) = \exp(+j\omega t)$ , so the Fourier Transform could be more generally written as

$$F(\omega) = \int_{-\infty}^{\infty} f(\tau) \psi^*(\tau) d\tau$$

where \* denotes complex conjugate. The test for orthogonality is done as follows

$$\int_{-\infty}^{\infty} \psi_m(\tau) \psi_n^*(\tau) d\tau = \begin{cases} k & m = n \\ 0 & m \neq n \end{cases}$$

For complex exponentials, because they are infinite in duration, one ends up with  $k=\infty$ , when  $m=n$  so it is necessary to define the orthogonality test in a different way :

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \psi_m(\tau) \psi_n^*(\tau) d\tau = \begin{cases} k & m = n \\ 0 & m \neq n \end{cases}$$

For **complex exponential**, this becomes:

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \exp(j\omega_m \tau) \exp(-j\omega_n \tau) d\tau = \lim_{T \rightarrow \infty} \frac{\sin(T[\omega_m - \omega_n]/2)}{T[\omega_m - \omega_n]/2} = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}$$

When the constant  $k=1$ , the function is said to be orthonormal.

Various types of signals can be analyzed with the Fourier Transform. If  $f(t)$  is periodic, then the amplitude spectral density clusters at discrete frequencies that are harmonics (integer multiples) of the fundamental frequency. One needs to invoke Dirac Delta functions if the Fourier Transform is used—otherwise Fourier series coefficients can be computed and same result can be obtained. If  $f(t)$  is deterministic and discrete, the discrete time Fourier Transform (DTFT) may be used to generate a periodic frequency response. If  $f(t)$  is assumed to be both periodic and discrete, then the discrete Fourier Transform (DFT), or its fast numeric equivalent the FFT may be applied to compute the spectrum. If  $f(t)$  is random, then in general one will have a difficult time of computing the Fourier Integral of the random 'data'. Hence treat the input as data and use an FFT, but the result of doing so is a random spectrum. This single random spectrum can give an idea of the frequency response, but in many instances it can be misleading. A better approach is to take the average of the random spectra. This leads to the formulation of power spectral density, which is an average over the FFT magnitude spectrum squared.

In certain signals, both random and deterministic, we are interested in the spectrum as a function of time in the signal. This suggests finding the spectrum over a limited time bin, moving the bin (sometimes with overlap, sometimes without), re-computing the spectrum,

and so on. This method is known as the short-time Fourier Transform (STFT), or the Gabor Transform.

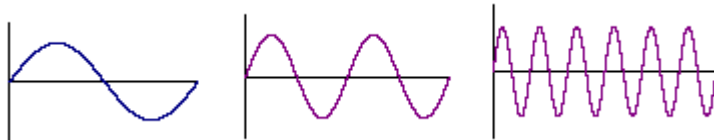
**Discrete Fourier Transform (DFT)** is an estimation of the Fourier Transform, which uses a finite number of sample points of the original signal to estimate the Fourier Transform of it. The order of computation cost for the DFT is in order of  $O(n^2)$ , where  $n$  is the length of the signal.

**Fast Fourier Transform (FFT)** is an efficient implementation of the Discrete Fourier Transform, which can be applied to the signal if the samples are uniformly spaced. FFT reduces the computation complexity to the order of  $O(n \log n)$  by taking advantage of self similarity properties of the DFT.

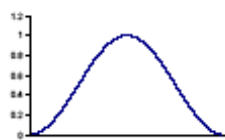
**If the input is a non-periodic signal**, the superposition of the periodic basis functions does not accurately represent the signal.

One way to overcome this problem is to extend the signal at both ends to make it periodic.

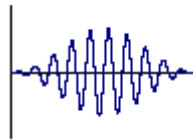
Another solution is to use **Windowed Fourier Transform (WFT)**. In this method the signal is multiplied with a window function ( Figure 2.2.2) prior to applying the Fourier transform. The window function localizes the signal in time by putting the emphasis in the middle of the window and attenuating the signal to zero towards both ends.



**Figure 1: A Set of Fourier basis functions**



**A window function**



**A windowed Signal**

**Figure 2**

### 3. Discrete Cosine Transform (DCT)

The discrete cosine transform (DCT) helps separate the image into parts (or spectral sub-bands) of differing importance (with respect to the image's visual quality). The DCT is similar to the discrete Fourier transform: it transforms a signal or image from the spatial domain to the frequency domain.

With an input image,  $A$ , the coefficients for the output “image,”  $B$ , are:

$$B(k_1, k_2) = \sum_{i=0}^{N_1-1} \sum_{j=0}^{N_2-1} 4 \cdot A(i, j) \cdot \cos \left[ \frac{\pi \cdot k_1}{2 \cdot N_1} \cdot (2 \cdot i + 1) \right] \cdot \cos \left[ \frac{\pi \cdot k_2}{2 \cdot N_2} \cdot (2 \cdot j + 1) \right]$$

The input image is  $N_2$  pixels wide by  $N_1$  pixels high;  $A(i,j)$  is the intensity of the pixel in row  $i$  and column  $j$ ;  $B(k_1,k_2)$  is the DCT coefficient in row  $k_1$  and column  $k_2$  of the DCT matrix. All DCT multiplications are real. This lowers the number of required multiplications, as compared to the discrete Fourier transform. The DCT input is an 8 by 8 array of integers. This array contains each pixel's gray scale level; 8 bit pixels have levels from 0 to 255. The output array of DCT coefficients contains integers; these can range from -1024 to 1023. For most images, much of the signal energy lies at low frequencies; these appear in the upper left corner of the DCT. The lower right values represent higher frequencies, and are often small - small enough to be neglected with little visible distortion. It is computationally easier to implement and more efficient to regard the DCT as a set of **basis functions** which given a known input array size (8 x 8) can be pre-computed and stored. This involves simply computing values for a convolution mask (8 x8 window) that get applied (sum values x pixel the window overlap with image apply window across all rows/columns of image). The values are simply calculated from the DCT formula. The 64 (8 x 8) DCT basis functions are there. Most software implementations use fixed point arithmetic. Some fast implementations approximate coefficients so all multiplies are shifts and adds.

#### 4. DCT Vs Fourier:

- DCT is similar to the Fast Fourier Transform (FFT), but can approximate lines well with fewer coefficients .
- DCT (Discrete Cosine Transform) is actually a *cut-down* version of the FFT i.e. it is only the **real** part of FFT .
- DCT Computationally simpler than FFT and much effective for Multimedia Compression.
- DCT is associated with very less MSE value in comparison to others.
- DCT has best information packing ability.
- DCT minimizes the block like appearance (blocking artifacts), that results when the boundaries between the sub-images become visible. But DFT gives rise to boundary discontinuities.

#### 5. Wavelet Transform:

Wavelet means 'small wave'. So wavelet analysis is about analyzing signal with short duration finite energy functions. They transform the signal under investigation into another representation which presents the signal in more useful form. This transformation of the signal is called Wavelet Transform [1,7,10] i.e. Wavelet Transforms are based on small waves, called wavelets, of varying frequency and limited duration. Unlike the Fourier transform, we have a variety of wavelets that are used for signal analysis. Choice of a particular wavelet depends on the type of application in hand. Wavelet Transforms provides time-frequency view i.e. provides both frequency as well as temporal (localization) information and exhibits multiresolution characteristics. Fourier is good for periodic or stationary signals and Wavelet is good for transients. Localization property allows wavelets to give efficient representation of transients. In Wavelet transforms a signal can be converted and manipulated while keeping resolution across the entire signal and still based in time i.e. Wavelets have special ability to examine signals simultaneously in both time and frequency. Wavelets are mathematical functions that satisfy certain criteria, like a zero mean, and are used for analyzing and representing signals or other functions. A set of dilations and Translations of a chosen mother wavelet is used for the spatial/frequency analysis of an input signal. The Wavelet Transform uses overlapping functions of variable size for analysis. The

overlapping nature of the transform alleviates the blocking artifacts, as each input sample contributes to several samples of the output. The variable size of the basis functions, in addition, leads to superior energy compaction and good perceptual quality of the decompressed image. Wavelets Transform is based on the concept of sub-band coding [1,7,12].

The current applications of wavelet include statistical signal processing, Image processing, climate analysis, financial time series analysis, heart monitoring, seismic signal de-noising, de-noising of astronomical images, audio and video compression, compression of medical image stacks, finger print analysis, fast solution of partial differential equations, computer graphics and so on.

#### **6. Wavelets Vs Fourier and DCT:**

- Fourier and DCT transforms convert a signal from time Vs amplitude to frequency Vs amplitude i.e. provides only frequency information and temporal information is lost during transformation process. But Wavelet transforms provide both frequency as well as temporal ( localization ) information.
- In Fourier and DCT basis functions are sinusoids (sine and cosine) and cosines respectively but in Wavelet Transform basis functions are various wavelets.
- Since Wavelet Transforms are both computationally efficient and inherently local (i.e. their basis functions are limited in duration), subdivision of original image before applying transformation is not required as required in DCT and others.
- The removal of subdivision step in Wavelet Transform eliminates the blocking artifact but FFT suffers from it. This property also characterizes DCT-based approximation, at higher compression ratios.
- Wavelets provide unconditional basis for large signal class. Wavelet coefficients drop sharply hence good for compression, de-noising, detection and recognition.
- Fourier is good for periodic or stationary signals. Wavelet is good for transients. Localization property allows wavelets to give efficient representation of transients.
- Wavelets have local description and separation of signal characteristics. Fourier puts localization information in the phase in a complicated way. STFT cannot give localization and orthogonality.
- Wavelets can be adjusted or adapted to application.
- Computation of wavelet coefficients is well suited to computer. No derivatives of integrals needed as required in Fourier and DCT and hence turn out to be a digital filter bank.