$$g(x,u) = x'Q_1x + x'q_2 + q_3 + u'R_1u + u'r_2 + 2x'Nu, \quad Q_1 = Q_1' > 0, R_1 = R_1' > 0$$

$$J = x'S_1x + x's_2 + s_3, \quad S_1 = S_1' > 0, \quad \frac{\partial J}{\partial x} = 2x'S_1 + s_2'$$

$$\min_u \left[g(x,u) + \frac{\partial J}{\partial x}(Ax + Bu + c) + \frac{\partial J}{\partial t} \right] = 0$$

$$\frac{\partial}{\partial u} = 2u'R_1 + r_2' + 2x'N + (2x'S_1 + s_2')B = 0$$

$$u^* = -R_1^{-1}\left[(N' + B'S_1)x + r_s\right], \quad r_s \equiv \frac{1}{2}(r_2 + B's_2)$$

$$x'Q_1x + x'q_2 + q_3 + \left[(N' + B'S_1)x' + r_s\right]' R_1^{-1}\left[(N' + B'S_1)x' + r_s\right] - \left[(N' + B'S)x' + r_s\right] R_1^{-1}r_2 + ...$$
$$-2x'NR_1^{-1}\left[(N' + B'S_1)x + r_s\right] + x'S_1Ax + x'A'S_1x + s_2'Ax - 2x'S_1BR_1^{-1}\left[(N' + B'S_1)x + r_s\right] + ...$$
$$-s_2'BR^{-1}\left[(N' + B'S_1)x + r_s\right] + 2x'S_1c + s_2'c + x'\dot{S}_1x + x's_2 + \dot{s}_3 = 0$$

$$-\dot{S}_1 = Q_1 + (N' + B'S_1)'R_1^{-1}(N' + B'S_1) - 2NR_1^{-1}(N' + B'S_1) + S_1A + A'S_1 - 2S_1BR_1^{-1}(N' + B'S_1)$$
$$= Q_1 - (N' + B'S_1)'R_1^{-1}(N' + B'S_1) + S_1A + A'S_1$$
$$-\dot{s}_2 = q_2 + 2(N' + B'S_1)'R_1^{-1}r_s - (N' + B'S_1)'R_1^{-1}r_2 + ...$$
$$-2NR_1^{-1}r_s + A's_2 - 2S_1BR_1^{-1}r_s - (N' + B'S_1)'R_1^{-1}B's_2 + 2S_1c$$
$$= q_2 - 2(N' + B'S_1)R_1^{-1}r_s + A's_2 + 2S_1c$$
$$-\dot{s}_3 = q_3 + r_s'R_1^{-1}r_s - r_s'R_1^{-1}r_2 - s_2'BR_1^{-1}r_s + s_2'c$$
$$= q_3 - r_s'R_1^{-1}r_s + s_2'c$$