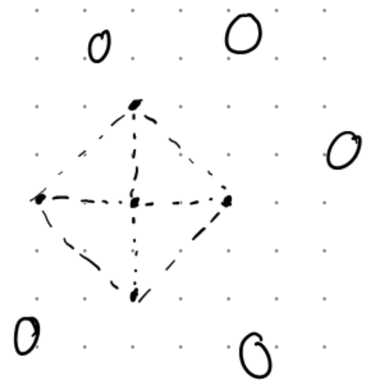
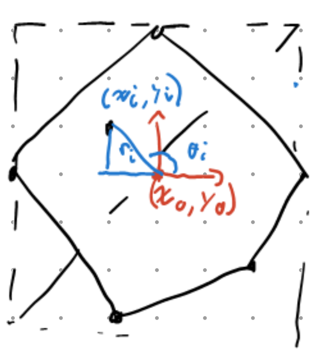
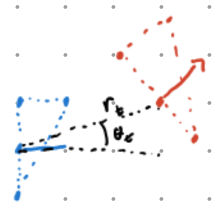




formation

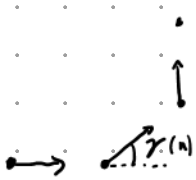
$$(x_i, y_i) = (r_i \cos\theta_i + x_0,\ r_i \sin\theta_i + y_0)$$

$$(x_2, y_2) = (r_t \cos\theta_t + x_1,\ r_t \sin\theta_t + y_1)$$



$$(x_i(n+1), y_i(n+1)) = (r_i \cos\theta_i + r_t(n)\cos\theta_t(n) + x_0(n),\ r_i \sin\theta_i + r_t(n)\sin\theta_t(n) + y_0(n))$$

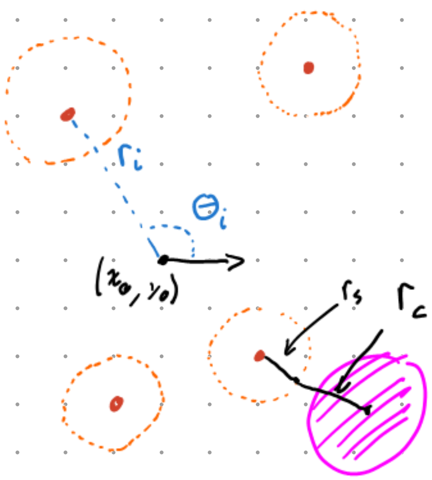
$$= (r_i \cos(\theta_i + \gamma(n)) + r_t(n)\cos\theta_t(n) + x_0(n),\ r_i \sin(\theta_i + \gamma(n)) + r_t(n)\sin\theta_t(n) + y_0(n))$$



= robots   = safe radius

every robot has some (r_i, θ_i) w.r.t the arbitrary center

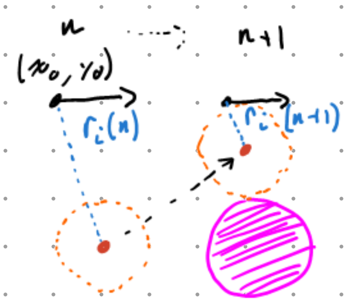
= obstacle



$$r_b = \min_{(i,j)} \left(r_{s_i} + r_{c_{ij}}\right) \quad \min_{(i,j)} \left(r_s + r_{c_{ij}}\right)$$

$r_c \triangleq$ critical radius (radius from obstacle to safe perimeter of robot

$r_b \triangleq$ bottleneck radius (minimum distance from robot to any known obstacle)



$$\left(x_i(n),\ y_i(n)\right) = \left(r_i(n)\cos\theta_i(n) + x_0(n),\ r_i \sin\theta_i(n) + y_0(n)\right)$$

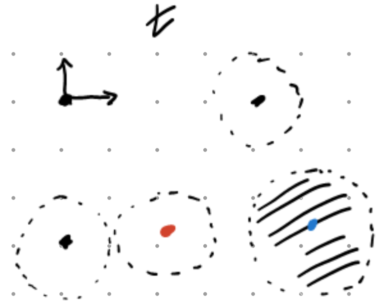
$$1 - \frac{r_i}{r_i^*} = \alpha$$

$$\begin{pmatrix} x_i(n+1) \\ y_i(n+1) \end{pmatrix} = \vec{O} \begin{pmatrix} x_i(n) \\ y_i(n) \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$J = \min_{u, \alpha} u^2 + \alpha$$

$$\gamma = \theta_i^* - \theta_i(t)$$

$$J = \min_{\alpha} \ \alpha + \gamma^2$$



$$\alpha \geq 1 - \frac{r_s}{r_i^*}$$

$$L_g h_s(x(t)) = g(x_i(t)) \ \frac{\partial h_s}{\partial x_i}(x_i(t))$$

parameters:

- $x_0(t+1)$
- $x_j(t) \quad \forall j \neq i$
- $O(t)$

note $\{x_j(t), O(t)\} \triangleq O_x(t,i)$

$$h_{s_i} = \log\left(\sum_{l=1}^{N} \exp\left(h_{s_{il}}\right)\right) \quad \leftarrow \quad h_{s_i} = \max_j\left(\left(r_{s_i} + r_{\sigma_j}\right) - \text{norm}\left(x_i(t) - \sigma_j(t)\right)\right)$$

where $\sigma_j(t) \in O_x(t,i)$

$r_{\sigma_j} \triangleq$ safe radius for $\sigma_j(t)$

known

need to find

$$\frac{\partial h_{s_i}}{\partial h_{s_{ij}}} \ \frac{\partial h_{s_{ij}}}{\partial x_i}$$

$$\frac{\partial h_{s_{ij}}}{\partial x_i} = \left(\frac{x_i - x_j}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}, \ \frac{y_i - y_j}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} \right)$$

$$-\frac{\partial}{\partial(x_i, y_i)}\left(\underbrace{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}_{\|x_i - \sigma_j\|}\right) = -\left(\frac{1}{2} \ \frac{2(x_i - x_j)}{\|x_i - \sigma_j\|}, \ \frac{1}{2} \ \frac{2(y_i - y_j)}{\|x_i - \sigma_j\|}\right)$$