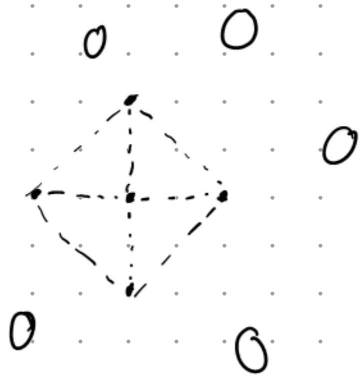
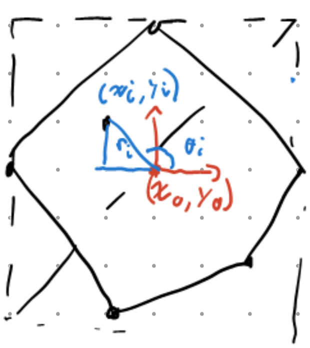
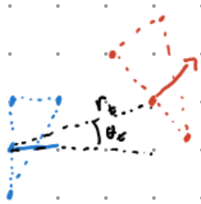




formation

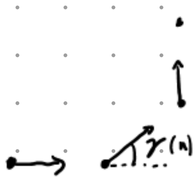
$$(x_i, y_i) = (r_i \cos\theta_i + x_0, r_i \sin\theta_i + y_0)$$

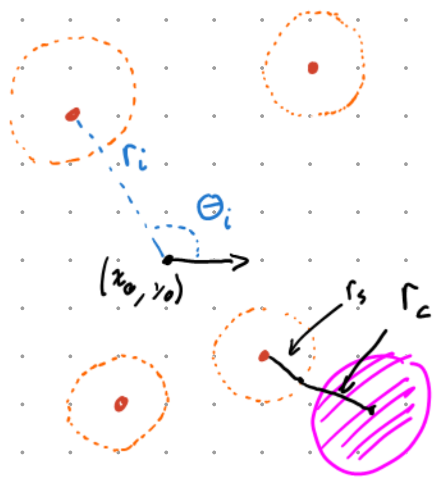
$$(x_2, y_2) = (r_t \cos\theta_t + x_1, r_t \sin\theta_t + y_1)$$



$$(x_i(n+1), y_i(n+1)) = (r_i \cos\theta_i + r_t(n)\cos\theta_t(n) + x_0(n), r_i \sin\theta_i + r_t(n)\sin\theta_t(n) + y_0(n))$$

$$= (r_i \cos(\theta_i + \gamma(n)) + r_t(n)\cos\theta_t(n) + x_0(n), r_i \sin(\theta_i + \gamma(n)) + r_t(n)\sin\theta_t(n) + y_0(n))$$





= robots = safe radius

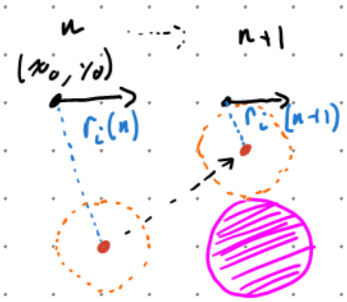
every robot has some (r_i, θ_i) w.r.t the aribitrary center

= obstacle

$$r_b = \min_{(i,j)} \left(r_{s_i} + r_{c_{ij}}\right) \quad \min_{(i,j)} \left(r_s + r_{c_{ij}}\right)$$

$r_c \triangleq$ critical radius (radius from obstacle to safe perimeter of robot

$r_b \triangleq$ bottleneck radius (minimum distance from robot to any known obstacle)

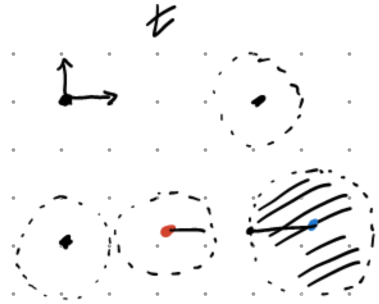


$$\left(x_i(n),\ y_i(n)\right) = \left(r_i(n)\cos\theta_i(n) + x_0(n),\ r_i \sin\theta_i(n) + y_0(n)\right)$$

$$1 - \frac{r_i}{r_i^*} = \alpha$$

$$\begin{pmatrix} x_i(n+1) \\ y_i(n+1) \end{pmatrix} = \vec{0} \begin{pmatrix} x_i(n) \\ y_i(n) \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$J = \min_{u, \alpha} u^2 + \alpha$$



$L_g h_s(x(t)) = g(x_i(t)) \, \frac{\partial h_s}{\partial x_i}(x_i(t))$

parameters:

- $x_o(t+1)$
- $x_j(t) \quad \forall j \neq i$
- $O(t)$

note $\{x_j(t), O(t)\} \triangleq O_x(t,i)$

$h_{s_i} = \log\left(\sum_{l=1}^{N} \exp\left(h_{s_{il}}\right)\right)$ $\leftarrow$ $h_{s_i} = \max_j\left(\left(r_{s_i} + r_{\sigma_j}\right) - \text{norm}\left(x_i(t) - \sigma_j(t)\right)\right)$

where $\sigma_j(t) \in O_x(t,i)$

$r_{\sigma_j} \triangleq$ safe radius for $\sigma_j(t)$

known — need to find

$\frac{\partial h_{s_i}}{\partial h_{s_{ij}}} \, \frac{\partial h_{s_{ij}}}{\partial x_i}$

$$-\frac{\partial}{\partial(x_i,y_i)}\left(\underbrace{\sqrt{(x_i-x_j)^2+(y_i-y_j)^2}}_{\|x_i-\sigma_j\|}\right) = -\left(\frac{1}{2}\,\frac{2(x_i-x_j)}{\|x_i-\sigma_j\|},\ \frac{1}{2}\,\frac{2(y_i-y_j)}{\|x_i-\sigma_j\|}\right)$$

$$= -\left(\frac{\tilde{x}_{i,w}-\tilde{x}_{j,w}}{\|\tilde{x}_{i,w}-\tilde{\sigma}_{j,w}\|},\ \frac{\tilde{y}_{i,w}-\tilde{y}_{j,w}}{\|\tilde{y}_{i,w}-\tilde{\sigma}_{j,w}\|}\right)$$

$x^*_{rel} = x^*_w - x_0$

$\tilde{x}_{rel} = \tilde{x}_w - x^*_w = \tilde{x}_w - r^*\begin{pmatrix}\cos(\theta^*+\gamma)\\ \sin(\theta^*+\gamma)\end{pmatrix} - x_0$

$\tilde{x}_w = \tilde{x}_{rel} + x^*_w$

$= \tilde{x}_{rel} + r^*\begin{pmatrix}\cos(\theta^*+\gamma)\\ \sin(\theta^*+\gamma)\end{pmatrix} + x_0$

$$= -\left(\frac{\tilde{x}_{i,rel}-\tilde{x}_{j,rel}}{\|\tilde{x}_{i,rel}-\tilde{\sigma}_{j,rel}\|},\ \frac{\tilde{y}_{i,rel}-\tilde{y}_{j,rel}}{\|\tilde{x}_{i,rel}-\tilde{\sigma}_{j,rel}\|}\right)$$

$$J = \min \ \frac{1}{2} u^T H u + F^T u$$

$$u = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x_i^*(t+1) = \begin{pmatrix} r_i^* \cos(\theta_i^* + \gamma) \\ r_i^* \sin(\theta_i^* + \gamma) \end{pmatrix} + \cancel{x_0(t+1)}$$

$$h_s(x_i) = e^{\tau(x_i)}$$

$$\frac{\partial h_s(x_i)}{\partial x_i} = e^{\tau(x_i)} \frac{\partial \tau(x_i)}{\partial x_i}$$

$$\tau(x_i) = \min \left(\|x_i - x_j\| - (r_{s_i} + r_{s_j}) \right)$$

$$\tau(x_i) = -\max \Big(\underbrace{(r_{s_i} + r_{s_j}) - \|x_i - x_j\|}_{K(x_i)} \Big) \geq 0$$

$$Au \leq b$$

$$L_g h_s(x) u + \delta h(x) \geq 0$$

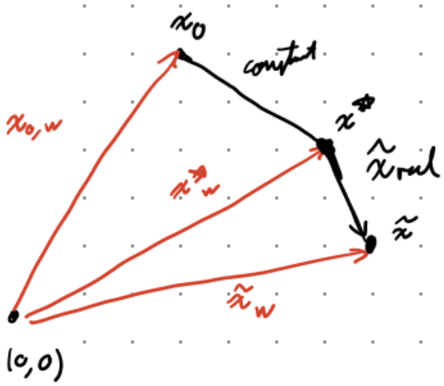
$$\alpha(h(x)) = \delta h(x)$$

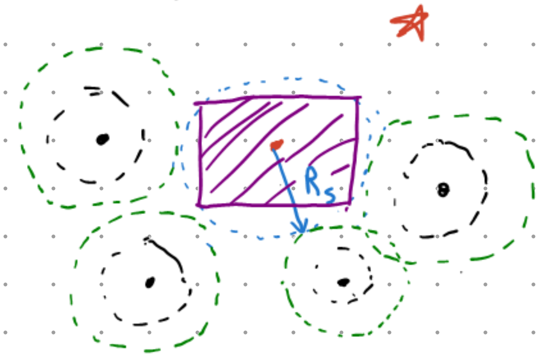
$$-L_g h_s(x) u - \delta h(x) \leq 0$$

$$-\delta \leq -0.0001$$

$$\delta > 0$$

$$-\frac{R_s\sqrt{2}}{2}$$





$$lb \leq \vec{x} \leq ub$$

$$R_s = \min\left(\|x_i - x_j\| - (r_s + r_{s,j})\right)$$

$$lb = \begin{pmatrix} -\frac{R_s\sqrt{2}}{2} + \tilde{x} \\ -\frac{R_s\sqrt{2}}{2} + \tilde{y} \end{pmatrix}$$

λ

$$ub = \begin{pmatrix} \frac{R_s\sqrt{2}}{2} + \tilde{x} \\ \frac{R_s\sqrt{2}}{2} + \tilde{y} \end{pmatrix}$$