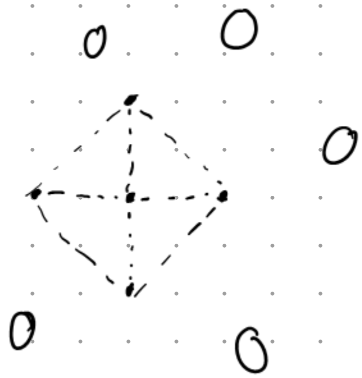
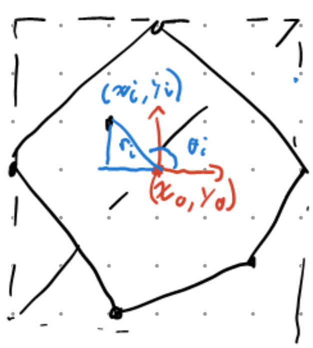
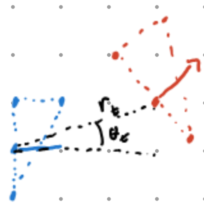




formation

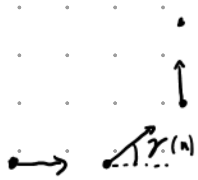
$$(x_i, y_i) = (r_i \cos\theta_i + x_0,\ r_i \sin\theta_i + y_0)$$

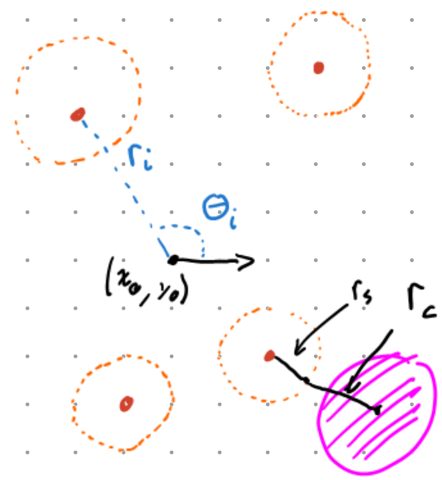
$$(x_2, y_2) = (r_t \cos\theta_t + x_1,\ r_t \sin\theta_t + y_1)$$



$$(x_i(n+1),\ y_i(n+1)) = (r_i \cos\theta_i + r_t(n)\cos\theta_t(n) + x_0(n),\ r_i \sin\theta_i + r_t(n)\sin\theta_t(n) + y_0(n))$$

$$= (r_i \cos(\theta_i + \gamma(n)) + r_t(n)\cos\theta_t(n) + x_0(n),\ r_i \sin(\theta_i + \gamma(n)) + r_t(n)\sin\theta_t(n) + y_0(n))$$





= robots

= safe radius

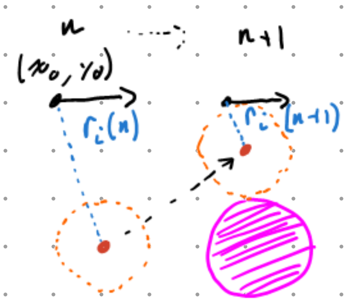
every robot has some (r_i, θ_i) w.r.t the arbitrary center

= obstacle

$$r_b = \min_{(i,j)} \left(r_{s_i} + r_{c_{ij}} \right) \quad \min_{(i,j)} \left(r_s + r_{c_{ij}} \right)$$

$r_c \triangleq$ critical radius (radius from obstacle to safe perimeter of robot

$r_b \triangleq$ bottleneck radius (minimum distance from robot to any known obstacle)

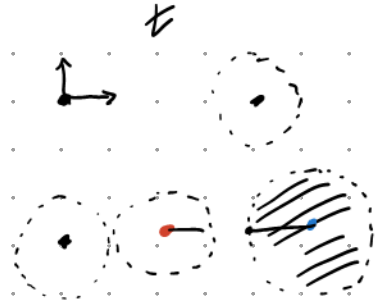


$$\left(x_i(n),\ y_i(n) \right) = \left(r_i(n) \cos \theta_i(n) + x_0(n), r_i \sin \theta_i(n) + y_0(n) \right)$$

$$1 - \frac{r_i}{r_i^*} = \alpha$$

$$\begin{pmatrix} x_i(n+1) \\ y_i(n+1) \end{pmatrix} = \vec{O} \begin{pmatrix} x_i(n) \\ y_i(n) \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$J = \min_{u, \alpha} u^2 + \alpha$$



$$L_g h_s(x(t)) = g(x_i(t)) \; \frac{\partial h_s}{\partial x_i}(x_i(t))$$

parameters:

- $x_0(t+1)$
- $x_j(t) \quad \forall j \neq i$
- $O(t)$

  note $\{x_j(t), O(t)\} \triangleq O_x(t;i)$

$$h_{s_i} = \log\left(\sum_{l=1}^{N} \exp\left(h_{s_{il}}\right)\right) \quad \leftarrow \quad h_{s_i} = \max_j\left(\left(r_{s_i} + r_{\sigma_j}\right) - \mathrm{norm}\left(x_i(t) - \sigma_j(t)\right)\right)$$

where $\sigma_j(t) \in O_x(t;i)$

$r_{\sigma_j} \triangleq$ safe radius for $\sigma_j(t)$

$$\underbrace{\frac{\partial h_{s_i}}{\partial h_{s_{ij}}}}_{\text{known}} \; \underbrace{\frac{\partial h_{s_{ij}}}{\partial x_i}}_{\text{need to find}}$$

$$\frac{\partial h_{s_{ij}}}{\partial x_i} = \left(\frac{x_i - x_j}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}, \; \frac{y_i - y_j}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}\right)$$

$$- \frac{\partial}{\partial(x_i, y_i)}\left(\underbrace{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}_{\|x_i - \sigma_j\|}\right) = -\left(\frac{1}{2}\,\frac{2(x_i - x_j)}{\|x_i - \sigma_j\|}, \; \frac{1}{2}\,\frac{2(y_i - y_j)}{\|x_i - \sigma_j\|}\right)$$

$$= -\left(\frac{\tilde{x}_{i,w} - \tilde{x}_{j,w}}{\|\tilde{x}_{i,w} - \tilde{\sigma}_{j,w}\|}, \; \frac{\tilde{y}_{i,w} - \tilde{y}_{j,w}}{\|\tilde{y}_{i,w} - \tilde{\sigma}_{j,w}\|}\right)$$

$x^*_{rel} = x^*_w - x_0$

$\tilde{x}_{rel} = \tilde{x}_w - x^*_w$

$$\tilde{x}_w = \tilde{x}_{rel} + x^*_w$$

$$= \tilde{x}_{rel} + r^*\begin{pmatrix}\cos(\theta^* + \gamma) \\ \sin(\theta^* + \gamma)\end{pmatrix} + x_0$$

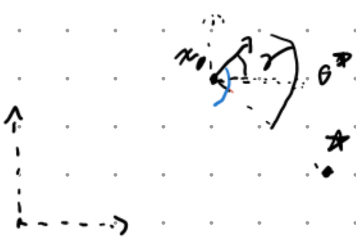
$$= -\left(\frac{\tilde{x}_{i,rel} - \tilde{x}_{j,rel}}{\|\tilde{x}_{i,rel} - \tilde{\sigma}_{j,rel}\|}, \; \frac{\tilde{y}_{i,rel} - \tilde{y}_{j,rel}}{\|\tilde{x}_{i,rel} - \tilde{\sigma}_{j,rel}\|}\right)$$

$$J = \min \ \frac{1}{2} u^T H u + F^T u$$

$$u = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x_i^*(t+1) = \begin{pmatrix} r_i^* \cos(\theta_i^* + \gamma) \\ r_i^* \sin(\theta_i^* + \gamma) \end{pmatrix} + \cancel{x_0(t+1)}$$

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



$$J = \min \ (\underbrace{x^* - x})^2$$

$$= \underset{r,\theta}{\arg\min} \ \frac{1}{2} \begin{pmatrix} x^* - x, & y^* - y \end{pmatrix} \overset{H}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} \begin{pmatrix} x^* - x \\ y^* - y \end{pmatrix}$$

$\begin{pmatrix} x^* \\ y^* \end{pmatrix}$ relative linearization

$$\left(\begin{pmatrix} r_i^* \cos(\theta_i^* + \gamma) \\ r_i^* \sin(\theta_i^* + \gamma) \end{pmatrix} - \begin{matrix} x_{rel} \\ y_{rel} \end{matrix} \right)^2$$

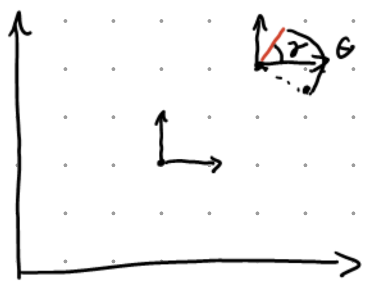
taylor series

$$J = \min G(x) \qquad S = \vec{0}$$

s.t

$$\cancel{L_f h(x)} + L_g h(x) u - \alpha(h(x)) \le 0 \Rightarrow \dot{x} = \cancel{f(x)} + g(x) u$$

$$\frac{\partial h(x)}{\partial x} - c\,h(x) \le 0$$



$$\begin{pmatrix} x_w \\ y_w \end{pmatrix} \begin{pmatrix} x_{rel} \\ y_{rel} \end{pmatrix} \begin{pmatrix} r_{rel} \\ \theta_{rel} \end{pmatrix}$$

$$\left(\begin{pmatrix} r^* \cos(\theta^* + \gamma) + \cancel{x_0} \\ r^* \sin(\theta^* + \gamma) + \cancel{y_0} \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \right)^2$$