PVMT 2022: Algebra/NT Tiebreakers

Problem 1

How many numbers less than 1000 have an odd number of divisors?

Problem 2

Find the maximum possible value of $3x + 4y$, given that $x^2 - 4x + y^2 - 6y = 12$.

Problem 3

Let the roots of the polynomial

$$x^3 + 4x^2 - 7x + 2$$

be a, b, and c. Find the magnitude of the expression:

$$\frac{a}{b} + \frac{a}{c} + \frac{b}{a} + \frac{b}{c} + \frac{c}{a} + \frac{c}{b}$$