Blue Matter, An Application Framework for Molecular Simulation on Blue Gene*

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Abstract

In this paper we describe the context, architecture, and challenges of Blue Matter, the application framework being developed in conjunction with the science effort within IBM's Blue Gene project. The study of the mechanisms behind protein folding and related topics can require long time simulations on systems with a wide range of sizes and the application supporting these studies must map efficiently onto a large range of parallel partition sizes to optimize scientific throughput for a particular study. The design goals for the Blue Matter architecture include separating the complexities of the parallel implementation on a particular machine from those of the scientific simulation as well as minimizing system environmental dependencies so that running an application within a low overhead kernel with minimal services is possible. We describe some of the parallel decompositions currently being explored that target the first member of the Blue Gene family, BG/L, and present simple performance models for these decompositions that we are using to prioritize our development work. Preliminary results indicate that the high performance networks on BG/L will allow us to use FFT-based techniques for periodic electrostatics with reasonable speedups on 512-1024 node count partitions even for systems with as few as 5000 atoms.

Key words:

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1 Introduction

In December 1999, IBM announced the start of a five year effort to build a massively parallel computer to be applied to the study of biomolecular phenomena such as protein folding.[1] The project has two goals: advancing our understanding of the mechanisms behind biologically important phenomena such as protein folding via large scale simulation and exploring novel ideas in massively parallel machine architecture and software. This project should enable biomolecular simulations that are orders of magnitude larger and longer than current technology permits. The first member of the Blue Gene machine family is Blue Gene/L, for which a 512 node prototype is projected to become available in the second half of 2003, with a fully operational machine scheduled for the 2004-2005 time frame. The machine architecture has evolved significantly since the inception of the project, but one aspect that has remained constant is the use of a three dimensional mesh interconnect topology, a feature that may require the application to be aware of the machine topology in order to achieve optimal performance. This has not been the case for other recent large parallel machines such as ASCI White. Another unusual feature of this machine is the availability of a second high performance interconnect network to provide low-latency and high bandwidth global broadcast and reduction capabilities. More details about the machine that are most relevant to an application developer are provided in the body of this paper and additional information can be found in a recent paper by the Blue Gene team[2]. The study of the mechanisms behind protein folding and related topics can require long time simulations on systems with a wide range of sizes and the application supporting these studies must map efficiently onto a large range of parallel partition sizes to optimize scientific throughput for a particular study. Typical system sizes of interest range from 5000 atoms (for a small peptide in water) through 30,000 atoms (for a folded 200-residue protein in water) up to 200,000 atoms or more (for a protein-membrane system or large unfolded protein in water).

To support the goals of the Blue Gene project related to protein science and to explore novel application programming approaches for massively parallel machine architectures in the context of a concrete problem, we have been working on an application framework, Blue Matter, which is currently focused on biomolecular simulation. It is hoped that the lessons learned and even some of the components developed can be reused in other application domains. One of the principal design goals for this framework is the effective logical separation of the complexities of programming a massively parallel machine from the complexities of biomolecular simulation through the definition of appropriate interfaces. Encapsulation of the semantics of the biomolecular simulation methodologies means that the application can track the evolution of the machine architecture and explorations of various parallel decomposition schemes can take place with minimal intervention from the domain experts on the team, who are also the end users of the Blue Matter application. Furthermore, we have decomposed the application into a core parallel engine with minimal system environmental requirements running on BG/L and a set of support modules providing setup, monitoring, and analysis functionality that can run on other host machines. Minimizing system environmental requirements for the core parallel engine enables the use of non-pre-emptive low overhead parallel operating system (OS) kernels[3] to enable scalability to thousands of nodes.

In order to prioritize our development efforts, we are exploring various mappings of the Blue Matter application onto the BG/L platform via simple models. One area of interest is the exploitation of the high performance networks available on Blue Gene/L in a way that minimizes potential software overheads. If we can find parallel decompositions that directly take advantage of the hardware communications facilities, we will reduce the software overheads that may limit the maximum partition for a fixed size problem, thereby maximizing throughput for a given molecular simulation.

1.1 Machine Overview

Blue Gene/L is a cellular architecture machine built of nodes containing a single application-specific integrated circuit (ASIC) and additional off-chip memory. Each ASIC has two IBM PowerPC processors with a projected clock speed of 700 MHz as well as cache memory and communications logic. The target configuration contains a maximum of 65,536 compute nodes with several communications networks. The networks that are of primary interest to application developers are the three-dimensional torus that connects each node to its six nearest neighbors with a link bandwidth of 175 MB/s bidirectional (2 bits/cycle), a physically separate global combining/broadcast tree with bandwidth of 350 MB/s (4 bits/cycle) and a 1.5 microsecond one-way latency on a 64K node partition, and a global barrier/interrupt network. BG/L can be electronically partitioned into multiple systems with a currently envisioned minimum size of 512 nodes, each of which has its own complete set of networks. The target peak performance of the full 65,536 node configuration of BG/L is 180/360 TeraFLOPs, where the lower number corresponds to scenarios in which one processor exclusively handles communication tasks, and the larger number assumes the application can take full advantage of both processors for computation.

1.2 Blue Gene Science

Understanding the physical basis of protein function is a central objective of molecular biology. Proteins function through internal motion and interaction with their environment. An understanding of protein motion at the atomic level has been pursued since the earliest simulations of their dynamics [4]. When simulations can connect to experimental results, the microscopic view of processes revealed by simulation acquire more credibility and the simulation results can help interpret the experimental data [5]. Improvements in computational

power and simulation methods have led to important progress in studies of protein structure, thermodynamics, and kinetics [4,6–8].

1.3 Methodological Implications for the Application

The validity of a molecular simulation depends strongly on the the quality of the force field, including the representation of water, the treatment of long-range intermolecular interactions, and the simulated environment of the protein. Validity comes at a cost, however, both in the computational expense of the simulation and the complexity of the molecular dynamics (MD) software.

Biological molecules are surrounded by water, so the representation of water in a simulation has a critical effect on its quality. Explicit solvent simulations represent the atoms of each of the water molecules around a protein, and calculate all of the intermolecular forces and dynamics associated with those atoms. The expense of explicit water is significant since the number of atoms contributed by the solvent can far exceed that of the protein itself. In order to avoid boundary effects in explicit water simulations, it is typical to treat the simulated system with periodic boundary conditions. Alternatively, implicit solvent models of various levels of sophistication have been used which typically represent the water as a continuum dielectric, optionally coupled with a cavity surface tension term to model the hydrophobic effect. Although implicit solvent models have produced important results in some areas, there are important cases where significant differences exist in the results obtained from an implicit as opposed to an explicit treatment of water[9].

A major feature of molecular simulation that poses an implementation challenge is the treatment of long-range interactions. For biological systems, this term arises from the long-ranged Coulombic interaction between charged amino acids or ions. A simplification is to only include the interactions of atoms within a short cutoff radius of each other, which reduces the computational burden, but in systems with charged groups or free charges this approach can lead to unphysical artifacts[10]. The periodic representations used for explicit solvent simulations can be treated correctly with the use of the Ewald summation technique[11] or related methods[12] which typically require the use of Fourier transform techniques. These techniques introduce global data dependencies and are correspondingly complex to parallelize efficiently. Nonetheless, this addition to the MD code is important for the accuracy of the results and these techniques, with their scalability consequences, are essential for large scale simulations that can match experiment.

Another feature in molecular simulation that demands added complexity is the specific ensemble being simulated, e.g. whether the simulated system is in a constant pressure or constant volume environment, and whether or not it is coupled to a heat bath. Supporting these capabilities requires additional code, communication, and added barriers to synchronize the execution of parallel tasks. In particular, the calculation of the instantaneous internal "pressure" inside the simulation cell requires a global reduction of all the interparticle forces projected along their displacements from the origin.

2 Application Architecture

2.1 Overview

Blue Matter is a software framework for performing biomolecular simulations primarily targeting parallel computing platforms containing thousands of nodes. At the highest level, the Blue Matter architecture specifies a modular decomposition and has been implemented as independent subprograms that cooperate via architected interfaces. By using concepts from generic programming[13] and defining appropriate interfaces, we have been working towards

a separation of the complexity of molecular dynamics simulation from the complexity of parallel programming with minimal impact on performance. The Blue Matter architecture requires infrastructure to support extensive regression and validation because of the aggressive and experimental nature of the computational platform we are targeting. As part of the effort to separate the molecular dynamics domain from the implementation of the framework, it is essential that functional correctness be established by validating the results of test simulations rather than by examining code.

Figure 1 shows the relationship between the major modules in the Blue Matter framework. A representation of the molecular system and its associated simulation parameters is created in a relational database as part of the set up process. This representation forms part of the experimental record which, along with information about specific simulation runs such as hardware platform type and configuration, can be accessed during analyses of the simulation data. A kernel generation module retrieves the data associated with a particular molecular system from the database and creates a Molecular System Definition (MSD) file containing C++ code which is then compiled and linked with framework source code libraries to produce an executable. While running on a suitable platform, the executable sends data to a host computer for consumption by analysis and monitoring modules via a raw datagram (RDG) stream.

The Blue Matter framework has been architected to allow exploration of multiple application decompositions with each implementation reusing the domain specific modules and semantics. Currently, this framework is specialized by selecting force field functional forms and simulation methods. Our strategy for separating MD complexity from parallel complexity includes factoring as much domain specific knowledge as possible into the setup phase. The framework is designed to manipulate domain function in very fine grained functional units, embodied by call-back functions that we refer to as User Defined Functions (UDFs), that encapsulate localized domain specific data transformations. The call-backs are resolved at

compile-time rather than run-time for performance reasons. Flexibly managing the granularity of the domain decomposition is important because of the range of node count and molecular system sizes we are targeting. Implementing a new parallel decomposition or communications pattern in this framework does not involve changing the UDFs or restating the simulation semantics. Using this generic infrastructure, we are able to support several force fields and map them onto drivers suitable to the computational platform available.

2.2 Set-up

The set-up phase of Blue Matter collects all the non-dynamic information required for a given simulation into a database and generates the source code specific to that run. First, the user runs a commercially available simulation package (e.g. CHARMm[14], Impact[15], AMBER[16], GROMOS[17]) to assign force field parameters for the molecular system. A package-specific filter then transforms the output into an XML file that represents the molecular system specification in a package-neutral form which is then loaded into a relational database. Relational constraints within the database schema maintain internal consistency of the molecular system where possible. The user then specifies simulation parameters that are stored in the relational database. This phase of set-up associates identifiers with both the inserted molecular system and the specific simulation parameters and allows us to maintain an audit path for our scientific and application development explorations including the ability to execute ad hoc queries to find molecular systems and parameter sets that have already been created.

Next, the kernel generation module (KGM) generates specialized source code for the simulation using information stored in the database. Since the KGM must recognize a wide variety of system specification parameters and create corresponding code, it must encapsulate the molecular dynamics domain knowledge. The KGM generates a customized molecular system

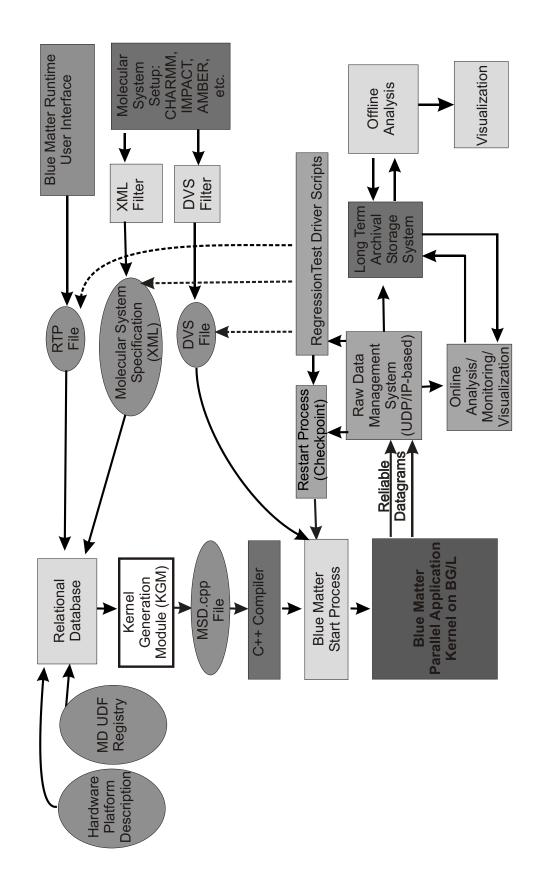


Fig. 1. Overview of components and high level data flow within the application.

definition (MSD) file containing parameters and C++ code containing only the desired code paths in the parallel kernel. Minimizing the lines of source code allows more aggressive and efficient optimization by the compiler. Although many parameters are built into the executable, others are provided at runtime through the dynamic variable set (DVS) file, such as positions and velocities.

2.3 Generic Molecular Dynamics Framework

Blue Matter encapsulates all molecular dynamics functions that transform data in User Defined Functions (UDFs). The term UDF comes from the database community where it refers to an encapsulated data transformation added to the built-in functions that may be used during a database query. Typically, these modules encapsulate domain specific knowledge at the most fine grained level possible. An example is the standard coulomb UDF, which operates on two charge sites and yields the force between them.

The UDF interface abstracts the calling context, allowing the framework flexibility in organizing access to parameters and dynamic variables. For example, this flexibility allows the UDF driver implementation to *tile* the invocations, an established technique for improving data locality[18]. In addition, the interface enables the UDF function to be aggressively inlined into the calling context.

UDF adapters and helpers are modules containing code that will be used by more than one UDF. An *adapter* is a module that implements the framework UDF interface and also wraps another UDF. A *helper* is a subroutine that is called by a UDF via standard C++ method invocation interfaces and is passed values from the immediate UDF context. Adapters allow generic code to be encapsulated. For example, the generation of a virial for pressure control can be added to a standard UDF by means of an adapter. Adapters are written as C++

template classes taking an existing UDF as a parameter.

UDFs represent pure data transformations that may be invoked from several drivers. The semantics of data movement are implemented in the selection of UDF drivers. For example, drivers exist to perform pairwise N^2 operations (NSQ) as well as to run lists of operands. Drivers used for a given simulation are selected during kernel generation along with the appropriate UDFs that appear as template parameters in the driver class. Different drivers may be written for different hardware platforms and/or load balancing strategies.

UDF drivers may be implemented in several ways while preserving the same semantics. For example, a set of drivers may support different NSQ parallel decompositions. The drivers themselves may be inherently parallel with an instance on every node, each executing only a fraction of the total number of UDF invocations. The implementation of the UDF driver is where specific knowledge of the target architecture is used to improve performance. In the case of the Cyclops architecture[1], which was the initial target hardware platform for the Blue Gene project, the drivers had to be implemented to issue enough independent work items to load 256 physical threads per chip. In the case of the BG/L architecture[2], it is important to feed the processor instruction pipeline and to enable reuse of framework recognizable intermediate values such as the distance between two atoms. Although these two hardware architectures are quite different in their requirements, the design of Blue Matter made it possible to adapt to both environments.

To facilitate early adoption of the experimental platforms which it targets, the Blue Matter framework has been designed to have minimal dependency on system software. The modularity of the framework allows much of the program to run on a stable server. To run the parallel core in a new node environment requires a C++ compiler, a minimal C++ runtime, the ability to send and receive datagrams to external machines, and a high performance communications mechanism.

Blue Matter can target two primary communications interfaces for inter-node messaging: MPI and active messaging. An explicit layer is most appropriate when the application is operating in distinct phases with relatively large grained parallelism. For large node counts, where scalability is bounded by the overhead of a very fine grained decomposition, we target a simple active message interface. For currently available machine configurations, the selection of communications library is handled at an interface layer by selecting either MPI or active message based collectives. For BG/L, we are integrating the framework drivers more directly with active messaging to enable lower overhead and the overlapping of communication and computation phases.

2.3.1 Online Monitoring/Analysis

The traditional view of a computational simulation has scientific code running on a computer, periodically dumping results to a file system for later analysis. Blue Matter breaks from this standard model by avoiding direct file i/o and instead communicating all results via packets over a reliable datagram connection to analysis tools listening on remote computers. The resulting packet stream is the raw datagram output (RDG). Outputting a packet stream offloads a great deal of the complexity of managing state and file formats from the kernel to analysis tools that are less performance-critical. The parallel kernel may emit packets out of sequence, and it is up is up to the host monitoring module to marshal a consistent set of packets for each time step, indicating the code is operating correctly and allowing well-defined parsing. Since the RDG stream is binary we retain the full precision of the results for comparison and analysis.

Analyzing the RDG stream involves a combination of online processing of the arriving packets and selective storing of pre-processed data for efficient offline analysis. For molecular simulations, the basic online analysis involves parsing the packet stream and determining the value of the energy terms at each time step, while storing the full set of atom positions and velocities at regular intervals. For each time step containing atom positions we calculate, as a pre-processing task, such features as the presence of hydrogen bonds, the secondary structure, radii of gyration, and more. The decision to calculate a term online and store the result during the run depends on the expense of the calculation and the size of the stored result.

The data stored during a computer simulation serve two distinct purposes:

- (1) If the time evolution of the simulation is important, one must record periodic snapshots of the simulation for later analysis since the final state of the simulation does not convey the dynamic path, or "trajectory," taken from the initial state. These snapshots need not include the full state of the system for later restart from that point, but should contain all the information needed for anticipated trajectory analysis. In molecular simulations, saving the trajectory is essential if the dynamic evolution of the system is central to the experiment.
- (2) Apart from the scientific needs of recording the system evolution there is also a practical need to store the full system state for an exact restart of the simulation in case of hardware or software error that either terminates the simulation or causes inaccuracy in the output results. In general, the state needed for restart may be much larger than the state relevant to analysis but, fortunately, molecular dynamics has relatively small state even for large systems, dominated by the positions and velocities of all the atoms (on the order of megabytes).

2.3.2 Post-Analysis and Data Management

Since Blue Matter relies on a database for the set up of each system to be run, the database can serve not only to define the simulation, but to track the simulation runs of many scientists

because the stream contains a run identifier that directly binds to its full set of input data in the database including: 1) the molecular system itself, 2) its compile-time parameters, and 3) its initial configuration, i.e. atomic positions and velocities, plus other state variables. With all the run information automatically tracked in the database without the need for human input, powerful SQL queries across many independent runs become possible. The result is a high-integrity single source of information in a database that serves as an experimental record to track all the simulations, their parameters, and their results.

2.3.3 Regression/Validation Infrastructure and Strategy

A molecular dynamics simulation should be as fast as possible without compromising the validity of the scientific results, and an important part of MD code development is a suite of tools and test cases that provide this validation[19]. Blue Matter relies on a combination of external regression tests that compare with other packages, internal regression tests that track code changes, and validation tests that confirm the scientific correctness of specific runs. To facilitate the specification and execution of these tests on a cluster of workstations we use an environment that allows a concise specification of a set of related runs, which makes it easier to create, modify, and run tests that vary many parameters, despite the need to compile and build each executable corresponding to each run. A more detailed description of these tests follows.

Since Blue Matter supports a variety of standard force fields and accepts input of molecular systems created by other MD packages, it is essential to confirm that Blue Matter duplicates the energy components and forces of those packages to a high degree of precision. Blue Matter uses the molecular systems and their corresponding energy components and force results of an extensive suite of blocked amino acids and peptides prepared with CHARMm, AMBER and Impact (OPLS).[20] We use the set up phase of each MD package to create a system in

the corresponding force field, then use Blue Matter to compute the energy terms and confirm the results agree with the external regression suite.

One of the main validation checks on the code is to verify the extent to which total energy is conserved. Although the total energy is not strictly constant in time, the width of the total energy distribution should vary quadratically with the time step size for dynamical integrators based on simple Verlet-type algorithms and the total energy should not display systematic drifts over time. This is a sensitive test of the correctness in the implementation of the dynamics. The procedure involves running the same molecular system for the same amount of simulation time, using a range of time step sizes, and calculating the root mean square deviation (RMSD) of the total energy for each run. A correct implementation will show RMSD's that vary quadratically with time step to a fraction of a percent. Since accumulated errors cause trajectories with different time steps to depart from one another in phase space, we use short runs of a fraction of a picosecond. Other validation tests include checking the density of water in a long constant pressure run with different water models, confirming the equipartition theorem applies when constraints such as rigid bonds are in use, confirming heat capacity estimates are consistent those implied by the slope of energy-temperature curves from temperature-controlled simulations, and simpler checks of momentum and energy conservation in long runs.

The internal regression tests provide immediate feedback on code development, but since Blue Matter outputs results in a lossless binary form, differences can appear due to innocuous code changes such as a reordering of terms in a summation. In order to be sensitive to the larger changes in results that indicate a bug, while insensitive to benign changes, a per-system tolerance is incorporated into the tests that scales with system size and the length of the regression test. This helps automate the testing process since only errors above threshold raise a flag that warrants inspection of the code.

3 Issues in Mapping the Application onto the Machine

3.1 Interconnect Topologies (Torus and Tree)

The replication unit for BG/L is a single "system-on-a-chip" application-specific integrated circuit (ASIC) to which additional commodity DRAM chips are added to give 256 MB of external memory per node. 512 of these replication units or "cells" will be be assembled into an 8 × 8 × 8 cubic lattice "building block" as shown in Figure 2. The surfaces of this "building block" (in effect, 6 sets of 8 × 8 wires) connect to simpler "link chips" that send and receive signals over the relatively long distance to the next "building block". A supervisory computer can set up the links to partition the set of "building blocks" into the sizes of lattice requested by the applications. The result is three dimensional torus wiring and the ability to route around blocks that are out of service for maintenance. The ASICs also have connections for a "tree" network. Each ASIC has a simple arithmetic-logic unit (ALU) for the tree, enabling the tree to support "add reduction" (where each ASIC provides a vector of numbers; the vectors being added element-wise to their partners at each node until a single vector reaches the root; the vector is then broadcast back to all the ASICs). "max", "or", "and", and "broadcast" are also supported by the tree ALU.

For the internal (torus and tree) networks, there is hardware link-level error checking and recovery; an application on the compute fabric is assured that a message sent from a node will arrive intact and exactly once at its destination. All links carry data in both directions at the same time. The torus links carry 2 bits per CPU clock cycle while the tree links carry 4 bits per CPU clock cycle. For both networks, data are forwarded as soon as possible, with a per-node latency equivalent to the time required to send a few bytes, unless congestion on the outgoing link (torus and tree) or a wait for partner data (tree reduction operations) forces data to be buffered. There is no need for software intervention in any point-to-point

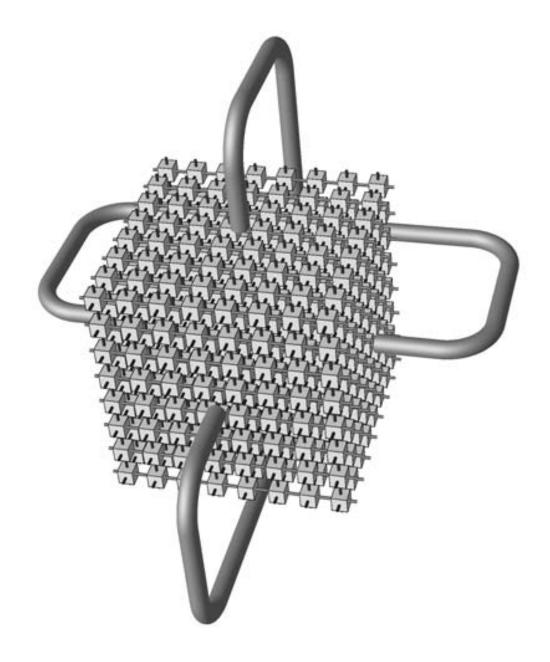


Fig. 2. View of a BG/L "brick" containing 512 nodes, showing the 3D torus network

 ${\it transmission.}$

3.2 Efficient use of the Floating Point unit

The Blue Gene/L CPU belongs to the IBM PowerPC family, and we use the IBM VisualAge compiler with a modified "back end" to generate code for it. Interaction with compiler developers has enabled better floating-point optimization for the current POWER3 architecture as well as for Blue Gene/L, which has resulted in a 30% improvement in the performance of Blue Matter when built to run on POWER3 machines.

On each ASIC there are 2 microprocessors. Each microprocessor is a PowerPC 440 microcontroller to which two copies of a PowerPC-type floating-point unit have been added. The CPU can run the normal PowerPC floating-point instruction set and supports additional instructions that operate on data in both floating-point units concurrently. Each floating-point unit has its own set of 32 registers at 64 bits wide. All floating-point instructions (except "divide") pass through a 5-cycle pipeline; provided there are no data dependencies, one floating-point instruction can be dispatched per clock cycle. The PPC440 can dispatch 2 instructions per clock cycle, thus peak floating-point performance would need 10 independent streams of data to work on, and the program logic would require a "multiply-add" operation at every stage. This would account for a "parallel multiply-add" instruction dispatched every clock cycle; the other dispatch slot in the clock cycle would be used for a load, store, integer, or branch op in support of the program logic to keep the floating-point pipeline busy and productive. It is straightforward to achieve this for a polynomial function f(x) that needs evaluation for 10 values of x, but for less regular code kernels, or poorly controlled vector lengths, the fraction of peak achieved depends on the skill of the programmer. The unoptimized case would dispatch only 1 instruction per clock cycle, not achieving any parallelization to use the second floating-point unit, and not exploiting the fused 'multiply' and 'add'; this would run at 5% of peak. We have already implemented some computational kernels, such as vector reciprocal square root, exponential, and inverse trigonometric functions, that have been set

up to exploit the double floating point unit by avoidance of conditional branches and other strategies.

3.3 Memory Hierarchy

Each CPU has a 32 KByte instruction cache and a 32 kByte data cache. The L1 line size is 32 bytes. There is no hardware support for keeping the L1 caches coherent between the CPUs. Associated with each CPU is a prefetcher, known as L2, that fetches 128 bytes at a time from L3. The ASIC contains 4 MBytes of DRAM as an L3 cache shared by both CPUs. The L1 can read 3 cache lines from L2 every 16 CPU clock cycles, and concurrently write 1 cache line every 2 CPU clock cycles. The L3 can keep up with this bandwidth from both CPUs.

The lack of coherency between the L1 data caches defines the ways in which both CPUs can be used together. Those we foresee are

- Structured application coding, for example using single-writer-single-reader queues and explicit memory synchronization points to move data between CPUs in a well-defined way even though L1 is not made coherent by hardware.
- CPUs used independently, as if they were on separate nodes joined by a network.
- Second CPU used as a Direct Memory Access (DMA) controller. The first CPU would perform all the computation, flush results as far as L3 cache, and set up DMA orders; the second CPU would copy data between memory and the network FIFO interfaces. A version of this mode would allow the second CPU to process active messages, where an incoming torus packet contains a function pointer and data, and the reception routine calls the function with the given data. Parallel loads/stores to registers in the double floating point unit request 16 bytes to be moved between registers and the relevant level in the

memory hierarchy every clock cycle.

• Compiler-driven loop partitioning. A compiler would spot a computationally-intensive loop (for example, 'evaluate the sine of each element of a vector'); assign part of the vector to each CPU; ensure that the relevant source data was flushed as far as L3; post the second processor to start work, then start work on its own portion; and synchronize with the second processor at completion.

3.4 Identifying Concurrency in the Application

As a starting point for any discussion about how to map an application onto a particular parallel architecture, it is essential to understand the fundamental limits to concurrency present in the algorithm. Of course, such estimates depend on the granularity of computation that one is willing to distribute. One way to view the application execution is as the materialization of a series of data structures that are transformed from one into the other by computation and/or communication. During a phase of computation, the number of possible data-independent operations determine the concurrency present during that phase of the application. A view of this sort representing the non-bonded force calculations in a molecular dynamics application using the Ewald technique is shown in Figure 3.

Molecular Dynamics proceeds as a sequence of simulation time steps. At each time step, forces on the atoms are computed; and then the equations of motion are integrated to update the velocities and positions of the atoms. At the start, corresponding to the leftmost box in Figure 3, the positions and velocities of all the particles are known and a data structure indexed by the particle identifier l can be distributed using any "hashing" function of l with the maximum possible number of hash slots equal to the number of particles, N. Both atom and volume based decompositions are possible.

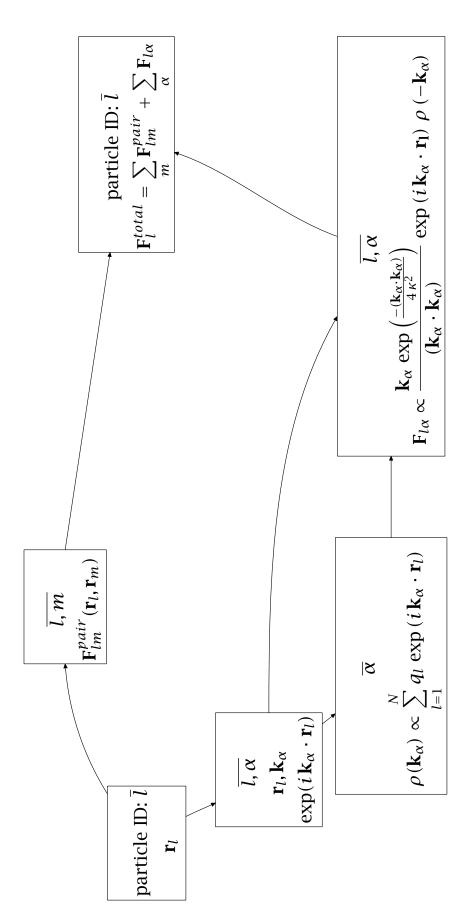


Fig. 3. Data dependencies of the major components of a molecular dynamics application using the Ewald Method

Following the upper branch of data dependencies, the next phase involves computation of all the pairwise forces between particles. This requires a data structure to be materialized that contains all pairs of particle positions so that the computation of the pair forces can take place. This structure can be indexed by a pair of particle identifiers l and m and indicates that distribution over a much larger number of hash slots, N^2 , is possible although the finite range cut-offs used for the pair potentials means that the actual number of non-vanishing pair computations is much smaller than N^2 . This is the phase of the application where an "interaction" decomposition is often used[21]. Next, a reduction is required to sum up the pairwise forces on each particle so that a data structure containing the total force on each particle, indexed by the particle identifier l can be created.

Along the lower branch of data dependencies, the calculation of the Ewald sum requires exponentials that are functions of particle position and \mathbf{k} -vector value. The \mathbf{k} -vector values are fixed and the number required is determined by the accuracy required in the Ewald sum. The data structure containing these values is indexed by the particle identifier l and the \mathbf{k} -vector index α . Next, components of the Fourier transform of the charge density need to be computed, which requires a reduction of the exponentials according to the \mathbf{k} -vector index α . The data structure containing the reciprocal space contributions to the force (indexed by particle identifier l and \mathbf{k} -vector index α) requires both the Fourier transform of the charge density and the exponentials computed earlier. These contributions must also be added into the total force on each particle. Finally, the data structure containing the total force on each particle, indexed by l, can be used to propagate the dynamics and give new values for the position and velocity of each particle.

The number of independent data items at each stage in this process is as follows:

 For stages that are partitionable by particle identifier, the number of independent data items is N, the number of particles.

- For the pair force computation stage, which is partitionable by pairs of particle identifiers, the theoretical limit on the number of data items is N^2 , but because of the finite range of the force expressions typically used in these computations, the actual number of independent computations possible is $\approx (1/2) N \rho (4/3) \pi r_c^3$ where ρ is the number density of atoms and r_c is the cutoff radius, beyond which the force between two particles vanishes.
- For the computations of the exponentials and the reciprocal space force that are partitionable by particle identifier l and k-vector index α, the number of independent data items is the product of the number of k-vectors required (typically a few hundred) and N, the number of particles.
- The computation of the Fourier transform of the charge density is partitionable by the
 k-vector index α, again giving a few hundred independent data items.

A similar analysis can be carried out for mesh-based Ewald techniques such as P3ME in which the solution of Poisson's equation is accelerated through use of the fast Fourier transform (FFT) after approximating the actual charge distribution by weights on a regularly spaced mesh. In the variant of P3ME that we will consider here, the *analytic* method, the meshed charge distribution is actually convolved with four kernels to give the electrostatic potential as well as the three components of the electric field within the simulation volume. This requires a single forward 3D-FFT followed by independent multiplications by each of the four kernels and then four inverse 3D-FFTs (one for each kernel). The parallel 3D-FFT itself involves significant communication that will be described below.

4 Case Studies of Parallel Decompositions and Scalability Projections

In this section we use relatively simple models to explore the scalability of various decomposition/parallelization schemes for molecular dynamics of macromolecular systems on BG/L. In studies of the scaling of fixed size problems to large numbers of nodes, there is no expec-

tation that ideal scaling will be observed.[22] However, for a specified problem size, useful parallel speed-ups should be achievable on a range of node counts with the upper limit on node count being determined by the portion of the computation with the least favorable scaling properties (Amdahl's law). Our intent is to use these model calculations to assist in making decisions about the relative viability of various approaches and to assess the balance between computation and communication within these approaches, not to make absolute performance predictions.

In the examples that follow, our focus is on communications patterns that map closely to the hardware capabilities of the BG/L architecture because we want to keep the potential for software overheads to a minimum. Some of the communications estimates used below are derived from work on a network simulator developed by members of the BG/L team while others, such as the capabilities of the tree network are taken directly from the designed capabilities of the hardware.[2] Cycle count estimates for computational kernels are taken from the output of the IBM C++ compiler that targets the BG/L architecture or from estimates of the time required for memory accesses. The assumptions built into the examples that follow are:

- System software overheads are neglected
- No overlap between computation and communication is utilized
- Only one CPU per node is available for computation
- Memory access times are neglected except where we have been able to identify them as dominating the time for computation (only for integration of the equations of motion).

Since all of our work requires accounting correctly for the effects of the long range interactions in the system, the first case study concerns the properties of a three-dimensional FFT on the BG/L architecture. The two other case studies, which deal with different mappings of molecular dynamics onto the BG/L hardware, both assume the same set of FFT-based

operations to support the P3ME method. Also, the projections made in these case studies neglect the contributions of bonded force calculations to the total computational burden since they represent relatively small fractions of the total and can be distributed well enough to keep their contribution to the per node computational burden small. The parameters used in the models are defined in Table 1. Although the actual machine has hardware support for partitioning down to a granularity of 512 nodes, we include smaller node counts in our model for illustrative purposes. Occasional "anomalies" visible in in graphs of the performance are due to the fact that since the machine is partitionable into multiples of eight nodes in each dimension, many node counts will be comprised of non-cubical partitions which affect the performance of the torus communications network.

In the model used to produce Figures 5 and 6, the contributions to $\tau_{p3me}(p, N_{mesh})$ comprise one forward and four inverse 3D real-valued FFTs to give the electrostatic potential and the three components of the electric field using the analytic method as described in Section 3.4.

4.1 Three Dimensional Fast Fourier Transform

The communication estimate for the 3D FFT assumes that three all-to-all communications within a processor plane or row are required for each FFT or its inverse. From simulations of the BG/L torus network, the following estimate for the time (in processor cycles) required for all-to-all communication of data that is distributed over a set of nodes in a line, plane, or volume is obtained[23]:

$$T_{all-to-all} = \frac{V_{received} \, \overline{N}_{hops}}{N_{links} \, BW \, f}$$

where $V_{received}$ is the volume of data received by each node, \overline{N}_{hops} is the average number of hops required (for a three dimensional torus where each dimension is p, $\overline{N}_{hops} = p/4$ for all-to-all in a line, $\overline{N}_{hops} = p/2$ for all-to-all in a plane, and $\overline{N}_{hops} = 3p/4$ for all-to-all in a

Parameter	Description			
$ au_{verlet}$	Execution time for dynamics propagation of a single site			
N_{sites}	Total number of sites			
r_c	Cut-off distance for non-bond forces			
ρ	Number density of system			
p	Node count			
$ au_{AllReduce}(N_{sites}, p)$	Execution time for AllReduce of forces			
$ au_{Globalize}(N_{sites},p)$	Execution time for position globalization			
N_i	Dimension of charge mesh in dimension i. $N_i = L_i/h$			
$ au_{non-bond}$	Execution time for pairwise non-bonded interactions			
$\tau_{p3me}(p,N_{mesh})$	Execution time for P ³ ME including FFT			

Table 1

Model Parameters

System Size (atoms)	5000	10,000	20,000	30,000
Cycle Count	3.097×10^9	3.454×10^9	4.168×10^9	4.881×10^{9}
Time (sec.)	4.42	4.93	5.95	6.97

Table 2

Single node cycle counts/time step and projected time/time step (at $700 \mathrm{MHz}$ nominal clock speed) for global force reduction method.

volume), N_{links} is the number of links available to each node (2 for linear communication, 4 for planar communication, and 6 for volumetric communication), BW is the raw bandwidth of the torus per link (2 bits per processor clock cycle), and f is the link utilization (assumed to be 80%). Note that the time required for all-to-all communication is independent of the dimensionality of the communication because of the compensating effects of the average hop count and the number of links available for the communication.

For the target scientific application, system sizes are such that a mesh size of $128 \times 128 \times 128$ is most common. For small node count systems, a "slab" decomposition of the FFT onto an array of processors is most efficient. However, this would only allow mapping of the FFT onto partitions with at most 128 nodes. In principle, there is plenty of work to distribute over a much larger number of nodes since if we assume that the 1D FFT is not to be parallelized, each stage in the 3D FFT requires N_{mesh}^2 1D FFT computations. This does require significantly more communication than a "slab" decomposition.

Since BG/L is a torus/mesh machine, it is natural to use a volume decomposition to map the 3D mesh domain onto the machine. Assuming that the domain mesh dimensions are $N_0 \times N_1 \times N_2$ and that the machine partition size is $p = p_0 \times p_1 \times p_2$ then each node will have responsibility for $(N_0/p_0) \times (N_1/p_1) \times (N_2/p_2)$ mesh points. Estimates of the various contributions to a $128 \times 128 \times 128$ 3D-FFT are shown in Figure 4 as a function of node count. For this choice of mesh size, there are $128 \times 128 = 16,384$ one-dimensional FFTs to be distributed over the nodes, which makes scaling the FFT beyond 16,384 nodes impossible without distributing the individual one-dimensional FFTs as well. Note that the computation dominates communication until the node count exceeds 512 and the FFT continues to speed up until hardware latencies begin to dominate for node counts in excess of 4096. These latencies become important because the amount of data sent from each node as part of the all-to-all communication drops as the node count increases.

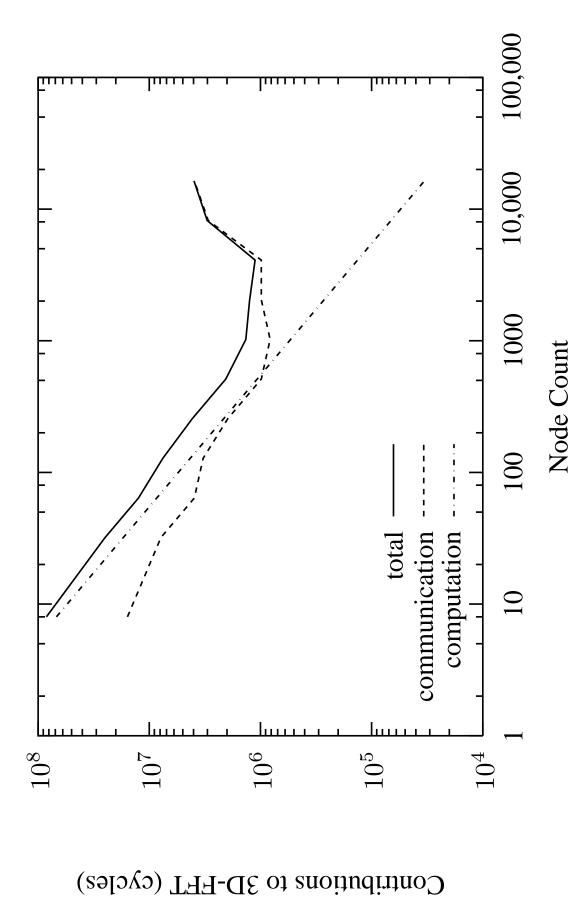


Fig. 4. Computation and communications estimates for a 3-dimensional FFT on a $128 \times 128 \times 128$ mesh. These estimates assume use nodes because at each phase in the 3D-FFT calculation, there are 128×128 individual one-dimensional FFTs to be distributed over the nodes. Also, communication dominates computation for node counts above 512 and hardware latencies actually decrease performance of the BG/L torus network to carry out the required communication. Note that for this mesh size, the FFT cannot scale beyond 16,384

with increasing node count above about 4000 nodes.

4.1.1 Global Force Reduction (GFR)

One simple parallel decomposition for real space molecular dynamics focuses on global force reduction. This allows the pairwise interactions to be arbitrarily partitioned amongst nodes to achieve load balance. Once per time step, all nodes participate in a summation all-reduction implemented over the global tree network (equivalent to standard global reduce followed by a broadcast) with each node contributing the results of its assigned interactions. Given access to net force on each particle and the current positions and velocities of each particle, each node can replicate the numerical integration of the equations of motion for each particle to give every node a copy of the updated full configuration of the system. Since each node has access to the full configuration of the system, this decomposition can work with versions of the P3ME method that use only a subset of the nodes for the FFT computation.

GFR has the following advantages:

- Load balancing of the non-bonded force calculation is straightforward.
- Double computation of the non-bonded forces can be avoided.
- P3ME can be supported in a number of decompositions including using only a subset of nodes for the P3ME portion of the computation since the computed forces are made visible on all nodes.

Drawbacks to GFR are:

- The replication of the dynamics propagation on all nodes represents non-parallelizable computation.
- The force reduction itself is non-scalable as implemented on the global tree.
- The implementation of a floating point reduction on the global tree network requires additional software overhead. In our model, two reductions are required—the first establishes a scale through a 16 bit/element comparison reduction while the second carries out a

summation reduction with scaled fixed point values (64 bits/element).

It is possible to reduce the replication of dynamics propagation by using some form of volume decomposition and Verlet lists. Using these facilities, each node need only propagate dynamics for atoms either homed on that node or within a cutoff distance (or within the guard zone when using Verlet lists).

4.1.2 Global Position Broadcast Decomposition

Another possible decomposition that leverages the capabilities of the global tree network on BG/L is based on a global position broadcast (GPB). Local force reductions will be required, but their near neighbor communications burden will be neglected in this model. A functional overview of the GPB class of decompositions is as follows. Once per time step the atom position vector will be bit-OR all-reduced (OR reduce, then broadcast) via the global tree network. However, for P3ME this method will also require near neighbor force reduction since atoms will generate forces from FFT mesh points on nodes other than their home node. Because of this, it is advantageous to maintain a 3D spatial mapping of atoms/fragment to nodes in BG/L. An issue with using near neighbor point-to-point messages to bring in the forces from the P3ME/FFT mesh points off node will be figuring out when all the forces have been received. To handle this, for each fragment homed on a node, a deterministic algorithm must yield an expectation of the exact number of forces to wait for. In this method, we partition both the force generating operations as well as the integration parts of the loop each node only propagates dynamics for those sites/fragments that are locally homed.

In this initial version of the model for global position broadcast, double computing of forces will be assumed as well as ideal load balancing of real-space non-bonded forces. The model corresponding to these assumptions is

$$T_{non-bond} = \frac{1}{p} N_{sites} \tau_{verlet}$$

$$+ \left(\frac{N_{sites}}{p}\right) \frac{4}{3} \pi r_c^3 \rho \tau_{non-bond}$$

$$+ \tau_{p3me}(p, N_{mesh})$$

$$+ \tau_{Globalize}(N_{sites}, p)$$

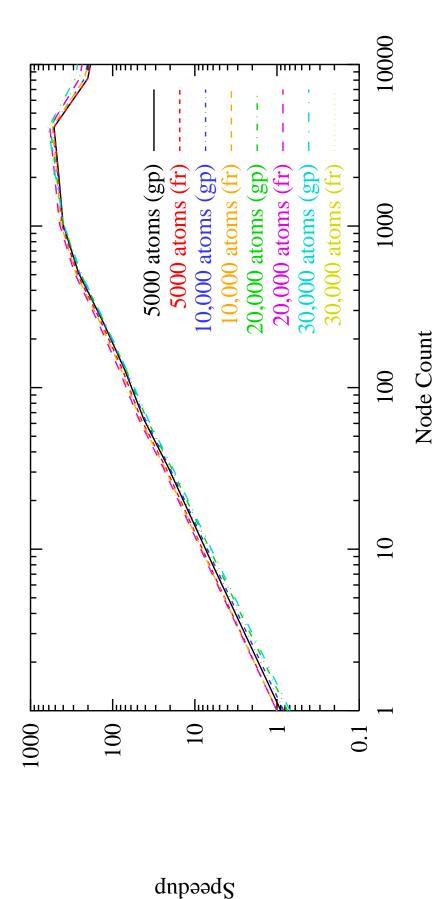
.

The speedups as a function of node count are shown in Figure 5 for a set of system sizes while the individual contributions to the total time required for the non-bond interactions in a system of 30,000 particles are shown in Figure 6.

4.2 Discussion

The data shown in Figure 5 indicate that in the range of system sizes of interest, both parallel decompositions discussed above have projected speedups of ≈ 250 at 512 nodes for a parallel efficiency of $\approx 50\%$. At sufficiently large partition sizes the communications required by the 3D FFT dominates the time step duration and eventually this communications cost becomes dominated by hardware latency (leading to the loss of parallel speedup seen in Figures 4, 5, and 6) because of the decreasing amount of data sent in each message. Because of this the tree-based globalization communication costs do not become an issue and the simple parallelization schemes considered here appear to be viable for node counts up to several thousand.

These projections were made from models constructed with the assumptions that no overlap is possible between communication and computation and that only one CPU on each node is available for computation. If we can implement these decompositions in ways that allow us to bypass any of these assumptions, it should be possible to increase the realized computational rate.



since the size of these contributions are a very small fraction of the computational burden as well susceptible to a reasonable degree of on a single node given in Table 2 since this is the fastest uniprocessor implementation. Note that the observed speedup is only weakly dependent on system size. These estimates assume ideal load balancing and do not include the contributions for bonded interactions Fig. 5. Speedup estimates for molecular dynamics on BG/L using two different parallelization schemes, position globalization (gp) and global force reduction (gfr). All speedups are computed with reference to the global force reduction implementation (for each system size) distribution. The molecular dynamics methodology assumed here is P3ME using a Verlet integrator.

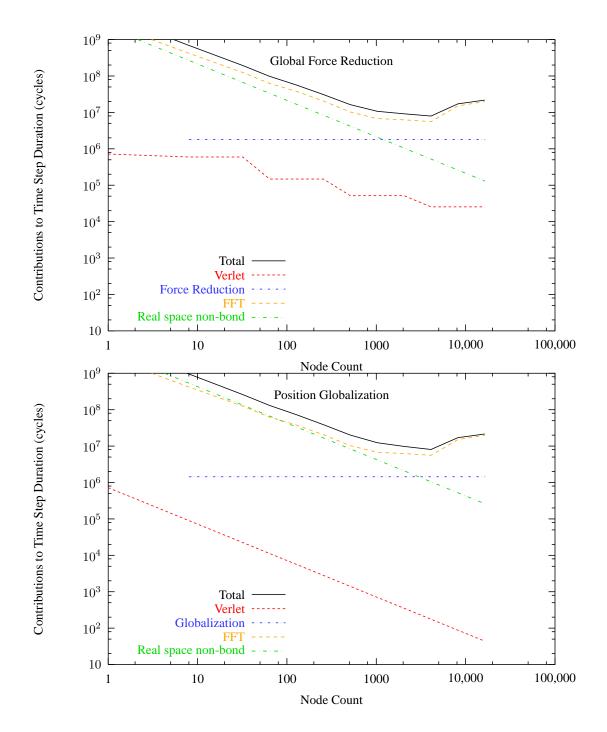


Fig. 6. An estimate of the contributions of various computational and communications operations to the total number of cycles required to carry out a time step on a 30,000 atom system is shown as a function of node count for two different parallelization schemes, position globalization (gp) and global force reduction (gfr). At very large node counts, the performance of both schemes is limited by the communications time required for the 3D FFT rather than the global position broadcast or force reduction.

5 Areas for Exploration

5.1 Multiple Time Step Integration

Any methods that increase the ratio of simulation time to computational effort through algorithmic means are extremely attractive candidates for investigation. The development of methods to integrate the equations of motion in a way that takes account of the multiple time scales present in a molecular dynamics simulation provides a means of reducing the number of times that expensive long range force evaluations need to take place during a fixed amount of simulation time. [24,25] Implementing these methods efficiently in the context of the parallel decompositions discussed above may require some modifications to the implementation of those decompositions, but the potential benefits in a parallel program are large [26] especially when an appropriate decomposition of the electrostatic forces is used in combination with an efficient algorithm for periodic electrostatics like P3ME. [25]

5.2 Alternatives for periodic electrostatics.

Several algorithms have been developed and extensively tested for efficient evaluation of electrostatic forces, but improving already existing and developing new algorithms remains an area of active research. These algorithms differ in the degree of implementation complexity, scalability with the number of particles, and suitability for parallelization as defined by scaling with the number of nodes used.

The particle-particle-mesh (P3ME) method[12] uses FFT methods to speed up calculation of the reciprocal space part. The P3ME method has good scalability with system size since the computational cost grows as $N \log N$, and this is currently the most popular approach used in MD packages. Unfortunately, parallelization of the P3ME method is limited

by the properties of the FFT algorithm, which involves significant communications between nodes.

Many algorithms can be classified as tree methods or multiple grid methods. Both approaches are similar in that they hierarchically separate interactions based on their spatial separation. The tree methods separate particles into clusters based on their relative distance, while the multi-grid methods use grids of different coarseness to approximate the smooth part of the potential between pairs of particles.

A well-known example of a tree method is the Fast Multipole Method[27], in which distant particles are grouped in clusters and the effect of distant clusters is approximated using multipole expansion. The scaling of this algorithm is O(N). A typical implementation of this algorithm is usually rather expensive, and therefore it becomes faster than the P3ME method only at relatively large system sizes.

Another method developed by Lekner[28] provides an alternative to Ewald summation and related techniques for handling electrostatic interactions between particles in a periodic box. The effect of all the periodic images is included in an effective pair-wise potential. The form of this potential allows a product decomposition to be applied[29]. The rate of convergence of the resulting series depends strongly on the ratio of the separation between particles in one of the dimensions and the periodic box size. The hierarchy of scales in this method is achieved by adding and subtracting images of charges to create periodic boundary conditions in a box of half the spatial extent in one of the dimensions. Then the procedure is repeated for several scales of box sizes. The scaling of this method with the particle number is $O(N \log N)$. Studies of the efficiency of this method show promising results in comparison with other methods in cases where the accuracy of the electrostatic interaction calculations is relatively high[30].

6 Summary and Conclusions

We have described the architecture of the Blue Matter framework for biomolecular simulation being developed as part of IBM's Blue Gene project as well as some of the features of the Blue Gene/L machine architecture that are relevant to application developers, particularly, issues and approaches associated with achieving efficient utilization of the double floating point unit. This work in collaboration with the IBM Toronto compiler team has not only improved code generation for BG/L, but has also led to significant improvements, 30% or more, in the performance of Blue Matter on the existing Power3 platform. In order to guide our development efforts and algorithmic investigations, we have explored two parallel decompositions that leverage the global tree interconnect on the BG/L platform as well as a distributed implementation of a three-dimensional FFT (required for the most widely used treatment of periodic electrostatics in molecular simulation) using the BG/L torus network through modeling.

Our estimates indicate that a 3D-FFT on a $128 \times 128 \times 128$ mesh remains computation dominated up to a partition size of 512 nodes and that significant speed-ups are realizable out to larger node counts. The two decompositions described here, position globalization and global force reduction, both rely on reductions implemented via BG/L's global tree. Both schemes have projected parallel efficiencies of about 50% out to 512 nodes for system sizes in the range of 5000 to 30,000 atoms and the time required to compute a single time step becomes dominated by communications latency effects in the 3D-FFT implementation for node counts above ≈ 4000 . Given that the simulation experiments under consideration require multiple independent or loosely coupled molecular simulations, we conclude that these results indicate that FFT-based particle mesh techniques are a reasonable first approach to implementing a molecular simulation application that targets BG/L. As hardware becomes available, we will be able to validate our estimates, assess the impact of system software

overheads, explore whether some overlap of communication and computation is possible, and assess the possibility of using both CPUs on a BG/L node for computation.

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